

Codebook- Team Far_Behind

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```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
using namespace std;
template<class T> ostream& operator<<(ostream &os, const
vector<T> &V) {
os << "["; for(auto v : V) os << v << " ";
return os << "];"}
template<class L, class R> ostream& operator<<(ostream &os, pair<L,R> P) {
return os << "(" << P.first << "," << P.second <<
"<< ")]";}
#define TRACE
#ifdef TRACE
#define trace(...) _f(#__VA_ARGS__, __VA_ARGS__)
template<typename Arg1>
void _f(const char* name, Arg1&& arg1){
cout << name << " : " << arg1 << std::endl;
}
template<typename Arg1, typename... Args>
```

```

void __f(const char* names, Arg1&& arg1, Args&&...<
args){
    const char* comma = strchr(names + 1, ',');cout.<
    write(names, comma - names) << " : " << arg1<<
    " | ";__f(comma+1, args...);
}
#else
#define trace(...) 1
#endif
#define ll long long
#define ld long double
#define vll vector<ll>
#define pll pair<ll,ll>
#define vpll vector<pll>
#define I insert
#define pb push_back
#define F first
#define S second
#define all(x) x.begin(),x.end()
#define endl "\n"
// const ll MAX=1e6+5;
// int mod=1e9+7;
inline int mul(int a,int b){return (a*1ll*b)%mod;}
inline int add(int a,int b){a+=b;if(a>=mod)a-=mod;<
return a;}
inline int sub(int a,int b){a-=b;if(a<0)a+=mod;<
return a;}
inline int power(int a,int b){int rt=1;while(b>0){<
if(b&1)rt=mul(rt,a);a=mul(a,a);b>>=1;}return rt<
;}
inline int inv(int a){return power(a,mod-2);}
inline void modadd(int &a,int b){a+=b;if(a>=mod)a<
-=mod;}
int main(){
    ios_base::sync_with_stdio(false);cin.tie(0);cout<
    .tie(0);cout<<setprecision(25);
}
// clock
clock_t clk = clock();
clk = clock() - clk;
((ld)clk)/CLOCKS_PER_SEC
// fastio
inline ll read() {
    ll n = 0; char c = getchar_unlocked();
    while (!('0' <= c && c <= '9')) c = <
    getchar_unlocked();
    while ('0' <= c && c <= '9')
        n = n * 10 + c - '0', c = getchar_unlocked<
        ();
    return n;
}
inline void write(ll a){
    register char c; char snum[20]; ll i=0;
    do{
        snum[i++] = a%10+48;

```

```

        a=a/10;
    }
    while(a!=0); i--;
    while(i>=0)
        putchar_unlocked(snum[i--]);
    putchar_unlocked('\n');
}
using getline, use cin.ignore()
// gp_hash_table
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
gp_hash_table<int, int> table; //cc_hash_table <
can also be used
//custom hash function
const int RANDOM = chrono::high_resolution_clock::<
now().time_since_epoch().count();
struct chash {
    int operator()(int x) { return hash<int>{}(x ^ <
RANDOM); }
};
gp_hash_table<int, int, chash> table;
//custom hash function for pair
struct chash {
    int operator()(pair<int,int> x) const { return<
x.first* 31 + x.second; }
};
// random
mt19937 rng(chrono::steady_clock::now().<
time_since_epoch().count());
uniform_int_distribution<int> uid(1,r);
int x=uid(rng);
//mt19937_64 rng(chrono::steady_clock::now().<
time_since_epoch().count());
// - for 64 bit unsigned numbers
vector<int> per(N);
for (int i = 0; i < N; i++)
    per[i] = i;
shuffle(per.begin(), per.end(), rng);
// string splitting
// this splitting is better than custom function(w<
.r.t time)
string line = "Ge";
vector<string> tokens;
stringstream check1(line);
string ele;
// Tokenizing w.r.t. space ' '
while(getline(check1, ele, ' '))
    tokens.push_back(ele);
//Ordered Sets
typedef tree<ll,null_type,less<ll>,rb_tree_tag,
tree_order_statistics_node_update> ordered_set;
ordered_set X;X.insert(1);X.insert(2);
*X.find_by_order(0)-> 1
*X.find_by_order(1)-> 2

```

```
(end(X)==X.find_by_order(2) -> true
//order_of_key(x) -># of elements < x
//For multiset use less_equal operator but
//it does support erase operations for multiset
```

1.2 C++ Sublime Build

```
{
  "cmd": ["bash", "-c", "g++ -std=c++11 -O3 '${file}' -o '${file_path}/${file_base_name}' && \
  gnome-terminal -- bash -c '\"${file_path}/${file_base_name}\" < input.txt > output.txt' "],
  "file_regex": "^(...[^:]*):([0-9]+):?([0-9]+)??:?(\.*)$",
  "working_dir": "${file_path}",
  "selector": "source.c++, source.cpp",
}
```

2 Data Structures

2.1 Fenwick

```
/*All indices are 1 indexed
*Range update and point query: maintain BIT of prefix sum of updates
-add val in [a,b] -> add val at a, -val at b
-value[a]=BITsum(a)+arr[a]
*Range update ,range query: maintain 2 BITs B1,B2
-add val in [a,b] -> B1:add val at a, -val at b+1 and in B2 -> Add val*(a-1) at a, -val*b at b+1
-sum[1,b]=B1sum(1,b)*b-B2sum(1,b)
-sum[a,b]=sum[1,b]-sum[1,a-1]*/
ll fen[MAX_N];
void update(ll p, ll val){
  for(ll i = p; i <= n; i += i & -i)
    fen[i] += val;
}
ll sum(ll p){
  ll ans = 0;
  for(ll i = p; i >= 1; i -= i & -i) ans += fen[i];
  return ans;
}
```

2.2 2D-BIT

```
/*All indices are 1 indexed.Increment value of cell (i,j) by val -> update(x,y,val)
*sum of rectangle [a,b]-[c,d] ->sum of rectangles [1,1]-[c,d],[1,1]-[c,b],[1,1][a,d] and [1,1]-[a,b] and use inclusion exclusion*/
ll bit[MAX][MAX];
void update(ll x, ll y, ll val){
  while(x < MAX){
    ll y1 = y;
    while(y1 < MAX){
      bit[x][y1] += val, y1 += (y1 & -y1);
      x += (x & -x);
    }
  }
```

```
}
ll sum(ll x, ll y){
  ll ans = 0;
  while(x > 0){
    ll y1 = y;
    while(y1 > 0){
      ans += bit[x][y1], y1 -= (y1 & -y1);
      x -= (x & -x);
    }
    return ans;
  }
```

2.3 Segment Tree

```
/*All arrays are 0 indexed. call bld(0,n-1,1)
upd(0,n-1,1,x,y,val) -> increase [x,y] by val
sum(0,n-1,1,x,y) -> sum [x,y]
array of size N -> segment tree of size 4*N*/
ll arr[N], st[N<<2], lz[N<<2];
void ppgt(ll l, ll r, ll id){
  if(l==r) return;
  ll m=l+r>>1;
  lz[id*2] += lz[id]; lz[id<<1|1] += lz[id];
  st[id<<1] += (m-l+1) * lz[id];
  st[id<<1|1] += (r-m) * lz[id]; lz[id] = 0;
}
void bld(ll l, ll r, ll id){
  if(l==r) { st[id] = arr[l]; return; }
  bld(l, l+r>>1, id*2); bld(l+r>>1, r, id*2+1);
  st[id] = st[id<<1] + st[id<<1|1];
}
void upd(ll l, ll r, ll id, ll x, ll y, ll val){
  if(l > y || r < x) return; ppgt(l, r, id);
  if(l >= x && r <= y){
    lz[id] += val; st[id] += (r-l+1) * val; return;
  }
  upd(l, l+r>>1, id<<1, x, y, val); upd((l+r>>1)+1, r, id<<1|1, x, y, val);
  st[id] = st[id<<1] + st[id<<1|1];
}
ll sum(ll l, ll r, ll id, ll x, ll y){
  if(l > y || r < x) return 0; ppgt(l, r, id);
  if(l >= x && r <= y) return st[id];
  return sum(l, l+r>>1, id<<1, x, y) + sum((l+r>>1)+1, r, id<<1|1, x, y);
}
```

2.4 Persistent Segment Tree

```
/*id of first node = 0. call build(0,n-1) first. afterwards call upd(0,n-1,previous id,i,val) to add val in ith number. It returns root of new segment tree after modification
*sum(0,n-1,id of root,l,r) -> sum of values in subarray l to r in tree rooted at id
**size of st,lc,rc >= N*2+(N+Q)*logN*/
const ll N=1e5+10;
ll arr[N], st[20*N], lc[20*N], rc[20*N], ids[N], cnt;
void build(ll l, ll r){
  if(l==r) { st[cnt]=arr[l]; ++cnt; return; }
  ll id = cnt++; lc[id] = cnt;
  build(l, l+r>>1);
  rc[id] = cnt; build((l+r>>1)+1, r);
}
```

```

    st[id] = st[lc[id]] + st[rc[id]];
11 upd(11 l, 11 r, 11 id, 11 x, 11 val){
    if(l == r)
        {st[cnt]=st[id]+val; ++cnt; return cnt-1;}
    11 myid = cnt++; 11 mid = l + r >> 1;
    if(x <= mid)
        rc[myid] = rc[id], lc[myid] = upd(1, mid, lc[id], x, val);
    else
        lc[myid] = lc[id], rc[myid] = upd(mid+1, r, rc[id], x, val);
    st[myid] = st[lc[myid]] + st[rc[myid]];
    return myid;}
11 sum(11 l, 11 r, 11 id, 11 x, 11 y){
    if (l > y || r < x) return 0;
    if (l >= x && r <= y) return st[id];
    return sum(l, l + r >> 1, lc[id], x, y) + sum((l + r >> 1) + 1, r, rc[id], x, y);}
11 gkth(11 l, 11 r, 11 id1, 11 id2, 11 k){
    if(l==r) return l; 11 mid = l+r>>1;
    11 a=st[lc[id2]]-(id1>=0?st[lc[id1]]:0);
    if(a >= k)
        return gkth(1, mid, (id1>=0?lc[id1]:-1), lc[id2], k);
    else
        return gkth(mid+1, r, (id1>=0?rc[id1]:-1), rc[id2], k-a);}
//kth largest num in range
int main(){
    11 n,m; v11 finalid(n); v11 v;
    loop : v.pb({arr[i], i}); sort(all(v));
    loop : finalid[v[i].second]=i;
    memset(arr, 0, sizeof(11)*N);
    arr[finalid[0]]++; build(0, n-1);
    loop: ids[i]=upd(0, n-1, ids[i-1], finalid[i], 1);
    while(m--){
        11 i,j,k; cin>>i>>j>>k; --i; --j;
        ans=gkth(0, n-1, (i==0?-1:ids[i-1]), ids[j], k);
        cout<<v[ans].F<<endl;}
}

```

2.5 DP Optimization

/*Split L size array into G intervals, minimizing the cost ($G \leq L$). The cost func. $C[i, j]$ satisfies: $C[a, b] + C[c, d] \leq C[a, d] + C[c, b]$ for $a \leq c \leq b \leq d$. (Q.E) & intuitively you can think that the c.f increases at a rate which is more than linear at all intervals.

So, if the c.f. satisfies Q.E., the following holds:

$F(g, l)$: min cost of splitting first l into g ivalds.

$F(g, l)$: $\min(F(g-1, k) + C(k+1, l))$ over all valid k.

$P(g, l)$: lowest position k s.t. it minimizes $F(g, l)$

$P(g, 0) \leq P(g, 1) \leq \dots \leq P(g, l)$; DivConq, $O(G \cdot L \cdot \log(L))$

$P(0, l) \leq P(1, l) \leq P(2, l) \dots \leq P(G-1, l) \leq P(G, l)$.

Knuth Opti, complexity $O(L \cdot L)$.

For div&conq, we calculate $P(g, l)$ for each g 1 by 1.

In each g, we calculate for mid-l and do recursively using the obtained upper and lower bounds. For knuth, we use $P(g, l-1) \leq P(g, l) \leq P(g+1, l)$, and fill our table in increasing l and decreasing g. In opt. BST type problems, use $bk[i][j-1] \leq bk[i][j] \leq bk[i+1][j]$

*/

// Code for Divide and Conquer Opti $O(G \cdot L \cdot \log(L))$:

