14 Applications of Trigonometry

14A Two-dimensional applications

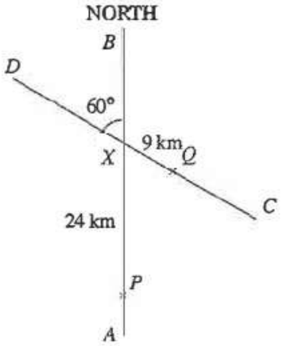
14A.1 HKCEE MA 1981(2/3) I 11

AB and CD are two straight roads intersecting at X. AB runs North and makes an angle of 60° with CD. At noon, two people P and Q are respectively 24 km and 9 km from X as shown in the figure. P walks at a speed of 4.5 km/h towards B and Q walks at a speed of 6 km/h towards D.

(a) Calculate the distance between P and Q at noon.

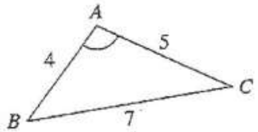
(b) What are the distances of P and Q from X at 4 p.m.?

(c) Calculate the bearing of Q from P at 4 p.m. to the nearest degree.



14A.2 HKCEE MA 1982(3) – I – 2

In the figure, $AB = 4$, $AC = 5$ and $BC = 7$.
Calculate $\angle A$ to the nearest degree.



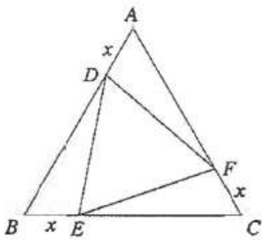
14A.3 HKCEE MA 1985(A/B) I 13

(To continue as **7C.1**.)

In the figure, ABC is an equilateral triangle. $AB = 2$. D, E, F are points on AB, BC, CA respectively such that $AD = BE = CF = x$.

(a) By using the cosine formula or otherwise, express DE^2 in terms of x.

(b) Show that the area of $\triangle DEF = \dfrac{\sqrt{3}}{4}(3x^2 - 6x + 4)$.



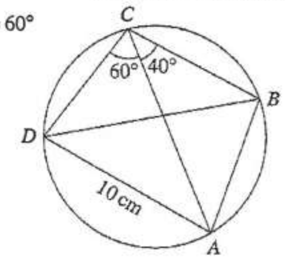
14A.4 HKCEE MA 1989 – I – 6

(Continued from **12A.7**.)

In the figure, $ABCD$ is a cyclic quadrilateral with $AD = 10$ cm, $\angle ACD = 60°$ and $\angle ACB = 40°$.

(a) Find $\angle ABD$ and $\angle BAD$.

(b) Find the length of BD in cm, correct to 2 decimal places.



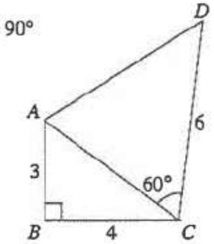
14A.5 HKCEE MA 1997 I 5

In the figure, ABC is a right angled triangle. $AB = 3$, $BC = 4$, $CD = 6$, $\angle ABC = 90°$ and $\angle ACD = 60°$. Find

(a) AC,

(b) AD,

(c) the area of $\triangle ACD$.



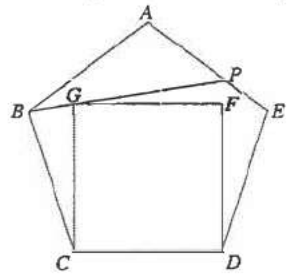
14A.6 HKCEE MA 2000 I 13

(Continued from **11A.11**.)

In the figure, $ABCDE$ is a regular pentagon and $CDFG$ is a square. BG produced meets AE at P.

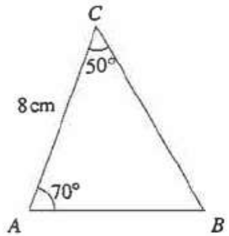
(a) Find $\angle BCG$, $\angle ABP$ and $\angle APB$.

(b) Using the fact that $\dfrac{AP}{\sin\angle ABP} = \dfrac{AB}{\sin\angle APB}$, or otherwise, determine which line segment, AP or PE, is longer.



14A.7 HKCEE MA 2001 I 9

In the figure, find AB and the area of $\triangle ABC$.



