Adaptive Control of Linear Time-Varying Plants: A New Model Reference Controller Structure

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Abstract—**In this paper we study the problem of developing a control law which can force the output of a linear time-varying plant to track the output of a stable linear time-invariant reference model. We first show that the standard model reference controller, used for linear time-invariant plants, cannot guarantee zero tracking error in general when the plant is time-varying. We then propose a new model reference controller which guarantees stability and zero tracking error for a general class of linear time-varying plants with known parameters. When the time-varying plant parameters are unknown but vary slowly with time, we show that the new controller can be combined with a suitable adaptive law so that all the signals in the closed loop remain bounded for any bounded initial conditions and the tracking error is small in the mean. The assumption of slow parameter variations in the adaptive case can be relaxed if some information about the frequency or the form of the fast varying parameters is available *a priori*. Such information can be incorporated in an appropriately designed adaptive law so that stability and improved tracking performance is guaranteed for a class of plants with fast varying parameters.**

I. INTRODUCTION

ALTHOUGH the natural working environment of adaptive controllers is time-varying (TV) plants, it is only recently that such a problem has been addressed with some success. In the early attempts [1], [2], the persistency of excitation (PE) of certain signals in the adaptive loop was employed to guarantee the exponential stability of the unperturbed error system and, eventually, the local stability of the closed-loop TV plant. Elsewhere, [3]–[7], the restriction of the type of time variations of the plant parameters also led to the conclusion that an adaptive controller can be used in the respective time-varying environment.

Recently, a few more general results have been obtained for linear time-varying (LTV) plants with slowly TV or *jump* parameters in the case of model reference adaptive control (MRAC) [8] and indirect adaptive control [9], [10]. The common key-point of those studies was the intuitive idea that a linear slowly TV system behaves, at each time instant, "almost" like an LTI one. The effect of the time variations was then expressed as a disturbance, not necessarily bounded, but small compared to the useful signals. This treatment led to the proof that the use of modified adaptive laws with certain robustness properties can guarantee signal boundedness and "small" tracking errors, without any PE requirements.

A closer examination of the results in [8] indicates that the MRAC problem is naturally decomposed into two parts. In the first, and more important part, one is faced with the model reference control (MRC) problem. That is, assuming that the TV parameters of the plant are known for all time, design a controller such that the input–output (I/O) operator of the closed-loop plant is equal to the I/O operator of the LTI reference model. In the second part, the adaptive laws, for the adjustment of the controller parameters, are to be chosen and the closed-loop stability is to be established. In [8] it was argued that the standard controller of [11] does not guarantee the existence of an *exact* solution to the MRC problem, when the plant is LTV. Instead, it was shown that, by assuming a separation between the time scale t of the plant states and the time scale $\rho = \mu t$, $\mu \in [0, 1]$ of the plant parameters, the I/O mismatch operator is stable for small μ. It was then shown that, despite this mismatch, the adaptive law of [12] guarantees the stability of the closed-loop system and "small" tracking error, provided that the parameters of the plant are slowly TV (i.e., μ is small). However, as a result of the I/O mismatch in the MRC case, the minimum margin of stability of the inverse plant was required to be known *a priori* for the design of the adaptive law. The inability of the standard MRC law to solve the MRC problem exactly in the TV case, even when the plant parameters are known and vary slowly with time, gave rise to the following questions.

i) Is there a control law which can force an LTV plant to behave, from an I/O point of view, like an LTI reference model even in the case of known plant parameters?

ii) If such a control law can be found, is the knowledge of the plant parameters necessary for stability and good tracking between the output of the plant and that of the reference model?

The purpose of this paper is to answer these two basic questions as well as other related ones for continuous-time plants. We start by first introducing the appropriate mathematical preliminaries which can be used to describe and analyze the I/O properties of continuous-time LTV systems in a rather similar way as in the case of LTI systems. The mathematical preliminaries developed in Section II are used in Section III to describe the plant and the control objective. In Section III we also demonstrate that the standard MRC law [11], [13], [12] cannot solve the MRC problem exactly in general and, therefore, is not appropriate for TV plants. In Section IV we present a new MRC structure which gives a complete answer to the first question. That is we develop a new control law which can force a general LTV plant to behave, from an I/O point of view, exactly as an LTI reference model, provided that the plant parameters are known for all time $t \geq 0$. The speed of variation of the plant parameters can be arbitrary, but finite, i.e., the plant parameters do not have to vary slowly with time. The properties of the new MRC structure are significant when compared to the standard one which, for stability, requires the plant parameters to vary sufficiently slowly with time and cannot guarantee zero tracking error even for arbitrarily slow TV parameters.

The second question ii) is answered in Section V where we examine whether the knowledge of the plant parameters is necessary for the stability of the closed-loop plant with the new MRC. We develop an adaptive law similar to that proposed in [8],

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[12] and use it to update the unknown parameters of the new controller. In contrast to the standard MRAC law used in [8], no *a priori* information about the stability margin of the "inverse" plant is required for implementation of the new one. When the unknown plant parameters are bounded, smooth functions which vary slowly with time, the new MRAC law guarantees boundedness for any bounded initial conditions and small in the mean tracking error. The assumption of slow parameter variations can be relaxed if some information about the frequencies or the form of the fast varying parameters is available *a priori*. As we show in Section V, such information can be incorporated in the new MRC law so that stability and tracking performance is considerably improved for a wider class of LTV plants at the expense of updating additional parameters.

The results of Sections III–V are illustrated with several examples and computer simulations and compared to those obtained by using the standard MRC structure.

II. Mathematical Preliminaries

In this section we present some definitions and lemmas which are used to describe the I/O properties of LTV systems in a very similar way as in the LTI case. More details on LTV systems are given in [14].

Definition 2.1: An LTV polynomial differential operator (PDO) of degree n is defined as $P(s, t) = a_0(t)s^n + a_1(t)s^{n-1} + \cdots + a_n(t)$ where $s \triangleq d/dt(\cdot)$; $a_i(t)$, $i = 0, 1, \cdots, n$ are bounded piecewise continuous functions of time, $a_0(t) \neq 0\ \forall t \geq 0$. When $a_0(t) = 1\ \forall t \geq 0$, $P(s, t)$ is referred to as a monic PDO.

The properties of the PDO follow directly from the rules of differentiation and are given in [15], [14].

Definition 2.2: An LTV polynomial integral operator (PIO) of order n is defined as the operator that maps the input u to the zero-state response (ZSR) of the ordinary differential equation (ODE) $P(s, t)y = u$ where $P(s, t)$ is a monic PDO of degree n. We will denote the PIO by $P^{-1}(s, t)$ and write

$$P^{-1}(s, t)u = c' \int_0^t \Phi(t, \tau)bu(\tau)\, d\tau \tag{2.1}$$

where $\Phi(\cdot, \cdot)$ is the state transition matrix (STM) corresponding to $P(s, t)y = u$, $c' = [1, 0, \cdots, 0]$, $b' = [0, \cdots, 0, 1]$.

Using Definition 2.2 the solution of the ODE $P(s, t)y = u$ (see [16], [17]) can be expressed in a compact form as

$$y(t) = \underbrace{P^{-1}(s, t)u}_{\text{ZSR}} + \underbrace{c'\Phi(t, 0)Y(0)}_{\text{ZIR}} \tag{2.2}$$

where $Y(0) = [y(0), y^{(1)}(0), \cdots, y^{(n-1)}(0)]'$ is the vector with the initial conditions. The first term in (2.2) is the ZSR and the second term is the zero input response (ZIR) of $P(s, t)y = u$ [17].

Definition 2.3 [18]: An LTV PIO, $P^{-1}(s, t)$, is exponentially stable (or uniformly asymptotically stable) with rate $-a_1$, $a_1 > 0$, if the STM $\Phi(t, \tau)$, associated with the linear ODE $P(s, t)y = u$ satisfies $\|\Phi(t, \tau)\| \leq k \exp[-a_1(t - \tau)]\ \forall t \geq \tau \geq 0$ for some positive constants k, a_1.

The coprimeness of two TV PDO's is defined in general as follows.

Definition 2.4: Two PDO's, $D(s, t)$, $N(s, t)$, with smooth coefficients, are right coprime for all $t \geq 0$ if there exist PDO's $Q(s, t)$, $P(s, t)$ such that $\forall t \geq 0$

$$Q(s, t)D(s, t) + P(s, t)N(s, t) = 1. \tag{2.3}$$

Note that the left coprimeness of two PDO's can be defined in a similar manner via the equation $D(s, t)Q(s, t) + N(s, t)P(s, t) = 1$.

The coprimeness of two TV PDO's can be checked in a similar way as in the LTI case by first extending the definition of the Sylvester matrix to the TV case and then examining its properties.

Definition 2.5: The right TV Sylvester matrix S_t^R of the PDO's $D(s, t)$, $N(s, t)$, of degree n, m, respectively, is defined as $S_t^R = [C_1, \cdots, C_m, B_1, \cdots, B_n]$ where C_i, B_j are vectors of the coefficients of $s^{m-i}D(s, t)$, $s^{n-j}N(s, t)$, respectively, expressed as $(n + m - 1)$-order PDO's by setting the higher order coefficients equal to zero.

