

Analysis of Carbon Reinforced Unidirectional Composites



Mechanics of Composites Term Project

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Abstract

Most natural things existing in the world is composite, from the veins of human body to the bark of tree. Structures made of these materials possesses the combination of required properties which for a pure material is impossible to possess. Recent years have seen rise in uses of composites due to its light weight and moderate strength. Simultaneously, research in this branch of engineering is attracting more and more investors which is further booming the uses of composites for cheaper products with much less material waste. The present investigation deals with the analysis of unidirectional random carbon fiber reinforced composite

Formulas for calculations:

1. Rule of mixtures: This formula is used by assuming that strains in the direction of fibers are same in the matrix material and fiber. It is also known as 'Voigt' model. This is generally used for calculating longitudinal modulus and corresponding poisons ratio. It is given by,

$$P = v_f P_f + v_m P_m$$

2. Inverse rule of mixtures: This formula is used by assuming iso-stress relationship i.e. assumes uniform stress in both fiber and matrix. It is used for calculating properties like transverse modulus, Shear modulus . This is also known as 'Reus' model. It is given by,

$$\frac{1}{P} = \frac{v_f}{P_f} + \frac{v_m}{P_m}$$

However, the above relationship deviates a lot with experimental data. Hence certain corrections are defined in order to obtain possibly accurate values.

Some of them are:

- 1) Halpin Tsai semi-Empirical relationship:

The Halpin Tsai relationships assume that the actual composite properties lie between the longitudinal and transverse law of mixtures relationships. The Halpin Tsai relationship introduces a "geometrical factor" to adjust the properties between these two extremes. This geometrical factor is associated with the shape and packing of the fibers and is a measure of the reinforcing efficiency of the fibers.

The general form of the Halpin Tsai relationship is given as,

$$\frac{M}{M_m} = \frac{1+\xi\eta v_f}{1-\eta v_f} \quad \text{where} \quad \eta = \frac{\frac{M_f}{M_m}-1}{\frac{M_f}{M_m}+\xi}$$

The value usually given for the geometrical factor for E22 is 2 and G12 is 1.

- 2) Chami's formula:

For properties which have non-linear relation with V_f , replace V_f with $\sqrt{V_f}$ after writing / eliminating V_m .

$$E_{22} = E_{33} = \frac{E_m}{\left[1 - \sqrt{v_f} \left(1 - \frac{E_m}{E_f}\right)\right]}$$

$$G_{12} = G_{13} = \frac{G_m}{\left[1 - \sqrt{\nu_f} \left(1 - \frac{G_m}{G_{f^{12}}}\right)\right]}$$

$$G_{23} = \frac{G_m}{\left[1 - \sqrt{\nu_f} \left(1 - \frac{G_m}{G_{f^{23}}}\right)\right]}$$

3) Stress partitioning: Stress in fiber and matrix are not equal . So, average stress in matrix is taken as the fraction (η_2) of average fiber stress. It is given by,

$$\frac{1}{E_2} = \frac{1}{(\nu_f + \eta_2 \nu_m)} \left[\frac{\nu_f}{E_f} + \frac{\eta_2 \nu_m}{E_m} \right]$$

The value η_2 is obtained by dividing average stress in matrix by average stress in fibers when loaded in transverse direction.

Methodology:

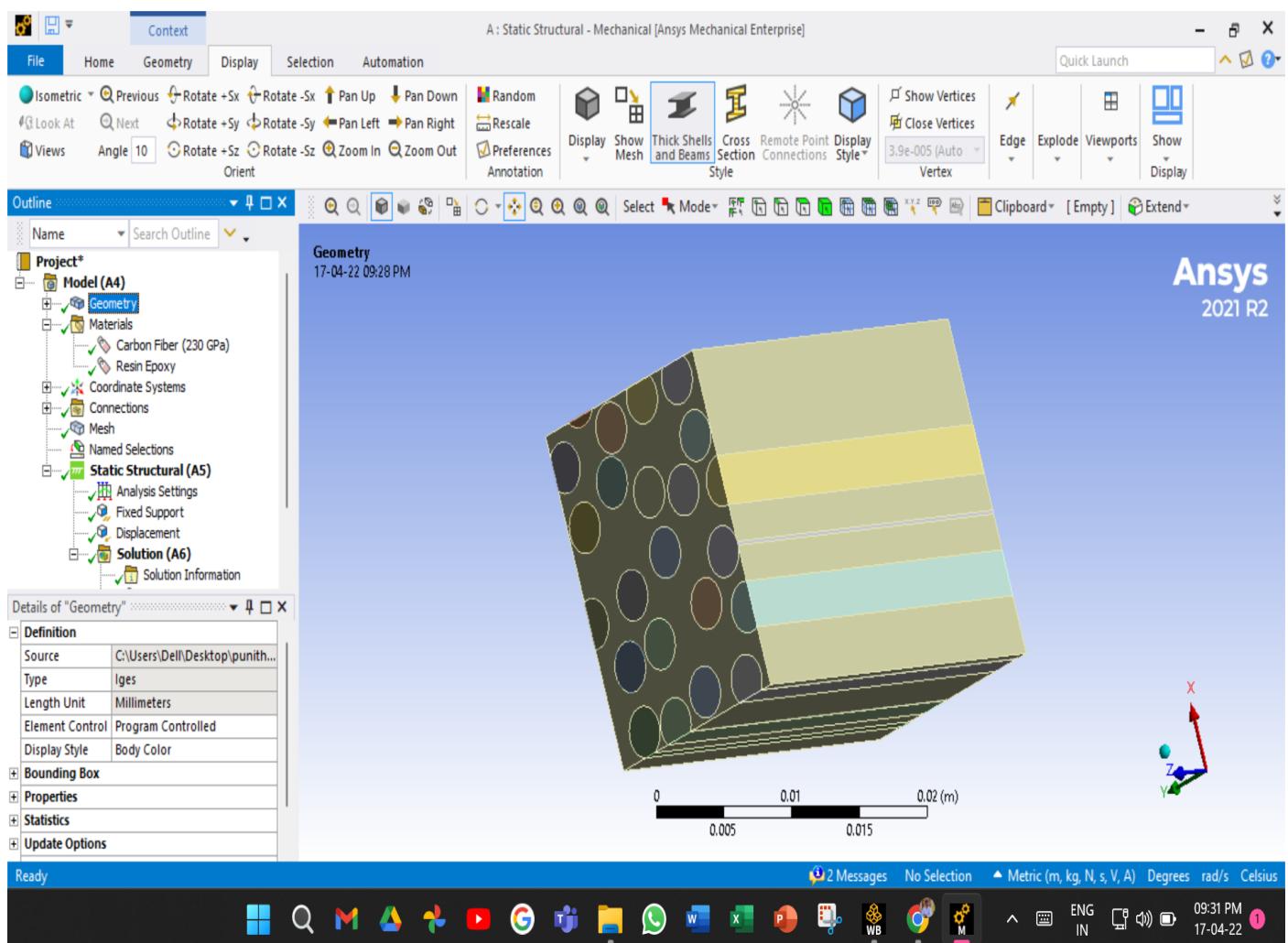
The properties of the composite are obtained by using both the theoretical and numerical methods. The theoretical method consists of using above defined relations correspondingly to certain properties. The numerical methods are utilized through the computer software ANSYS, by defining the boundary conditions with respect to each property as shown in the figure below.

FEM Procedure and Boundary Conditions:

1. Model of Composite: The matrix and fibers are designed and assembled in Creo parametric. The final assembly is imported into ANSYS as. igs format file.

Dimensions of composite:

- a) Length of composite : 15 mm
- b) Width of composite : 20 mm
- c) Thickness of composite : 30 mm
- d) Radius of fiber : 2 mm
- e) No. of fibers : 23
- f) Volume fraction of fiber : 0.46
- g) Volume fraction of Matrix : 0.54



2. Assigning the materials:

The fibers and matrix are assigned with selected materials, whose properties are obtained from the engineering data in ANSYS. Carbon fiber and Resin epoxy are assigned to the fiber and matrix respectively. The properties of the materials are as shown below.

Matrix: Material used for matrix is Epoxy Resin

Fibers: Material used for fiber is Carbon Fiber

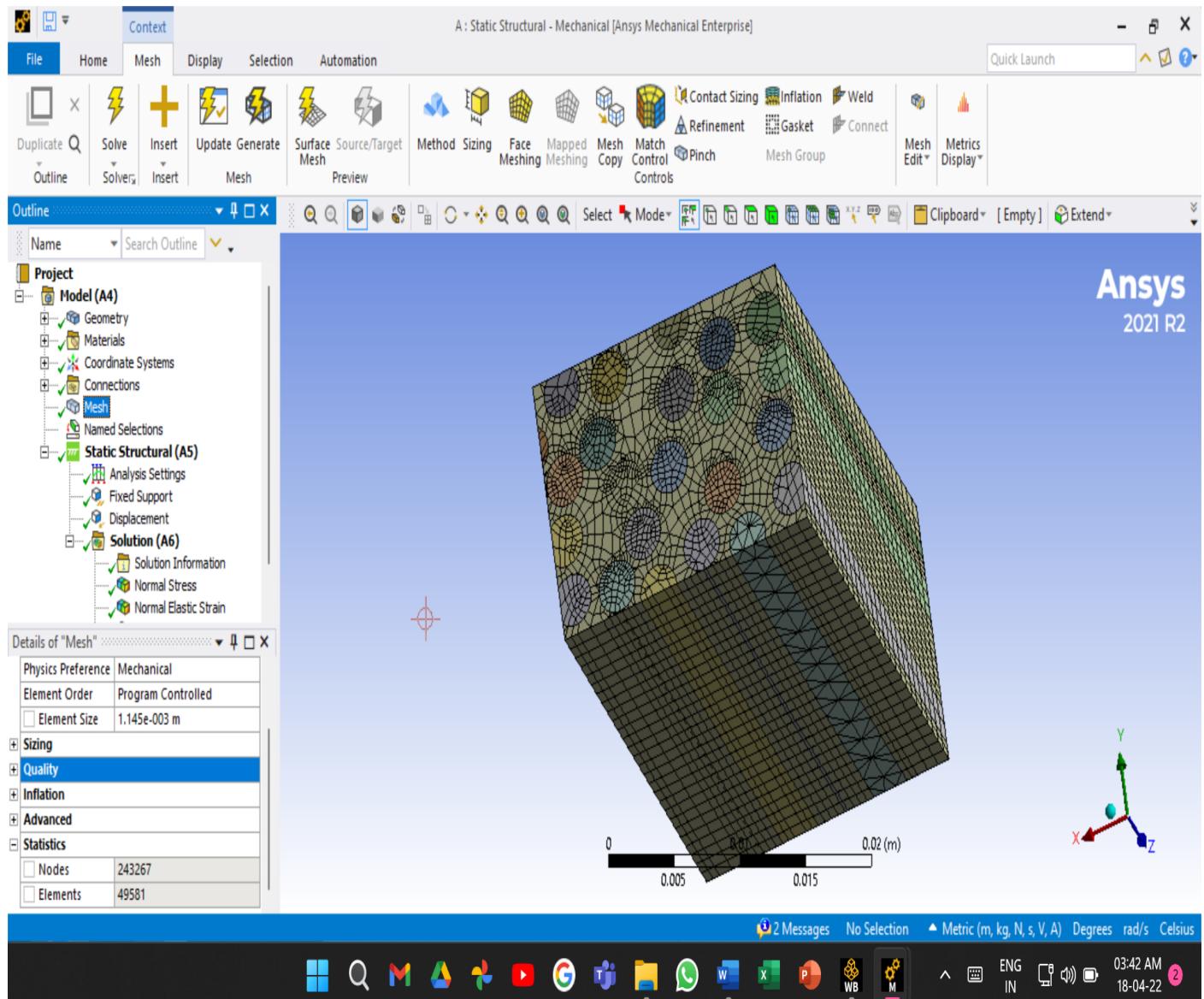
This screenshot shows the ANSYS Mechanical Enterprise interface with the 'Materials' tab selected. The left pane displays the project structure under 'Model (A4)'. In the 'Materials' section, 'Resin Epoxy' is selected. The right pane shows the 'Engineering Data: Material View' for 'Resin Epoxy'. The 'Structural' properties are listed as follows:

