



Quant Session 5





Quant 5 – Live Session

Permutations & Combinations and Probability

You must have solved each of the questions extremely thoroughly before the live class

Also, check the Answerkey, and think hard on each question

Standard directions for all Data Sufficiency Questions:

Mark:

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- D. EACH statement ALONE is sufficient.
- E. Statements (1) and (2) TOGETHER are not sufficient.

2. How many different 5-person teams can be formed from a group of x individuals?
 1. If there had been $x + 2$ individuals in the group, exactly 126 different 5-person teams could have been formed.
 2. If there had been $x + 1$ individuals in the group, exactly 56 different 3-person teams could have been formed.

3. Right triangle PQR is to be constructed in the xy -plane so that the right angle is at P and PR is parallel to the x -axis. The x and y coordinates of P , Q , and R are integers that satisfy the inequalities $-4 \leq x \leq 5$ and $6 \leq y \leq 16$. How many different triangles with these properties could be constructed?

A. 110 B. 1100 C. 9900 D. 10000 E. 12100

4. If x is to be selected at random from T , what is the probability that $\frac{x}{4} - 5 \leq 0$?

(1) T is a set of 8 integers.

(2) T is contained in the set of integers from 1 to 25, inclusive.

5. A magician has five animals in his magic hat: 3 doves and 2 rabbits. If he pulls two animals out of the hat at random, what is the chance that he will have a matched pair?
- A. $\frac{2}{5}$ B. $\frac{3}{5}$ C. $\frac{1}{5}$ D. $\frac{1}{2}$ E. $\frac{7}{5}$

6. If two of the four expressions $x + y$, $x + 5y$, $x - y$, and $5x - y$ are chosen at random, what is the probability that their product will be of the form of $x^2 - (by)^2$, where b is an integer?
- A. $1/2$
 - B. $1/3$
 - C. $1/4$
 - D. $1/5$
 - E. $1/6$

7. If an integer n is to be selected at random from 1 to 100, inclusive, what is probability $n(n+1)$ will be divisible by 4?

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

E. $\frac{3}{4}$

8. A photographer will arrange 6 people of 6 different heights for photograph by placing them in two rows of three so that each person in the first row is standing in front of someone in the second row. The heights of the people within each row must increase from left to right, and each person in the second row must be taller than the person standing in front of him or her. How many such arrangements of the 6 people are possible?

(A) 5

(B) 6

(C) 9

(D) 24

(E) 36

9. A palindrome is a number which reads the same when read forward as it does when read backward. How many 5-digit palindromes are there?
- (A) 720 (B) 800 (C) 890 (D) 900 (E) 950

10. Of the three-digit positive integers whose three digits are all different and nonzero, how many are odd integers greater than 700?
- (A) 84 (B) 91 (C) 100 (D) 105 (E) 243

11. Of the three-digit integers greater than 700, how many have two digits that are equal to each other and the remaining digit different from the other two?
- (A) 90 (B) 82 (C) 80 (D) 45 (E) 36

12. How many integers between 0 and 1570 have a prime tens digit and a prime units digit?
- (A) 295 (B) 252 (C) 236 (D) 96 (E) 76

13. A credit card number has 6 digits (between 1 to 9). The first two digits are 12 in that order, the third digit is bigger than 6, the fourth is divisible by 3 and the fifth digit is 3 times the sixth. How many different credit card numbers exist?
- A. 27 B. 36 C. 72 D. 112 E. 422

14. If a committee of 3 people is to be selected from among 5 married couples so that the committee does not include two people who are married to each other, how many such committees are possible?

- A. 20 B. 40 C. 50 D. 80 E. 120

15. A company plans to assign identification numbers to its employees. Each number is to consist of four different digits from 0 to 9, inclusive, except that the first digit cannot be 0. How many different identification numbers are possible?

(A) 3,024

(B) 4,536

(C) 5,040

(D) 9,000

(E) 10,000

16. A basket contains 5 apples, of which 1 apple is spoiled and the rest are good. If you select 2 apples simultaneously and at random, what is the probability that the 2 apples will include the spoiled apple?

(A) $1/5$

(B) $3/10$

(C) $2/5$

(D) $1/2$

(E) $3/5$

17. Six mobsters have arrived at the theater for the premiere of the film “Goodbuddies.” One of the mobsters, Frankie, is an informer, and he's afraid that another member of his crew, Joey, is on to him. Frankie, wanting to keep Joey in his sights, insists upon standing behind Joey in line at the concession stand. How many ways can the six arrange themselves in line such that Frankie’s requirement is satisfied?

6

24

120

360

720

18. If the probability of rain on any given day in Chicago during the summer is 50%, independent of what happens on any other day, what is the probability of having exactly 3 rainy days from July 4 through July 8, inclusive?