```

11 C[8111]; 11 sums[8111];
11 F[8111][8111]; // optimal value
int P[8111][8111]; // optimal position.
// note first val. in arrays is for no. of groups
11 cost(int i, int j) { // cost function
    return i > j ? 0 : (sums[j] - sums[i-1]) * (j - i + 1);
}
/*fill(g, l1, l2, p1, p2) calcs. P[g][l1] and F[g][l1] for
l1 <= l <= l2, with the knowledge that p1 <= P[g][l1] <= p2
*/
void fill(int g, int l1, int l2, int p1, int p2) {
    if (l1 > l2) return; int lm = (l1 + l2) >> 1;
    11 nv=INF, nv1=-1;
    for (int k = p1; k <= min(lm-1, p2); k++) {
        11 new_cost = F[g-1][k] + cost[k+1][lm];
        if (nv > new_cost) { nv=new_cost; nv1 = k; }
    }
    P[g][lm]=nv1; F[g][lm]=nv;
    fill(g, l1, lm-1, p1, P[g][lm]);
    fill(g, lm+1, l2, P[g][lm], p2);
}
int main() { // example call
    for(i=0; i<=n; i++) F[0][i]=INF;
    for(i=0; i<=k; i++) F[i][0]=0;
    F[0][0]=0;
    for(i=1; i<=k; i++) fill(i, 1, n, 0, n);
}
// Code for Knuth Optimization  $O(L \cdot L)$  :-
11 dp[8002][802];
int a[8002], s[8002][802];
11 sum[8002];
// index strats from 1
11 run(int n, int m) {
    memset(dp, 0xff, sizeof(dp)); dp[0][0] = 0;
    for (int i = 1; i <= n; ++i) {
        sum[i] = sum[i-1] + a[i];
        int maxj = min(i, m), mk; 11 mn = INF;

```

```

for (int k = 0; k < i; ++k) {
    if (dp[k][maxj - 1] >= 0) {
        ll tmp = dp[k][maxj - 1] +
            (sum[i] - sum[k]) * (i - k);
        if (tmp < mn) {
            mn = tmp; mk = k; }
    }
}
dp[i][maxj] = mn; s[i][maxj] = mk;
for (int j = maxj - 1; j >= 1; --j) {
    ll mn = INF; int mk;
    for (ll k = s[i - 1][j]; k <= s[i][j + 1]; ++k) {
        if (dp[k][j - 1] >= 0) {
            ll tmp = dp[k][j - 1] + (sum[i] - sum[k]) * (i - k);
            if (tmp < mn) {mn = tmp; mk = k; } }
    }
    dp[i][j] = mn; s[i][j] = mk;
}
} return dp[n][m];
}
// call -> run(n, min(n,m))

```

3 Flows and Matching

3.1 General Matching

```

/*Given any directed graph, finds maximal matching
Vertices-0-indexed, O(n^3) per call to edmonds*/
vll adj[MAXN]; int p[MAXN], base[MAXN], match[MAXN];
int lca(int n, int u, int v){
    vector<bool> used(n);
    for (;;) {
        u = base[u]; used[u] = true;
        if (match[u] == -1) break; u = p[match[u]];
    }
    for (;;) {
        v = base[v]; if (used[v]) return v;
        v = p[match[v]];
    }
}
void mark_path(vector<bool> &blo, int u, int b, int child){
    for (; base[u] != b; u = p[match[u]]){
        blo[base[u]] = true; blo[base[match[u]]] = true;
        p[u] = child; child = match[u];
    }
}
int find_path(int n, int root) {
    vector<bool> used(n);
    for (int i = 0; i < n; ++i)
        p[i] = -1, base[i] = i;
    used[root] = true;
    queue<int> q; q.push(root);
    while (!q.empty()) {
        int u = q.front(); q.pop();
        for (int j = 0; j < (int)adj[u].size(); j++) {
            int v = adj[u][j];

```

```

            if (base[u] == base[v] || match[u] == v) continue;
            if (v == root || (match[v] != -1 && p[match[v]] != -1)) {
                int curr_base = lca(n, u, v);
                vector<bool> blossom(n);
                mark_path(blossom, u, curr_base, v);
                mark_path(blossom, v, curr_base, u);
                for (int i = 0; i < n; i++) {
                    if (blossom[base[i]]) {
                        base[i] = curr_base;
                        if (!used[i]) used[i] = true, q.push(i);
                    }
                }
            }
            else if (p[v] == -1) {
                p[v] = u;
                if (match[v] == -1) return v;
                v = match[v]; used[v] = true; q.push(v);
            }
        }
    }
    return -1;
}
int edmonds(int n){
    for (int i = 0; i < n; i++) match[i] = -1;
    for (int i = 0; i < n; i++) {
        if (match[i] == -1) {
            int u, pu, ppu;
            for (u = find_path(n, i); u != -1; u = ppu) {
                pu = p[u]; ppu = match[pu];
                match[u] = pu; match[pu] = u;
            }
        }
        int matches = 0;
        for (int i = 0; i < n; i++)
            if (match[i] != -1) matches++;
        return matches/2;
    }
    u--; v--; adj[u].pb(v); adj[v].pb(u);
    cout << edmonds(n) * 2 << endl;
    for (int i = 0; i < n; i++) {
        if (match[i] != -1 && i < match[i]) {
            cout << i + 1 << " " << match[i] + 1 << endl;
        }
    }
}

```

3.2 Global Mincut

```

/*finds min weighted cut in undirected graph in
O(n^3), Adj Matrix, 0-indexed vertices
output-(min cut value, nodes in half of min cut)*/
typedef vector<int> VI;
typedef vector<VI> VVI;
const int INF = 1000000000;
pair<int, VI> GetMinCut(VVI &weights) {

```



```

int N = weights.size();
VI used(N), cut, best_cut;
int best_weight = -1;
for (int phase = N-1; phase >= 0; phase--) {
    VI w = weights[0];
    VI added = used;
    int prev, last = 0;
    for (int i = 0; i < phase; i++) {
        prev = last; last = -1;
        for (int j = 1; j < N; j++)
            if (!added[j] && (last == -1 || w[j] > w[last]))
                last = j;
        if (i == phase-1) {
            for (int j=0; j<N; j++)
                weights[prev][j] += weights[last][j];
            for (int j=0; j<N; j++)
                weights[j][prev] = weights[j][last];
            used[last] = true; cut.push_back(last);
            if (best_weight == -1 || w[last] < best_weight)
                best_weight = w[last];
            best_cut = cut, best_weight = w[last];
        }
        else {
            for (int j = 0; j < N; j++)
                w[j] += weights[last][j];
            added[last] = true;
        }
    }
}
return make_pair(best_weight, best_cut);
}
VVI weights(n, VI(n));
pair<int, VI> res = GetMinCut(weights);

```

3.3 Hopcroft Matching

```

// O(m * \sqrt{n})
struct graph {
    int L, R; // 0-indexed vertices
    vector<vector<int>> adj;
    graph(int L, int R) : L(L), R(R), adj(L+R) {}
    void add_edge(int u, int v) {
        adj[u].pb(v+L); adj[v+L].pb(u);
    }
    int maximum_matching() {
        vector<int> level(L), mate(L+R, -1);
        function<bool(void)> levelize = [&]() { // BFS
            queue<int> Q;
            for (int u = 0; u < L; ++u) {
                level[u] = -1;
                if (mate[u] < 0) level[u] = 0, Q.push(u);
            }
            while (!Q.empty()) {
                int u = Q.front(); Q.pop();

```

```

                for (int w: adj[u]) {
                    int v = mate[w];
                    if (v < 0) return true;
                    if (level[v] < 0)
                        level[v] = level[u] + 1, Q.push(v);
                }
            }
            return false;
        };
        function<bool(int)> augment = [&](int u) { // ←
            DFS
            for (int w: adj[u]) {
                int v = mate[w];
                if (v < 0 || (level[v] > level[u] && augment(v))) {
                    mate[u] = w; mate[w] = u; return true;
                }
            }
            return false;
        };
        int match = 0;
        while (levelize())
            for (int u = 0; u < L; ++u)
                if (mate[u] < 0 && augment(u)) ++match;
        return match;
    }; // L-left size, R-right size
    graph g(L,R); g.add_edge(u,v); g.maximum_matching(
    );

```

3.4 Dinic

```

/*O(min(fm,mn^2)), for any unit capacity network
O(m*sqrt(n)), in practice it is pretty fast for ←
any
bipartite network, **vertices are 1-indexed**
e=(u,v), e.flow represent effective flow from u to ←
v
(i.e f(u->v) - f(v->u))
*use int if possible(ll could be slow in dinic)
To put lower bound on edge capacities form a new
graph G' with source s' and t' for each edge u->v
in G with cap (low, high), replace it with
s'->v with low, v->t' with low
u->v with high - low*/
struct edge {ll x, y, cap, flow;};
struct DinicFlow {
    // *** change inf accordingly *****
    const ll inf = (1e18);
    vector<edge> e; vll cur, d;
    vector<vll> adj; ll n, source, sink;
    DinicFlow() {}
    DinicFlow(ll v) {
        n = v; cur = vll(n+1);
        d = vll(n+1); adj = vector<vll>(n+1);
    }
    void addEdge(ll from, ll to, ll cap) {
        edge e1 = {from, to, cap, 0};

```

```

edge e2 = {to, from, 0, 0};
adj[from].pb(e.size()); e.pb(e1);
adj[to].pb(e.size()); e.pb(e2);
}
ll bfs() {
    queue<ll> q;
    for(ll i = 0; i <= n; ++i) d[i] = -1;
    q.push(source); d[source] = 0;
    while(!q.empty() and d[sink] < 0) {
        ll x = q.front(); q.pop();
        for(ll i = 0; i < (ll)adj[x].size(); ++i){
            ll id = adj[x][i], y = e[id].y;
            if(d[y]<0 and e[id].flow < e[id].cap){
                q.push(y); d[y] = d[x] + 1;
            }
        }
    }
    return d[sink] >= 0;
}
ll dfs(ll x, ll flow) {
    if(!flow) return 0;
    if(x == sink) return flow;
    for(; cur[x] < (ll)adj[x].size(); ++cur[x]) {
        ll id = adj[x][cur[x]], y = e[id].y;
        if(d[y] != d[x] + 1) continue;
        ll pushe=dfs(y,min(flow,e[id].cap-e[id].flow←
        ));
        if(pushed) {
            e[id].flow += pushed; e[id^1].flow -= ←
            pushed;
            return pushed;
        }
    }
    return 0;
}
ll maxFlow(ll src, ll snk) {
    this->source = src; this->sink = snk;
    ll flow = 0;
    while(bfs()) {
        for(ll i = 0; i <= n; ++i) cur[i] = 0;
        while(ll pushed = dfs(source, inf))
            flow += pushed;
    }
    return flow;
}
};

```

3.5 Ford Fulkerson

```

/*O(f*m)*/ ll n; // number of vertices
ll cap[N][N]; // adj matrix for cap
vll adj[N]; // *** vertices are 0-indexed ***
// adj list of the corresponding undirected(*imp*)
ll INF = (1e18);
ll snk,cnt; //cnt for vis, no need to initialize ←
vis

```

```

vector<ll> par, vis;
ll dfs(ll u,ll curf){
    vis[u] = cnt; if(u == snk) return curf;
    if(adj[u].size() == 0) return 0;
    for(ll j=0;j<5;j++){ // random for good aug.
        ll a = rand()%(adj[u].size()); ll v = adj[u][a←
        ];
        if(vis[v]==cnt || cap[u][v] == 0) continue;
        par[v] = u;
        ll f = dfs(v,min(curf, cap[u][v]));
        if(vis[snk] == cnt) return f;
    }
    for(auto v : adj[u]){
        if(vis[v] == cnt || cap[u][v] == 0) continue;
        par[v] = u;
        ll f = dfs(v,min(curf, cap[u][v]));
        if(vis[snk] == cnt) return f;
    }
    return 0;
}
ll maxflow(ll s, ll t) {
    snk = t; ll flow = 0; cnt++;
    par = vll(n,-1); vis = vll(n,0);
    while(ll new_flow = dfs(s,INF)){
        flow += new_flow; cnt++;
        ll cur = t;
        while(cur != s){
            ll prev = par[cur];
            cap[prev][cur] -= new_flow;
            cap[cur][prev] += new_flow;
            cur = prev;
        }
    }
    return flow;
}

```

3.6 MCMF

```

/*Works for -ve costs, doesn't work for -ve cycles
O(min(E^2 *V log V, E logV * flow))
**INF is used in both flow_type and cost_type*/
const ll INF = 1e9; // vertices are 0-indexed
struct graph {
    typedef ll flow_type; // **** flow type ****
    typedef ll cost_type; // **** cost type ****
    struct edge {
        int src, dst;
        flow_type cap, flow;
        cost_type cost;
        size_t rev;};
    vector<edge> edges;
    void add_edge(int s, int t, flow_type cap, ←
        cost_type cost) {
        adj[s].pb({s,t,cap,0,cost,adj[t].size()});
        adj[t].pb({t,s,0,0,-cost,adj[s].size()-1});
    }
}

```