Property	Value
Young's Modulus	3.78e+09 Pa
Poisson's Ratio	0.35
Bulk Modulus	4.2e+09 Pa
Shear Modulus	1.4e+09 Pa
Tensile Yield Strength	5.46e+07 Pa

This screenshot shows the ANSYS Mechanical Enterprise interface with the 'Materials' tab selected. The left pane displays the project structure under 'Model (A4)'. In the 'Materials' section, 'Carbon Fiber (230 GPa)' is selected. The right pane shows the 'Engineering Data: Material View' for 'Carbon Fiber (230 GPa)'. The 'Structural' properties are listed as follows:

Property	Value
Young's Modulus X direction	2.3e+11 Pa
Young's Modulus Y direction	2.3e+10 Pa
Young's Modulus Z direction	2.3e+10 Pa
Poisson's Ratio XY	0.2
Poisson's Ratio YZ	0.4
Poisson's Ratio XZ	0.2
Shear Modulus XY	9e+09 Pa
Shear Modulus YZ	8.2143e+09 Pa
Shear Modulus XZ	9e+09 Pa

Meshing of composite:

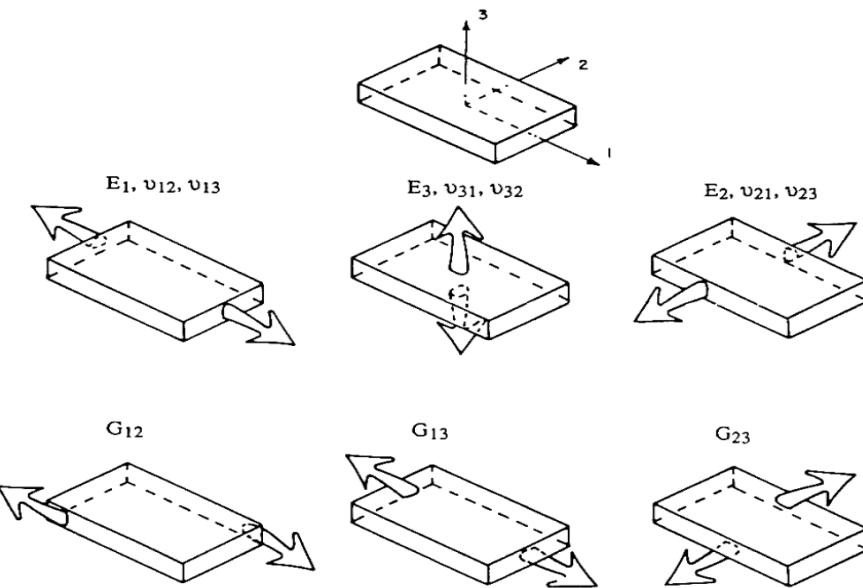


Meshing Size: 1.145 mm

Nodes: 243267

Elements: 49581

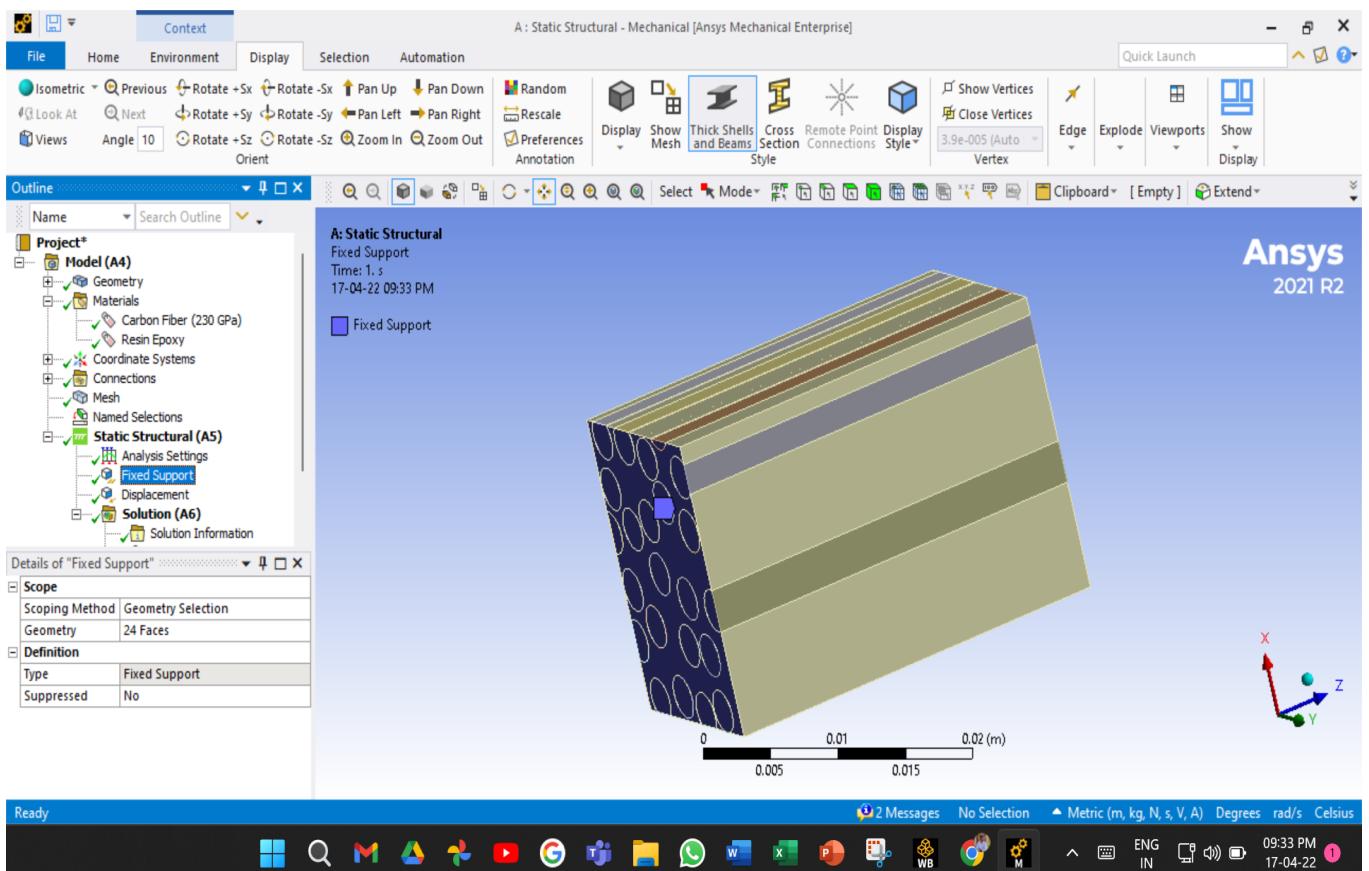
Calculating E_1 :

