Codebook- Team Far_Behind
IIT Delhi, India

Ayush Ranjan, Naman Jain, Manish Tanwar

Ideas
Analysis Complexity Carefully, Relate To Theory, First Find Solution for small sub-problems, Div and Conq , Brute force and observe, (+1,-1), Dominator Tree for Directed Graph, use fenwick, Monte Carlo, Summation Interchange, Clever Optimization of brute force(binary search/ignore), Try solving problem backwards

Tree Ideas
a. LCA b. DSU on trees (possibly make greater depth as heavy child) c. Centroid Decomposition d. HLD e. Euler Tour (dfs order/bfs order/any other ordering) f. Pass some structure(set/array/..) in dfs (segment/fenwick) g. Reachability Tree (construction using dsu) h. Dominator Tree (directed graphs) i. Biconnectivity j. DFS tree

1 Syntax

1.1 Template

```cpp
#include<bits/stdc++.h>
#include<ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
using namespace std;
#define ll long long
//increase stack size
#pragma comment(linker, "/STACK:16777216")
ll mxm() {return LLONG_MIN;}
template<typename... Args>
ll mxm(ll a, Args... args) { return max(a,mxm(args...)); }
ll mnm() {return LLONG_MAX;}
template<typename... Args>
ll mnm(ll a, Args... args) { return min(a,mnm(args...)); }
template<class T> ostream& operator<<(ostream& os,vector<T> V){
  os<<"[ ";for(auto v:V)os<<v<<" ";return os<<"]";}
template<class L,class R> ostream& operator<<(ostream& os,pair<L,R> P){
  return os<<"("<<P.first<<","<<P.second<<")";}
#define TRACE
#ifdef TRACE
#define trace(...) __f(#__VA_ARGS__,__VA_ARGS__)
template<typename Arg1>
void __f(const char* name,Arg1&& arg1){
  cout<<name<<" : "<<arg1<<endl;}
template <typename Arg1,typename... Args>
void __f(const char* names,Arg1&& arg1,Args&&... args){
  const char* comma=strchr(names+1,',');cout.write(names,comma-names)<<" : "<<arg1<<" | ";__f(comma+1,args...);}
#else
#define trace(...) 1
#endif
#define ld long double
#define pll pair<ll,ll>
#define pii pair<int,int>
#define vll vector<ll>
#define vi vector<int>
#define vpll vector<pll>
#define I insert
#define F first
#define S second
```

```cpp
#define pb push_back
#define endl "\n"
#define all(x)  x.begin(),x.end()
// const int mod=1e9+7;
// 128 bit: __int128
inline int add(int a,int b){a+=b;if(a>=mod)a-=mod;↩
   return a;}
inline int sub(int a,int b){a-=b;if(a<0)a+=mod;↩
   return a;}
inline int mul(int a,int b){return (a*1ll*b)%mod;}
inline int power(int a,int b){int rt=1;while(b>0){↩
   if(b&1)rt=mul(rt,a);a=mul(a,a);b>>=1;}return rt↩
   ;}
inline int inv(int a){return power(a,mod-2);}
int main()
{
  ios_base::sync_with_stdio(false);cin.tie(0);cout↩
     .tie(0);cout<<setprecision(25);
}
// clock
clock_t clk = clock();
clk = clock() - clk;
((ld)clk)/CLOCKS_PER_SEC
// fastio
inline ll read() {
    ll n = 0; char c = getchar_unlocked();
    while (!('0' <= c && c <= '9')) c = ↩
       getchar_unlocked();
    while ('0' <= c && c <= '9')
        n = n * 10 + c - '0', c = getchar_unlocked↩
           ();
    return n;
}
inline void write(ll a){
    register char c; char snum[20]; ll i=0;
    do{
        snum[i++]=a%10+48;
        a=a/10;
    }
    while(a!=0); i--;
    while(i>=0)
        putchar_unlocked(snum[i--]);
    putchar_unlocked('\n');
}
// gp_hash_table
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
gp_hash_table<int, int> table;  //cc_hash_table ↩
   can also be used
//custom hash function
const int RANDOM = chrono::high_resolution_clock::↩
   now().time_since_epoch().count();
struct chash {
    int operator()(int x) { return hash<int>{}(x ^↩
        RANDOM); }
};
gp_hash_table<int, int, chash> table;
//custom hash function for pair
struct chash {
    int operator()(pair<int,int> x) const { return↩
        x.first* 31 + x.second; }
};
// random
mt19937 rng(chrono::steady_clock::now().↩
   time_since_epoch().count());
uniform_int_distribution<int> uid(l,r);
int x=uid(rng);
//mt19937_64 rng(chrono::steady_clock::now().↩
   time_since_epoch().count());
 // - for 64 bit unsigned numbers
vector<int> per(N);
for (int i = 0; i < N; i++)
    per[i] = i;
shuffle(per.begin(), per.end(), rng);
// string splitting
// this splitting is better than custom function(w↩
   .r.t time)
using getline, use cin.ignore()
string line = "Ge";
vector <string> tokens;
stringstream check1(line);
string ele;
// Tokenizing w.r.t. space ' '
while(getline(check1, ele, ' '))
tokens.push_back(ele);
//Ordered Sets
typedef tree<ll,null_type,less<ll>,rb_tree_tag,
tree_order_statistics_node_update> ordered_set;
ordered_set X;X.insert(1);X.insert(2);
*X.find_by_order(0)-> 1
*X.find_by_order(1)-> 2
(end(X)==X.find_by_order(2) -> true
//order_of_key(x) -> # of elements < x
//For multiset use less_equal operator but
//it does support erase operations for multiset
```

1.2 C++ Sublime Build

```json
{
  "cmd": ["bash", "-c", "g++ -std=c++11 -O3 '${↩
     file}' -o '${file_path}/${file_base_name}' && ↩
      gnome-terminal -- bash -c '\"${file_path}/${↩
     file_base_name}\" < input.txt >output.txt'  "↩
     ],
  "file_regex": "^(..[^:]*):([0-9]+):?([0-9]+)?:? ↩
     (.*)$",
  "working_dir": "${file_path}",
  "selector": "source.c++, source.cpp",
```

```
}
```

2 Data Structures

2.1 Fenwick

```
/*All indices are 1 indexed
*Range update and point query: maintain BIT of prefix sum of updates
-add val in [a,b] -> add val at a,-val at b+1
-value[a]=BITsum(a)+arr[a]
*Range update ,range query: maintain 2 BITs B1,B2
-add val in [a,b] -> B1:add val at a,-val at b+1 and in B2 -> Add val*(a-1) at a, -val*b at b+1
-sum[1,b]=B1sum(1,b)*b-B2sum(1,b)
-sum[a,b]=sum[1,b]-sum[1,a-1]*/
ll fen[MAX_N];
void update(ll p,ll val){
  for(ll i = p;i <= n;i += i & -i)
    fen[i] += val;}
ll sum(ll p){
  ll ans = 0;
  for(ll i = p;i;i -= i & -i) ans += fen[i];
  return ans;}
```

2.2 2D-BIT

```
/*All indices are 1 indexed.Increment value of cell (i,j) by val -> update(x,y,val)
*sum of rectangle [a,b]-[c,d] ->sum of rectangles [1,1]-[c,d],[1,1]-[c,b],[1,1][a,d] and [1,1]-[a,b] and use inclusion exclusion*/
ll bit[MAX][MAX];
void update(ll x , ll y, ll val){
  while( x < MAX ){
    ll y1 = y;
    while( y1 < MAX )
      bit[x][y1]+=val , y1 += ( y1 & -y1 );
    x += (x & -x);}
}
ll sum(ll x , ll y){
  ll ans = 0;
  while( x > 0 ){
    ll y1 = y;
    while( y1 > 0 )
      ans+=bit[x][y1] , y1 -= ( y1 & -y1 );
    x -= (x & -x);}
  return ans;}
```

2.3 Segment Tree

```
/*All arrays are 0 indexed. call bld(0,n-1,1)
upd(0,n-1,1,x,y,val) -> increase [x,y] by val
sum(0,n-1,1,x,y) -> sum [x,y]
array of size N -> segment tree of size 4*N*/
ll arr[N],st[N<<2], lz[N<<2];
void ppgt(ll l, ll r,ll id){
  if(l==r) return;
  ll m=l+r>>1;
  lz[id*2]+=lz[id];lz[id<<1|1]+=lz[id];
  st[id << 1] += (m - l + 1) * lz[id];
  st[id<<1|1]+=(r-m)*lz[id];lz[id] = 0;}
void bld(ll l,ll r,ll id){
  if(l==r) { st[id] = arr[l]; return; }
  bld(l,l+r>>1,id*2);bld(l+r+1>>1,r,id*2+1);
  st[id] = st[ id << 1] + st[id << 1 | 1];}
void upd(ll l,ll r,ll id,ll x,ll y,ll val){
  if (l > y || r < x ) return;ppgt(l, r, id);
  if (l >= x && r <= y ) {
    lz[id]+=val;st[id]+=(r-l+1)*val; return;}
  upd(l,l + r >> 1,id << 1, x, y, val);upd((l + r >> 1) + 1,r ,id << 1 | 1,x, y, val);
  st[id] = st[id << 1] + st[ id << 1 | 1];}
ll sum(ll l,ll r,ll id,ll x,ll y){
  if (l > y || r < x ) return 0;ppgt(l, r, id);
  if (l >= x && r <= y ) return st[id];
  return sum(l, l + r >> 1,id << 1, x, y) + sum((l + r >> 1 ) + 1,r ,id << 1 | 1,x, y);}
```

2.4 Persistent Segment Tree

```
/*id of first node = 0. call build(0,n-1) first. afterwards call upd(0,n-1,previous id,i,val) to add val in ith number. It returns root of new segment tree after modification
*sum(0,n-1,id of root,l,r) -> sum of values in subarray l to r in tree rooted at id
**size of st,lc,rc >= N*2+(N+Q)*logN*/
const ll N=1e5+10;
ll arr[N],st[20*N],lc[20*N],rc[20*N],ids[N],cnt;
void build(ll l,ll r){
  if(l==r) {st[cnt]=arr[l];++cnt;return;}
  ll id = cnt++;lc[id] = cnt;
  build ( l, l+r >>1);
  rc[id] = cnt; build( (l + r >> 1) + 1, r);
  st[id] = st[lc[id]] + st[rc[id]];}
ll upd(ll l,ll r,ll id,ll x,ll val){
  if(l == r)
    {st[cnt]=st[id]+val;++cnt;return cnt-1;}
  ll myid = cnt++; ll mid = l + r >>1;
  if(x <= mid)
    rc[myid] = rc[id],lc[myid] = upd(l, mid, lc[id], x, val);
  else
    lc[myid] = lc[id],rc[myid] = upd(mid+1, r, rc[id], x, val);
  st[myid] = st[lc[myid]] + st[rc[myid]];
  return myid;}
ll sum(ll l,ll r,ll id,ll x,ll y){
  if (l > y || r < x ) return 0;
  if (l >= x && r <= y ) return st[id];
  return sum(l, l + r >> 1,lc[id], x, y) + sum((l + r >> 1 ) + 1,r ,rc[id],x, y);}
```

```
ll gkth(ll l,ll r,ll id1,ll id2,ll k){
  if(l==r) return l;ll mid = l+r>>1;
  ll a=st[lc[id2]]-(id1>=0?st[lc[id1]]:0);
  if(a >= k)
    return gkth(l, mid ,(id1>=0?lc[id1]:-1), lc[id2], k);
  else
    return gkth(mid+1, r,(id1>=0?rc[id1]:-1), rc[id2], k-a);}
//kth largest num in range
int main(){
  ll n,m;vll finalid(n);vpll v;
  loop : v.pb({arr[i],i});sort(all(v));
  loop : finalid[v[i].second]=i;
  memset(arr,0,sizeof(ll)*N);
  arr[finalid[0]]++;build(0,n-1);
  loop:ids[i]=upd(0,n-1,ids[i-1],finalid[i],1);
  while(m--){
  ll i,j,k;cin>>i>>j>>k;--i;--j;
  ans=gkth(0,n-1,(i==0?-1:ids[i-1]),ids[j],k);
  cout<<v[ans].F<<endl;}
}
```