```

int n; vector<vector<edge>> adj;
graph(int n) : n(n), adj(n) {}
pair<flow_type, cost_type> min_cost_max_flow(int s, int t) {
    flow_type flow = 0;
    cost_type cost = 0;
    for (int u = 0; u < n; ++u) // initialize
        for (auto &e: adj[u]) e.flow = 0;
    vector<cost_type> p(n, 0);
    auto rcost = [&](edge e)
    {return e.cost+p[e.src]-p[e.dst];};
    for (int iter = 0; ; ++iter) {
        vector<int> prev(n, -1); prev[s] = 0;
        vector<cost_type> dist(n, INF); dist[s] = 0;
        if (iter == 0) { // use Bellman-Ford to
            // remove negative cost edges
            vector<int> count(n); count[s] = 1;
            queue<int> que;
            for (que.push(s); !que.empty(); ) {
                int u = que.front(); que.pop();
                count[u] = -count[u];
                for (auto &e: adj[u]) {
                    if (e.cap > e.flow && dist[e.dst] > ←
                        dist[e.src] + rcost(e)) {
                        dist[e.dst] = dist[e.src]+rcost(e);
                        prev[e.dst] = e.rev;
                        if (count[e.dst] <= 0) {
                            count[e.dst] = -count[e.dst] + 1;
                            que.push(e.dst);
                        }
                    }
                }
            }
        }
        for (int i=0; i<n; i++) p[i] = dist[i];
        continue; // added last 2 lines
    } else { // use Dijkstra
        typedef pair<cost_type, int> node;
        priority_queue<node, vector<node>, greater<←
            node>> que;
        que.push({0, s});
        while (!que.empty()) {
            node a = que.top(); que.pop();
            if (a.S == t) break;
            if (dist[a.S] > a.F) continue;
            for (auto e: adj[a.S]) {
                if (e.cap > e.flow && dist[e.dst] > a.←
                    F + rcost(e)) {
                    dist[e.dst] = dist[e.src]+rcost(e);
                    prev[e.dst] = e.rev;
                    que.push({dist[e.dst], e.dst});
                }
            }
        }
    }
}

```

```

if (prev[t] == -1) break;
for (int u = 0; u < n; ++u)
    if (dist[u] < dist[t]) p[u] += dist[u] - ←
        dist[t];
function<flow_type(int, flow_type)> augment =←
    [&](int u, flow_type cur) {
        if (u == s) return cur;
        edge &r = adj[u][prev[u]], &e = adj[r.dst←
            ][r.rev];
        flow_type f = augment(e.src, min(e.cap - e←
            .flow, cur));
        e.flow += f; r.flow -= f;
        return f;
    };
flow_type f = augment(t, INF);
flow += f;
cost += f * (p[t] - p[s]);
}
return {flow, cost};
};

```

3.7 MinCost Matching

```

/*O(n^3) solves 1000x1000 problems in around 1s
cost[i][j] = cost for pairing Li with Rj
Lmate[i]=index of R node that L node i pairs with
Rmate[j]=index of L node that R node j pairs with
cost[i][j] may be +ve or -ve. To perform
maximization, simply negate the cost[][] matrix.*/
typedef ll cost_type;
typedef vector<cost_type> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;
cost_type MCM(const VVD &cost, VI &Lmate, VI &←
    Rmate) {
    int n = int(cost.size()); VD u(n), v(n);
    for (int i = 0; i < n; i++) {
        u[i] = cost[i][0];
        for (int j = 1; j < n; j++) u[i] = min(u[i], ←
            cost[i][j]);
    }
    for (int j = 0; j < n; j++) {
        v[j] = cost[0][j] - u[0];
        for (int i = 1; i < n; i++) v[j] = min(v[j], ←
            cost[i][j] - u[i]);
    }
    Lmate = VI(n, -1); Rmate = VI(n, -1);
    int mated = 0;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            if (Rmate[j] != -1) continue;
            if (fabs(cost[i][j]-u[i]-v[j]) < 1e-10){
                //**** change this comparision if double cost ****

```



```

    Lmate[i]=j; Rmate[j]=i; mated++; break;
}
}
}
VD dist(n); VI dad(n); VI seen(n);
while (mated < n) {
    int s = 0;
    while (Lmate[s] != -1) s++;
    fill(dad.begin(), dad.end(), -1);
    fill(seen.begin(), seen.end(), 0);
    for (int k = 0; k < n; k++)
        dist[k] = cost[s][k] - u[s] - v[k];
    int j = 0;
    while (true) {
        j = -1;
        for (int k = 0; k < n; k++) {
            if (seen[k]) continue;
            if (j == -1 || dist[k] < dist[j]) j = k;
        }
        seen[j] = 1;
        if (Rmate[j] == -1) break;
        const int i = Rmate[j];
        for (int k = 0; k < n; k++) {
            if (seen[k]) continue;
            const cost_type new_dist = dist[j] + cost[←
                i][k] - u[i] - v[k];
            if (dist[k] > new_dist) {
                dist[k] = new_dist;
                dad[k] = j;
            }
        }
    }
    for (int k = 0; k < n; k++) {
        if (k == j || !seen[k]) continue;
        const int i = Rmate[k];
        v[k] += dist[k] - dist[j];
        u[i] -= dist[k] - dist[j];
        u[s] += dist[j];
        while (dad[j] >= 0) {
            const int d = dad[j];
            Rmate[j]=Rmate[d]; Lmate[Rmate[j]]=j; j=d;
            Rmate[j] = s; Lmate[s] = j; mated++;
        }
        cost_type value = 0;
        for (int i = 0; i < n; i++)
            value += cost[i][Lmate[i]];
        return value;
    }
}

```

4 Geometry

4.1 Geometry

```

//do not read input in double format
#define PI acos(-1)

```

```

//atan2(y,x) slope of line (0,0)->(x,y) in radian ←
(-PI,PI]
// to convert to degree multiply by 180/PI
ld INF = 1e100;
ld EPS = 1e-9;
inline bool eq(ld a,ld b) {return fabs(a-b)<EPS;}
inline bool lt(ld a,ld b) {return a+EPS<b;}
inline bool gt(ld a,ld b) {return a>b+EPS;}
inline bool le(ld a,ld b) {return lt(a,b)||eq(a,b)←
};
inline bool ge(ld a,ld b) {return gt(a,b)||eq(a,b)←
};
struct pt {
    ld x, y;
    pt() {}
    pt(ld x, ld y) : x(x), y(y) {}
    pt(const pt &p) : x(p.x), y(p.y) {}
    pt operator + (const pt &p)
    const { return pt(x+p.x, y+p.y); }
    pt operator - (const pt &p)
    const { return pt(x-p.x, y-p.y); }
    pt operator * (ld c)
    const { return pt(x*c, y*c ); }
    pt operator / (ld c)
    const { return pt(x/c, y/c ); }
    bool operator < (const pt &p)
    const {return lt(y,p.y)|| (eq(y,p.y)&&lt(x,p.x))←
};
    bool operator > (const pt &p)
    const { return p<pt(x,y); }
    bool operator <= (const pt &p)
    const { return !(pt(x,y)>p); }
    bool operator >= (const pt &p)
    const { return !(pt(x,y)<p); }
    bool operator == (const pt &p)
    const { return (pt(x,y)<=p)&&(pt(x,y)>=p); }
};
ld dot(pt p,pt q) {return p.x*q.x+p.y*q.y;}
ld dist2(pt p, pt q) {return dot(p-q,p-q);}
ld dist(pt p,pt q) {return sqrt(dist2(p,q));}
ld norm2(pt p) {return dot(p,p);}
ld norm(pt p) {return sqrt(norm2(p));}
ld cross(pt p, pt q) { return p.x*q.y-p.y*q.x;}
ostream &operator<<(ostream &os, const pt &p) {
    return os << "(" << p.x << "," << p.y << ")"; }
istream& operator >> (istream &is, pt &p){
    return is >> p.x >> p.y;}
//returns 0 if a,b,c are collinear,1 if a->b->c is←
cw and -1 if ccw
int orient(pt a,pt b,pt c)
{
    pt p=b-a,q=c-b;double cr=cross(p,q);

```

```

if(eq(cr,0))return 0;if(lt(cr,0))return 1;return←
-1;}
// rotate a point CCW or CW around the origin
pt RotateCCW90(pt p) { return pt(-p.y,p.x); }
pt RotateCW90(pt p) { return pt(p.y,-p.x); }
pt RotateCCW(pt p, ld t) { //rotate by t degree ccw
return pt(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*←
cos(t)); }
// project point c onto line (not segment) through←
a and b assuming a != b
pt ProjectPointLine(pt a, pt b, pt c) {
return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);}
// project point c onto line segment through a and←
b (closest point on line segment)
pt ProjectPointSegment(pt a, pt b, pt c) {
ld r = dot(b-a,b-a); if (eq(r,0)) return a; //a ←
and b are same
r = dot(c-a, b-a)/r;if (lt(r,0)) return a; //c on←
left of a
if (gt(r,1)) return b; return a + (b-a)*r;}
// compute dist from c to segment between a and b
ld DistancePointSegment(pt a, pt b, pt c) {
return sqrt(dist2(c, ProjectPointSegment(a, b, c←
)));}
// compute dist from c to line between a and b
ld DistancePointLine(pt a, pt b, pt c) {
return sqrt(dist2(c, ProjectPointLine(a, b, c))←
);}
// determine if lines from a to b and c to d are ←
parallel or collinear
bool LinesParallel(pt a, pt b, pt c, pt d) {
return eq(cross(b-a, c-d),0); }
bool LinesCollinear(pt a, pt b, pt c, pt d) {
return LinesParallel(a, b, c, d) && eq(cross(a-b←
, a-c),0) && eq(cross(c-d, c-a),0);}
// determine if line segment from a to b ←
intersects with line segment from c to d
bool SegmentsIntersect(pt a, pt b, pt c, pt d) {
if (LinesCollinear(a, b, c, d)) {
//a->b and c->d are collinear and have one ←
point common
if(eq(dist2(a,c),0)||eq(dist2(a,d),0)||eq(←
dist2(b,c),0)||eq(dist2(b,d),0))
return true;
if(gt(dot(c-a,c-b),0)&&gt;(dot(d-a,d-b),0)&&gt;(←
dot(c-b,d-b),0)) return false;
return true;}
if(gt(cross(d-a,b-a)*cross(c-a,b-a),0)) return ←
false; //c,d on same side of a,b
if(gt(cross(a-c,d-c)*cross(b-c,d-c),0))
return false; //a,b on same side of c,d
return true;}
// compute intersection of line passing through a ←
and b

```

```

// with line passing through c and d, assuming that←
**unique** intersection exists;
//for segment intersection, check if segments ←
intersect first
pt ComputeLineIntersection(pt a,pt b,pt c,pt d){
b=b-a;d=c-d;c=c-a; //lines must not be collinear
assert(gt(dot(b, b),0)&&gt;(dot(d, d),0));
return a + b*cross(c, d)/cross(b, d);}
//returns true if point a,b,c are collinear and b ←
lies between a and c
bool between(pt a,pt b,pt c){
if(!eq(cross(b-a,c-b),0))return 0; //not ←
collinear
return le(dot(b-a,b-c),0);
}
//compute intersection of line segment a-b and c-d
pt ComputeSegmentIntersection(pt a,pt b,pt c,pt d)←
{
if(!SegmentsIntersect(a,b,c,d))return {INF,INF};←
//don't intersect
//if collinear then infinite intersection points←
, this returns any one
if(LinesCollinear(a,b,c,d)){if(between(a,c,b))←
return c;if(between(c,a,d))return a;return b;}
return ComputeLineIntersection(a,b,c,d);
}
// compute center of circle given three points - *←
a,b,c shouldn't be collinear
pt ComputeCircleCenter(pt a,pt b,pt c){
b=(a+b)/2;c=(a+c)/2;
return ComputeLineIntersection(b,b+RotateCW90(a-←
b),c,c+RotateCW90(a-c));}
//point in polygon using winding number -> returns←
0 if point is outside
//winding number>0 if point is inside and equal to←
0 if outside
//draw a ray to the right and add 1 if side goes ←
from up to down and -1 otherwise
bool PointInPolygon(const vector<pt> &p,pt q){
int n=p.size(),windingNumber=0;
for(int i=0;i<n;++i){
if(eq(dist2(q,p[i]),0)) return 1; //q is a ←
vertex
int j=(i+1)%n;
if(eq(p[i].y,q.y)&&eq(p[j].y,q.y)) { //i,i+1 ←
vertex is vertical
if(le(min(p[i].x,p[j].x),q.x)&&le(q.x,max(p[←
i].x, p[j].x))) return 1;} //q lies on ←
boundary
else {
bool below=lt(p[i].y,q.y);
if(below!=lt(p[j].y,q.y)) {
auto orientation=orient(q,p[j],p[i]);
if(orientation==0) return 1; //q lies on ←
boundary i->j
}
}
}
}

```

