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MATH2600H
Test-2

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Problem 0)

Problem 1) $A = \{w, x\}$

$B = \{y\}$

$A \times B$

$\{w, x\} \times \{y\}$

relations from $A \to B$

~~$\{w, y\}$~~ $\{(w, y), (x, y)\}$

no. of functions from A to B $= |B|^{|A|}$

$f: A \to B$

$= (1)^2$

$|B| = 1$

$= \boxed{1}$

$|A| = 2$

Problem 2) $f: R \times R^+ \to R$

$f(x, y) = |x| + 2y$.

(a) Is f one-one?

let (x_1, y_1) and $(x_2, y_2) \in R \times R^+$

and suppose f is one to one

$\Rightarrow f(x_1, y_1) = f(x_2, y_2)$

$|x_1| + 2y_1 = |x_2| + 2y_2$

$\because y_1, y_2 \in R^+$

$\therefore 2y_1, 2y_2 \in R^+$

but $x_1, x_2 \in R$

$\Rightarrow |x_1|, |x_2| \in R^+$

so for f is one to one

$|x_1| = |x_2|$

$\Rightarrow \pm x_1 = \pm x_2$

$\boxed{x_1 \neq -x_2}$

one positive and one negative ~~of~~ x will yield same image but x_1 & x_2 will be different & not equal.

$\therefore$ f is not one to one.

(b) Is f onto?

If f is onto, every element in R has a pre-image ordered pair in $R \times R^+$

$$f(x, y) = |x| + 2y$$

we know $y \in R^+$ and even though $x \in R$, $|x| \in R^+$ i.e. $|x|$ and y are always positive.

$$|x| \geq 0 \quad y \geq 0$$

$$\Rightarrow |x| + 2y \geq 0$$

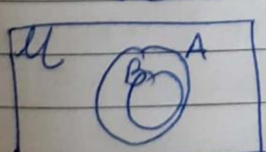
$$\Rightarrow |x| + 2y \in R^+$$

and thus not all of 'R' which is the range.

$\therefore$ f is not onto.

Problem 3) $A \subseteq U$ $B \subseteq U$

let's suppose $B \subseteq A$



$$A \cup B = A$$

$$\Rightarrow P(A \cup B) = P(A)$$

$$\Rightarrow P(A \cup B) \neq P(A) \cup P(B)$$

as we would be adding P(B) twice

$\therefore$ $P(A) \cup P(B) \neq P(A \cup B)$

$P(A) \cup P(B) = P(A \cup B)$ does not hold true always but only when A and B are mutually exclusive.

Problem 4) A → set of functions from Z → R.

$R = \{(f, g), \forall x \in Z, f(x) - g(x) = c, c \in Z\}$

Show that R is an equivalence relation on A.

⇒ To show R is an equivalence relation on A, we need to prove R is reflexive, symmetric and transitive.

Reflexive :

let's see we R is reflexive for a belonging to set A

$aRa = a(x) - a(x) \quad \forall x \in Z$

$= 0 \Rightarrow 0 \in Z$

∴ R is reflexive.

Symmetric :

~~for~~ let a and b be functions of set A ∈ A

$(a, b) \in R$

let, $a(x) - b(x) = k$ where $k \in Z$

then $b(x) - a(x) = -k$, $-k \in Z$

So, $(b, a) \in R$

∴ R is symmetric.

Transitive :

let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow a(x) - b(x) = k$ — (1) & $b(x) - c(x) = l$ — (2)

$k, l \in Z$

(1) + (2)

$a(x) - b(\cancel{x}) + b(\cancel{x}) - c(x) = k + l$

$a(x) - c(x) = k + l$

$\because k, l \in Z \Rightarrow k + l \in Z$

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$\therefore (a,c) \in R$

$\therefore R$ is transitive

$\because R$ is reflexive, symmetric & transitive

$\therefore R$ is an equivalence relation.

hence proved.

Problem 5) heights of blocks available

$\Rightarrow$ 1cm, 3cm, 4cm

b_n = number of ways to construct a tower of height n cm.