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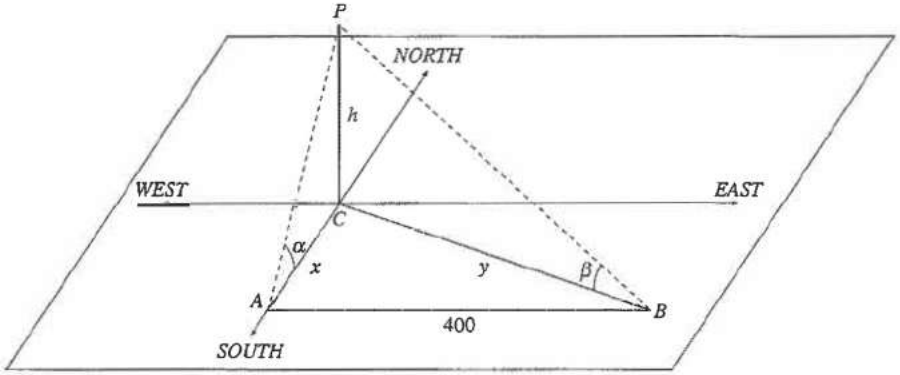
14B Three-dimensional applications

14B.1 HKCEE MA 1980(1/1*/3) – I – 9



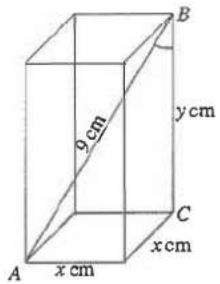
In the figure, PC represents a vertical object of height h metres. From a point A, south of C, the angle of elevation of P is α. From a point B, 400 metres east of A, the angle of elevation of P is β. AC and BC are x metres and y metres respectively.

(a) (i) Express x in terms of h and α.
    (ii) Express y in terms of h and β.
(b) If $\alpha = 60°$ and $\beta = 30°$, find the value of h correct to 3 significant figures.

14B.2 HKCEE MA 1982(1/2/3) I 8

The figure represents the framework of a cuboid made of iron wire. It has a square base of side x cm and a height of y cm. The length of the diagonal AB is 9 cm. The total length of wire used for the framework (including the diagonal AB) is 69 cm.

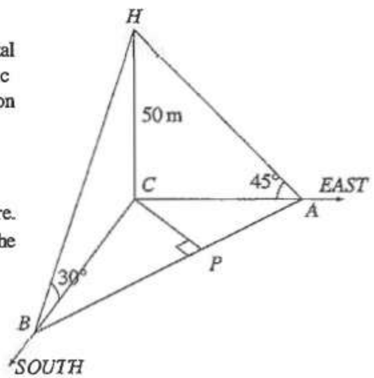
(a) Find all the values of x and y.
(b) Hence calculate $\angle ABC$ to the nearest degree for the case in which $y > x$.



14B.3 HKCEE MA 1983(A/B) I – 13

In the figure, A, B and C are three points on the same horizontal ground. HC is a vertical tower 50 m high. A and B are respectively due east and due south of the tower. The angles of elevation of H observed from A and B are respectively 45° and 30°.

(a) Find the distance between A and B.
(b) P is a point on AB such that $CP \perp AB$.
    (i) Find the distance between C and P to the nearest metre.
    (ii) Find the angle of elevation of H observed from P to the nearest degree.

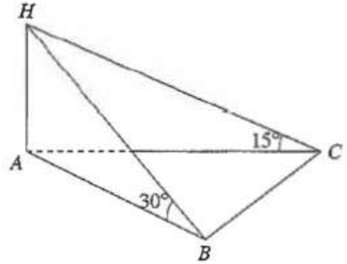


14B.4 HKCEE MA 1984(A/B) – I 13

In the figure, A, B and C lie in a horizontal plane. $AC = 20$ m. HA is a vertical pole. The angles of elevation of H from B and C are 30° and 15° respectively.

(In this question, give your answers correct to 2 decimal places.)

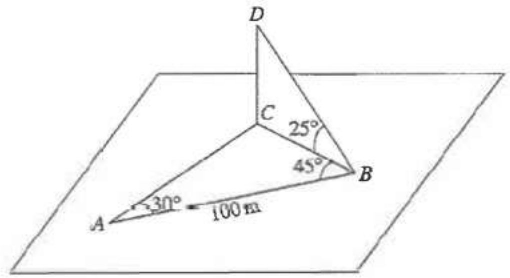
(a) (i) Find, in m, the length of the pole HA.
    (ii) Find, in m, the length of AB.
(b) If A, B and C lie on a circle with AC as diameter,
    (i) find, in m, the distance between B and C;
    (ii) find, in m^2, the area of $\triangle ABC$.



14B.5 HKCEE MA 1985(A/B) – I – 8

In the figure, A, B and C are three points in a horizontal plane. $AB = 100$ m, $\angle CAB = 30°$, $\angle ABC = 45°$.