```

        if(below==(orientation>0)) windingNumber+=←
        below?1:-1;}}}}
    return windingNumber==0?0:1;
}
// determine if point is on the boundary of a ←
// polygon
bool PointOnPolygon(const vector<pt> &p,pt q) {
    for (int i = 0; i < p.size(); i++)
        if (eq(dist2(ProjectPointSegment(p[i],p[(i+1)%←
        p.size()]),q),q),0)) return true;
    return false;}
// Compute area or centroid of any polygon (←
// coordinates must be listed in cw/ccw
// fashion.The centroid is often known as center of←
// gravity/mass
ld ComputeSignedArea(const vector<pt> &p) {
    ld ans=0;
    for(int i = 0; i < p.size(); i++) {
        int j = (i+1) % p.size();
        ans+=cross(p[i],p[j]);
    } return ans / 2.0;}
ld ComputeArea(const vector<pt> &p) {
    return fabs(ComputeSignedArea(p));
}
// compute intersection of line through points a ←
// and b with
// circle centered at c with radius r > 0
vector<pt> CircleLineIntersection(pt a, pt b, pt c←
    , ld r) {
    vector<pt> ret;
    b = b-a;a = a-c;
    ld A = dot(b, b),B = dot(a, b),C = dot(a, a) - r←
    *r,D = B*B - A*C;
    if (lt(D,0)) return ret; //don't intersect
    ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
    if (gt(D,0)) ret.push_back(c+a+b*(-B-sqrt(D))/A)←
    ;
    return ret;}
// compute intersection of circle centered at a ←
// with radius r
// with circle centered at b with radius R
vector<pt> CircleCircleIntersection(pt a, pt b, ld←
    r, ld R) {
    vector<pt> ret;
    ld d = sqrt(dist2(a, b)),d1=dist2(a,b);
    pt inf(INF,INF);
    if(eq(d1,0)&&eq(r,R)){ret.pb(inf);return ret;}//←
    circles are same return (INF,INF)
    if(gt(d,r+R) || lt(d+min(r, R),max(r, R)) ) ←
    return ret;
    ld x = (d*d-R*R+r*r)/(2*d),y = sqrt(r*r-x*x);
    pt v = (b-a)/d;
    ret.push_back(a+v*x + RotateCCW90(v)*y);
    if (gt(y,0)) ret.push_back(a+v*x - RotateCCW90(v←
    )*y);

```

```

    return ret;}
//compute centroid of simple polygon by dividing ←
//it into disjoint triangles
//and taking weighted mean of their centroids (←
//Jerome)
pt ComputeCentroid(const vector<pt> &p) {
    pt c(0,0),inf(INF,INF);
    ld scale = 6.0 * ComputeSignedArea(p);
    if(p.empty())return inf;//empty vector
    if(eq(scale,0))return inf;//all points on ←
    straight line
    for (int i = 0; i < p.size(); i++){
        int j = (i+1) % p.size();
        c = c + (p[i]+p[j])*cross(p[i],p[j]);}
    return c / scale;}
// tests whether or not a given polygon (in CW or ←
// CCW order) is simple
bool IsSimple(const vector<pt> &p) {
    for (int i = 0; i < p.size(); i++) {
        for (int k = i+1; k < p.size(); k++) {
            int j = (i+1) % p.size();
            int l = (k+1) % p.size();
            if (i == l || j == k) continue;
            if (SegmentsIntersect(p[i], p[j], p[k], p[l]←
            ))
                return false;}}
    return true;}
/*point in convex polygon
****bottom left point must be at index 0 and top ←
is the index of upper right vertex
****if not call make_hull once*/
bool pointinConvexPolygon(vector<pt> poly,pt point←
    , int top) {
    if (point < poly[0] || point > poly[top]) return←
    0;//0 for outside and 1 for on/inside
    auto orientation = orient(point, poly[top], poly←
    [0]);
    if (orientation == 0) {
        if (point == poly[0] || point == poly[top]) ←
        return 1;
        return top == 1 || top + 1 == poly.size() ? 1 ←
        : 1;//checks if point lies on boundary when
        //bottom and top points are adjacent
    } else if (orientation < 0) {
        auto itRight = lower_bound(poly.begin() + 1, ←
        poly.begin() + top, point);
        return orient(itRight[0], point, itRight[-1])←
        <=0;
    } else {
        auto itLeft = upper_bound(poly.rbegin(), poly.←
        rend() - top-1, point);
        return (orient(itLeft == poly.rbegin() ? poly←
        [0] : itLeft[-1], point, itLeft[0]))<=0;
    }
}

```

```

}
/*maximum distance between two points in convexy ←
polygon using rotating calipers
make sure that polygon is convex. if not call ←
make_hull first*/
ld maxDist2(vector<pt> poly) {
    int n = poly.size();
    ld res=0;
    for (int i = 0, j = n < 2 ? 0 : 1; i < j; ++i)
        for (; j = j+1 % n) {
            res = max(res, dist2(poly[i], poly[j]));
            if (gt(cross(poly[j+1 % n] - poly[j], poly[i] ←
+1] - poly[i]), 0)) break;
        }
    return res;
}
//Line polygon intersection: check if given line ←
intersects any side of polygon
//if yes then line intersects. If no, then check ←
if its midpoint is inside polygon
//if midpoint is inside then line is inside else ←
outside
// compute distance between point (x,y,z) and ←
plane ax+by+cz=d
ld DistancePointPlane(ld x,ld y,ld z,ld a,ld b,ld ←
c,ld d)
{ return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);}

```

4.2 Convex Hull

```

pt firstpoint;
//for sorting points in ccw(counter clockwise) ←
direction w.r.t firstpoint (leftmost and ←
bottommost)
bool compare(pt x,pt y){
    ll o=orient(firstpoint,x,y);
    if(o==0)return lt(x.x+x.y,y.x+y.y);
    return o<0;}
/*poi->input points, hull->empty vector
returns hull in ccw order with min points*/
void make_hull(vector<pt>& poi,vector<pt>& hull){
    pair<ld,ld> bl={INF,INF};
    ll n=poi.size();ll ind;
    for(ll i=0;i<n;i++){
        pair<ld,ld> pp={poi[i].y,poi[i].x};
        if(pp<bl){
            ind=i;bl={poi[i].y,poi[i].x};}
    }
    swap(bl.F,bl.S);firstpoint=pt(bl.F,bl.S);
    vector<pt> cons;
    for(ll i=0;i<n;i++){
        if(i==ind)continue;cons.pb(poi[i]);}
    sort(cons.begin(),cons.end(),compare);
    hull.pb(firstpoint);ll m;
    for(auto z:cons){

```

```

if(hull.size()<=1){hull.pb(z);continue;}
pt pr,ppr;bool fl=true;
while((m=hull.size())>=2){
    pr=hull[m-1];ppr=hull[m-2];
    ll ch=orient(ppr,pr,z);
    if(ch==-1){break;}
    else if(ch==1){hull.pop_back();continue;}
    else {
        ld d1,d2;
        d1=dist2(ppr,pr);d2=dist2(ppr,z);
        if(gt(d1,d2)){fl=false;break;}else {hull.←
pop_back();}
    }
    if(fl){hull.push_back(z);}
}
return;
}

```

4.3 Li Chao Tree

```

/*All pair of lines must not intersect at more
than 1 points*/
void add_l(pt nw,int v=1,int l=0,int r=maxn) {
    int m = (l + r) / 2;
    bool lef = f(nw, l) < f(line[v], l);
    bool mid = f(nw, m) < f(line[v], m);
    if(mid) swap(line[v], nw);
    if(r - l == 1) return;
    else if(lef != mid) add_line(nw, 2 * v, l, m);
    else add_line(nw, 2 * v + 1, m, r);}
int get(int x,int v=1,int l=0,int r=maxn) {
    int m=(l+r)/2;
    if(r - l == 1) return f(line[v], x);
    else if(x < m)
        return min(f(line[v],x),get(x,2*v,l,m));
    else
        return min(f(line[v],x),get(x,2*v+1,m,r));}

```

4.4 Convex Hull Trick

```

/*maintains upper convex hull of lines ax+b and ←
gives minimum value at a given x
to add line ax+b: sameoldcht.addline(a,b), to get ←
min value at x: sameoldcht.getbest(x)
to get maximum value at x add -ax-b as lines ←
instead of ax+b and use -sameoldcht.getbest(x)*/
const int N = 1e5 + 5;
int n,a[N],b[N];ll dp[N];
struct line{
    ll a , b;double xleft;bool type;
    line(ll _a , ll _b){a = _a;b = _b;type = 0;}
    bool operator < (const line &other) const{
        if(other.type){return xleft < other.xleft;}
        return a > other.a;}
};

```



```

double meet(line x , line y){
    return 1.0 * (y.b - x.b) / (x.a - y.a);}
struct cht{
    set <line> hull;
    cht() {hull.clear();}
    typedef set < line > :: iterator ite;
    bool hasleft(ite node){
        return node != hull.begin();}
    bool hasright(ite node){
        return node != prev(hull.end());}
    void updateborder(ite node){
        if(hasright(node)){line temp = *next(node);
            hull.erase(temp);
            temp.xleft=meet(*node,temp);
            hull.insert(temp);}
        if(hasleft(node)){line temp = *node;
            temp.xleft = meet(*prev(node) , temp);
            hull.erase(node);hull.insert(temp);}
        else{
            line temp = *node;hull.erase(node);
            temp.xleft = -1e18;hull.insert(temp);}
    }
    bool useless(line left,line middle,line right){
        double x = meet(left,right);
        double y = x * middle.a + middle.b;
        double ly = left.a * x + left.b;
        return y > ly;}
    bool useless(ite node){
        if(hasleft(node) && hasright(node)){return
            useless(*prev(node),*node,*next(node));}
        return 0;}
    void addline(ll a , ll b){
        line temp = line(a , b);
        auto it = hull.lower_bound(temp);
        if(it != hull.end() && it -> a == a){
            if(it -> b > b){hull.erase(it);}
            else return;}
        hull.insert(temp);it = hull.find(temp);
        if(useless(it)){hull.erase(it);return;}
        while(hasleft(it) && useless(prev(it))){
            hull.erase(prev(it));}
        while(hasright(it) && useless(next(it))){
            hull.erase(next(it));}
        updateborder(it);}
    ll getbest(ll x){
        if(hull.empty())return 1e18;
        line query(0 , 0);
        query.xleft = x;query.type = 1;
        auto it = hull.lower_bound(query);
        it = prev(it);
        return it -> a * x + it -> b;}
};
cht sameoldcht;
int main(){

```

```

    sameoldcht.addline(b[1] , 0);
    dp[i] = sameoldcht.getbest(a[i]);
    sameoldcht.addline(b[i] , dp[i]);}

```

5 Trees

5.1 BlockCut Tree

```

// Take care it is 0 indexed --
struct BiconnectedComponents {
    struct Edge {
        int from, to;
    };
    struct To {
        int to; int edge;
    };
    vector<Edge> edges; vector<vector<To> > g;
    vector<int> low, ord, depth;
    vector<bool> isArtic;vll edgeColor;
    vector<int> edgeStack;
    int colors; int dfsCounter;
    void init(int n) {
        edges.clear();
        g.assign(n, vector<To>());
    }
    void addEdge(int u, int v) {
        if(u > v) swap(u, v); Edge e = { u, v };
        int ei = edges.size(); edges.push_back(e);
        To tu = { v, ei }, tv = { u, ei };
        g[u].push_back(tu); g[v].push_back(tv);
    }
    void run() {
        int n = g.size(), m = edges.size();
        low.assign(n, -2); ord.assign(n, -1);
        depth.assign(n, -2);isArtic.assign(n,false);
        edgeColor.assign(m, -1); edgeStack.clear();
        colors = 0;
        for(int i = 0; i < n; ++ i) if(ord[i] == -1) {
            dfsCounter = 0;
            dfs(i);
        }
    }
private:
    void dfs(int i) {
        low[i] = ord[i] = dfsCounter ++;
        for(int j=0;j<(int)g[i].size();++j) {
            int to = g[i][j].to, ei = g[i][j].edge;
            if(ord[to] == -1) {
                depth[to] = depth[i] + 1;
                edgeStack.push_back(ei);
                dfs(to);
                low[i] = min(low[i], low[to]);
            }
            if(low[to] >= ord[i]) {
                if(ord[i] != 0 || j >= 1)
                    isArtic[i] = true;
                while(!edgeStack.empty()) {

```



```

        int fi=edgeStack.back();
        edgeStack.pop_back();
        edgeColor[fi] = colors;
        if(fi == ei) break;
    } ++colors;
}
} else if(depth[to] < depth[i] - 1) {
    low[i] = min(low[i], ord[to]);
    edgeStack.push_back(ei);
}
}
};

```

5.2 Bridge Tree