The motivation of Definition 2.5 becomes clear when the equation $Q(s, t)D(s, t) + P(s, t)N(s, t) = A(s, t)$ with deg $[A(s, t)] \leq n + m - 1$ is put in the form $S_t^R \boldsymbol{x} = \boldsymbol{a}$ where $\boldsymbol{x}$, $\boldsymbol{a}$ are vectors of the coefficients of $Q(s, t)$, $P(s, t)$, and $A(s, t)$, respectively.

Lemma 2.1: Two monic PDO's with smooth coefficients are right coprime $\forall t \geq 0$ iff their right TV Sylvester matrix is nonsingular $\forall t \geq 0$.

The proof of Lemma 2.1 is given in Appendix A.

Definition 2.6: Two monic PDO's with smooth coefficients are strongly (uniformly) right coprime if their right TV Sylvester matrix S_t^R satisfies $|\det(S_t^R)| \geq c$; $\forall t \geq 0$ for some constant $c > 0$.

Remark 2.1: From Definition 2.6 and the properties of the determinants it follows that if $D(s, t)$, $N(s, t)$ are strongly right coprime PDO's and $k_1(t)$, $k_2(t)$ are smooth functions of time such that $|k_1(t)|$, $|k_2(t)| \geq c > 0\ \forall t$, then $k_1(t)D(s, t)$, $k_2(t)N(s, t)$ are strongly right coprime. ■

Using the notion of PDO's and PIO's we can express the ZSR of more general LTV differential equations than $P(s, t)y = u$ in a compact form. This is achieved by introducing the notion of an LTV I/O operator which is a combination of PDO's and PIO's and has properties analogous to those of a transfer function in LTI systems. We consider the following general LTV system:

$$\dot{z} = A(t)z + b(t)u, \quad y = c'(t)z + d(t)u; \quad z(0) = z_0 \tag{2.4}$$

where $A(t)$, $b(t)$, $c(t)$, $d(t)$ are bounded, piecewise continuous functions of time.

Definition 2.7: The operator G that maps the input u to the ZSR of the differential equation (2.4) is defined as $Gu = c'(t) \int_0^t \Phi(t, \tau)b(\tau)u(\tau)\, d\tau + d(t)u$. We refer to G as the proper LTV I/O operator of (2.4). When $d(t) = 0\ \forall t \geq 0$, we refer to G as the strictly proper LTV I/O operator of (2.4).

The I/O operator G may be expressed as a combination of PDO's and PIO's; for example, the I/O operator G_1 of the system described by the differential equation $D(s, t)x = u$, $y = K(t)N(s, t)x$ where x is an internal state, $u, y \in \boldsymbol{R}^1$ is the input and the output of the system, respectively, $K(t)$ is a scalar function of t, and $D(s, t)$, $N(s, t)$ are monic PDO's of degree n, m, respectively, is given by $G_1(s, t) = K(t)N(s, t)D^{-1}(s, t)$.

Lemma 2.2: Let $P^{-1}(s, t)$ be an exp. stable PIO with rate $-a_1 < 0$ and consider the LTV systems with I/O pairs (y, x), $(\bar{y}, x)$ and I/O operators $P^{-1}(s, t)P(s, t)$, $P(s, t)P^{-1}(s, t)$, respectively. Then, $y = x + \epsilon_t$ and $\bar{y} = x + \epsilon_t$ where ϵ_t denotes exp. decaying to zero terms with rate at least $-a_1$, which depend on the initial conditions $x^{(i)}(0)$, $y^{(i)}(0)$.

The proof of Lemma 2.2 follows immediately by applying Definition 2.3 on the respective ODE's and is omitted.

Lemma 2.2 shows that if $P^{-1}(s, t)$ is exp. stable, the effect of setting $PP^{-1} = P^{-1}P = I$ is the appearance of exp. decaying to zero terms ϵ_t in the solution of the ODE's associated with these operators. Thus, Lemma 2.2 extends the properties of pole-zero cancellation in the LTI case, to the LTV one.

Remark 2.2: To assess the stability of proper LTV I/O operators, described by multiple combinations of PDO's and PIO's, we note that from Definitions 2.2 and 2.3 it follows that such an I/O operator is exp. stable if all the PIO's in its description are exp. stable. For example, if $P^{-1}(s, t)$, $D^{-1}(s, t)$ are exp. stable PIO's, then a proper LTV I/O operator described by $P^{-1}(s, t)N(s, t)D^{-1}(s, t)$ is exp. stable. ■

III. LTV Plant and Control Objective

Consider a single-input single-output (SISO) LTV plant that can be described by either one of the following forms of differential equations:

$$\text{P}-1: \quad \begin{array}{l} D_p(s,t)[x_p(t)] = u_p(t); \qquad X_p(0) = X_0 \\ y_p(t) = k_p(t)N_p(s,t)[x_p(t)] \end{array} \tag{3.1}$$

$$\text{P}-2: \quad \hat{D}_p(s,t)[y_p(t)] = \hat{k}_p(t)\hat{N}_p(s,t)[u_p(t)]; \qquad Y_p(0) = Y_0 \tag{3.2}$$

where $x_p \in \boldsymbol{R}^1$ is an internal state; $u_p, y_p \in \boldsymbol{R}^1$ are the input and the output of the plant, respectively; $D_p(s,t)$, $N_p(s,t)$, $\hat{D}_p(s,t)$, $\hat{N}_p(s,t)$ are monic LTV PDO's and $k_p(t)$, $\hat{k}_p(t) \neq 0 \ \forall t \geq 0$; $X_p(0) = [x_p(0), \cdots, x_p^{(n-1)}(0)]'$, $Y_p(0) = [y_p(0), \cdots, y_p^{(n-1)}(0)]'$ are the initial conditions for (3.1), (3.2), respectively. We will assume that the plant parameters, i.e., $k_p(t)$ or $\hat{k}_p(t)$ and the coefficients of the PDO's in (3.1) or (3.2), are uniformly bounded, smooth functions of t.

The control objective is to design a control law $u_p(t)$ such that the output $y_p(t)$ of the plant tracks the output $y_m(t)$ of the LTI reference model

$$y_m = W_m(s)r = k_m D_m^{-1}(s)r \tag{3.3}$$

where $D_m(s)$ is a monic Hurwitz polynomial $k_m > 0$ and $r(t)$ is a uniformly bounded reference input signal. In order to achieve such an objective, we make the following assumptions.

S.1: n, m are constant and known.

S.2: $D_p(s,t)$, $N_p(s,t)$ are strongly right coprime PDO's $\forall t \geq 0$.

S.3: The sign of $k_p(t)$ [or $\hat{k}_p(t)$] is constant and known. Without further loss of generality we will assume that $k_p(t) > 0$ (or $\hat{k}_p(t) > 0$) $\forall t \geq 0$. Furthermore, the range of $k_p(t)$ [or $\hat{k}_p(t)$] is a subset of a closed interval on the real axis that does not contain 0.

S.4: $N_p^{-1}(s,t)$ [or $\hat{N}_p^{-1}(s,t)$] is an exp. stable PIO with rate bounded from above by $-\alpha$, for some $\alpha > 0$.

S.5: $D_m(s)$ is designed so that $\deg[D_m(s)] = n^* \triangleq n - m$.

Remark 3.1: As shown in [21], a general LTV system (2.4) with smooth parameters can be transformed in the form P-1 iff it is uniformly controllable $\forall t$ and, by using dual arguments, in the form P-2 iff it is uniformly observable $\forall t$. Furthermore, it can be shown that the coprimeness assumption S.2 is satisfied iff the plant in the form P-1 is uniformly strongly observable $\forall t \geq 0$ (see [14] for details). Hence, in view of S.3, S.4, the LTV plant is required to be uniformly strongly observable and stabilizable. Thus, the lack of need for any coprimeness requirements on the PDO's of P-2 can be explained since an LTV plant in the form P-2 is already uniformly strongly observable due to the particular structure of the ODE and uniformly strongly stabilizable by S.3, S.4. ■

Assumptions S.1–S.5 are the extensions of those in the MRC of LTI plants, to the TV case. When the plant parameters are constant, the controller of [13] can be used to meet the control objectives exactly, both in the case of known and unknown plant parameters. When the plant parameters are TV, the standard MRC law cannot, in general, meet the control objective exactly, for either one of the representations P-1 or P-2, even in the case of known, slowly TV plant parameters [8]. We demonstrate this inability of the standard MRC law by using the following example.

