$$= \frac{\sum_{i=1} S_i}{376}$$

Simulation Model 2

# A Simple Inventory System


# Conceptual Model


<u>Policies:</u>

- Inventory review is periodic
- Items are ordered, if necessary, only at review times
- Maximum level: S
- Minimum level: s

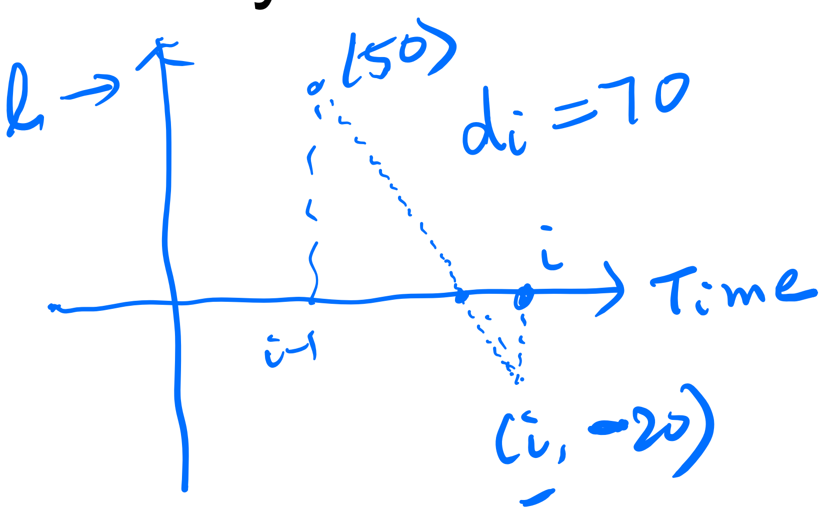
s S

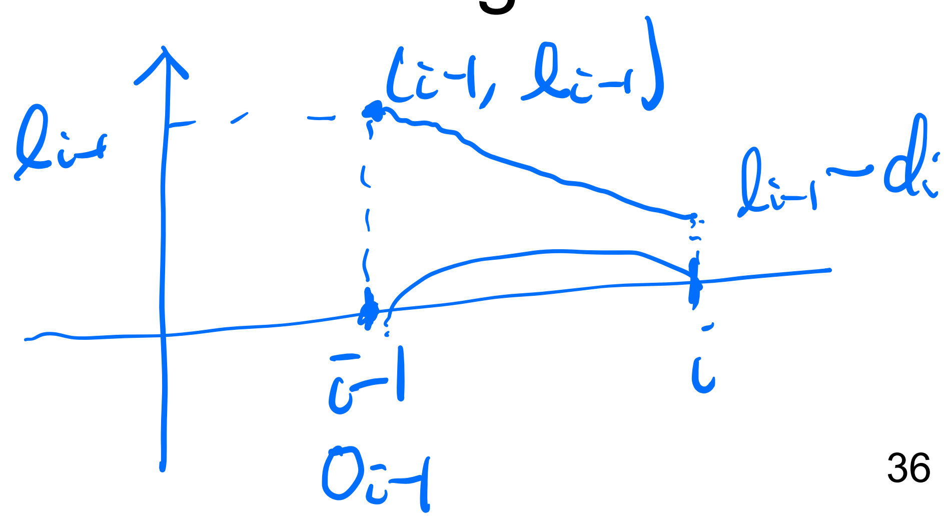
- Back ordering is possible
- No delivery lag
- Both Initial and terminal inventory level are S

