Analysis of Slinky Levitation

Mark Graham, Department of Physics and Astronomy, University of Alabama, PO Box 870324, Tuscaloosa, AL 35487-0324; markgraham@mindspring.com

An amazing phenomenon is described by Martin Gardner in the February 2000 "Physics Trick of the Month."[1] When a Slinky™ spring is hung by one end such that it extends by its own weight (one will have to stand on a chair to allow it to fully extend) and then released, the lower end will not fall for a moment but will hang briefly suspended in the air. This levitation effect is even more dramatic if a small object, such as a plastic cup, is attached to the end of the Slinky.

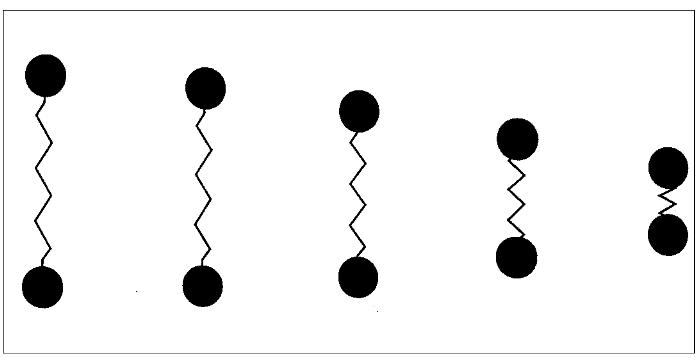

Fig. 1a: The motion of masses at the ends of a spring is simulated with *Interactive Physics* program (www.krev.com/products/ip_sim.html). Nearly one quarter of an oscillation period is shown.

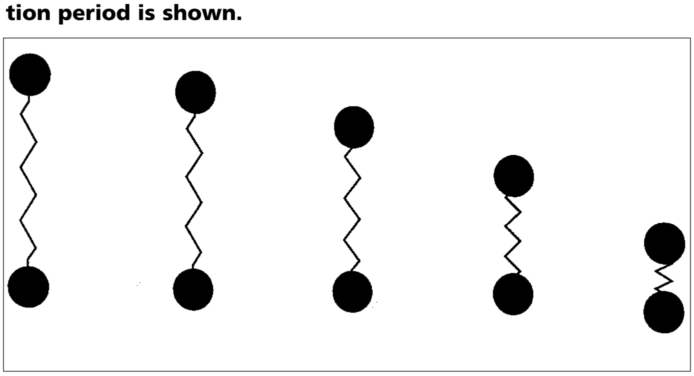

Fig. 1b: With gravity acting, the center of mass falls at nearly the same rate as the lower mass is drawn upward. For the first quarter period, the lower mass remains nearly at rest.

Gardner explains that the system's center of gravity falls with the acceleration of gravity when released. How precisely is it that the falling of the center of mass is cancelled by the contraction of the lower end? How long can this effect endure? The falling spring has been analyzed by M.G. Calkin as a continuous medium using partial differential equations.[2] With appropriate idealization and approximation, the levitation effect may be explained with high school level physics.

Let's idealize the system as a massless spring of zero length with equal masses attached to each end. In the absence of gravity, if the system were released from being stretched, it would undergo symmetrical motion about the center of mass as shown by Fig. 1a. We can split the system in half, with each half undergoing sinusoidal motion. In anticipation of halving the system, denote the total spring constant K/2 and the total mass 2M. Thus, the spring constant and mass of a half are just K and M, respectively. The motion from the time of release ($t = 0$) is given by $y = -A\cos\omega t$, with A the distance from the end of the stretched system to the center, and $\omega = (K/M)^{1/2}$. A Slinky is a "tightly wound spring,"[2] meaning the unstretched coils touch each other, so harmonic motion will only last for a relatively small time (about one quarter of the period). A Taylor expansion of the cosine function gives, to second order, $y = -A\cos\omega t \approx -A + A(\omega t)^2/2$, which upon substituting for ω is $y \approx A - AKt^2/2M$.

Now we consider gravity and hold the upper mass fixed, allowing the lower mass to hang at rest at its equilibrium position. From Hooke's law, the displacement from the center of the spring is $A = Mg/K$, which also corresponds to

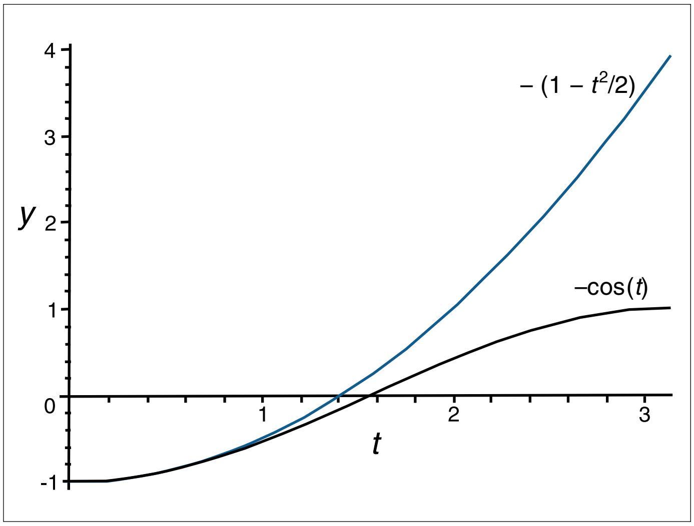

Fig. 2: The function $y = -\cos(t)$ is approximated well for the first quarter period by $y = -(1 - t^2/2)$.

the amplitude of the oscillation that begins when the spring is released. Upon substituting for A in the second order term above, we have $y \approx A - gt^2/2$. This expression gives the approximate location of the lower mass with respect to the center of the spring. But the center of mass falls according to $y_{cm} = gt^2/2$. The displacement of the lower end of the spring with respect to the stationary origin is $y + y_{cm} \approx A$. In other words, the end remains nearly motionless for a short while, about one quarter period (Fig. 1b). If the motion is graphed (see Fig. 2), one can see that to second order, the displacement due to harmonic oscillation is approximately equal to that due to freefall until the higher order terms become significant and turn the cosine function around. Analysis of Gardner's trick illustrates much of the philosophy of solving professional physics problems; idealization and approximation are crucial to success, along with appreciating the parameters within which the assumptions are meaningful.

Acknowledgments

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References

1. Martin Gardner, "A Slinky problem," *Phys. Teach.* **38**, 78 (Feb. 2000).
2. M.G. Calkin, "Motion of a falling spring," *Am. J. Phys.* **61**, 261-264 (March 1993).