```

vll tree[N],g[N]; //edge list rep. of graph
ll U[M],V[M],vis[N],arr[N],T,dsu[N];
bool isbridge[M]; // if i'th edge is a bridge edge
ll adj(ll u,ll e) {
    return U[e]^V[e]^u;
}
ll f(ll x) {
    return dsu[x]=(dsu[x]==x?f(dsu[x]));
}
void merge(ll a,ll b) {
    dsu[f(a)]=f(b);
}
ll dfs0(ll u,ll edge) { //mark bridges
    vis[u]=1;
    arr[u]=T++;
    ll dbe = arr[u];
    for(auto e : g[u]) {
        ll w = adj(u,e);
        if(!vis[w]) dbe = min(dbe,dfs0(w,e));
        else if(e!=edge) dbe = min(dbe,arr[w]);
    }
    if(dbe==arr[u] && edge!=-1) isbridge[edge]=true;
    else if(edge!=-1) merge(U[edge],V[edge]);
    return dbe;
}
void buildBridgeTree(ll n,ll m) {
    for(ll i=1; i<=n; i++) dsu[i]=i;
    for(ll i=1; i<=n; i++) if(!vis[i]) dfs0(i,-1);
    for(ll i=1; i<=m; i++)
        if(f(U[i])!=f(V[i])) {
            tree[f(U[i])].pb(f(V[i]));
            tree[f(V[i])].pb(f(U[i]));
        }
}
ll n,m;
for(i=1;i<=m;i++)
    cin>>U[i]>>V[i]; g[U[i]].pb(i); g[V[i]].pb(i);
buildBridgeTree(n,m);

```

5.3 Dominator Tree

```

/*g:adjacency matrix (directed graph).
tree rooted at node 1. call dominator().
tree: undirected graph of dominators rooted
at node 1 (use visit times while doing dfs)*/
const int N = int(2e5)+10;
vi g[N],tree[N],rg[N],bucket[N];
int sdом[N],par[N],dom[N],dsu[N],label[N];
int arr[N],rev[N],T;
int Find(int u,int x=0){
    if(u==dsu[u])return x?-1:u;
    int v = Find(dsu[u],x+1);
    if(v<0)return u;
    if(sdom[label[dsu[u]]] < sdom[label[u]])
        label[u] = label[dsu[u]];
    dsu[u] = v;
    return x?v:label[u];
}
void Union(int u,int v) {dsu[v]=u;}
void dfs0(int u){
    T++;arr[u]=T;rev[T]=u;
    label[T]=T;sdom[T]=T;dsu[T]=T;
    for(int i=0;i<g[u].size();i++){
        int w = g[u][i];
        if(!arr[w])dfs0(w),par[arr[w]]=arr[u];
        rg[arr[w]].pb(arr[u]);
    }
}
void dominator(){
    dfs0(1);int n=T;
    for(int i=n;i>=1;i--){
        for(int j=0;j<rg[i].size();j++){
            sdом[i] = min(sdom[i],sdom[Find(rg[i][j])]);
            if(i>1)bucket[sdom[i]].pb(i);
            for(int j=0;j<bucket[i].size();j++){
                int w = bucket[i][j];
                int v = Find(w);
                if(sdom[v]==sdom[w])dom[w]=sdom[w];
                else dom[w] = v;
            }
            if(i>1)Union(par[i],i);
        }
        for(int i=2;i<=n;i++){
            if(dom[i]!=sdom[i]) dom[i]=dom[dom[i]];
            tree[rev[i]].pb(rev[dom[i]]);
            tree[rev[dom[i]]].pb(rev[i]);
        }
    }
}

```

5.4 Bridges Online

```

vector<int> par(MAX), dsu_2ecc(MAX), dsu_cc(MAX), ←
    dsu_cc_size(MAX);
int bridges,lca_iteration;
vector<int> last_visit(MAX);
void init(int n) {
    lca_iteration = 0;
    for (int i=0; i<n; ++i) {

```

```

    dsu_2ecc[i] = i; dsu_cc[i] = i;
    dsu_cc_size[i] = 1; par[i] = -1;
    last_visit[i]=0;
} bridges = 0;
}
int find_2ecc(int v) { // 2-edge connected comp.
    if (v == -1) return -1;
    return dsu_2ecc[v] == v ? v : dsu_2ecc[v] = ←
        find_2ecc(dsu_2ecc[v]);
}
int find_cc(int v) { // connected comp.
    v = find_2ecc(v);
    return dsu_cc[v] == v ? v : dsu_cc[v] = find_cc(←
        dsu_cc[v]);
}
void make_root(int v) {
    v = find_2ecc(v);
    int root = v; int child = -1;
    while (v != -1) {
        int p = find_2ecc(par[v]);
        par[v] = child; dsu_cc[v] = root;
        child = v; v = p;
    }
    dsu_cc_size[root] = dsu_cc_size[child];
}
vector<int> path_a, path_b;
void merge_path (int a, int b) {
    ++lca_iteration;
    int lca = -1;
    while (lca == -1) {
        if (a != -1) {
            a = find_2ecc(a); path_a.push_back(a);
            if (last_visit[a] == lca_iteration) lca = a;
            last_visit[a] = lca_iteration; a=par[a];
        }
        if (b != -1) {
            b = find_2ecc(b); path_b.push_back(b);
            if (last_visit[b] == lca_iteration) lca = b;
            last_visit[b] = lca_iteration; b = par[b];
        }
    }
    for (int v : path_a) {
        dsu_2ecc[v] = lca; if (v == lca) break;
        --bridges;
    }
    for (int v : path_b) {
        dsu_2ecc[v] = lca; if (v == lca) break;
        --bridges;
    }
    path_a.clear(); path_b.clear();
}
void add_edge(int a, int b) {
    a = find_2ecc(a); b = find_2ecc(b);
    if (a == b) return;
    int ca = find_cc(a); int cb = find_cc(b);
    if (ca != cb) { ++bridges;
        if (dsu_cc_size[ca] > dsu_cc_size[cb]) {
            swap(a, b); swap(ca, cb);

```

```

        make_root(a); par[a] = dsu_cc[a] = b;
        dsu_cc_size[cb] += dsu_cc_size[a];
    } else { merge_path(a, b); }
}

```

5.5 HLD

```

/*v : adj matrix of tree.clear v[i],hdc[i]=0,i=-1 ←
before every run,clear ord and curc=0*/
vll v[MAX],ord;
ll par[MAX],noc[MAX],hdc[MAX],curc,posinch[MAX],←
len[MAX],ti=-1,sta[MAX],en[MAX],subs[MAX],level[←
MAX];
ll st[4*MAX],lazy[4*MAX],n;
void dfs(ll x){
    subs[x]=1;
    for(auto z:v[x]){
        if(z!=par[x]){par[z]=x;level[z]=level[x]+1;
            dfs(z);subs[x]+=subs[z];
        }}
void makehld(ll x){
    if(hdc[curc]==0){hdc[curc]=x;len[curc]=0;}
    noc[x]=curc;posinch[x]=++len[curc];
    ll a,b,c;a=b=0;ord.pb(x);sta[x]=++ti;
    for(auto z:v[x]){ if(z==par[x]) continue;
        if(subs[z]>b){b=subs[z];a=z;}
    }
    if(a!=0)makehld(a);
    for(auto z:v[x]){if(z==par[x]||z==a) continue;←
        curc++;makehld(z);}
    en[x]=ti;}
inline void upd(ll x,ll y){//update path a->b
    ll a,b,c,d;
    while(x!=y){a=hdc[noc[x]],b=hdc[noc[y]];
        if(a==b){
            if(level[x]>level[y])swap(x,y);c=sta[x],d=←
                sta[y];
            //lca=a;
            update(1,0,n-1,c+1,d);return;}
        if(level[a]>level[b])swap(a,b),swap(x,y);
        //update on seg tree
        update(1,0,n-1,sta[b],sta[y]);y=par[b];}}
int main(){
    loop: v[i].clear(),hdc[i]=0,ti=-1;
    ord.clear(),curc=0;
    level[1]=0;par[1]=0;curc=1;dfs(1);makehld(1);
    while(q--){cin>>a>>b;upd(a,b);ll ans=sumq(1,0,←
        n-1,0,n-1);}
}

```

5.6 LCA

```

int lca(int a,int b){
    if(level[a]>level[b])swap(a,b);
    int d=level[b]-level[a];
    for(int i=0;i<LOGN;i++){if(d&(1<<i))

```

```

        b=DP[i][b];
    if(a==b) return a;
    for(int i=LOGN-1;i>=0;i--)
        if(DP[i][a]!=DP[i][b])
            a=DP[i][a], b=DP[i][b];
    return DP[0][a];}

```

5.7 Centroid Decomposition

```

/*nx: max nodes, par: parents of nodes in centroid tree,
  timstamp: timestamps of nodes when they became centroids,
  ssize, vis: subtree size and visit times in dfs,
  tim: timestamp iterator
dist[nx]: dist[i][0][j]=no. of nodes at dist k in subtree of i
in centroid tree
dist[i][j][k]=no. of nodes at distance k in jth child of i
in centroid tree *** (use adj while doing dfs instead of adj1) ***
preprocess: stores all values in dist array*/
const int nx=1e5;
vector<int> adj[nx], adj1[nx];
int par[nx], timstamp[nx], ssize[nx], vis[nx];
int tim=1;
vector<int> cntorder; // centroids in order
vector<vector<int>> > dist[nx];
int dfs(int root){
    vis[root]=tim;
    int t=0;
    for(auto i: adj[root]){
        if(!timstamp[i]&&vis[i]<tim)t+=dfs(i);}
    ssize[root]=t+1; return t+1;}
int dfs1(int root, int n){
    vis[root]=tim; pair<int, int> mxc={0, -1};
    bool poss=true;
    for(auto i: adj[root]){
        if(!timstamp[i]&&vis[i]<tim)
            poss&=(ssize[i]<=n/2), mxc=max(mxc, {ssize[i], i});}
    if(poss&&(n-ssize[root])<=n/2) return root;
    return dfs1(mxc.second, n);}
int findc(int root){
    dfs(root);
    int n=ssize[root]; tim++;
    return dfs1(root, n);}
void cntordercom(int root, int p){
    int cnt=findc(root);
    cntorder.pb(cnt);
    timstamp[cnt]=tim++; par[cnt]=p;
    if(p>=0) adj1[p].pb(cnt);
    for(auto i: adj[cnt])
        if(!timstamp[i])
            cntordercom(i, cnt);}
void dfs2(int root, int nod, int j, int dst){
    if(dist[root][j].size()==dst) dist[root][j].pb(0);
    vis[nod]=tim; dist[root][j][dst]++;

```

```

    for(auto i: adj[nod]){
        if((timstamp[i]<=timstamp[root])|| (vis[i]==vis[nod])) continue;
        vis[i]=tim; dfs2(root, i, j, dst+1);}
}
void preprocess(){
    for(int i=0; i<cntorder.size(); i++){
        int root=cntorder[i];
        vector<int> temp;
        dist[root].pb(temp); temp.pb(0); ++tim;
        dfs2(root, root, 0, 0);
        int cnt=0;
        for(int j=0; j<adj[root].size(); j++){
            int nod=adj[root][j];
            if(timstamp[nod]<timstamp[root]) continue;
            dist[root].pb(temp); ++tim;
            dfs2(root, nod, ++cnt, 1);}
    }
}

```

6 Maths

6.1 Chinese Remainder Theorem

```

/*x=rem[i]%mods[i] for any mods
input: rem->remainder, mods->moduli
output: (x%lcm of mods, lcm), -1 if infeasible*/
ll LCM(ll a, ll b) { return a / __gcd(a, b) * b; }
ll normalize(ll x, ll mod)
{x %= mod; if (x < 0) x += mod; return x; }
struct GCD_type { ll x, y, d; };
GCD_type ex_GCD(ll a, ll b){
    if (b == 0) return {1, 0, a};
    GCD_type pom = ex_GCD(b, a % b);
    return {pom.y, pom.x - a / b * pom.y, pom.d};
}
p11 CRT(vll &rem, vll &mods){
    ll n=rem.size(), ans=rem[0], lcm=mods[0];
    for(ll i=1; i<n; i++){
        auto pom=ex_GCD(lcm, mods[i]);
        ll x1=pom.x, d=pom.d;
        if((rem[i]-ans)%d!=0) return {-1, 0};
        ans=normalize(ans+x1*(rem[i]-ans)/d%(mods[i]/d),
            i]/d)*lcm, lcm*mods[i]/d);
        lcm=LCM(lcm, mods[i]); // you can save time by replacing above lcm * n[i] / d by lcm = lcm * n[i] / d
    }
    return {ans, lcm};
}

```

6.2 Discrete Log

```

/*Discrete Log , Baby-Step Giant-Step , e-maxx
The idea is to make two functions, f1(p), f2(q)
and find p, q s.t. f1(p) = f2(q) by storing all

```