# Specification Model

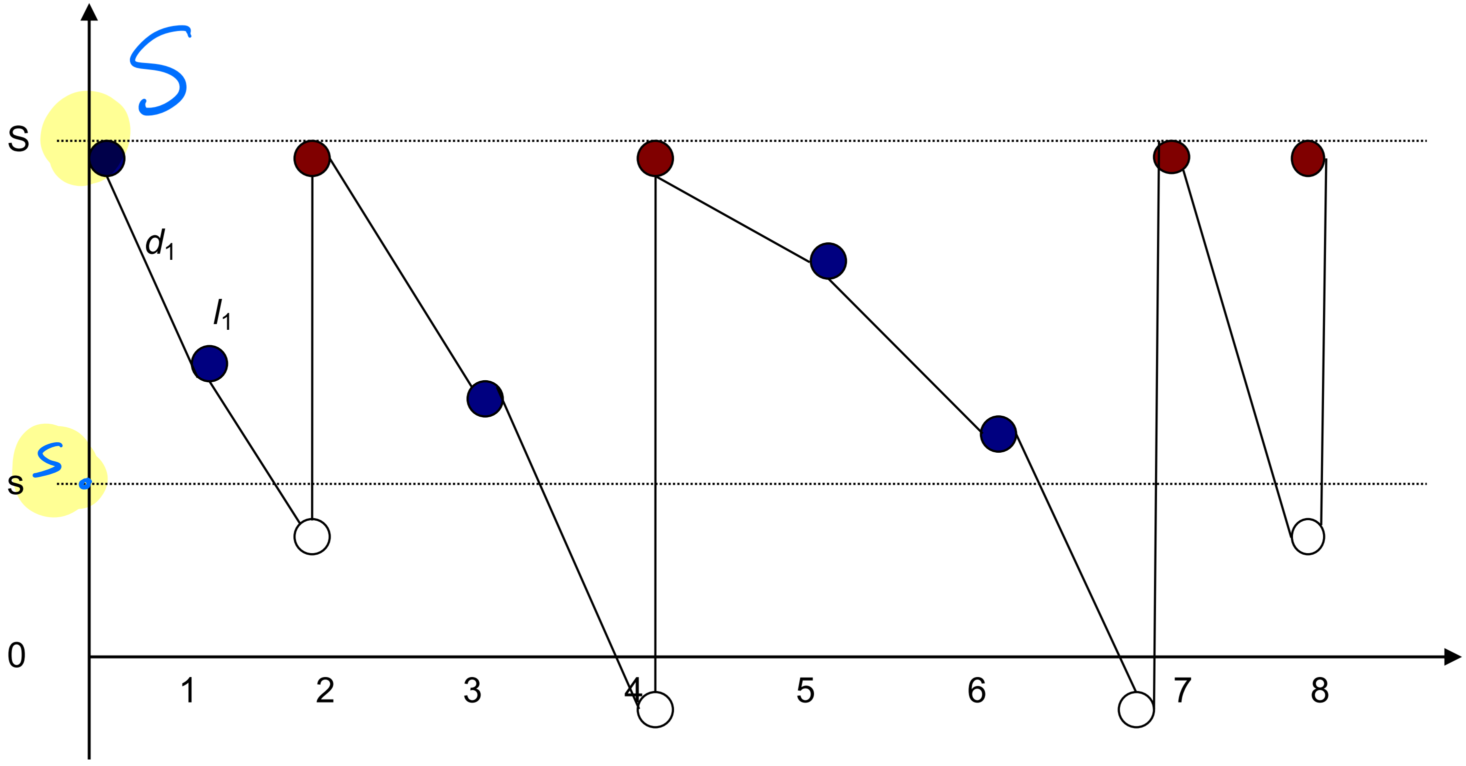
$l_{i-1}$

- Time begins at $t = 0$
- Review times are $t = 0, 1, 2, \ldots$
- $l_{i-1}$ : inventory level at beginning of *i* th interval
- $o_{i-1}$ : amount ordered at time $t = i - 1$, ($o_{i-1} >= 0$)
- $d_i$ : demand quantity during *i* th interval, ($d_i >= 0$)
- Inventory at end of interval can be negative