2.5 DP Optimization

```
/*Split L size array into G intervals, minimizing
the cost (G<=L).The cost func. C[i,j] satisfies:
C[a,b]+C[c,d]<=C[a,d]+C[c,b] for a<=c<=b<=d.(Q.E)
& intuitively you can think that the c.f increases
at a rate which is more than linear at all intervals.
So, if the c.f. satisfies Q.E., the following holds:
F(g,l):min cost of spliting first l into g ivals.
F(g,l): min(F(g-1,k)+C(k+1,l)) over all valid k.
P(g,l): lowest position k s.t. it minimizes F(g,l).
P(g,0)<=P(g,1)<=.....<=P(g,l); DivConq, O(G.L.log(L))
P(0,l)<=P(1,l)<=P(2,l)....<=P(G-1,l)<=P(G,l).
Knuth Opti, complexity O(L.L).
For div&conq, we calculate P(g,l) for each g 1 by 1.
In each g, we calculate for mid-l and do recursively
using the obtained upper and lower bounds.For knuth,
we use P(g,l-1)<=P(g,l)<=P(g+1,l), and fill our table
in increasing l and decreasing g. In opt. BST type
problems, use  bk[i][j-1]<= bk[i][j] <=bk[i+1][j] */
// Code for Divide and Conquer Opti O(G.L.log(L)):-
ll C[8111]; ll sums[8111];
ll F[811][8111];   // optimal value
int P[811][8111];  // optimal position.
// note first val. in arrays is for no. of groups
ll cost(int i, int j) { // cost function
  return i > j ? 0 :(sums[j]-sums[i-1])*(j-i+1);
}
/*fill(g,l1,l2,p1,p2) calcs. P[g][l] and F[g][l] for
l1<=l<= l2,with the knowledge that p1<=P[g][l]<=p2 */
void fill(int g, int l1, int l2, int p1, int p2) {
  if (l1 > l2) return; int lm = (l1 + l2) >> 1;
  ll nv=INF,nv1=-1;
  for (int k = p1; k <= min(lm-1,p2); k++) {
    ll new_cost = F[g-1][k] + cost[k+1][lm];
    if (nv > new_cost) { nv=new_cost; nv1 = k; }
  }
  P[g][lm]=nv1; F[g][lm]=nv;
  fill(g, l1, lm-1, p1, P[g][lm]);
  fill(g, lm+1, l2, P[g][lm], p2);
}
int main() { // example call
  for(i=0;i<=n;i++)F[0][i]=INF;
  for(i=0;i<=k;i++)F[i][0]=0;
  F[0][0]=0;
  for(i=1;i<=k;i++)fill(i,1,n,0,n);
}
// Code for Knuth Optimization O(L.L) :-
ll dp[8002][802];
int a[8002],s[8002][802];
ll  sum[8002];
// index strats from 1
ll run(int n,int m) {
  memset(dp,0xff,sizeof(dp)); dp[0][0] = 0;
  for (int i = 1; i <= n; ++i) {
    sum[i] = sum[i - 1] + a[i];
    int maxj = min(i, m), mk; ll mn = INF;
    for (int k = 0; k < i; ++k) {
      if (dp[k][maxj - 1] >= 0) {
        ll tmp = dp[k][maxj - 1] +
          (sum[i] - sum[k]) * (i - k);
        if (tmp < mn) {
          mn = tmp;mk = k; }
      }
    }
    dp[i][maxj] = mn; s[i][maxj] = mk;
    for (int j = maxj - 1; j >= 1; --j) {
      ll mn = INF; int mk;
      for(ll k=s[i - 1][j]; k<=s[i][j + 1];++k){
        if (dp[k][j - 1] >= 0) {
          ll tmp =dp[k][j - 1]+(sum[i]-sum[k])*(i-k);
          if (tmp < mn) {mn = tmp;mk = k;} }
      }
      dp[i][j] = mn; s[i][j] = mk;
    }
  } return dp[n][m];
```

```
}
// call -> run(n, min(n,m))
```

3 Flows and Matching

3.1 General Matching

```
/*Given any directed graph, finds maximal matching
  Vertices-0-indexed, O(n^3) per call to edmonds*/
vll adj[MAXN]; int p[MAXN], base[MAXN], match[MAXN];
int lca(int n, int u, int v){
  vector<bool> used(n);
  for (;;) {
    u = base[u]; used[u] = true;
    if (match[u] == -1) break; u = p[match[u]];}
  for (;;) {
    v = base[v]; if (used[v]) return v;
    v = p[match[v]];}}
void mark_path(vector<bool> &blo,int u,int b,int child){
  for (; base[u] != b; u = p[match[u]]){
    blo[base[u]] = true; blo[base[match[u]]] = true;
    p[u] = child; child = match[u];}}
int find_path(int n, int root) {
  vector<bool> used(n);
  for (int i = 0; i < n; ++i)
    p[i] = -1, base[i] = i;
  used[root] = true;
  queue<int> q; q.push(root);
  while(!q.empty()) {
    int u = q.front(); q.pop();
    for (int j = 0; j < (int)adj[u].size(); j++) {
      int v = adj[u][j];
      if(base[u]==base[v] || match[u]==v)continue;
      if(v==root||(match[v]!=-1 && p[match[v]]!=-1)){
        int curr_base = lca(n, u, v);
        vector<bool> blossom(n);
        mark_path(blossom, u, curr_base, v);
        mark_path(blossom, v, curr_base, u);
        for(int i = 0; i < n; i++){
          if(blossom[base[i]]){
            base[i] = curr_base;
            if(!used[i]) used[i] = true, q.push(i);
          }
        }
      }else if (p[v] == -1){
        p[v] = u;
        if (match[v] == -1) return v;
        v=match[v]; used[v]=true; q.push(v);
      }
    }
  }
  return -1;}
int edmonds(int n){
  for(int i=0;i<n;i++) match[i] = -1;
  for(int i = 0; i < n; i++){
    if (match[i] == -1) {
      int u, pu, ppu;
      for (u = find_path(n, i); u != -1; u = ppu) {
        pu = p[u]; ppu = match[pu];
        match[u] = pu; match[pu] = u;
      }
    }
  }
  int matches = 0;
  for (int i = 0; i < n; i++)
    if (match[i] != -1) matches++;
  return matches/2;
}
u--; v--; adj[u].pb(v); adj[v].pb(u);
cout << edmonds(n) * 2 << endl;
for (int i = 0; i < n; i++) {
    if (match[i] != -1 && i < match[i]) {
        cout << i + 1 << " " << match[i] + 1 << endl;
    }
}
```

3.2 Global Mincut

```
/*finds min weighted cut in undirected graph in
O(n^3), Adj Matrix, 0-indexed vertices
output-(min cut value, nodes in half of min cut)*/
typedef vector<int> VI;
typedef vector<VI> VVI;
const int INF = 1000000000;
pair<int, VI> GetMinCut(VVI &weights) {
  int N = weights.size();
  VI used(N), cut, best_cut;
  int best_weight = -1;

  for (int phase = N-1; phase >= 0; phase--) {
    VI w = weights[0];
    VI added = used;
    int prev, last = 0;
    for (int i = 0; i < phase; i++) {
      prev = last; last = -1;
      for (int j = 1; j < N; j++)
      if(!added[j] && (last == -1 || w[j] > w[last]))
          last = j;
      if (i == phase-1) {
        for(int j=0;j<N;j++)
          weights[prev][j] += weights[last][j];
        for(int j=0;j<N;j++)
          weights[j][prev] = weights[prev][j];
```

```
        used[last] = true; cut.push_back(last);
        if (best_weight==-1 || w[last]<best_weight)
          best_cut = cut, best_weight = w[last];
      }
      else {
        for (int j = 0; j < N; j++)
        w[j] += weights[last][j];
        added[last] = true;
      }
    }
  }
  return make_pair(best_weight, best_cut);
}
VVI weights(n, VI(n));
pair<int, VI> res = GetMinCut(weights);
```

3.3 Hopcroft Matching

```
// O(m * \sqrt{n})
struct graph {
  int L, R; // 0-indexed vertices
  vector<vector<int>> adj;
  graph(int L, int R) : L(L), R(R), adj(L+R) {}
  void add_edge(int u, int v) {
    adj[u].pb(v+L); adj[v+L].pb(u);}
  int maximum_matching(){
    vector<int> level(L), mate(L+R, -1);
    function<bool(void)> levelize = [&]() { // BFS
      queue<int> Q;
      for (int u = 0; u < L; ++u) {
        level[u] = -1;
        if (mate[u] < 0) level[u] = 0, Q.push(u);
      }
      while (!Q.empty()) {
        int u = Q.front(); Q.pop();
        for (int w: adj[u]) {
          int v = mate[w];
          if (v < 0) return true;
          if (level[v] < 0)
            level[v] = level[u] + 1, Q.push(v);
        }
      }
      return false;
    };
    function<bool(int)> augment = [&](int u) { // DFS
      for (int w: adj[u]) {
        int v = mate[w];
        if (v<0 || (level[v]>level[u] &&augment(v)){
          mate[u] = w; mate[w] = u; return true;
        }
      }
      return false;
    };
    int match = 0;
    while (levelize())
      for (int u = 0; u < L; ++u)
        if (mate[u] < 0 && augment(u)) ++match;
    return match;
  }
}; // L-left size, R->right size
graph g(L,R); g.add_edge(u,v); g.maximum_matching();
```

3.4 Dinic

```
/*O(min(fm,mn^2)), for any unit capacity network
O(m*sqrt(n)), in practice it is pretty fast for any
bipartite network, **vertices are 1-indexed**
e=(u,v), e.flow represent effective flow from u to v
(i.e f(u->v) - f(v->u))
*use int if possible(ll could be slow in dinic)
1). To put lower bound on edge capacities form a new
graph G' with source s' and t' for each edge u->v
 in G with cap (low, high), replace it with
s'->v with low, u->t' with low
u->v with high - low
2). To convert circulation with edge lower bounds
to circulation without edge lower bounds
old=> e=u->v,l(e)<=f(e)<=c(e), d(u), d(v).
new=> d'(u)=d(u)+l(e), d'(v)=d(v)-l(e), c'(e)=c(e)-l(e))*/
struct edge {ll x, y, cap, flow;};
struct DinicFlow {
  // *** change inf accordingly *****
  const ll inf = (1e18);
  vector <edge> e; vll cur, d;
  vector<vll> adj; ll n, source, sink;
  DinicFlow() {}
  DinicFlow(ll v) {
    n = v; cur = vll(n+1);
    d = vll(n+1); adj = vector<vll>(n+1);}
  void addEdge(ll from, ll to, ll cap) {
    edge e1 = {from, to, cap, 0};
    edge e2 = {to, from, 0, 0};
    adj[from].pb(e.size()); e.pb(e1);
    adj[to].pb(e.size()); e.pb(e2);
  }
  ll bfs() {
    queue <ll> q;
    for(ll i = 0; i <= n; ++i) d[i] = -1;
    q.push(source); d[source] = 0;
    while(!q.empty() and d[sink] < 0) {
      ll x = q.front(); q.pop();
      for(ll i = 0; i < (ll)adj[x].size(); ++i){
        ll id = adj[x][i], y = e[id].y;
        if(d[y]<0 and e[id].flow < e[id].cap){
```

```
                q.push(y); d[y] = d[x] + 1;
            }
        }
    }
    return d[sink] >= 0;
  }
  ll dfs(ll x, ll flow) {
    if(!flow) return 0;
    if(x == sink) return flow;
    for(;cur[x] < (ll)adj[x].size(); ++cur[x]) {
      ll id = adj[x][cur[x]], y = e[id].y;
      if(d[y] != d[x] + 1) continue;
      ll pushe=dfs(y,min(flow,e[id].cap-e[id].flow↩
        ));
      if(pushed) {
        e[id].flow += pushed; e[id^1].flow -= ↩
          pushed;
        return pushed;
      }
    }
    return 0;
  }
  ll maxFlow(ll src, ll snk) {
    this->source = src; this->sink = snk;
    ll flow = 0;
    while(bfs()) {
      for(ll i = 0; i <= n; ++i) cur[i] = 0;
      while(ll pushed = dfs(source, inf))
        flow += pushed;
    }
    return flow;
  }
};
```

3.5 Ford Fulkerson

```
/*O(f*m)*/ ll n; // number of vertices
ll cap[N][N]; // adj matrix for cap
vll adj[N]; // *** vertices are 0-indexed ***
// adj list of the corresponding undirected(*imp*)
ll INF = (1e18);
ll snk,cnt;//cnt for vis, no need to initialize ↩
  vis
vector<ll> par, vis;
ll dfs(ll u,ll curf){
  vis[u] = cnt; if(u == snk) return curf;
  if(adj[u].size() == 0) return 0;
  for(ll j=0;j<5;j++){ // random for good aug.
    ll a = rand()%(adj[u].size()); ll v = adj[u][a↩
      ];
    if(vis[v]==cnt || cap[u][v] == 0) continue;
    par[v] = u;
    ll f = dfs(v,min(curf, cap[u][v]));
    if(vis[snk] == cnt) return f;
  }
  for(auto v : adj[u]){
    if(vis[v] == cnt || cap[u][v] == 0) continue;
    par[v] = u;
    ll f = dfs(v,min(curf, cap[u][v]));
    if(vis[snk] == cnt) return f;
  }
  return 0;
}
ll maxflow(ll s, ll t) {
  snk = t; ll flow = 0; cnt++;
  par = vll(n,-1); vis = vll(n,0);
  while(ll new_flow = dfs(s,INF)){
    flow += new_flow; cnt++;
    ll cur = t;
    while(cur != s){
      ll prev = par[cur];
      cap[prev][cur] -= new_flow;
      cap[cur][prev] += new_flow;
      cur = prev;
    }
  }
  return flow;
}
```

