Codebook- Team Far_Behind
IIT Delhi, India

Ayush Ranjan, Naman Jain, Manish Tanwar

Contents

1 Syntax

1.1 Template

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
using namespace std;
typedef tree<pair<ll,ll> ,null_type,less<pair<ll, ↩
   ll> >,rb_tree_tag,↩
   tree_order_statistics_node_update> ordered_set;

template<class T> ostream& operator<<(ostream &os,↩
    vector<T> V) {
 os << "[ "; for(auto v : V) os << v << " "; ↩
    return os << "]";}
template<class L, class R> ostream& operator<<(↩
   ostream &os, pair<L,R> P) {
 return os << "(" << P.first << "," << P.second <<↩
     ")";}

#define TRACE
#ifdef TRACE
#define trace(...) __f(#__VA_ARGS__, __VA_ARGS__)
template <typename Arg1>
void __f(const char* name, Arg1&& arg1){
 cout << name << " : " << arg1 << std::endl;
}
template <typename Arg1, typename... Args>
```

```
void __f(const char* names, Arg1&& arg1, Args&&...↩
    args){
 const char* comma = strchr(names + 1, ',');cout.↩
    write(names, comma - names) << " : " << arg1<<"↩
     | ";__f(comma+1, args...);
}
#else
#define trace(...) 1
#endif

#define ll long long
#define ld long double
#define vll vector<ll>
#define pll pair<ll,ll>
#define vpll vector<pll>
#define I insert
#define pb push_back
#define F first
#define S second
#define endl "\n"
const ll MAX=1e6+5;
// int mod=1e9+7;
inline int mul(int a,int b){return (a*1ll*b)%mod;}
inline int add(int a,int b){a+=b;if(a>=mod)a-=mod;↩
   return a;}
inline int sub(int a,int b){a-=b;if(a<0)a+=mod;↩
   return a;}
inline int power(int a,int b){int rt=1;while(b>0){↩
   if(b&1)rt=mul(rt,a);a=mul(a,a);b>>=1;}return rt↩
   ;}
inline int inv(int a){return power(a,mod-2);}
inline void modadd(int &a,int &b){a+=b;if(a>=mod)a↩
   -=mod;}

int main(){
 ios_base::sync_with_stdio(false);cin.tie(0);cout.↩
    tie(0);cout<<setprecision(25);

}
```

1.2 Clock

```
clock_t clk = clock();
    // code goes here
clk = clock() - clk;
cout << "Time Elapsed: " << fixed << setprecision↩
   (10) << ((long double)clk)/CLOCKS_PER_SEC << "\n↩
   ";
```

1.3 C++ Sublime Build

```
{
  "cmd": ["bash", "-c", "g++ -std=c++11 -O3 '${↩
     file}' -o '${file_path}/${file_base_name}' && ↩
      gnome-terminal -- bash -c '\"${file_path}/${↩
     file_base_name}\" < input.txt >output.txt' "],
  "file_regex": "^(..[^:]*):([0-9]+):?([0-9]+)?:? ↩
     (.*)$",
  "working_dir": "${file_path}",
  "selector": "source.c++, source.cpp",
}
```

1.4 Fast IO

```
/*getchar_unlocked and putchar_unlocked doesn't ↩
   work in windows (Codeforces);replace them with ↩
   getchar and putchar*/
inline ll read()
{
    ll n = 0;
    char c = getchar_unlocked();
    while (!('0' <= c && c <= '9'))
    {
        c = getchar_unlocked();
    }
    while ('0' <= c && c <= '9')
    {
        n = n * 10 + c - '0';
        c = getchar_unlocked();
    }
    return n;
}
inline void write(ll a)
{
    register char c;
    char snum[20];
    ll i=0;
    do
    {
        snum[i++]=a%10+48;
        a=a/10;
    }
    while(a!=0);
    i=i-1;
    while(i>=0)
```

```
        putchar_unlocked(snum[i--]);
    putchar_unlocked('\n');
}
// although getline(cin, string) with IOS is ↩
    better than this
// time taken => scanf("%[^\n]s",ch) < getline < ↩
    reading char by char < char by char with ↩
    getchar_unlocked
inline void fastRead_string(char *str)
{
    char c = 0;
    int i = 0;

    while (c < 33)
        c = getchar_unlocked();

    while (c != '\n') {
        str[i] = c;
        c = getchar_unlocked();
        i = i + 1;
    }
    // getchar_unlocked returns -1 on EOF
    str[i] = '\0';
}
// use
char s[100];
fastRead_string(s);
printf("%s\n", s);
```

1.5 GP Hash Table

```
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
gp_hash_table<int, int> table; //cc_hash_table can↩
    also be used
//custom hash function
const int RANDOM = chrono::high_resolution_clock::↩
    now().time_since_epoch().count();
struct chash {
    int operator()(int x) { return hash<int>{}(x ^↩
        RANDOM); }
};
gp_hash_table<int, int, chash> table;
//custom hash function for pair
struct chash {
    int operator()(pair<int,int> x) const { return↩
        x.first* 31 + x.second; }
};
```

1.6 Ordered Set

```
#include<bits/stdc++.h>
using namespace std;
#define ll long long
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
typedef tree<pair<ll,ll> ,null_type,less<pair<ll, ↩
    ll> >,rb_tree_tag,↩
    tree_order_statistics_node_update> ordered_set;
ordered_set X;
X.insert(1);
X.insert(2);
cout<<*X.find_by_order(0)<<endl; // 1
cout<<*X.find_by_order(1)<<endl; // 2
cout<<(end(X)==X.find_by_order(2))<<endl; // true
//order_of_key(x) returns number of elements ↩
    strictly less than x in ordered_set
cout<<X.order_of_key(-5)<<endl;  // 0
cout<<X.order_of_key(1)<<endl;   // 0
cout<<X.order_of_key(3)<<endl;   // 2
//For multiset use less_equal operator but it does↩
     support erase operations for multiset
```

1.7 Random Shuffle

```
#include<bits/stdc++.h>
using namespace std;
const int N = 3000000;
int main() {
    mt19937 rng(chrono::steady_clock::now().↩
        time_since_epoch().count());
    vector<int> permutation(N);
    for (int i = 0; i < N; i++)
        permutation[i] = i;
    shuffle(permutation.begin(), permutation.end()↩
        , rng);
}
```

1.8 String Splitting

```
// this splitting is better than custom function(w.r.t time)
string line = "GeeksForGeeks is a must try";
// Vector of string to save tokens
vector <string> tokens;
// stringstream class check1
stringstream check1(line);
string ele;
// Tokenizing w.r.t. space ' '
while(getline(check1, ele, ' '))
{
    tokens.push_back(ele);
}
```

2 Data Structures

2.1 Fenwick

```
ll n;
ll fen[MAX_N];
void update(ll p,ll val){
 for(ll i = p;i <= n;i += i & -i)
  fen[i] += val;
}
ll sum(ll p){
 ll ans = 0;
 for(ll i = p;i;i -= i & -i)
  ans += fen[i];
 return ans;
}
```

3 Flows

3.1 Ford Fulkerson

