Note: There are three questions out of 50 marks. Give all your answers and discussions in a pdf file. Submit both the answer file and all programs on Blackboard by 11:59pm on the due date.

1. [15 marks] People arrive at a newspaper stand with an interarrival time that is exponentially distributed with a mean of 0.5 minute. 58% of the people buy just the morning paper, while 22% buy the morning paper and a Wall Street Journal. The remainder buys only the Wall Street Journal. One clerk handles the Wall Street Journal sales, while another clerk handles morning paper sales. A person buying both goes to the Wall Street Journal clerk. The time it takes to serve a customer is 40+4 seconds for all transactions.

   (a) Write a GPSS program to simulate this system and collect statistics on queues for each type of transactions. Simulation for 2 hours.
   (b) Suggest ways for making the system more efficient. Use simulation results to support your suggestions.

2. [15 marks] You are the IS manager for a growing "Web Mall". Business has been so good that the rate of requests arriving into the single Web server exceeds the service rate of these requests. In other words, your Web server is overflowing, and you are losing real customers. You plan to install a second mirrored Web server and distribute the incoming requests between the two identical servers. You have found three request distribution products on the market. They are:

   LoadSplit - A device that randomly assigns an incoming request to one of two
   Servers (50% to each server).
   TurnTaker - A device that alternates incoming requests to the two servers.
   ShortQ - A device that sends incoming requests to the server with the shortest
   queue length.

   It is intuitively obvious that **ShortQ** will result in the lowest delay for processing a request. However, **ShortQ** is a very expensive product and you need to justify its purchase to your manager.

   Assume that a Web server can process 10 requests per second with a deterministic distribution (i.e., each request takes exactly 100 milliseconds to process). Assume also the interarrival times for requests are exponentially distributed with mean 55 milliseconds. Evaluate performance of the three products: **LoadSplit**, **TurnTaker**, and **ShortQ** by simulating 100 requests. Fill in the following comparison form. What product would you recommend and why? (hint: the modulo division of A by B A@B returns the integer remainder).

Performance Measure	LoadSplit	TurnTaker	ShortQ
Server Utilization - Server 1			
Server Utilization - Server 2			
Number of request served - Server 1			
Number of request served - Server 2			
Average number in queue – queue for server 1			
Average number in queue – queue for server 2			
Mean delay time - queue for server 1			
Mean delay time - queue for server 2			
Mean Response Time – Server 1 (total waiting time for each request)			
Mean Response Time – Server 2			

3. [20 marks] **Random number generation**: A Lehmer random-number generator is defined by

$$x_{i+1} = g(x_i) = ax_i \bmod 251, \; i = 0, 1, 2, 3, \ldots$$

(a) How many full-period multipliers?
(b) Find all the full-period multipliers (show your program).
(c) Write a computer program to generate random numbers in the interval (0, 1) with the maximum period. The seed (x_0) should be input at run time from the user.
(d) Run your program to generate 100 random variates with a reasonable seed.
(e) Using the frequency test discussed in class to determine if the generator is uniform with a level of significance of 0.01.
(f) Apply the Gap Test with the interval (0.2, 0.5) to determine if the random numbers are independent with a level of significance of 0.01.

Chi-Square Distribution values for $\alpha=0.01$ with ν degrees of freedom:

ν	1	2	3	4	5	6	7	8	9	10	11	12
$\chi^2_{0.01}$	6.63	9.21	11.3	13.3	15.1	16.8	18.5	20.1	21.7	23.2	24.7	26.2