3.6 MCMF

```
/*Works for -ve costs, doesn't work for -ve cycles
O(min(E^2 *V log V, E logV * flow))
**INF is used in both flow_type and cost_type*/
const ll INF = 1e9;  // vertices are 0-indexed
struct graph {
  typedef ll flow_type; // **** flow type ****
  typedef ll cost_type; // **** cost type ****
  struct edge {
    int src, dst;
    flow_type cap, flow;
    cost_type cost;
    size_t rev;};
  vector<edge> edges;
  void add_edge(int s, int t, flow_type cap, ↩
     cost_type cost) {
    adj[s].pb({s,t,cap,0,cost,adj[t].size()});
    adj[t].pb({t,s,0,0,-cost,adj[s].size()-1});
  }
  int n; vector<vector<edge>> adj;
  graph(int n) : n(n), adj(n) { }
  pair<flow_type, cost_type> min_cost_max_flow(int↩
      s, int t) {
    flow_type flow = 0;
    cost_type cost = 0;
    for (int u = 0; u < n; ++u) // initialize
      for (auto &e: adj[u]) e.flow = 0;
    vector<cost_type> p(n, 0);
    auto rcost = [&](edge e)
    {return e.cost+p[e.src]-p[e.dst];};
    for (int iter = 0; ; ++iter) {
```

```
    vector<int> prev(n, -1); prev[s] = 0;
    vector<cost_type> dist(n, INF); dist[s] = 0;
    if (iter == 0) {// use Bellman-Ford to
      // remove negative cost edges
      vector<int> count(n); count[s] = 1;
      queue<int> que;
      for (que.push(s); !que.empty(); ) {
        int u = que.front(); que.pop();
        count[u] = -count[u];
        for (auto &e: adj[u]) {
          if (e.cap > e.flow && dist[e.dst] > ↩
              dist[e.src] + rcost(e)) {
            dist[e.dst] = dist[e.src]+rcost(e);
            prev[e.dst] = e.rev;
            if (count[e.dst] <= 0) {
              count[e.dst] = -count[e.dst] + 1;
              que.push(e.dst);
            }
          }
        }
      }
      for(int i=0;i<n;i++) p[i] = dist[i];
      continue; // added last 2 lines
    } else { // use Dijkstra
      typedef pair<cost_type, int> node;
      priority_queue<node, vector<node>, greater↩
          <node>> que;
      que.push({0, s});
      while (!que.empty()) {
        node a = que.top(); que.pop();
        if (a.S == t) break;
        if (dist[a.S] > a.F) continue;
        for (auto e: adj[a.S]) {
          if (e.cap > e.flow && dist[e.dst] > a.↩
              F + rcost(e)) {
            dist[e.dst] = dist[e.src]+rcost(e);
            prev[e.dst] = e.rev;
            que.push({dist[e.dst], e.dst});
          }
        }
      }
    }
    if (prev[t] == -1) break;
    for (int u = 0; u < n; ++u)
      if (dist[u] < dist[t]) p[u] += dist[u] - ↩
          dist[t];
    function<flow_type(int,flow_type)> augment =↩
        [&](int u, flow_type cur) {
      if (u == s) return cur;
      edge &r = adj[u][prev[u]], &e = adj[r.dst↩
          ][r.rev];
      flow_type f = augment(e.src, min(e.cap - e↩
          .flow, cur));
      e.flow += f; r.flow -= f;
      return f;
    };
    flow_type f = augment(t, INF);
    flow += f;
    cost += f * (p[t] - p[s]);
  }
  return {flow, cost};
 }
};
```

3.7 MinCost Matching

```
/*O(n^3) solves 1000x1000 problems in around 1s
cost[i][j] = cost for pairing Li with Rj
Lmate[i]=index of R node that L node i pairs with
Rmate[j]=index of L node that R node j pairs with
cost[i][j] may be +ve or -ve. To perform
maximization,simply negate the cost[][] matrix.*/
typedef ll cost_type;
typedef vector<cost_type> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;
cost_type MCM(const VVD &cost, VI &Lmate, VI &↩
    Rmate) {
  int n = int(cost.size()); VD u(n),v(n);
  for (int i = 0; i < n; i++) {
    u[i] = cost[i][0];
    for (int j = 1; j < n; j++) u[i] = min(u[i], ↩
        cost[i][j]);
  }
  for (int j = 0; j < n; j++) {
    v[j] = cost[0][j] - u[0];
    for (int i = 1; i < n; i++) v[j] = min(v[j], ↩
        cost[i][j] - u[i]);
  }
  Lmate = VI(n, -1); Rmate = VI(n, -1);
  int mated = 0;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
      if (Rmate[j] != -1) continue;
      if (fabs(cost[i][j]-u[i]-v[j]) < 1e-10){
//**** change this comparision if double cost ****
        Lmate[i]=j; Rmate[j]=i; mated++; break;
      }
    }
  }
  VD dist(n); VI dad(n); VI seen(n);
  while (mated < n) {
    int s = 0;
    while (Lmate[s] != -1) s++;
    fill(dad.begin(), dad.end(), -1);
    fill(seen.begin(), seen.end(), 0);
    for (int k = 0; k < n; k++)
      dist[k] = cost[s][k] - u[s] - v[k];
    int j = 0;
```

```
    while (true) {
      j = -1;
      for (int k = 0; k < n; k++) {
        if (seen[k]) continue;
        if (j == -1 || dist[k] < dist[j]) j = k;
      }
      seen[j] = 1;
      if (Rmate[j] == -1) break;
      const int i = Rmate[j];
      for (int k = 0; k < n; k++) {
        if (seen[k]) continue;
        const cost_type new_dist = dist[j] + cost[↩
           i][k] - u[i] - v[k];
        if (dist[k] > new_dist) {
          dist[k] = new_dist;
          dad[k] = j;
        }
      }
    }
    for (int k = 0; k < n; k++) {
      if (k == j || !seen[k]) continue;
      const int i = Rmate[k];
      v[k] += dist[k]-dist[j];
      u[i] -= dist[k]-dist[j];}
    u[s] += dist[j];
    while (dad[j] >= 0) {
      const int d = dad[j];
      Rmate[j]=Rmate[d]; Lmate[Rmate[j]]=j; j=d;}
    Rmate[j] = s; Lmate[s] = j; mated++;
  }
  cost_type value = 0;
  for (int i = 0; i < n; i++)
    value += cost[i][Lmate[i]];
  return value;
}
```

4 Geometry

4.1 Geometry

```
//do not read input in double format
#define PI acos(-1)
//atan2(y,x) slope of line (0,0)->(x,y) in radian ↩
    (-PI,PI]
// to convert to degree multiply by 180/PI
ld INF = 1e100;
ld EPS = 1e-9;
inline bool eq(ld a,ld b) {return fabs(a-b)<EPS;}
inline bool lt(ld a,ld b) {return a+EPS<b;}
inline bool gt(ld a,ld b) {return a>b+EPS;}
inline bool le(ld a,ld b) {return lt(a,b)||eq(a,b)↩
    ;}
inline bool ge(ld a,ld b) {return gt(a,b)||eq(a,b)↩
    ;}
struct pt {
  ld x, y;
  pt() {}
  pt(ld x, ld y) : x(x), y(y) {}
  pt(const pt &p) : x(p.x), y(p.y)    {}
  pt operator + (const pt &p)
  const { return pt(x+p.x, y+p.y); }
  pt operator - (const pt &p)
  const { return pt(x-p.x, y-p.y); }
  pt operator * (ld c)
  const { return pt(x*c,   y*c  ); }
  pt operator / (ld c)
  const { return pt(x/c,   y/c  ); }
  bool operator < (const pt &p)
  const {return lt(y,p.y)||(eq(y,p.y)&&lt(x,p.x))↩
     ;}
  bool operator > (const pt &p)
  const{ return p<pt(x,y);}
  bool operator <= (const pt &p)
  const{ return !(pt(x,y)>p);}
  bool operator >= (const pt &p)
  const{ return !(pt(x,y)<p);}
  bool operator == (const pt &p)
  const{ return (pt(x,y)<=p)&&(pt(x,y)>=p);}
};
ld dot(pt p,pt q) {return p.x*q.x+p.y*q.y;}
ld dist2(pt p, pt q) {return dot(p-q,p-q);}
ld dist(pt p,pt q) {return sqrt(dist2(p,q));}
ld norm2(pt p) {return dot(p,p);}
ld norm(pt p) {return sqrt(norm2(p));}
ld cross(pt p, pt q) { return p.x*q.y-p.y*q.x;}
ostream &operator<<(ostream &os, const pt &p) {
  return os << "(" << p.x << "," << p.y << ")";}
istream& operator >> (istream &is, pt &p){
  return is >> p.x >> p.y;}
//returns 0 if a,b,c are collinear,1 if a->b->c is↩
    cw and -1 if ccw
int orient(pt a,pt b,pt c)
{
  pt p=b-a,q=c-b;double cr=cross(p,q);
  if(eq(cr,0))return 0;if(lt(cr,0))return 1;return↩
      -1;}
// rotate a point CCW or CW around the origin
pt RotateCCW90(pt p)   { return pt(-p.y,p.x); }
pt RotateCW90(pt p)   { return pt(p.y,-p.x); }
pt RotateCCW(pt p, ld t) {//rotate by t degree ccw
return pt(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*↩
   cos(t)); }
// project point c onto line (not segment) through↩
    a and b assuming a != b
pt ProjectPointLine(pt a, pt b, pt c) {
  return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);}
// project point c onto line segment through a and↩
    b (closest point on line segment)
pt ProjectPointSegment(pt a, pt b, pt c) {
```

```
  ld r = dot(b-a,b-a); if (eq(r,0)) return a;//a and b are same
  r = dot(c-a, b-a)/r;if (lt(r,0)) return a;//c on left of a
  if (gt(r,1)) return b; return a + (b-a)*r;}
// compute dist from c to segment between a and b
ld DistancePointSegment(pt a, pt b, pt c) {
  return sqrt(dist2(c, ProjectPointSegment(a, b, c)));}
// compute dist from c to line between a and b
ld DistancePointLine(pt a, pt b, pt c) {
  return sqrt(dist2(c, ProjectPointLine(a, b, c)));}
// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(pt a, pt b, pt c, pt d) {
  return eq(cross(b-a, c-d),0); }
bool LinesCollinear(pt a, pt b, pt c, pt d) {
  return LinesParallel(a, b, c, d) && eq(cross(a-b, a-c),0) && eq(cross(c-d, c-a),0);}
// determine if line segment from a to b intersects with line segment from c to d
bool SegmentsIntersect(pt a, pt b, pt c, pt d) {
  if (LinesCollinear(a, b, c, d)) {
    //a->b and c->d are collinear and have one point common
    if(eq(dist2(a,c),0)||eq(dist2(a,d),0)||eq(dist2(b,c),0)||eq(dist2(b,d),0))
       return true;
    if(gt(dot(c-a,c-b),0)&&gt(dot(d-a,d-b),0)&&gt(dot(c-b,d-b),0)) return false;
    return true;}
  if(gt(cross(d-a,b-a)*cross(c-a,b-a),0)) return false;//c,d on same side of a,b
  if(gt(cross(a-c,d-c)*cross(b-c,d-c),0))
  return false;//a,b on same side of c,d
  return true;}
// compute intersection of line passing through a and b
// with line passing through c and d,assuming that **unique** intersection exists;
//*for segment intersection,check if segments intersect first
pt ComputeLineIntersection(pt a,pt b,pt c,pt d){
  b=b-a;d=c-d;c=c-a;//lines must not be collinear
  assert(gt(dot(b, b),0)&&gt(dot(d, d),0));
  return a + b*cross(c, d)/cross(b, d);}
//returns true if point a,b,c are collinear and b lies between a and c
bool between(pt a,pt b,pt c){
  if(!eq(cross(b-a,c-b),0))return 0;//not collinear
  return le(dot(b-a,b-c),0);
}
//compute intersection of line segment a-b and c-d
```

```
pt ComputeSegmentIntersection(pt a,pt b,pt c,pt d){
  if(!SegmentsIntersect(a,b,c,d))return {INF,INF};//don't intersect
  //if collinear then infinite intersection points, this returns any one
  if(LinesCollinear(a,b,c,d)){if(between(a,c,b))return c;if(between(c,a,d))return a;return b;}
  return ComputeLineIntersection(a,b,c,d);
}
// compute center of circle given three points - *a,b,c shouldn't be collinear
pt ComputeCircleCenter(pt a,pt b,pt c){
  b=(a+b)/2;c=(a+c)/2;
  return ComputeLineIntersection(b,b+RotateCW90(a-b),c,c+RotateCW90(a-c));}
//point in polygon using winding number -> returns 0 if point is outside
//winding number>0 if point is inside and equal to 0 if outside
//draw a ray to the right and add 1 if side goes from up to down and -1 otherwise
bool PointInPolygon(const vector<pt> &p,pt q){
  int n=p.size(),windingNumber=0;
  for(int i=0;i<n;++i){
    if(eq(dist2(q,p[i]),0)) return 1;//q is a vertex
    int j=(i+1)%n;
    if(eq(p[i].y,q.y)&&eq(p[j].y,q.y)) {//i,i+1 vertex is vertical
      if(le(min(p[i].x,p[j].x),q.x)&&le(q.x,max(p[i].x, p[j].x))) return 1;}//q lies on boundary
    else {
      bool below=lt(p[i].y,q.y);
      if(below!=lt(p[j].y,q.y)) {
        auto orientation=orient(q,p[j],p[i]);
        if(orientation==0) return 1;//q lies on boundary i->j
        if(below==(orientation>0)) windingNumber+=below?1:-1;}}}
  return windingNumber==0?0:1;
}
// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<pt> &p,pt q) {
  for (int i = 0; i < p.size(); i++)
    if (eq(dist2(ProjectPointSegment(p[i],p[(i+1)%p.size()],q),q),0)) return true;
  return false;}
// Compute area or centroid of any polygon (coordinates must be listed in cw/ccw
//fashion.The centroid is often known as center of gravity/mass
ld ComputeSignedArea(const vector<pt> &p) {
```

```cpp
  ld ans=0;
  for(int i = 0; i < p.size(); i++) {
    int j = (i+1) % p.size();
    ans+=cross(p[i],p[j]);
  } return ans / 2.0;}
ld ComputeArea(const vector<pt> &p) {
  return fabs(ComputeSignedArea(p));
}
// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<pt> CircleLineIntersection(pt a, pt b, pt c, ld r) {
  vector<pt> ret;
  b = b-a;a = a-c;
  ld A = dot(b, b),B = dot(a, b),C = dot(a, a) - r*r,D = B*B - A*C;
  if (lt(D,0)) return ret;  //don't intersect
  ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
  if (gt(D,0)) ret.push_back(c+a+b*(-B-sqrt(D))/A);
  return ret;}
// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<pt> CircleCircleIntersection(pt a, pt b, ld r, ld R) {
  vector<pt> ret;
  ld d = sqrt(dist2(a, b)),d1=dist2(a,b);
  pt inf(INF,INF);
  if(eq(d1,0)&&eq(r,R)){ret.pb(inf);return ret;}//circles are same return (INF,INF)
  if(gt(d,r+R) || lt(d+min(r, R),max(r, R)) ) return ret;
  ld x = (d*d-R*R+r*r)/(2*d),y = sqrt(r*r-x*x);
  pt v = (b-a)/d;
  ret.push_back(a+v*x + RotateCCW90(v)*y);
  if (gt(y,0)) ret.push_back(a+v*x - RotateCCW90(v)*y);
  return ret;}
ld CircleCircleIntersectionArea(pt c1,ld r1,pt c2,ld r2){
  if(lt(r1,r2))swap(c1,c2),swap(r1,r2);
  ld d=dist2(c1,c2),d1=dist(c1,c2);
  if(le(r1+r2,d1))return 0;
  if(ge(r1-r2,d1))return PI*r2*r2;
  ld alfa=acos((d+r1*r1-r2*r2)/(2*d1*r1));
  ld beta=acos((d+r2*r2-r1*r1)/(2*d1*r2));
  return alfa*r1*r1+beta*r2*r2-sin(2*alfa)*r1*r1/2-sin(2*beta)*r2*r2/2;}
//compute centroid of simple polygon by dividing it into disjoint triangles
//and taking weighted mean of their centroids (Jerome)
pt ComputeCentroid(const vector<pt> &p) {
  pt c(0,0),inf(INF,INF);
  ld scale = 6.0 * ComputeSignedArea(p);
  if(p.empty())return inf;//empty vector
  if(eq(scale,0))return inf;//all points on straight line
  for (int i = 0; i < p.size(); i++){
    int j = (i+1) % p.size();
    c = c + (p[i]+p[j])*cross(p[i],p[j]);}
  return c / scale;}
// tests whether or not a given polygon (in CW or CCW order) is simple
bool IsSimple(const vector<pt> &p) {
  for (int i = 0; i < p.size(); i++) {
    for (int k = i+1; k < p.size(); k++) {
      int j = (i+1) % p.size();
      int l = (k+1) % p.size();
      if (i == l || j == k) continue;
      if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
        return false;}}
  return true;}
/*point in convex polygon
****bottom left point must be at index 0 and top is the index of upper right vertex
****if not call make_hull once*/
bool pointinConvexPolygon(vector<pt> poly,pt point, int top) {
  if (point < poly[0] || point > poly[top]) return 0;//0 for outside and 1 for on/inside
  auto orientation = orient(point, poly[top], poly[0]);
  if (orientation == 0) {
    if (point == poly[0] || point == poly[top]) return 1;
    return top == 1 || top + 1 == poly.size() ? 1 : 1;//checks if point lies on boundary when
    //bottom and top points are adjacent
  } else if (orientation < 0) {
    auto itRight = lower_bound(poly.begin() + 1, poly.begin() + top, point);
    return orient(itRight[0], point, itRight[-1])<=0;
  } else {
    auto itLeft = upper_bound(poly.rbegin(), poly.rend() - top-1, point);
    return (orient(itLeft == poly.rbegin() ? poly[0] : itLeft[-1], point, itLeft[0]))<=0;
  }
}
/*maximum distance between two points in convexy polygon using rotating calipers
make sure that polygon is convex. if not call make_hull first*/
ld maxDist2(vector<pt> poly) {
```

```
  int n = poly.size();
  ld res=0;
  for (int i = 0, j = n < 2 ? 0 : 1; i < j; ++i)
    for (;; j = j+1 %n) {
        res = max(res, dist2(poly[i], poly[j]));
      if (gt(cross(poly[j+1 % n] - poly[j],poly[i+1] - poly[i]),0)) break;
  }
  return res;
}
//Line polygon intersection: check if given line intersects any side of polygon
//if yes then line intersects. If no, then check if its midpoint is inside polygon
//if midpoint is inside then line is inside else outside
// compute distance between point (x,y,z) and plane ax+by+cz=d
ld DistancePointPlane(ld x,ld y,ld z,ld a,ld b,ld c,ld d)
{ return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);}
```

