CS450 Homework one: Sensitivity Subtraction

Xiuyi Qin

January 21, 2019

1. Answer1:

   The condition number:

   $$\kappa(f) = \left| \frac{\Delta f(x,y) \times (x,y)}{(\Delta x, \Delta y) \times f(x,y)} \right| \tag{1}$$

   So:

   $$\kappa(f) = \frac{|\Delta x + x - (\Delta y + y) - (x - y)|(|x| + |y|)}{(|\Delta x| + |\Delta y|)(|x - y|)} \tag{2}$$

   Since:

   $$|x - y| \approx \epsilon, |x| + |y| \approx 1 \tag{3}$$

   And:

   $$|\Delta x + x - (\Delta y + y) - (x - y)| = |\Delta x - \Delta y| \leq |\Delta x| + |\Delta y| \tag{4}$$

   Therefore:

   $$\kappa(f) = \frac{|\Delta x + x - (\Delta y + y) - (x - y)|}{(|\Delta x| + |\Delta y|)\epsilon} \leq \frac{(|\Delta x| + |\Delta y|)}{(|\Delta x| + |\Delta y|)\epsilon} = \frac{1}{\epsilon} \tag{5}$$

2. Answer2: Since ϵ is a really small number, we can say $\frac{1}{\epsilon}$ is a really big number. In that case, we could say the sensitivity of subtraction is large or this problem is ill-condition ed and sensitive.