# Inventory levels



# The Sequences

## $l_i$, $o_i$, $d_i$ , $\{i=0, 1, 2, \ldots\}$

Time

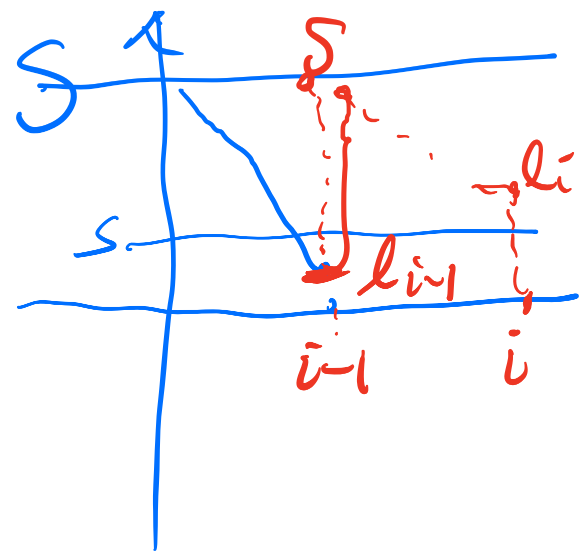
$l_1 \quad l_2 \quad l_3 \quad \ldots$

$o_1 \quad o_2 \quad o_3 \quad \ldots$

$d_1 \quad d_2 \quad d_3 \quad \ldots$

- At $t = i - 1$: $l_{i-1}$ , $o_{i-1}$
- $l_{i-1} >= s$ → no order is placed
- $l_{i-1} < s$ → replenished to S
- At the end of the *ith* interval → Items are delivered immediately, inventory diminished by $d_i$.

$$o_{i-1} = \begin{cases} 0, & \text{if } l_{i-1} \geq s \\ S - l_{i-1}, & \text{if } l_{i-1} < s \end{cases}$$




$$l_i = l_{i-1} + o_{i-1} - d_i$$

# Time Evolution of Inventory Level

```
I0 = S;      /* the initial inventory level is S */
i = 0;
while (more demand to process )
{
    i++;
    if (Ii-1 < s)
        { oi-1 = S - Ii-1; }
    else
        {oi-1 = 0;}
    di = GetDemand();
    Ii = Ii-1 + oi-1 - di;
}
n = i;
on = S - In;
In = S;             /* the terminal inventory level is S */
return I1, I2, . . . , In and o1, o2, . . . , on;
```

$l_0 = 100$ $S = 100$

$s = 30$

$\rightarrow i = 2,\ i = 3$

$i = 2,\ i - 1 = 1$

$O_1 = 100 - (-10)$

$= 110$

$l_1 \geq 30$ False

$O_0 = 0$

For input $d_2 = 50$

$l_1 = l_0 + O_0 - d_1 = 100 + 0 - 60 = 40$

$l_2 = l_1 + O_1 - d_2 = 40 + 0 - 50 = -10$

$d_3 = 30$

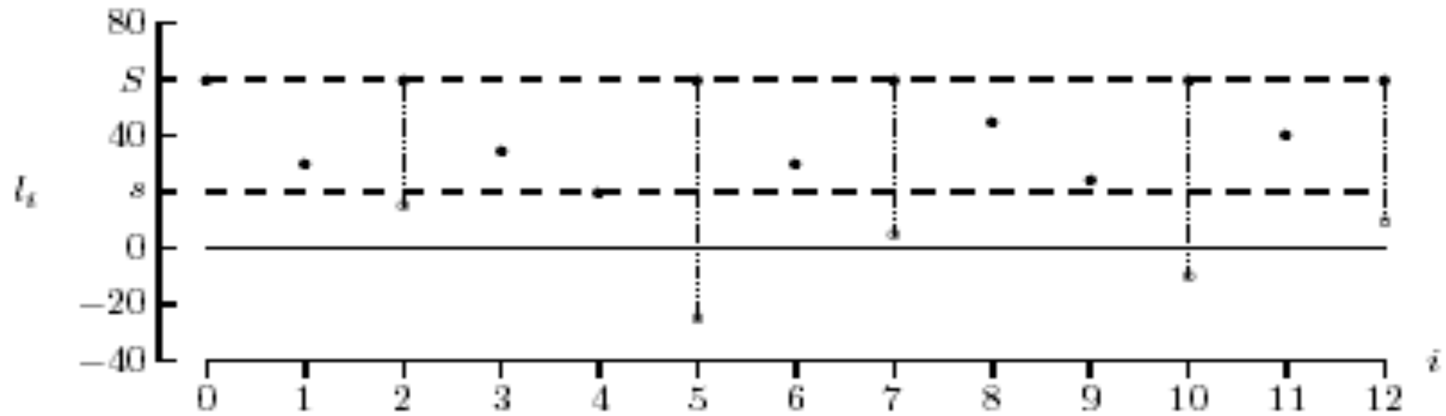
$l_3 = l_2 + O_2 - d_3 = -10 + 110 - 30$

$= 70$

# Sample Demands

Let (s, S) = (20, 60) and consider n = 12 time intervals:

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| i : | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| di : | 30 | 15 | 25 | 15 | 45 | 30 | 25 | 15 | 20 | 35 | 20 | 30 |



# Output Statistics

- Average demand and average order
- For Example data

$$\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$$

$$\bar{d} = \bar{o} = \frac{305}{12} \approx 25.42$$

(items per time interval)

$$\bar{o} = \frac{1}{n}\sum_{i=1}^{n} o_i$$

# Flow Balance

- Over the simulated period, all demand is satisfied.