```
// running time - O(f*m) (f -> flow routed)
const ll N = 3e3;
ll n; // number of vertices
ll capacity[N][N]; // adj matrix for capacity
vll adj[N]; // adj list of the corresponding undirected graph(***imp***)
// E = {1-2,2->3,3->2}, adj list should be => {1->2,2->1,2->3,3->2}
// *** vertices are 0-indexed ***
ll INF = (1e18);
ll snk, cnt; // cnt for vis, no need to initialize vis
vector<ll> par, vis;
ll dfs(ll u,ll curr_flow){
 vis[u] = cnt; if(u == snk) return curr_flow;
 if(adj[u].size() == 0) return 0;
 for(ll j=0;j<5;j++){ // random for good augmentation(**sometimes take time**)
        ll a = rand()%(adj[u].size());
        ll v = adj[u][a];
        if(vis[v] == cnt || capacity[u][v] == 0) continue;
        par[v] = u;
        ll f = dfs(v,min(curr_flow, capacity[u][v])); if(vis[snk] == cnt) return f;
    }
    for(auto v : adj[u]){
     if(vis[v] == cnt || capacity[u][v] == 0) continue;
     par[v] = u;
     ll f = dfs(v,min(curr_flow, capacity[u][v])); if(vis[snk] == cnt) return f;
    }
    return 0;
}
ll maxflow(ll s, ll t) {
    snk = t; ll flow = 0; cnt++;
    par = vll(n,-1); vis = vll(n,0);
    while(ll new_flow = dfs(s,INF)){
     flow += new_flow; cnt++;
     ll cur = t;
     while(cur != s){
      ll prev = par[cur];
      capacity[prev][cur] -= new_flow;
      capacity[cur][prev] += new_flow;
      cur = prev;
     }
    }
    return flow;
}
```

3.2 Dinic

```cpp
// Time: O(m*n^2) and for any unit capacity ↩
   network O(m * n^1/2)
// TIme: O(min(fm,mn^2)) (f: flow routed)
// (so for bipartite matching as well)
// In practice it is pretty fast for any bipartite↩
    network
// I/O:     n -> vertice; DinicFlow net(n);
//          for(z : edges) net.addEdge(z.F,z.S,cap↩
   );
//          max flow = maxFlow(s,t);
// e=(u,v), e.flow represents the effective flow ↩
   from u to v
// (i.e f(u->v) - f(v->u)), vertices are 1-indexed
struct edge {
    ll x, y, cap, flow; };

struct DinicFlow {
    // *** change inf accordingly *****
    const ll inf = (1e18);
    vector <edge> e;
    vector <ll> cur, d;
    vector < vector <ll> > adj;
    ll n, source, sink;

    DinicFlow() {}

    DinicFlow(ll v) {
        n = v;
        cur = vector <ll> (n + 1);
        d = vector <ll> (n + 1);
        adj = vector < vector <ll> > (n + 1);
    }

    void addEdge(ll from, ll to, ll cap) {
        edge e1 = {from, to, cap, 0};
        edge e2 = {to, from, 0, 0};
        adj[from].push_back(e.size()); e.push_back↩
           (e1);
        adj[to].push_back(e.size()); e.push_back(↩
           e2);
    }

    ll bfs() {
        queue <ll> q;
        for(ll i = 0; i <= n; ++i) d[i] = -1;
        q.push(source); d[source] = 0;
        while(!q.empty() and d[sink] < 0) {
            ll x = q.front(); q.pop();
            for(ll i = 0; i < (ll)adj[x].size(); ↩
               ++i) {
                ll id = adj[x][i], y = e[id].y;
                if(d[y] < 0 and e[id].flow < e[id↩
                   ].cap) {
                    q.push(y); d[y] = d[x] + 1;
                }
            }
        }
        return d[sink] >= 0;
    }

    ll dfs(ll x, ll flow) {
        if(!flow) return 0;
        if(x == sink) return flow;
        for(;cur[x] < (ll)adj[x].size(); ++cur[x])↩
            {
            ll id = adj[x][cur[x]], y = e[id].y;
            if(d[y] != d[x] + 1) continue;
            ll pushed = dfs(y, min(flow, e[id].cap↩
               - e[id].flow));
            if(pushed) {
                e[id].flow += pushed;
                e[id ^ 1].flow -= pushed;
                return pushed;
            }
        }
        return 0;
    }

    ll maxFlow(ll src, ll snk) {
        this->source = src; this->sink = snk;
        ll flow = 0;
        while(bfs()) {
            for(ll i = 0; i <= n; ++i) cur[i] = 0;
            while(ll pushed = dfs(source, inf)) {
                flow += pushed;
            }
        }
        return flow;
    }
};
```

3.3 Edmond Karp

```cpp
// running time - O(n*m^2)
// The matrix capacity stores the capacity for ↩
   every pair of vertices. adj
// is the adjacency list of the undirected graph, ↩
   since we have also to use
// the reversed of directed edges when we are ↩
   looking for augmenting paths.
// The function maxflow will return the value of ↩
   the maximal flow. During
// the algorithm the matrix capacity will actually↩
    store the residual capacity
// of the network. The value of the flow in each ↩
   edge will actually no stored,
// but it is easy to extent the implementation - ↩
   by using an additional matrix
```

```
// - to also store the flow and return it.
const ll N = 3e3;
ll n; // number of vertices
ll capacity[N][N]; // adj matrix for capacity
vll adj[N]; // adj list of the corresponding ↩
   undirected graph(***imp***)
// E = {1-2,2->3,3->2}, adj list should be => ↩
   {1->2,2->1,2->3,3->2}
// *** vertices are 0-indexed ***
ll INF = (1e18);
ll bfs(ll s, ll t, vector<ll>& parent) {
    fill(parent.begin(), parent.end(), -1);
    parent[s] = -2;
    queue<pair<ll, ll>> q;
    q.push({s, INF});

    while (!q.empty()) {
        ll cur = q.front().first;
        ll flow = q.front().second;
        q.pop();
        for (ll next : adj[cur]) {
            if (parent[next] == -1 && capacity[cur↩
               ][next]) {
                parent[next] = cur;
                ll new_flow = min(flow, capacity[↩
                   cur][next]);
                if (next == t)
                    return new_flow;
                q.push({next, new_flow});
            }
        }
    }
    return 0;
}
ll maxflow(ll s, ll t) {
    ll flow = 0; ll new_flow;
    vector<ll> parent(n);
    while (new_flow = bfs(s, t, parent)) {
        flow += new_flow;
        ll cur = t;
        while (cur != s) {
            ll prev = parent[cur];
            capacity[prev][cur] -= new_flow;
            capacity[cur][prev] += new_flow;
            cur = prev;
        }
    }
    return flow;
}
```

3.4 Push Relabel

```
// Adjacency list implementation of FIFO push ↩
   relabel maximum flow
// with the gap relabeling heuristic.  This ↩
   implementation is
// significantly faster than straight Ford-↩
   Fulkerson.  It solves
// random problems with 10000 vertices and 1000000↩
    edges in a few
// seconds, though it is possible to construct ↩
   test cases that
// achieve the worst-case.
// Time: O(V^3)
// I/O:- addEdge(),src,snk ** vertices are 0-↩
   indexed **
//      - To obtain the actual flow values, look at↩
    all edges with
//        capacity > 0 (zero capacity edges are ↩
   residual edges).
struct edge {
  ll from, to, cap, flow, index;
  edge(ll from, ll to, ll cap, ll flow, ll index) ↩
     :
    from(from), to(to), cap(cap), flow(flow), ↩
       index(index) {}
};

struct PushRelabel {
  ll n;
  vector<vector<edge> > G;
  vector<ll> excess;
  vector<ll> dist, active, count;
  queue<ll> Q;

