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TOTAL POINTS: /10

Trent University MATH 2600 - Discrete Structures
Instructor: Aras Erzurumluo˘glu

Assignment 5 (due 4:00 pm on Wednesday, April 8th, 2020)

READ ME: When attempting the problems you are allowed to consult your lecture notes, textbooks, the internet, etc. However, you are not allowed to copy each other's work (not even partially!).
You are expected to write your solutions in full detail and using a precise mathematical language. You will lose points for imprecise solutions.
Late assignments may not be accepted.

Problem 1) (1 point): What is the smallest number n such that if any n positive integers are chosen at random, at least 3 among them give the same remainder when divided by 11?

Problem 2) (2 points): What is the coefficient of x13y10 in the expansion of $(x-3y)^{23}$?

DOMS Page No.
Date / /

MATH 2600H

Assignment - 05

Problem 1) In the worst case scenario, there can be $11+11=22$ numbers such that at least 2 among them give the same remainder when divided by 11. (Because, for ex, from numbers 1-11, 11th number is always divisible by 11). Therefore, $22+1 = 23$ numbers guarantee that at least 3 among them give the same remainder when divided by 11.

Problem 2) $x^{13}y^{10}$

considering binomial coefficient $\{(x-3y)^{23}\}$

$$(a+b)^n = \sum_{k=0}^{n} \cdot a^{n-k} \cdot b^k$$

$$\therefore (x-3y)^{23} = \sum_{k=0}^{23} \cdot (x)^{23-k} \cdot (-3y)^k \cdot \binom{n}{k}$$

so for $x^{13} \cdot y^{10}$ $\boxed{k=10}$ $23-k=13$ $\boxed{n=23}$

$$= \binom{23}{10} x^{13} \cdot (-3 \cdot y)^{10}$$

$$= \binom{23}{10} \cdot (-3)^{10} \cdot \left[x^{13} y^{10}\right]$$

$$\therefore \text{coefficient of } x^{13} y^{10} = \boxed{\binom{23}{10} (-3)^{10}}$$

$$= {}^{23}C_{10} (-3)^{10}$$

$$= \frac{23!}{10!\,13!} (-3)^{10}$$

Problem 3) (2 points): How many integer solutions are there to the equation x1 + x2 + x3 + x4 + x5 = 12, where x1 ≥ 4,x2 ≥ −3,x3 ≥ −1,x4 ≥ 0,x5 ≥−1?

Problem 4) (2 points): Define a graph G as follows: • The vertex set of G is the set of all 3-element subsets of S = {1,2,3, 4,5,6,7,8,9}. (For example, {1,3,7} is a vertex of G.) • Two vertices A and B are adjacent in G if and only if |A∩B| = 1. How many vertices are there in G? How many edges are there in G?

Page No.

Date / /

Problem 3)

$x_1 + x_2 + x_3 + x_4 + x_5 = 12$

$x_1 \geq 4 \;,\; x_2 \geq -3 \;,\; x_3 \geq -1 \;,\; x_4 \geq 0 \;,\; x_5 \geq -1$

$x_1 = x_1 + 4 \qquad (x_1 \geq 0)$

$x_2 = x_2 - 3 \qquad (x_2 \geq 0)$

$x_3 = x_3 - 1 \qquad (x_3 \geq 0)$

$x_4 = x_4 + 0 \qquad (x_4 \geq 0)$

$x_5 = x_5 - 1 \qquad (x_5 \geq 0)$

$(x_1+4) + (x_2-3) + (x_3-1) + (x_4+0) + (x_5-1) = 12$

$x_1 + x_2 + x_3 + x_4 + x_5 - 1 = 12$

$\underbrace{x_1 + x_2 + x_3 + x_4 + x_5 = 13}$

$x_1, x_2, x_3, x_4, x_5 \geq 0$

$n = 13$

$r = 5$

$\binom{n+r-1}{r-1} = \binom{13+5-1}{5-1} = \binom{17}{4} = \boxed{{}^{17}C_4}$

$= \dfrac{17!}{4!\,12!} = \dfrac{17 \times 16 \times 15 \times 14 \times 13}{4 \times 3 \times 2 \times 1}$

$= \boxed{30,940}$ integer solution

Problem 4) graph G

$S = \{1,2,3,4,5,6,7,8,9\}$

$|S| = 9$

- vertex is a set of $\boxed{\text{3 element}}$ subset of S.
- A & B adjacent $\Leftrightarrow |A \cap B| = 1$

number of vertices in G $= \boxed{\binom{9}{3}} = \dfrac{9!}{3!\,6!} = \dfrac{9 \times 8 \times 7}{3 \times 2}$

$= \boxed{84}$ vertices

Problem 5) (1 point): How many integers between 358 and 843 are divisible by 5 or 7? (Note that "between" may or may not be inclusive of 358 and 843, it won't change the answer.)