Example 3.1: Consider the LTV plant $[s^2 + a_1(t)s + a_2(t)]y = u$ where $a_1(t)$, $a_2(t)$ are TV parameters.[1] The plant output y is required to track the output of the reference model $[s^2 + 3s + 2]y_m = r$ where r is the reference input signal. The standard MRC law [11] is given by

$$\dot{\omega}_1 = -\omega_1 + u, \ \dot{\omega}_2 = -\omega_2 + y; \ u = \theta_1\omega_1 + \theta_2\omega_2 + \theta_3 y + r \tag{3.4}$$

[1] Note that either forms P-1 or P-2 can describe the plant used in this example.

where θ_1, θ_2, θ_3 are the scalar controller parameters to be chosen for model-plant I/O matching. Using the properties of the PDO's and PIO's the matching condition can be written as

$$(s+1-\theta_1)(s+1)^{-1}[s^2 + a_1(t)s + a_2(t)](s+1) - [\theta_2 + \theta_3(s+1)] = (s^2+3s+2)(s+1). \tag{3.5}$$

Since PDO's with TV parameters do not commute w.r.t. multiplication (3.5) cannot be solved, in general, for θ_1, θ_2, θ_3 directly as it is done in the LTI case [11]. Despite this difficulty, we can solve (3.5) pointwise in time to obtain the following expressions: $\bar{\theta}_1^* = a_1(t) - 3$, $\bar{\theta}_2^* = a_1^2(t) - 4a_1(t) - a_1(t)a_2(t) + 3a_2(t) + 3$, $\bar{\theta}_3^* = 4a_1(t) + a_2(t) - a_1^2(t) - 5$. Using $\bar{\theta}_i^*$ in the control law (3.4), the output y of the closed-loop plant is expressed as

$$y_p = (s^2+3s+2)^{-1}r + L(s,t)r \tag{3.6}$$

where the mismatch operator $L(s,t)$ is of the form

$$L(s,t) = -[(s+1)(s^2+3s+2) + X(s,t)]^{-1}X(s,t)(s^2+3s+2)^{-1}$$
$$X(s,t) = (a_1 - 3)[\dot{a}_1 + (s+1)^{-1}(\dot{a}_2 - \dot{a}_1 - \ddot{a}_1)]. \tag{3.7}$$

It follows that for small $\dot{a}_1(t)$, $\ddot{a}_1(t)$, $\dot{a}_2(t)$ the mismatch operator $L(s,t)$ is proper, exp. stable, and $L(s,t)r$ is $O[\dot{a}_1(t), \ddot{a}_1(t), \dot{a}_2(t)]$. Due to the time variation of the plant parameters, the closed-loop I/O operator cannot be made exactly equal to that of the reference model with the standard MRC law. By using an approximate solution of (3.5) (e.g., the pointwise one) in the known parameter case, we can guarantee stability and a small tracking error provided that the plant parameters vary slowly with time, i.e., $\dot{a}_1(t)$, $\ddot{a}_1(t)$, $\dot{a}_2(t)$ are small [8]. When the plant parameters are unknown, it is shown in [8] (for plants in the form P-1) that the standard MRC law can be combined with a suitable adaptive law to update the controller parameters and guarantee global stability for slowly TV plant parameters. In this case, however, the implementation of the adaptive law requires the *a priori* knowledge of an upper bound for the exponential rate $-\alpha$ of $N_p^{-1}(s,t)$. ■

In the following section we propose a new MRC structure which is applicable to LTV plants in either one of the forms P-1 or P-2 and discuss its advantages over the standard one. The new MRC law guarantees stability and exact I/O matching between the LTV plant and the LTI reference model for plant parameters with arbitrary but finite speed of variations.

IV. A New Controller Structure

The new controller structure is shown in Fig. 1 and is described as follows. The plant input u_p is taken as

$$u_p = g'\omega + c_0 r: \ \dot{\omega}_1 = F\omega_1 + \theta_1 u_p; \ \dot{\omega}_2 = F\omega_2 + \theta_2 y_p; \ \omega_3 = \theta_3 y_p \tag{4.1}$$

where $\omega = [\omega_1', \omega_2', \omega_3]'$ is a $(2n-1)$-dimensional auxiliary vector, $\theta = [\theta_1', \theta_2', \theta_3]'$ is a $(2n-1)$-dimensional parameter vector, c_0 is a scalar parameter, $F \in \boldsymbol{R}^{(n-1)\times(n-1)}$ is a stable matrix, and $g = [q', q', 1]'$ is a constant vector such that (q', F) is an observable pair.

The difference between the new MRC structure shown in Fig. 1 and that of [8], [11], [12] is that in the new controller the auxiliary signal ω is obtained by filtering the parameter vectors θ_1, θ_2 scaled by u_p and y_p, respectively, and u_p is calculated as the sum of $c_0(t)r$ and the inner product of ω and a constant vector. In the standard MRC structure, however, ω is formed by filtering u_p and y_p alone and u_p is calculated as the sum of $c_0(t)r$ and the inner product of ω and the parameter vector $\theta(t)$. As we will show later in this section the central property of the new MRC is that the I/O operator $y_p \mapsto u_p$ can be expressed as $\bar{N}_1^{-1}\bar{N}_2$ where $\bar{N}_i$ are

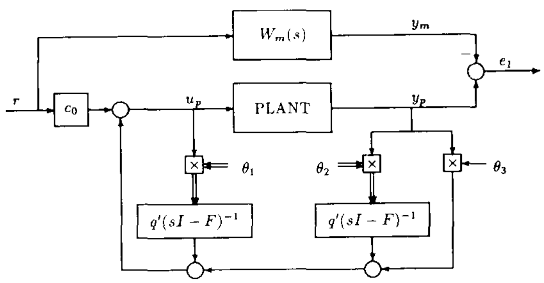

Fig. 1. New MRC structure.

arbitrary $(n-1)$-order PDO's, with $\bar{N}_1$ monic; such a description is not possible for the standard MRC due to the noncommutativity of the TV PDO's and PIO's (see Example 3.1). It will thus be possible to determine the coefficients of $\bar{N}_i$ and θ_i from the solution of a Diophantine equation, as in [23], so that the control objective is satisfied for arbitrary, smooth plant parameter variations.

The stability and tracking properties of the new MRC law, when applied to the LTV plant represented by either P-1 or P-2 form, is given by the following theorem.

Theorem 4.1: Consider a plant represented by either one of the forms given by (3.1), (3.2) whose parameters are known, bounded, smooth functions of t that satisfy Assumptions S.1–S.4. Then there exist bounded $\theta^*(t): \boldsymbol{R}^+ \mapsto \boldsymbol{R}^{(2n-1)}$ and $c_0^*(t): \boldsymbol{R}^+ \mapsto \boldsymbol{R}$ so that, for $\theta(t) = \theta^*(t)$ and $c_0(t) = c_0^*(t)$ in (4.1), the closed-loop TV plant (3.1) [or (3.2)], (4.1) is internally stable and its I/O operator $r \mapsto y_p$ is equal to that of the TI reference model (3.3).

Proof of Theorem 4.1:

a) Plant P-1: From (4.1) we obtain

$$u_p = D^{-1}(s)N_1(s, t)u_p + D^{-1}(s)N_2(s, t)y_p + c_0(t)r$$

$$N_1(s, t) = [Q_1(s), \cdots, Q_{n-1}(s)]\theta_1(t);$$

$$N_2(s, t) = [Q_1(s), \cdots, Q_{n-1}(s)]\theta_2(t) + D(s)\theta_3(t)$$

$$D(s) = \det\,(sI-F); \quad D^{-1}(s)[Q_1(s), \cdots, Q_{n-1}(s)] = q'(sI-F)^{-1}. \tag{4.2}$$

Using (4.2) and (3.1) we get

$$y_p = k_p(t)N_p(s, t)\{[D(s) - N_1(s, t)]D_p(s, t) - N_2(s, t)k_p(t)N_p(s, t)\}^{-1}D(s)c_0(t)r. \tag{4.3}$$

To satisfy the MRC objective the I/O operator $r \mapsto y_p$ should be made equal to $W_m(s)$, i.e.,

$$[D(s) - N_1(s, t)]D_p(s, t) - N_2(s, t)k_p(t)N_p(s, t) = D(s)c_0(t)D_m(s)k_p(t)N_p(s, t)k_m^{-1}. \tag{4.4}$$

The TV Diophantine equation (4.4) can be solved for $c_0(t)$ and the unknown coefficients of $\bar{N}_1(s, t) \triangleq D(s) - N_1(s, t)$ and $N_2(s, t)$ by choosing $c_0(t) = c_0^*(t) \triangleq k_m/k_p(t)$ and solving an algebraic system of linear equations $\hat{S}_{t1}X_1 = B_1$ where X_1 is the vector of the unknown coefficients of $\bar{N}_1(s, t)$ and $N_2(s, t)$. B_1 is the vector of the coefficients of $D(s)k_p^{-1}(t)D_m(s)k_p(t)N_p(s, t) - s^{n-1}D_p(s, t)$ and $\hat{S}_{t1} = [\Lambda_1, \cdots, \Lambda_{n-1}, \Pi_1, \cdots, \Pi_n]$ where Λ_i, Π_j consist of the coefficients of the PDO's $s^{n-1-i}D_p(s, t)$ and $s^{n-j}k_p(t)N_p(s, t)$, respectively, expressed as $(n-2)$-order PDO's. The matrix $\hat{S}_{t1}$ can be decomposed so that $\det[\hat{S}_{t1}] = \det[L_t]\det[S_t^R]$ where S_t^R is the right TV Sylvester matrix of $D_p(s, t)$ and $k_p(t)N_p(s, t)$ and L_t is an $(n-m-1) \times (n-m-1)$ lower triangular matrix with ones in the diagonal. Since, by S.2, S.3, and Remark 2.1, $D_p(s, t)$, $k_p(t)N_p(s, t)$ are strongly right coprime $\forall t \geq 0$, we have $|\det(\hat{S}_{t1})| \geq c$ for some $c > 0$ and $\forall t \geq 0$ and thus, (4.4) has a unique bounded solution.

