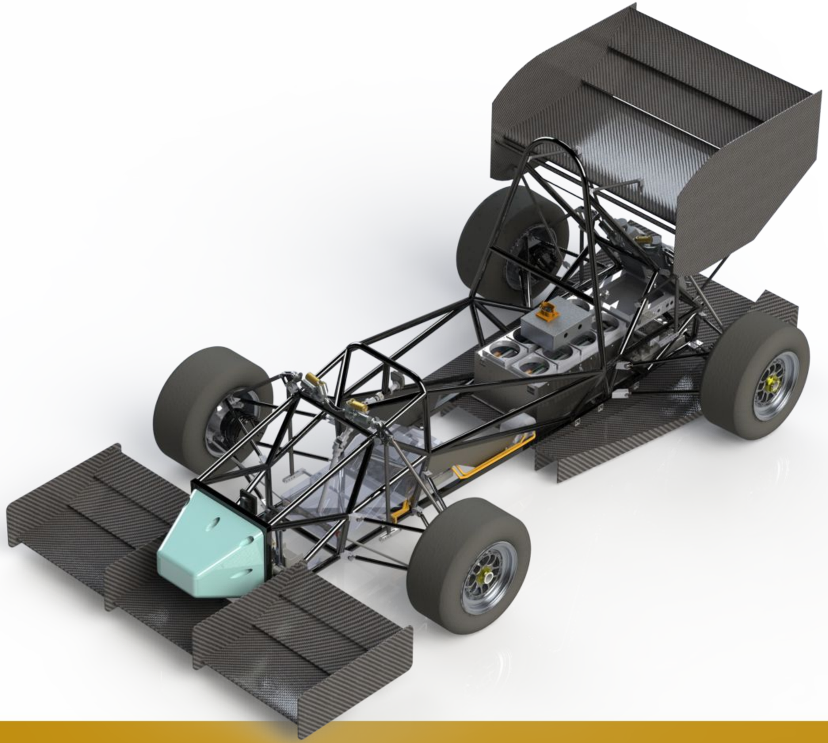
Torque Vectoring Overview

Purdue Electric Racing ‘22 Controls System

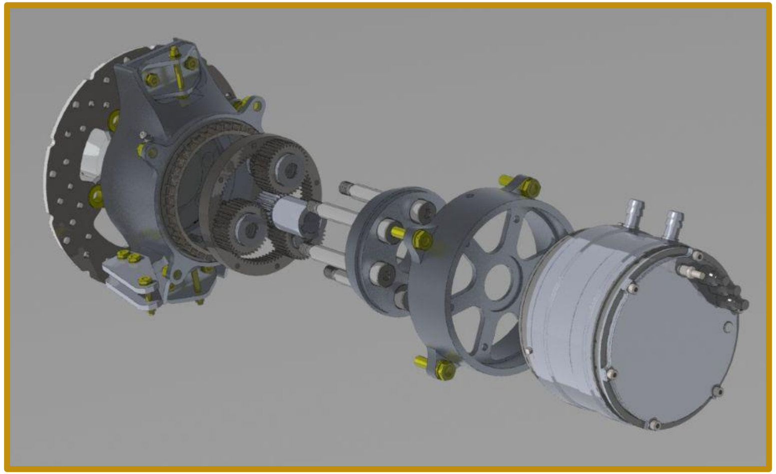


Desired Meeting Outcomes

- Conceptual validation of torque vectoring model
  - Objective statement
  - Boundary Conditions
  - Yaw Control
- Mathematical validation of torque vectoring model
  - Reference Yaw
  - Upper & Lower torque boundaries
  - Torque to Force conversion

Motivation

To develop a control scheme for a 4WD hub motor system in Matlab/Simulink to be validated and implemented on PER '22.