4.2 Convex Hull

```
pt firstpoint;
//for sorting points in ccw(counter clockwise) direction w.r.t firstpoint (leftmost and bottommost)
bool compare(pt x,pt y){
  ll o=orient(firstpoint,x,y);
  if(o==0)return lt(x.x+x.y,y.x+y.y);
  return o<0;}
/*poi->input points, hull->empty vector
returns hull in ccw order with min points*/
void make_hull(vector<pt>& poi,vector<pt>& hull){
  pair<ld,ld> bl={INF,INF};
  ll n=poi.size();ll ind;
  for(ll i=0;i<n;i++){
    pair<ld,ld> pp={poi[i].y,poi[i].x};
    if(pp<bl){
      ind=i;bl={poi[i].y,poi[i].x};}
  }
  swap(bl.F,bl.S);firstpoint=pt(bl.F,bl.S);
  vector<pt> cons;
  for(ll i=0;i<n;i++){
    if(i==ind)continue;cons.pb(poi[i]);}
  sort(cons.begin(),cons.end(),compare);
  hull.pb(firstpoint);ll m;
  for(auto z:cons){
    if(hull.size()<=1){hull.pb(z);continue;}
    pt pr,ppr;bool fl=true;
    while((m=hull.size())>=2){
      pr=hull[m-1];ppr=hull[m-2];
      ll ch=orient(ppr,pr,z);
      if(ch==-1){break;}
      else if(ch==1){hull.pop_back();continue;}
      else {
        ld d1,d2;
        d1=dist2(ppr,pr);d2=dist2(ppr,z);
        if(gt(d1,d2)){fl=false;break;}else {hull.pop_back();}
      }
    }
    if(fl){hull.push_back(z);}
  }
  return;
}
```

4.3 Li Chao Tree

```
/*All pair of lines must not intersect at more
than 1 points*/
void add_l(pt nw,int v=1,int l=0,int r=maxn) {
   int m = (l + r) / 2;
   bool lef = f(nw, l) < f(line[v], l);
   bool mid = f(nw, m) < f(line[v], m);
   if(mid) swap(line[v], nw);
   if(r - l == 1) return;
   else if(lef != mid) add_line(nw, 2 * v, l, m);
   else add_line(nw, 2 * v + 1, m, r);}
int get(int x,int v=1,int l=0,int r=maxn) {
  int m=(l+r)/2;
  if(r - l == 1) return f(line[v], x);
   else if(x < m)
    return min(f(line[v],x),get(x,2*v,l,m));
   else
    return min(f(line[v],x),get(x,2*v+1,m,r));}
```

4.4 Convex Hull Trick

```
/*maintains upper convex hull of lines ax+b and gives minimum value at a given x
to add line ax+b: sameoldcht.addline(a,b), to get min value at x: sameoldcht.getbest(x)
to get maximum value at x add -ax-b as lines instead of ax+b and use -sameoldcht.getbest(x)*/
const int N = 1e5 + 5;
int n,a[N],b[N];ll dp[N];
struct line{
  ll a , b;double xleft;bool type;
  line(ll _a , ll _b){a = _a;b = _b;type = 0;}
  bool operator < (const line &other) const{
    if(other.type){return xleft < other.xleft;}
    return a > other.a;}
};
double meet(line x , line y){
  return 1.0 * (y.b - x.b) / (x.a - y.a);}
struct cht{
  set <line> hull;
  cht() {hull.clear();}
  typedef set < line > :: iterator ite;
```

```
  bool hasleft(ite node){
    return node != hull.begin();}
  bool hasright(ite node){
    return node != prev(hull.end());}
  void updateborder(ite node){
    if(hasright(node)){line temp = *next(node);
      hull.erase(temp);
      temp.xleft=meet(*node,temp);
      hull.insert(temp);}
    if(hasleft(node)){line temp = *node;
      temp.xleft = meet(*prev(node) , temp);
      hull.erase(node);hull.insert(temp);}
    else{
      line temp = *node;hull.erase(node);
      temp.xleft = -1e18;hull.insert(temp);}
  }
  bool useless(line left,line middle,line right){
    double x = meet(left,right);
    double y = x * middle.a + middle.b;
    double ly = left.a * x + left.b;
    return y > ly;}
  bool useless(ite node){
    if(hasleft(node) && hasright(node)){return
      useless(*prev(node),*node,*next(node));}
    return 0;}
  void addline(ll a , ll b){
    line temp = line(a , b);
    auto it = hull.lower_bound(temp);
    if(it != hull.end() && it -> a == a){
      if(it -> b > b){hull.erase(it);}
      else return;}
    hull.insert(temp);it = hull.find(temp);
    if(useless(it)){hull.erase(it);return;}
    while(hasleft(it) && useless(prev(it))){
      hull.erase(prev(it));}
    while(hasright(it) && useless(next(it))){
      hull.erase(next(it));}
    updateborder(it);}
  ll getbest(ll x){
    if(hull.empty())return 1e18;
    line query(0 , 0);
    query.xleft = x;query.type = 1;
    auto it = hull.lower_bound(query);
    it = prev(it);
    return it -> a * x + it -> b;}
};
cht sameoldcht;
int main(){
  sameoldcht.addline(b[1] , 0);
    dp[i] = sameoldcht.getbest(a[i]);
    sameoldcht.addline(b[i] ,dp[i]);}
```

5 Trees

5.1 BlockCut Tree

```
// Take care it is 0 indexed -_-
struct BiconnectedComponents {
  struct Edge {
    int from, to;
  };
  struct To {
    int to; int edge;
  };
  vector<Edge> edges; vector<vector<To> > g;
  vector<int> low, ord, depth;
  vector<bool> isArtic;vll edgeColor;
  vector<int> edgeStack;
  int colors; int dfsCounter;
  void init(int n) {
    edges.clear();
    g.assign(n, vector<To>());
  }
  void addEdge(int u, int v) {
    if(u > v) swap(u, v); Edge e = { u, v };
    int ei = edges.size(); edges.push_back(e);
    To tu = { v, ei }, tv = { u, ei };
    g[u].push_back(tu); g[v].push_back(tv);
  }
  void run() {
    int n = g.size(), m = edges.size();
    low.assign(n, -2); ord.assign(n, -1);
    depth.assign(n, -2);isArtic.assign(n,false);
    edgeColor.assign(m, -1); edgeStack.clear();
    colors = 0;
    for(int i = 0; i < n; ++ i) if(ord[i] == -1) {
      dfsCounter = 0;
      dfs(i);
    }
  }
private:
  void dfs(int i) {
    low[i] = ord[i] = dfsCounter ++;
    for(int j=0;j<(int)g[i].size();++j) {
      int to = g[i][j].to, ei = g[i][j].edge;
      if(ord[to] == -1) {
        depth[to] = depth[i] + 1;
        edgeStack.push_back(ei);
        dfs(to);
        low[i] = min(low[i], low[to]);
        if(low[to] >= ord[i]) {
          if(ord[i] != 0 || j >= 1)
            isArtic[i] = true;
          while(!edgeStack.empty()) {
            int fi=edgeStack.back();
            edgeStack.pop_back();
            edgeColor[fi] = colors;
            if(fi == ei) break;
```

```
        } ++colors;
      }
    }else if(depth[to] < depth[i] - 1) {
      low[i] = min(low[i], ord[to]);
      edgeStack.push_back(ei);
    }
  }
 }
};
```

5.2 Bridge Tree

```
vll tree[N],g[N];//edge list rep. of graph
ll U[M],V[M],vis[N],arr[N],T,dsu[N];
bool isbridge[M]; // if i'th edge is a bridge edge
ll adj(ll u,ll e) {
  return U[e]^V[e]^u;
}
ll f(ll x) {
  return dsu[x]=(dsu[x]==x?x:f(dsu[x]));
}
void merge(ll a,ll b) {
  dsu[f(a)]=f(b);
}
ll dfs0(ll u,ll edge) { //mark bridges
  vis[u]=1;
  arr[u]=T++;
  ll dbe = arr[u];
  for(auto e : g[u]) {
    ll w = adj(u,e);
    if(!vis[w])dbe = min(dbe,dfs0(w,e));
    else if(e!=edge)dbe = min(dbe,arr[w]);
  }
  if(dbe==arr[u] && edge!=-1)isbridge[edge]=true;
  else if(edge!=-1)merge(U[edge],V[edge]);
  return dbe;
}
void buildBridgeTree(ll n,ll m) {
  for(ll i=1; i<=n; i++)dsu[i]=i;
  for(ll i=1; i<=n; i++)if(!vis[i])dfs0(i,-1);
  for(ll i=1; i<=m; i++)
    if(f(U[i])!=f(V[i])) {
      tree[f(U[i])].pb(f(V[i]));
      tree[f(V[i])].pb(f(U[i]));
    }
}
ll n,m;
for(i=1;i<=m;i++)
  cin>>U[i]>>V[i]; g[U[i]].pb(i); g[V[i]].pb(i);
buildBridgeTree(n,m);
```

