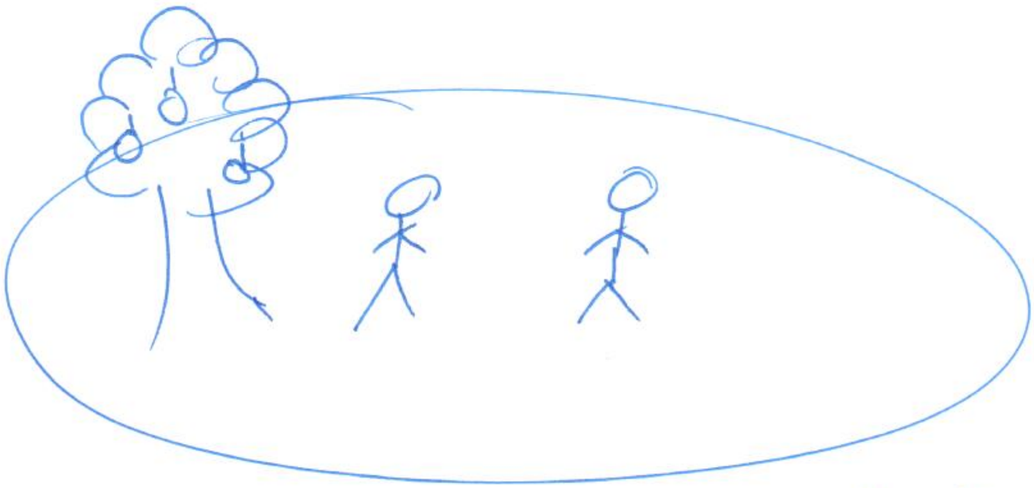
Examples: (Knights and Knaves)



On the Island of Knights and Knaves,
there are two types of people:
knights : they always speak the truth
knaves : they always lie.

You hear the following conversation:

Person A says: I'm a knight if B is a knave.

Person B says: I'm a knave and A is a knight.

Determine (if possible) if A and B is a knight or knave.

p : A is a knight

q : B is a knight

A said : $\neg q \to p$

B said : $\neg q \wedge p$

A: I'm a knight if B is a knave

B: I'm a knave and A is a knight

		A said	B said
p	q	$\neg q \to p$	$\neg q \wedge p$
T	T	T	F
T	F	T	T
F	T	T	F
F	$\boxed{F}$	F	$\boxed{F}$

~~$p \to \neg q$~~

~~A is a knight implies B is a knave~~

In a solution to the knight & knaves problem we have the following condition to be met:

the value of p has to match the value of the 3rd column (what A says).

similarly, the value of q has to match the value of the 4th column (what B said).

We see that the only time where the values in columns 1 & 3 match and also where the values in columns 2 & 4 match is when the truth value assignments are as in Row 4.

So, we conclude that

p is F and q is F is the only answer.

p is F $\Rightarrow$ A is a knave

q is F $\Rightarrow$ B is a knave

Another example: This time

A says: I'm a knight if and only if B is a knave

B says: I'm a knave

p: A is a knight

q: B is a knight

p	q	A says $p \Leftrightarrow \neg q$	$\neg q$
T	T	F	F
T	F	T	T
F	T	T	F
F	F	F	T

We don't see any "good" row, so this puzzle has no solution.

Consistency

Consider a set $\{p, q, r, \ldots\}$ of compound propositions in the variables $x, y, z, \ldots$. (That is, each compound proposition is constructed using some of the propositions $x, y, z, \ldots$ together with some logical connectives.)

Then, the set $\{p, q, r, \ldots\}$ of compound propositions is said to be **consistent** if there exists a truth assignment for $x, y, z, \ldots$ which makes the compound propositions $\{p, q, r, \ldots\}$ <u>all true at the same time</u>.

We can check the consistency of a set of compound propositions using a truth table:

1) Make a column for each variable $x, y, z, \ldots$ and each compound proposition $\{p, q, r, \ldots\}$.

2) Check if there is a row where all compound propositions $p, q, r, \ldots$ are T (true).

Example: Is the set of three system requirements given below consistent?

A) Whenever the system software is being upgraded, users cannot access the file system.

B) If users can access the file system, then they can save new files.

C) If users cannot save new files, then the system software is not being upgraded.

Let x: "the system software is being upgraded",

y: "users can access the file system",

z: "users can save new files".

A: $x \to \neg y$

B: $y \to z$

C: $\neg z \to \neg x$

contra-positive of $x \to z$

			A	B	C
x	y	z	$x \to \neg y$	$y \to z$	$\neg z \to \neg x$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

all three requirements true

Since we have at least one scenario where all three system requirements have value T, we conclude that the set of requirements is consistent.