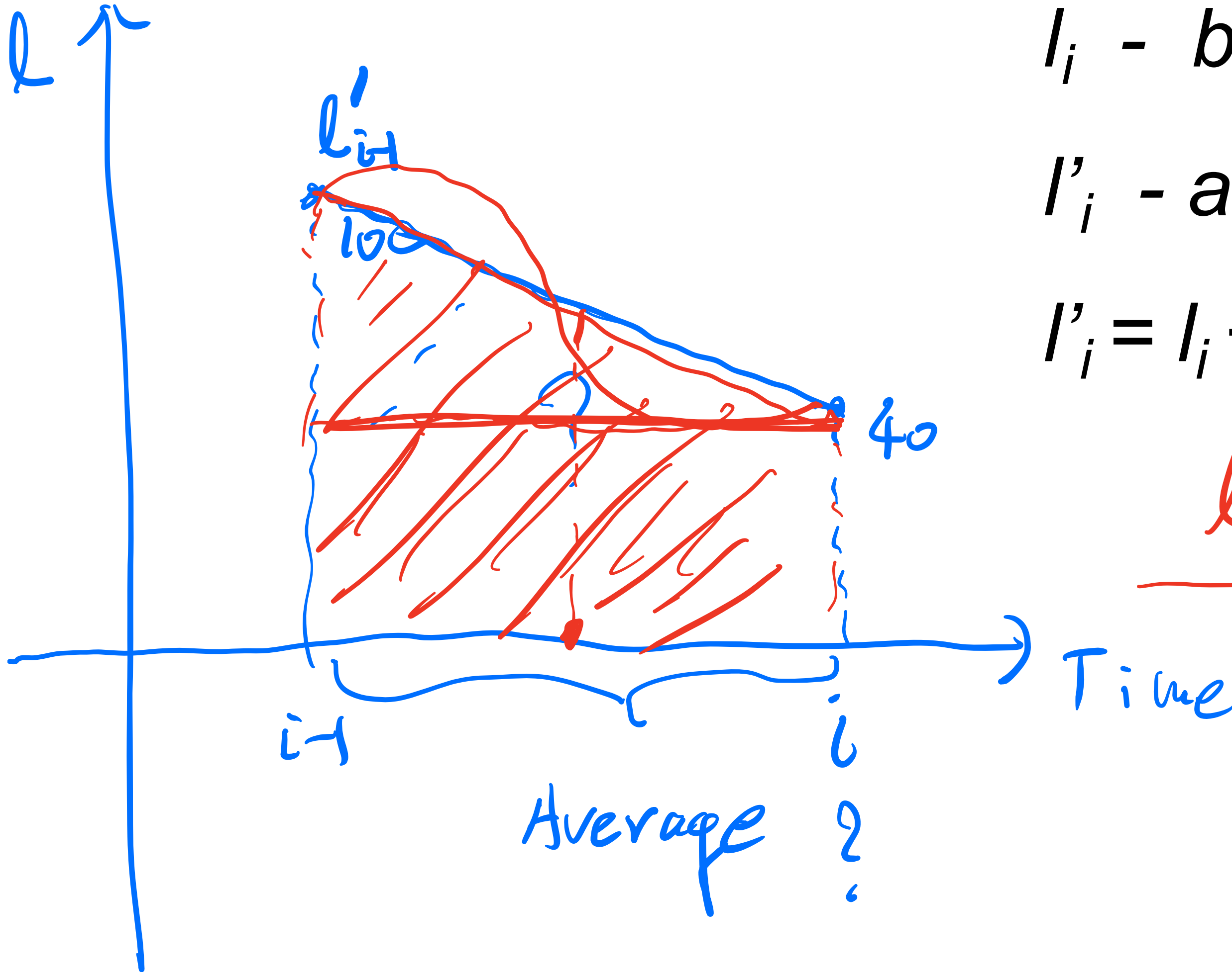
starting inventory(S) = ending inventory(S)