To obtain the desired MRC parameters $\theta^*(t)$ from the coefficients of $\bar{N}_1(s, t)$, $N_2(s, t)$ we use (4.2) with $\theta_3^*(t)$ being the leading coefficient of $N_2(s, t)$; then

$$[Q_1(s), \cdots, Q_{n-1}(s)]\theta_i^*(t) = \hat{N}_i(s, t), \qquad i = 1, 2 \tag{4.5}$$

$$\hat{N}_1(s, t) = N_1(s, t); \; \hat{N}_2(s, t) = N_2(s, t) - D(s)\theta_3^*(t). \tag{4.6}$$

Using the properties of differentiation we can rewrite the RHS of (4.5) as a PDO in the form $\{s^j\hat{n}_{ji}\}$ and the LHS as a PDO in the form $\{s^j q_j'\theta_i(t)\}$ where $j = 0, 1, \cdots, n-2$, $i = 1, 2$ and q_j is a vector containing the coefficients of s^{n-1-j} of $Q_j(s)$. Letting $Q \triangleq [q_1', \cdots, q_{n-1}']'$ and equating the coefficients of equal powers of s (4.5) is written as $\theta_i^*(t) = Q^{-1}\hat{n}_i$ where $\hat{n}_i$ is the vector of the coefficients $\{\hat{n}_{ji}\}$ and Q^{-1} exists due to the observability of (q', F) [19]. Furthermore, it follows that a sufficient condition for $\theta^*(t)$ to be l-times differentiable is that the plant parameters should be at least $(l + \max[3n-m-2, 4n-m-4])$-times differentiable. This completes the proof of Theorem 4.1 for the form P-1.

b) Plant P-2: Since the controller is the same for both forms P-1 and P-2 of the plant, equations (4.2) of part a) are applicable to this case as well; using (4.2) with (3.2) we obtain

$$\{[D(s) - N_1(s, t)]\hat{N}_p^{-1}(s, t)\hat{k}_p^{-1}(t)\hat{D}_p(s, t) - N_2(s, t)\}y_p = D(s)c_0(t)r. \tag{4.7}$$

The PDO $[D(s) - N_1(s, t)]$ is monic of degree $n-1$ with coefficients to be determined and $\hat{N}_p(s, t)$ is also monic of degree $m \leq n-1$. We can therefore set

$$D(s) - N_1(s, t) = \tilde{N}_1(s, t)\hat{N}_p(s, t) \tag{4.8}$$

where $\tilde{N}_1(s, t)$ is some monic PDO of degree $n-m-1$ to be determined. Using (4.8) in (4.7) and the Assumption S.4 that $\hat{N}_p^{-1}(s, t)$ is exp. stable, the closed-loop I/O operator $r \mapsto y_p$ is expressed as $\{\tilde{N}_1(s, t)\hat{k}_p^{-1}(t)\hat{D}_p(s, t) - N_2(s, t)\}^{-1}D(s)c_0(t)r$. Hence, the MRC objective is satisfied if

$$\tilde{N}_1(s, t)\hat{k}_p^{-1}(t)\hat{D}_p(s, t) - N_2(s, t) = D(s)c_0(t)D_m(s)/k_m. \tag{4.9}$$

Choosing $c_0(t) = c_0^*(t) \triangleq k_m/\hat{k}_p(t)$, (4.9) can be solved for $N_2(s, t)$ and $\tilde{N}_1(s, t)$ since both PDO's in the LHS and RHS of (4.9) are of the same degree $(2n-m-1)$ and their leading coefficient is $\hat{k}_p^{-1}(t)$. The solution for the unknown PDO coefficients is found, as in part a), by solving a system of linear algebraic equations $\hat{S}_{t2}X_2 = B_2$. In this case, however, $\det(\hat{S}_{t2}) = \hat{k}_p^{-n+m+1}(t)$, and by S.3, (4.9) has a unique bounded solution which can be obtained by a simple forward substitution. The PDO $N_1(s, t)$ is then found by substituting the solution of (4.9) in (4.8) and $\theta^*(t)$ is obtained from $N_1(s, t)$, $N_2(s, t)$ as in part a). In this case, it follows from (4.8), (4.9) that a sufficient condition for $\theta^*(t)$ to be l-times differentiable is that the plant parameters are $(l + \max[2n-2, 3n-4])$-times differentiable. △△

As in the LTI case [11], in the proof of Theorem 4.1 we assume that all initial conditions are equal to zero. Since in the proof of Theorem 4.1 no unstable PIO was cancelled with a corresponding PDO, the internal stability of the closed-loop plant is guaranteed. If we account for any nonzero initial conditions, then by following the same procedure as in the proof of Theorem 4.1 the closed-loop plant response may be expressed as $y_p = W_m(s)r + \epsilon_t$ where ϵ_t is an exp. decaying to zero term which depends on the initial conditions and its rate depends on $W_m(s)$, $D^{-1}(s)$, and $N_p^{-1}(s, t)$ [or $\hat{N}_p^{-1}(s, t)$].

The new MRC law guarantees exact I/O matching between the closed-loop plant and the LTI reference model independent of the speed of variation of the plant parameters, i.e., the plant does not have to be slowly TV. In contrast, the standard MRC law not only requires the plant to be slowly TV [8] but also fails to guarantee exact I/O matching between the closed-loop plant and the

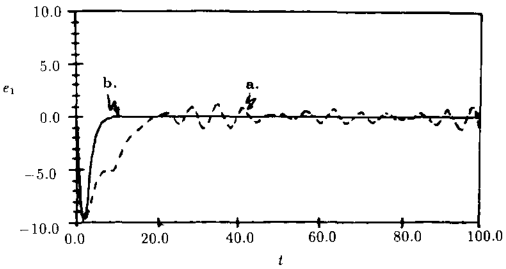

Fig. 2. MRC tracking error response. Known plant parameters; $\mu = 0.1$. (a) Standard MRC law. (b) New MRC law.

reference model in general, even for sufficiently slow TV plant parameters.

Let us now demonstrate the properties of the new MRC law by the following example.

Example 4.1: Let us apply the new MRC structure to the example presented in Section III, i.e., consider the control law

$$\dot{\omega}_1 = -\omega_1 + \theta_1 u,\ \dot{\omega}_2 = -\omega_2 + \theta_2 y;\ u = \omega_1 + \omega_2 + \theta_3 y + r. \quad (4.10)$$

From (4.9) we have that $y_p = y_m$, $\forall r$ if θ_i are selected to satisfy

$$(s+1-\theta_1)[s^2 + a_1(t)s + a_2(t)] - [\theta_2 + (s+1)\theta_3] = (s^2+3s+2)(s+1). \quad (4.11)$$

Comparing (4.11) to (3.5) it is clear that in the former no PIO appears either on the left or the right-hand side and, therefore, no commutativity problem arises. Choosing

$$\theta_1^* = a_1(t) - 3;\ \theta_3^* = 4a_1(t) + a_2(t) - a_1^2(t) - 5 + \dot{a}_1(t)$$

$$\theta_2^* = a_1^2(t) - 4a_1(t) - a_1(t)a_2(t) + 3a_2(t) + 3 - 5\dot{a}_1(t) + 2a_1(t)\dot{a}_1(t) - \ddot{a}_1(t) \quad (4.12)$$

(4.11) is satisfied and the I/O operator of the closed-loop plant is equal to that of the reference model for any bounded, smooth functions $a_1(t)$, $a_2(t)$.

Let us now simulate the response of the plant used in Example 3.1 with the standard and the new MRC for $r = 10 \sin t$, $a_1(t) = -6$ and $a_2(t) = 2 \sin \mu t$. The controller parameters for the standard MRC structure (3.4) are obtained as in Example 3.1 and for the new MRC structure (4.10) are calculated from (4.12). When $\mu = 0.1$ the standard MRC results to a bounded but nonzero tracking error [Fig. 2(a)] while for the new MRC the tracking error converges to zero [Fig. 2(b)]. Increasing the value of μ to 1, however, the tracking error for the standard MRC grows unbounded with time, as shown in Fig. 3(a), but the new MRC still results to a tracking error that converges to zero [Fig. 3(b)]. The unboundedness of the response of the closed-loop plant with the standard MRC is due to the large value of μ which results to an unstable mismatch operator $L(s, t)$. We should note that for this example the solution $\theta^*(t)$ for the new MRC structure is the same as the pointwise solution $\bar{\theta}^*(t)$ for the standard MRC structure, i.e., $\theta^*(t) = \bar{\theta}^*(t)$ when $a_1(t) = constant$ in Example 3.1. This demonstrates that the exact model-plant matching achieved by the new MRC law is a characteristic of the new structure and not only the specific choice of $\theta^*(t)$. ■

The calculation of the desired controller parameter vector $\theta^*(t)$, whose existence is established by Theorem 4.1 is possible, via rather tedious but quite straightforward calculations, only in the ideal case when the plant parameters are known for all time. In a more realistic situation, the parameters of the TV plant are unknown and, therefore, $\theta^*(t)$ should be estimated on line by a suitable adaptive law, so that the stability and tracking properties of the adaptive closed-loop plant are as close as possible to those in the known parameter case. We pose and analyze this problem in Section V.