  PushRelabel(ll n) : n(n), G(n), excess(n), dist(↩
     n), active(n), count(2*n) {}

  void addEdge(ll from, ll to, ll cap) {
    G[from].push_back(edge(from, to, cap, 0, G[to↩
       ].size()));
    if (from == to) G[from].back().index++;
    G[to].push_back(edge(to, from, 0, 0, (ll)G[↩
       from].size() - 1));
  }

  void enqueue(ll v) {
    if (!active[v] && excess[v] > 0) { active[v] =↩
        true; Q.push(v); }
  }

  void push(edge &e) {
    ll amt = min(excess[e.from], e.cap - e.flow);
    if (dist[e.from] <= dist[e.to] || amt == 0) ↩
       return;
    e.flow += amt;
    G[e.to][e.index].flow -= amt;
    excess[e.to] += amt;
    excess[e.from] -= amt;
    enqueue(e.to);
```

```
  }
  void gap(ll k) {
    for (ll v = 0; v < n; v++) {
      if (dist[v] < k) continue;
      count[dist[v]]--; dist[v] = max(dist[v], n+1);
      count[dist[v]]++; enqueue(v);
    }
  }
  void relabel(ll v) {
    count[dist[v]]--;
    dist[v] = 2*n;
    for (ll i = 0; i < G[v].size(); i++)
      if (G[v][i].cap - G[v][i].flow > 0)
    dist[v] = min(dist[v], dist[G[v][i].to] + 1);
    count[dist[v]]++;
    enqueue(v);
  }
  void discharge(ll v) {
    for (ll i = 0; excess[v] > 0 && i < G[v].size(); i++) push(G[v][i]);
    if (excess[v] > 0) {
      if (count[dist[v]] == 1)  gap(dist[v]);
      else relabel(v);
    }
  }
  ll getMaxFlow(ll s, ll t) {
    count[0] = n-1;
    count[n] = 1;
    dist[s] = n;
    active[s] = active[t] = true;
    for (ll i = 0; i < G[s].size(); i++) {
      excess[s] += G[s][i].cap;
      push(G[s][i]);
    }
    while (!Q.empty()) {
      ll v = Q.front(); Q.pop();
      active[v] = false; discharge(v);
    }
    ll totflow = 0;
    for (ll i = 0; i < G[s].size(); i++) totflow += G[s][i].flow;
    return totflow;
  }
};
```

3.5 MCMF

```
// MCMF Theory:
// 1.  If a network with negative costs had no negative cycle it is possible to transform it into one with nonnegative
//     costs. Using Cij_new(pi) = Cij_old + pi(i) - pi(j), where pi(x) is shortest path from s to x in network with an
//     added vertex s. The objective value remains the same (z_new = z + constant). z(x) = sum(cij*xij)
//     (x->flow, c->cost, u->cap, r->residual cap).
// 2.  Residual Network: cji = -cij, rij = uij-xij, rji = xij.
// 3.  Note: If edge (i,j),(j,i) both are there then residual graph will have four edges b/w i,j (pairs of parellel edges).
// 4.  let x* be a feasible soln, its optimal iff residual network Gx* contains no negative cost cycle.
// 5.  Cycle Cancelling algo => Complexity O(n*m^2*U*C) (C->max abs value of cost, U->max cap) (m*U*C iterations).
// 6.  Succesive shortest path algo => Complexity O(n^3 * B) / O(nmBlogn)(using heap in Dijkstra)(B -> largest supply node).

//Works for negative costs, but does not work for negative cycles
//Complexity: O(min(E^2 *V log V, E logV * flow))
// to use -> graph G(n), G.add_edge(u,v,cap,cost), G.min_cost_max_flow(s,t)
// ********* INF is used in both flow_type and cost_type so change accordingly
const ll INF = 999999999;
// vertices are 0-indexed
struct graph {
  typedef ll flow_type; // **** flow type ****
  typedef ll cost_type; // **** cost type ****
  struct edge {
    int src, dst;
    flow_type capacity, flow;
    cost_type cost;
    size_t rev;
  };
  vector<edge> edges;
  void add_edge(int src, int dst, flow_type cap, cost_type cost) {
    adj[src].push_back({src, dst, cap, 0, cost, adj[dst].size()});
    adj[dst].push_back({dst, src, 0, 0, -cost, adj[src].size()-1});
  }
  int n;
  vector<vector<edge>> adj;
```

```
  graph(int n) : n(n), adj(n) { }

  pair<flow_type, cost_type> min_cost_max_flow(int↩
      s, int t) {
    flow_type flow = 0;
    cost_type cost = 0;

    for (int u = 0; u < n; ++u) // initialize
      for (auto &e: adj[u]) e.flow = 0;

    vector<cost_type> p(n, 0);

    auto rcost = [&](edge e) { return e.cost + p[e↩
       .src] - p[e.dst]; };
    for (int iter = 0; ; ++iter) {
      vector<int> prev(n, -1); prev[s] = 0;
      vector<cost_type> dist(n, INF); dist[s] = 0;
      if (iter == 0) { // use Bellman-Ford to ↩
         remove negative cost edges
        vector<int> count(n); count[s] = 1;
        queue<int> que;
        for (que.push(s); !que.empty(); ) {
          int u = que.front(); que.pop();
          count[u] = -count[u];
          for (auto &e: adj[u]) {
            if (e.capacity > e.flow && dist[e.dst]↩
                > dist[e.src] + rcost(e)) {
              dist[e.dst] = dist[e.src] + rcost(e)↩
                 ;
              prev[e.dst] = e.rev;
              if (count[e.dst] <= 0) {
                count[e.dst] = -count[e.dst] + 1;
                que.push(e.dst);
              }
            }
          }
        }
        for(int i=0;i<n;i++) p[i] = dist[i]; // ↩
           added it
        continue;
      } else { // use Dijkstra
        typedef pair<cost_type, int> node;
        priority_queue<node, vector<node>, greater↩
           <node>> que;
        que.push({0, s});
        while (!que.empty()) {
          node a = que.top(); que.pop();
          if (a.S == t) break;
          if (dist[a.S] > a.F) continue;
          for (auto e: adj[a.S]) {
            if (e.capacity > e.flow && dist[e.dst]↩
                > a.F + rcost(e)) {
              dist[e.dst] = dist[e.src] + rcost(e)↩
                 ;
              prev[e.dst] = e.rev;
              que.push({dist[e.dst], e.dst});
            }
          }
        }
      }
      if (prev[t] == -1) break;

      for (int u = 0; u < n; ++u)
        if (dist[u] < dist[t]) p[u] += dist[u] - ↩
           dist[t];

      function<flow_type(int,flow_type)> augment =↩
          [&](int u, flow_type cur) {
        if (u == s) return cur;
        edge &r = adj[u][prev[u]], &e = adj[r.dst↩
           ][r.rev];
        flow_type f = augment(e.src, min(e.↩
           capacity - e.flow, cur));
        e.flow += f; r.flow -= f;
        return f;
      };
      flow_type f = augment(t, INF);
      flow += f;
      cost += f * (p[t] - p[s]);
    }
    return {flow, cost};
  }
};
```

4 Geometry

4.1 Convex Hull

```
// code credits(PT struct) -->> https://github.com↩
   /jaehyunp/stanfordacm/blob/master/code/Geometry.↩
   cc

double INF = 1e100;
double EPS = 1e-9;

struct PT {
  double x, y;
  PT() {}
  PT(double x, double y) : x(x), y(y) {}
  PT(const PT &p) : x(p.x), y(p.y)    {}
  PT operator + (const PT &p)  const { return PT(x↩
     +p.x, y+p.y); }
  PT operator - (const PT &p)  const { return PT(x↩
     -p.x, y-p.y); }
  PT operator * (double c)     const { return PT(x↩
     *c,   y*c  ); }
```