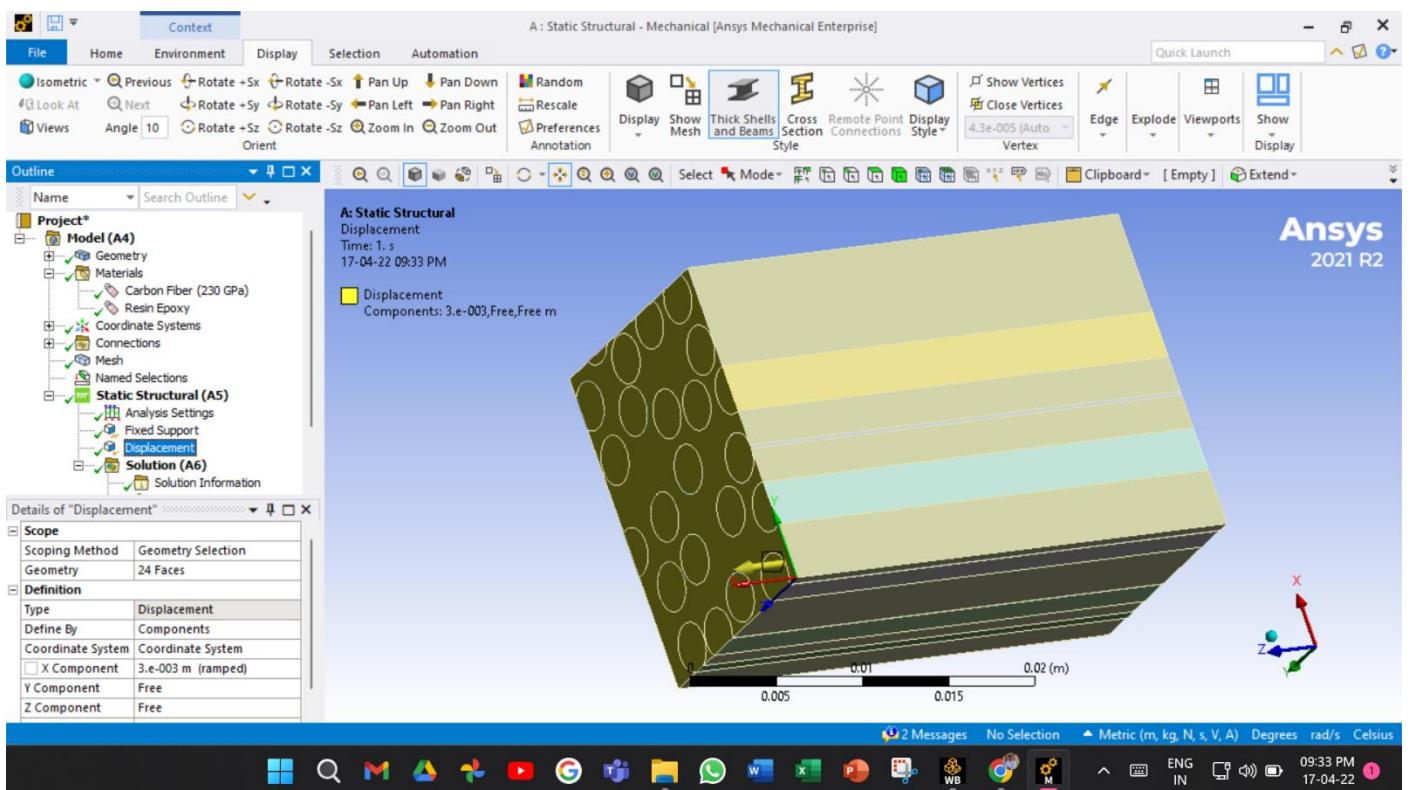


For iso-strain condition:

To Find E_1

- Fixed Y-Z plane at $X=-20$ mm
- 0.003 m displacement of Y-Z plane at $X=0$ mm





Normal stress:

Minimum normal stress	2.841e+008 Pa
Maximum normal stress	3.093e+010 Pa
Average normal stress	1.266e+010 Pa

Normal Elastic strain :

Minimum normal strain	6.712e-002 m/m
Maximum normal strain	0.202 m/m
Average normal strain	9.995e-002 m/m

Poison's ratio(θ):

$$\theta_{12} = -\left(\frac{\text{Transverse elastic strain}}{\text{Normal elastic strain}}\right)$$

$$= -\left(\frac{-2.347 \times 10^{-2}}{9.995 \times 10^{-2}}\right)$$

$$= 0.234$$

Longitudinal Stiffness:

$$E_{11} = \frac{\text{Average Normal stress}}{\text{Average Normal strain}}$$
$$= \frac{(1.266 * 10^{10})}{9.995 * 10^{-2}}$$
$$= 126.62 \text{ GPa}$$

Analytical calculation:

Longitudinal Stiffness:

$$E_{11} = V_f * E_f + V_m * E_m$$
$$= 0.46 * 230 + 0.54 * 3.78$$
$$= 107.84 \text{ GPa}$$

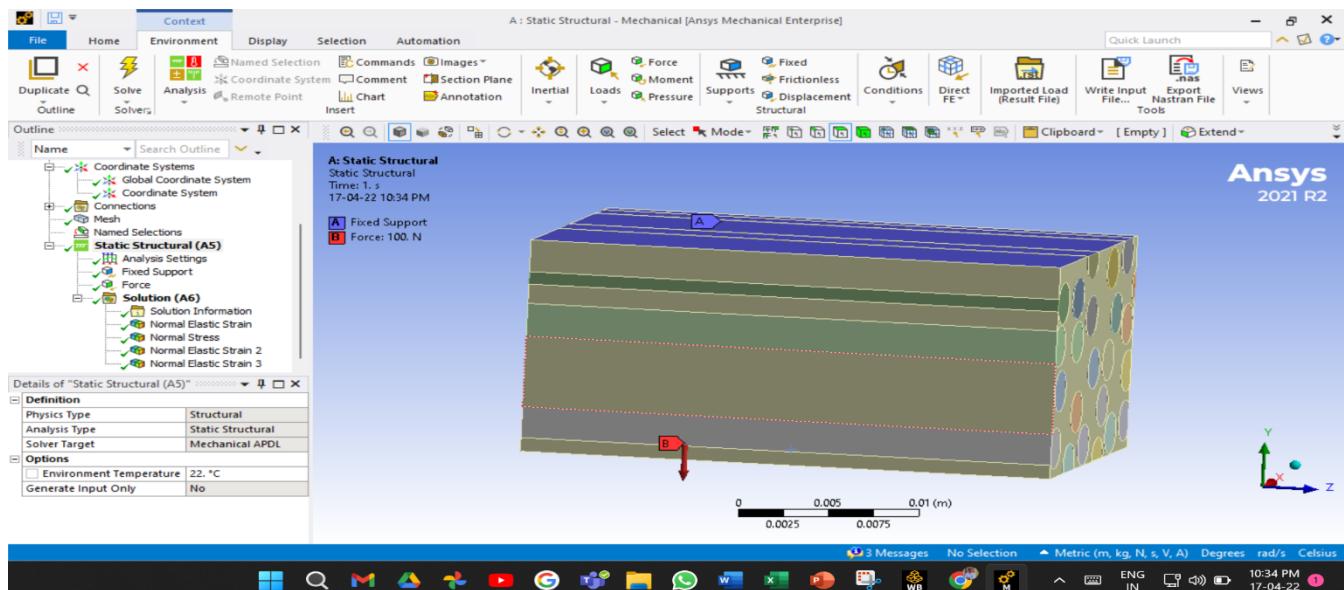
Poison's ratio(ϑ):

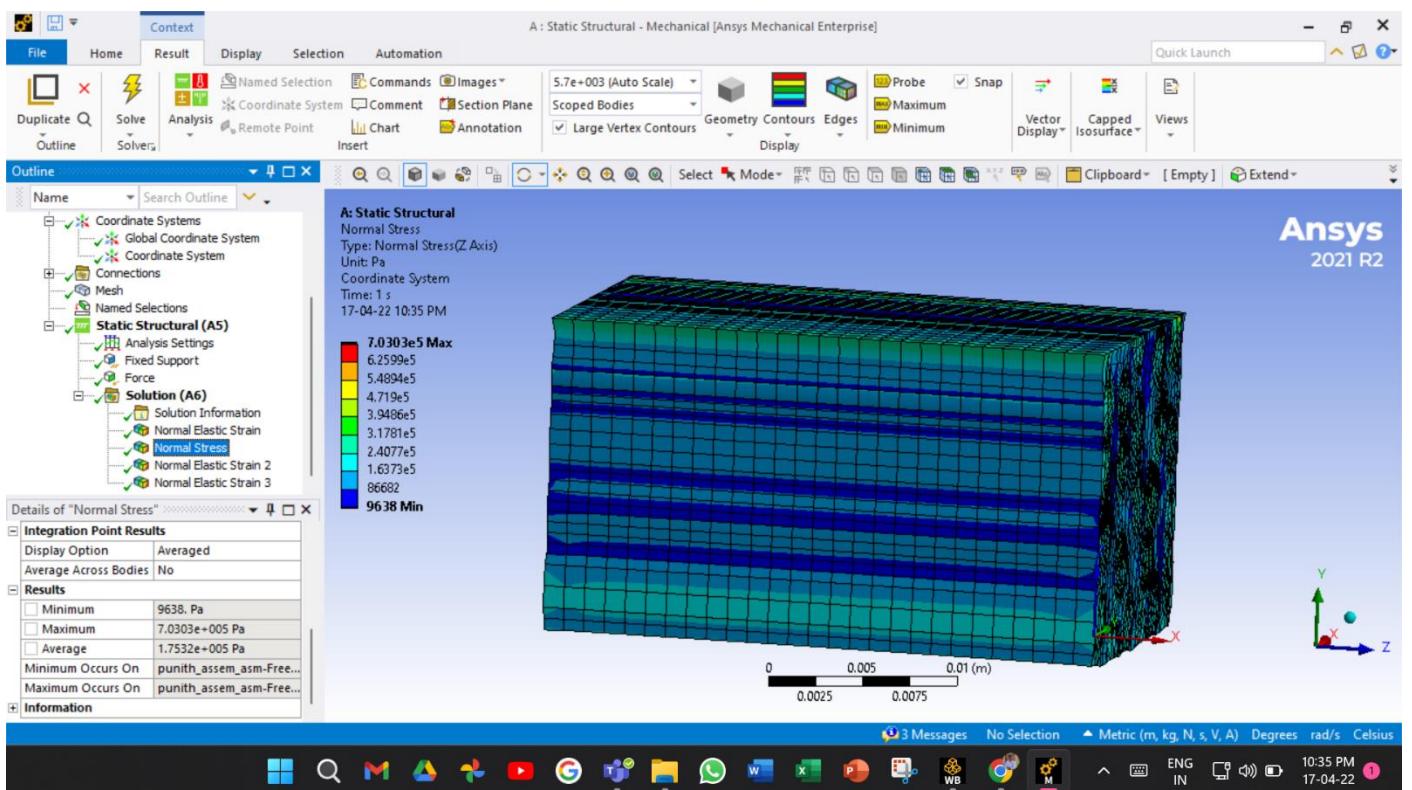
$$\vartheta_{12} = V_m * \vartheta_m + V_f * \vartheta_f$$
$$= 0.4869 * 0.54 + 0.4131 * 0.46$$
$$= 0.254$$

Calculating E_2 :

For iso-stress condition:

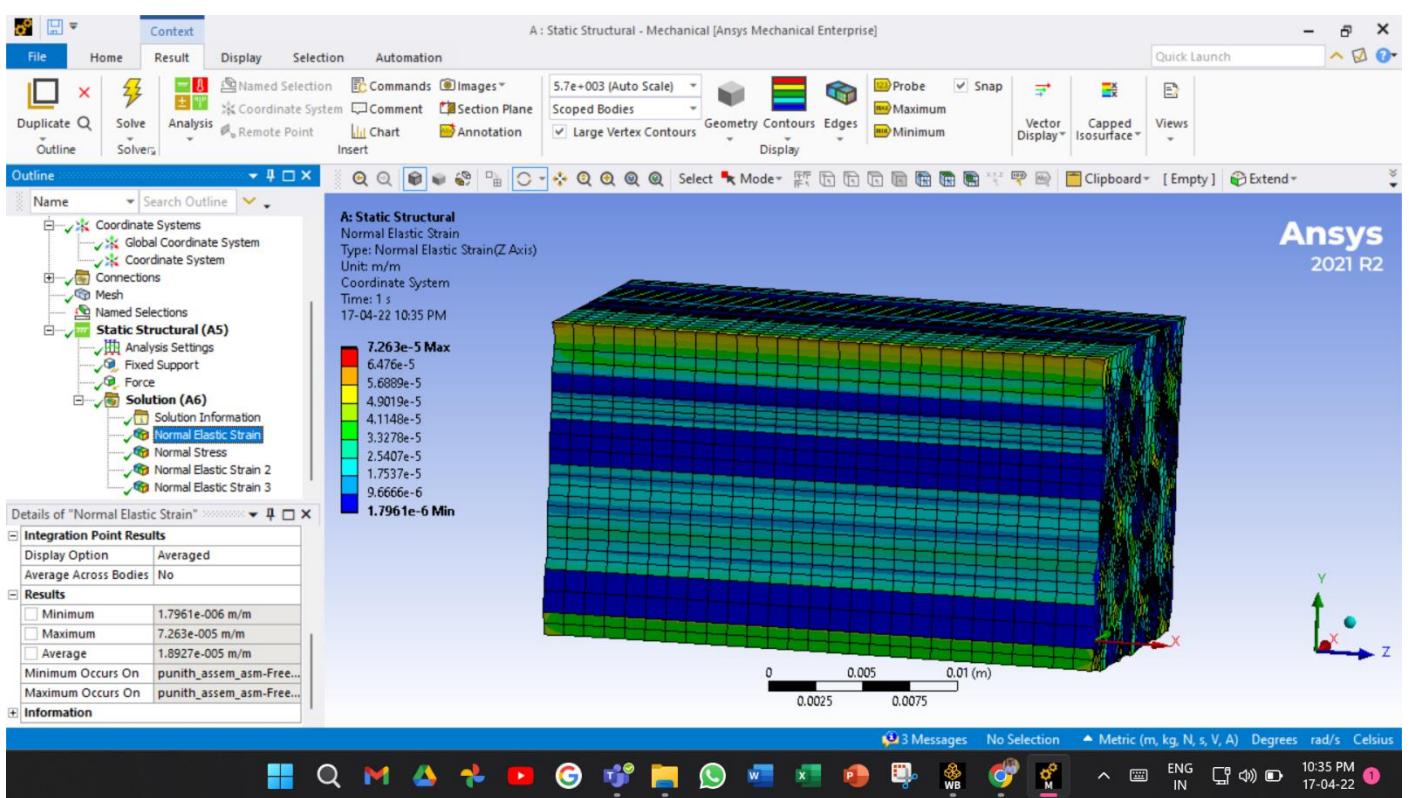
- Fixed X-y plane at Y=0 mm
- 100 N force in -Z direction





Normal stress:

Minimum normal stress	9638 Pa
Maximum normal stress	57.030 e+05 Pa
Average normal stress	1.7532 e+05 Pa



Normal Elastic strain:

Minimum normal strain	1.796 e-06 m/m
Maximum normal strain	7.263 e-05 m/m
Average normal strain	1.892 e-05 m/m

Transverse Stiffness:

$$E_{22} = \frac{\text{Average Normal stress}}{\text{Average Normal strain}}$$

$$= \frac{(1.753 * 10^5)}{1.892 * 10^{-5}} = 9.26 \text{ GPa}$$

Analytical calculation:

Transverse Stiffness:

$$\frac{1}{E_{22}} = \frac{V_f}{E_f} + \frac{V_m}{E_m}$$

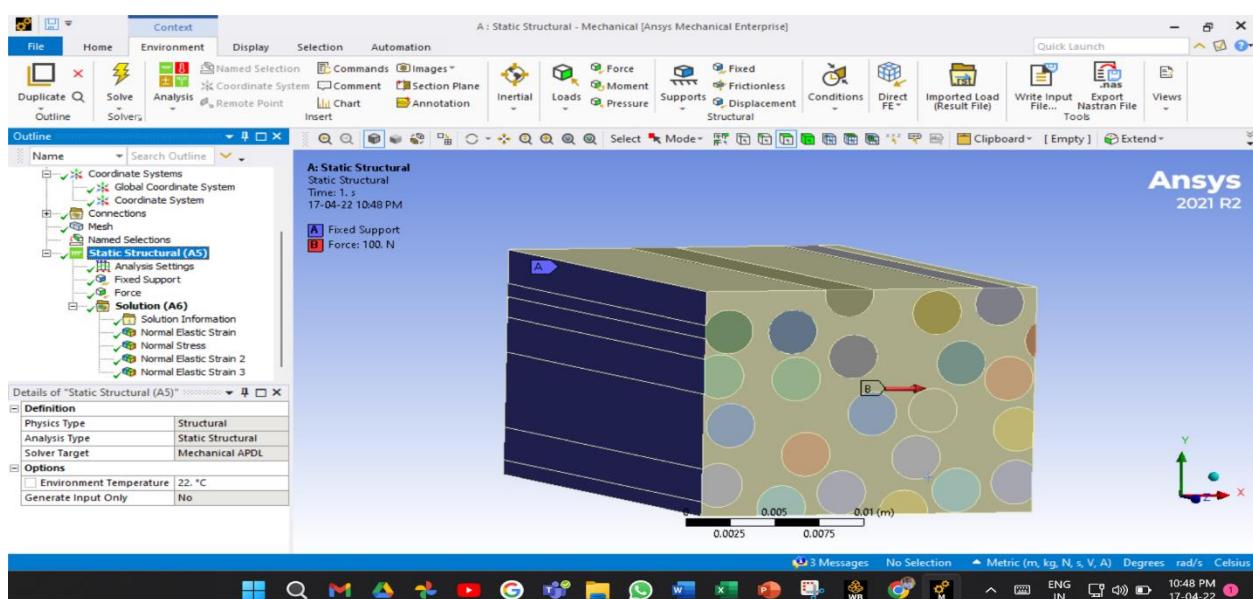
$$= \frac{0.4131}{230} + \frac{0.4869}{3.78}$$

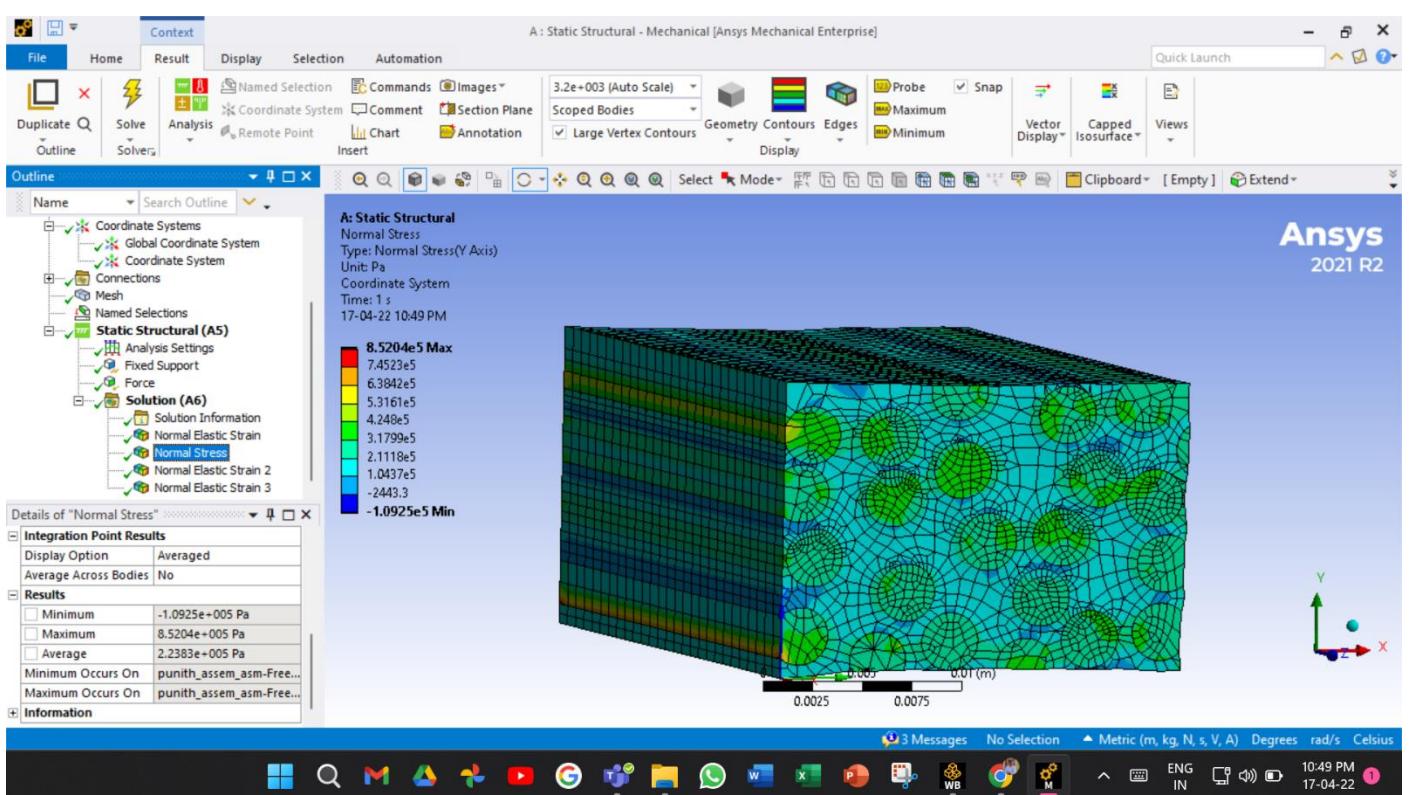
$$E_2 = 6.9 \text{ GPa}$$

Calculating E_3 :

For iso-stress condition:

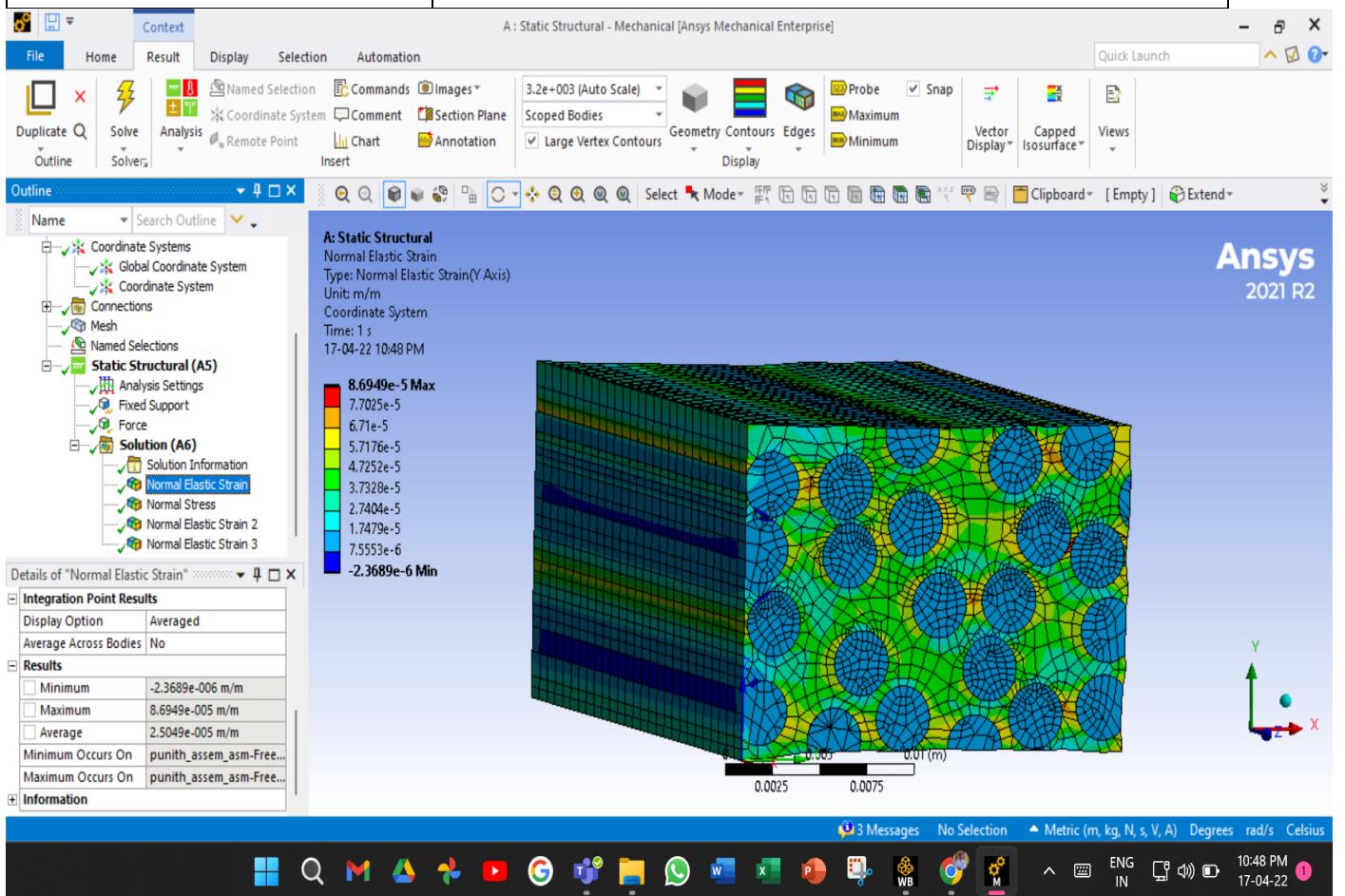
- A. Fixed X-Z plane
- B. 100 N force in Y- direction





Normal stress:

Minimum normal stress	-1.092e+005 Pa
Maximum normal stress	8.520e+005 Pa
Average normal stress	2.238e+005 Pa



Normal Elastic strain:

Minimum normal strain	-2.368 e-06 m/m
Maximum normal strain	8.694 e-05 m/m
Average normal strain	2.504 e-5 m/m

Transverse Stiffness:

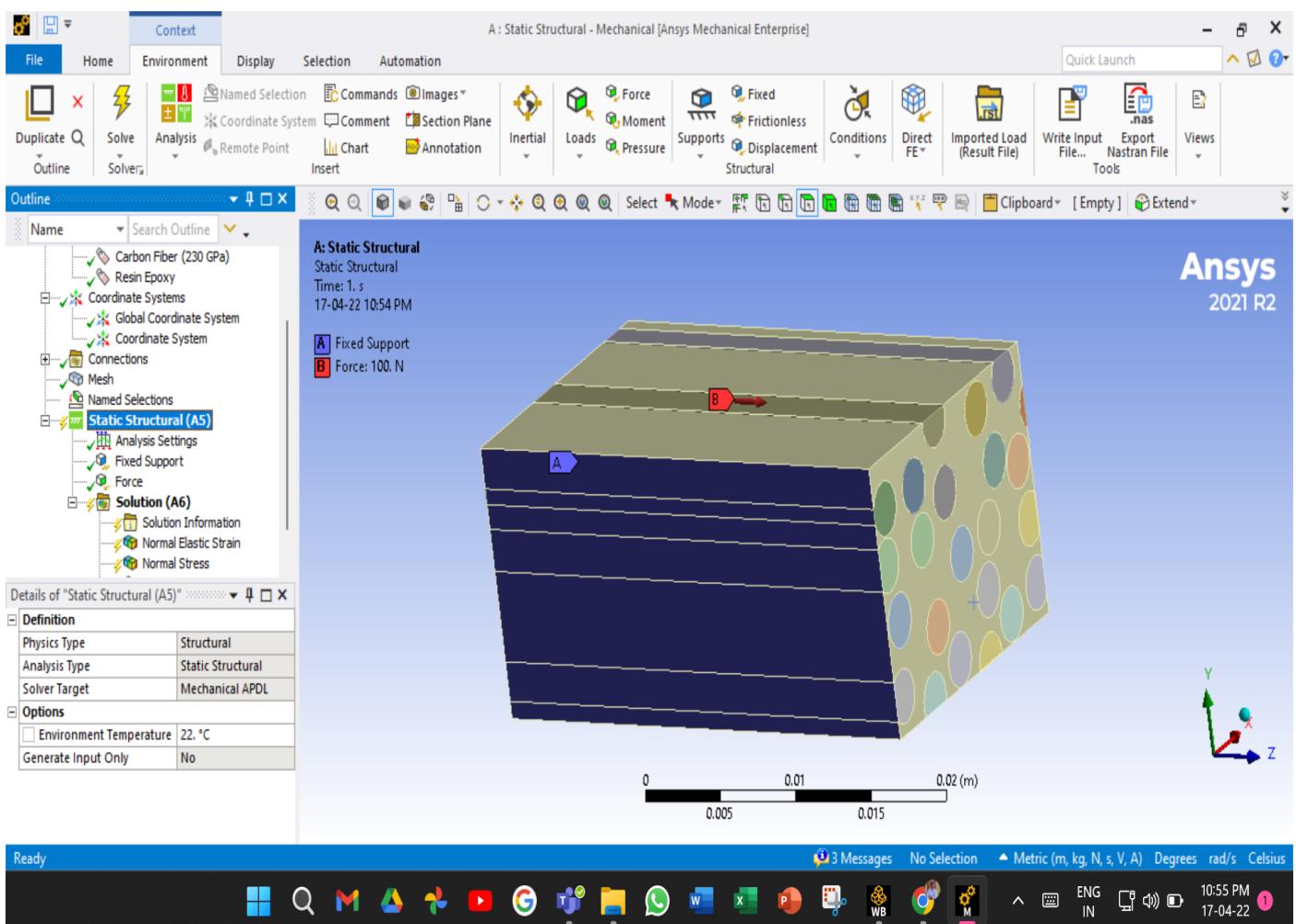
$$E_{33} = \frac{\text{Average Normal stress}}{\text{Average Normal strain}} = \frac{(2.238 * 10^5)}{2.504 * 10^{-5}} = 8.93 \text{ GPa}$$

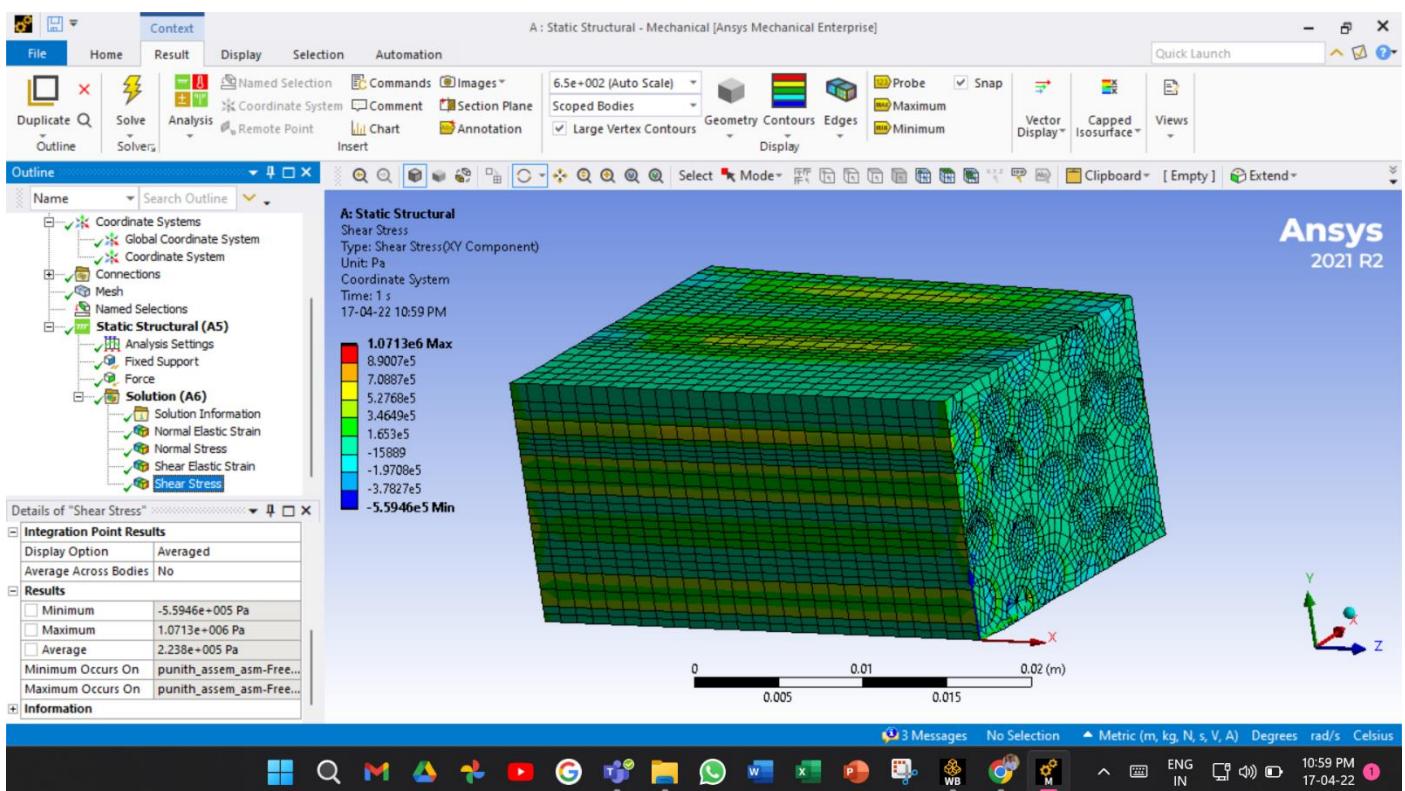
For Shear loads

Loads and Boundary Condition:

Calculating $G_{12}=G_{xy}$

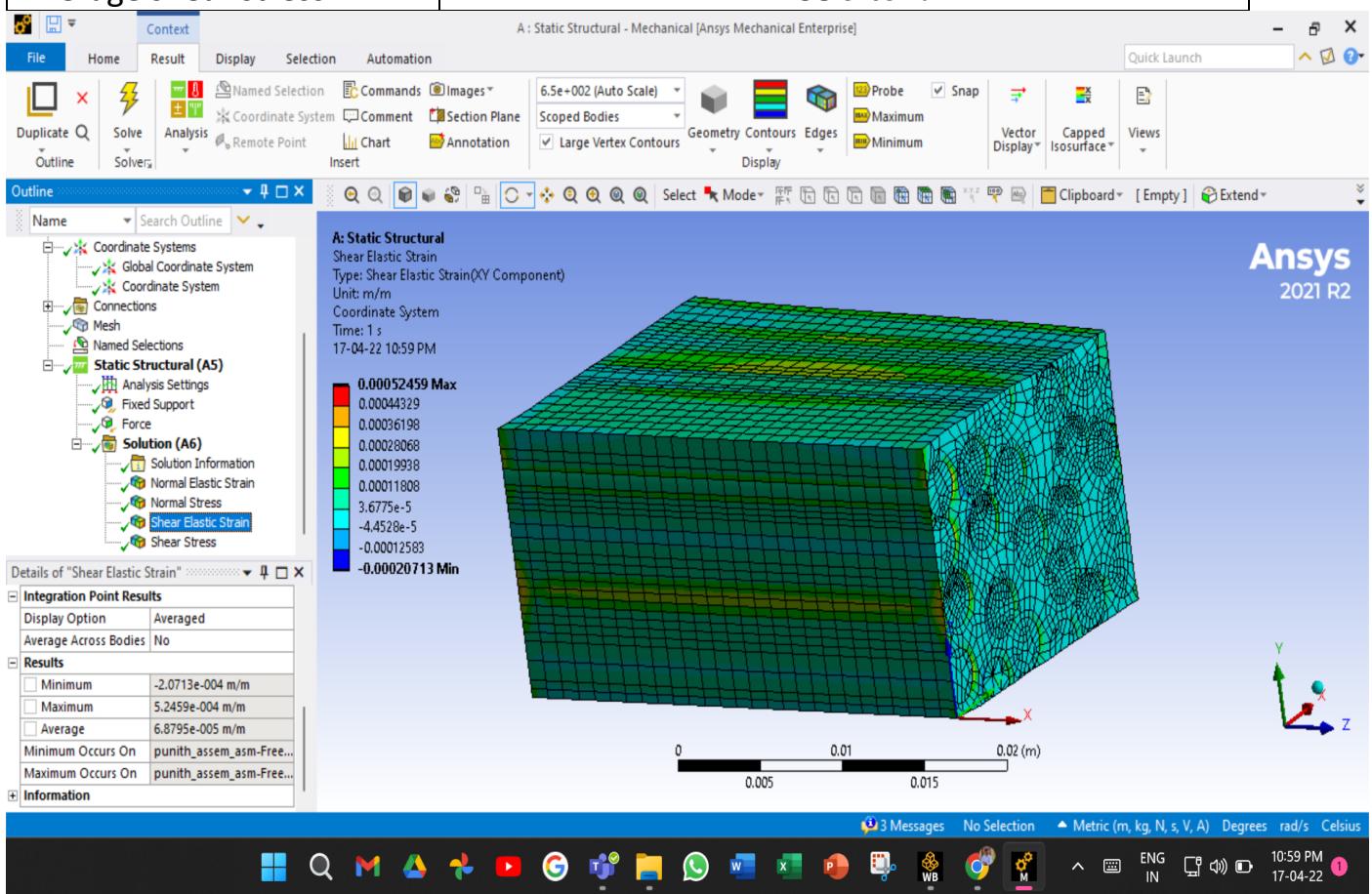
1. Plane XZ is fixed
2. 100 N force on X plane





Shear stress:

Minimum shear stress	-5.594 e+05Pa
Maximum shear stress	1.071 e+06 Pa
Average shear stress	2.238 e+05 Pa



Shear Elastic strain:

Minimum shear strain	-2.071 e-04 m/m
Maximum shear strain	5.245 e-04 m/m
Average shear strain	6.879 e-05 m/m

Longitudinal Stiffness:

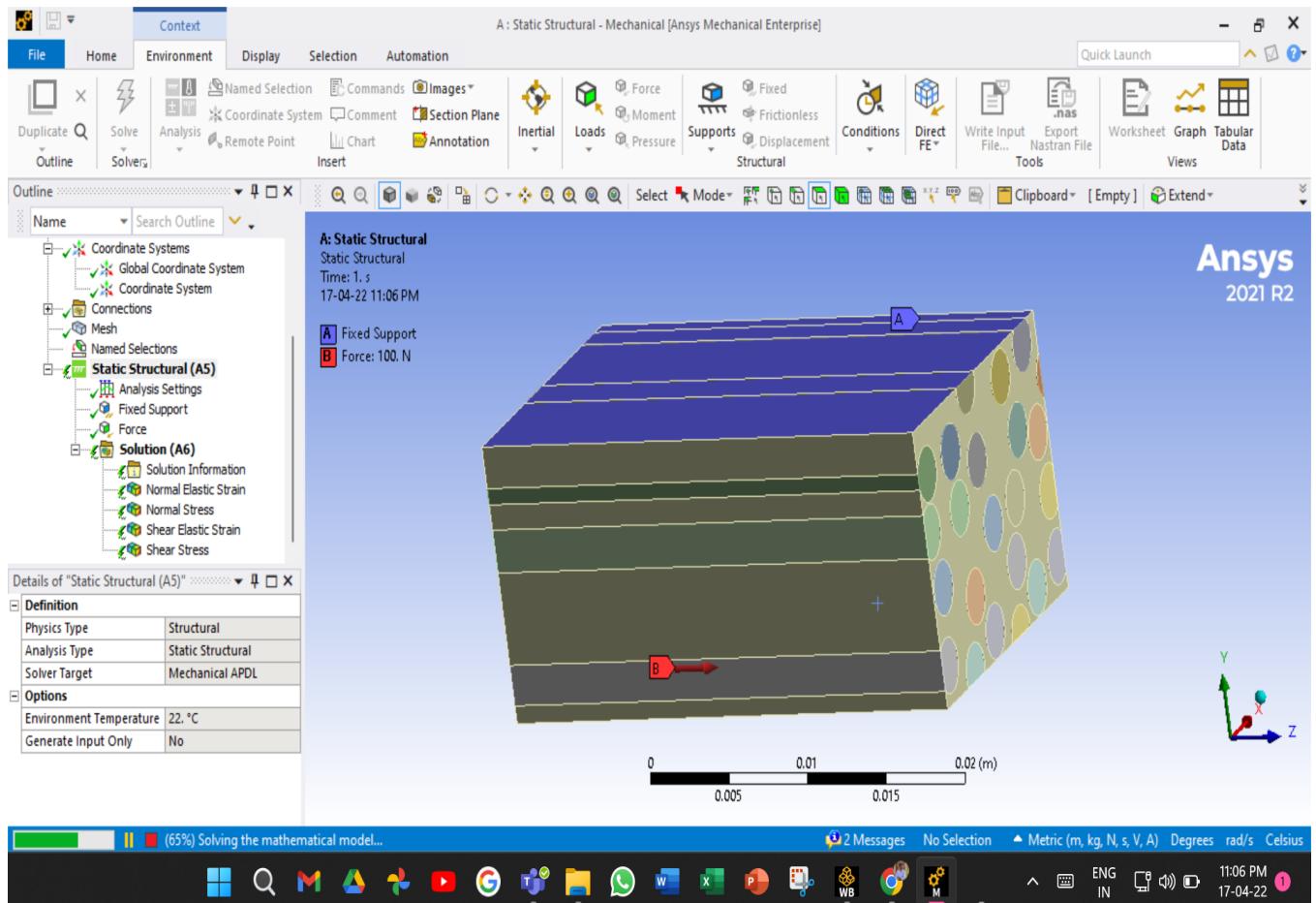
$$G_{xy} = \frac{\text{shear stress}}{\text{shear strain}}$$