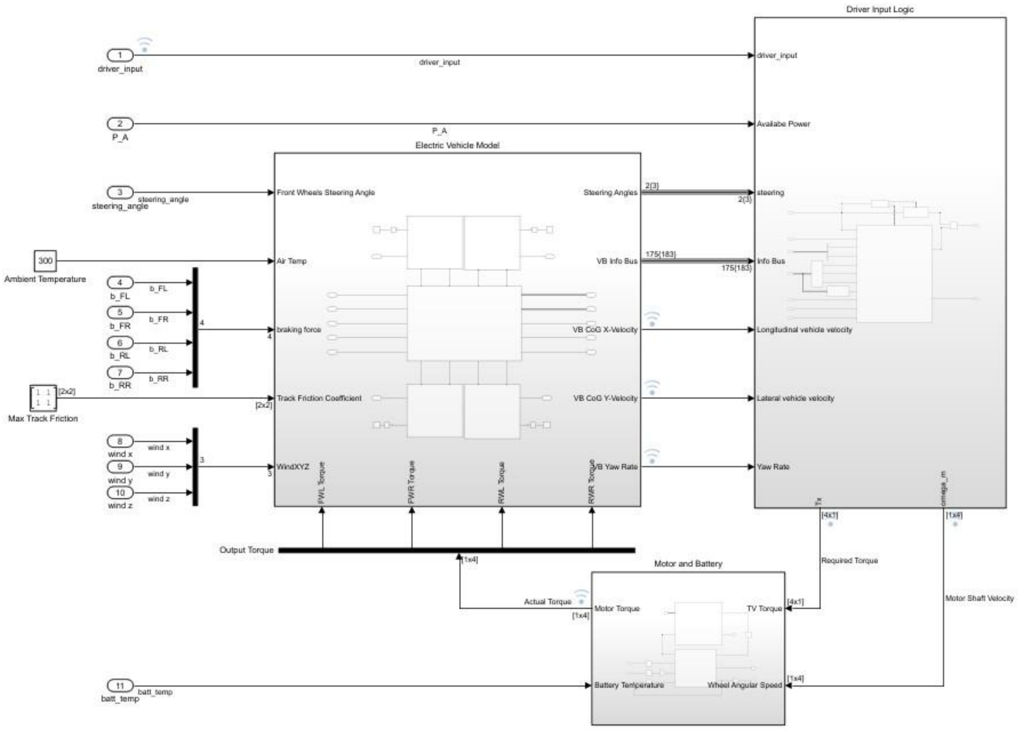


Development

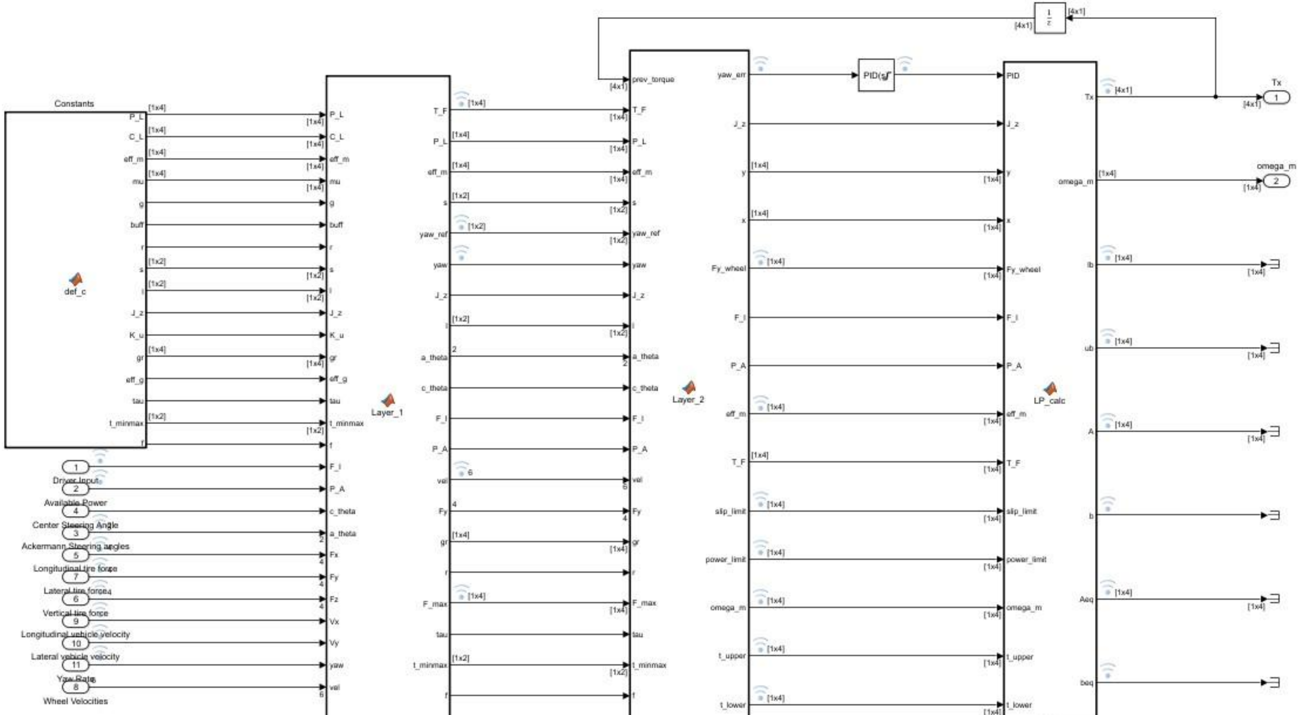
Research

- Forze Hydrogen Racing 4WD torque vectoring model (Anton Stoop)
- Race Car Vehicle Dynamics (Milliken & Milliken)
- Matlab + Simulink vehicle body control schemes (Matlab documentation)

Development - Lap Sim Validation



Development - Block Diagram



Development - PID

- PI Controller, with fixed proportional and integral gains
- Controls the yaw rate based on reference, $\dot{\varphi}_{ref}$
- Error: $e = \dot{\varphi}_{ref} - \dot{\varphi}_{real}$
- Output: Yaw moment, $\propto \ddot{\varphi}$

Development - Upper & Lower Boundaries

Slip Boundary:

- Torque must be less than the corresponding maximum longitudinal force on the ideal traction circle

Power Boundary:

- Per motor power input must not exceed power input limit (15kW)

Motor Boundary:

- Per motor torque request must not exceed the maximum and minimum torque each motor can achieve by the next time step

Development - Acceleration Statement

Objective:

- Maximize total driving force

Subject to:

- Total power input to motors must not exceed total power available
- Moment about vehicle CG must be equal to PID
- Torque per motor must stay between an upper and lower bounds, based on slip, power, and motor defined limits.

Pause:

- Driving force as the objective
- Lateral Forces influencing yaw acceleration
- Constraining torque output
- Multiple different optimization statements
- Validation during Simulation

Development - Braking Statement

Objective:

- Maximize total braking force

Subject to:

- Total power input to battery must not exceed total power available
- Moment about vehicle CG must be equal to PID
- Torque per motor must stay between an upper and lower bounds, based on slip, power, and motor defined limits.

Project Difficulties

- Determining friction limits of tires
- Detect if slip does occur, how to react
- Tire characterization
- Make system resilient against input error

Thanks to GM:

Tamim Noor

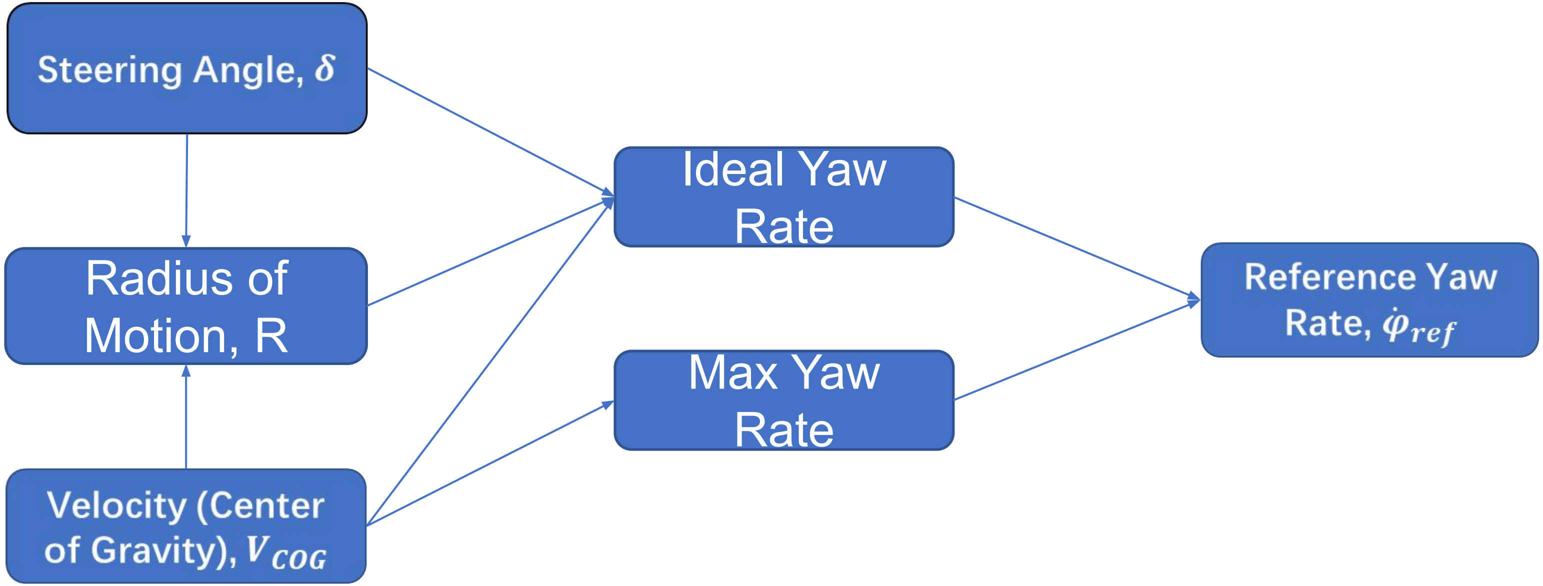
Ehsan Asadi

Patrick Monsere

Thanks to PER members:

Demetrius Gulewicz	David Farrell	Adam Busch
Wasif Islam	Tao Sun	Zach Ellis
Ruhaan Joshi	Dawson Moore	Jacob Kosiba
James Hofstadler	Ethan Brown	

Development: Generating Reference Yaw Rate



Development - Generate Reference Yaw

$$\dot{\psi}_{desired} = \frac{V_{cog}\theta_c}{(l_f+l_r)+(K_u V_{cog}^2)}$$

$$\dot{\psi}_{max} = \frac{\sigma\mu g}{V_{cog}+b}$$

$$\dot{\psi}_{ref} = min(\dot{\psi}_{desired}, \dot{\psi}_{max})$$

- Desired yaw rate is a function of driver input steering angle.
- Maximum yaw rate is limited by tire road friction coefficient. Simplified to exclude sideslip angle, a contribution factor is used to account for that simplification