5.3 Dominator Tree

```
/*g:adjacency matrix (directed graph).
tree rooted at node 1. call dominator().
tree: undirected graph of dominators rooted
at node 1 (use visit times while doing dfs)*/
const int N = int(2e5)+10;
vi g[N],tree[N],rg[N],bucket[N];
int sdom[N],par[N],dom[N],dsu[N],label[N];
int arr[N],rev[N],T;
int Find(int u,int x=0){
  if(u==dsu[u])return x?-1:u;
  int v = Find(dsu[u],x+1);
  if(v<0)return u;
  if(sdom[label[dsu[u]]] < sdom[label[u]])
    label[u] = label[dsu[u]];
  dsu[u] = v;
  return x?v:label[u];}
void Union(int u,int v) {dsu[v]=u;}
void dfs0(int u){
  T++;arr[u]=T;rev[T]=u;
  label[T]=T;sdom[T]=T;dsu[T]=T;
  for(int i=0;i<g[u].size();i++){
    int w = g[u][i];
    if(!arr[w])dfs0(w),par[arr[w]]=arr[u];
    rg[arr[w]].pb(arr[u]);
  }
}
void dominator(){
  dfs0(1);int n=T;
  for(int i=n;i>=1;i--){
    for(int j=0;j<rg[i].size();j++)
      sdom[i] = min(sdom[i],sdom[Find(rg[i][j])]);
    if(i>1)bucket[sdom[i]].pb(i);
    for(int j=0;j<bucket[i].size();j++){
      int w = bucket[i][j];
      int v = Find(w);
      if(sdom[v]==sdom[w])dom[w]=sdom[w];
      else dom[w] = v;}
    if(i>1)Union(par[i],i);
  }
  for(int i=2;i<=n;i++){
    if(dom[i]!=sdom[i]) dom[i]=dom[dom[i]];
    tree[rev[i]].pb(rev[dom[i]]);
    tree[rev[dom[i]]].pb(rev[i]);
  }
}
```

5.4 Bridges Online

```
vector<int> par(MAX), dsu_2ecc(MAX), dsu_cc(MAX), ↩
   dsu_cc_size(MAX);
int bridges,lca_iteration;
vector<int> last_visit(MAX);
void init(int n) {
  lca_iteration = 0;
  for (int i=0; i<n; ++i) {
    dsu_2ecc[i] = i; dsu_cc[i] = i;
    dsu_cc_size[i] = 1; par[i] = -1;
    last_visit[i]=0;
  } bridges = 0;
```

```
}
int find_2ecc(int v) {  // 2-edge connected comp.
  if (v == -1) return -1;
  return dsu_2ecc[v] == v ? v : dsu_2ecc[v] = ↩
     find_2ecc(dsu_2ecc[v]);
}
int find_cc(int v) {  // connected comp.
  v = find_2ecc(v);
  return dsu_cc[v] == v ? v : dsu_cc[v] = find_cc(↩
     dsu_cc[v]);
}
void make_root(int v) {
  v = find_2ecc(v);
  int root = v;int child = -1;
  while (v != -1) {
    int p = find_2ecc(par[v]);
    par[v] = child; dsu_cc[v] = root;
    child = v; v = p;
  }
  dsu_cc_size[root] = dsu_cc_size[child];
}
vector<int> path_a, path_b;
void merge_path (int a, int b) {
  ++lca_iteration;
  int lca = -1;
  while (lca == -1) {
    if (a != -1) {
      a = find_2ecc(a); path_a.push_back(a);
      if (last_visit[a] == lca_iteration) lca = a;
      last_visit[a] = lca_iteration; a=par[a];
    }
    if (b != -1) {
      b = find_2ecc(b); path_b.push_back(b);
      if (last_visit[b] == lca_iteration) lca = b;
      last_visit[b] = lca_iteration; b = par[b];
    }
  }
  for (int v : path_a) {
    dsu_2ecc[v] = lca; if (v == lca)break;
    --bridges; }
  for (int v : path_b) {
    dsu_2ecc[v] = lca; if (v == lca)break;
    --bridges;}
  path_a.clear();path_b.clear();
}
void add_edge(int a, int b) {
  a = find_2ecc(a); b = find_2ecc(b);
  if (a == b) return;
  int ca = find_cc(a); int cb = find_cc(b);
  if (ca != cb) { ++bridges;
    if (dsu_cc_size[ca] > dsu_cc_size[cb]) {
      swap(a, b); swap(ca, cb);}
    make_root(a); par[a] = dsu_cc[a] = b;
    dsu_cc_size[cb] += dsu_cc_size[a];
  } else { merge_path(a, b);}
}
```

5.5 HLD

```
/*v : adj matrix of tree.clear v[i],hdc[i]=0,i=-1 ↩
   before every run,clear ord and curc=0*/
vll v[MAX],ord;
ll par[MAX],noc[MAX],hdc[MAX],curc,posinch[MAX],↩
   len[MAX],ti=-1,sta[MAX],en[MAX],subs[MAX],level[↩
   MAX];
ll st[4*MAX],lazy[4*MAX],n;
void dfs(ll x){
    subs[x]=1;
    for(auto z:v[x]){
    if(z!=par[x]){par[z]=x;level[z]=level[x]+1;
      dfs(z);subs[x]+=subs[z];
    }}}
void makehld(ll x){
    if(hdc[curc]==0){hdc[curc]=x;len[curc]=0;}
    noc[x]=curc;posinch[x]=++len[curc];
    ll a,b,c;a=b=0;ord.pb(x);sta[x]=++ti;
    for(auto z:v[x]){   if(z==par[x])continue;
    if(subs[z]>b){b=subs[z];a=z;}
    }
    if(a!=0)makehld(a);
    for(auto z:v[x]){if(z==par[x]||z==a)continue;↩
       curc++;makehld(z);}
    en[x]=ti;}
inline void upd(ll x,ll y){//update path a->b
  ll a,b,c,d;
  while(x!=y){a=hdc[noc[x]],b=hdc[noc[y]];
    if(a==b){
      if(level[x]>level[y])swap(x,y);c=sta[x],d=↩
         sta[y];
      //lca=a;
      update(1,0,n-1,c+1,d);return;}
    if(level[a]>level[b])swap(a,b),swap(x,y);
    //update on seg tree
    update(1,0,n-1,sta[b],sta[y]);y=par[b];}}
int main(){
    loop: v[i].clear(),hdc[i]=0,ti=-1;
    ord.clear(),curc=0;
    level[1]=0;par[1]=0;curc=1;dfs(1);makehld(1);
    while(q--){cin>>a>>b;upd(a,b);ll ans=sumq(1,0,↩
       n-1,0,n-1);}
}
```

5.6 LCA

```
int lca(int a,int b){
  if(level[a]>level[b])swap(a,b);
  int d=level[b]-level[a];
  for(int i=0;i<LOGN;i++)if(d&(1<<i))
      b=DP[i][b];
  if(a==b)return a;
  for(int i=LOGN-1;i>=0;i--)
    if(DP[i][a]!=DP[i][b])
      a=DP[i][a],b=DP[i][b];
```

```
    return DP[0][a];}
```

5.7 Centroid Decompostion

```
/*nx:max nodes,par:parents of nodes in centroid ↩
    tree,timstmp: timestamps of nodes when they ↩
    became centroids,ssize,vis: subtree size and ↩
    visit times in dfs,tim: timestamp iterator
dist[nx]: dist[i][0][j]=no. of nodes at dist k in ↩
    subtree of i in centroid tree
dist[i][j][k]=no. of nodes at distance k in jth ↩
    child of i in centroid tree ***(use adj while ↩
    doing dfs instead of adj1)***
preprocess: stores all values in dist array*/
const int nx=1e5;
vector<int> adj[nx],adj1[nx];
int par[nx],timstmp[nx],ssize[nx],vis[nx];
int tim=1;
vector<int> cntrorder;//centroids in order
vector<vector<int> > dist[nx];
int dfs(int root){
  vis[root]=tim;
  int t=0;
  for(auto i:adj[root]){
    if(!timstmp[i]&&vis[i]<tim)t+=dfs(i);}
  ssize[root]=t+1;return t+1;}
int dfs1(int root,int n){
  vis[root]=tim;pair<int,int> mxc={0,-1};
  bool poss=true;
  for(auto i:adj[root]){
    if(!timstmp[i]&&vis[i]<tim)
      poss&=(ssize[i]<=n/2),mxc=max(mxc,{ssize[i],↩
          i});;}
  if(poss&&(n-ssize[root])<=n/2)return root;
  return dfs1(mxc.second,n);}
int findc(int root){
  dfs(root);
  int n=ssize[root];tim++;
  return dfs1(root,n);}
void cntrdecom(int root,int p){
  int cntr=findc(root);
  cntrorder.pb(cntr);
  timstmp[cntr]=tim++;par[cntr]=p;
  if(p>=0)adj1[p].pb(cntr);
  for(auto i:adj[cntr])
    if(!timstmp[i])
      cntrdecom(i,cntr);}
void dfs2(int root,int nod,int j,int dst){
  if(dist[root][j].size()==dst)dist[root][j].pb(0)↩
      ;
  vis[nod]=tim;dist[root][j][dst]+=1;
  for(auto i:adj[nod]){
    if((timstmp[i]<=timstmp[root])||(vis[i]==vis[↩
        nod]))continue;
    vis[i]=tim;dfs2(root,i,j,dst+1);}
}
void preprocess(){
  for(int i=0;i<cntrorder.size();i++){
    int root=cntrorder[i];
    vector<int> temp;
    dist[root].pb(temp);temp.pb(0);++tim;
    dfs2(root,root,0,0);
    int cnt=0;
    for(int j=0;j<adj[root].size();j++){
      int nod=adj[root][j];
      if(timstmp[nod]<timstmp[root])
        continue;
      dist[root].pb(temp);++tim;
      dfs2(root,nod,++cnt,1);}
  }
}
```

6 Maths

6.1 Chinese Remainder Theorem

```
/*x=rem[i]%mods[i] for any mods
intput: rem->remainder,mods->moduli
output: (x%lcm of mods,lcm),-1 if infeasible*/
ll LCM(ll a, ll b) { return a /__gcd(a, b) * b; }
ll normalize(ll x,ll mod)
{x %= mod; if (x < 0) x += mod; return x; }
struct GCD_type { ll x, y, d; };
GCD_type ex_GCD(ll a, ll b){
    if (b == 0) return {1, 0, a};
    GCD_type pom = ex_GCD(b, a % b);
    return {pom.y, pom.x - a / b * pom.y, pom.d};
}
pll CRT(vll &rem,vll &mods){
    ll n=rem.size(),ans=rem[0],lcm=mods[0];
    for(ll i=1;i<n;i++){
        auto pom=ex_GCD(lcm,mods[i]);
        ll x1=pom.x,d=pom.d;
        if((rem[i]-ans)%d!=0)return {-1,0};
        ans=normalize(ans+x1*(rem[i]-ans)/d%(mods[↩
            i]/d)*lcm,lcm*mods[i]/d);
        lcm=LCM(lcm,mods[i]); // you can save time↩
            by replacing above lcm * n[i] /d by lcm↩
            = lcm * n[i] / d}
    return {ans,lcm};
}
```

6.2 Discrete Log

```
/*Discrete Log , Baby-Step Giant-Step , e-maxx
The idea is to make two functions, f1(p) ,f2(q)
and find p,q s.t. f1(p) = f2(q) by storing all
possible values of f1, and checking for q. In
this case a^(x) = b (mod m) is solved by subst.
x by p.n-q , where n is choosen optimally.*/
/*returns a soln. for a^(x) = b (mod m) for
```

```
given a,b,m; -1 if no. soln; O(sqrt(m).log(m))
use unordered_map to remove log factor.
IMP : works only if a,m are co-prime.
But can be modified.*/
int solve (int a, int b, int m) {
    int n = (int) sqrt (m + .0) + 1;
    int an = 1;
    for (int i=0; i<n; ++i)
        an = (an * a) % m;
    map<int,int> vals;
    for (int i=1, cur=an; i<=n; ++i) {
        if (!vals.count(cur))
            vals[cur] = i;
        cur = (cur * an) % m;
    }
    for (int i=0, cur=b; i<=n; ++i) {
        if (vals.count(cur)) {
            int ans = vals[cur] * n - i;
            if (ans < m) return ans;
        }
        cur = (cur * a) % m;
    }
    return -1;
}
```

