COIS 4470H - Winter 2023

Assignment 4 - Due: Friday, April 7th

1. [5 marks] Let X be a discrete random variable with the possible values x=1, 2, …, n.

   (a) If the pdf of X is $f(x)=ax$, what is the value of a (as a function of n)?
   (b) What are the cdf, mean and standard deviation of X?

Hint: $\sum_{x=1}^{n} x = \frac{n(n+1)}{2}$, $\sum_{x=1}^{n} x^2 = \frac{n(n+1)(2n+1)}{6}$, $\sum_{x=1}^{n} x^3 = \frac{n^2(n+1)^2}{4}$

2. [10 marks] A sequence of random variates following exponential distribution with mean μ can be generated using the formula:

$$X=-\mu ln(1-u),$$

k=1/mu
f(x) = ke^-kx

xf(x) = kxe^-kx

where u is a random variate uniformly distributed between 0 and 1. Find the cdf, pdf, mean and variance of the random variable X.

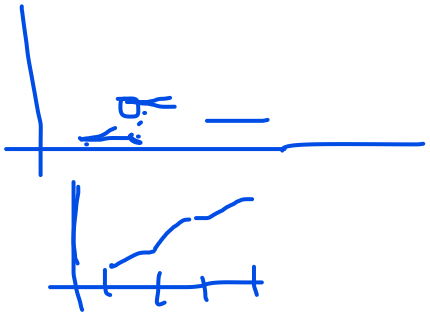
3. [10 marks] A continuous random variable has the following pdf:

$$f(x)=\begin{cases}\frac{x}{2}-\frac{1}{2}, & 1\le x\le 3\\ 0, & \text{elsewhere}\end{cases}$$

   (a) Draw the graph of the pdf.
   (b) Find the cdf and draw the graph of the cdf.
   (c) What are the mean and variance of the random variable?
   (b) Develop a generator that generates the random variates using the Inverse Transformation Method.

4. [10 marks] A continuous random variable X has the following pdf:

$$f(x)=\begin{cases}0.1, & 1\le x\le 2,\\ 0.5, & 2<x\le 3,\\ 0.4, & 3<x\le 4,\\ 0, & elsewhere.\end{cases}$$



   (a) Draw the graph of the pdf.
   (b) Find the cdf and draw the graph of the cdf.
   (c) What are the mean and variance of the random variable?
   (b) Develop a generator that generates the random variates using the Inverse Transformation Method.

x = ax mod m

x = ax + c mod m

c = seed

5. [5 marks] At Tony and Cleo's bakery, one kind of birthday cake is offered, It takes 15 minutes to decorate this particular cake, and the job is performed by one particular baker. In fact, it is all this baker does. What mean time between arrivals (exponentially distributed) can be accepted if the mean length of the queue for decorating is not to exceed five cakes?

6. [10 marks] A classic example of a discrete-time finite-state birth-death process is based on a game of chance played as follows. Two players, *A* and *B*, start with a fixed amount of money, *k* (dollars), with *a* belonging to *A* and *k-a* to *B*. On each play of the game, either *A* will win one dollar (from *B*) with probability λ, independent of the result of previous games, or *B* will win one dollar with probability $1-\lambda$. Play continuous until one player has no more money. Let *P(n)* be the steady-state probability that player *A* has *n* dollars. Assume that $k=10$ and $\lambda=0.51$.

   (a) Draw the state transition diagram for this process.
   (b) Give the balance equations.
   (c) Find P(n) for n=0, 1, …, 10.
   (d) Comment.