average demand ( $\bar{d}$ ) = average items per order ( $\bar{o}$ )

average “flow” of items *in* = average “flow” of items *out*

- The inventory system is ***flow balanced***

# Inventory Level as a Function of Time - *I(t)*




$I_i$ - *before review*

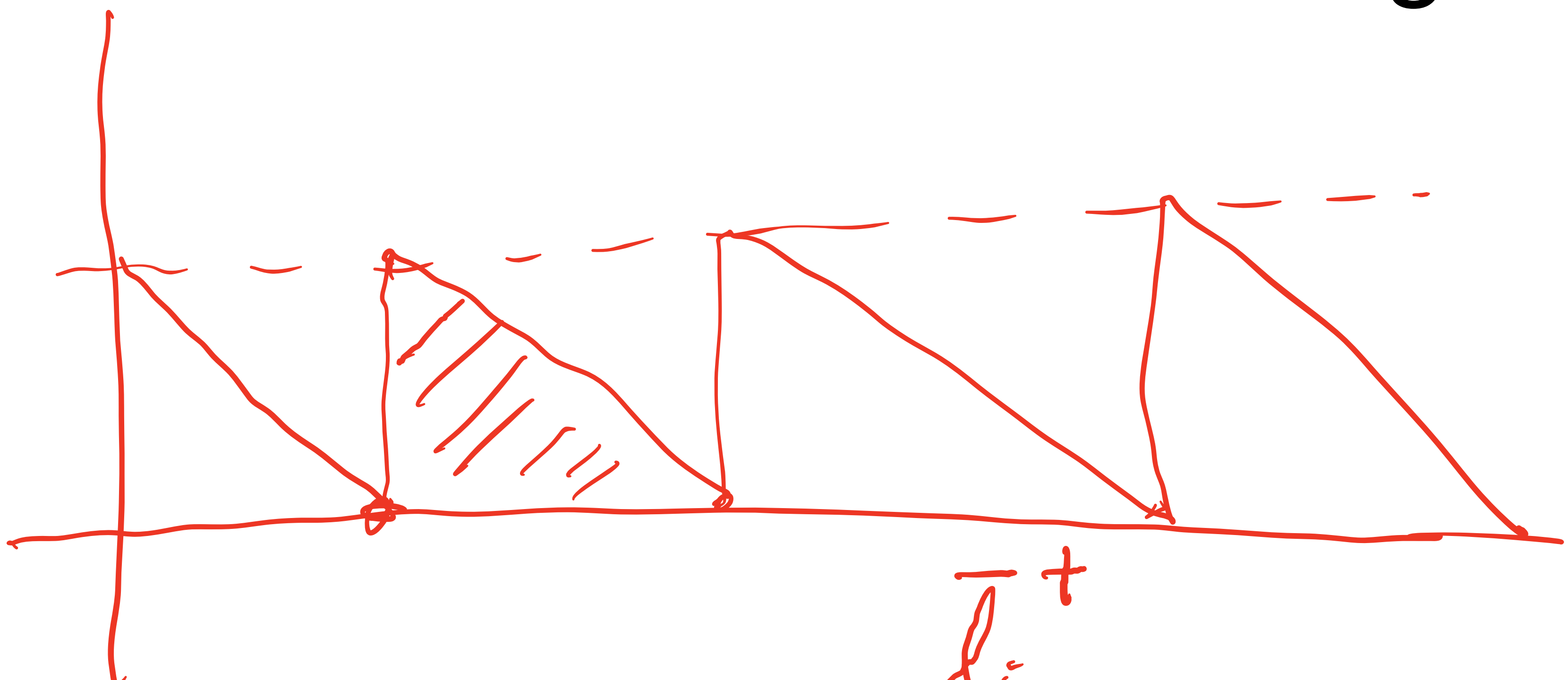
$I'_i$ - *after review*

$I'_i = I_i + o_i$

$$\frac{l'_{i-1} + l_i}{2}$$

Constant Demand Rate – Linear function