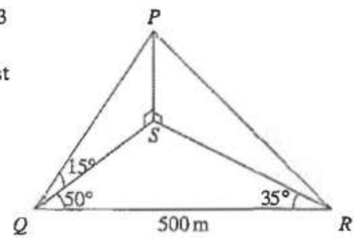
(a) Find BC and AC, in metres, correct to 1 decimal place.
(b) D is a point vertically above C. From B, the angle of elevation of D is 25°.
    (i) Find CD, in metres, correct to 1 decimal place.
    (ii) X is a point on AB such that $CX \perp AB$.
        (1) Find CX, in metres, correct to 1 decimal place.
        (2) Find the angle of elevation of D from X, correct to the nearest degree.



14B.6 HKCEE MA 1986(A/B) I 10

In the figure, Q, R and S are three points on the same horizontal plane. $QR = 500$ m, $\angle SQR = 50°$ and $\angle QRS = 35°$. P is a point vertically above S. The angle of elevation of P from Q is 15°.

(a) Find the distance, in metres, from P to the plane, correct to 3 significant figures.
(b) Find the angle of elevation of P from R, correct to the nearest degree.



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14 Applications of Trigonometry

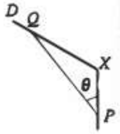
14A Two-dimensional applications

14A.1 HKCEE MA 1981(2/3) – I – 11

(a) Distance at noon $= \sqrt{24^2+9^2-2\cdot 24\cdot 9\cos 60^\circ}$
$= 21$ (km)

(b) At 4 p.m.,
Distance travelled by $P = 4.5\times 4 = 18$ (km)
$\Rightarrow\ PX = 24-18 = 6$ (km)
Distance travelled by $Q = 6\times 4 = 24$ (km)
$\Rightarrow\ QX = 24-9 = 15$ (km) [Q has gone past X.]
$\therefore$ Distance at 4 p.m. $\sqrt{6^2+15^2-2\cdot 6\cdot 15\cos 60^\circ}$
$= \sqrt{171} = 13.1$ (km, 3 s.f.)

(c) $\theta = \cos^{-1}\dfrac{(\sqrt{171})^2+6^2-15^2}{2(\sqrt{171})(6)} = 96.59^\circ$
$\therefore$ Bearing $= 360^\circ - 96.59^\circ$
$= 263^\circ$
or N97°W (nearest deg)



14A.2 HKCEE MA 1982(3) – I – 2

$\angle A = \cos^{-1}\dfrac{4^2+5^2-7^2}{2\cdot 4\cdot 5} = 102^\circ$ (nearest deg)

14A.3 HKCEE MA 1985(A/B) – I – 13

(a) $DE^2 = BD^2+BE^2-2\cdot BD\cdot BE\cos\angle B$
$= (2-x)^2+x^2-\cancel{2}(2-x)(x)\cancel{\cos 60^\circ}$
$= 3x^2-6x+4$

(b) Area of $\triangle DEF = \dfrac{1}{2}DE\cdot DE\sin 60^\circ$
$= \dfrac{1}{2}(3x^2-6x+4)\cdot\dfrac{\sqrt{3}}{2}$
$= \dfrac{\sqrt{3}}{4}(3x^2-6x+4)$
$= \dfrac{3\sqrt{3}}{4}\left(x^2-2x+\dfrac{4}{3}\right)$
$= \dfrac{3\sqrt{3}}{4}\left(x^2-2x+1+\dfrac{1}{3}\right)$
$= \dfrac{3\sqrt{3}}{4}(x-1)^2+\dfrac{\sqrt{3}}{4}$
$\therefore$ Minimum area is attained when $x = 1$.

(c) $\dfrac{3\sqrt{3}}{4}(x-1)^2+\dfrac{\sqrt{3}}{4} < \dfrac{\sqrt{3}}{3}$
$(x-1)^2 \le \dfrac{1}{9}$
$\dfrac{-1}{3} \le x-1 \le \dfrac{1}{3}\ \Rightarrow\ \dfrac{2}{3} \le x \le \dfrac{4}{3}$

14A.4 HKCEE MA 1989 – I – 6

(a) $\angle ABD = \angle ACD = 60^\circ$ ($\angle$s in the same segment)
$\angle BAD = 180^\circ - (60^\circ+40^\circ)$ (opp. $\angle$s, cyclic quad.)
$= 80^\circ$

(b) $\dfrac{BD}{\sin\angle BAD} = \dfrac{AD}{\sin\angle ABD}$
$BD = \dfrac{10\sin 80^\circ}{\sin 60^\circ} = 11.37$ (cm, 2 d.p.)

