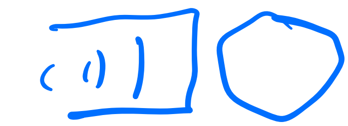
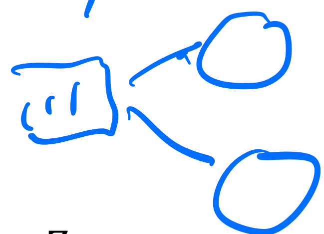
Queueing Models

M/M/1



M/M/2



Notations: A/B/C/K/m/Z

- A: the interarrival distribution
- B: the service time distribution
- C: the number of servers
- K: the system capacity
- m: the number in the source
- Z: the queue discipline

A short notation A/B/C has the assumptions for K, m, Z:

- K: system capacity is infinite (no limit).
- m: customer source is infinite
- Z: the queue discipline is FIFO

For A and B

M: exponential interarrival or service time distribution.

D: deterministic (constant) interarrival or service time distribution.

H_k: k-stage hyper-exponential interarrival or service time distribution.

E_k: Erlang-*k* interarrival or service time distribution.

G: general interarrival or service time distribution.

- M/M/1 A=M, B=M, C=1

  single server queue with exponential interarrival or service times.

- M/M/3/10 → c = 3, K=10

  a 3-server queueing system with exponential interarrival or service times and capacity of 10 customers.

- M/M/m/K → C= m

  a m-server queueing system with exponential interarrival or service times and capacity of K customers.

- M/M/1/k/N A=M, B=M, C=1 K=∞

  an infinite capacity (K= ∞), finite population model with 1 server and exponential interarrival or service times.

- The queue discipline could be: FIFO, LIFO, Priority, SIRO (Service in random order).

<u>Steady-state single-server queue statistics</u>

System State: number of customers in the system

Steady State:

“flow out” rate = “flow in” rate for each state

Birth-Death Models

M/M/1 $\lambda(n)$

<u>Basic Steps</u>

(1) Draw the State Transition Diagram

M/M/2

(2) Obtain the balance equations

(3) Solve the balance equations

$$G = \sum_{n=0}^{\infty} \prod_{i=0}^{n-1} \frac{\lambda(i)}{\mu(i+1)} \quad \text{then} \quad P(n) = \frac{1}{G} \prod_{i=0}^{n-1} \frac{\lambda(i)}{\mu(i+1)}$$

P(n): is the equilibrium distribution of the number of customers in the system.

(4) $\bar{N}$: Mean number in the system

$P(0)$ $P(1)$ $P(2)$ . . .
0 1 2 . . .

$$\bar{N} = \sum_{n=0}^{\infty} nP(n)$$

$\lambda(0)$ $\lambda(1)$ $\lambda(2)$ . . .

(5) $\bar{\lambda} = \sum_{n=0}^{\infty} \lambda(n) P(n)$ - mean arrival rate

(6) $\bar{T} = \frac{\bar{N}}{\bar{\lambda}}$ - Little's Result

Response Time $= \frac{\bar{N}}{\bar{\lambda}}$

<u>M/M/1 Queue</u>:

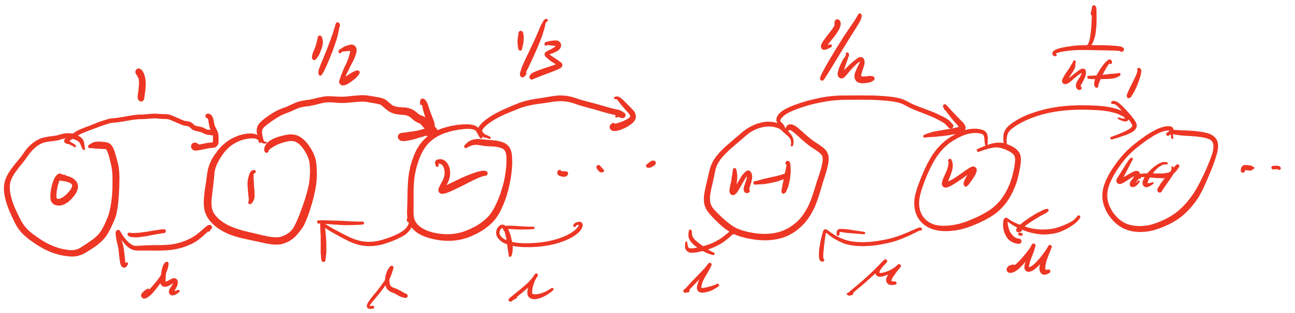
$$\bar{N} = \frac{\rho}{1-\rho}, \quad \bar{T} = \frac{\bar{N}}{\lambda} = \frac{1}{\mu(1-\rho)}$$

$\rho = \frac{\lambda}{\mu}$

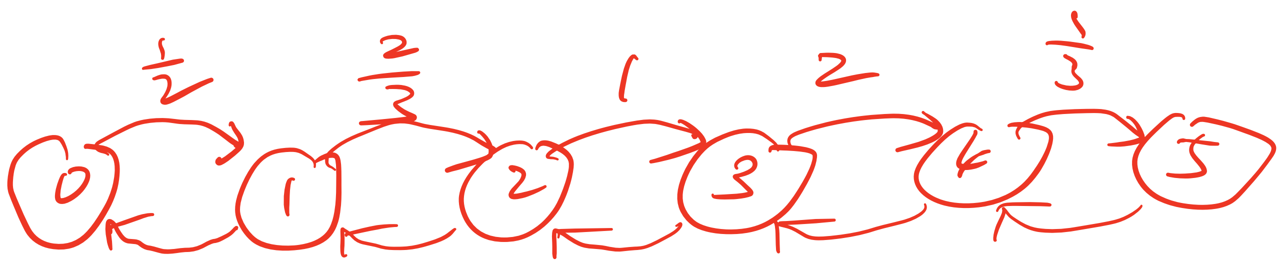
<u>M/M/2 Queue</u>:

$$\bar{N} = \frac{4\rho}{4-\rho^2}, \quad \bar{T} = \frac{4}{\mu(4-\rho^2)}$$

$$\begin{cases} \lambda(n) = \dfrac{1}{n+1}, & n = 0, 1, 2, \dots \\ \mu(n) = \mu, & n = 1, 2, 3 \dots \end{cases}$$



$$\lambda(0) = \frac{1}{0+1} = 1$$



$$\lambda(n) = \begin{cases} \frac{1}{2} & n=0 \\ \frac{2}{3} & n=1 \\ 1 & n=2 \\ 2 & n=3 \\ \frac{1}{3} & n=4 \\ 0 & n=5 \end{cases} \qquad \mu(n) = \frac{5}{7}, \; n=1, \dots 5$$