Example: How many edges are there in a graph with 5 vertices and degree sequence (2,3,3,4,4)?

degree sum $= 2+3+3+4+4 = 16$

By Handshake Theorem, $16 = 2|E|$

$\Rightarrow |E| = \frac{16}{2} = 8$

There are 8 edges

Example: A graph with 8 edges has twice as many vertices of degree 3 as there are vertices of degree 2 (and no other vertices). How many vertices are of degree 2?

$|V_3|$ 8 edges

\# of vertices of degree 3: $2x$

\# of " " degree 2: x

$|V_2|$

Find # of vertices of degree 2.

By Handshake theorem, the degree sum is $8 \cdot 2 = 16$.

$16 = 3(2x) + 2(x)$

$16 = 6x + 2x = 8x \Rightarrow x = 2$ $\Rightarrow$ So there are 2 vertices of degree 2 and 4 vertices of degree 3.

Corollary: Every graph has an even number of vertices of odd degree. (odd vertices: vertices of odd degree)

(even vertices: vertices of even degree).

Proof:

By the Handshake Theorem, $\sum_{u \in V} \deg_G(u) = 2|E|$

Then $\underbrace{\sum_{\substack{u \in V \\ u \text{ odd vertex}}} \deg_G(u)}_{\text{each vertex contributes an odd number to the sum}} + \underbrace{\sum_{\substack{u \in V \\ u \text{ even vertex}}} \deg_G(u)}_{\text{even number (1)}} = \underbrace{2|E|}_{\text{two times something this is an even number (2)}}$

each vertex contributes an odd number to the sum, and the sum has to be even because (1) and (2) are even.

So there has to be an even number of vertices here (even number of odd [illegible]

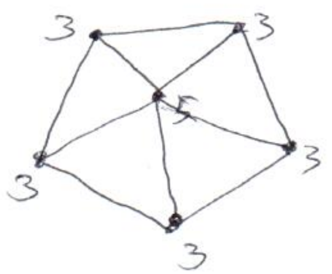
Exercise: Does there exist a simple graph with the following degree sequence?

If so, draw a picture of such a graph. If not, explain why.

(i): (1, 2, 2, 4, 5, 5)

no such (simple) graph exists because by the corollary ~~requires that~~ there must be an even number of odd-degree vertices. This degree sequence has 3 vertices of odd degree.

(ii): (3, 3, 3, 3, 3, 5)



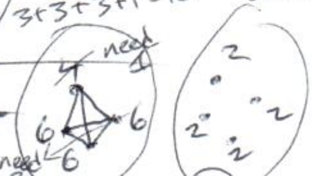
Yes. There it is.

(iii): (0, 2, 2, 3, 4, 5) 6 vertices

There's a vertex v of degree 5 so v is adjacent with all other vertices, but there's a vertex of degree 0, which cannot be adjacent to v. So, this doesn't work.

need 3+3+3+1=10

has 2+2+2+2 =8



NO! ~~(1,1,3,3,4,6,6,6)~~

~~(1,1,3,3,4,6,6,6)~~

similar idea group 4,6,6,6 vertices

(2,2,2,2,4,6,6,6) Too many edges left to join to 2,2,2,2

~~There not enough~~

NO! need 4+4+4=12 more degree

(2,2,2,2,2,6,6,6) ?

degree 6 vertices

can contribute 2+2+2+2+2=10 degree

Some Special Graphs

A graph G is said to be k-**regular** if each vertex in G has degree k.

2-regular

3-regular

Let G and H be simple graphs. We say that H is a **subgraph** of G if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$.

Example: Let G be a graph with vertex set $V = \{a, b, c, d\}$ and edge set $E = \{ab, ac, ad, bc\}$. Find all subgraphs of the graph G with exactly 3 vertices and 2 edges.

subgraph:
vertices: a, b, c (1)
a, b, d (2)
a, c, d (3)
b, c, d (4)

(1) a, b, c
edges ab, ac G_1
ab, bc G_2
ac, bc G_3

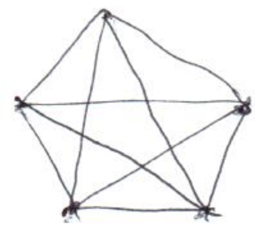
(2) a, b, d : vertices
edges ab, ad G_4

(3) a, c, d : vertices
edges : ac, ad G_5

(4) b, c, d : vertices
no subgraph with 2 edges

A **complete graph** K_n:

K_5



n vertices
each vertex adjacent to all others

K_n has how many edges? $\binom{n}{2} = \frac{n \cdot (n-1)}{2}$

each vertex has degree $n-1$

A **path** P_n of length n:

A **cycle** C_n of length n: