

COIS 4470H Assignment 3

Question 1: GPSS 01

Language: GPSS

Interarrival Time = RVEXPO(1,30) #in seconds

Service Time = 40_4 seconds

- a. ≤ 0.58 go to Morning Paper Guy. Everyone else goes to Wall Street Journal Guy (even who are buying both papers)

≡ A3Q1a.gps X		≡ A3Q1A.LIS	
≡ A3Q1a.gps			
1		SIMULATE	
2		REALLOCATE	COM,20000
3			
4		GENERATE	RVEXPO(1,30)
5		TRANSFER	.58,WALL,MORN
6			
7	MORN	QUEUE	LINE1
8		SEIZE	VENDOR1
9		DEPART	LINE1
10			
11		ADVANCE	40,4
12		RELEASE	VENDOR1
13		TERMINATE	
14			
15	WALL	QUEUE	LINE2
16		SEIZE	VENDOR2
17		DEPART	LINE2
18			
19		ADVANCE	40,4
20		RELEASE	VENDOR2
21		TERMINATE	
22			
23		GENERATE	7200
24		TERMINATE	1
25			
26		START	1
27		END	

--AVG-UTIL-DURING--									
FACILITY	TOTAL	AVAIL	UNAVL	ENTRIES	AVERAGE	CURRENT	PERCENT	SEIZING	PREEMPTING
	TIME	TIME	TIME		TIME/XACT	STATUS	AVAIL	XACT	XACT
VENDOR1	0.683			124	39.642	AVAIL		215	
VENDOR2	0.498			91	39.435	AVAIL		218	

QUEUE	MAXIMUM	AVERAGE	TOTAL	ZERO	PERCENT	AVERAGE	\$AVERAGE	QTABLE	CURRENT
	CONTENTS	CONTENTS	ENTRIES	ENTRIES	ZEROS	TIME/UNIT	TIME/UNIT	NUMBER	CONTENTS
LINE1	4	0.660	127	29	22.8	37.441	48.521		3
LINE2	4	0.219	93	49	52.7	16.972	35.873		2

RANDOM	ANTITHETIC	INITIAL	CURRENT	SAMPLE	CHI-SQUARE
STREAM	VARIATES	POSITION	POSITION	COUNT	UNIFORMITY
1	OFF	100000	100656	656	0.72

Vendor1 = Morning Paper Guy

Vendor2 = Wall Street Journal Guy

Vendor	Number of People/ Contents	Time
Morning Paper Guy	124	Total: 0.683 Average: 39.642
Wall Street Journal	91	Total: 0.498 Average: 39.435

Queue	Number of People/ Contents	Time
Line1 (Morning)	Total: 127 Max: 4 Average: 0.660	Average Time: 37.441
Line2 (Wall Street)	Total: 93 Max: 4 Average: 0.219	Average Time: 16.972

b. How to make the system more efficient?

As we can see, the average time in the Queue for the Morning Paper Vendor, is very high as compared to Wall Street Journal Guy. Since the probability of people buying morning paper is 58, which is almost 60% - that is a high probability.

We know that Wall Street Journal Vendor has both Morning Paper and Wall Street Journals. Therefore, To optimize the system, we can implement the method where the incoming customers for morning papers are DISTRIBUTED among the two vendors. That is, the customer

who wants to buy the morning paper will be sent to the Wall Street Journal Vendor if the Wall Street Journal has the shorter queue

≡ A3Q1a.gps			≡ A3Q1b.gps X	≡ A3Q1B.LIS
≡ A3Q1b.gps				
3				
4		REAL &U15		
5		LET &U15=0		
6				
7		REAL &U30		
8		LET &U30=0		
9				
10		GENERATE RVEXPO(1,30)		
11		TRANSFER .58,WALL,MORN		
12				
13	MORN	TEST LE &U15,&U30,WALL		
14		QUEUE LINE1		
15		BLET &U15=&U15+1		
16				
17		SEIZE VENDOR1		
18		DEPART LINE1		
19		BLET &U15=&U15-1		
20				
21		ADVANCE 40,4		
22		RELEASE VENDOR1		
23				
24		TERMINATE		
25				
26	WALL	QUEUE LINE2		
27		BLET &U30=&U30+1		
28				
29		SEIZE VENDOR2		
30		DEPART LINE2		
31		BLET &U30=&U30-1		
32				
33		ADVANCE 40,4		
34		RELEASE VENDOR2		
35		TERMINATE		
36				
37		GENERATE 7200		
38		TERMINATE 1		
39				
40		START 1		
41		END		

--AVG-UTIL-DURING--										
FACILITY	TOTAL TIME	AVAIL TIME	UNAVL TIME	ENTRIES	AVERAGE TIME/XACT	CURRENT STATUS	PERCENT AVAIL	SEIZING XACT	PREEMPTING XACT	
VENDOR1	0.602			109	39.764	AVAIL		221		
VENDOR2	0.611			111	39.660	AVAIL		220		
QUEUE	MAXIMUM CONTENTS	AVERAGE CONTENTS	TOTAL ENTRIES	ZERO ENTRIES	PERCENT ZEROS	AVERAGE TIME/UNIT	AVERAGE TIME/UNIT	QTABLE NUMBER	CURRENT CONTENTS	
LINE1	2	0.215	109	47	43.1	14.230	25.018		0	
LINE2	3	0.404	111	38	34.2	26.195	39.831		0	
RANDOM STREAM	ANTITHETIC VARIATES	INITIAL POSITION	CURRENT POSITION	SAMPLE COUNT	CHI-SQUARE UNIFORMITY					
1	OFF	100000	100661	661	0.77					

As we see, the average time for the Morning Paper line decreased drastically.

One more thing that we could do, the very obvious thing is add another vendor. This way we have 3 servers now so that would decrease the total time. But having one more vendor depends on 'budget' realistically speaking. So, the above solution would be better option to make it efficient.

Question 2: GPSS 02

Performance Measure	LoadSplit	TurnTaker	ShortQ
Server Utilization – Server 1	Total = 0.770 Avg time/Xact = 100.000	Total = 0.822 Avg time/Xact = 100.00	Total = 0.934 Avg time/Xact = 98.765
Server Utilization – Server 2	Total = 0.910 Avg time/Xact = 98.801	Total = 0.859 Avg time/Xact = 98.472	Total = 0.857 Avg time/Xact = 100.000
Number of request served - Server 1	46	49	52
Number of request served - Server 2	54	51	48
Average number in queue – queue for server 1	2.010	2.482	0.990
Average number in queue – queue for server 2	1.936	2.843	0.613
Mean delay time - queue for server 1	239.981	269.082	102.747
Mean delay time - queue for server 2	195.947	325.884	71.509
Mean Response Time – Server 1 (total waiting time for each request)	339.981	369.082	201.512
Mean Response Time – Server 2	294.748	424.356	171.509

The obvious recommendation would be the “ShortQ”. The server utilization depends on the shorter queue length, which means, there is not going to be an unnecessary crowd and load on one server just because of some ‘random number probability’.

Taking the statistics and performance of the simulation into consideration to support my recommendation –

- The server utilization for both the servers are ‘equal’ or similar. This means, that both servers are being used in a similar way and just one server does NOT have excess load
- The number of requests served by both the Web servers are again, similar – (51 and 49). Since we have assumed that all the requests take exactly 100.00 time for processing therefore, then this similarity indicates that both servers’ utilization is very close
- Average number in queue and the mean delay time as you can see makes a BIG difference. The requests are being serviced at a much faster rate as compared to other servers
- Overall, The mean response time is the lowest for ShortQ and hence the best approach

Load Split

≡ A3Q2_L.gps	≡ A3Q2LO X	≡ A3Q2_T.gps	≡
≡ A3Q2LO			
1			Server 1 is 46
2			Server 2 is 54
3			

--AVG-UTIL-DURING--										
FACILITY	TOTAL TIME	AVAIL TIME	UNAVL TIME	ENTRIES	AVERAGE TIME/XACT	CURRENT STATUS	PERCENT AVAIL	SEIZING XACT	PREEMPTING XACT	
SERVER1	0.770			46	100.000	AVAIL				
SERVER2	0.910			55	98.801	AVAIL		104		
QUEUE	MAXIMUM CONTENTS	AVERAGE CONTENTS	TOTAL ENTRIES	ZERO ENTRIES	PERCENT ZEROS	AVERAGE TIME/UNIT	\$AVERAGE TIME/UNIT	QTABLE NUMBER	CURRENT CONTENTS	
LINE1	8	2.010	50	10	20.0	239.981	299.976		4	
LINE2	6	1.936	59	7	11.9	195.947	222.324		4	
RANDOM	ANTITHETIC	INITIAL	CURRENT	SAMPLE	CHI-SQUARE					
STREAM	VARIATES	POSITION	POSITION	COUNT	UNIFORMITY					
1	OFF	100000	100219	219	0.76					

Turn Taker

		Server 1 is	49
		Server 2 is	51

--AVG-UTIL-DURING--										
FACILITY	TOTAL	AVAIL	UNAVL	ENTRIES	AVERAGE	CURRENT	PERCENT	SEIZING	PREEMPTING	
	TIME	TIME	TIME		TIME/XACT	STATUS	AVAIL	XACT	XACT	
SERVER1	0.822			49	100.000	AVAIL				
SERVER2	0.859			52	98.473	AVAIL		93		
QUEUE	MAXIMUM	AVERAGE	TOTAL	ZERO	PERCENT	AVERAGE	\$AVERAGE	QTABLE	CURRENT	
	CONTENTS	CONTENTS	ENTRIES	ENTRIES	ZEROS	TIME/UNIT	TIME/UNIT	NUMBER	CONTENTS	
LINE1	9	2.482	55	5	9.1	269.082	295.990		6	
LINE2	10	2.843	52	6	11.5	325.884	368.391		0	
RANDOM	ANTITHETIC	INITIAL	CURRENT	SAMPLE	CHI-SQUARE					
STREAM	VARIATES	POSITION	POSITION	COUNT	UNIFORMITY					
1	OFF	100000	100108	108	0.39					

ShortQ

☰ A3Q2_T.LIS X		☰ A3Q2SO X			
☰ A3Q2SO					
1			Server 1 is	52	
2			Server 2 is	48	
3					

--AVG-UTIL-DURING--										
FACILITY	TOTAL	AVAIL	UNAVL	ENTRIES	AVERAGE	CURRENT	PERCENT	SEIZING	PREEMPTING	
	TIME	TIME	TIME		TIME/XACT	STATUS	AVAIL	XACT	XACT	
SERVER1	0.934			53	98.765	AVAIL		101		
SERVER2	0.857			48	100.000	AVAIL				
QUEUE	MAXIMUM	AVERAGE	TOTAL	ZERO	PERCENT	AVERAGE	\$AVERAGE	QTABLE	CURRENT	
	CONTENTS	CONTENTS	ENTRIES	ENTRIES	ZEROS	TIME/UNIT	TIME/UNIT	NUMBER	CONTENTS	
LINE1	4	0.990	54	8	14.8	102.747	120.616		1	
LINE2	3	0.613	48	15	31.2	71.509	104.013		0	
RANDOM	ANTITHETIC	INITIAL	CURRENT	SAMPLE	CHI-SQUARE					
STREAM	VARIATES	POSITION	POSITION	COUNT	UNIFORMITY					
1	OFF	100000	100103	103	0.37					

Question 3: Random Number Generator

Lehmer random-number generator

$$x_{i+1} = g(x) = ax_i \bmod 251 \quad i=0,1,2,3,\dots$$

$m = 251$

a. Using Theorem: "If m is prime and p_1, p_2, \dots, p_r are the (unique) prime factors of $m - 1$ "

$$m-1 = 250 = 2 \cdot 5^3$$

$$\text{number of full-period multipliers} = (p_1-1)(p_2-1)(m-1)/(p_1 \cdot p_2)$$

$$= (2-1)(5-1)(251-1)/(2 \cdot 5)$$

$$= (250 \cdot 4)/10 = 100$$

There are **100** full – period multipliers

b. Program:

```
PS C:\Users\punya\Documents\Priyam\2023 Winter\COIS 4470H\Assignments\A3>
PS C:\Users\punya\Documents\Priyam\2023 Winter\COIS 4470H\Assignments\A3> python3.8 A3Q3_FPM.py > A3_all_multipliers.txt
PS C:\Users\punya\Documents\Priyam\2023 Winter\COIS 4470H\Assignments\A3>
```

A3_all_multipliers.txt - Notepad

File Edit Format View Help

m = 251

Number of multipliers: 100

6
11
14
18
19
24
26
29
30
33
34
37
42
43
44
46
53
54
55

Please check entire output in attached file A3_all_multipliers.txt

c. Output File: A3_random_numbers.txt

Generating random numbers in range (0,1)

$x_0 = 3$

Please enter seed 'x0' value: 3
Generating random numbers uniformly from (0,1):

0.07171314741035857
0.4302788844621514
0.5816733067729084
0.4900398406374502
0.9402390438247012
0.6414342629482072
0.848605577689243
0.09163346613545817
0.549800796812749
0.29880478087649404
0.7928286852589641
0.7569721115537849
0.5418326693227091
0.250996015936255
0.5059760956175299
0.035856573705179286
0.2151394422310757
0.2908366533864542
0.7450199203187251
0.4701195219123506
0.8207171314741036
0.9243027888446215
0.545816733067729
0.2749003984063745
0.649402390438247
0.896414342629482
0.3784860557768924
0.27091633466135456
0.6254980079681275
0.7529880478087649
0.5179282868525896
0.10756972111553785
0.6454183266932271
0.8725099601593626
0.2350597609561753
0.4103585657370518
0.46215139442231074
0.7729083665338645
0.6374501992031872
0.8247011952191236
0.9482071713147411
0.6892430278884463
0.13545816733067728
0.8127490039840638
0.8764940239043825
0.2589641434262948
0.5537848605577689
0.32270916334661354

d. Output File: A3_100RN

Making a change to code so 100 random variates are generated

```
def generate_random_numbers(seed: int, a: int, m: int, n: int) -> list:
    numbers = list()
    x = seed

    for i in range(n):
        x = (x*a) % m
        numbers.append(x/m)
        if x == seed: break

    return numbers

if __name__ == '__main__':
    x0 = int(input("\n Please enter seed 'x0' value: "))
    n = 100
    m = 251
    a = get_period_multiplier(m)

    random_numbers = generate_random_numbers(x0,a,m,100)

    print("Generating random numbers uniformly from (0,1): \n")
    for num in random_numbers:
        print(num)
```

```
Please enter seed 'x0' value: 3
Generating random numbers uniformly from (0,1):
```

```
0.07171314741035857
0.4302788844621514
0.5816733067729084
0.4900398406374502
0.9402390438247012
0.6414342629482072
0.848605577689243
0.09163346613545817
0.549800796812749
0.29880478087649404
0.7928286852589641
0.7569721115537849
0.5418326693227091
0.250996015936255
0.5059760956175299
0.035856573705179286
0.2151394422310757
0.2908366533864542
0.7450199203187251
0.4701195219123506
0.8207171314741036
0.9243027888446215
0.545816733067729
0.2749003984063745
0.649402390438247
0.896414342629482
0.3784860557768924
0.27091633466135456
0.6254980079681275
0.7529880478087649
0.5179282868525896
0.40756078144553705
```

e. Chi squared test

Chi squared statistic value generated = 12.2

For alpha = 0.01 and s = 10, (degree of freedom = 9) chi critical value = 21.7

12.2 is less than 21.7, therefore, TRUE all values generated are uniform

I anyways implemented the calculation of this in the program itself. As you see, it says "True" for being uniformly generated

```
Please enter seed 'x0' value: 3

chi squared statistic value : 12.2
Generating random numbers uniformly from (0,1)

Using chi-squared test, Are the random numbers uniformly distributives: True
0.07171314741035857
0.4302788844621514
0.5816733067729084
0.4900398406374502
0.9402390438247012
0.6414342629482072
0.848605577689243
0.09163346613545817
0.549800796812749
0.29880478087649404
0.7928286852589641
0.7569721115537849
0.5418326693227091
0.250996015936255
0.5059760956175299
0.035856573705179286
0.2151394422310757
0.2908366533864542
0.7450199203187251
0.4701195219123506
0.8207171314741036
0.9243027888446215
0.545816733067729
0.2749003984063745
0.649402390438247
0.896414342629482
0.3784860557768924
0.27091633466135456
0.6254980079681275
0.7529880478087649
0.5179282868525896
0.10756972111553785
0.6454183266932271
0.8725099601593626
0.2350597609561753
0.4103585657370518
0.46215139442231074
0.7729083665338645
```

f. GAP Test (0.2, 0.5)

Output File: A3_GAP_Out.txt

Random numbers are independent :)

```
Please enter seed 'x0' value: 3

Random Numbers in range: 32

Number of gaps : 31

Printing Gap length and fo:

{1: 8, 5: 1, 3: 3, 2: 6, 0: 9, 6: 1, 8: 2, 11: 1}

Printing fe values:

[6.51, 1.5630509999999993, 3.1898999999999993, 4.556999999999999, 9.299999999999999, 1.0941356999999996, 0.5361264929999997, 0.18389138700899988]

X2 chi : 8.648806634645176

Table value: 52.19139483319193

Random Numbers are independent!
Generating random numbers uniformly from (0,1)

0.07
0.43
0.58
0.49
0.94
0.64
0.85
0.09
0.55
0.3
0.79
0.76
0.54
0.25
0.51
0.04
0.22
0.29
0.75
0.47
0.82
0.92
0.55
0.27
0.65
0.9
0.38
```