6.3 NTT

```
/**a*b%mod if a%mod*b%mod results in overflow:
  ll mulmod(ll a, ll b, ll mod) {ll res = 0;
    while (a!=0){if(a&1)(res+=b)%=mod;a>>=1;(b<<=1)%=mod;}
    return res;}
P=A*B A[0]=coeff of x^0
x = a1 mod p1, x = a2 mod p2 => x=((a1*(m2^-1)%m1)*m2+(a2*(m1^-1)%m2)*m1)%m1m2
***max_base=x (s.t. if mod=c*(2^k)+1 then x<=k and 2^x >= nearest power of 2 of 2*n)
root=primitive_root^((mod-1)/(2^max_base))
For P=A*A use square function
635437057,11|639631361,6|985661441&998244353,3*/
#define chinese(a1,m1,invm2m1,a2,m2,invm1m2) ((a1 *1ll* invm2m1 % m1 * 1ll*m2 + a2 *1ll* invm1m2 % m2 * 1ll*m1) % (m1 *1ll* m2))
int mod;//reset mod everytime
int base = 1;
vll roots = {0, 1},rev = {0, 1};
int max_base=18,root=202376916;
void ensure_base(int nbase) {
  if (nbase <= base) return;
  rev.resize(1 << nbase);
  for (int i = 0; i < (1 << nbase); i++) {
    rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (nbase - 1));}
  roots.resize(1 << nbase);
  while (base < nbase) {
    int z = power(root, 1 << (max_base - 1 - base));
    for (int i = 1 << (base - 1); i < (1 << base); i++) {
      roots[i << 1] = roots[i];
      roots[(i << 1) + 1] = mul(roots[i], z);
    }
    base++;
  }
}
void fft(vll &a) {
  int n = (int) a.size();
  int zeros = __builtin_ctz(n);
  ensure_base(zeros);
  int shift = base - zeros;
  for (int i = 0; i < n; i++) {
    if (i < (rev[i] >> shift)) {
    swap(a[i], a[rev[i] >> shift]);}
  }
  for (int k = 1; k < n; k <<= 1) {
    for (int i = 0; i < n; i += 2 * k) {
    for (int j = 0; j < k; j++) {
      int x = a[i + j];
      int y = mul(a[i + j + k], roots[j+k]);
      a[i + j] = x + y - mod;
      if (a[i + j] < 0) a[i + j] += mod;
      a[i + j + k] = x - y + mod;
      if(a[i+j+k]>=mod) a[i + j + k] -= mod;}
    }
  }
}
vll multiply(vll a, vll b,int eq = 0) {
  int need = (int) (a.size() + b.size() - 1);
  int nbase = 0;
  while ((1 << nbase) < need) nbase++;
  ensure_base(nbase);
  int sz = 1 << nbase;
  a.resize(sz);b.resize(sz);fft(a);
  if (eq) b = a; else fft(b);
  int inv_sz = inv(sz);
  for (int i = 0; i < sz; i++)
    a[i] = mul(mul(a[i], b[i]), inv_sz);
  reverse(a.begin() + 1, a.end());
  fft(a);a.resize(need);return a;
}
vll square(vll a) {return multiply(a, a, 1);}
```

6.4 Online FFT

```
//f[i]=sum over j from 0 to i-1 f[j]*g[i-1-j]
//handle f[0] and g[0] separately
const int nx=131072;int f[nx],g[nx];
void onlinefft(int a,int b,int c,int d){
  vector<int> v1,v2;
  v1.pb(f+a,f+b+1);v2.pb(g+c,g+d+1); vector<int> res=multiply(v1,v2);
  for(int i=0;i<res.size();i++)
```

```
    if(a+c+i+1<nx) f[a+c+i+1]=add(f[a+c+i+1],res[i↩
        ]);}
void precal(){
  g[0]=1;
  for(int i=1;i<nx;i++)
    g[i]=power(i,i-1);
  f[1]=1;
  for(int i=1;i<=100000;i++){
    f[i+1]=add(f[i+1],g[i]);f[i+1]=add(f[i+1],f[i↩
        ]);
    f[i+2]=add(f[i+2],mul(f[i],g[1]));f[i+3]=add(f↩
        [i+3],mul(f[i],g[2]));
    for(int j=2;i%j==0&&j<nx;j=j*2)
      onlinefft(i-j,i-1,j+1,2*j);}
}
```

6.5 Langrange Interpolation

```
/* Input :
Degree of polynomial: k
Polynomial values at x=0,1,2,3,...,k

Output :
Polynomial value at x
Complexity: O(degree of polynomial)
Works only if the points are equally spaced
*/
ll lagrange(vll& v , int k, ll x,int mod){
    if(x <= k) return v[x];
    ll inn = 1; ll den = 1;
    for(int i = 1;i<=k;i++){
        inn = (inn*(x - i))%mod;
        den = (den*(mod - i))%mod;
    }
    inn = (inn*inv(den % mod))%mod;
    ll ret = 0;
    for(int i = 0;i<=k;i++){
        ret = (ret + v[i]*inn)%mod;
        ll md1 = mod - ((x-i)*(k-i))%mod;
        ll md2 = ((i+1)*(x-i-1))%mod;
        if(i!=k)
        inn = (((inn*md1)%mod)*inv(md2 % mod))%mod↩
            ;
    } return ret;
}
```

6.6 Matrix Struct

```
struct matrix{
  ld B[N][N], n;
  matrix(){n = N; memset(B,0,sizeof B);}
  matrix(int _n)
    {n = _n; memset(B, 0, sizeof B);}
  void iden(){
    for(int i = 0; i < n; i++) B[i][i] = 1;}
  void operator += (matrix M){
    for(int i = 0; i < n; i++)
      for(int j = 0; j < n; j++)
        B[i][j]=add(B[i][j],M.B[i][j]);}
  void operator -= (matrix M){}
  void operator *= (ld b){}
  matrix operator - (matrix M){}
  matrix operator + (matrix M){
    matrix ret = (*this); ret += M; return ret;}
  matrix operator * (matrix M){
    matrix ret = matrix(n); memset(ret.B, 0, ↩
        sizeof ret.B);
    for(int i = 0; i < n; i++)
      for(int j = 0; j < n;j++)
        for(int k = 0; k < n; k++)
          ret.B[i][j] = add(ret.B[i][j], mul(B[i][↩
              k], M.B[k][j]));
    return ret;}
  matrix operator *= (matrix M){*this=((*this)*M)↩
      ;}
  matrix operator * (int b){
    matrix ret =(*this);ret *= b; return ret;}
  vector<double> multiply(const vector<double> & v↩
      ) const{
    vector<double> ret(n);
    for(int i = 0; i < n; i++)
      for(int j = 0; j < n; j++)
        ret[i] += B[i][j] * v[j];
    return ret;
  }
};
```

6.7 nCr(Non Prime Modulo)

```
// calculates nCr, for small
// non-prime modulo, and (very) big n,r.
ll phimod;
vll pr,prn;vll fact;
ll power(ll a,ll x,ll mod){
  ll ans=1;
  while(x){
    if((1LL)&(x))ans=(ans*a)%mod;
    a=(a*a)%mod;x>>=1LL;
  }
  return ans;
}
// prime factorization of x.
// pr-> prime ; prn -> it's exponent
void getprime(ll x){
    pr.clear();prn.clear();
    ll i,j,k;
    for(i=2;(i*i)<=x;i++){
      k=0;while((x%i)==0){k++;x/=i;}
      if(k>0){pr.pb(i);prn.pb(k);}
    }
    if(x!=1){pr.pb(x);prn.pb(1);}
    return;
}
// factorials are calculated ignoring
```

```
//  multiples of p.
void primeproc(ll p,ll pe){   // p , p^e
    ll i,d;
    fact.clear();fact.pb(1);d=1;
    for(i=1;i<pe;i++){
      if(i%p){fact.pb((fact[i-1]*i)%pe);}
      else {fact.pb(fact[i-1]);}
    }
    return;
}
// again note this has ignored multiples of p
ll Bigfact(ll n,ll mod){
  ll a,b,c,d,i,j,k;
  a=n/mod;a%=phimod;a=power(fact[mod-1],a,mod);
  b=n%mod;a=(a*fact[b])%mod;
  return a;
}
// Chinese Remainder Thm.
vll crtval,crtmod;
ll crt(vll &val,vll &mod){
  ll a,b,c,d,i,j,k;b=1;
  for(ll z:mod)b*=z;
  ll ans=0;
  for(i=0;i<mod.size();i++){
    a=mod[i];c=b/a;
    d=power(c,(((a/pr[i])*(pr[i]-1))-1),a);
    c=(c*d)%b;c=(c*val[i])%b;ans=(ans+c)%b;
  }
  return ans;
}
// calculate for prime powers and
// take crt. For each prime power,
// first ignore multiples of p,
// and then do recursively, calculating
// the powers of p separately.
ll Bigncr(ll n,ll r,ll mod){
  ll a,b,c,d,i,j,k;ll p,pe;
  getprime(mod);ll Fnum=1;ll Fden;
  crtval.clear();crtmod.clear();
  for(i=0;i<pr.size();i++){
    Fnum=1;Fden=1;
    p=pr[i];pe=power(p,prn[i],1e17);
    primeproc(p,pe);
    a=1;d=0;
    phimod=(pe*(p-1LL))/p;
    ll n1=n,r1=r,nr=n-r;
    while(n1){
      Fnum=(Fnum*(Bigfact(n1,pe)))%pe;
      Fden=(Fden*(Bigfact(r1,pe)))%pe;
      Fden=(Fden*(Bigfact(nr,pe)))%pe;
      d+=n1-(r1+nr);
      n1/=p;r1/=p;nr/=p;
    }
    Fnum=(Fnum*(power(Fden,(phimod-1LL),pe)))%pe;
    if(d>=prn[i])Fnum=0;
    else Fnum=(Fnum*(power(p,d,pe)))%pe;
    crtmod.pb(pe);crtval.pb(Fnum);
  }
  // you can just iterate instead of crt
  // for(i=0;i<mod;i++){
  //   bool cg=true;
  //   for(j=0;j<crtmod.size();j++){
  //     if(i%crtmod[j]!=crtval[j])cg=false;
  //   }
  //   if(cg)return i;
  // }
  return crt(crtval,crtmod);
}
```

6.8 Primitive Root Generator

```
/*To find generator of U(p),we check for all
g in [1,p]. But only for powers of the
form phi(p)/p_j, where p_j is a prime factor of
phi(p). Note that p is not prime here.
Existence , if one of these : 1. p = 1,2,4
2. p = q^k , where q -> odd prime.
3. p = 2.(q^k) , where q-> odd prime
Note that a.g^(phi(p)) = 1 (mod p)
b.there are phi(phi(p)) generators if exists.
Finds "a" generator of U(p), multiplicative group
of integers mod p. Here calc_phi returns the ↩
   toitent
function for p. O(Ans.log(phi(p)).log(p)) +
time for factorizing phi(p). By some theorem,
Ans = O((log(p))^6). Should be fast generally.*/
int generator (int p) {
  vector<int> fact;
  int phi = calc_phi(p),  n = phi;
  for (int i=2; i*i<=n; ++i)
    if (n % i == 0) {
      fact.push_back (i);
      while (n % i == 0)
        n /= i; }
  if (n > 1)fact.push_back (n);
  for (int res=2; res<=p; ++res) {
    if(gcd(res,p)!=1)continue;
    bool ok = true;
    for (size_t i=0; i<fact.size() && ok; ++i)
      ok &= powmod (res, phi / fact[i], p) != 1;
    if (ok) return res;
  }
  return -1;
}
```

6.9 Math Miscellaneous

```
int gcd(int a,int b,int &x,int &y) {
  if (a == 0) {x = 0; y = 1;return b;}
  int x1,y1,d = gcd(b%a, a, x1, y1);
  x = y1 - (b / a) * x1;y = x1;return d;}
```

```
int g (int n) {return n^(n >> 1);}//nth Gray code
int rev_g (int g) {//index of gray code g
  int n = 0;for (; g; g >>= 1)n ^= g;return n;}
```

6.10 Group Theory

```
x^2 = n mod (p). Existence - n^((p-1)/2) == 1 -> there is a soln.,
else == -1, no solution.
Finding sqrt. in some Z mod p :
Cipollas Algorithm.
Find an 'a' (randomly) , s.t. a^2-n doesnt has a sqrt.
Adjoin it to the field. Take (a+sqrt(a^2-n))^((p+1)/2).
Do all operations mod p, ans will be integer.
Cipollas Algo works only when mod is prime.
[Remember (a+b)^p = a^p + b^p (mod p) = a + b (mod p)]
For non-prime :
x^2 = n (mod m).
Soln. -> Compute it modulo prime powers and take CRT.
For prime powers :
We have a solution x0 mod p. We use it to find a solution (mod p^2),
then (p^3) and so on. For p^2 : x^2 = n (mod p^2); We want x to
reduce to x0 mod p. So x=x0+p*x1. Square it. x0^2+2*x0*x1*p = n mod (p^2).
Calculate x1.This can be extended to find for greater powers of p.
But the inverse may not exist always which may give a problem.
But then no solution or all solutions. This is called Hensel's Lifting.
This can also be extended to find f(x) = 0 mod p^2, if we have a
soln. for f(x) = 0 (mod p). Get something in f'(x).
```

