Periodic Forcing of the Swift-Hohenberg Equation in Time

Punit Gandhi,* Cédric Beaume, and Edgar Knobloch

Department of Physics, University of California, Berkeley CA 94720, USA

(Dated: May 1, 2014)

Abstract

Systems with a periodic forcing in time abound! We use the generalized Swift-Hohenberg equation with a quadratic-cubic nonlinearity as test-bed for studying localized pattern formation in such systems with a periodic forcing in time. We apply a sinisiodal linear forcing to the SHE and study the dependence of localization on the amplitude, oscillation period, and offset of the forcing. As one might expect, the region of existence of stable localized solutions dramatically decreases as the system is "jiggled." The parameter space within the pinning region of the constant forcing case, however, is partitioned into regions of growth, stability, and decay with an unexpected structure.

* punit_gandhi@berkeley.edu

I. INTRODUCTION

A. motivation for periodic time forcing in pattern formation

B. motivation for SHE

C. Description for SHE

D. Numerical Methods

E. paper outline

II. CONSTANT FORCING IN TIME

A. free energy and SHE

B. bistability and maxwell point

C. snakes and ladders structure within pinning region

D. front speed just outside pinning region

E. Eckhaus instability and connection of snaking branch to different period periodic branch

F. Description of simple toy model of SHE and nucleations??

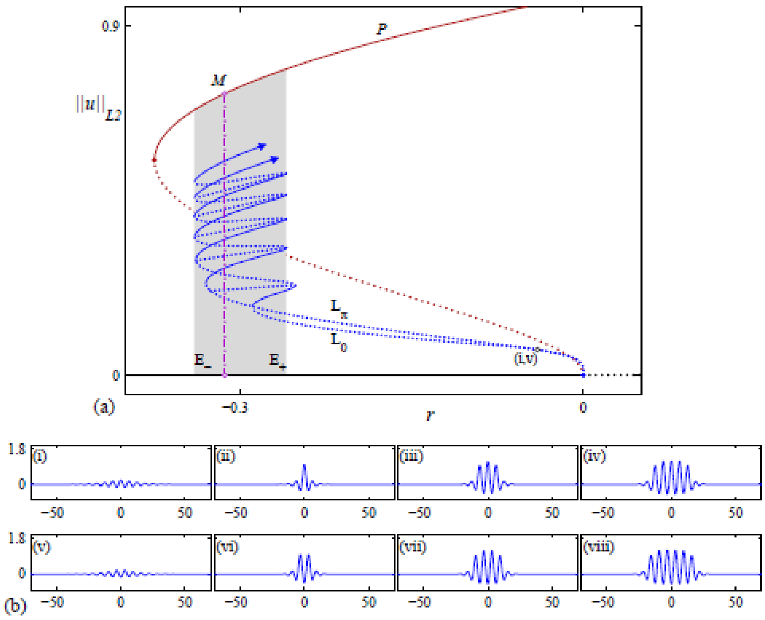

FIG. 1: This figure was taken from Burke[] (a) Bifurcation diagram showing the snakes-and-ladders structure of localized states. Away from the origin the snaking branches L_0 and L_- are contained within the snaking region (shaded) betweenE_- and E_+, where $r(E_-) \approx -0.3390$ and $r(E_+) \approx -0.2593$. Solid (dotted) lines indicate stable (unstable) states. In addition, the Maxwell point M, occurring at $r(M) \approx -0.3128$ is indicated with a vertical dash-dot line. The saddle node bifurcation that creates the stable periodic state occurs at $r < r(SN_P) \approx -0.3744$, defining the left edge of the bistability region. We will also find it useful to define the center of the snaking region C, which corresponds to the forcing parameter $r(C) \approx -0.2992$. (b) Sample localized profiles $u(x) : (i - iv)$ lie on L_0, near onset and at the 1st, 3rd, and 5th saddle-nodes from the bottom, respectively; (v-viii) lie on L_-, near onset and at the 1st, 3rd, and 5th saddle-nodes, respectively. Parameters: $b = 1.8$.

III. PERIODIC FORCING IN TIME

A. schematic of solution structure and regions will oscillate through

B. description of some behaviors exhibited (growing, decaying ,stable, etc..)

C. descrption of ways to visualize solutions (Xcm,Vcm, slices of phase space we will use, etc..)

IV. EFFECT OF SMALL OSCILLATIONS ON THE FRONT SPEED NEAR THE EDGE OF THE PINNING REGION

A. graph of numerical results - we don't really have this yet

B. asymptotic calculation and comparison to numerical result

V. STABILITY, GROWTH, AND DECAY OF LOCALIZED SOLUTIONS UNDER LARGE OSCILLATIONS

A. Stable oscillations of the solution

1. stable region for $\rho = .1, .8, .6$

2. ρ vs r_0, Tosc=100

B. Growth and decay

1. big detailed figure of nucleations per oscillation

2. stability lines and avoided crossings?

3. simple model interpretaion

C. some asymptotic calculations???

VI. PERSISTENCE OF DEFECTS DUE TO OSCILLATIONS

A. show solutions of quasistable defect connecting to both 39 and 40 period solution as well as stable defect

B. graph of regions where for each case

C. Some kind of explanaition (Eckaus instability and delayed bifurcations?)

VII. CONCLUSION

A. summarize results

B. future directions