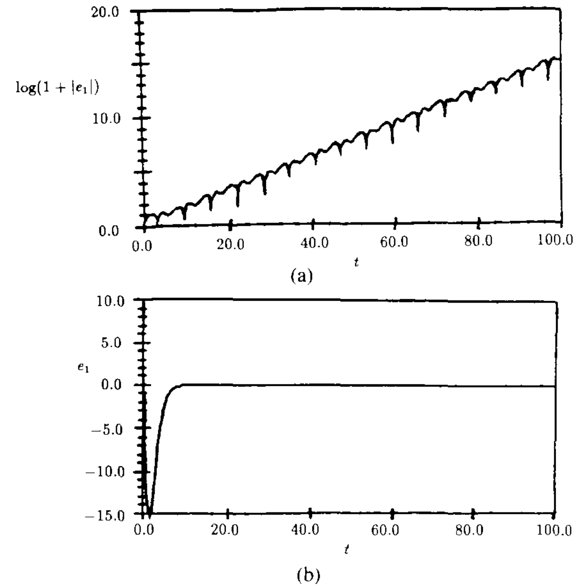

Fig. 3. MRC tracking error response. Known plant parameters; $\mu = 1$. (a) Standard MRC law: unbounded response due to fast parameter variations. (b) New MRC law.

V. MRAC FOR TV PLANTS

In this section we establish that the knowledge of the TV plant parameters is not necessary for the implementation and stability of the new MRC law, developed in Section IV, by designing an appropriate adaptive law to estimate the unknown parameters $\theta^*(t)$, $c_0^*(t)$.

Let us first consider the parameter vector $\Theta^*(t) = [\theta^{*\prime}(t), c_0^*(t)]'$ and define $\Theta(t) = [\theta'(t), c_0(t)]'$ to be the estimate of $\Theta^*(t)$ at time t. Without loss of generality we can assume that the structure of the time variations of $\Theta^*(t)$ is described by the following model:

$$\left.\begin{aligned} \theta^*(t) &= \theta_0^*(t) + H_1(t)\theta_1^*(t) + \cdots + H_l(t)\theta_l^*(t) \\ c_0^*(t) &= \hat{c}_0^*(t)h_0^{-1}(t) \end{aligned}\right\} \Leftrightarrow \Theta^*(t) = \hat{H}(t)\hat{\Theta}^*(t) \quad (5.1)$$

where $\hat{\Theta}^*(t) = [\theta_0^{*\prime}(t), \cdots, \theta_l^{*\prime}(t), \hat{c}_0^*(t)]'$ is an unknown, possibly TV vector $\hat{H}(t) = \text{diag}\,([I, H_1(t), \cdots, H_l(t)], h_0^{-1}(t))$ with $H_i(t): \boldsymbol{R}^+ \mapsto \boldsymbol{R}^{2n-1 \times 2n-1}$ being known TV matrices whose entries are smooth uniformly bounded functions of time t; $h_0(t): \boldsymbol{R}^+ \mapsto \boldsymbol{R}^+$ is a known smooth function of time t which arises when the high frequency gain $k_p(t)$ is of the form $k_p(t) = \bar{k}_p(t)h_0(t)$ with $h_0(t) > 0\ \forall t \geq 0$ such that Assumption S.3 is satisfied. The reason for using (5.1) is as follows. When $\Theta^*(t)$ contains fast TV elements, it cannot be estimated by a general adaptive law [20] due to the finite speed of adaptation. If, however, these fast elements are of known structure and can be incorporated in $\hat{H}(t)$ s.t. $\hat{\Theta}^*(t)$ is slowly TV and $\hat{H}(t)$ is known, then the estimate $\Theta(t)$ of $\Theta^*(t)$ can be obtained from

$$\Theta(t) = \hat{H}(t)\hat{\Theta}(t) \quad (5.2)$$

where $\hat{\Theta}(t)$ is the estimate of $\hat{\Theta}^*(t)$ at time t. We note that the knowledge of the complete TV structure of $\Theta^*(t)$ implies that in (5.1) $\hat{H}(t)$ is known and $\hat{\Theta}^*$ is constant. Allowing $\hat{\Theta}^*$ to be TV, (5.1) can be used to describe partial or approximate knowledge of the TV structure of $\Theta^*(t)$ as well; the case of $\Theta^*(t)$ with completely unknown TV structure (no *a priori* information) is also covered by (5.1) by taking $\hat{H}(t) = I$.

The TV structure of the control parameter vector $\Theta^*(t)$ can be found from the TV structure of the plant parameters by using (4.4) for plants in the form P-1 or (4.9) for plants in the form P-2 and the corresponding algebraic equations. For plants in the form P-2, due to the simpler form of the design equations, we can always find the complete structure of $\Theta^*(t)$ when the complete structure of the plant parameters is available. For example, consider an nth order TV plant with deg $[\hat{N}_p(s, t)] = m$ and $\hat{k}_p(t) = 1$[2] whose parameters vary as sinusoids with a known frequency w_0 and unknown amplitude. Using Theorem 4.1, $\theta^*(t)$ may then be expressed as $\theta^*(t) = \theta_0^* + \Sigma_1^{n-m}\theta_{1i}^* \sin(iw_0 t) + \Sigma_1^{n-m}\theta_{2i}^* \cos(iw_0 t)$ where θ_0^*, θ_{ji}^* are constant vectors and w_0 is any finite constant. For plants in the form P-1, however, the nonlinear dependence of $\theta^*(t)$ on the plant parameters may cause θ_0^*, θ_{ji}^* to be TV.

Employing the certainty equivalence principle [20], we replace $\Theta^*(t)$ in the new MRC law (4.1) with the estimate $\Theta(t)$, obtained from (5.2) and propose the following adaptive law for updating $\hat{\Theta}(t)$:

$$\dot{\hat{\Theta}} = -\Gamma\frac{\epsilon_1 Z}{m^2} - \Gamma\sigma\hat{\Theta}; \quad \dot{\hat{\psi}}_0 = -\gamma\frac{\epsilon_1\xi}{m^2} - \gamma\sigma_1\hat{\psi}_0 \tag{5.3}$$

where $\hat{\psi}_0(t)$ is the estimate of $\hat{\psi}_0^*(t) \triangleq 1/\hat{c}_0^*(t)$ at time t and

$$\dot{m} = -\delta_0 m + \delta_1(|u_p| + |y_p| + 1); \qquad m(0) \geq \delta_1/\delta_0$$

$$\sigma = \begin{cases} 0 & \text{if } \|\hat{\Theta}\| < M_0 \\ \sigma_0(\|\hat{\Theta}\|/M_0 - 1) & \text{if } M_0 \leq \|\hat{\Theta}\| \leq 2M_0 \\ \sigma_0 & \text{if } \|\hat{\Theta}\| > 2M_0 \end{cases} \tag{5.4}$$

$$\epsilon_1 = y_p - y_m + \hat{\psi}_0\xi; \quad \xi = \hat{\Theta}'Z - W_m(s)h_0(t)u_p;$$
$$Z = W_m(s)[h_0(t)\hat{G}'(s)U\hat{H}(t)]' \tag{5.5}$$

$$\hat{G}'(s) = [D^{-1}(s)[Q_1(s), \cdots, Q_{n-1}(s)],$$
$$D^{-1}(s)[Q_1(s), \cdots, Q_{n-1}(s)], 1, 1]$$

$$U = \text{diag}\,(\underbrace{u_p, \cdots, u_p}_{n-1}, \underbrace{y_p, \cdots, y_p}_{n-1}, y_p, r) \tag{5.6}$$

where $\Gamma = \Gamma' > 0$; $D(s)$, $Q_i(s)$ are as in (4.2); $\sigma_0, \gamma, \delta_1 > 0$ are design parameters; σ_1 is the same as σ with $\hat{\Theta}$, M_0 replaced by $\hat{\psi}_0$, M_1, and $M_0, M_1, \delta_0 > 0$ are chosen so that

$$M_0 > \|\hat{\Theta}^*(t)\|; \; M_1 > |\hat{\psi}_0^*(t)|; \; \delta_0 + \delta_2 \leq q_0 \tag{5.7}$$

for some $\delta_2 > 0$ and by designing q_0, $W_m(s)$ and the matrix F of the filters (4.1) so that the poles of $W_m(s - q_0)$ and the eigenvalues of $F + q_0 I$ are stable [12]. In contrast to the standard MRAC law [8], in this case no *a priori* information is needed about the stability margin of the "inverse" plant for the design of δ_0. The form of the adaptive law (5.3), however, is similar to the ones used in [12], [8] with the standard MRC structure generalized to incorporate the available *a priori* knowledge of the structure of $\Theta^*(t)$.

The following theorem defines the class of TV plants for which the controller (4.1), (5.2)–(5.7) guarantees the boundedness of all signals in the closed loop and the smallness in the mean of the residual tracking error.

Theorem 5.1: Let $\|(d/dt)\hat{\Theta}^*(t)\| \leq \mu$, $\forall t \geq 0$ where $\mu \geq 0$. Then for all $\mu \in [0, \mu^*)$ and some $\mu^* > 0$ all the signals in the closed-loop plant (3.1) or (3.2) with the controller (4.1), (5.2)–(5.7) are bounded for any bounded initial conditions and the tracking error $e_1 \triangleq y_p - y_m$ belongs to the residual set

$$D_e = \left\{ e_1 : \lim_{T\to\infty} \sup_{T>0} \frac{1}{T}\int_{t_0}^{t_0+T} |e_1(t)|^2\, dt \leq \mu C \right\} \tag{5.8}$$

where $C > 0$ is a constant.

[2] Taking $\hat{k}_p(t)$ to be TV will only result in similar but more complicated expressions.