```

possible values of f1, and checking for q. In
this case  $a^x \equiv b \pmod m$  is solved by subst.
x by p.n-q, where n is choosen optimally.*/
/*returns a soln. for  $a^x \equiv b \pmod m$  for
given a,b,m; -1 if no. soln; 0(sqrt(m).log(m))
use unordered_map to remove log factor.
IMP : works only if a,m are co-prime.
But can be modified.*/
int solve (int a, int b, int m) {
    int n = (int) sqrt (m + .0) + 1;
    int an = 1;
    for (int i=0; i<n; ++i)
        an = (an * a) % m;
    map<int,int> vals;
    for (int i=1, cur=an; i<=n; ++i) {
        if (!vals.count(cur))
            vals[cur] = i;
        cur = (cur * an) % m;
    }
    for (int i=0, cur=b; i<=n; ++i) {
        if (vals.count(cur)) {
            int ans = vals[cur] * n - i;
            if (ans < m) return ans;
        }
        cur = (cur * a) % m;
    }
    return -1;
}

```

6.3 NTT

```

/*a*b%mod if a%mod*b%mod results in overflow:
ll mulmod(ll a, ll b, ll mod) {ll res = 0;
    while (a!=0){if(a&1)(res+=b)%=mod;a>>=1;(b<=1)%=mod;}
    return res;}
P=A*B A[0]=coeff of x^0
x = a1 mod p1, x = a2 mod p2 => x=((a1*(m2^-1)%m1)+
    *m2+(a2*(m1^-1)%m2)*m1)%m1m2
***max_base=x (s.t. if mod=c*(2^k)+1 then x<=k and
    2^x >= nearest power of 2 of 2*n)
root=primitive_root^((mod-1)/(2^max_base))
For P=A*A use square function
635437057,11|639631361,6|985661441&998244353,3*/
#define chinese(a1,m1,inv2m1,a2,m2,inv1m2) ((a1 <=
    *1ll* inv2m1 % m1 * 1ll*m2 + a2 *1ll* inv1m2 % m2 * 1ll*m1) % (m1 *1ll* m2))
int mod;//reset mod everytime
int base = 1;
vll roots = {0, 1}, rev = {0, 1};
int max_base=18, root=202376916;
void ensure_base(int nbase) {
    if (nbase <= base) return;
    rev.resize(1 << nbase);
    for (int i = 0; i < (1 << nbase); i++) {
        rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (base - 1));
    }
}

```

```

roots.resize(1 << nbase);
while (base < nbase) {
    int z = power(root, 1 << (max_base - 1 - base));
    for (int i = 1 << (base - 1); i < (1 << base); i++) {
        roots[i << 1] = roots[i];
        roots[(i << 1) + 1] = mul(roots[i], z);
    }
    base++;
}
void fft(vll &a) {
    int n = (int) a.size();
    int zeros = __builtin_ctz(n);
    ensure_base(zeros);
    int shift = base - zeros;
    for (int i = 0; i < n; i++) {
        if (i < (rev[i] >> shift)) {
            swap(a[i], a[rev[i] >> shift]);
        }
    }
    for (int k = 1; k < n; k <= 1) {
        for (int i = 0; i < n; i += 2 * k) {
            for (int j = 0; j < k; j++) {
                int x = a[i + j];
                int y = mul(a[i + j + k], roots[j+k]);
                a[i + j] = x + y - mod;
                if (a[i + j] < 0) a[i + j] += mod;
                a[i + j + k] = x - y + mod;
                if (a[i+j+k] >= mod) a[i + j + k] -= mod;
            }
        }
    }
}
vll multiply(vll a, vll b, int eq = 0) {
    int need = (int) (a.size() + b.size() - 1);
    int nbase = 0;
    while ((1 << nbase) < need) nbase++;
    ensure_base(nbase);
    int sz = 1 << nbase;
    a.resize(sz); b.resize(sz); fft(a);
    if (eq) b = a; else fft(b);
    int inv_sz = inv(sz);
    for (int i = 0; i < sz; i++)
        a[i] = mul(mul(a[i], b[i]), inv_sz);
    reverse(a.begin() + 1, a.end());
    fft(a); a.resize(need); return a;
}
vll square(vll a) {return multiply(a, a, 1);}

```

6.4 Online FFT

```

//f[i]=sum over j from 0 to i-1 f[j]*g[i-1-j]
//handle f[0] and g[0] separately
const int nx=131072; int f[nx], g[nx];
void onlinefft(int a, int b, int c, int d){
    vector<int> v1,v2;
}

```

```

v1.pb(f+a,f+b+1);v2.pb(g+c,g+d+1); vector<int> <-
    res=multiply(v1,v2);
for(int i=0;i<res.size();i++)
    if(a+c+i+1<nx) f[a+c+i+1]=add(f[a+c+i+1],res[i<-
    ]));}
void precal(){
    g[0]=1;
    for(int i=1;i<nx;i++)
        g[i]=power(i,i-1);
    f[1]=1;
    for(int i=1;i<=100000;i++){
        f[i+1]=add(f[i+1],g[i]);f[i+1]=add(f[i+1],f[i<-
        ]));
        f[i+2]=add(f[i+2],mul(f[i],g[1]));f[i+3]=add(f<-
        [i+3],mul(f[i],g[2]));
        for(int j=2;i%j==0&&j<nx;j=j*2)
            onlinefft(i-j,i-1,j+1,2*j);}
}

```

6.5 Langrange Interpolation

```

/* Input :
Degree of polynomial: k
Polynomial values at x=0,1,2,3,...,k
Output :
Polynomial value at x
Complexity: O(degree of polynomial)
Works only if the points are equally spaced
*/
ll lagrange(vll& v , int k, ll x,int mod){
    if(x <= k) return v[x];
    ll inn = 1; ll den = 1;
    for(int i = 1;i<=k;i++){
        inn = (inn*(x - i))%mod;
        den = (den*(mod - i))%mod;
    }
    inn = (inn*inv(den % mod))%mod;
    ll ret = 0;
    for(int i = 0;i<=k;i++){
        ret = (ret + v[i]*inn)%mod;
        ll md1 = mod - ((x-i)*(k-i))%mod;
        ll md2 = ((i+1)*(x-i-1))%mod;
        if(i!=k)
            inn = (((inn*md1)%mod)*inv(md2 % mod))%mod<-
    } return ret;
}

```

6.6 Matrix Struct

```

struct matrix{
    ld B[N][N], n;
    matrix(){n = N; memset(B,0,sizeof B);}
    matrix(int _n)
    {n = _n; memset(B, 0, sizeof B);}
    void iden(){
        for(int i = 0; i < n; i++) B[i][i] = 1;}
}

```

```

void operator += (matrix M){
    for(int i = 0; i < n; i++)
        for(int j = 0; j < n; j++)
            B[i][j]=add(B[i][j],M.B[i][j]);}
void operator -= (matrix M){
void operator *= (ld b){
matrix operator - (matrix M){
matrix operator + (matrix M){
    matrix ret = (*this); ret += M; return ret;}
matrix operator * (matrix M){
    matrix ret = matrix(n); memset(ret.B, 0, <-
        sizeof ret.B);
    for(int i = 0; i < n; i++)
        for(int j = 0; j < n;j++)
            for(int k = 0; k < n; k++)
                ret.B[i][j] = add(ret.B[i][j], mul(B[i][<-
                k], M.B[k][j]));
    return ret;}
matrix operator *= (matrix M){*this=((*this)*M)<-
    };}
matrix operator * (int b){
    matrix ret =(*this);ret *= b; return ret;}
vector<double> multiply(const vector<double> & v<-
    ) const{
    vector<double> ret(n);
    for(int i = 0; i < n; i++)
        for(int j = 0; j < n; j++)
            ret[i] += B[i][j] * v[j];
    return ret;
}
};

```

6.7 nCr(Non Prime Modulo)

```

// calculates nCr, for small
// non-prime modulo, and (very) big n,r.
ll phimod;
vll pr,prn;vll fact;
ll power(ll a,ll x,ll mod){
    ll ans=1;
    while(x){
        if((1LL)&(x))ans=(ans*a)%mod;
        a=(a*a)%mod;x>>=1LL;
    }
    return ans;
}
// prime factorization of x.
// pr-> prime ; prn -> it's exponent
void getprime(ll x){
    pr.clear();prn.clear();
    ll i,j,k;
    for(i=2;(i*i)<=x;i++){
        k=0;while((x%i)==0){k++;x/=i;}
        if(k>0){pr.pb(i);prn.pb(k);}
    }
    if(x!=1){pr.pb(x);prn.pb(1);}
}

```



```

    return;
}
// factorials are calculated ignoring
// multiples of p.
void primeproc(ll p,ll pe){    // p , p^e
    ll i,d;
    fact.clear();fact.pb(1);d=1;
    for(i=1;i<pe;i++){
        if(i%p){fact.pb((fact[i-1]*i)%pe);}
        else {fact.pb(fact[i-1]);}
    }
    return;
}
// again note this has ignored multiples of p
ll Bigfact(ll n,ll mod){
    ll a,b,c,d,i,j,k;
    a=n/mod;a%=phimod;a=power(fact[mod-1],a,mod);
    b=n%mod;a=(a*fact[b])%mod;
    return a;
}
// Chinese Remainder Thm.
vll crtval,crtmod;
ll crt(vll &val,vll &mod){
    ll a,b,c,d,i,j,k;b=1;
    for(ll z:mod)b*=z;
    ll ans=0;
    for(i=0;i<mod.size();i++){
        a=mod[i];c=b/a;
        d=power(c,(((a/pr[i])*(pr[i]-1))-1),a);
        c=(c*d)%b;c=(c*val[i])%b;ans=(ans+c)%b;
    }
    return ans;
}
// calculate for prime powers and
// take crt. For each prime power,
// first ignore multiples of p,
// and then do recursively, calculating
// the powers of p separately.
ll Bigncr(ll n,ll r,ll mod){
    ll a,b,c,d,i,j,k;ll p,pe;
    getprime(mod);ll Fnum=1;ll Fden;
    crtval.clear();crtmod.clear();
    for(i=0;i<pr.size();i++){
        Fnum=1;Fden=1;
        p=pr[i];pe=power(p,prn[i],1e17);
        primeproc(p,pe);
        a=1;d=0;
        phimod=(pe*(p-1LL))/p;
        ll n1=n,r1=r,nr=n-r;
        while(n1){
            Fnum=(Fnum*(Bigfact(n1,pe)))%pe;
            Fden=(Fden*(Bigfact(r1,pe)))%pe;
            Fden=(Fden*(Bigfact(nr,pe)))%pe;
            d+=n1-(r1+nr);
            n1/=p;r1/=p;nr/=p;
        }
    }
}

```

```

Fnum=(Fnum*(power(Fden,(phimod-1LL),pe)))%pe;
if(d>=prn[i])Fnum=0;
else Fnum=(Fnum*(power(p,d,pe)))%pe;
crtmod.pb(pe);crtval.pb(Fnum);
}
// you can just iterate instead of crt
// for(i=0;i<mod;i++){
//     bool cg=true;
//     for(j=0;j<crtmod.size();j++){
//         if(i%crtmod[j]!=crtval[j])cg=false;
//     }
//     if(cg)return i;
// }
return crt(crtval,crtmod);
}

```

6.8 Primitive Root Generator

/*To find generator of $U(p)$, we check for all g in $[1,p]$. But only for powers of the form $\phi(p)/p_j$, where p_j is a prime factor of $\phi(p)$. Note that p is not prime here. Existence, if one of these: 1. $p = 1, 2, 4$
2. $p = q^k$, where $q \rightarrow$ odd prime.
3. $p = 2 \cdot (q^k)$, where $q \rightarrow$ odd prime
Note that $a.g^{(\phi(p))} = 1 \pmod{p}$
b. there are $\phi(\phi(p))$ generators if exists.
Finds "a" generator of $U(p)$, multiplicative group of integers mod p . Here `calc_phi` returns the \leftarrow toitent
function for p . $O(\text{Ans}.\log(\phi(p)).\log(p)) +$ time for factorizing $\phi(p)$. By some theorem, $\text{Ans} = O((\log(p))^6)$. Should be fast generally.*/

```

int generator(int p) {
    vector<int> fact;
    int phi = calc_phi(p), n = phi;
    for (int i=2; i*i<=n; ++i)
        if (n % i == 0) {
            fact.push_back(i);
            while (n % i == 0)
                n /= i;
        }
    if (n > 1) fact.push_back(n);
    for (int res=2; res<=p; ++res) {
        if(gcd(res,p)!=1) continue;
        bool ok = true;
        for (size_t i=0; i<fact.size() && ok; ++i)
            ok &= powmod(res, phi / fact[i], p) != 1;
        if (ok) return res;
    }
    return -1;
}

```

6.9 Math Miscellaneous