```
  PT operator / (double c)     const { return PT(x/c,   y/c  ); }
};
double dot(PT p, PT q)     { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q)   { return dot(p-q,p-q); }
double dist(PT p, PT q)    { return sqrt(dist2(p,q)); }
double cross(PT p, PT q)   { return p.x*q.y-p.y*q.x; }
//print a point
ostream &operator<<(ostream &os, const PT &p) {
  return os << "(" << p.x << "," << p.y << ")";
}
//point of reference for making hull (leftmost and bottommost)
PT firstpoint;
//Returns 0 is x,y,z lie on a line, 1 is x->y->z is ccw direction and 2 if x->y->z is cw
ll orient(PT x,PT y,PT z){
 PT p,q;
 p=y-x;q=z-y;ld cr=cross(p,q);
 if(abs(cr)<EPS){return 0;}
 else if(cr>0)return 1;
 return 2;
}
//for sorting points in ccw(counter clockwise) direction w.r.t firstpoint (leftmost and bottommost)
bool compare(PT x,PT y){
 if(orient(firstpoint,x,y)!=2)return true;return false;
}
/*takes as input a vector of points containing input points and an empty vector for making hull
the points forming convex hull are pushed in vector hull
returns hull containing minimum number of points in ccw order
****remove EPS for making integer hull
*/
void make_hull(vector<PT>& poi,vector<PT>& hull)
{
 pair<ld,ld> bl={INF,INF};
 ll n=poi.size();ll ind;
 for(ll i=0;i<n;i++){
  pair<ld,ld> pp={poi[i].y,poi[i].x};
  if(pp<bl){
   ind=i;bl={poi[i].y,poi[i].x};
  }
 }
 swap(bl.F,bl.S);firstpoint=PT(bl.F,bl.S);
 vector<PT> cons;
 for(ll i=0;i<n;i++){
  if(i==ind)continue;cons.pb(poi[i]);
 }
 sort(cons.begin(),cons.end(),compare);
 hull.pb(firstpoint);ll m;
 for(auto z:cons){
  if(hull.size()<=1){hull.pb(z);continue;}
  PT pr,ppr;bool fl=true;
  while((m=hull.size())>=2){
   pr=hull[m-1];ppr=hull[m-2];
   ll ch=orient(ppr,pr,z);
   if(ch==1){break;}
   else if(ch==2){hull.pop_back();continue;}
   else {
    ll d1,d2;
    d1=dist2(ppr,pr);d2=dist2(ppr,z);
    if(d1>d2){fl=false;break;}else {hull.pop_back();}
   }
  }
  if(fl){hull.push_back(z);}
 }
 return;
}
```

4.2 Convex Hull Trick

```
/*
maintains upper convex hull of lines ax+b and gives minimum value at a given x
to add line ax+b: sameoldcht.addline(a,b), to get min value at x: sameoldcht.getbest(x)
to get maximum value at x add -ax-b as lines instead of ax+b and use -sameoldcht.getbest(x)
*/
const int N = 1e5 + 5;
int n;
int a[N];
int b[N];
long long dp[N];
struct line{
    long long a , b;
    double xleft;
    bool type;
    line(long long _a , long long _b){
        a = _a;
        b = _b;
        type = 0;
    }
    bool operator < (const line &other) const{
```

```
        if(other.type){
            return xleft < other.xleft;
        }
        return a > other.a;
    }
};
double meet(line x , line y){
    return 1.0 * (y.b - x.b) / (x.a - y.a);
}
struct cht{
    set < line > hull;
    cht(){
        hull.clear();
    }
    typedef set < line > :: iterator ite;
    bool hasleft(ite node){
        return node != hull.begin();
    }
    bool hasright(ite node){
        return node != prev(hull.end());
    }
    void updateborder(ite node){
        if(hasright(node)){
            line temp = *next(node);
            hull.erase(temp);
            temp.xleft = meet(*node , temp);
            hull.insert(temp);
        }
        if(hasleft(node)){
            line temp = *node;
            temp.xleft = meet(*prev(node) , temp);
            hull.erase(node);
            hull.insert(temp);
        }
        else{
            line temp = *node;
            hull.erase(node);
            temp.xleft = -1e18;
            hull.insert(temp);
        }
    }
    bool useless(line left , line middle , line ↩
       right){
        double x = meet(left , right);
        double y = x * middle.a + middle.b;
        double ly = left.a * x + left.b;
        return y > ly;
    }
    bool useless(ite node){
        if(hasleft(node) && hasright(node)){
            return useless(*prev(node) , *node , *↩
               next(node));
        }
        return 0;
    }
    void addline(long long a , long long b){
        line temp = line(a , b);
        auto it = hull.lower_bound(temp);
        if(it != hull.end() && it -> a == a){
            if(it -> b > b){
                hull.erase(it);
            }
            else{
                return;
            }
        }
        hull.insert(temp);
        it = hull.find(temp);
        if(useless(it)){
            hull.erase(it);
            return;
        }
        while(hasleft(it) && useless(prev(it))){
            hull.erase(prev(it));
        }
        while(hasright(it) && useless(next(it))){
            hull.erase(next(it));
        }
        updateborder(it);
    }
    long long getbest(long long x){
        if(hull.empty()){
            return 1e18;
        }
        line query(0 , 0);
        query.xleft = x;
        query.type = 1;
        auto it = hull.lower_bound(query);
        it = prev(it);
        return it -> a * x + it -> b;
    }
};
cht sameoldcht;
int main()
{
    scanf("%d" , &n);
    for(int i = 1 ; i <= n ; ++i){
        scanf("%d" , a + i);
    }
    for(int i = 1 ; i <= n ; ++i){
        scanf("%d" , b + i);
    }
    sameoldcht.addline(b[1] , 0);
    for(int i = 2 ; i <= n ; ++i){
        dp[i] = sameoldcht.getbest(a[i]);
        sameoldcht.addline(b[i] , dp[i]);
    }
    printf("%lld\n" , dp[n]);
}
```

5 Trees

5.1 LCA

```
const int N = int(1e5)+10;
const int LOGN = 20;
set<int> g[N];
int level[N];
int DP[LOGN][N];
int n,m;
/*----------- Pre-Processing ------------*/
/* Code Cridits : Tanuj Khattar codeforces ↩
   submission */
void dfs0(int u)
{
 for(auto it=g[u].begin();it!=g[u].end();it++)
  if(*it!=DP[0][u])
  {
   DP[0][*it]=u;
   level[*it]=level[u]+1;
   dfs0(*it);
  }
}
void preprocess()
{
 level[0]=0;
 DP[0][0]=0;
 dfs0(0);
 for(int i=1;i<LOGN;i++)
  for(int j=0;j<n;j++)
   DP[i][j] = DP[i-1][DP[i-1][j]];
}
int lca(int a,int b)
{
 if(level[a]>level[b])swap(a,b);
 int d = level[b]-level[a];
 for(int i=0;i<LOGN;i++)
  if(d&(1<<i))
   b=DP[i][b];
 if(a==b)return a;
 for(int i=LOGN-1;i>=0;i--)
  if(DP[i][a]!=DP[i][b])
   a=DP[i][a],b=DP[i][b];
 return DP[0][a];
}
int dist(int u,int v)
{
 return level[u] + level[v] - 2*level[lca(u,v)];
}
```

5.2 Centroid Decompostion