$b_1 = 1$

$b_3 = 2$ for $n \geqslant 5$

4
1+1+1+1
3+1
1+3

$b_4 = 4$ the towers would be a combinatio of the blocks.

$$\therefore \boxed{b_n = b_{n-1} + b_{n-3} + b_{n-4}}$$

Since till n=4, they n=1, n=3, n=4 can be built using one block of that available block.

Problem 6) integer $n \geq 1$

$4^{n+1} + 5^{2n-1}$ is divisible by 21

$P(n) = 4^{n+1} + 5^{2n-1}$ is divisible by 21

BI: Show that $P(n_0) = P(1)$ is true

$P(1) = 4^{1+1} + 5^{2(1)-1}$

$= 4^2 + 5$

$= 16 + 5$

$= 21$ is divisible by 21

$\therefore P(n_0) = P(1)$ is true

Thus BI is completed

IS: Suppose $P(K)$ is true for $K \geq 1$ and show that $P(K+1)$ follows

So, we suppose

$P(K) = 4^{K+1} + 5^{2K-1}$ is divisible by 21

$P(K+1) = 4^{(K+1)+1} + 5^{2(K+1)-1}$ is divisible by 21.

$$4^{(K+1)+1} + 5^{2(K+1)-1}$$
$$= 4 \cdot 4^{(K+1)} + 5^{2K+2-1}$$
$$= 4 \cdot 4^{K+1} + 5^2 \cdot 5^{2K-1}$$
$$= 4 \cdot 4^{K+1} + 25 \cdot 5^{2K-1}$$
$$= 4 \cdot 4^{K+1} + (21+4) \cdot 5^{2K-1}$$
$$= 4 \cdot 4^{K+1} + 21 \cdot 5^{2K-1} + 4 \cdot 5^{2K-1}$$
$$= 4 \cdot \underbrace{(4^{K+1} + 5^{2K-1})} + \underbrace{21 \cdot 5^{2K-1}}$$

by $P(K)$, divisible by 21 — divisible by 21

addition of 2 terms, divisibly by 21 is also divisible by 21

$\therefore P(K+1)$ is divisible by 21

Hence, IS is completed.

So, BI and IS are completed and hence we prove the proposition for all integers $n \geq 1$

Problem 7) 8 letters into 8 envelopes but ~~on~~ exactly one letters go into the correct envelope.

This is possible, to put any ~~(i)~~ i letter in correct envelope is same as derangements of remaining of $(n-i)$ letters

$\Rightarrow D_{n-i}$

$\Rightarrow \frac{1}{n!} \binom{n}{i} D_{n-i}$

So, since we want exactly one letter to go in correct envelope. $(n=8,\ i=1)$

$\frac{1}{8!} \binom{8}{1} D_7$

$= \frac{1}{8!} \times 8 D_7$

$= \boxed{\frac{1}{7!} D_7}$

Problem 8) relation R on set A is cyclic.

To show that if R is reflexive and cyclic, then it is symmetric and transitive.

reflexive $\Rightarrow aRa,\ a \in R$

cyclic $\Rightarrow ((a,b) \in R) \wedge ((b,c) \in R) \rightarrow (c,a) \in R$

Symmetric

to prove symmetric if $\boxed{(a,b) \in R}$ then $(b,a) \in R$

$(a,b) \in R$

since R is reflexive $\Rightarrow (a,a) \in R$ & $(b,b) \in R$

since R is cyclic

~~$(a,a) \in R$ if $((b,b) \in R) \wedge ((a,a) \in R) \rightarrow (b,a) \in$~~

if $((a,a) \in R) \wedge ((b,b) \in R) \rightarrow \boxed{(b,a) \in R}$

hence proved R is symmetric

Transitive

to prove transitive if $((a,b) \in R)$ and $((b,c) \in R)$ then $(a,c) \in R$.

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$\because$ R is cyclic,
$((a,b) \in R) \land ((b,c) \in R) \rightarrow (c,a) \in R$
and we proved R is symmetric
$\Rightarrow (c,a) \in R \longrightarrow \boxed{(a,c) \in R}$

$\therefore$ R is transitive.

Hence proved if R is reflexive & cyclic then R is symmetric & transitive.