$P_6 = \underset{\text{36 options}}{\underline{36}}\ \underline{36}\ \underline{36}\ \underline{36}\ \underline{36}\ \underline{36} - \underbrace{\underline{26}\ \underline{26}\ \underline{26}\ \underline{26}\ \underline{26}\ \underline{26}}_{\#\ 6 \text{ character strings with no digits}}$

$P_6 = 36^6 - 26^6$

$P_7 = 36^7 - 26^7$

$P_8 = 36^8 - 26^8$

So there are $(36^6 - 26^6) + (36^7 - 26^7) + (36^8 - 26^8)$ acceptable passwords.

Product Rule: Suppose that a procedure can be broken down into a sequence of k tasks $T_1, T_2, \ldots, T_k$, where for each $i \in \{1, \ldots, k\}$ there are n_i ways to carry out task T_i after the tasks $T_1, T_2, \ldots, T_{i-1}$ are completed. Then there are $n_1 n_2 \ldots n_k$ ways to carry out the procedure.

Example: A Canadian postal code is a six character-long string (with a space between the third and the fourth characters) where the first, third and fifth characters are letters and the second, fourth and sixth characters are digits (such as: K1N 6N5 or H0H 0H0).

How many Canadian postal codes can be formed considering that

- Postal codes do not include the letters D, F, I, O, Q or U, and
- The first character also does not make use of the letters W or Z.

26 letters in total
6 are forbidden
first character
W and Z also forbidden

$\underset{\text{letter}}{\underline{18}}\ \underset{\text{digit}}{\underline{10}}\ \underset{\ell}{\underline{20}}\ \underset{d}{\underline{10}}\ \underset{\ell}{\underline{20}}\ \underset{d}{\underline{10}}$

There are 26 letters in the English alphabet.

There are 10 digits.

The first character is one of the 18 letters (any letter except for D, F, I, O, Q, U, W, Z).

The second character is one of the 10 digits $(0, 1, \ldots, 9)$.

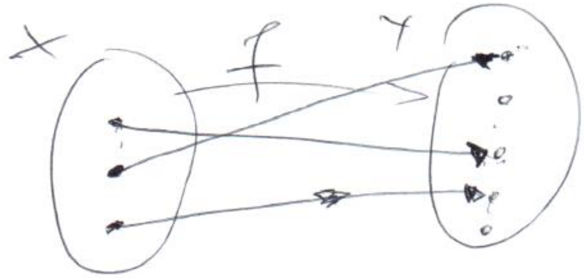
The third character is one of the 20 letters (any letter except for D, F, I, O, Q, U).

The fourth character is one of the 10 digits $(0, 1, \ldots, 9)$.

The fifth character is one of the 20 letters (any letter except for D, F, I, O, Q, U).

The sixth character is one of the 10 digits $(0, 1, \ldots, 9)$.

So under these conditions one can form $18 \cdot 10 \cdot 20 \cdot 10 \cdot 20 \cdot 10 = 7200000$ Canadian postal codes. (According to Statistics Canada, currently an estimated 830000 active postal codes exist.)



$f : X \to Y$

one-to-one

Example: How many one-to-one functions are there from a set with $\boxed{3}$ elements to a set with 5 elements?

$5 \cdot 4 \cdot 3$ We have $\boxed{3}$ factors

$5-3+1$

$n \quad r$

Theorem: The number of one-to-one functions from a set with r elements to a set with n elements is

r factors in the product

- $n(n-1)(n-2)\ldots(n-r+1)$ if $n \geq r$
- 0 if $n < r$.

This theorem is actually a special case of a more general concept.

Defn: An r-**permutation** of a set of n elements is an ordered selection of r elements taken from the set of n elements.

The number of r-permutations of a set of n elements is denoted $P(n, r)$.

$P(7,3) = 7 \cdot 6 \cdot 5$ (3 factors)

So we have the following result. $P(11,4) = 11 \cdot 10 \cdot 9 \cdot 8$ (4 factors)

Theorem: If n and r are integers and $1 \leq r \leq n$, then the number of r-permutations of a set of n elements is given by the formula

$$P(n, r) = n(n-1)(n-2)\cdots(n-r+1).$$

$P(13,5) = ?$

$13 \cdot 12 \cdot 11 \cdot 10 \cdot 9$

(5 factors)

Equivalently,

$$P(14,3) = \frac{14!}{(14-3)!} = 14 \cdot 13 \cdot 12 \qquad \boxed{P(n, r) = \frac{n!}{(n-r)!}}$$

$$= \frac{13!}{8!}$$

$$\frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot \cancel{8} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{8} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$$

if $r = n$, then

$$P(n,n) = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$$

Note that the number of n-permutations of a set of n elements is just $n!$.

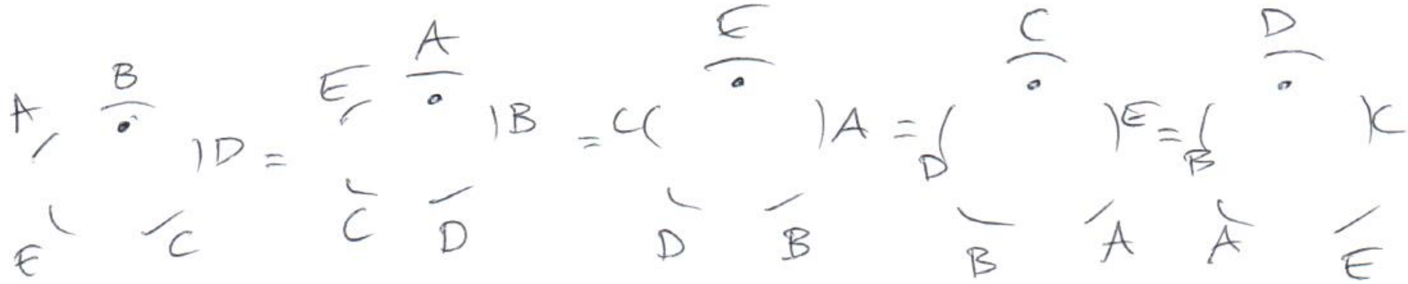
$$P(n,n) = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$$

Example: A group of 5 friends wants to take a selfie. If they line up in a row, how many different configurations are there?

$\underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$ $\quad 5! = 120$

$P(5,5)$

How many different configurations are there if they are seated at a round table?



BDCEA ABDCE EABDC CEABD DCEAB

all 5 correspond to the "same" circular arrangement.

Theorem: The number of circular arrangements of n objects is

$$\boxed{\frac{n!}{n} = (n-1)!}$$

$$\frac{n!}{n} = \frac{\cancel{n}(n-1)(n-2)\cdots 2\cdot 1}{\cancel{n}}$$

$= (n-1)(n-2)\cdots\cdot 2\cdot 1$

$= (n-1)!$

Among $n!$ linear arrangements there are n which correspond to each distinct circular arrangement. Then, there are $\frac{n!}{n} = (n-1)!$ distinct circular arrangements