Full Name:
Student Number:

TOTAL POINTS: /10

Trent University MATH 2600 - Discrete Structures
Instructor: Aras Erzurumluoğlu

Assignment 3 (due noon on Monday, March 2nd, 2020)

READ ME: Please print this page, write down your name and student number, and attach this page as the cover page to your homework solutions.

When attempting the problems you are allowed to consult any resources such as the lecture notes, textbooks, the internet, etc. In particular, you are encouraged to collaborate and brainstorm with your classmates. However, you are not allowed to copy each other's work (not even partially!).

Each student is required to **submit** a separate solution set by the deadline **to the assignment box in the Mathematics department that has my name on it.**

You are expected to write your solutions in full detail and using a precise mathematical language. You will lose points for imprecise solutions.

Late assignments will not be accepted except for extreme situations.

Problem 1) (2 points): Let A, B, C be finite sets. Prove rigorously that $(B \cap C) \times A = (B \times A) \cap (C \times A)$.

(To prove this identity "rigorously" you need to show that any element from $(B \cap C) \times A$ is also contained in $(B \times A) \cap (C \times A)$ and vice versa.)

Problem 2) (1 point): Let A, B, C be three subsets of a universal set U. Use set identities to show that $\overline{(\overline{A} - B)} \cap C = (C \cap A) \cup (C \cap B)$.

(To earn points, you are required to use set identities; do not only draw Venn diagrams.)

Problem 3) (3 points): Let $f : \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R} \times \mathbb{R}^+$ be a function given as $f(x, y) = (|x|, 2^y)$ (where $\mathbb{R}^+$ is the set of positive real numbers).

Answer the following questions, and explain your answers.

(a) Is f one-to-one?

(b) Is f onto?

(c) Is f a bijection? Is f invertible?

Problem 4) (1 point): Prove that there does not exist a pair of subsets S and T of $\mathbb{Z}$ such that $S \times T = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (2, 5)\}$.

Problem 5) (1 point): Prove by contraposition that if $n^4 + 1$ is even, then n is odd.

Problem 6) (2 points): Let $g : X \to Y$ and $f : Y \to Z$ be functions. If f and $f \circ g$ are onto functions, is it then always true that g is an onto function?

(If your answer is yes, then you should give a proof; otherwise you need to find a concrete counterexample.)