6.11 Gaussian Elimination

```
/*O(n^3)
(1) solving systems of linear equations (AX=B)
(2) inverting matrices (AX=I)
(3) computing determinants of square matrices
INPUT: a[][] = an nxn mat, b[][] = an nxm mat
 A MUST BE INVERTIBLE!
OUTPUT: X = an nxm matrix (stored in b[][])
       A^{-1} = an nxn matrix (stored in a[][])
       returns determinant of a[][]*/
ll GaussJordan(vvll &a, vvll &b) {
  const int n = a.size(), m = b[0].size();
  int n1 = a[0].size();
  vll irow(n),icol(n),ipiv(n,0);ll det = 1;
  for (int i=0;i<n1;i++) {
    int pj=-1,pk=-1;
    for(int j=0;j<n;j++)
      if(!ipiv[j])
        for(int k=0;k<n1;k++)
          if(!ipiv[k])
            if(pj==-1||fabs(a[j][k])>fabs(a[pj][pk]))){pj=j;pk=k;}
    // if(fabs(a[pj][pk])<EPS){return 0;}  // uncomment in double version
    ipiv[pk]++;
    swap(a[pj],a[pk]);swap(b[pj],b[pk]);
    if(a[pk][pk]==0)return 0;  // comment in double version
    if(pj!=pk)det = mul(det,mod-1);   // det*=-1;
    irow[i]=pj;icol[i]=pk;
    ll c=inv(a[pk][pk]);det = mul(det,a[pk][pk]);
    a[pk][pk]=1;
    for(int p=0;p<n1;p++) a[pk][p] = mul(a[pk][p],c);
    for(int p=0;p<m;p++) b[pk][p] = mul(b[pk][p], c);
    for(int p=0;p<n;p++){
      if(p!=pk){
        c=a[p][pk];
        a[p][pk]=0;
        for(int q=0;q<n1;q++) a[p][q] = sub(a[p][q], mul(a[pk][q],c));
        for(int q=0;q<m;q++) b[p][q] = sub(b[p][q], mul(b[pk][q],c));
      }
    }
  }
  // comment below if not square matrix .
  for (int p=n-1;p>=0;p--)
    if(irow[p]!=icol[p]){
      for(int k=0;k<n;k++)
        swap(a[k][irow[p]],a[k][icol[p]]);
    }
  return det;
}
```

6.12 Inclusion-Exclusion

```
//pr is current product, sign is mobius value, i is index in list of primes
//p is list of primes under consideration
void rec(int i,int pr,int sign)
{
  int m=n/pr;
  ans+=sign*m;
  for(int j=i;j<L && p[j]<=m;j++)
    rec(j+1,pr*p[j],-sign);
}
```

7 Strings

7.1 Hashing Theory

```
If order not imp. and count/frequency imp. use this as hash fn:-
( (a_1+h)^k + (a_2+h)^k + (a_3+h)^k + (a_4+h)^k ) % p. Select : h,k,p
Alternate:
((x)^(a_1)+(x)^(a_2)+...+(x)^(a_k))%mod where x and mod are fixed and a_1...a_k is an unordered set
```

7.2 Manacher

```
/*Same idea as Z_fn,O(n)
[l,r]: rightmost detected subpalindrom(with max r)
len of odd length palindrom centered around that
char(e.g abac for 'b' returns 2(not 3))*/
vll manacher_odd(string s){
  ll n = s.length(); vll d1(n);
  for(ll i = 0, l = 0, r = -1;i<n;i++){
    d1[i] = 1;
    if(i <= r){ // use prev val
      d1[i] = min(r-i+1,d1[l+r-i]);}
    while(i+d1[i] < n && i-d1[i] >= 0 && s[i+d1[i]] == s[i-d1[i]])
    d1[i]++; // trivial matching
    if(r < i+d1[i]-1) l=i-d1[i]+1, r=i+d1[i]-1;
  }
  return d1;}
//even lens centered around (bb is centered around the later 'b')
vll manacher_even(string s){
  ll n = s.length(); vll d2(n);
  for(ll i = 0, l = 0, r = -1;i<n;i++){
    d2[i] = 0;
    if(i <= r){
        d2[i] = min(r-i+1,d2[l+r+1-i]);}
    while(i+d2[i] < n && i-d2[i]-1 >= 0 && s[i+d2[i]] == s[i-d2[i]-1]) d2[i]++;
    if(d2[i] > 0 && r < i+d2[i]-1)
      l=i-d2[i], r=i+d2[i]-1;
  }
  return d2;
}
// Other mtd : To do both things in one pass,
// add special char e.g string "abc" => "$a$b$c$"
```

7.3 Trie

```
const ll AS = 26; // alphabet size
ll go[MAX][AS]; ll cnt[MAX];ll cn=0;
// cn -> index of next new node
// convert all strings to vll
ll newNode() {
  for(ll i=0;i<AS;i++)
    go[cn][i]=-1;
  return cn++;
}
// call newNode once **** before adding anything **
void addTrie(vll &x) {
  ll v = 0;
  cnt[v]++;
  for(ll i=0;i<x.size();i++){
    ll y=x[i];
    if(go[v][y]==-1)
      go[v][y]=newNode();
    v=go[v][y];
    cnt[v]++;
  }
}
// returns count of substrings with prefix x
ll getcount(vll &x){
  ll v=0;
  for(i=0;i<x.size();i++){
    ll y=x[i];
    if(go[v][y]==-1)
      go[v][y]=newNode();
    v=go[v][y];
  }
  return cnt[v];
}
```

7.4 Z-algorithm

```
/*[l,r]->rightmost segment match(with max r)
Time : O(n)(asy. behavior), Proof:each itr of
inner while loop make r pointer advance to right,
App:1)Search substring(text t,pat p)s=p+ '$' + t.
3) String compression(s=t+t+..+t, then find |t|)
2) Number of distinct substrings (in O(n^2))
(useful when appending or deleting characters
online from the end or beginning)*/
vector<ll> z_function(string s) {
  ll n = (ll) s.length();
  vector<ll> z(n);
  for (ll i=1, L=0, R=0; i<n; ++i) {
    if (i <= R) // use previous z val
      z[i] = min (R - i + 1, z[i - L]);
    while (i + z[i] < n && s[z[i]] == s[i + z[i]])
      ++z[i]; // trivial matching
    if (i + z[i] - 1 > R) L = i, R = i + z[i] - 1;
    // update rightmost segment matched
  }
  return z;
}
```

7.5 Aho Corasick

```
const int K = 26;
// remember to set K
struct Vertex {
```

```
  int next[K]; bool leaf = false;
  int p = -1;char pch;
  int link = -1; int go[K];
  Vertex(int p=-1, char ch='$') : p(p), pch(ch) {
    fill(begin(next), end(next), -1);
    fill(begin(go), end(go), -1);
  }
};
vector<Vertex> aho(1);
void add_string(string const& s) {
  int v = 0;
  for (char ch : s) {
    int c = ch - 'a';
    if (aho[v].next[c] == -1) {
      aho[v].next[c] = aho.size();
      aho.emplace_back(v, ch);
    }
    v = aho[v].next[c];
  } aho[v].leaf = true;
}
int go(int v, char ch);
int get_link(int v) {
  if (aho[v].link == -1) {
    if (v==0 || aho[v].p==0)aho[v].link = 0;
    else aho[v].link =
      go(get_link(aho[v].p),aho[v].pch);
  }
  return aho[v].link;
}
int go(int v, char ch) {
  int c = ch - 'a';
  if (aho[v].go[c] == -1) {
    if (aho[v].next[c] != -1)
      aho[v].go[c] = aho[v].next[c];
    else
      aho[v].go[c] = v == 0 ? 0 : go(get_link(v), ↩
          ch);
  }
  return aho[v].go[c];
}
```

7.6 KMP

```
/*Time:O(n)(j increases n times(& j>=0) only so
asy. O(n)), pi[i] = length of longset prefix of
s ending at i
app.: search substring,
# of different substrings(O(n^2)),
3) String compression(s = t+t+...+t,
then find |t|, k=n-pi[n-1],if k|n)
4) Building Automaton(Gray Code Example)*/
vector<ll> prefix_function(string s) {
  ll n = (ll)s.length(); vll pi(n);
  for (ll i = 1; i < n; i++) {
    ll j = pi[i-1];
    while (j > 0 && s[i] != s[j]) j = pi[j-1];
    if (s[i] == s[j]) j++;
    pi[i] = j;
  }
  return pi;}
//searching s in t, returns all occurences(indices
vector<ll> search(string s,string t){
  vll pi = prefix_function(s);
  ll m = s.length(); vll ans; ll j = 0;
  for(ll i=0;i<t.length();i++){
    while(j > 0 && t[i] != s[j])
        j = pi[j-1];
    if(t[i] == s[j]) j++;
    if(j == m) ans.pb(i-m+1);
  } // if ans empty then no occurence
  return ans;}
```

7.7 Palindrome Tree

```
ll par[MAX];  // stores index of parent node
ll suli[MAX]; // stores index of suffix link
ll len[MAX];  /* stores len of largest
 pallindrome ending at that node */
ll child[MAX][30]; // stores the children of the ↩
   node
ll nodeno[MAX];
/*-----------------------------------------
index 0 - root "-1"
index 1 - root  "0"
therefore node of s[i] is i+2
initialize all child[i][j] to -1
-----------------------------------------------↩
   */
void eer_tree(string s){
  ll a,b,c,d,i,j,k,e,f;
  suli[1]=0;suli[0]=0;len[1]=0;len[0]=-1;
  ll n=s.length();
  for(i=0;i<n+10;i++)
    for(j=0;j<30;j++)child[i][j]=-1;
  ll cur=1;d=1;
  for(i=0;i<s.size();i++){
    ++d;
    while(true){
      a=i-1-len[cur];
      if(a>=0){
        if(s[a]==s[i]){
          if(child[cur][(ll)(s[i]-'a')]==-1){
            par[d]=cur;child[cur][(ll)(s[i]-'a')]=↩
               d;
            len[d]=len[cur]+2;cur=d;
          }
          else{
            par[d]=cur;len[d]=len[cur]+2;
            cur=child[cur][(ll)(s[i]-'a')];
          }
          break;
        }
      }
```

```
        if(cur==0)break;
        cur=suli[cur];
      }
      nodeno[d] = cur;
      if(cur!=d)continue;
      if(len[d]==1)suli[d]=1;
      else{
        c=suli[par[d]];
        while(child[c][(ll)(s[i]-'a')]==-1){
          if(c==0)break;
          c=suli[c];
        }
        suli[d]=child[c][(ll)(s[i]-'a')];
      }
    }
}
```

7.8 Suffix Array

```
/*Sorted array of suffixes = sorted array of ↩
   cyclic
shifts of string+$.We consider a prefix of len. 2^↩
   k
of the cyclic, in the kth iteration.String of len.
2^k->combination of 2 strings of len. 2^(k-1), ↩
   whose
order we know, from previous iteration.Just radix
sort on pair for next iteration.
Time :- O(nlog(n) + alphabet). Applications :-
Finding the smallest cyclic shift;Finding a ↩
   substring
in a string; Comparing two substrings of a string;
Longest common prefix of two substrings; Number of
different substrings. */
//returns permutation of indices in sorted order ↩
   ***
vector<ll> sort_cyclic_shifts(string const& s) {
  ll n = s.size();
  const ll alphabet = 256;
//change the alphabet size accordingly and ↩
   indexing
  vector<ll> p(n), c(n), cnt(max(alphabet, n), 0);
//  p:sorted ord. of 1-len prefix of each cyclic
//  shift index. c:class of a index
//  pn:same as p for kth iteration . ||ly cn.
  for (ll i = 0; i < n; i++)
    cnt[s[i]]++;
  for (ll i = 1; i < alphabet; i++)
    cnt[i] += cnt[i-1];
  for (ll i = 0; i < n; i++)
    p[--cnt[s[i]]] = i;
  c[p[0]] = 0;
  ll classes = 1;
  for (ll i = 1; i < n; i++) {
    if (s[p[i]] != s[p[i-1]])
      classes++;
    c[p[i]] = classes - 1;
  }
  vector<ll> pn(n), cn(n);
  for (ll h = 0; (1 << h) < n; ++h) {
    for (ll i = 0; i < n; i++) {  //sorting w.r.t
      pn[i] = p[i] - (1 << h);  //second part.
      if (pn[i] < 0)
        pn[i] += n;
    }
    fill(cnt.begin(), cnt.begin() + classes, 0);
    for (ll i = 0; i < n; i++)
      cnt[c[pn[i]]]++;
    for (ll i = 1; i < classes; i++)
      cnt[i] += cnt[i-1];
// sorting w.r.t. 1st(more significant) part
    for (ll i = n-1; i >= 0; i--)
      p[--cnt[c[pn[i]]]] = pn[i];
    cn[p[0]] = 0; classes = 1;
// determining new classes in sorted array.
    for (ll i = 1; i < n; i++) {
      pll cur={c[p[i]],c[(p[i]+(1<<h))%n]};
      pll prev={c[p[i-1]],c[(p[i-1]+(1<<h))%n]};
      if (cur != prev) ++classes;
      cn[p[i]] = classes - 1;
    }
    c.swap(cn); }
  return p;
}
vector<ll> suffix_array_construction(string s) {
  s += "$";
  vector<ll> sorted_shifts = sort_cyclic_shifts(s)↩
   ;
  sorted_shifts.erase(sorted_shifts.begin());
  return sorted_shifts; }
// For comp. two substr. of len. l starting at i,j↩
   .
// k - 2^k > l/2. check the first 2^k part, if ↩
   equal,
// check last 2^k part. c[k] is the c in kth iter
//of S.A construction.
int compare(int i, int j, int l, int k) {
  pll a = {c[k][i],c[k][(i+l-(1 << k))%n]};
  pll b = {c[k][j],c[k][(j+l-(1 << k))%n]};
  return a == b ? 0 : a < b ? -1 : 1; }
/*lcp[i]=len. of lcp of ith & (i+1)th suffix in ↩
   the SA
1.Consider suffixes in decreasing order of length.
2.Let p = s[i....n]. It will be somewhere in the S↩
   .A.
We determine its lcp = k. 3.Then lcp of q=s[(i+1)↩
   ..n]
will be atleast k-1 coz 4.remove the first char of↩
   p
and its successor in the S.A. These are suffixes ↩
   with
lcp k-1. 5.But note that these 2 may not be ↩
   consecutive
```

```
in S.A.But lcp of str. in b/w have to be also >= k-1.*/
vll lcp_cons(string const& s, vector<ll> const& p) {
  ll n = s.size();
  vector<ll> rank(n, 0);
  for (ll i = 0; i < n; i++)
    rank[p[i]] = i;
  ll k = 0; vector<ll> lcp(n-1, 0);
  for (ll i = 0; i < n; i++) {
    if (rank[i] == n - 1) {
      k = 0; continue; }
    ll j = p[rank[i] + 1];
    while(i+k<n && j+k<n && s[i+k]==s[j+k]) k++;
    lcp[rank[i]] = k; if (k) k--; }
  return lcp;
}
```

