Full Name:
Student Number:

TOTAL POINTS: /10

Trent University MATH 2600 - Discrete Structures
Instructor: Aras Erzurumluoğlu

Assignment 4 (due noon on Monday, March 16th, 2020)

READ ME: Please print this page, write down your name and student number, and attach this page as the cover page to your homework solutions.

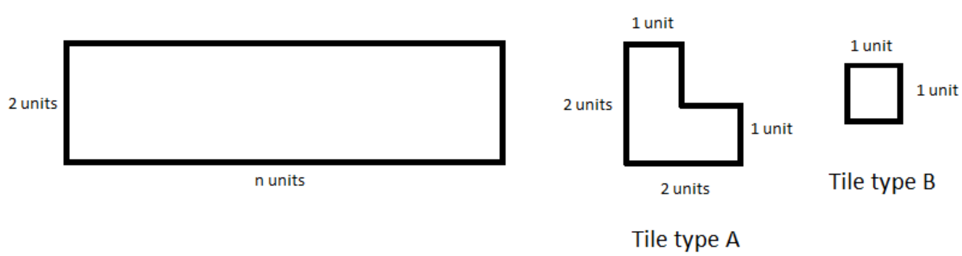
When attempting the problems you are allowed to consult any resources such as the lecture notes, textbooks, the internet, etc. In particular, you are encouraged to collaborate and brainstorm with your classmates. However, you are not allowed to copy each other's work (not even partially!).

Each student is required to **submit** a separate solution set by the deadline **to the assignment box in the Mathematics department that has my name on it.**

You are expected to write your solutions in full detail and using a precise mathematical language. You will lose points for imprecise solutions.

Late assignments will not be accepted except for extreme situations.

Problem 1) (2 points): We want to pave a $2 \times n$ walkway using two types of tiles. (See the picture below – type A tile may be rotated to get different configurations.) Let a_n be the number of such tilings.



(a) Find a_1, a_2 and a_3.

(b) Find a linear homogeneous recurrence relation for the sequence a_n. Explain your work. Do **not** solve the recurrence relation. (**Hint**: You are looking for a recurrence relation of order 3.)

Problem 2) (2 points): Prove that $\forall n \geq 2 \in \mathbb{Z}$

$$\prod_{j=2}^{n}(1 - \frac{1}{j^2}) = \frac{n+1}{2n}.$$

Problem 3) (2 points): Prove that for all positive integers n,

$$\sum_{i=1}^{n}(-1)^i i^2 = \frac{(-1)^n n(n+1)}{2}.$$

Problem 4) (2 points): Prove that every amount of postage $n \geq 12$ cents can be made from 4-cent and 5-cent stamps. (**Hint:** You might need to prove the statement separately for $n = 12, 13, 14, 15$ before attempting an induction proof.)

Problem 5) (2 points): Solve the recurrence relation $a_n = 11a_{n-1} - 28a_{n-2}$ with the initial conditions $a_0 = 5$ and $a_1 = 6$. (Show all your work.)