```
/*
nx:maximum number of nodes
adj:adjacency list of tree,adj1: adjacency list of↩
    centroid tree
par:parents of nodes in centroid tree,timstmp: ↩
   timestamps of nodes when they became centroids (↩
   helpful in comparing which of the two nodes ↩
   became centroid first)
ssize,vis:utility arrays for storing subtree size ↩
   and visit times in dfs
tim: utility for doing dfs (for deciding which ↩
   nodes to visit)
cntrorder: centroids stored in order in which they↩
    were formed
dist[nx]: vector of vectors with dist[i][0][j]=↩
   number of nodes at distance of k in subtree of i↩
    in centroid tree and dist[i][j][k]=number of ↩
   nodes at distance k in jth child of i in ↩
   centroid tree ***(use adj while doing dfs ↩
   instead of adj1)***
dfs: find subtree sizes visiting nodes starting ↩
   from root without visiting already formed ↩
   centroids
dfs1: root- starting node, n- subtree size ↩
   remaining after removing centroids -> returns ↩
   centroid in subtree of root
preprocess: stores all values in dist array
*/
const int nx=1e5;
vector<int> adj[nx],adj1[nx]; //adj is adjacency ↩
   list of tree and adj1 is adjacency list for ↩
   centroid tree
int par[nx],timstmp[nx],ssize[nx],vis[nx];//par is↩
    parent of each node in centroid tree,ssize is ↩
   subtree size of each node in centroid tree,vis ↩
   and timstmp are auxillary arrays for visit times↩
    in dfs- timstmp contains nonzero values only ↩
   for centroids
int tim=1;
vector<int> cntrorder;//contains list of centroids↩
    generated (in order)
vector<vector<int> > dist[nx];
int dfs(int root)
{
 vis[root]=tim;
 int t=0;
 for(auto i:adj[root])
 {
  if(!timstmp[i]&&vis[i]<tim)
   t+=dfs(i);
 }
 ssize[root]=t+1;return t+1;
}
```

```
int dfs1(int root,int n)
{
 vis[root]=tim;pair<int,int> mxc={0,-1};bool poss=↩
     true;
 for(auto i:adj[root])
 {
  if(!timstmp[i]&&vis[i]<tim)
   poss&=(ssize[i]<=n/2),mxc=max(mxc,{ssize[i],i})↩
       ;
 }
 if(poss&&(n-ssize[root])<=n/2)return root;
 return dfs1(mxc.second,n);
}
int findc(int root)
{
 dfs(root);
 int n=ssize[root];tim++;
 return dfs1(root,n);
}
void cntrdecom(int root,int p)
{
 int cntr=findc(root);
 cntrorder.push_back(cntr);
 timstmp[cntr]=tim++;
 par[cntr]=p;
 if(p>=0)adj1[p].push_back(cntr);
 for(auto i:adj[cntr])
  if(!timstmp[i])
   cntrdecom(i,cntr);
}
void dfs2(int root,int nod,int j,int dst)
{
 if(dist[root][j].size()==dst)dist[root][j].↩
     push_back(0);
 vis[nod]=tim;
 dist[root][j][dst]+=1;
 for(auto i:adj[nod])
 {
  if((timstmp[i]<=timstmp[root])||(vis[i]==vis[nod↩
      ]))continue;
  vis[i]=tim;dfs2(root,i,j,dst+1);
 }
}
void preprocess()
{
 for(int i=0;i<cntrorder.size();i++)
 {
  int root=cntrorder[i];
  vector<int> temp;
  dist[root].push_back(temp);
  temp.push_back(0);
  ++tim;
  dfs2(root,root,0,0);
  int cnt=0;
  for(int j=0;j<adj[root].size();j++)
  {
   int nod=adj[root][j];
   if(timstmp[nod]<timstmp[root])
    continue;
   dist[root].push_back(temp);
   ++tim;
   dfs2(root,nod,++cnt,1);
  }
 }
}
```

---

6 Maths

6.1 Chinese Remainder Theorem

```
#include<bits/stdc++.h>
using namespace std;
#define ll long long
/*solves system of equations x=rem[i]%mods[i] for ↩
   any mod (need not be coprime)
intput:vector of remainders and moduli
output: pair of answer(x%lcm of modulo) and lcm of↩
    all the modulo (returns -1 if it is ↩
   inconsistent)*/
ll GCD(ll a, ll b) { return (b == 0) ? a : GCD(b, ↩
   a % b); }
inline ll LCM(ll a, ll b) { return a / GCD(a, b) *↩
    b; }
inline ll normalize(ll x, ll mod) { x %= mod; if (↩
   x < 0) x += mod; return x; }
struct GCD_type { ll x, y, d; };
GCD_type ex_GCD(ll a, ll b)
{
    if (b == 0) return {1, 0, a};
    GCD_type pom = ex_GCD(b, a % b);
    return {pom.y, pom.x - a / b * pom.y, pom.d};
}
pair<ll,ll> CRT(vector<ll> &rem,vector<ll> &mods)
{
    ll n=rem.size();
    ll ans=rem[0];
    ll lcm=mods[0];
    for(ll i=1;i<n;i++)
    {
        auto pom=ex_GCD(lcm,mods[i]);
        ll x1=pom.x;
        ll d=pom.d;
        if((rem[i]-ans)%d!=0)return {-1,0};
```

```
        ans=normalize(ans+x1*(rem[i]-ans)/d%(mods[i]/d)*lcm,lcm*mods[i]/d);
        lcm=LCM(lcm,mods[i]); // you save time by replacing above lcm * n[i] /d by lcm = lcm * n[i] / d
    }
    return {ans,lcm};
}
```

6.2 Discrete Log

```
// Discrete Log , Baby-Step Giant-Step , e-maxx
// The idea is to make two functions,
// f1(p) , f2(q) and find p,q s.t.
// f1(p) = f2(q) by storing all possible values of f1,
// and checking for q. In this case a^(x) = b (mod m) is
// solved by substituting x by p.n-q , where
// is choosen optimally , usually sqrt(m).

// credits : https://cp-algorithms.com/algebra/discrete-log.html
// returns a soln. for a^(x) = b (mod m)
// for given a,b,m . -1 if no. soln.
// complexity : O(sqrt(m).log(m))
// use unordered_map to remove log factor.
// IMP : works only if a,m are co-prime. But can be modified.
int solve (int a, int b, int m) {
    int n = (int) sqrt (m + .0) + 1;

    int an = 1;
    for (int i=0; i<n; ++i)
        an = (an * a) % m;

    map<int,int> vals;
    for (int i=1, cur=an; i<=n; ++i) {
        if (!vals.count(cur))
            vals[cur] = i;
        cur = (cur * an) % m;
    }

    for (int i=0, cur=b; i<=n; ++i) {
        if (vals.count(cur)) {
            int ans = vals[cur] * n - i;
            if (ans < m)
                return ans;
        }
        cur = (cur * a) % m;
    }
    return -1;
}
```

6.3 NTT

```
/*
Kevin's different Code: https://s3.amazonaws.com/codechef_shared/download/Solutions/JUNE15/tester/MOREFB.cpp
****There is no problem that FFT can solve while this NTT cannot
 Case1: If the answer would be small choose a small enough NTT prime modulus
 Case2: If the answer is large(> ~1e9) FFT would not work anyway due to precision issues
 In Case2 use NTT. If max_answer_size=n*(largest_coefficient^2)
 So use two or three modulus to solve it
****Compute a*b%mod if a%mod*b%mod would result in overflow in O(log(a)) time:
 ll mulmod(ll a, ll b, ll mod) {
     ll res = 0;
     while (a != 0) {
         if (a & 1) res = (res + b) % m;
         a >>= 1;
         b = (b << 1) % m;
     }
     return res;
 }
Fastest NTT (can also do polynomial multiplication if max coefficients are upto 1e18 using 2 modulus and CRT)
How to use:
P=A*B
Polynomial1 = A[0]+A[1]*x^1+A[2]*x^2+..+A[n-1]*x^n-1
Polynomial2 = B[0]+B[1]*x^1+B[2]*x^2+..+B[n-1]*x^n-1
P=multiply(A,B)
A and B are not passed by reference because they are changed in multiply function
For CRT after obtaining answer modulo two primes p1 and p2:
x = a1 mod p1, x = a2 mod p2 => x=((a1*(m2^-1)%m1)*m2+(a2*(m1^-1)%m2)*m1)%m1m2
*** Before each call to multiply:
 set base=1,roots={0,1},rev={0,1},max_base=x (such that if mod=c*(2^k)+1 then x<=k and 2^x is greater than equal to nearest power of 2 of 2*n)
```

