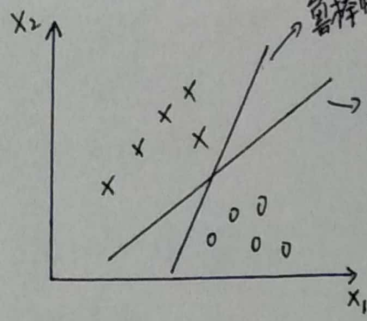
Support Vector Machine.

SVM有三宝：间隔 对偶 核技巧

SVM：hard-margin SVM

soft-margin SVM

kernel SVM



要找到一条最中间的超平面，让它离样本点距离都足够大.

$f(w) = \text{sign}(w^Tx+b)$. $\longrightarrow$ 判别模型.

$\{(x_i, y_i)\}_{i=1}^{N}$ $x_i \in \mathbb{R}^p$, $y_i \in \{-1, 1\}$

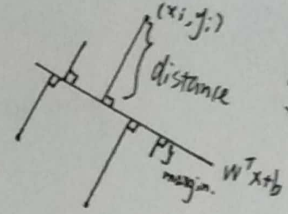
最大间隔分类器.

$\max \text{margin}(w,b)$

$$\text{s.t.} \begin{cases} w^Tx_i+b>0, & y_i=+1 \\ w^Tx_i+b<0, & y_i=-1 \end{cases}$$

$\Rightarrow y_i(w^Tx+b)>0, \ i=1,2,\cdots N$

$\rightarrow$ s.t. $y_i(w^Tx_i+b)>0$, for $\forall i=1,\cdots,N$



N个样本点，N个distance.

$\Downarrow$

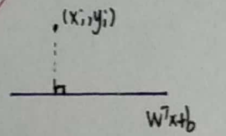
最小的distance

$\|$

margin

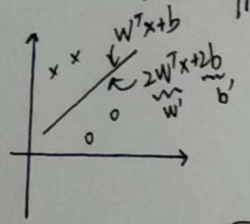
$$\text{margin}(w,b) = \min_{\substack{w,b \\ x_i \\ i=1,\cdots N}} \text{distance}(w,b,x_i)$$

$$= \min_{\substack{w,b,x_i \\ i=1,\cdots,N}} \frac{1}{\|w\|}|w^Tx_i+b|$$



$$\text{distance} = \frac{1}{\|w\|}|w^Tx_i+b|$$

$$\Rightarrow \begin{cases} \max\limits_{w,b} \min\limits_{\substack{x_i \\ i=1,\cdots,N}} \dfrac{1}{\|w\|}|w^Tx_i+b| \\ \text{s.t.} \ y_i(w^Tx_i+b)>0 \end{cases}$$



$$\Rightarrow \exists r>0, \ \text{s.t.} \min_{\substack{x_i \\ y_i \\ i=1,\cdots N}} y_i(w^Tx_i+b)=r$$

$$\max_{w,b} \min_{\substack{x_i \\ i=1,\cdots,N}} \frac{1}{\|w\|} y_i(w^Tx_i+b) = \max_{w,b} \frac{1}{\|w\|} \min_{\substack{x_i \\ i=1,\cdots N}} y_i(w^Tx_i+b)$$

$r=1$

$$\Rightarrow \begin{cases} \max\limits_{w,b} \dfrac{1}{\|w\|} \\ \text{s.t.} \ \min y_i(w^Tx_i+b)=1 \end{cases}$$

$\rightarrow y_i(w^Tx_i+b) \geq 1, i=1,\cdots N$

凸优化问题.

convex opimization

QP

$$\Rightarrow \boxed{\begin{cases} \min\limits_{w,b} \dfrac{1}{2}w^Tw \ \rightarrow \text{二次} \\ \text{s.t.} \ \underbrace{y_i(w^Tx_i+b) \geq 1}_{\text{N个约束}} \ \text{for} \ \forall i=1,\cdots,N \end{cases}}$$