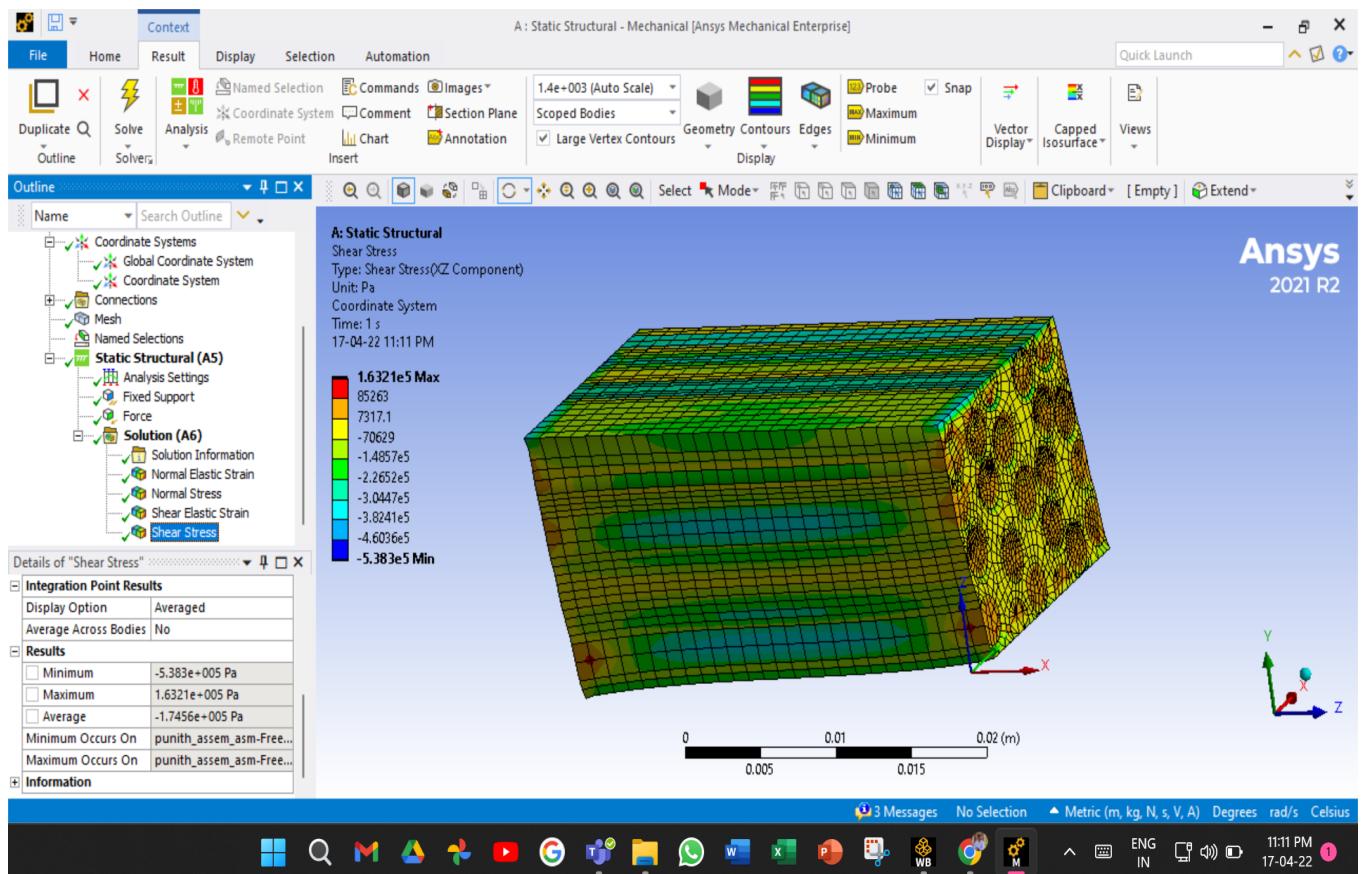
$$= \frac{(2.238 \times 10^5)}{(6.879 \times 10^{-5})} = 3.253 \text{ GPa}$$

Calculating G₁₃=G_{xz}

Loads and Boundary Condition:

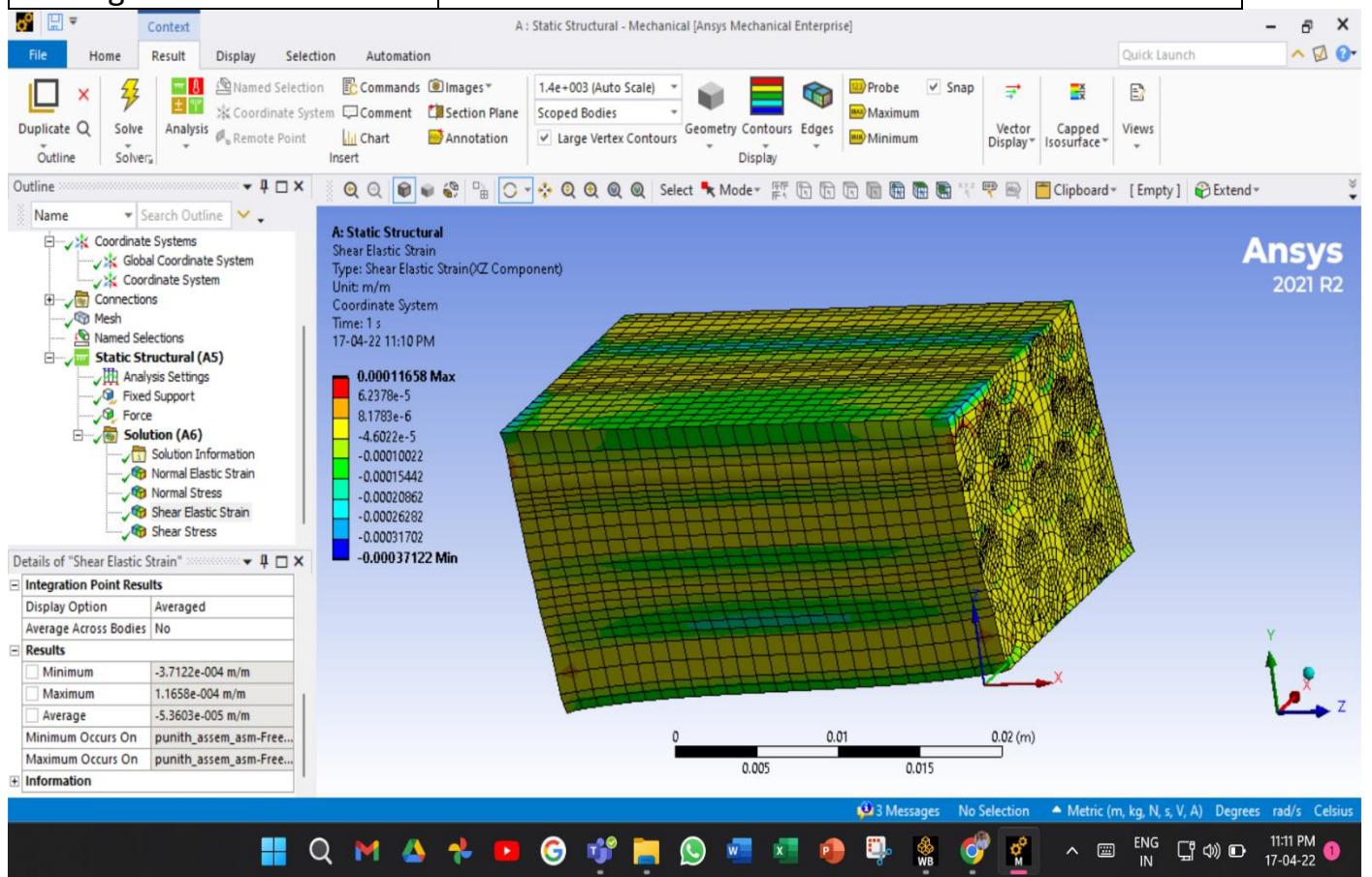
1. Plane XY is fixed
2. 100 N force on X plane





Shear stress:

Minimum shear stress	-5.383 e+05 Pa
Maximum shear stress	1.6321 e+05 Pa
Average shear stress	-1.745 e+05 Pa



Shear Elastic strain:

Minimum shear strain	-3.712 e-04 m/m
Maximum shear strain	1.165 e-04 m/m
Average shear strain	-5.360 e-05 m/m

Longitudinal Stiffness:

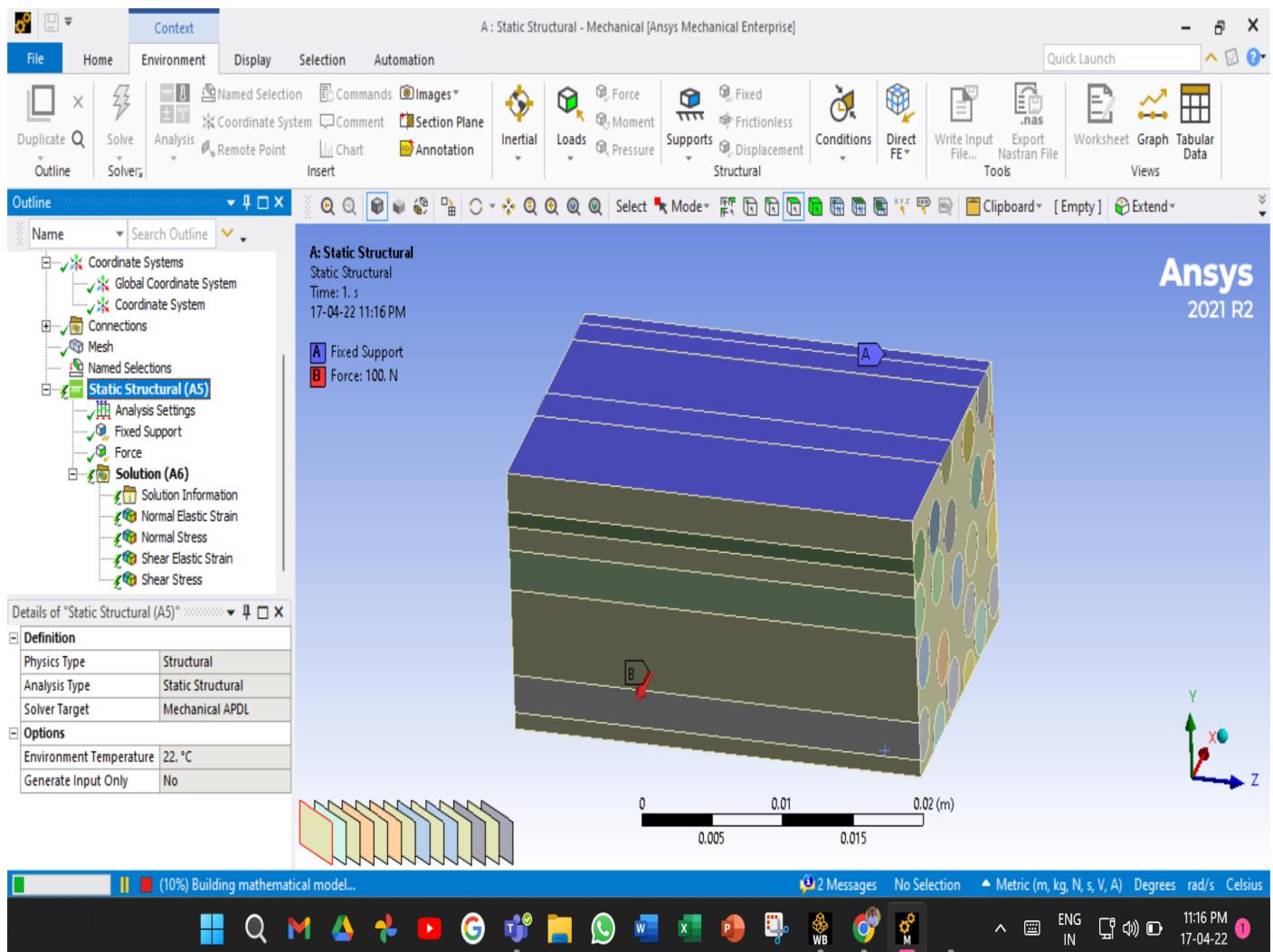
$$G_{xz} = \frac{\text{shear stress}}{\text{shear strain}}$$