```
 root=primitive_root^((mod-1)/(2^max_base))
 For P=A*A use square function
Some useful modulo and examples
mod1=463470593 = 1768*2^18+1 primitive root = 3 => max_base=18,root=3^1768
mod2=469762049 = 1792*2^18+1 primitive root = 3 => max_base=18,root=3^1792
(mod1^-1)%mod2=313174774 (mod2^-1)%mod1=154490124
Some prime modulus and primitive root
  635437057   11
     639631361   6
     645922817   3
     648019969   17
     666894337   5
     683671553   3
     710934529   17
     715128833   3
     740294657   3
     754974721   11
     786432001   7
     799014913   13
     824180737   5
     880803841   26
     897581057   3
     899678209   7
     918552577   5
     924844033   5
     935329793   3
     943718401   7
     950009857   7
     962592769   7
     975175681   17
     985661441   3
     998244353   3
*/
//x = a1 mod m1, x = a2 mod m2, invm2m1 = (m2^-1)%m1, invm1m2 = (m1^-1)%m2, gives x%m1*m2
#define chinese(a1,m1,invm2m1,a2,m2,invm1m2) ((a1 *1ll* invm2m1 % m1 * 1ll*m2 + a2 *1ll* invm1m2 % m2 * 1ll*m1) % (m1 *1ll* m2))
int mod;//reset mod everytime with required modulus
inline int mul(int a,int b){return (a*1ll*b)%mod;}
inline int add(int a,int b){a+=b;if(a>=mod)a-=mod;return a;}
inline int sub(int a,int b){a-=b;if(a<0)a+=mod;return a;}
inline int power(int a,int b){int rt=1;while(b>0){if(b&1)rt=mul(rt,a);a=mul(a,a);b>>=1;}return rt;}
inline int inv(int a){return power(a,mod-2);}
inline void modadd(int &a,int &b){a+=b;if(a>=mod)a-=mod;}

int base = 1;
vector<int> roots = {0, 1};
vector<int> rev = {0, 1};
int max_base=18; //x such that 2^x|(mod-1) and 2^x>max answer size(=2*n)
int root=202376916;  //primitive root^((mod-1)/(2^max_base))
void ensure_base(int nbase) {
 if (nbase <= base) {
   return;
 }
 assert(nbase <= max_base);
 rev.resize(1 << nbase);
 for (int i = 0; i < (1 << nbase); i++) {
   rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (nbase - 1));
 }
 roots.resize(1 << nbase);
 while (base < nbase) {
   int z = power(root, 1 << (max_base - 1 - base));
   for (int i = 1 << (base - 1); i < (1 << base); i++) {
     roots[i << 1] = roots[i];
     roots[(i << 1) + 1] = mul(roots[i], z);
   }
   base++;
 }
}
void fft(vector<int> &a) {
 int n = (int) a.size();
 assert((n & (n - 1)) == 0);
 int zeros = __builtin_ctz(n);
 ensure_base(zeros);
 int shift = base - zeros;
 for (int i = 0; i < n; i++) {
   if (i < (rev[i] >> shift)) {
     swap(a[i], a[rev[i] >> shift]);
   }
 }
 for (int k = 1; k < n; k <<= 1) {
   for (int i = 0; i < n; i += 2 * k) {
     for (int j = 0; j < k; j++) {
       int x = a[i + j];
       int y = mul(a[i + j + k], roots[j + k]);
       a[i + j] = x + y - mod;
       if (a[i + j] < 0) a[i + j] += mod;
       a[i + j + k] = x - y + mod;
       if (a[i + j + k] >= mod) a[i + j + k] -= mod;
     }
   }
 }
}
vector<int> multiply(vector<int> a, vector<int> b, int eq = 0) {
 int need = (int) (a.size() + b.size() - 1);
 int nbase = 0;
 while ((1 << nbase) < need) nbase++;
 ensure_base(nbase);
 int sz = 1 << nbase;
 a.resize(sz);
```

```
 b.resize(sz);
 fft(a);
 if (eq) b = a; else fft(b);
 int inv_sz = inv(sz);
 for (int i = 0; i < sz; i++) {
   a[i] = mul(mul(a[i], b[i]), inv_sz);
 }
 reverse(a.begin() + 1, a.end());
 fft(a);
 a.resize(need);
 return a;
}
vector<int> square(vector<int> a) {
 return multiply(a, a, 1);
}
```

6.4 Langrange Interpolation

```
/* Input :
Degree of polynomial: k
Polynomial values at x=0,1,2,3,...,k

Output :
Polynomial value at x
Complexity: O(degree of polynomial)
Works only if the points are equally spaced
*/
ll lagrange(vll& v , int k, ll x,int mod)
{
    if(x <= k)
        return v[x];
    ll inn = 1;
    ll den = 1;
    for(int i = 1;i<=k;i++)
    {
        inn = (inn*(x - i))%mod;
        den = (den*(mod - i))%mod;
    }
    inn = (inn*inv(den % mod))%mod;
    ll ret = 0;
    for(int i = 0;i<=k;i++){
        ret = (ret + v[i]*inn)%mod;
        ll md1 = mod - ((x-i)*(k-i))%mod;
        ll md2 = ((i+1)*(x-i-1))%mod;
        if(i!=k)
            inn = (((inn*md1)%mod)*inv(md2 % mod))↩
               %mod;
    }
    return ret;
}
```

6.5 Matrix Struct

```
struct matrix{
    ld B[N][N], n;
    matrix(){n = N; memset(B,0,sizeof B);}
    matrix(int _n){
        n = _n; memset(B, 0, sizeof B);
    }
    void iden(){
     for(int i = 0; i < n; i++)
      B[i][i] = 1;
    }
    void operator += (matrix M){
        for(int i = 0; i < n; i++)
         for(int j = 0; j < n; j++)
          B[i][j] = add(B[i][j], M.B[i][j]);
    }
    void operator -= (matrix M){
        for(int i = 0; i < n; i++)
         for(int j = 0; j < n; j++)
          B[i][j] = sub(B[i][j], M.B[i][j]);
    }
    void operator *= (ld b){
        for(int i = 0; i < n; i++)
         for(int j = 0; j < n; j++)
          B[i][j] = mul(b, B[i][j]);
    }
    matrix operator - (matrix M){
        matrix ret = (*this);
        ret -= M; return ret;
    }
    matrix operator + (matrix M){
        matrix ret = (*this);
        ret += M; return ret;
    }
    matrix operator * (matrix M){
        matrix ret = matrix(n); memset(ret.B, 0, ↩
           sizeof ret.B);
        for(int i = 0; i < n; i++)
            for(int j = 0; j < n;j++)
                for(int k = 0; k < n; k++){
                    ret.B[i][j] = add(ret.B[i][j],↩
                        mul(B[i][k], M.B[k][j]));
                }
        return ret;
    }
    matrix operator *= (matrix M){ *this = ((*this↩
       ) * M);}
    matrix operator * (int b){
```

