Exercise5

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Exercise 4.2. 一平面波以 $\theta = 45°$ 从真空入射到 $\varepsilon_r = 2$ 的介质，电场强度垂直于入射面，求反射系数和折射系数。

Solution. 根据定义

$$R = \left|\frac{E'}{E}\right|^2$$
$$T = \left|\frac{E''}{E}\right|^2$$

在 E 垂直于入射面时还有 $\dfrac{E'}{E} = \dfrac{\sqrt{\varepsilon_1}\cos\theta - \sqrt{\varepsilon_2}\cos\theta''}{\sqrt{\varepsilon_1}\cos\theta + \sqrt{\varepsilon_2}\cos\theta''}$，$\dfrac{E''}{E} = \dfrac{2\sqrt{\varepsilon_1}\cos\theta}{\sqrt{\varepsilon_1}\cos\theta + \sqrt{\varepsilon_2}\cos\theta''}$。

由 $\dfrac{\sin\theta}{\sin\theta''} = \sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}} = \sqrt{2}$ 得 $\sin\theta'' = \dfrac{1}{2}$，因此 $\sin\theta = \cos\theta = \dfrac{\sqrt{2}}{2}$，$\cos\theta'' = \dfrac{\sqrt{3}}{2}$，还有 $\varepsilon_1 = \varepsilon_0$，$\varepsilon_2 = 2\varepsilon_0$。根据这些求出 $R = \dfrac{2-\sqrt{3}}{2+\sqrt{3}}$，$T = \dfrac{2\sqrt{3}}{2+\sqrt{3}}$ □

Exercise 4.3. 可见光由水入射到空气，入射角为 60°，证明会发生全反射，求折射波沿表面传播的像速度和透入空气的深度。$\lambda_0 = 6.28 \times 10^{-5}$，$n = 1.33$.

Solution. $\sin\theta = \dfrac{\sqrt{3}}{2} > n$，所以会发生全反射。

由 $k''\sin\theta'' = k''_x = k_x = k\sin\theta$ 得 $k'' = kn$，$v = \dfrac{\omega''}{k''} = \dfrac{\omega}{nk}$，又 $\omega = ck$，故 $v = \dfrac{c}{n}$.

?????????? □

Exercise 4.6. 平面电磁波垂直入射到金属表面，证明金属内部电磁波全部转化为焦耳热。

Solution. $\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}$，$\omega = \dfrac{1}{2}(\boldsymbol{E}\cdot\boldsymbol{D} + \boldsymbol{H}\cdot\boldsymbol{B})$。如果 $f(t) = f_0 e^{-i\omega t}$，$g(t) = g_0 e^{-i\omega t + i\phi}$，则其平均值定义为 $\overline{fg} = \dfrac{1}{2}\mathrm{Re}\,[f^* g]$。

该问中 $\boldsymbol{E} = \boldsymbol{E}_0 e^{-\alpha z} e^{i(\beta z - \omega t)}$，$\boldsymbol{H} = \dfrac{1}{\omega\mu}\boldsymbol{k} \times \boldsymbol{E} = \dfrac{1}{\omega\mu}(\alpha i + \beta)\boldsymbol{e}_k \times \boldsymbol{E}$，得到

$$\begin{aligned}
\boldsymbol{S} &= \boldsymbol{E} \times \boldsymbol{H} \\
&= \boldsymbol{E} \times \left[\frac{1}{\omega\mu}(\alpha i + \beta)\boldsymbol{e}_k \times \boldsymbol{E}\right] \\
&= (\alpha i + \beta) e^{i(\beta z - \omega t)} \frac{E_0^2}{\omega\mu} e^{-2\alpha z} e^{i(\beta z - \omega t)} \hat{\boldsymbol{e}}_z
\end{aligned}$$

$$\overline{S} = \frac{1}{2}\mathrm{Re}\left[(\alpha i + \beta) e^{i(\beta z - \omega t)} \frac{E_0^2}{\omega\mu} e^{-2\alpha z} e^{-i(\beta z - \omega t)}\right] = \frac{\beta E_0^2}{2\omega\mu} e^{-\alpha z}$$