# Average Inventory Level
## - no back-ordering



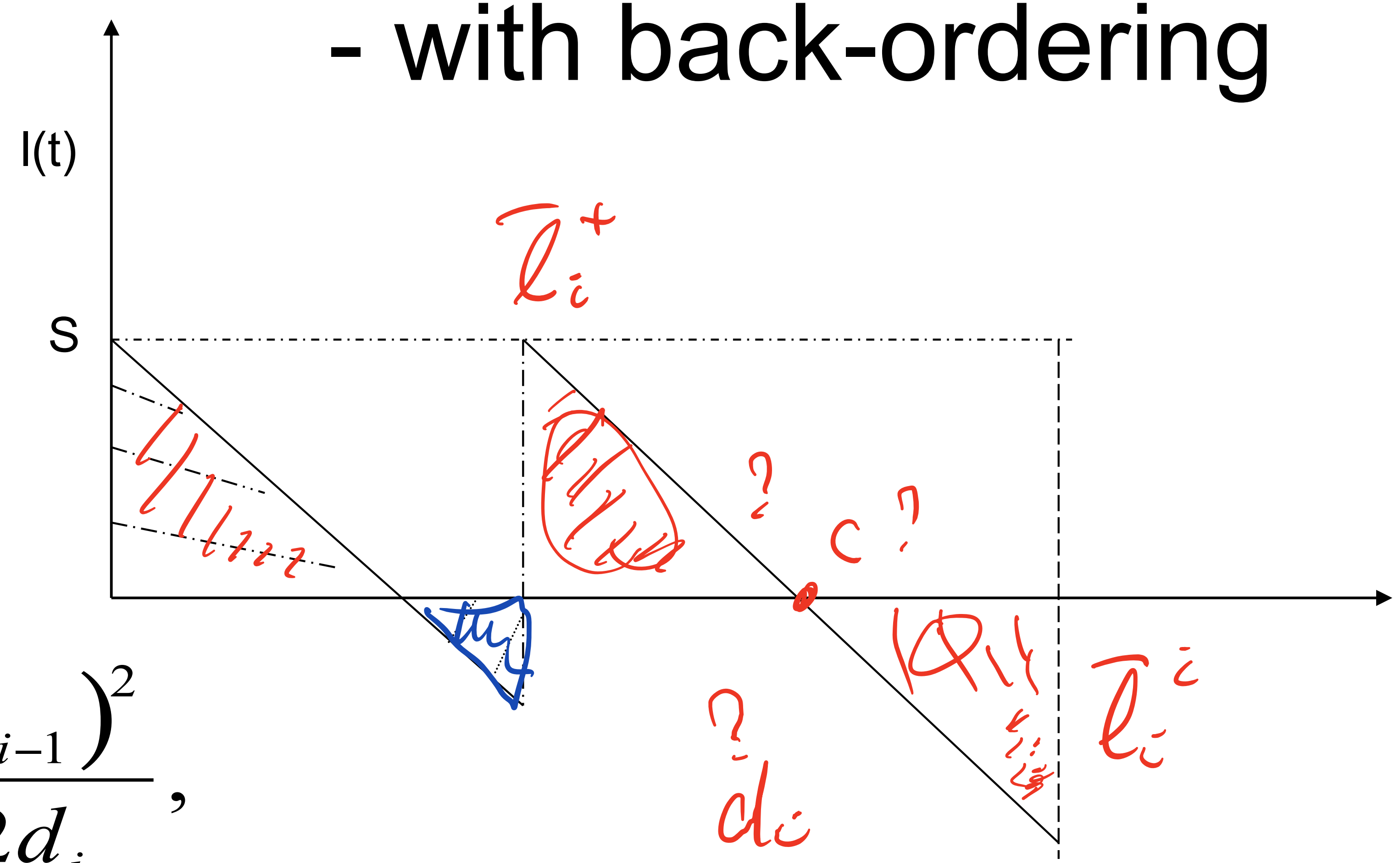
$\bar{l}_i^{+}$

$$\bar{l}_i^{+} = \frac{l'_{i-1} + (l'_{i-1} - d)}{2} = l'_{i-1} - \frac{1}{2} d_i,$$

$$\bar{l}_i^{-} = 0.$$

$\bar{l}_i^{-} = 0$

# Average Inventory Level
# - with back-ordering




$$\bar{l}_i^{\,+} == \frac{(l'_{i-1})^2}{2d_i},$$

$$\bar{l}^{\,-}_{\;i} = \frac{(d_i - l'_{i-1})^2}{2d_i}.$$