```
        matrix ret = (*this); ret *= b; return ret;
    }
    vector<double> multiply(const vector<double> &v) const{
     vector<double> ret(n);
     for(int i = 0; i < n; i++)
      for(int j = 0; j < n; j++){
       ret[i] += B[i][j] * v[j];
      }
     return ret;
    }
};
```

6.6 nCr(Non Prime Modulo)

```
// sandwich, jtnydv25 video
// calculates nCr, for small
// non-prime modulo, and (very) big n,r.
ll phimod;
vll pr,prn;vll fact;
ll power(ll a,ll x,ll mod){
 ll ans=1;
 while(x){
  if((1LL)&(x))ans=(ans*a)%mod;
  a=(a*a)%mod;x>>=1LL;
 }
 return ans;
}
// prime factorization of x.
// pr-> prime ; prn -> it's exponent
void getprime(ll x){
  pr.clear();prn.clear();
  ll i,j,k;
  for(i=2;(i*i)<=x;i++){
   k=0;while((x%i)==0){k++;x/=i;}
   if(k>0){pr.pb(i);prn.pb(k);}
  }
  if(x!=1){pr.pb(x);prn.pb(1);}
  return;
}
// factorials are calculated ignoring
//  multiples of p.
void primeproc(ll p,ll pe){   // p , p^e
  ll i,d;
  fact.clear();fact.pb(1);d=1;
  for(i=1;i<pe;i++){
   if(i%p){fact.pb((fact[i-1]*i)%pe);}
   else {fact.pb(fact[i-1]);}
  }
  return;
}
// again note this has ignored multiples of p
ll Bigfact(ll n,ll mod){
 ll a,b,c,d,i,j,k;
 a=n/mod;a%=phimod;a=power(fact[mod-1],a,mod);
 b=n%mod;a=(a*fact[b])%mod;
 return a;
}
// Chinese Remainder Thm.
vll crtval,crtmod;
ll crt(vll &val,vll &mod){
 ll a,b,c,d,i,j,k;b=1;
 for(ll z:mod)b*=z;
 ll ans=0;
 for(i=0;i<mod.size();i++){
  a=mod[i];c=b/a;
  d=power(c,(((a/pr[i])*(pr[i]-1))-1),a);
  c=(c*d)%b;c=(c*val[i])%b;ans=(ans+c)%b;
 }
 return ans;
}
// calculate for prime powers and
// take crt. For each prime power,
// first ignore multiples of p,
// and then do recursively, calculating
// the powers of p separately.
ll Bigncr(ll n,ll r,ll mod){
 ll a,b,c,d,i,j,k;ll p,pe;
 getprime(mod);ll Fnum=1;ll Fden;
 crtval.clear();crtmod.clear();
 for(i=0;i<pr.size();i++){
  Fnum=1;Fden=1;
  p=pr[i];pe=power(p,prn[i],1e17);
  primeproc(p,pe);
  a=1;d=0;
  phimod=(pe*(p-1LL))/p;
  ll n1=n,r1=r,nr=n-r;
  while(n1){
   Fnum=(Fnum*(Bigfact(n1,pe)))%pe;
   Fden=(Fden*(Bigfact(r1,pe)))%pe;
   Fden=(Fden*(Bigfact(nr,pe)))%pe;
   d+=n1-(r1+nr);
   n1/=p;r1/=p;nr/=p;
  }
  Fnum=(Fnum*(power(Fden,(phimod-1LL),pe)))%pe;
  if(d>=prn[i])Fnum=0;
  else Fnum=(Fnum*(power(p,d,pe)))%pe;
  crtmod.pb(pe);crtval.pb(Fnum);
 }
 // you can just iterate instead of crt
 // for(i=0;i<mod;i++){
 //  bool cg=true;
 //  for(j=0;j<crtmod.size();j++){
```

```
//    if(i%crtmod[j]!=crtval[j])cg=false;
//   }
//   if(cg)return i;
// }
return crt(crtval,crtmod);
}
```

6.7 Primitive Root Generator

```
/*To find generator of U(p),we check for all
  g in [1,p]. But only for powers of the
  form phi(p)/p_j, where p_j is a prime factor of
  phi(p). Note that p is not prime here.

  Existence , if one of these :
  1. p = 1,2,4
  2. p = q^k , where q -> odd prime.
  3. p = 2.(q^k) , where q-> odd prime

  Note that a.g^(phi(p)) = 1 (mod p)
            b.there are phi(phi(p)) generators if ↩
               exists.
*/
// Finds "a" generator of U(p),
// multiplicative group of integers mod p.
// here calc_phi returns the toitent function for ↩
   p
// Complexity : O(Ans.log(phi(p)).log(p)) + time ↩
    for factorizing phi(p).
// By some theorem, Ans = O((log(p))^6). Should be↩
    fast generally.
int generator (int p) {
    vector<int> fact;
    int phi = calc_phi(p),  n = phi;
    for (int i=2; i*i<=n; ++i)
        if (n % i == 0) {
            fact.push_back (i);
            while (n % i == 0)
                n /= i;
        }
    if (n > 1)
        fact.push_back (n);

    for (int res=2; res<=p; ++res) {
        if(gcd(res,p)!=1)continue;
        bool ok = true;
        for (size_t i=0; i<fact.size() && ok; ++i)
            ok &= powmod (res, phi / fact[i], p) ↩
               != 1;
        if (ok)  return res;
    }
    return -1;
}
```

7 Strings

7.1 Hashing Theory

```
If order not imp. and count/frequency imp. use ↩
   this as hash fn:-
( (a_1+h)^k + (a_2+h)^k + (a_3+h)^k + (a_4+h)^k ) ↩
   % p. Select : h,k,p
Alternate:
((x)^(a_1)+(x)^(a_2)+...+(x)^(a_k))%mod where x ↩
   and mod are fixed and a_1...a_k is an unordered ↩
   set
```

7.2 Manacher

```
// Same idea as Z_algo, Time : O(n)
// [l,r] represents : boundaries of rightmost ↩
   detected subpalindrom(with max r)
// takes string s and returns a vector of lengths ↩
   of odd length palindrom
// centered around that char(e.g abac for 'b' ↩
   returns 2(not 3))
vll manacher_odd(string s){
    ll n = s.length(); vll d1(n);
    for(ll i = 0, l = 0, r = -1;i<n;i++){
        d1[i] = 1;
        if(i <= r){
            d1[i] = min(r-i+1,d1[l+r-i]); // use ↩
               prev val
        }
        while(i+d1[i] < n && i-d1[i] >= 0 && s[i+↩
           d1[i]] == s[i-d1[i]]) d1[i]++; // ↩
           trivial matching
        if(r < i+d1[i]-1) l=i-d1[i]+1, r=i+d1[i↩
           ]-1; // update r
    }
    return d1;
}
```

```
// takes string s and returns vector of lengths of↩
    even length ...
// (it's centered around the right middle char, bb↩
    is centered around the later 'b')
vll manacher_even(string s){
    ll n = s.length(); vll d2(n);
    for(ll i = 0, l = 0, r = -1;i<n;i++){
        d2[i] = 0;
        if(i <= r){
            d2[i] = min(r-i+1,d2[l+r+1-i]);
        }
        while(i+d2[i] < n && i-d2[i]-1 >= 0 && s[i↩
           +d2[i]] == s[i-d2[i]-1]) d2[i]++;
        if(d2[i] > 0 && r < i+d2[i]-1) l=i-d2[i], ↩
           r=i+d2[i]-1;
    }
    return d2;
}
// Other mtd : To do both things in one pass, add ↩
   special char e.g string "abc" => "$a$b$c$"
```

7.3 Suffix Array

```
//code credits - https://cp-algorithms.com/string/↩
   suffix-array.html
/*Theory :-
Sorted array of suffixes = sorted array of cyclic ↩
   shifts of string+$.
We consider a prefix of len. 2^k of the cyclic, in↩
    the kth iteration.
And find the sorted order, using values for (k-1)↩
   th iteration and
kind of radix sort. Could be thought as some kind ↩
   of binary lifting.
String of len. 2^k -> combination of 2 strings of ↩
   len. 2^(k-1), whose
order we know. Just radix sort on pair for next ↩
   iteration.

