Full Name:
Student Number:

TOTAL POINTS: /15

Trent University MATH 2600 - Discrete Structures - Winter 2020

Test 2

READ ME: Any attempts for cheating on graded work will be dealt with according to the university policies. Show all your work. Explain your solutions when appropriate.

Problem 0) (0 points): Use this space to draw a picture. No restriction on the theme.

Problem 1) (2 points): Let $A = \{w, x\}$ and $B = \{y\}$. Write down all relations from A to B. How many functions are there from A to B?

Problem 2) (2 points): Let $f : \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$ be a function given as $f(x, y) = |x| + 2y$ (where $\mathbb{R}^+$ is the set of positive real numbers).

Answer the following questions, and explain your answers.

(a) Is f one-to-one?

(b) Is f onto?

Problem 3) (2 points): Let A and B be any two subsets of a universal set $\mathcal{U}$. Does $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$ always hold? If your answer is "yes", then prove it; otherwise show a counterexample.

Problem 4) (3 points): Let A be the set of all functions from $\mathbb{Z}$ to $\mathbb{R}$. We define a relation $\mathcal{R}$ on A as $\mathcal{R} = \{(f, g)$ such that for all $x \in \mathbb{Z}, f(x) - g(x) = c$ for some constant $c \in \mathbb{Z}\}$. Show that $\mathcal{R}$ is an equivalence relation on A.

Problem 5) (1 point): You have a set of building-blocks which contains blocks of heights 1, 3 and 4 centimeters. (Other dimensions irrelevant.) You are constructing towers by piling blocks directly on top of one another. (A tower of height 7 cm could be obtained using seven blocks of height 1; one block of height 3 and one block of height 4; 2 blocks of height 3 and one block of height 1; etc.)

Let b_n be the number of ways to construct a tower of height n cm using blocks from the set. Assume that there is an unlimited supply of blocks of each size. Find a recurrence relation for b_n. (You are not required to solve the recurrence relation.)

Problem 6) (2 points): Use mathematical induction to show that for all integers $n \geq 1$, $4^{n+1} + 5^{2n-1}$ is divisible by 21.

Problem 7) (1 point): Let d_n be the number of derangements of n elements. Express the answer to the following problem in terms of d_n (for the appropriate value of n).

A machine inserts 8 letters into 8 envelopes randomly (one in each). In how many ways can the machine insert the letters into envelopes so that exactly one of the 8 letters go into the correct envelope?

Problem 8) (2 points): A relation $\mathcal{R}$ on a set A is said to be *cyclic* if the following implication holds:

$$\big((a, b) \in \mathcal{R}\big) \wedge \big((b, c) \in \mathcal{R}\big) \rightarrow (c, a) \in \mathcal{R}.$$

Show that if $\mathcal{R}$ is reflexive and cyclic, then it is symmetric and transitive.