The proof of Theorem 5.1 follows similar steps as those in [12]. It is, however, modified due to the different structure of the controller and the form of the plant and is given in Appendix B.

Remark 5.1: When $\|(d/dt)\hat{\Theta}^*(t)\| = 0$, $\forall t \geq t_0 \geq 0$. Theorem 5.1 can be used to show that $e_1 \to 0$ as $t \to \infty$. That is, exact tracking is achieved with the new MRAC law despite the fact that $\hat{H}(t)$ and therefore the overall plant is TV. ■

Remark 5.2: If no *a priori* information about the TV structure of $\Theta^*(t)$ is available, i.e., in (5.1) $\hat{H}(t) = I$ and $\hat{\Theta}^*(t) = \Theta^*(t)$, Theorem 5.1 requires $\Theta^*(t)$ to be slowly TV. From the proof of Theorem 4.1 it follows that this requirement is satisfied if $q + 1$ derivatives of the plant parameters are $O(\mu)$ small where $q = \max[3n - m - 2, 4n - m - 4]$ or $\max[2n - 2, 3n - 4]$ for plants in the form P-1 or P-2, respectively, i.e., if the plant is slowly TV. ■

The significance of Theorem 5.1 is that it allows a wide class of TV plants to be adaptively controlled in a stable manner and with a zero or "small" residual tracking error. The plants in this class may also possess fast TV parameters at the expense of updating additional controller parameters and requiring some *a priori* knowledge about the speed and the structure of the fast parameter variations.[3] In other words, the crucial parameter for the stability and tracking properties of the new MRAC is the speed of variation of the unstructured (unknown) part of the desired controller parameters and not the speed of the parameter variations of the overall plant. It should be pointed out that this result relies on the existence of θ^*, c_0^* such that the I/O operator of the closed-loop plant is equal to that of the reference model. As explained in Section IV, such a property is guaranteed by using the new MRC structure, but not the standard one. Consequently, the stability and tracking properties of the standard MRAC will depend on the overall speed of the plant parameter variations, even if their structure is known.

Example 5.1: Let us now demonstrate the results of Theorem 5.1 in the case of structured parameter variations by using the Example 3.1. Assume that $a_1(t) = A_1$ is constant and unknown and $a_2(t) = A_2 \sin wt$ where A_2 is unknown but w is known. Then, using Theorem 4.1, the desired control parameter vector $\theta^* = [\theta_1^*, \theta_2^*, \theta_3^*]'$ can be expressed as $\theta_1^* = \theta_{10}^*$, $\theta_2^* = \theta_{20}^* + \theta_{21}^* \sin wt$, $\theta_3^* = \theta_{30}^* + \theta_{31}^* \sin wt$ where $\theta_{10}^* = A_1 - 3$, $\theta_{20}^* = (A_1 - 4)A_1 + 3$, $\theta_{21}^* = (3 - A_1)A_2$, $\theta_{30}^* = (4 - A_1)A_1 - 5$, and $\theta_{31}^* = A_2$. The estimate $\theta(t) = [\theta_1(t), \theta_2(t), \theta_3(t)]'$ of the desired parameter vector θ^* is obtained from

$$\theta_1(t) = \hat{\theta}_{10}(t); \; \theta_2(t) = \hat{\theta}_{20}(t) + \hat{\theta}_{21}(t)\sin wt;$$
$$\theta_3(t) = \hat{\theta}_{30}(t) + \hat{\theta}_{31}(t)\sin wt \tag{5.9}$$

where $\hat{\Theta} = [\hat{\theta}_{10}, \hat{\theta}_{20}, \hat{\theta}_{30}, \hat{\theta}_{21}, \hat{\theta}_{31}]'$ is the estimate of $\hat{\Theta}^* = [\theta_{10}^*, \theta_{20}^*, \theta_{30}^*, \theta_{21}^*, \theta_{31}^*]'$ and is updated by

$$\dot{\hat{\Theta}} = -100\epsilon_1 Z/m^2 - \sigma\hat{\Theta}; \; \epsilon_1 = y + \hat{\Theta}'Z - (s^2 + 3s + 2)^{-1}u \tag{5.10}$$

$$Z = (s^2 + 3s + 2)^{-1}[(s+1)^{-1}u, (s+1)^{-1}y,$$
$$y, (s+1)^{-1}y \sin wt, y \sin wt]' \tag{5.11}$$

and m, σ are as in (5.4) with $\delta_0 = 0.9$, $\delta_1 = 0.5$, $\sigma_0 = 0.1$, and $M_0 = 120$. The control input is generated by (4.10) with $\theta_1, \theta_2, \theta_3$ obtained from (5.9).

We apply the new adaptive control law (4.10), (5.9)–(5.11) to the plant of Example 3.1 with $a_1(t) = -6$ and $a_2(t) = 2 \sin t$ and initial conditions $\hat{\Theta}(0) = [-9.6, 72.96, -74.96, 21.2, 2.2]$, reflecting a 10 percent error on the plant parameters. The plant output is required to track the output of the reference model of Example 3.1 for $r = 10 \sin t$. As shown in Fig. 4(a) the new MRC law guarantees zero residual tracking error even though $w = 1$ in this case so that the plant is not "slowly" TV.

For reasons of comparison, let us now incorporate the same *a*

[3] In the case of unstructured parameter variations, both the new and the standard MRAC require the same number of adjustable parameters.

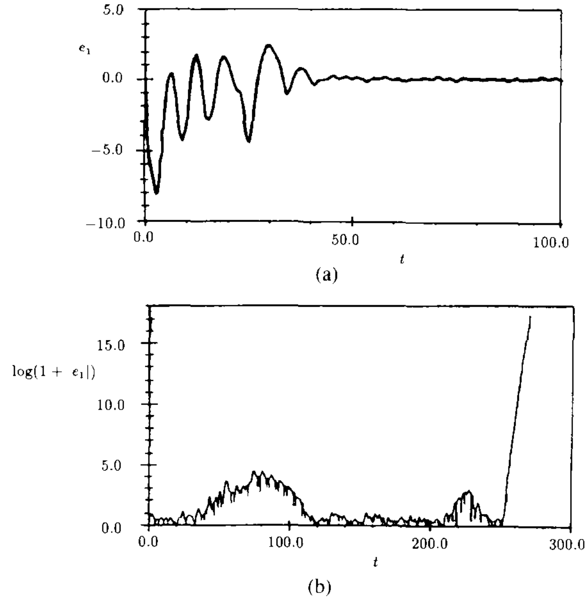

Fig. 4. MRAC tracking error response. Structured parameter variations; $w = 1$. (a) New MRAC. (b) Standard MRAC: unbounded response due to fast parameter variations.

priori knowledge of the structure of the plant parameters $a_1(t)$, $a_2(t)$ in the standard MRAC scheme. Following the procedure of Example 3.1, the value of $\bar{\theta}^*$ for pointwise matching, in this case, is exactly the same as θ^* due to the fact that a_1 is constant. Using the *a priori* knowledge about w and the structure of $\bar{\theta}^*$, the same adaptive law and signals as in (5.9) with the exception of $\hat{Z}$ now given by $\hat{Z} = (s^2 + 3s + 2)^{-1}[(s + 1)^{-1}u, (s + 1)^{-1}y, y, \sin wt(s + 1)^{-1}y, y \sin wt]'$ can be used to update the parameters of the standard MRC law (3.4). Using the same design parameters and initial conditions as before, the standard MRAC law (3.4), (5.9) is applied to the same plant with $w = 1$. Due to the different controller structure and the fast plant-parameter variations, however, the tracking error eventually grows unbounded as shown in Fig. 4(b). ■

VI. Conclusion

In this paper we consider the MRC and MRAC problem of LTV plants. The plant parameters are assumed to be bounded, smooth functions of time which satisfy the usual assumptions of MRC for LTI plants, extended to the TV case. We first show that the standard MRC law for LTI plants does not guarantee zero tracking error in general when the plant parameters are TV. We then propose a new MRC law and show that it guarantees stability and zero tracking error, provided that the TV plant parameters are known. The case of unknown plant parameters is also investigated by combining the new MRC structure with a suitable adaptive law for adjusting the controller parameters. In contrast to the standard MRAC used in [8], the new MRAC requires no *a priori* knowledge of the stability margin of the inverse plant. Furthermore, the new MRAC guarantees global stability and good tracking properties for a wide class of LTV plants with smooth parameters. This class includes plants with unknown slowly TV parameters as well as fast TV parameters with known structure.

The development of other controller structures, such as pole placement and structures derived as the solution of the linear quadratic problem [24] for LTV plants, are topics for future research.

Appendix A

Proof of Lemma 2.1

Let $D(s, t)$, $N(s, t)$ be monic PDO's of degrees n, m, respectively, and S_t^R be the corresponding right TV Sylvester matrix.

(If): Straightforward from Definitions 2.5 and 2.4.