热功率密度 $p = \boldsymbol{J}\cdot\boldsymbol{E} = \sigma E^2 = \sigma e^{i(\beta z - \omega t)} \dfrac{E_0^2}{\omega\mu} e^{-2\alpha z} e^{i(\beta z - \omega t)}$，得

$$\overline{p} = \frac{1}{2}\sigma E_0^2 e^{-2\alpha z}$$

单位面积的热功率

$$P = \int_0^{+\infty} \frac{1}{2}\sigma E_0^2 e^{-2\alpha z}\,\mathrm{d}z = \frac{1}{2}\sqrt{\frac{\sigma}{2\omega\mu}}E_0^2$$

单位时间进入导体的能流

$$\overline{S}|_{z=0} = \frac{1}{2}\sqrt{\frac{\sigma}{2\omega\mu}}E_0^2$$

□

Exercise 4.9. 无限长矩形导波管，在 $z=0$ 处被一块垂直插入的理想导体平板完全封闭，求 $z=-\infty$ 到 $z=0$ 这段可能存在的波模。

Solution. 导波管内满足方程

$$\begin{cases} \nabla^2\boldsymbol{E} + k^2\boldsymbol{E} = 0 \\ \nabla\cdot\boldsymbol{E} = 0 & \text{(每一处)} \\ \hat{\boldsymbol{e}}_n \times \boldsymbol{E} = 0 & \text{(边界处)} \end{cases}$$

后两个条件又可以推出

$$\begin{cases} E^y = E^z = \dfrac{\partial E^x}{\partial x} = 0 & (x=0,\ a) \\ E^x = E^z = \dfrac{\partial E^y}{\partial y} = 0 & (y=0,\ a) \\ E^x = E^y = \dfrac{\partial E^z}{\partial z} = 0 & (z=0) \end{cases}$$

方程 $\nabla^2 E + k^2 E = 0$ 的通解为

$$E(x,y,z) = (C_x\cos k_x x + D_x \sin k_x x)(C_y\cos k_y y + D_y\sin k_y y)(C_z\cos k_z z + D_z\sin k_z z)$$

由 $E^x = 0$ $(y=0,\ a,\ z=0)$ 得到 $C_y = 0,\ k_y = \dfrac{n\pi}{a},\ k_x = \dfrac{m\pi}{a},\ C_z = 0$，由 $\left.\dfrac{\partial E^x}{\partial x}\right|_{y=0,\ a} = 0$ 得 $D_x = 0$。可以写出

$$E^x = A_1 \cos\frac{m\pi}{a}x\sin\frac{n\pi}{a}y\sin k_z z$$

同样的方法求出

$$E^y = A_2\sin\frac{m\pi}{a}x\cos\frac{n\pi}{a}y\sin k_z z$$
$$E^z = A_3\sin\frac{m\pi}{a}x\sin\frac{n\pi}{a}y\cos k_z z$$

要求 $k_x^2 + k_y^2 + k_z^2 = \omega^2\mu\varepsilon \Rightarrow k_z = \sqrt{\omega^2\mu\varepsilon - \dfrac{m^2\pi^2}{a^2} - \dfrac{n^2\pi^2}{a^2}}$，以及 $k_xA_1 + k_yA_2 + k_zA_3 = 0$。 □

Exercise 4.11. 写出矩形导波管内磁场 $\boldsymbol{H}$ 满足的方程及边界条件。

Solution. $\nabla\times\boldsymbol{H} = \dfrac{\partial\boldsymbol{D}}{\partial t}$, $\nabla\times\boldsymbol{E} = -\dfrac{\partial\boldsymbol{B}}{\partial t}$，所以

$$-\nabla^2\boldsymbol{H} = \nabla(\nabla\cdot\boldsymbol{H}) - \nabla^2\boldsymbol{H} = \nabla\times(\nabla\times\boldsymbol{H}) = \omega^2\mu\varepsilon\boldsymbol{H}$$