```

int gcd(int a,int b,int &x,int &y) {

```

```

if (a == 0) {x = 0; y = 1;return b;}
int x1,y1,d = gcd(b%a, a, x1, y1);
x = y1 - (b / a) * x1;y = x1;return d;}
int g (int n) {return n^(n >> 1);} //nth Gray code
int rev_g (int g) { //index of gray code g
int n = 0;for (; g; g >= 1)n ^= g;return n;}

```

6.10 Group Theory

$x^2 = n \pmod{p}$. Existence - $n^{((p-1)/2)} == 1 \rightarrow$ there is a soln.,
 $else == -1$, no solution.
 Finding sqrt. in some $\mathbb{Z} \pmod{p}$:
 Cipollas Algorithm.
 Find an 'a' (randomly) , s.t. $a^2 - n$ doesn't have a sqrt.
 Adjoin it to the field. Take $(a + \sqrt{a^2 - n})^{((p+1)/2)}$.
 Do all operations mod p, ans will be integer.
 Cipollas Algo works only when mod is prime.
 [Remember $(a+b)^p = a^p + b^p \pmod{p} = a + b \pmod{p}$]
 For non-prime :
 $x^2 = n \pmod{m}$.
 Soln. \rightarrow Compute it modulo prime powers and take CRT.
 For prime powers :
 We have a solution $x_0 \pmod{p}$. We use it to find a solution $\pmod{p^2}$,
 then (p^3) and so on. For p^2 : $x^2 = n \pmod{p^2}$;
 We want x to reduce to $x_0 \pmod{p}$. So $x = x_0 + p \cdot x_1$. Square it. $x_0^2 + 2 \cdot x_0 \cdot x_1 = n \pmod{p^2}$.
 Calculate x_1 . This can be extended to find for greater powers of p.
 But the inverse may not exist always which may give a problem.
 But then no solution or all solutions. This is called Hensel's Lifting.
 This can also be extended to find $f(x) = 0 \pmod{p^2}$, if we have a soln. for $f(x) = 0 \pmod{p}$. Get something in $f'(x)$

7 Strings

7.1 Hashing Theory

If order not imp. and count/frequency imp. use this as hash fn:-
 $((a_1+h)^k + (a_2+h)^k + (a_3+h)^k + (a_4+h)^k) \pmod{p}$. Select : h, k, p
 Alternate:
 $((x)^{(a_1)} + (x)^{(a_2)} + \dots + (x)^{(a_k)}) \pmod{\text{mod}}$ where x and mod are fixed and $a_1 \dots a_k$ is an unordered set

7.2 Manacher

```

/*Same idea as Z_fn, O(n)
[l,r]: rightmost detected subpalindrom (with max r)
len of odd length palindrom centered around that
char (e.g abac for 'b' returns 2 (not 3))*/
vll manacher_odd(string s){
ll n = s.length(); vll d1(n);
for(ll i = 0, l = 0, r = -1; i < n; i++){
d1[i] = 1;
if(i <= r){ // use prev val
d1[i] = min(r-i+1, d1[l+r-i]);}
while(i+d1[i] < n && i-d1[i] >= 0 && s[i+d1[i]] == s[i-d1[i]])
d1[i]++; // trivial matching
if(r < i+d1[i]-1) l=i-d1[i]+1, r=i+d1[i]-1;
}
return d1;}
//even lens centered around (bb is centered around the later 'b')
vll manacher_even(string s){
ll n = s.length(); vll d2(n);
for(ll i = 0, l = 0, r = -1; i < n; i++){
d2[i] = 0;
if(i <= r){
d2[i] = min(r-i+1, d2[l+r+1-i]);}
while(i+d2[i] < n && i-d2[i]-1 >= 0 && s[i+d2[i]] == s[i-d2[i]-1]) d2[i]++;
if(d2[i] > 0 && r < i+d2[i]-1)
l=i-d2[i], r=i+d2[i]-1;
}
return d2;}
// Other mtd : To do both things in one pass,
// add special char e.g string "abc" => "$a$b$c$"

```

7.3 Trie

```

const ll AS = 26; // alphabet size
ll go[MAX][AS]; ll cnt[MAX]; ll cn=0;
// cn -> index of next new node
// convert all strings to vll
ll newNode() {
for(ll i=0; i<AS; i++)
go[cn][i]=-1;
return cn++;
}
// call newNode once **** before adding anything
**
void addTrie(vll &x) {
ll v=0;
cnt[v]++;
for(ll i=0; i<x.size(); i++){
ll y=x[i];
if(go[v][y]==-1)
go[v][y]=newNode();
v=go[v][y];
}
}

```

```

    cnt[v]++;
}
// returns count of substrings with prefix x
ll getcount(vll &x){
    ll v=0;
    for(i=0;i<x.size();i++){
        ll y=x[i];
        if(go[v][y]==-1)
            go[v][y]=newNode();
        v=go[v][y];
    }
    return cnt[v];
}

```

7.4 Z-algorithm

```

/*[l,r]->rightmost segment match(with max r)
Time : O(n)(asy. behavior), Proof:each itr of
inner while loop make r pointer advance to right,
App:1)Search substring(text t,pat p)s=p+ '$' + t.
3) String compression(s=t+t+...+t, then find |t|)
2) Number of distinct substrings (in O(n^2))
(useful when appending or deleting characters
online from the end or beginning)*/
vector<ll> z_function(string s) {
    ll n = (ll) s.length();
    vector<ll> z(n);
    for (ll i=1, L=0, R=0; i<n; ++i) {
        if (i <= R) // use previous z val
            z[i] = min (R - i + 1, z[i - L]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]])
            ++z[i]; // trivial matching
        if (i + z[i] - 1 > R) L = i, R = i + z[i] - 1;
        // update rightmost segment matched
    }
    return z;
}

```

7.5 Aho Corasick

```

const int K = 26;
// remember to set K
struct Vertex {
    int next[K]; bool leaf = false;
    int p = -1; char pch;
    int link = -1; int go[K];
    Vertex(int p=-1, char ch='$') : p(p), pch(ch) {
        fill(begin(next), end(next), -1);
        fill(begin(go), end(go), -1);
    }
};
vector<Vertex> aho(1);
void add_string(string const& s) {
    int v = 0;
    for (char ch : s) {

```

```

        int c = ch - 'a';
        if (aho[v].next[c] == -1) {
            aho[v].next[c] = aho.size();
            aho.emplace_back(v, ch);
        }
        v = aho[v].next[c];
    }
    aho[v].leaf = true;
}
int go(int v, char ch);
int get_link(int v) {
    if (aho[v].link == -1) {
        if (v==0 || aho[v].p==0) aho[v].link = 0;
        else aho[v].link =
            go(get_link(aho[v].p), aho[v].pch);
    }
    return aho[v].link;
}
int go(int v, char ch) {
    int c = ch - 'a';
    if (aho[v].go[c] == -1) {
        if (aho[v].next[c] != -1)
            aho[v].go[c] = aho[v].next[c];
        else
            aho[v].go[c] = v == 0 ? 0 : go(get_link(v), ch);
    }
    return aho[v].go[c];
}

```

7.6 KMP

```

/*Time:O(n)(j increases n times(& j>=0) only so
asy. O(n)), pi[i] = length of longest prefix of
s ending at i
app.: search substring,
# of different substrings(O(n^2)),
3) String compression(s = t+t+...+t,
then find |t|, k=n-pi[n-1],if k|n)
4) Building Automaton(Gray Code Example)*/
vector<ll> prefix_function(string s) {
    ll n = (ll)s.length(); vll pi(n);
    for (ll i = 1; i < n; i++) {
        ll j = pi[i-1];
        while (j > 0 && s[i] != s[j]) j = pi[j-1];
        if (s[i] == s[j]) j++;
        pi[i] = j;
    }
    return pi;
}
//searching s in t, returns all occurrences(indices
vector<ll> search(string s,string t){
    vll pi = prefix_function(s);
    ll m = s.length(); vll ans; ll j = 0;
    for(ll i=0;i<t.length();i++){
        while(j > 0 && t[i] != s[j])
            j = pi[j-1];

```

```

    if(t[i] == s[j]) j++;
    if(j == m) ans.pb(i-m+1);
} // if ans empty then no occurrence
return ans;}

```

7.7 Palindrome Tree

```

const ll MAX=1e5+15;
ll par[MAX]; // stores index of parent node
ll sul_i[MAX]; // stores index of suffix link
ll len[MAX]; // stores len of largest
               palindrome ending at that node */
ll child[MAX][30]; // stores the children of the ←
                    node
/*-----
index 0 - root "-1"
index 1 - root "0"
therefore node of s[i] is i+2
initialize all child[i][j] to -1
-----*/
void eer_tree(string s){
    ll a,b,c,d,i,j,k,e,f;
    sul_i[1]=0;sul_i[0]=0;len[1]=0;len[0]=-1;
    ll n=s.length();
    for(i=0;i<n+10;i++)
        for(j=0;j<30;j++) child[i][j]=-1;
    ll cur=1;d=1;
    for(i=0;i<s.size();i++){
        ++d;
        while(true){
            a=i-1-len[cur];
            if(a>=0){
                if(s[a]==s[i]){
                    if(child[cur][((ll)(s[i]-'a'))]==-1){
                        par[d]=cur;child[cur][((ll)(s[i]-'a'))]=←
                            d;
                        len[d]=len[cur]+2;cur=d;
                    }
                    else{
                        par[d]=cur;len[d]=len[cur]+2;
                        cur=child[cur][((ll)(s[i]-'a'))];
                    }
                    break;
                }
            }
            if(cur==0) break;
            cur=sul_i[cur];
        }
        if(cur!=d) continue;
        if(len[d]==1) sul_i[d]=1;
        else{
            c=sul_i[par[d]];
            while(child[c][((ll)(s[i]-'a'))]==-1){
                if(c==0) break;
                c=sul_i[c];
            }
            sul_i[d]=child[c][((ll)(s[i]-'a'))];
        }
    }
}

```

```

    }
}
}

```

7.8 Suffix Array

```

/*Sorted array of suffixes = sorted array of ←
   cyclic
   shifts of string+$. We consider a prefix of len. 2^←
   k
   of the cyclic, in the kth iteration. String of len.
   2^k→combination of 2 strings of len. 2^(k-1), ←
   whose
   order we know, from previous iteration. Just radix
   sort on pair for next iteration.
Time :- O(nlog(n) + alphabet). Applications :-
Finding the smallest cyclic shift; Finding a ←
   substring
   in a string; Comparing two substrings of a string;
Longest common prefix of two substrings; Number of
different substrings. */
//returns permutation of indices in sorted order ←
***
vector<ll> sort_cyclic_shifts(string const& s) {
    ll n = s.size();
    const ll alphabet = 256;
    //change the alphabet size accordingly and ←
    indexing
    vector<ll> p(n), c(n), cnt(max(alphabet, n), 0);
    // p:sorted ord. of 1-len prefix of each cyclic
    // shift index. c:class of a index
    // pn:same as p for kth iteration . ||ly cn.
    for (ll i = 0; i < n; i++)
        cnt[s[i]]++;
    for (ll i = 1; i < alphabet; i++)
        cnt[i] += cnt[i-1];
    for (ll i = 0; i < n; i++)
        p[--cnt[s[i]]] = i;
    c[p[0]] = 0;
    ll classes = 1;
    for (ll i = 1; i < n; i++) {
        if (s[p[i]] != s[p[i-1]])
            classes++;
        c[p[i]] = classes - 1;
    }
    vector<ll> pn(n), cn(n);
    for (ll h = 0; (1 << h) < n; ++h) {
        for (ll i = 0; i < n; i++) { //sorting w.r.t
            pn[i] = p[i] - (1 << h); //second part.
            if (pn[i] < 0)
                pn[i] += n;
        }
        fill(cnt.begin(), cnt.begin() + classes, 0);
        for (ll i = 0; i < n; i++)
            cnt[c[pn[i]]]++;
        for (ll i = 1; i < classes; i++)

```

```

    cnt[i] += cnt[i-1];
// sorting w.r.t. 1st(more significant) part
    for (ll i = n-1; i >= 0; i--)
        p[--cnt[c[pn[i]]]] = pn[i];
    cn[p[0]] = 0; classes = 1;
// determining new classes in sorted array.
    for (ll i = 1; i < n; i++) {
        pll cur={c[p[i]],c[(p[i]+(1<<h))%n]};
        pll prev={c[p[i-1]],c[(p[i-1]+(1<<h))%n]};
        if (cur != prev) ++classes;
        cn[p[i]] = classes - 1;
    }
    c.swap(cn); }
return p;
}

vector<ll> suffix_array_construction(string s) {
    s += "$";
    vector<ll> sorted_shifts = sort_cyclic_shifts(s);
    sorted_shifts.erase(sorted_shifts.begin());
    return sorted_shifts; }
// For comp. two substr. of len. l starting at i,j
// k - 2^k > l/2. check the first 2^k part, if
// equal,
// check last 2^k part. c[k] is the c in kth iter
// of S.A construction.
int compare(int i, int j, int l, int k) {
    pll a = {c[k][i],c[k][(i+1-(1 << k))%n]};
    pll b = {c[k][j],c[k][(j+1-(1 << k))%n]};
    return a == b ? 0 : a < b ? -1 : 1; }
/*lcp[i]=len. of lcp of ith & (i+1)th suffix in
the SA
1.Consider suffixes in decreasing order of length.
2.Let p = s[i....n]. It will be somewhere in the S
A.
We determine its lcp = k. 3.Then lcp of q=s[(i+1)
.n]
will be atleast k-1 coz 4.remove the first char of
and its successor in the S.A. These are suffixes
with
lcp k-1. 5.But note that these 2 may not be
consecutive
in S.A.But lcp of str. in b/w have to be also >= k
-1.*/
vll lcp_cons(string const& s, vector<ll> const& p)
{
    ll n = s.size();
    vector<ll> rank(n, 0);
    for (ll i = 0; i < n; i++)
        rank[p[i]] = i;
    ll k = 0; vector<ll> lcp(n-1, 0);
    for (ll i = 0; i < n; i++) {
        if (rank[i] == n - 1) {
            k = 0; continue; }

```