(Only If): Assume that $D(s, t)$, $N(s, t)$ satisfy Definition 2.4 for some PDO's $Q_0(s, t)$, $P_0(s, t)$ of degree $\hat{m}$, $\hat{n}$ ($\hat{n} - \hat{m} = n - m$). Also, let $D_1(s, t)$, $N_1(s, t)$ be PDO's of degree n_1, m_1 such that $N_1(s, t)D(s, t) + D_1(s, t)N(s, t) = 0$. Note that such PDO's exist, whether unique or not, with $n_1 \le n$, $m_1 \le m$, and $n - m = n_1 - m_1$. Without loss of generality, we can assume that $\hat{m} < m_1$; $\hat{n} < n_1$. Furthermore, we can select $\lambda_k(t)$, p such that

$$Q_{k+1}(s, t) = sQ_k(s, t) + \lambda_k(t)s^pN_1(s, t);$$
$$P_{k+1}(s, t) = sP_k(s, t) + \lambda_k(t)s^pD_1(s, t) \quad \text{(A.1)}$$

where $Q_{k+1}(s, t)$, $P_{k+1}(s, t)$ are of order $\le m - 1$, $n - 1$, respectively, for $k = 0, 1, \cdots, n + m - 1$. Multiplying (2.3) by s^k from the left and using (A.1), it follows that

$$Q_k(s, t)D(s, t) + P_k(s, t)N(s, t) = s^k;$$
$$k = 0, 1, \cdots, n+m-1. \quad \text{(A.2)}$$

Using Definition 2.5 we can write (A.2) as $S_t^R X = I$ where X is a matrix of the coefficients of $Q_k(s, t)$, $P_k(s, t)$. Hence, det $[S_t^R] \neq 0$. To complete the proof, we note that the coprimeness of two PDO's is not affected if they are left multiplied by smooth, nonzero functions of time. △△

Appendix B

Theorem 5.1: Proof of Stability

Throughout the proof we will use the properties of the normalizing signal m given in [8, Lemma 4.1] and [12, Lemma 3.1]. Briefly, these properties can be stated as follows. The normalizing signal m bounds any signal that can be written as the output of a, possibly TV, filter with uniformly bounded parameters whose STM decays at least as fast as $\exp[-\delta_0(t - \tau)]$ and its input is bounded from above by $|y_p| + |u_p|$. The same result holds if the input to the filter is bounded from above by $|y_p| + |u_p| + m$ provided that its STM decays at least as fast as $\exp[-(\delta_0 + \delta_2)(t - \tau)]$ where $\delta_2 < \delta_0$ is a positive constant.

Step 1—The Augmented Error: By letting $\phi = \hat{\Theta}(t) - \hat{\Theta}^*(t)$, $\tilde{\psi}_0 = \hat{\psi}_0(t) - \hat{\psi}_0^*(t)$, the tracking error e_1, the plant input u_p, and the augmented (prediction) error ϵ_1 (5.5) can written as

$$e_1 = W_m(s)[\hat{\psi}_0^*(t)h_0(t)\hat{G}'(s)U\hat{H}(t)\hat{\phi}]; \quad u_p = \hat{G}'(s)U\hat{H}(t)\hat{\Theta} \tag{B.1}$$

$$\begin{aligned}\epsilon_1 = {} & \hat{\psi}_0^*(t)\phi'Z + \tilde{\psi}_0\xi + \hat{\psi}_0^*(t)W_m(s) \\ & \cdot [W_m^{-1}(s)\hat{\Theta}^{*\prime}(t)Z - h_0(t)\hat{G}'(s)U\hat{H}(t)\hat{\Theta}^*(t)] + W_m(s) \\ & \cdot [(\hat{\psi}_0^*(t) - D_m(s)\hat{\psi}_0^*(t)D_m^{-1}(s))h_0(t)\hat{G}'(s)U\hat{H}(t)\hat{\phi}].\end{aligned} \tag{B.2}$$

By writing the partial fraction expansion for $W_m(s)$ and the elements of $\hat{G}(s)$[4] and noting that $(s + a)x(t)(s + a)^{-1} = x(t) + \dot{x}(t)(s + a)^{-1}$, the assumption $\|(d/dt)\hat{\Theta}^*(t)\| \le \mu$ implies that the augmented error can be expressed as

$$\epsilon_1 = \hat{\psi}_0^*(t)\phi'Z + \tilde{\psi}_0\xi + \mu\eta \tag{B.3}$$

where by [8, Lemma 4.1], η/m is bounded.

Step 2—Boundedness of the Parameter Estimates: Using (B.3) the adaptive law (5.3) can be expressed as

$$\begin{aligned}\dot{\phi} &= -\Gamma\hat{\psi}_0^*(t)\frac{\phi'ZZ}{m^2} - \Gamma\frac{\tilde{\psi}_0\xi Z}{m^2} - \mu\Gamma\frac{\eta}{m}\frac{Z}{m} - \Gamma\sigma\hat{\Theta} - \frac{d}{dt}\hat{\Theta}^*(t) \\ \dot{\tilde{\psi}}_0 &= -\gamma\frac{\tilde{\psi}_0\xi\xi}{m^2} - \gamma\hat{\psi}_0^*(t)\frac{\phi'Z\xi}{m^2} - \mu\gamma\frac{\eta}{m}\frac{\xi}{m} - \gamma\sigma_1\hat{\psi}_0 - \frac{d}{dt}\hat{\psi}_0^*(t).\end{aligned} \tag{B.4}$$

[4] For simplicity, we assume that $W_m(s)$, $\hat{G}(s)$ have distinct poles.

To analyze (B.4) we consider the positive definite function $V = (\tilde{\psi}_0^*(t)\phi'\Gamma^{-1}\phi + \tilde{\psi}_0^2/\gamma)/2$. Taking the time derivative of V along (B.4) and using the fact that $\|(d/dt)\hat{\Theta}^*(t)\| \leq \mu$[5] and η/m is bounded, independent of the boundedness of ϕ, $\tilde{\psi}_0$, we get after some straightforward calculations

$$\dot{V} \leq -\frac{1}{2}\frac{[\tilde{\psi}_0^*(t)\phi' Z + \tilde{\psi}_0\xi]^2}{m^2} - \tilde{\psi}_0^*(t)\sigma\hat{\Theta}'\phi + \sigma_1\tilde{\psi}_0\tilde{\psi}_0$$
$$-\frac{1}{2}\left[\frac{|\tilde{\psi}_0^*(t)\phi' Z + \tilde{\psi}_0\xi|}{m} - \frac{\mu|\eta|}{m}\right]^2 + \mu^2\frac{k_1}{2}$$
$$+\mu k_2\tilde{\psi}_0^*(t)\|\phi\| + \mu k_3|\tilde{\psi}_0| \tag{B.5}$$

where k_1, k_2, k_3 are the respective bounds of $(\eta/m)^2$, $\Gamma^{-1}d\hat{\Theta}^*(t)/dt$, and $\gamma^{-1}d\tilde{\psi}_0^*(t)/dt$. From the definition of σ, σ_1, (5.4), we have that $\sigma\hat{\Theta}'\phi \geq 0$, $\sigma_1\tilde{\psi}_0\tilde{\psi}_0 \geq 0$ and for $\|\hat{\Theta}\| \geq 2M_0$, $|\tilde{\psi}_0| \geq 2M_1$ we obtain $\sigma\Theta'\phi > 0$, $\sigma_1\tilde{\psi}_0\tilde{\psi}_0 > 0$, and $-\sigma\hat{\Theta}'\phi + \mu k_2\|\phi\| \leq -\sigma_0\phi'\phi/2 + \sigma_0\|\hat{\Theta}^*(t)\|^2 + \mu^2k_2^2/\sigma_0$. A similar inequality can be also obtained for $\sigma_1\tilde{\psi}_0\tilde{\psi}_0 + \mu k_3|\tilde{\psi}_0|$. Since $\tilde{\psi}_0^*(t) > 0$ (B.5) implies that $\dot{V} < 0$ whenever $V > V_0$ for some $V_0 > 0$. Hence, [25], V and therefore ϕ, $\tilde{\psi}_0$ are uniformly bounded. Furthermore, from (B.5) we obtain

$$\int_t^{t+T}\frac{(\tilde{\psi}_0^*(t)\phi' Z + \tilde{\psi}_0\xi)^2}{m^2} + \sigma\tilde{\psi}_0^*(t)\hat{\Theta}'\phi + \sigma_1\tilde{\psi}_0\tilde{\psi}_0\,d\tau \leq g_0 + \mu g_1 T; \qquad \forall t \geq t_0 \geq 0,\ T \geq 0 \tag{B.6}$$

where $g_0 = \sup_t |V(t) - V(t+T)|$ and $g_1 = \mu k_1 + k_2 \sup_t \tilde{\psi}_0^*(t)\|\phi\| + k_3 \sup_t |\tilde{\psi}_0|$.

Step 3—Smallness in the Mean of Normalized Error Signals: The third step of our analysis is to show that the boundedness of the parameter estimates and (B.6) imply that $\tilde{\psi}_0^*(t)h_0(t)\hat{G}'(s)U\hat{H}(t)\phi/m$ is also small in the mean.