(A) $1/32$

(B) $2/25$

(C) $5/16$

(D) $8/25$

(E) $3/4$

19. A small, experimental plane has three engines, one of which is redundant. That is, as long as two of the engines are working, the plane will stay in the air. Over the course of a typical flight, there is a $\frac{1}{3}$ chance that engine one will fail. There is a 75% probability that engine two will work. The third engine works only half the time. What is the probability that the plane will crash in any given flight?

- (A) $\frac{7}{12}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{7}{24}$ (E) $\frac{17}{24}$

20. At a certain car dealership, the 40 vehicles equipped with air conditioning represent 80% of all cars available for sale. Among all the cars, there are 15 convertibles, 14 of which are equipped with an air-conditioning system. If a customer is willing to purchase either a convertible or a car equipped with air conditioning, what is the probability that a randomly selected vehicle will fit customer specifications?

21/50

31/50

37/50

41/50

47/50

21. Sammy has x flavors of candies with which to make goody bags for Frank's birthday party. Sammy tosses out y flavors, because he doesn't like them. How many different 10-flavor bags can Sammy make from the remaining flavors? (It doesn't matter how many candies are in a bag, only how many flavors).
1. If Sammy had thrown away 2 additional flavors of candy, he could have made exactly 3,003 different 10-flavor bags.
 2. $x = y + 17$

22. The 4 sticks in a complete bag of Pick-Up Sticks are all straight-line segments of negligible width, but each has a different length: 1 inch, 2 inches, 3 inches, and 4 inches, respectively. If Tommy picks a stick at random from each of 3 different complete bags of Pick-Up Sticks, what is the probability that Tommy CANNOT form a triangle from the 3 sticks?
- (A) $11/32$ (B) $13/32$ (C) $15/32$ (D) $17/32$ (E) $19/32$

23. Two integers between 1 and 100, inclusive, each randomly and independently chosen, are either added or multiplied, with an equal chance of either operation. What is the probability that the result is even?

(A) $1/3$

(B) $1/2$

(C) $5/8$

(D) $2/3$

(E) $7/8$

24. Each of three investments has a 20% chance of becoming worthless within a year of purchase, independently of what happens to the other two investments. If Simone invests an equal sum in each of these three investments on January 1, the approximate chance that by the end of the year, she loses no more than $\frac{1}{3}$ of her original investment is
- (A) 90% (B) 80% (C) 70% (D) 60% (E) 40%

25. Every trading day, the price of CF Corp stock either goes up by \$1 or goes down by \$1 with equal likelihood. At the end of 5 trading days, what is the probability that the price of CF Corp stock is up by exactly \$3 from its initial price?

(A) $1/16$

(B) $1/8$

(C) $5/32$

(D) $9/32$

(E) $3/8$

26. The Simplastic language has only 2 unique vowels and 3 unique consonants. Every noun in Simplastic has the structure CVCVC, where C stands for a consonant and V stands for a vowel. How many different nouns are possible in Simplastic?
- (A) 9 (B) 12 (C) 36 (D) 72 (E) 108

27. Each of four different locks has a matching key. The keys are randomly reassigned to the locks. What is the probability that exactly two of the keys fit the locks to which they are reassigned?
- (A) $1/8$ (B) $1/6$ (C) $1/4$ (D) $3/8$ (E) $1/2$

28. A football team has 99 players. Each player has a uniform number from 1 to 99 and no two players share the same number. When football practice ends, all the players run off the field one-by-one in a completely random manner. What is the probability that the first four players off the field will leave in order of increasing uniform numbers (e.g., #2, then #6, then #67, then #72, etc)?

(A) $1/64$

(B) $1/48$

(C) $1/36$

(D) $1/24$

(E) $1/16$

29. A group of four women and three men have tickets for seven adjacent seats in one row of a theatre. If the three men will not sit in three adjacent seats, how many possible different seating arrangements are there for these 7 theatre-goers?

- (A) $7! - 2!3!2!$ (B) $7! - 4!3!$ (C) $7! - 5!3!$ (D) $7 \times 2!3!2!$ (E) $2!3!2!$

30. A random 10-letter code is to be formed using the letters A, B, C, D, E, F, G, H, I and I (only the “I” will be used twice). What is the probability that a code that has the two I’s adjacent to one another will be formed?

(A) $1/10$

(B) $1/8$

(C) $1/5$

(D) $1/4$

(E) $1/2$

31. Mike recently won a contest in which he will have the opportunity to shoot free throws in order to win \$10,000. In order to win the money Mike can either shoot 1 free throw and make it, or shoot 3 free throws and make at least 2 of them. Mike occasionally makes shots and occasionally misses shots. He knows that his probability of making a single free throw is p , and that this probability doesn't change. Would Mike have a better chance of winning if he chose to attempt 3 free throws?