```

        ll j = p[rank[i] + 1];
        while (i+k<n && j+k<n && s[i+k]==s[j+k]) k++;
        lcp[rank[i]] = k; if (k) k--; }
    return lcp;
}

```

7.9 Suffix Tree

```

const int N=1000000, // set it more than 2*(len. of string)
string str; // input string for which the suffix tree is being built
int chi[N][26],
lef[N], // left...
rig[N], // ...and right boundaries of the substring of a which correspond to incoming edge
par[N], // parent of the node
sfli[N], // suffix link
tv,tp,la,
ts; // the number of nodes
void ukkadd(int c) {
    suff++;
    if (rig[tp]<tp){
        if (chi[tp][c]==-1){chi[tp][c]=ts;lef[ts]=la;
            par[ts++]=tp;tp=sfli[tp];tp=rig[tp]+1;goto suff;}
        tv=chi[tp][c];tp=lef[tp];
    }
    if (tp==-1 || c==str[tp]-'a')tp++;
    else {
        lef[ts]=lef[tp]; rig[ts]=tp-1; par[ts]=par[tp];
        chi[ts][str[tp]-'a']=tp; chi[ts][c]=ts+1;
        lef[ts+1]=la; par[ts+1]=ts; lef[tp]=tp; par[tp]=ts;
        chi[par[ts]][str[lef[ts]]-'a']=ts; ts+=2;
        tv=sfli[par[ts-2]]; tp=lef[ts-2];
        while (tp <= rig[ts-2]) {
            tv=chi[tp][str[tp]-'a']; tp+=rig[tp]-lef[tp]+1;}
        if (tp == rig[ts-2]+1) sfli[ts-2]=tv; else sfli[ts-2]=ts;
        tp=rig[tp]-(tp-rig[ts-2])+2;goto suff;
    }
}

void build() {
    ts=2; tv=0; tp=0;
    ll ss = str.size();ss*=2;ss+=15;
    fill(rig,rig+ss,(int)str.size()-1);
    // initialize data for the root of the tree
    sfli[0]=1; lef[0]=-1; rig[0]=-1;
    lef[1]=-1; rig[1]=-1; for (ll i=0;i<ss;i++)
        fill (chi[i], chi[i]+27,-1);
    fill(chi[1],chi[1]+26,0);
    // add the text to the tree, letter by letter

```



```

    for (la=0; la<(int)str.size(); ++la)
        ukkadd (str[la]-'a');
}

```

7.10 Suffix Automaton

```

struct state {
    int len, link;
    map<char, int> next;
};
const int MAXLEN = 200005;
state st[MAXLEN];
int sz, last;
void sa_init() {
    st[0].len = 0;
    st[0].link = -1;
    sz++;
    last = 0;
}
void sa_extend(char c) {
    int cur = sz++;
    st[cur].len = st[last].len + 1;
    int p = last;
    while (p != -1 && !st[p].next.count(c)) {
        st[p].next[c] = cur;
        p = st[p].link;
    }
    if (p == -1) {
        st[cur].link = 0;
    } else {
        int q = st[p].next[c];
        if (st[p].len + 1 == st[q].len) {
            st[cur].link = q;
        } else {
            int clone = sz++;
            st[clone].len = st[p].len + 1;
            st[clone].next = st[q].next;
            st[clone].link = st[q].link;
            while (p != -1 && st[p].next[c] == q) {
                st[p].next[c] = clone;
                p = st[p].link;
            }
            st[q].link = st[cur].link = clone;
        }
    }
    last = cur;
}
void build(string &x){
    sz=0;
    for(ll i=0;i<3*x.length()+15;i++)
    {
        st[i].next.clear();
        st[i].len=0;st[i].link=0;
    }
    sa_init();
    for(ll i=0;i<x.size();i++) sa_extend(x[i]);
}

```

```

}

```

Möbius Function

$$\mu(n) = \begin{cases} 0 & n \text{ not squarefree} \\ 1 & n \text{ squarefree w/ even no. of prime factors} \\ -1 & n \text{ squarefree w/ odd no. of prime factors} \end{cases}$$

Note that $\mu(a)\mu(b) = \mu(ab)$ for a, b relatively prime

$$\text{Also } \sum_{d|n} \mu(d) = \begin{cases} 1 & \text{if } n = 1 \\ 0 & \text{otherwise} \end{cases}$$

Möbius Inversion If $g(n) = \sum_{d|n} f(d)$ for all $n \geq 1$, then $f(n) = \sum_{d|n} \mu(d)g(n/d)$ for all $n \geq 1$.

Burnside's Lemma (Text)

The number of orbits of a set X under the group action G equals the average number of elements of X fixed by the elements of G .

Here's an example. Consider a square of $2n$ times $2n$ cells. How many ways are there to color it into X colors, up to rotations and/or reflections? Here, the group has only 8 elements (rotations by 0, 90, 180 and 270 degrees, reflections over two diagonals, over a vertical line and over a horizontal line). Every coloring stays itself after rotating by 0 degrees, so that rotation has X^{4n^2} fixed points. Rotation by 180 degrees and reflections over a horizontal/vertical line split all cells in pairs that must be of the same color for a coloring to be unaffected by such rotation/reflection, thus there exist X^{2n^2} such colorings for each of them. Rotations by 90 and 270 degrees split cells in groups of four, thus yielding X^{n^2} fixed colorings. Reflections over diagonals split cells into $2n$ groups of 1 (the diagonal itself) and $2n^2 - n$ groups of 2 (all remaining cells), thus yielding $X^{2n^2 - n + 2n} = X^{2n^2 + n}$ unaffected colorings. So, the answer is $(X^{4n^2} + 3X^{2n^2} + 2X^{n^2} + 2X^{2n^2 + n})/8$. Every tree with n vertices has $n - 1$ edges.

Trees-Kraft inequality:

If the depths of the leaves of a binary tree are $d_1 \dots d_n$: $\sum_{i=1}^n 2^{-d_i} \leq 1$, and equality holds only if every internal node has 2 sons.

Euler Tour:

- Undirected graph iff All vertices have even degree, all non-zero degree vertices are in a single connected component.(Can be decomposed into edge-disjoint cycles)
- Directed graph iff For each vertex in-degree=out-degree, all non-zero degree vertices are in a single strongly connected component.(Decomposable into directed-edge disjoint cycles)

Euler Trail:

- Undirected graph iff exactly 0 or 2 vertices have odd degree, single connected component(consider zero degree vertices only).
- Directed graph iff at most one vertex has (out-degree)-(in-degree) = 1, at most one vertex has (in-degree)-(out-degree) = 1, every other vertex has equal in-degree and out-degree, and all of its vertices with nonzero degree belong to a single connected component of the underlying undirected graph.

Master method:

$$T(n) = aT(n/b) + f(n), \quad a \geq 1, b > 1$$

If $\exists \epsilon > 0$ such that $f(n) = O(n^{\log_b a - \epsilon})$ then $T(n) = \Theta(n^{\log_b a})$.

If $f(n) = \Theta(n^{\log_b a})$ then $T(n) = \Theta(n^{\log_b a} \log_2 n)$.

If $\exists \epsilon > 0$ such that $f(n) = \Omega(n^{\log_b a + \epsilon})$, and $\exists c < 1$ such that $af(n/b) \leq cf(n)$ for large n , then $T(n) = \Theta(f(n))$.

Probability:

Variance, standard deviation: $\text{Var}[X] = E[X^2] - E[X]^2$

Poisson distribution:

Normal (Gaussian) distribution:

$$\Pr[X = k] = \frac{e^{-\lambda} \lambda^k}{k!}, \quad E[X] = \lambda \quad \left| \quad p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}, \quad E[X] = \mu.$$

The “coupon collector”: We are given a random coupon each day, and there are n different types of coupons. The distribution of coupons is uniform. The expected number of days to pass before we to collect all n types is nH_n .

Miscellaneous:

1. Radius of inscribed circle for Right Angle Tringle: $\frac{AB}{A+B+C}$
2. Law of cosine: $c^2 = a^2 + b^2 - 2ab \cos C$
3. Area of a triangle: Area: $A = \frac{1}{2}hc = \frac{1}{2}ab \sin C = \frac{c^2 \sin A \sin B}{2 \sin C}$.
4. $\det A = \sum_{\pi} \prod_{i=1}^n \text{sign}(\pi) a_{i, \pi(i)}$, for Permanents remove sign.
5. Perfect Numbers: x is an even perfect number iff $x = 2^{n-1}(2^n - 1)$ and $2^n - 1$ is prime.
6. Wilson's theorem: n is a prime iff $(n-1)! \equiv -1 \pmod n$.
7. If graph G is planar then $n - m + f = 2$, so $f \leq 2n - 4$, $m \leq 3n - 6$. Any planar graph has a vertex with degree ≤ 5 .
8. Dirichlet power series: $A(x) = \sum_{i=1}^{\infty} \frac{a_i}{i^x}$
9. Coefficient of x^r in $(1-x)^{-n}$ is $\binom{n+r-1}{r}$.

For Bipartite Graphs

1. Min-edge cover(me) = Max-independent set(mi) (G has no isolated vertex).
2. Min-vertex cover(mv) = Max matching(mm) $mi + mv = |V|$, $mi \geq \frac{|V|}{2}$
3. Min-edge cover subgraph is a combination of star graphs.
4. Min Vertex cover : In residual graph of max flow pick all the vertices in L(left bipartite) not reachable from s (whose edges are cut) and lly in R reachable from s .

5. Min-edge cover(no isolated vertex) : Find max matching, take all those edges, for vertices not covered take any edge.

$$\frac{x}{(1-x)^2} = \sum_{i=0}^{\infty} ix^i, \quad \ln(1+x) = \sum_{i=1}^{\infty} (-1)^{i+1} \frac{x^i}{i}, \quad \frac{x}{1-x-x^2} = \sum_{i=0}^{\infty} F_i x^i$$

$$(x_1 + x_2 + \dots + x_k)^n = \sum_{c_1+c_2+\dots+c_k=n} \frac{n!}{c_1!c_2!\dots c_k!} x_1^{c_1} x_2^{c_2} \dots x_k^{c_k}$$

$$\int \tanh x dx = \ln |\cosh x|, \quad \int \coth x dx = \ln |\sinh x|, \quad \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right|,$$

$$\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln \left(x + \sqrt{a^2 + x^2} \right) \quad (a > 0), \quad \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} \quad (a > 0),$$

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a} \quad (a > 0)$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} \quad (a > 0)$$

Fibonacci:

1. $F_{-i} = (-1)^{i-1} F_i$, $F_i = \frac{1}{\sqrt{5}} (\phi^i - \hat{\phi}^i)$
2. Cassini's identity: $F_{i+1} F_{i-1} - F_i^2 = (-1)^i$ for $i > 0$,
3. Addictive Rule: $F_{n+k} = F_k F_{n+1} + F_{k-1} F_n$, $F_{2n} = F_n F_{n+1} + F_{n-1} F_n$
4. Every integer n has a unique representation $n = F_{k_1} + F_{k_2} + \dots + F_{k_m}$, where $k_i \geq k_{i+1} + 2$ for $1 \leq i < m$ and $k_m \geq 2$.

Primes

$\forall (a, b)$, The largest prime smaller than 10^a is $p = 10^a - b$

(1,3), (2,3), (3,3), (4,27), (5,9), (6,17), (7,9), (8,11), (9,63), (10,33),
(11,23), (12,11), (13,29), (14,27), (15,11), (16,63), (17,3), (18,11)

Ideas

Div and Conq, Brute force and observe, (+1,-1), Dominator Tree for Directed Graph, use fenwick, Monte Carlo, Summation Interchange, Clever Optimization of brute force(binary search/ignore)

C++ Sublime Build Extra

```
{
  "cmd" : ["g++ -std=c++11 $file_name -o ↵
           $file_base_name && timeout 4s ./$file_base_name<↵
           input.txt>output.txt"],
  "selector" : "source.c",
  "shell": true,
  "working_dir" : "$file_path"
}
```