Special Relativity

1 Spacetime

In 1905, Albert Einstein proposed his special theory of relativity. The core of it form the following two postulates:

> The laws of mechanics and electrodynamics hold in all inertial frames of reference.

> The speed of light in vacuum is constant, no matter the speed of the observer or the source.

The first consequence of this is, that we need to redefine what "at the same time" means. Only when a observer standing in the middle of two events sees them at the same time are they actually synchronized. This however only holds if the events are in the same inertial frame. In a moving inertial frame (relative to the first observer) the events won't happen at the same time.

Because of this, we cannot consider space and time as absolute and separate from each other anymore. Every inertial frame of reference has its own time and length scale. For a representation spacetime diagrams are used. These diagrams are called *Minkowski-diagrams* and if our frame only moves in x direction, they look like this:

2 Time dilatation and Length contraction

If we have a system that moves with constant speed relative to the observers system we notice the following effects:

> **Time dilatation:** If a clock next to the observer passes time t, then the observer can see a time t_0 pass on a clock in the moving system. It holds that
>
> $$\Delta t = \gamma \Delta t_0.$$

> **Length contraction:** Since the speed of the system stays constant we also have to adapt the length l_0 measured in the resting frame. The contracted length l in the moving system is
>
> $$l = \frac{l_0}{\gamma}.$$

In both cases, γ is the Lorentz factor, which is defined as follows:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Because this factor is very close to 1 for velocities less than about 10% of the speed of light, we only deal with relativistic effects for $v > 0.1c$.

3 Some paradoxons

One of the most famous paradoxes is the twin Paradoxon. Assume your (potentially inexistent) twin boards a super fast rocket to visit Alpha Centauri (our nearest star). From earth the twin should be younger when he returns, however your twin could claim the opposite when we consider his frame of reference.

This Paradoxon is resolved by the fact, that your twin had to change his inertial frame of reference at some point to return to earth. It can be shown that he indeed will end up younger.

Another classic is a hypertrain. Because of length contraction, the trains wheels shouldn't be on the track for an outside observer but on them for a passenger. this is absurd and the reason for it lies in the fact, that contraction only happens parallel to $\vec{v}$.

4 Coordinate transformations

Let's consider a train moving along the x-axis with an outside observer standing at $x = 0$. The train shallst move with velocity v. Let's see how we can transform coordinates $\vec{r}$ of the resting observer into $\vec{r'}$, the coordinates of someone riding the train who passes $x = 0$ at $t = 0$.

In classical mechanics we would have

> **Galilei transformation:**
>
> $$\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}$$

In relativity, this would be absurd as this would allow for velocities greater than c. Thus, under special relativity, the transformation becomes

Lorentz transformation:

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - \frac{v}{c^2}x)$$

We now also have the possibility to add velocities. Consider a system moving with velocity v relative to a stationary observer. If a person in the moving system sees an object with velocity u', the stationary observer sees the object with velocity u as follows:

Relativistic velocity addition

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}} \text{ and } u' = \frac{u - v}{1 - \frac{uv}{c^2}}.$$

Another interesting effect is when changing frames is the Doppler effect (I didnt think about it at EuPhO, F). If a sender distances himself from a receiver with velocity v (can also be negative) then the frequency of the signal sent with f_0 changes to f as follows:

Doppler effect

$$f = f_0\sqrt{\frac{c - v}{c + v}}.$$