14A.5 HKCEE MA 1997 – I – 5

(a) $AC = \sqrt{3^2+4^2} = 5$

(b) $AD = \sqrt{5^2+6^2-2\cdot 5\cdot 6\cos 60^\circ} = \sqrt{31}$ (= 5.57, 3.s.f)

(c) Area $= \dfrac{1}{2}(5)(6)\sin 60^\circ = \dfrac{15\sqrt{3}}{2}$ (= 13.0, 3 s.f.)

14A.6 HKCEE MA 2000 – I – 13

(a) $\angle A = \angle ABC = \angle BCD$ (given)
$= (5-2)180^\circ \div 5$ ($\angle$ sum of polygon)
$= 108^\circ$
$\angle GCD = 90^\circ$ (property of square)
$\Rightarrow\ \angle BCG = 108^\circ - 90^\circ = 18^\circ$
$BC = CD = CG$ (given)
$\angle GBC = \angle BGC$ (base $\angle$s, isos. $\triangle$)
In $\triangle BCG$, $\angle GBC = (180^\circ - \angle BCG)\div 2$ ($\angle$ sum of $\triangle$)
$= 81^\circ$
$\angle ABP = 108^\circ - 81^\circ = 27^\circ$
$\angle APB = 180^\circ - \angle A - \angle ABP = 45^\circ$ ($\angle$ sum of $\triangle$)

(b) $AP = \dfrac{\sin\angle ABP}{\sin\angle APB}AB = \dfrac{\sin 27^\circ}{\sin 45^\circ}AB = 0.642AB$
$PE = AB \quad AP = (1-0.642)AB = 0.358AB < AP$
i.e. AP is longer.

14A.7 HKCEE MA 2001 – I – 9

$\dfrac{AB}{\sin 50^\circ} = \dfrac{8}{\sin(180^\circ-50^\circ-70^\circ)}$
$\Rightarrow\ AB = 7.0764 = 7.08$ (cm, 3 s.f.)
$\therefore$ Area $= \dfrac{1}{2}(8)(7.0764)\sin 70^\circ = 26.6$ (cm², 3 s.f.)

14B Three-dimensional applications

14B.1 HKCEE MA 1980(1/1*/3) – I – 9

(a) (i) In $\triangle PAC$, $x = \dfrac{h}{\tan\alpha}$

(ii) In $\triangle PBC$, $y = \dfrac{h}{\tan\beta}$

(b) In $\triangle ABC$,
$x^2+400^2 = y^2$
$\left(\dfrac{h}{\tan 60^\circ}\right)^2+160000 = \left(\dfrac{h}{\tan 30^\circ}\right)^2$
$\dfrac{h^2}{3}+160000 = 3h^2$
$h^2 = 60000\ \Rightarrow\ h = 245$ (3 s.f.)

14B.2 HKCEE MA 1982(1/2/3) – I – 8

(a) $8x+4y+9 = 69\ \Rightarrow\ y = 15-2x$
$AC^2 = 9^2-y^2\ \Rightarrow\ 2x^2 = 81-y^2$
$2x^2 = 81-(15-2x)^2$
$x^2-10x+24 = 0\ \Rightarrow\ x = 4$ or 6
When $x = 4$, $y = 15-2(4) = 7$
When $x = 6$, $y = 15 \quad 2(6) = 3$

(b) $\angle ABC = \cos^{-1}\dfrac{y}{9} = \cos^{-1}\dfrac{7}{9} = 39^\circ$ (nrst deg)

14B.3 HKCEE MA 1983(A/B) – I – 13

(a) In $\triangle ACH$, $AC = \dfrac{50}{\tan 45^\circ} = 50$ (m)
In $\triangle BCH$, $BC = \dfrac{50}{\tan 30^\circ} = 50\sqrt{3}$ (m)
In $\triangle ABC$, $AB = \sqrt{(50)^2+(50\sqrt{3})^2} = 100$ (m)

(b) (i) $\dfrac{AC\cdot BC}{2} = \dfrac{CP\cdot AB}{2}$ (= Area of $\triangle ABC$)
$\Rightarrow\ CP = \dfrac{(50)(50\sqrt{3})}{100} = 25\sqrt{3} = 43.3$ (m, 3 s.f.)

(ii) Required $\angle = \angle HPC = \tan^{-1}\dfrac{HC}{CP} = 49^\circ$ (nrst deg)

14B.4 HKCEE MA 1984(A/B) – I – 13

(a) (i) In $\triangle ACH$,
$HA = 20\tan 15^\circ = 5.25898 = 5.36$ (m, 2 d.p.)

