Quant Session 4

Quant 4 – Live Session

Algebra

Functions, Graphs, Coordinate Geometry, Sequences, Series, Progressions, Quadratic Equations, Symbols

You must have solved each of the questions extremely thoroughly before the live class

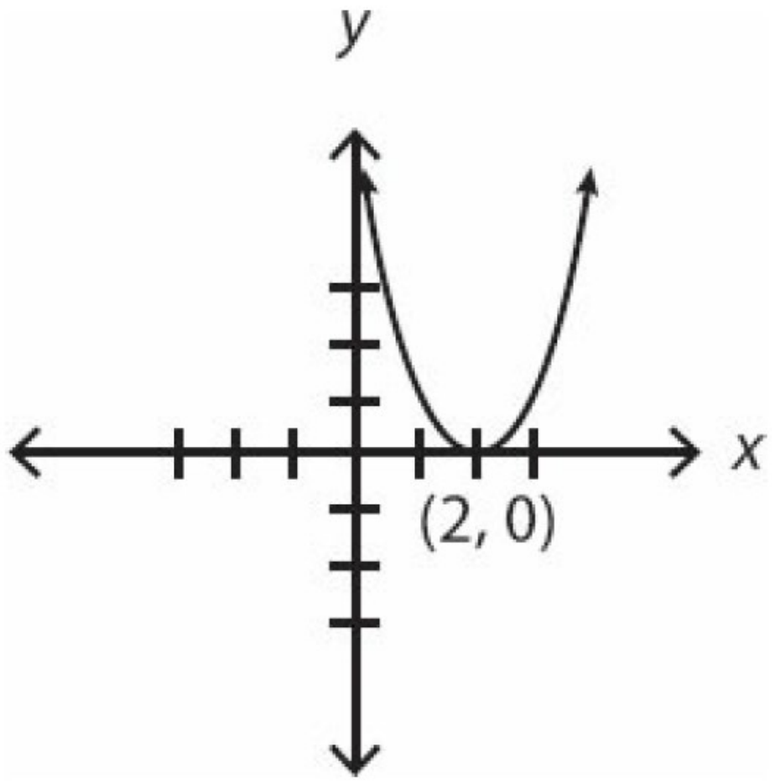
Also, check the Answerkey, and think hard on each question

1. If $f(x) = x^3 - \frac{1}{x^3}$, then $f(x) + f\left(\frac{1}{x}\right) = ?$

A. 0
B. 1
C. −1
D. 2
E. 3

2. If the equation of the parabola in the coordinate plane is $y = (x - h)^2 + k$ and $(-3, n)$ is a point on the parabola, what is the value of n?

A. 10
B. 15
C. 20
D. 25
E. 30



3.

If $f(x) = \frac{125}{x^3}$, what is the value of $f(5x) * f(\frac{x}{5})$ in terms of f(x)?

A. $(f(x))^2$

B. $f(x^2)$

C. $(f(x))^3$

D. $f(x^3)$

E. $f(125x)$

4.

If f is the function defined by $f(x) = x^2 * (1 - x)^2$ for all x, then f(1-x) =

A. f(x)
B. f(x)^2
C. 1 - f(x)
D. (1-x)*f(x)
E. f(1) - f(x)

5.

In which quadrant is there no solution for $y \geq 2x + 1$ and $y > \frac{1}{2} x - 1$?

A. I
B. II
C. III
D. IV
E. II and IV

6. If $f(x) = x^3 + \sqrt{x}$ and $g(x) = 4x - 3$, what is $f(g(3))$?

A. 459
B. 712
C. 732
D. 624
E. 551

7.

For which of the following functions f is f(x) = f(1-x) for all x?

A. $f(x) = 1 - x$

B. $f(x) = 1 - x^2$

C. $f(x) = x^2 - (1 - x)^2$

D. $f(x) = x^2 * (1 - x)^2$

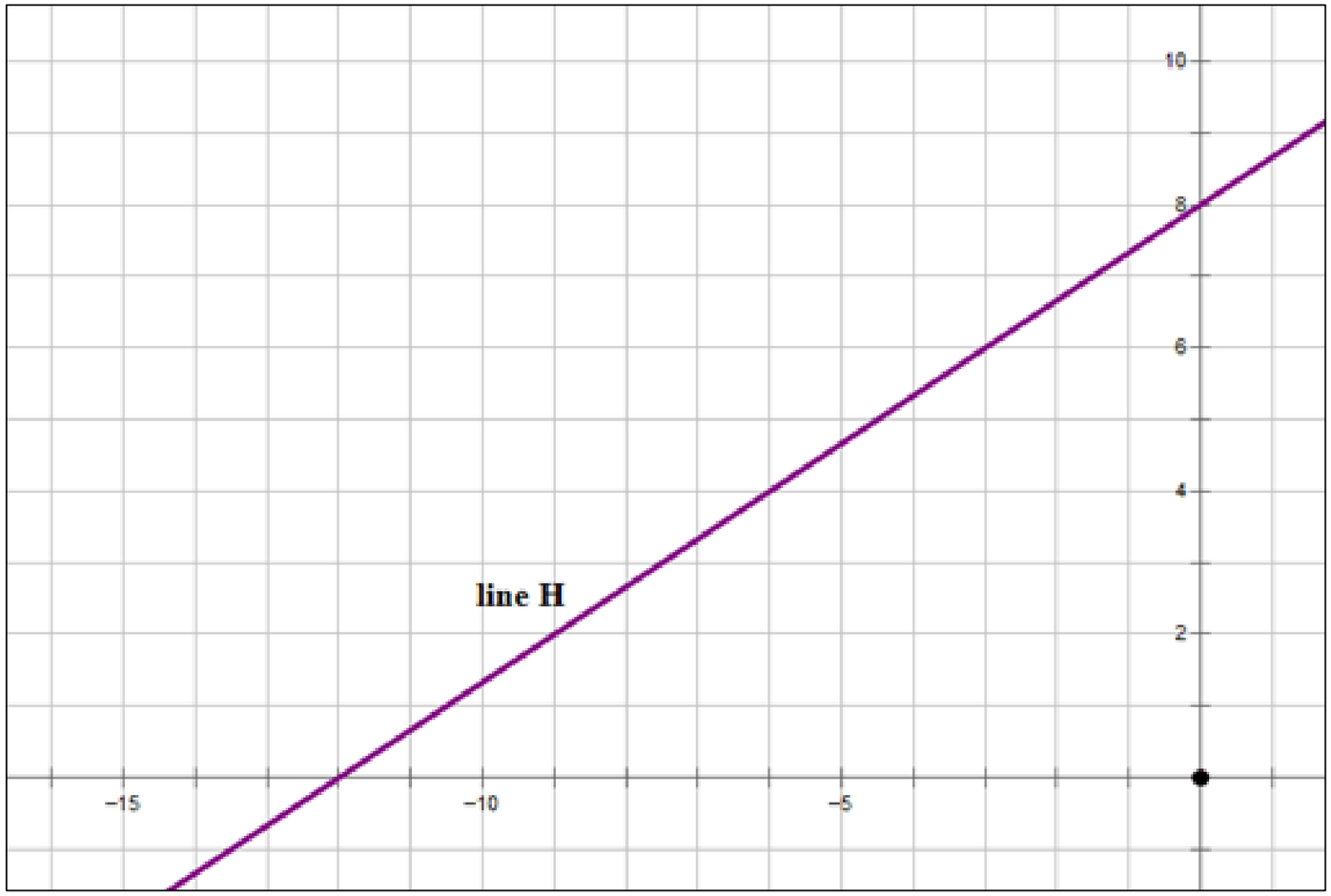
E. $f(x) = \frac{x}{(1-x)}$

8.

The graph shows line H. Line J (not shown) does not pass through the first quadrant. Which of the following could be true?

I. line J is perpendicular to line H
II. line J is parallel to line H
III. line J intersect line H in the third quadrant

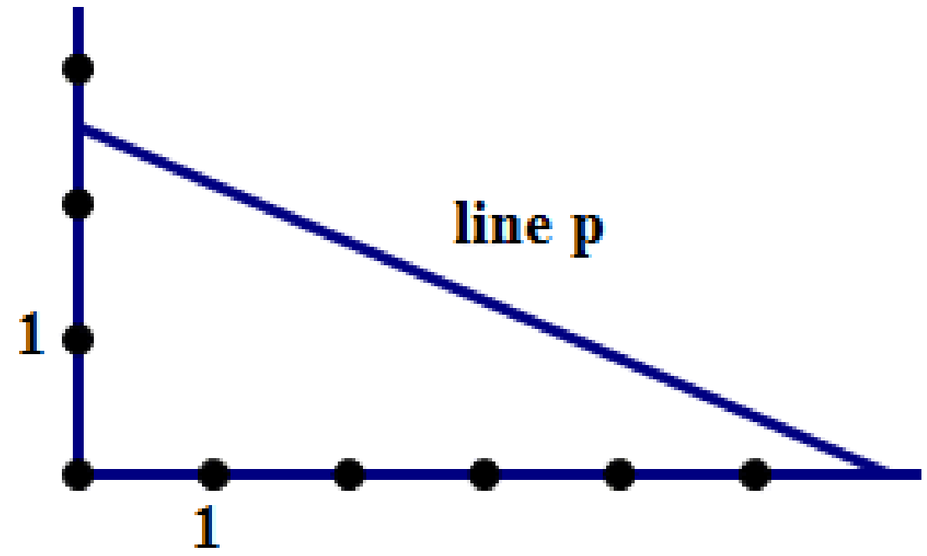
(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) I, II, and III



9.

Among the following options, which of the following is the best possible equation of line p?

(A) 3x + 7y = 18
(B) 7x + 3y = 18
(C) 3x – 7y = 18
(D) 7x – 3y = 18
(E) 3x + 7y = –18



10.

If $f(x) = 3x^2 - 5x - 4$, then $f(-2x)$ is equal to

A. $2f(-x)$
B. $-f(x)$
C. $4f(x)$
D. $-4f(x)$
E. None of these

11.

Which of the following could be the equation of the parabola in the coordinate plane above?

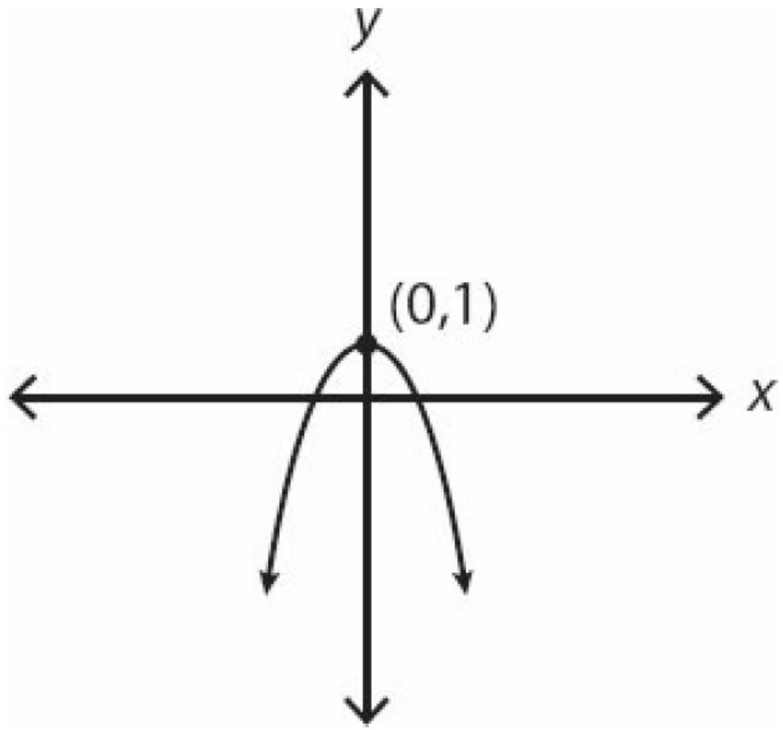
(A) $y = -x - 1$

(B) $y = x^2 + 1$

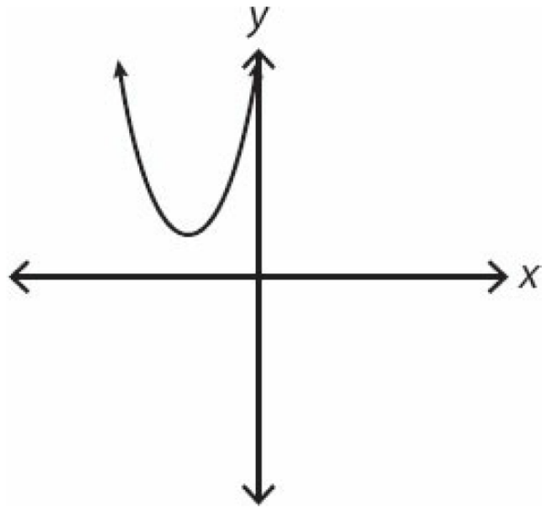
(C) $y = -x^2 - 1$

(D) $y = -x^2 + 1$

(E) $y = -(x - 1)^2$



12.



Which of the following could be the equation of the parabola in the coordinate plane above?

(A) $y = x^2 + 3$

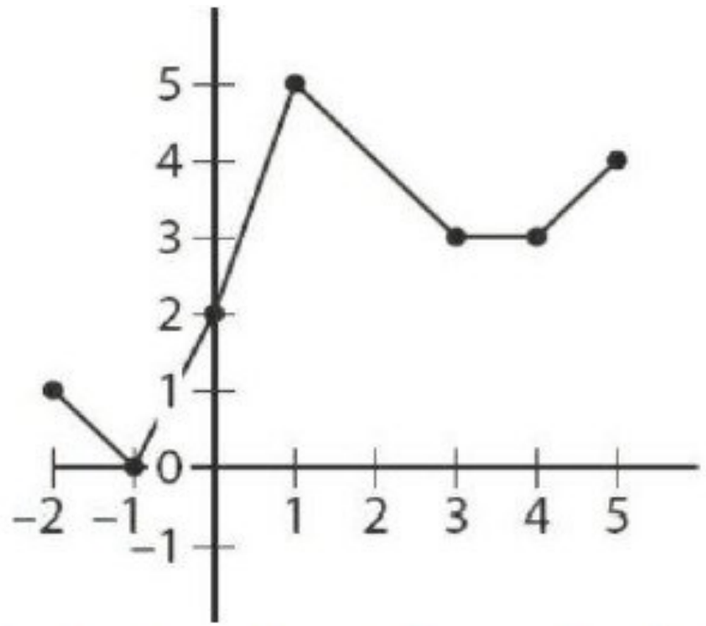
(B) $y = (x - 3)^2 + 3$

(C) $y = (x + 3)^2 - 3$

(D) $y = (x - 3)^2 - 3$

(E) $y = (x + 3)^2 + 3$

13.

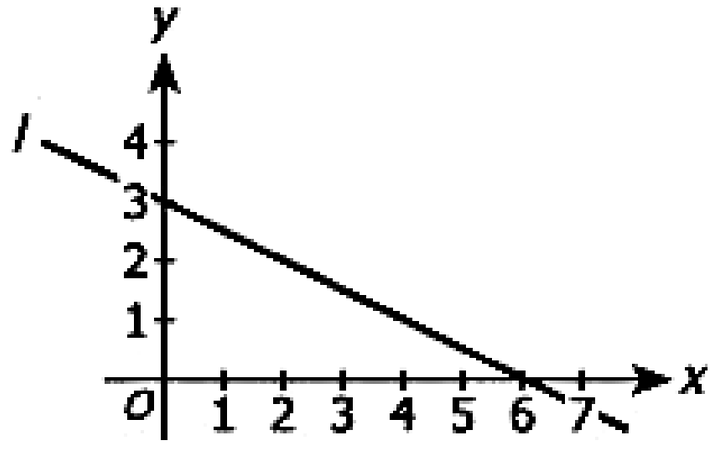


The plot above shows the graph of function f(x). For what integer value of x in the interval shown does f(x) = x + 1?

A. −2
B. −1
C. 0
D. D. 1
E. 2

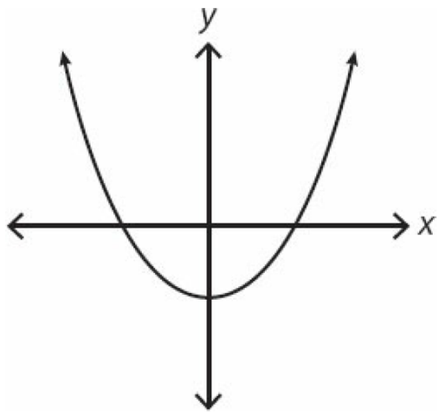
14.

All points (x, y) that lie below the line *l*, shown above, satisfy which of the following inequalities?



A. $y < 2x + 3$
B. $y < -2x + 3$
C. $y < -x + 3$
D. $y < \frac{1}{2}x + 3$
E. $y < -\frac{1}{2}x + 3$

15.



Which of the following could be the equation of the figure above?

(A) $y = x - 3$

(B) $y = 2x^2 - x$

(C) $y = x^2 - 3$

(D) $y = x^2 + 3$

(E) $y = x^3 - 3$

16.

What is the sum of all the even integers between 99 and 401?

(A) 10,100
(B) 20,200
(C) 37,750
(D) 40,200
(E) 45,150

17.

The sum of the fourth and twelfth term of an arithmetic progression is 20. What is the sum of the first 15 terms of the arithmetic progression?

A. 300
B. 120
C. 150
D. 170
E. 270

18.

The n_{th} term, t_n, of a certain sequence is defined as $t_n = t_{n-1} + 4$. If $t_1 = -11$, then $t_{82} =$

(A) 313
(B) 317
(C) 320
(D) 321
(E) 340

19.

If the first, third and thirteenth terms of an arithmetic progression are in geometric progression, and the sum of the fourth and seventh terms of this arithmetic progression is 40, find the first term of the sequence?

A. 2
B. 3
C. 4
D. 5
E. 6

20.

When a certain tree was first planted, it was 4 feet tall, and the height of the tree increased by a constant amount each year for the next 6 years. At the end of the 6th year, the tree was 1/5 taller than it was at the end of the 4th year. By how many feet did the height of the tree increased each year?

3/10 2/5 ½ 2/3 6/5

21.

What is the sixtieth term in the following sequence? 1, 2, 4, 7, 11, 16, 22...

(A) 1,671 (B) 1,760 (C) 1,761 (D) 1,771 (E) 1,821

22.

In an increasing sequence of 5 consecutive even integers, the sum of the second, third, and fourth integer is 132. What is the sum of the first and last integers?

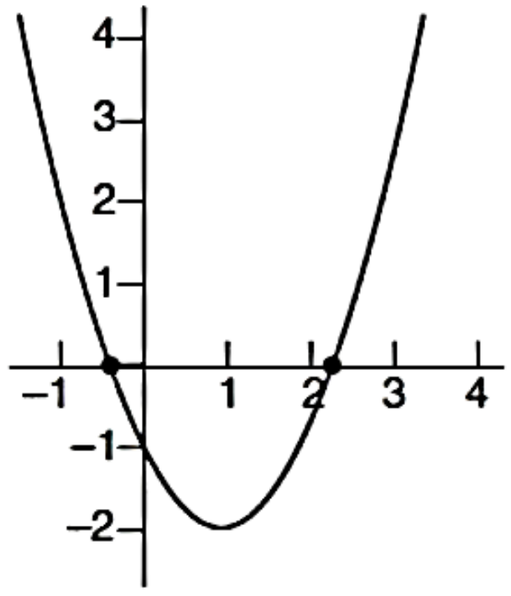
84 86 88 90 92

23.

In the arithmetic sequence $t_1, t_2, t_3 \ldots t_n$
$t_1=23$ and $t_n = t_{n-1}-3$ for each $n > 1$. What is the value of n when $t_n=-4$?

A) −1
B) 7
C) 14
D) 10
E) 20

24.



A graph of the function $g(x)$ is shown above. $g(x)$ is defined by which of the following equations?

A. $g(x) = (x+1)^2 - 2$

B. $g(x) = (x-1)^2 - 2$

C. $g(x) = (x+1)^2 + 2$

D. $g(x) = (x-1)^2 + 2$

E. $g(x) = (x-2)^2 + 1$

25.

Mitchell plans to work at a day camp over the summer. Each week, he will be paid according to the following schedule: at the end of the first week, he will receive $1. At the end of each subsequent week, he will receive $1, plus an additional amount equal to the sum of all payments he's received in previous weeks. How much money will Mitchell be paid in total during the summer, if he works for the entire duration of the 8–week–long camp?

A. 126
B. 127
C. 252
D. 254
E. 255

26.

For every integer k from 1 to 10, inclusive the "k"th term of a certain sequence is given by $(-1)^{(k+1)} * (\frac{1}{2^k})$. If T is the sum of the first 10 terms in the sequence, then T is

A. Greater than 2
B. Between 1 and 2
C. Between 1/2 and 1
D. Between 1/4 and 1/2
E. Less than 1/4

27.

The value of $1/2+(1/2)^2+(1/2)^3+...+(1/2)^{20}$ is between?

A. 1/2 and 2/3
B. 2/3 and ¾
C. 3/4 and 9/10
D. 9/10 and 10/9
E. 10/9 and 3/2

28.

If the ratio of the sum of the first 6 terms of a G.P. to the sum of the first 3 terms of the G.P. is 9, what is the common ratio of the G.P?

A. 3
B. 1/3
C. 2
D. 9
E. 19

29.

A certain club has exactly 5 new members at the end of its first week. Every subsequent week, each of the previous week's new members (and only these members) brings exactly x new members into the club. If y is the number of new members brought into the club during the twelfth week, which of the following could be y?

(A) $\sqrt[12]{5}$ (B) $3^{11}5^{11}$ (C) $3^{12}5^{12}$ (D) $3^{11}5^{12}$ (E) 60^{12}

30. In a certain sequence, every term after the first is determined by multiplying the previous term by an integer constant greater than 1. If the fifth term of the sequence is less than 1000, what is the maximum number of positive integer values possible for the first term?

A) 60
B) 61
C) 62
D) 63
E) 64

31.

The sequence A is defined by the following relationship: $A_n = A_{n-1} + (-1)^{n+1}(n^2)$ for all integer values of $n > 1$. If $A_1 = 1$, what is $A_{15} - A_{13}$?

(A) 14
(B) 29
(C) 169
(D) 196
(E) 421

32.

The y intercept of a line L is 4. If the slope of L is negative, which of the following could be the x intercept of L? **(only one answer is correct)**

I. –1

II. 0

III. 6

33.

An (x, y) coordinate pair is to be chosen at random from the xy–plane. What is the probability that $y \geq |x|$?

A. 1/10
B. 1/8
C. 1/6
D. 1/5
E. 1/4

34.For every point (a, b) lying on line 1, point (b, −a) lies on line 2. If the equation of line 1 is y = 2x + 1, what is the equation of line 2?

A. y = (1/2) + (x/2)
B. 2y = 1 − x
C. (x + y) / 2 = −1
D. y = (x / 2) − 1
E. X = 2y + 1

35.

In sequence Q, the first number is 3, and each subsequent number in the sequence is determined by doubling the previous number and then adding 2. How many times does the digit 8 appear in the unit's digit of the first 10 terms of the sequence?

A. 6
B. 7
C. 8
D. 9
E. 4

36.

$$A_n = \frac{1}{n(n+1)}$$

What is the sum of the first 100 elements of A_n?

A. 1/101
B. 51/101
C. 101/100
D. 100/101

37.

G(x) is defined as the product of all even integers k such that $0 < k \leq x$. For example: $G(14) = 2 \times 4 \times 6 \times 8 \times 10 \times 12 \times 14$. If G(y) is divisible by 4^{11}, what is the smallest possible value for y?

A. 22

B. 24

C. 28

D. 32

E. 44

38.

A sequence is defined by $b_n = (b_{n-1})^2 + 1$ and $b_4 = 101$.
All terms in the sequence are positive. What is the value of b_1?

(A) $\sqrt{2}$
(B) 2
(C) $\sqrt{3}$
(D) 3
(E) $\sqrt{10}$

39.

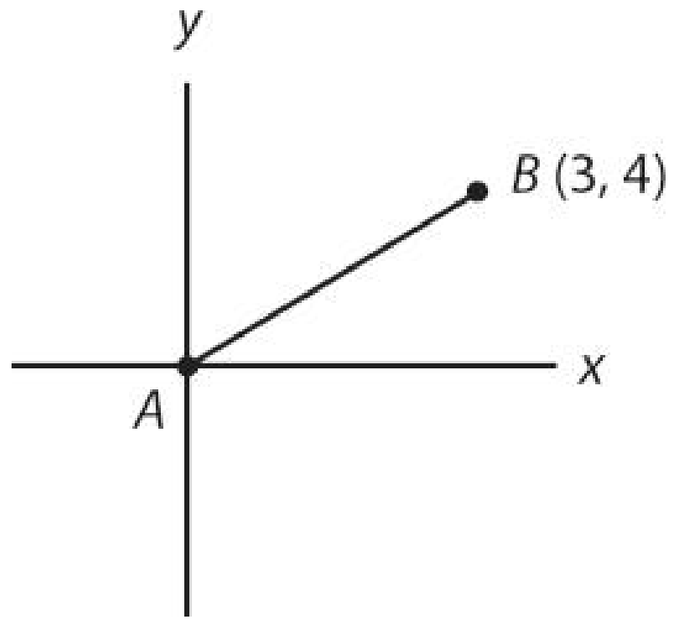
If each term in the sum $a_1 + a_2 + \ldots a_n$ is either 7 or 77 and the sum equals 350, which of the following could be the value of n?

A. 38
B. 39
C. 40
D. 41
E. 42

40.If equation |x/2| + |y/2| = 5 encloses a certain region on the coordinate plane, what is the area of this region?

20 50 100 200 400

41. In the coordinate plane, point C is not displayed. If the length of line segment BC is twice the length of line segment AB, which of the following could not be the coordinates of point C?



A. (−5, −2)
B. (9, 12)
C. (10, 11)
D. (11, 10)
E. (13, 4)

42.

For what value of 'm' will the quadratic equation $x^2 - mx + 4 = 0$ have real and equal roots?

A. 16
B. 8
C. 2
D. −4
E. 7

43.

Which of the following equations has $1 + \sqrt{2}$ as one of its roots?

A. $x^2 + 2x - 1 = 0$

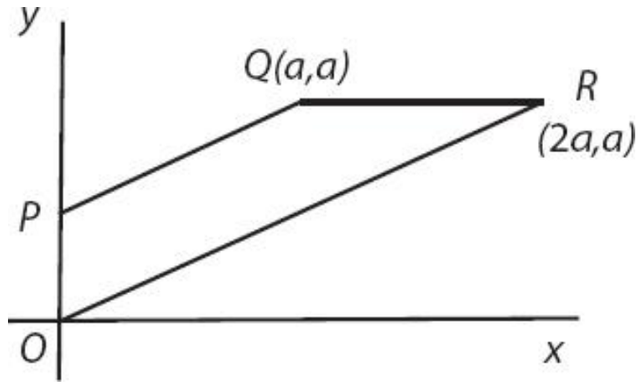
B. $x^2 - 2x + 1 = 0$

C. $x^2 + 2x + 1 = 0$

D. $x^2 - 2x - 1 = 0$

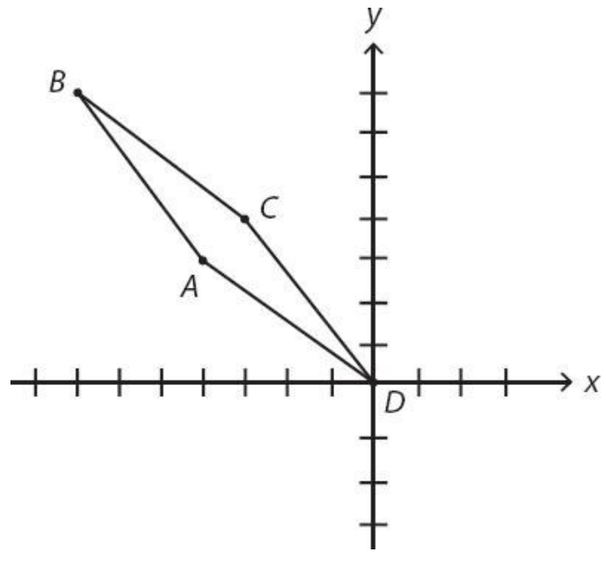
E. $x^2 - x - 1 = 0$

44. Trapezoid OPQR has one vertex at the origin. What is the area of OPQR?



A. $a^2/4$
B. $a^2/2$
C. $3a^2/4$
D. $3a^2/2$
E. $2a^2$

45. Rhombus ABCD lies in the xy–plane, as shown in the figure. The coordinates of point C are (–3, 4) and the coordinates of point B are (–7, 7). What is the area of the ABCD?



A. 1

B. $2\sqrt{7}$

C. 7

D. 8

E. $7\sqrt{2}$

46.

The graph of which of the following equations have two intersections with axis–x?

(A) $y=x^2+2x+3$

(B) $y=2x^2-3x+6$

(C) $y=-3x^2-5x+7$

(D) $y=2x^2-4x+2$

(E) $y=4x^2+2x+1$

47.

If $x^2 - 4x = 5$, then one possible value of $x - 4 =$?

A. -9

B. -5

C. -3

D. -1

E. 5

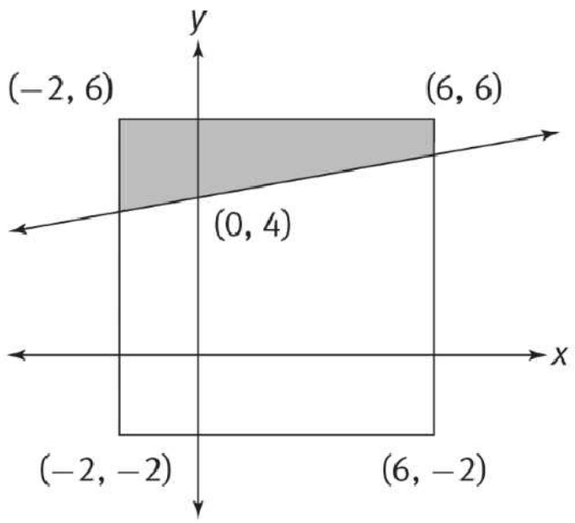
48.

Three of the four vertices of a rectangle ABCD in the xy–coordinate plane are A (– 5, 1), B (– 4, 4), and C (8, 0). What is the fourth vertex D?

(– 4.5, 2.5) (– 4, 5) (6, –2) (7, –3) (10, 1)

49.

A square is drawn in the coordinate plane with its vertices at the points (– 2, –2), (– 2, 6), (6, 6), and (6, –2), and a non–vertical line is drawn that passes through the point (0, 4). The portion of the coordinate plane that lies within the square, but above the line, is then shaded as shown above. If A is the area of the shaded region in square units, which of the following specifies all the possible values of A?



A. $8 \le A \le 16$
B. $8 \le A < 48$
C. $24 \le A < 48$
D. $8 < A \le 32$
E. $24 < A \le 32$

50.

$y = x^2 + bx + 256$ cuts the x axis at (h, 0) and (k, 0). If h and k are integers, what is the least value of b?

A. −32
B. −256
C. −255
D. −257
E. 0

51.

If $x > 0$, how many integer values of (x, y) will satisfy the equation $5x + 4|y| = 55$?

A. 3
B. 6
C. 5
D. 4
E. Infinitely many

Standard directions for all Data Sufficiency Questions:

Mark:

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are not sufficient.

52.

If the operation ^ is one of the four arithmetic operations addition, subtraction, multiplication, and division, is (6 ^ 2) ^ 4 = 6 ^ (2 ^ 4)?
(1) 3 ^ 2 > 3
(2) 3 ^ 1 = 3

53. If the operation * is defined for all integers a and b by a * b = a + b – ab, which of the following statements could be true? **More than one answers may be correct!**

I. a*b = b*a

II. a*0 = a

III. (a*b)*c = a*(b*c)

54.

For any numbers a and b, a * b = a + b – ab. If a * b = 0, which of the following CANNOT be a value of b?

A. 2
B. 1
C. 0
D. –1
E. 3

55. The line represented by the equation $y = 4 - 2x$ is the perpendicular bisector of line segment RP. If R has the coordinates (4, 1), what are the coordinates of point P?

A. (0, –1)

B. (–2, 2)

C. (0, 1)

D. (–4, 1)

E. (2, 7)

56.
The symbol * represents one of the four arithmetic operations: addition, subtraction, multiplication, and division.

Is (5 * 6) * 2 = 5 * (6 * 2)?

(1) 5 * 6 = 6 * 5
(2) 2 * 0 = 2

57. In the addition table given, what is the value of m + n?

A. –19
B. 4
C. 5
D. 6
E. 22

+	*x*	*y*	*z*
4	1	–5	*m*
e	7	*n*	10
f	2	–4	5

58. Is the hundredth digit of decimal d greater than 5?

(1) The tenth digit of 10d is 7

(2) The thousandth digit of d/10 is 7

59.What is the tens digit of the positive integer r?

(1) The tens digit of r/10 is 3.
(2) The hundreds digit of 10r is 6.

60. What is the result when x is rounded to the nearest hundredth?

(1) When x is rounded to the nearest thousandth the result is 0.455
(2) The thousandth digit of x is 5

1. A	16.C	31.B	46.C
2. D	17.C	32.III only	47.B
3. A	18.A	33.E	48.D
4. A	19.A	34.B	49.B
5. D	20.D	35.D	50.D
6. C	21.D	36.D	51.C
7. D	22.C	37.B	52.A
8. D	23.D	38.A	53.All three
9. A	24.B	39.C	54.B
10.E	25.E	40.D	55.A
11.D	26.D	41.C	56.A
12.E	27.D	42.D	57.C
13.B	28.C	43.D	58.D
14.E	29.D	44.C	59.B
15.C	30.C	45.C	60.C