Development - Upper & Lower Boundaries

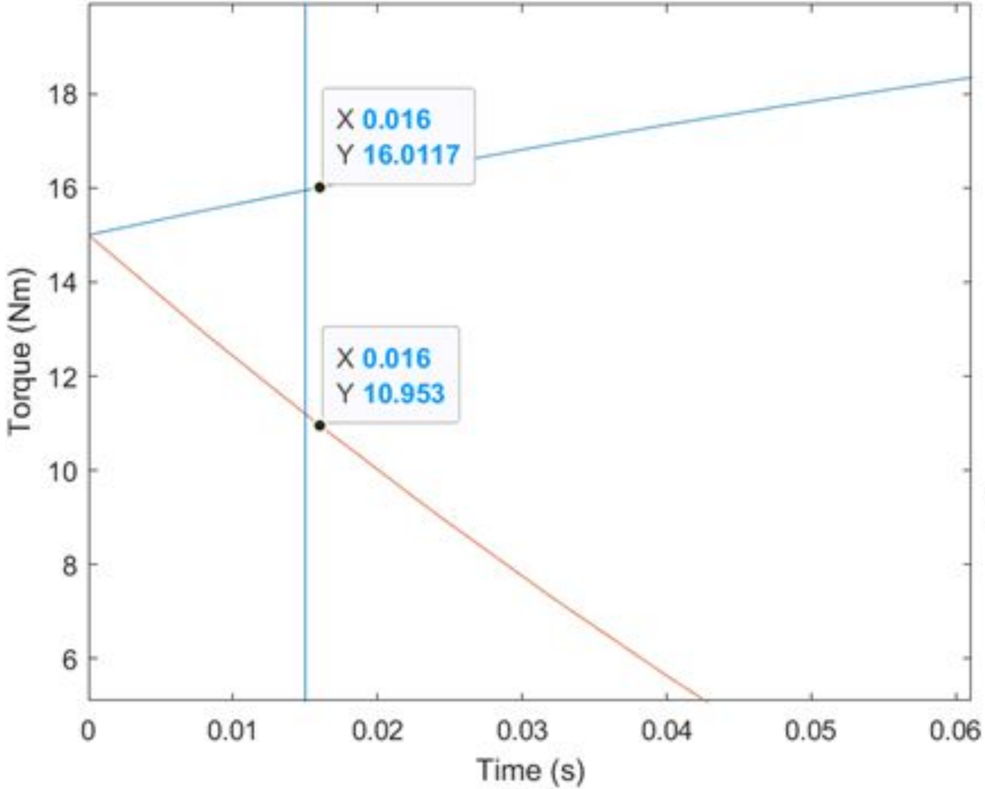
$$\vec{T}_{SL} = \frac{\sqrt{\mu \vec{F}_z^2 - \vec{F}_y^2}}{\vec{C}}$$

$$\vec{T}_{PL} = \frac{\vec{P}_L \odot \vec{\eta}_{motor}}{\vec{\omega}_{motor}}$$

$$\vec{T}_{ML} = \pm 25[1 - e^{\frac{-1}{\tau f}}] + [\vec{T}_{x,prev} \odot e^{\frac{-1}{\tau f}}]$$

- Torque must be less than the corresponding maximum longitudinal force on the current ideal traction circle
- Per motor power input must not exceed power input limit (15kW)
- Per motor torque request must not exceed the maximum and minimum torque each motor can achieve in the next time step

Development - Motor Boundaries



- Per motor torque request must not exceed the maximum and minimum torque each motor can achieve in the next time step

$$\vec{T}_{ML} = \pm 25[1 - e^{\frac{-1}{\tau f}}] + [\vec{T}_{x,prev} \odot e^{\frac{-1}{\tau f}}]$$

Development - Torque to Force

- Moment equation uses force at tires
- Controller output is torque request for each wheel
- Need to link motor shaft torque to driving force

$$\vec{C} = \frac{1}{r_{tire}} (\vec{gr} \odot \vec{\eta}_{gearbox})$$

Development - Optimization Statement

Maximize:

$$\vec{T_x} \cdot \vec{C}$$

Subject to:

$$\vec{T_x} \cdot \left(\frac{\vec{\omega_m}}{\vec{\eta}_{motors}}\right) \leq P_A$$

$$(\vec{C} \odot \vec{K_1}) \cdot \vec{T_x} = -(\vec{K_2} \cdot \vec{F_y}) + PID$$

$$\vec{lb} \leq \vec{T_x} \leq \vec{ub}$$

- Maximize total driving force
- Total power input must not exceed total power available
- Moment about vehicle CG must be equal to PID
- Torque per motor must stay between an upper and lower bounds, based on slip, power, and motor defined limits.

Basic Results