(1) $p < 0.7$

(2) $p > 0.6$

32. A certain bag of gemstones is composed of two-thirds diamonds and one-third rubies. If the probability of randomly selecting two diamonds from the bag, without replacement, is $\frac{5}{12}$, what is the probability of selecting two rubies from the bag, without replacement?
- (A) $\frac{5}{36}$ (B) $\frac{5}{24}$ (C) $\frac{1}{12}$ (D) $\frac{1}{6}$ (E) $\frac{1}{4}$

33. If $p^2 - 13p + 40 = q$, and p is a positive integer between 1 and 10, inclusive, what is the probability that $q < 0$?

- (A) $1/10$ (B) $1/5$ (C) $2/5$ (D) $3/5$ (E) $3/10$

34. There is a 10% chance that it won't snow all winter long. There is a 20% chance that schools will not be closed all winter long. What is the greatest possible probability that it will snow and schools will be closed during the winter?

(A) 55%

(B) 60%

(C) 70%

(D) 72%

(E) 80%

35. If 40 percent of all students at College X have brown hair and 70 percent of all students at College X have blue eyes, what is the difference between the minimum and the maximum probability of picking a student from College X who has neither brown hair nor blue eyes?
- (A) 0.2 (B) 0.3 (C) 0.4 (D) 0.6 (E) 0.7

37. Each factor of 210 is inscribed on its own plastic ball and all of the balls are placed in a jar. If a ball is randomly selected from the jar, what is the probability that the ball is inscribed with a multiple of 42?

$\frac{1}{16}$

$\frac{5}{42}$

$\frac{1}{8}$

$\frac{3}{16}$

$\frac{1}{4}$

38. If a jury of 12 people is to be selected randomly from a pool of 15 potential jurors, and the jury pool consists of $\frac{2}{3}$ men and $\frac{1}{3}$ women, what is the probability that the jury will comprise at least $\frac{2}{3}$ men?

$\frac{24}{91}$

$\frac{45}{91}$

$\frac{2}{3}$

$\frac{67}{91}$

$\frac{84}{91}$

39. A family consisting of one mother, one father, two daughters and a son is taking a road trip in a sedan. The sedan has two front seats and three back seats. If one of the parents must drive and the two daughters refuse to sit next to each other, how many possible seating arrangements are there?
- 28 32 48 60 120

40. In a room filled with 7 people, 4 people have exactly 1 sibling in the room and 3 people have exactly 2 siblings in the room. If two individuals are selected from the room at random, what is the probability that those two individuals are NOT siblings?

$5/21$

$3/7$

$4/7$

$5/7$

$16/21$

41. In how many ways can a four-letter password be chosen, using the letters A , B , C , D , E , and F , such that at least one letter is repeated within the password?

(A) 720

(B) 864

(C) 900

(D) 936

(E) 1,296

42. Gita, Hussain, Inge, Jeong, Karen, and Leila are seated in a row of six chairs. How many seating arrangements are possible if Gita cannot sit next to Inge and Jeong must sit next to Leila?
- (A) 288 (B) 240 (C) 144 (D) 120 (E) 96

43. On Tuesday, Kramer purchases exactly 3 new shirts, 2 new sweaters, and 4 new hats. On the following day and each subsequent day thereafter, Kramer wears one of his new shirts together with one of his new sweaters and one of his new hats. Kramer avoids wearing the exact same combination of shirt, sweater, and hat for as long as possible. On which day is this no longer possible?

Tuesday

Wednesday

Thursday

Friday

Saturday

44. A dice with its faces numbered 1 through 6 is rolled twice, first landing on a and then landing on b . If any roll of the dice yields an equal chance of landing on any of the numbers 1 through 6, what is the probability that $a + b$ is prime?

(A) 0

(B) $1/12$

(C) $5/12$

(D) $7/18$

(E) $4/9$

45. Leila is playing a carnival game in which she is given 4 chances to throw a ball through a hoop. If her chance of success on each throw is $\frac{1}{5}$, what is the chance that she will succeed on at least 3 of the throws?

$\frac{1}{625}$

$\frac{3}{625}$

$\frac{16}{625}$

$\frac{17}{625}$

$\frac{19}{625}$

1. B	24.A
2. D	25.C
3. C	26.E
4. E	27.C
5. A	28.D
6. E	29.C
7. C	30.C
8. A	31.B
9. D	32.C
10.B	33.B
11.C	34.E
12.B	35.B
13.A	36.B
14.D	37.C
15.B	38.D
16.C	39.B
17.D	40.E
18.C	41.D
19.D	42.C
20.D	43.E
21.D	44.C
22.C	45.D
23.C	