(ii) In $\triangle ABH$,
$AB = \dfrac{HA}{\tan 30^\circ} = 9.28203 = 9.28$ (m, 2 d.p.)

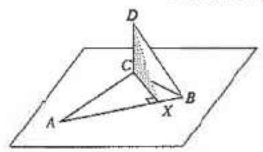
(b) Given: $\angle ABC = 90^\circ$ ($\angle$ in semi-circle)
(i) $BC = \sqrt{AC^2-AB^2} = 17.71564 = 17.72$ (m, 2 d.p.)
(ii) Area $= \dfrac{1}{2}AB\cdot BC = 82.22$ m² (2 d.p.)

14B.5 HKCEE MA 1985(A/B) – I – 8

(a) In $\triangle ABC$, $\dfrac{BC}{\sin 30^\circ} = \dfrac{100}{\sin(180^\circ-30^\circ-45^\circ)} = \dfrac{AC}{\sin 45^\circ}$
$\Rightarrow\ BC = 51.76381 = 51.8$ (m, 1 d.p.)
$AC = 73.20508 = 73.2$ (m, 1 d.p.)

(b) (i) In $\triangle BCD$, $CD = BC\tan 25^\circ = 24.13789$
$= 24.1$ (m, 1 d.p.)

(ii)



(1) In $\triangle CXB$, $CX = BC\sin 45^\circ = 36.60254$
$= 36.6$ (m, 1 d.p.)

(2) Required $\angle = \angle DXC$
$= \tan^{-1}\dfrac{CD}{CX} = 33^\circ$ (nrst deg)

14B.6 HKCEE MA 1986(A/B) – I – 10

(a) In $\triangle QRS$, $\dfrac{QS}{\sin 35^\circ} = \dfrac{500}{\sin(180^\circ-50^\circ-35^\circ)}$
$\Rightarrow\ QS = 287.88370$ (m)
$\therefore$ In $\triangle PQS$,
Required distance $= PS = QS\tan 15^\circ$
$= 77.13821 = 77.1$ (m, 3 s.f.)

(b) In $\triangle QRS$, $\dfrac{RS}{\sin 50^\circ} = \dfrac{500}{\sin 95^\circ}\ \Rightarrow\ RS = 384.48530$ (m)
$\therefore$ In $\triangle PRS$, Required $\angle = \angle PRS = = \tan^{-1}\dfrac{PS}{RS}$
$= 11^\circ$ (nrst deg)

14B.7 HKCEE MA 1987(A/B) – I – 11

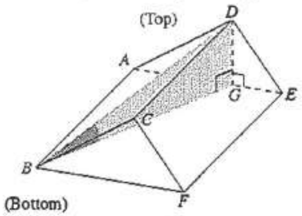
(a) In $\triangle ADE$. $AE = \sqrt{3^2+2^2-2\cdot 3\cdot 2\cos 80^\circ}$
$= 3.30397 = 3.304$ (cm, 3 d.p.)

(b) In $\triangle ADE$, $\angle DAE = \cos^{-1}\dfrac{AE^2+3^2-2^2}{2\cdot AE\cdot 3}$
$= 36.59365^\circ = 36.594^\circ$ (3 d.p.)

(c) In $\triangle ADG$, $DG = 3\sin\angle DAE$
$= 1.7884077 = 1.788$ (cm, 3 d.p.)

(d) In $\triangle ABD$, $BD = \sqrt{3^2+3^2}$
$= \sqrt{18} = 4.243$ (cm, 3 d.p.)

(e)



Required $\angle = \angle DBG = \sin^{-1}\dfrac{DG}{BD} = 24.931^\circ$ (3 d.p.)

14B.8 HKCEE MA 1988 – I – 13

(a) In $\triangle ABH$, $HB = \dfrac{3}{\tan\theta}$
In $\triangle DCK$, $KC = \dfrac{2}{\tan\theta}$

(b) (i) $S_1 = \dfrac{(2+3)(6)}{2} = 15$ (m²)

(ii) $S_2 = \dfrac{(\frac{3}{\tan\theta}+\frac{2}{\tan\theta})(6)}{2} = \dfrac{15}{\tan\theta}$ (m²)
$\therefore\ \dfrac{S_1}{S_2} = \dfrac{15}{\frac{15}{\tan\theta}} = \tan\theta$

(c)

Let P be the foot of perpendicular from K to BH.
$PK = 6$ m, $PH = \dfrac{3}{\tan 30^\circ} - \dfrac{2}{\tan 30^\circ} = \sqrt{3}$ (m)
$\therefore\ HK = \sqrt{PK^2+PH^2} = \sqrt{39}$ (m)

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