From the definition of ξ in (5.5) and following a similar procedure as in Step 1 it can be shown that $\xi = \phi' Z - W_m(s)h_0(t)\hat{G}'(s)U\hat{H}(t)\phi + \mu\eta_4$ where η_4/m is bounded. Letting $\bar{\phi} = [(\tilde{\psi}_0^*(t) + \tilde{\psi}_0)\phi', -\tilde{\psi}_0]'$, $\bar{Z} = [Z', W_m(s)h_0(t)\hat{G}'(s)U\hat{H}(t)\phi]'$ we get $\tilde{\psi}_0^*(t)\phi' Z + \tilde{\psi}_0\xi = \bar{\phi}'\bar{Z} + \mu\eta_4$. Hence, (B.6) can be written as

$$\int_t^{t+T}\frac{(\bar{\phi}'\bar{Z})^2}{m^2} + \sigma\tilde{\psi}_0^*(t)\hat{\Theta}'\phi + \sigma_1\tilde{\psi}_0\tilde{\psi}_0\,d\tau \leq g_0 + \mu g_1 T; \qquad \forall t \geq t_0 \geq 0,\ T \geq 0 \tag{B.7}$$

where, without loss of generality, we use the same constants to avoid unnecessary proliferation of symbols. As in [12, Appendix C], we can show that (B.7) implies

$$\int_t^{t+T}\frac{|\bar{\phi}' W_m^{-1}(s)\bar{Z}|}{m}\,d\tau \leq \frac{\gamma_0}{\epsilon_0^2} + \left(\mu\frac{\gamma_1}{\epsilon_0^2} + \gamma_2\epsilon_0^l\right)T; \qquad \forall t \geq t_0 \geq 0,\ T \geq 0 \tag{B.8}$$

where l, γ_0, γ_1, γ_2 are positive constants and $\epsilon_0 \in (0, 1]$ is an arbitrary constant. From the definition of $\bar{\phi}$, $\bar{Z}$ we have that

$$\bar{\phi}' W_m^{-1}(s)\bar{Z} = \tilde{\psi}_0^*(t)h_0(t)\hat{G}'(s)U\hat{H}(t)\phi + \Delta(t) \tag{B.9}$$

where $\Delta(t) = \tilde{\psi}_0 h_0(t)\{\phi'[\hat{G}'(s)U\hat{H}(t)]' - \hat{G}'(s)U\hat{H}(t)\phi\}$. Furthermore, we can show as in Step 1 that

$$\frac{|\Delta(\tau)|}{m(\tau)} \leq c_0\exp[-\delta_0(\tau - t_0)] + c_1 \cdot \int_{t_0}^{\tau}\|\dot{\phi}\|\exp\left[-\frac{\delta_2}{2}(\tau - s)\right]ds \tag{B.10}$$

where c_0, $c_1 > 0$ are constants. Since $\dot{\phi}$ is bounded and $\|\dot{\phi}\| \leq \gamma_3\sqrt{\epsilon_0} + \mu\gamma_4$ for some constants γ_3, $\gamma_4 > 0$ whenever the integrant of the LHS of (B.6) is less than ϵ_0, we can obtain an inequality similar to (B.8) for $\|\phi\|$. Thus, from (B.9), (B.10), and (B.8) it follows that

$$\int_t^{t+T}\frac{|\tilde{\psi}_0^*(t)h_0(t)\hat{G}'(s)U\hat{H}(t)\phi|}{m}\,d\tau \leq \frac{\gamma_0}{\epsilon_0^2} + \left(\mu\frac{\gamma_1}{\epsilon_0^2} + \gamma_2\epsilon_0^l\right)(t - t_0); \qquad \forall t \geq t_0 \geq 0,\ T \geq 0 \tag{B.11}$$

for some positive constants l, γ_0, γ_1, γ_2 and $\epsilon_0 \in (0, 1]$ being an arbitrary constant.

Step 4—Boundedness of All Signals and Tracking Performance: The input to the plant u_p can be expressed in terms of $\tilde{\psi}_0^*(t)h_0(t)\hat{G}'(s)U\hat{H}(t)\phi$ as

$$u_p = D_p(s, t)N_p^{-1}(s, t)k_p^{-1}(t)D_m^{-1}(s)k_p(t) \cdot [r + \tilde{\psi}_0^*(t)h_0(t)\hat{G}'(s)U\hat{H}(t)\phi] \tag{B.12}$$

$$u_p = \hat{N}_p^{-1}(s, t)\hat{k}_p^{-1}(t)\hat{D}_p(s, t)D_m^{-1}(s)\hat{k}_p(t) \cdot [r + \tilde{\psi}_0^*(t)h_0(t)\hat{G}'(s)U\hat{H}(t)\phi]. \tag{B.13}$$

Since $N_p^{-1}(s, t)$, $\hat{N}_p^{-1}(s, t)$, $D_m^{-1}(s)$ are exp. stable PIO's and the plant parameters are bounded, there exist constants α_1, β_0, β_1, $\beta_2 > 0$ such that

$$|u_p| \leq \beta_0\exp(-\alpha_1 t) + \beta_1 \cdot \int_0^t \exp[-\alpha_1(t - \tau)][|\tilde{\psi}_0^*(t)h_0(t)\hat{G}'(s)U\hat{H}(t)\phi| + |r|]\,d\tau + \beta_2[|\tilde{\psi}_0^*(t)h_0(t)\hat{G}'(s)U\hat{H}(t)\phi| + |r|] \tag{B.14}$$

(P-1 and P-2)

$$|y_p| \leq \beta_0\exp(-\delta_0 t) + \beta_1 \cdot \int_0^t \exp[-\delta_0(t - \tau)][|\tilde{\psi}_0^*(t)h_0(t)\hat{G}'(s)U\hat{H}(t)\phi| + |r|]\,d\tau. \tag{B.15}$$

Since Θ is bounded the throughput term $|\tilde{\psi}_0^*(t)h_0(t)\hat{G}'(s) \cdot U\hat{H}(t)\phi|$ in the first inequality can also be expressed in terms of filtered values of $|u_p|$, $|y_p|$ and $|\tilde{\psi}_0^*(t)h_0(t)\hat{G}'(s)U\hat{H}(t)\phi|$ and, after some straightforward calculations, can be shown to satisfy a similar inequality as $|y_p|$ with δ_0 replaced by $\delta = \min(\alpha_1, \delta_0)$. Applying the Bellman–Gronwall Lemma [18] on the sum $W = |y_p| + |u_p| + \lambda m$, where λ is an arbitrary positive constant and m is defined by (5.4), we finally obtain[6]

$$W \leq \beta_0 + \beta_1\int_{t_0}^t \exp[-\delta(t - \tau)] \cdot \left[\frac{|\tilde{\psi}_0^*(t)h_0(t)\hat{G}'(s)U\hat{H}(t)\phi|}{\lambda m} + \lambda\right]W(\tau)\,d\tau. \tag{B.16}$$

From (B.11) and (B.16), W and all the signals in the closed loop will be bounded if

$$\delta - \beta_1[\lambda + \mu\gamma_1/\lambda\epsilon_0^2 + \gamma_2\epsilon_0^l/\lambda] > 0. \tag{B.17}$$

The inequality (B.17) can always be satisfied for some fixed λ, ϵ_0 and all $\mu \in [0, \mu^*)$ for some $\mu^* > 0$. For example, by taking $\lambda = \delta/3\beta_1$, $\epsilon_0 = \min[1, (\delta^2/9\beta_1^2\gamma_2)^{1/l}]$, $\mu^* = \lambda\epsilon_0^2\delta/3\beta_1\gamma_1$ the inequality (B.17) is satisfied for all $\mu \in [0, \mu^*)$ and therefore, u_p,

[5] By Assumption S.3 we also have that $|(d/dt)\tilde{\psi}_0^*(t)|$ is $O(\mu)$.

[6] Again, without loss of generality, we use the same constants where the subscript 0 indicates dependence on the initial conditions and subscripts 1, 2, etc., indicate dependence on signal bounds and system parameters.

y_p, m and all the other signals in the closed loop will be bounded for any bounded initial conditions.

To show the performance properties of the new MRAC, we express $\bar\phi'\bar Z$ as $W_m(s)W_m^{-1}(s)\bar\phi' W_m(s)W_m^{-1}(s)\bar Z$ and working as in Step 1 we obtain

$$\bar\phi'\bar Z = W_m(s)\bar\phi' W_m^{-1}(s)\bar Z + \sum_k R_k(s)\dot{\bar\phi}'\hat R_k(s) W_m^{-1}(s)\bar Z \quad (B.18)$$

where $R_k(s)$, $\hat R_k(s)$ are proper, stable filters whose poles are those of $W_m(s)$. Furthermore, from (5.4) we have that $\sigma^2\hat\Theta'\hat\Theta \le \sigma_0(1 + \|\hat\Theta^*(t)\|/M_0 - \|\hat\Theta^*(t)\|)\sigma\hat\Theta'\phi$ (and similarly for $\hat\psi_0$) which, together with (B.6) yield

$$\int_t^{t+T} \dot\phi'\dot\phi + \dot\phi_0^2\, d\tau \le g_0 + \mu g_1 T; \qquad \forall t \ge t_0 \ge 0,\ T \ge 0. \quad (B.19)$$

From (B.9), (B.1), and (B.18) it follows that the tracking error satisfies $e_1^2 \le K\{[\bar\phi'\bar Z]^2 + [W_m(s)\Delta(t)]^2 + \Sigma_k[R_k(s)\dot{\bar\phi}'\hat R_k(s) W_m^{-1}(s)\bar Z]^2\}$ where K is a positive constant. Hence, since for $\mu \in [0, \mu^*)$ m is bounded, integrating the last expression and using (B.7), the Schwarz inequality, and (B.19) we finally obtain (5.8) which concludes the proof of Theorem 5.1. △△

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