7.9 Suffix Tree

```
const int N=1000000, // set it more than 2*(len. of string)
string str; // input string for which the suffix tree is being built
int chi[N][26],
lef[N], // left...
rig[N], // ...and right boundaries of the substring of a which correspond to incoming edge
par[N], // parent of the node
sfli[N], // suffix link
tv,tp,la,
ts; // the number of nodes
void ukkadd(int c) {
  suff:;
  if (rig[tv]<tp){
    if (chi[tv][c]==-1){chi[tv][c]=ts;lef[ts]=la;
    par[ts++]=tv;tv=sfli[tv];tp=rig[tv]+1;goto suff;}
    tv=chi[tv][c];tp=lef[tv];
  }
  if (tp==-1 || c==str[tp]-'a')tp++;
  else {
    lef[ts]=lef[tv]; rig[ts]=tp-1; par[ts]=par[tv];
    chi[ts][str[tp]-'a']=tv; chi[ts][c]=ts+1;
    lef[ts+1]=la; par[ts+1]=ts; lef[tv]=tp; par[tv]=ts;
    chi[par[ts]][str[lef[ts]]-'a']=ts;  ts+=2;
    tv=sfli[par[ts-2]]; tp=lef[ts-2];
    while (tp <= rig[ts-2]) {
      tv=chi[tv][str[tp]-'a']; tp+=rig[tv]-lef[tv]+1;}
    if (tp == rig[ts-2]+1) sfli[ts-2]=tv; else sfli[ts-2]=ts;
    tp=rig[tv]-(tp-rig[ts-2])+2;goto suff;
  }
}
void build() {
  ts=2; tv=0; tp=0;
  ll ss = str.size();ss*=2;ss+=15;
  fill(rig,rig+ss,(int)str.size()-1);
  // initialize data for the root of the tree
  sfli[0]=1; lef[0]=-1; rig[0]=-1;
  lef[1]=-1; rig[1]=-1; for(ll i=0;i<ss;i++)
  fill (chi[i], chi[i]+26,-1);
  fill(chi[1],chi[1]+26,0);
  // add the text to the tree, letter by letter
  for (la=0; la<(int)str.size(); ++la)
  ukkadd (str[la]-'a');
}
```

7.10 Suffix Automaton

```
struct state {
  int len, link;
  map<char, int> next;
};
const int MAXLEN = 200005;
state st[MAXLEN];
int sz, last;
void sa_init() {
  st[0].len = 0;
  st[0].link = -1;
  sz++;
  last = 0;
}
void sa_extend(char c) {
  int cur = sz++;
  st[cur].len = st[last].len + 1;
  int p = last;
  while (p != -1 && !st[p].next.count(c)) {
    st[p].next[c] = cur;
    p = st[p].link;
  }
  if (p == -1) {
    st[cur].link = 0;
  } else {
    int q = st[p].next[c];
    if (st[p].len + 1 == st[q].len) {
      st[cur].link = q;
    } else {
      int clone = sz++;
      st[clone].len = st[p].len + 1;
      st[clone].next = st[q].next;
      st[clone].link = st[q].link;
      while (p != -1 && st[p].next[c] == q) {
        st[p].next[c] = clone;
        p = st[p].link;
      }
      st[q].link = st[cur].link = clone;
    }
```

```
    }
    last = cur;
}
void build(string &x){
    sz=0;
    for(ll i=0;i<3*x.length()+15;i++)
    {
        st[i].next.clear();
        st[i].len=0;st[i].link=0;
    }
    sa_init();
    for(ll i=0;i<x.size();i++)sa_extend(x[i]);
}
```

Möbius Function

$\mu(n) = \begin{cases} 0 & n \text{ not squarefree} \\ 1 & n \text{ squarefree w/ even no. of prime factors} \\ -1 & n \text{ squarefree w/ odd no. of prime factors} \end{cases}$

Note that $\mu(a)\mu(b) = \mu(ab)$ for a,b relatively prime

Also $\sum_{d|n} \mu(d) = \begin{cases} 1 & \text{if } n = 1 \\ 0 & \text{otherwise} \end{cases}$

Möbius Inversion If $g(n) = \sum_{d|n} f(d)$ for all $n \geq 1$, then $f(n) = \sum_{d|n} \mu(d) g(n/d)$ for all $n \geq 1$.

Burnside's Lemma (Text)

The number of orbits of a set X under the group action G equals the average number of elements of X fixed by the elements of G.

Here's an example. Consider a square of $2n$ times $2n$ cells. How many ways are there to color it into X colors, up to rotations and/or reflections? Here, the group has only 8 elements (rotations by 0, 90, 180 and 270 degrees, reflections over two diagonals, over a vertical line and over a horizontal line). Every coloring stays itself after rotating by 0 degrees, so that rotation has X^{4n^2} fixed points. Rotation by 180 degrees and reflections over a horizonal/vertical line split all cells in pairs that must be of the same color for a coloring to be unaffected by such rotation/reflection, thus there exist X^{2n^2} such colorings for each of them. Rotations by 90 and 270 degrees split cells in groups of four, thus yielding X^{n^2} fixed colorings. Reflections over diagonals split cells into $2n$ groups of 1 (the diagonal itself) and $2n^2 - n$ groups of 2 (all remaining cells), thus yielding $X^{2n^2-n+2n} = X^{2n^2+n}$ unaffected colorings. So, the answer is $(X^{4n^2} + 3X^{2n^2} + 2X^{n^2} + 2X^{2n^2+n})/8$. Every tree with n vertices has $n-1$ edges.

Trees-Kraft inequality:

If the depths of the leaves of a binary tree are $d_1 \ldots d_n$: $\sum_{i=1}^{n} 2^{-d_i} \leq 1$, and equality holds only if every internal node has 2 sons.

Euler Tour:

- Undirected graph iff All vertices have even degree, all non-zero degree vertices are in a single connected component.(Can be decomposed into edge-disjoint cycles)
- Directed graph iff For each vertex in-degree=out-degree, all non-zero degree vertices are in a single strongly connected component.(Decomposible into directed-edge disjoint cycles)

Euler Trail:

- Undirdected graph iff exactly 0 or 2 vertices have odd degree, single connected component(consider zero degree vertices only).
- Directed graph iff at most one vertex has (out-degree)-(in-degree) = 1, at most one vertex has (in-degree)-(out-degree) = 1, every other vertex has equal in-degree and out-degree, and all of its vertices with nonzero degree belong to a single connected component of the underlying undirected graph.

Master method:

$$T(n) = aT(n/b) + f(n), \quad a \geq 1, b > 1$$

If $\exists \varepsilon > 0$ such that $f(n) = O(n^{\log_b a - \varepsilon})$ then $T(n) = \Theta(n^{\log_b a})$.
If $f(n) = \Theta(n^{\log_b a})$ then $T(n) = \Theta(n^{\log_b a} \log_2 n)$.
If $\exists \varepsilon > 0$ such that $f(n) = \Omega(n^{\log_b a + \varepsilon})$, and $\exists c < 1$ such that $af(n/b) \leq cf(n)$ for large n, then $T(n) = \Theta(f(n))$.

Probability:

Variance, standard deviation: $Var[X] = E[X^2] - E[X]^2$

Poisson distribution:	Normal (Gaussian) distribution:
$\Pr[X=k] = \frac{e^{-\lambda}\lambda^k}{k!}, \quad E[X] = \lambda$	$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}, \quad E[X] = \mu.$

The "coupon collector": We are given a random coupon each day, and there are n different types of coupons. The distribution of coupons is uniform. The expected number of days to pass before we to collect all n types is nH_n.

Miscellaneous:

1. Radius of inscribed circle for Right Angle Tringle: $\frac{AB}{A+B+C}$
2. Law of cosine: $c^2 = a^2 + b^2 - 2ab\cos C$
3. Area of a triangle: Area: $A = \frac{1}{2}hc = \frac{1}{2}ab\sin C = \frac{c^2 \sin A \sin B}{2 \sin C}$.
4. $\det A = \sum_\pi \prod_{i=1}^{n} \text{sign}(\pi) a_{i,\pi(i)}$, for Permanents remove sign.
5. Perfect Numbers: x is an even perfect number iff $x = 2^{n-1}(2^n - 1)$ and $2^n - 1$ is prime.
6. Wilson's theorem: n is a prime iff $(n-1)! \equiv -1 \mod n$.
7. If graph G is planar then $n - m + f = 2$, so $f \leq 2n - 4, \quad m \leq 3n - 6$. Any planar graph has a vertex with degree ≤ 5.
8. Dirichlet power series: $A(x) = \sum_{i=1}^{\infty} \frac{a_i}{i^x}$

9. Coefficient of x^r in $(1-x)^{-n}$ is $\binom{n+r-1}{r}$.
10. Stirling numbers (1st kind): $s(n,2)=(n-1)!H_{n-1}, \quad s(n,k)=(n-1)s(n-1,k)+s(n-1,k-1)$
11. Stirling numbers (2nd kind): $S(n,k)=kS(n-1,k)+S(n-1,k-1)$
 $S(n,k)=\frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n, \quad S(n,m)=\sum_k\binom{n}{k}S(k+1,m+1)(-1)^{n-k}$
12. Catalan Numbers: Binary trees with $n+1$ vertices $\quad C_n=\frac{1}{n+1}\binom{2n}{n}$

For Bipartite Graphs

1. Min-edge cover(me) = Max-independent set(mi) (G has no isolated vertex).
2. Min-vertex cover(mv) = Max matching(mm) $\quad mi+mv=|V|, \quad mi\geq\frac{|V|}{2}$
3. Min-edge cover subgraph is a combination of star graphs.
4. Min Vertex cover : In residual graph of max flow pick all the vertices in L(left bipartite) not reachable from s(whose edges are cut) and lly in R reachable from s.
5. Min-edge cover(no isolated vertex) : Find max matching, take all those edges, for vertices not covered take any edge.

$$(1)\quad \sum_{i=1}^{n} i^m=\frac{1}{m+1}\left[(n+1)^{m+1}-1-\sum_{i=1}^{n}\left((i+1)^{m+1}-i^{m+1}-(m+1)i^m\right)\right]$$

$$(2)\quad \frac{x}{(1-x)^2}=\sum_{i=0}^{\infty} ix^i, \quad (3)\quad \ln(1+x)=\sum_{i=1}^{\infty}(-1)^{i+1}\frac{x^i}{i}, \quad (4)\quad \frac{x}{1-x-x^2}=\sum_{i=0}^{\infty}F_ix^i$$

$$(5)\quad (x_1+x_2+\ldots+x_k)^n=\sum_{c_1+c_2+\ldots+c_k=n}\frac{n!}{c_1!\ldots c_k!}x_1^{c_1}x_2^{c_2}\ldots x_k^{c_k}, (6)\quad \frac{1}{(1-x)^{n+1}}=\sum_{i=0}^{\infty}\binom{i+n}{i}x^i$$

$$(1)\quad \int\tanh x\,dx=\ln|\cosh x|, \quad (2)\quad \int\coth x\,dx=\ln|\sinh x|, (3)\quad \int\frac{dx}{a^2-x^2}=\frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|,$$

$$(4)\quad \int\frac{dx}{\sqrt{a^2+x^2}}=\ln\left(x+\sqrt{a^2+x^2}\right)\quad (a>0), (5)\quad \int\frac{dx}{a^2+x^2}=\frac{1}{a}\arctan\frac{x}{a}\quad (a>0),$$

$$(6)\quad \int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\arcsin\frac{x}{a}\quad (a>0)\quad (7)\quad \int\frac{dx}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}\quad (a>0)$$

Fibonacci:

1. $F_{-i}=(-1)^{i-1}F_i, \quad F_i=\frac{1}{\sqrt{5}}\left(\phi^i-\hat{\phi}^i\right)$
2. Cassini's identity: $F_{i+1}F_{i-1}-F_i^2=(-1)^i$ for $i>0$,
3. Additive Rule: $F_{n+k}=F_kF_{n+1}+F_{k-1}F_n, \quad F_{2n}=F_nF_{n+1}+F_{n-1}F_n$
4. Every integer n has a unique representation $n=F_{k_1}+F_{k_2}+\cdots+F_{k_m}$, where $k_i\geq k_{i+1}+2$ for $1\leq i<m$ and $k_m\geq 2$.

Primes

$\forall(a,b)$, The largest prime smaller than 10^a is $p=10^a-b$

$(1,3)$, $(2,3)$, $(3,3)$, $(4,27)$, $(5,9)$, $(6,17)$, $(7,9)$, $(8,11)$, $(9,63)$, $(10,33)$, $(11,23)$, $(12,11)$, $(13,29)$, $(14,27)$, $(15,11)$, $(16,63)$, $(17,3)$, $(18,11)$