得方程

$$\begin{cases} \nabla^2\boldsymbol{H} + k^2\boldsymbol{H} = 0 \\ \nabla\cdot\boldsymbol{H} = 0 & \text{(每一处)} \\ \hat{\boldsymbol{e}}_n \times \boldsymbol{H} = \boldsymbol{a} & \text{(边界处)} \end{cases}$$

□

Exercise 4.12. 论证矩形导波管不存在 TM_{m0}，TM_{0n} 波。

Solution. 导波管内满足方程

$$\begin{cases} \nabla^2 \boldsymbol{E} + k^2 \boldsymbol{E} = 0 & \\ \nabla \cdot \boldsymbol{E} = 0 & (\text{每一处}) \\ \hat{\boldsymbol{e}}_n \times \boldsymbol{E} = 0 & (\text{边界处}) \end{cases}$$

在 $z = 0$ 面没有约束时，得通解

$$\begin{cases} E^x = A_1 \cos k_x x \sin k_y y e^{ik_z z} \\ E^y = A_2 \sin x_x x \cos k_y y e^{ik_z z} \\ E^z = A_3 \sin k_x x \sin k_y y e^{ik_z z} \end{cases}$$

由 $\boldsymbol{H} = -\dfrac{i}{\omega\mu} \nabla \times \boldsymbol{E}$

$$\begin{aligned} H^x &= -\frac{i}{\omega\mu} \epsilon^{ij1} \partial_i E_j \\ &= -\frac{i}{\omega\mu} \left[A_1 k_y \sin k_x x \cos k_y y e^{ik_z z} - ik_z z A_2 \sin k_x x \cos k_y y e^{ik_z z} \right] \end{aligned}$$

同理有

$$\begin{aligned} H^y &= -\frac{i}{\omega\mu} \left[iA_1 k_z - A_3 k_x \right] \cos k_x x \sin k_y y e^{ik_z z} \\ H^z &= -\frac{i}{\omega\mu} \left[A_2 k_x - A_1 k_y \right] \cos k_x x \cos k_y y e^{ik_z z} \end{aligned}$$

需要 $k^2 = \omega^2 \mu\varepsilon = k_x^2 + k_y^2 + k_z^2$，$k_x = \dfrac{m\pi}{a}$，$k_y = \dfrac{n\pi}{b}$，$A_1 k_x + A_2 k_y + ik_z A_3 = 0$。

TM 波 $H^z = 0 \Rightarrow A_2 k_x - A_1 k_y = 0$

TM_{m0} 波：$n = 0 \Rightarrow k_y = 0,\ A_2 = 0 \Rightarrow H^x = H^y = H^z = 0$。

TM_{0n} 波：$m = 0 \Rightarrow k_x = 0, A_1 = 0 \Rightarrow H^x = H^y = H^z = 0$。

□

Exercise 4.13. 频率为 $30 \times 10^9\mathrm{Hz}$ 的微波，在 $0.7 \times 0.4\mathrm{cm}$ 的矩形导波管内能以什么波模传播，在 $0.7 \times 0.6\mathrm{cm}$ 呢。

Solution. 截至频率 $\omega_{cmn} = \dfrac{\pi}{\mu\varepsilon} \sqrt{\dfrac{m^2}{a^2} + \dfrac{n^2}{b^2}} = \pi c \sqrt{\dfrac{m^2}{a^2} + \dfrac{n^2}{b^2}}$

可计算出得一种情况可以传播 $\mathrm{TE}_{00\sim55}$，TE_{65}，第二种情况可以传播 $\mathrm{TE}_{00\sim99}$，$\mathrm{TE}_{9,10}$，$\mathrm{TE}_{10,9}$ □