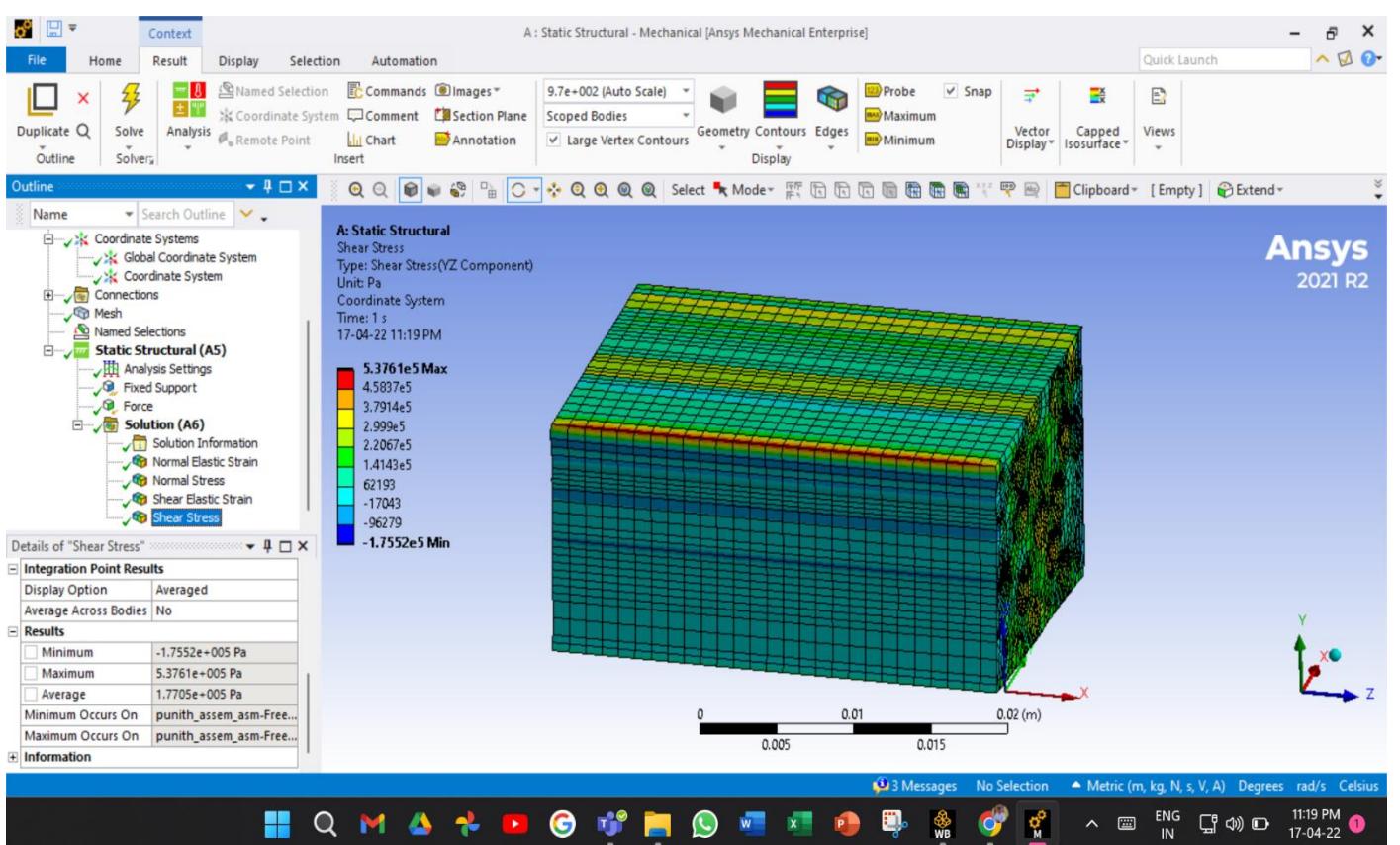
$$= \frac{(-1.745 \times 10^5)}{(-5.360 \times 10^{-5})} = 3.255 \text{ GPa}$$

Calculating G23=Gyz

Loads and Boundary Condition:

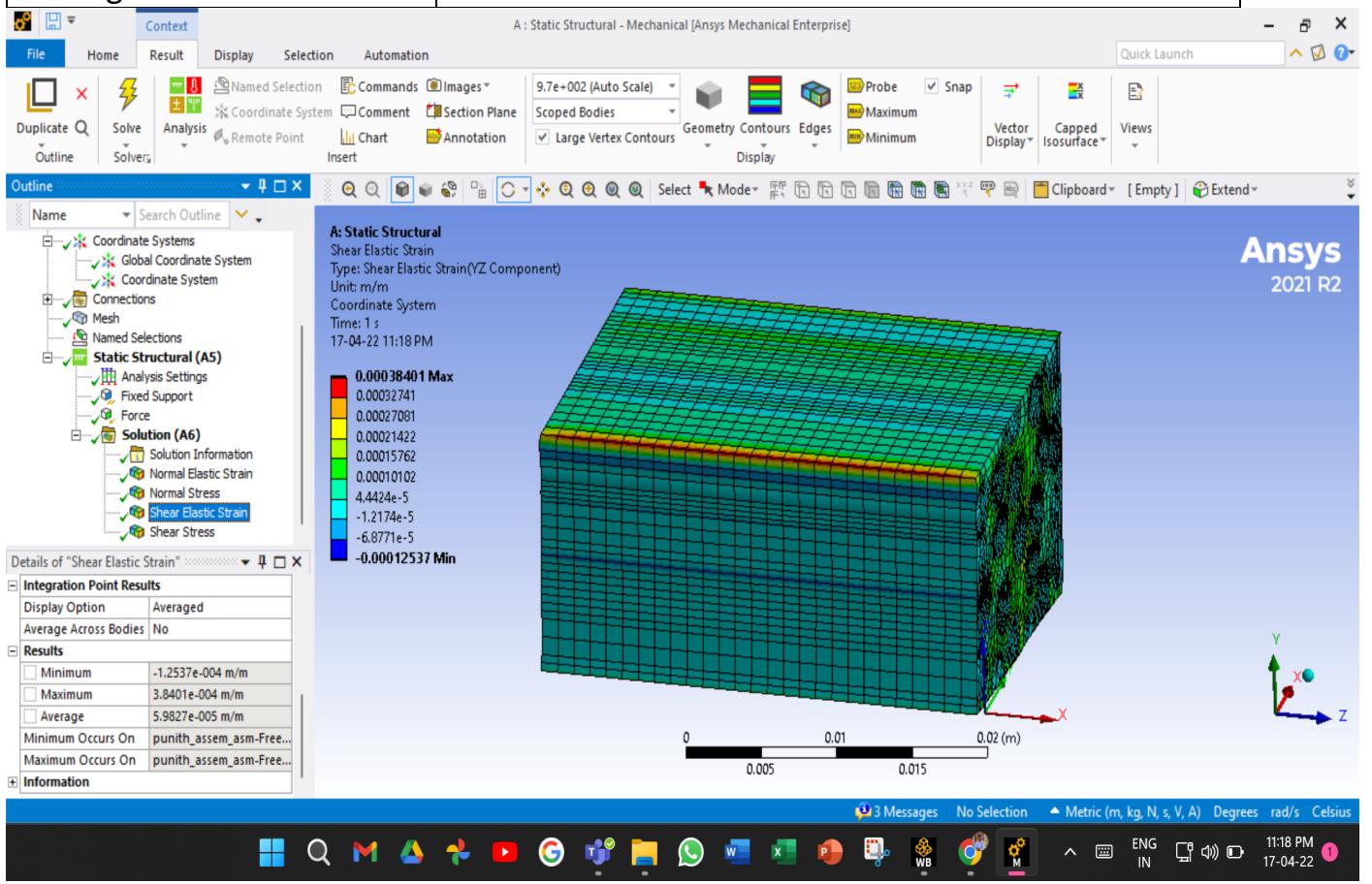
1. Plane XY is fixed at
2. 100 N force on -Y plane





Shear stress:

Minimum shear stress	-1.755 e+05 Pa
Maximum shear stress	5.376 e+05 Pa
Average shear stress	1.770 e+05 Pa



Shear Elastic strain:

Minimum shear strain	-1.253 e-04 m/m
Maximum shear strain	3.840 e-04 m/m
Average shear strain	5.982 e-05 m/m

Longitudinal Stiffness:

$$\begin{aligned} G_{yz} &= \frac{\text{shear stress}}{\text{shear strain}} \\ &= \frac{(1.770 \times 10^5)}{(5.982 \times 10^{-5})} \\ &= 2.958 \text{ GPa} \end{aligned}$$

Conclusion

Using iso-stress and iso-strain assumptions for obtaining properties of the composite material. By creating the random composite material model and applied the force or displacement in certain directions to obtain stresses and strain in those directions and indirectly obtaining the properties material. From our results through software tool, the properties of material are almost similar to the values obtained through iso-stress and iso-strain assumption calculation. And also, from our analysis we were able to notice that $E_{22} = E_{33}$ and $G_{12}=G_{13}$.