Applications :-
Finding the smallest cyclic shift;Finding a ↩
   substring in a string;
Comparing two substrings of a string;Longest ↩
   common prefix of two substrings;
Number of different substrings.

*/
vector<ll> sort_cyclic_shifts(string const& s) {
    ll n = s.size();
    const ll alphabet = 256;
    //********* change the alphabet size ↩
       accordingly and indexing ******************
        vector<ll> p(n), c(n), cnt(max(alphabet, n↩
           ), 0);
    // p -> sorted order of 1-len prefix of each ↩
       cyclic shift index.
    // c -> class of a index
    // pn -> same as p for kth iteration . ||ly cn↩
       .
    for (ll i = 0; i < n; i++)
        cnt[s[i]]++;
    for (ll i = 1; i < alphabet; i++)
        cnt[i] += cnt[i-1];
    for (ll i = 0; i < n; i++)
        p[--cnt[s[i]]] = i;
    c[p[0]] = 0;
    ll classes = 1;
    for (ll i = 1; i < n; i++) {
        if (s[p[i]] != s[p[i-1]])
            classes++;
        c[p[i]] = classes - 1;
    }
        vector<ll> pn(n), cn(n);
    for (ll h = 0; (1 << h) < n; ++h) {
        for (ll i = 0; i < n; i++) {  // sorting w↩
           .r.t second part.
            pn[i] = p[i] - (1 << h);
            if (pn[i] < 0)
                pn[i] += n;
        }
        fill(cnt.begin(), cnt.begin() + classes, ↩
           0);
        for (ll i = 0; i < n; i++)
            cnt[c[pn[i]]]++;
        for (ll i = 1; i < classes; i++)
            cnt[i] += cnt[i-1];
        for (ll i = n-1; i >= 0; i--)
            p[--cnt[c[pn[i]]]] = pn[i];    // ↩
               sorting w.r.t first (more ↩
               significant) part.
        cn[p[0]] = 0;
        classes = 1;
        for (ll i = 1; i < n; i++) {  // ↩
           determining new classes in sorted array.
            pair<ll, ll> cur = {c[p[i]], c[(p[i] +↩
                (1 << h)) % n]};
            pair<ll, ll> prev = {c[p[i-1]], c[(p[i↩
               -1] + (1 << h)) % n]};
            if (cur != prev)
                ++classes;
            cn[p[i]] = classes - 1;
        }
        c.swap(cn);
    }
    return p;
}
```

```
vector<ll> suffix_array_construction(string s) {
    s += "$";
    vector<ll> sorted_shifts = sort_cyclic_shifts(↩
       s);
    sorted_shifts.erase(sorted_shifts.begin());
    return sorted_shifts;
}
// For comparing two substring of length l ↩
   starting at i,j.
// k - 2^k > l/2. check the first 2^k part, if ↩
   equal,
// check last 2^k part. c[k] is the c in kth iter ↩
   of S.A construction.
int compare(int i, int j, int l, int k) {
    pair<int, int> a = {c[k][i], c[k][(i+l-(1 << k↩
       ))%n]};
    pair<int, int> b = {c[k][j], c[k][(j+l-(1 << k↩
       ))%n]};
    return a == b ? 0 : a < b ? -1 : 1;
}
/*
Kasai's Algo for LCP construction :
Longest Common Prefix for consecutive suffixes in ↩
   suffix array.
lcp[i]=length of lcp of ith and (i+1)th suffix in ↩
   the susffix array.
1. Consider suffixes in decreasing order of length↩
   .
2. Let p = s[i....n]. It will be somewhere in the ↩
   S.A.We determine its lcp = k.
3. Then lcp of q=s[(i+1)....n] will be atleast k↩
   -1. Why?
4. Remove the first char of p and its successor in↩
    the S.A. These are suffixes with lcp k-1.
5. But note that these 2 may not be consecutive in↩
    S.A. But however lcp of strings in
   b/w have to be also atleast k-1.
*/
vector<ll> lcp_construction(string const& s, ↩
   vector<ll> const& p) {
    ll n = s.size();
    vector<ll> rank(n, 0);
    for (ll i = 0; i < n; i++)
        rank[p[i]] = i;

    ll k = 0;
    vector<ll> lcp(n-1, 0);
    for (ll i = 0; i < n; i++) {
        if (rank[i] == n - 1) {
            k = 0;
            continue;
        }
        ll j = p[rank[i] + 1];
        while (i + k < n && j + k < n && s[i+k] ==↩
            s[j+k])
            k++;
        lcp[rank[i]] = k;
        if (k)
            k--;
    }
    return lcp;
}
```

7.4 Trie

```
const ll AS = 26; // alphabet size
ll go[MAX][AS];
ll cnt[MAX];ll cn=0;
// cn -> index of next new node
// convert all strings to vll
ll newNode() {
 for(ll i=0;i<AS;i++)
  go[cn][i]=0;
 return cn++;
}
// call newNode once *******
// before adding anything **
void addTrie(vll &x) {
 ll v = 0;
 cnt[v]++;
 for(ll i=0;i<x.size();i++){
  ll y=x[i];
  if(go[v][y]==-1)
   go[v][y]=newNode();
  v=go[v][y];
  cnt[v]++;
 }
}
// returns count of substrings with prefix x
ll getcount(vll &x){
 ll v=0;
 for(i=0;i<x.size();i++){
  ll y=x[i];
  if(go[v][y]==-1)
   go[v][y]=newNode();
  v=go[v][y];
 }
 return cnt[v];
}
```

7.5 Z-algorithm

```
// [l,r] -> indices of the rightmost segment match
// (the detected segment that ends rightmost(with ↩
   max r))
// 2 cases -> 1st. i <= r : z[i] is atleast min(r-↩
   i+1,z[i-l]), then match trivially
// 2nd. o.w compute z[i] with trivial matching
// update l,r
// Time : O(n)(asy. behavior), Proof : each ↩
   iteration of inner while loop make r pointer ↩
   advance to right,
// Applications:    1) Search substring(text t,↩
   pattern p) s = p + '$' + t.
// 3) String compression(s = t+t+...+t, then find ↩
   |t|)
// 2) Number of distinct substrings (in O(n^2))
// (useful when appending or deleting characters ↩
   online from the end or beginning)
vector<ll> z_function(string s) {
    ll n = (ll) s.length();
    vector<ll> z(n);
    for (ll i = 1, l = 0, r = 0; i < n; ++i) {
        if (i <= r)
            z[i] = min (r - i + 1, z[i - l]); // ↩
               use previous z val
        while (i + z[i] < n && s[z[i]] == s[i + z[↩
           i]]) // trivial matching
            ++z[i];
        if (i + z[i] - 1 > r)
            l = i, r = i + z[i] - 1; // update ↩
               rightmost segment matched
    }
    return z;
}
```