DOMS Page No.
Date / /

now, we want $|A \cap B| = 1$ for an edge / A & B vertices to be adjacent.

So, we can take 1 common element from vertices A & B in 9 ways ($\because$ there are 9 vertices to choose from).
Once we have 1 common element, we can choose other 2 elements for A in $\binom{8}{2}$ ways and other 2 elements for B in $\binom{6}{2}$ ways.

So total number of ways of selecting 2 vertices with only one element in common, remembering that order doesn't matter

$$= \frac{1}{2} \times 9 \times \binom{8}{2} \times \binom{6}{2}$$

$$= \frac{1}{2} \times 9 \times \frac{8!}{2!\,6!} \times \frac{6!}{2!\,4!}$$

$$= \frac{1}{\cancel{2}} \times 9 \times \frac{\cancel{8} \times 7}{\cancel{2}} \times \frac{6 \times 5}{\cancel{2}}$$

$$= \cancel{8}\ 9 \times 7 \times 6 \times 5$$

$$= 1890$$

$\therefore$ number of edges $= \boxed{1890}$

Problem 5) no. of integers between 358 and 843 not including 358 and 843

$$= 843 - 358 - 1$$
$$= 484$$

by inclusion-exclusion principle
let T be set of integers b/w 358 and 843 that are divisible by 5 or 7, we want to find $|T|$.

~~all integers in~~ $|T|$
$|T_5| \rightarrow$ divisible by 5.
$|T_7| \rightarrow$ all integers in $|T|$ divisible by 7.
$|T| = |T_5| + |T_7| - |T_5 \cap T_7| \longrightarrow$ to find

$|T_5 \cap T_7| = |T_{5,7}| =$ integers in $|T|$ divisible by 5 or 7.

$|T_5| = \frac{484}{5} = 96$

$|T_7| = \frac{484}{7} = 69$

$|T_{5,7}| = \frac{484}{35} = 13$

$\therefore |T| = 96 + 69 - 13$
$= \boxed{152} \rightarrow$ all integers b/w 358 and 843 divisible by 5 or 7.

Problem 6) degree sequence $(2, 2, 3, 3, 3, 6, 6, 7, 9, 9)$
no. of vertices = 10
the highest degree = 9
$\Rightarrow$ ~~it with~~ this vertex, with degree 9 so it is adjacent with all other vertices.
There is another vertex with degree 9 so it is also adjacent with all other vertices.
Therefore vertices with degree 2 will be adjacent to the 2 vertices with degree 9.
let's say degree of vertex $V_1 = 2$
$V_2 = 2$
$V_3 = 3$
$V_4 = 3$
$V_5 = 3$
$V_6 = 6$
$V_7 = 6$
$V_8 = 7$
$V_9 = 9$
$V_{10} = 9$

Problem 6) (2 points): Does there exist a simple graph with degree sequence (2,2,3,3,3,6,6,7,9,9)?

now if we focus on vertex V_8, it has degree 7.
V_8 is adjacent already to V_9 & V_{10} degree 9.

then $7-2=5$

V_8 needs to be adjacent to 5 more vertices.

So, let's say it is adjacent to V_7, V_6, V_5, V_4, V_3.

This way, V_{10} is adjacent to all 9.
V_9 is adjacent to all 9.
V_8 is adjacent to 7 vertices

~~now~~ degree 2, V_1, V_2 is adjacent to V_9, V_{10}.
V_3, V_4, V_5 have degree 3 and they are adjacent to V_8, V_9, V_{10}.

Only V_6 and V_7 needs vertices to be adjacent & complete there degree.

For vertex V_7 with degree 6

it is adjacent to V_9, V_{10}, V_8.
but it still needs $(6-3)=3$ more vertices to be adjacent to.

So it would be adjacent to V_6, V_5, V_4
but no other vertex instead of V_6 can be adjacent as V_5, V_4, V_3, V_2, V_1 are already adjacent to vertices as per their degrees.

Therefore, this doesn't work.

Thus this degree sequence (2,2,3,3,3,6,6,7,9,9) is not valid.