Chapter 1

Introduction

1.1 Basics

Definition 1.1.1

(1) S is a null semigroup if $\forall x, y \in S(xy = 0)$;

(2) S is a left zero semigroup if $\forall x, y \in S(xy = x)$, the definition for right zero semigroup is obvious;

(3) $I \subset S$ is a proper ideal if $\{0\} \subset I \subsetneq S$ and $IS \subset S \wedge SI \subset S$;

(4) given a set X, the full transformation semigroup is defined as $(\mathrm{End}_{\mathsf{Set}}(X), \circ)$, where $\circ$ refers the composition of functions;

(5) a morphism $S \xrightarrow[\mathsf{Smg}]{\phi} \mathrm{End}(X)$ is a *representation* of S, and φ is faithful if it is injective;

(6) a semigroup S is a rectangular band if $\forall a, b \in S(aba = a)$;

(7) $\langle \{a\} \rangle_{\mathrm{smg}}$ is called a *monogenic semigroup*.