Support Vector Problem

求 $\min \mathcal{L}(w,b,\lambda)$

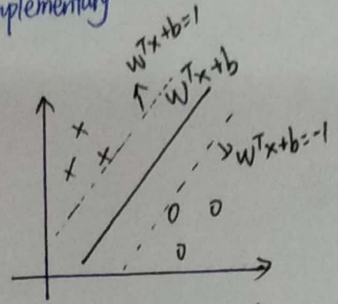
$$\begin{cases} \frac{\partial \mathcal{L}}{\partial b} \triangleq 0 \Rightarrow \sum_{i=1}^{N} \lambda_i y_i = 0 \\ \frac{\partial \mathcal{L}}{\partial w} \triangleq 0 \Rightarrow w = \sum_{i=0}^{N} \lambda_i y_i x_i \end{cases}$$

原、对偶问题具有强对偶性 $\Leftrightarrow$ 满足KKT条件.

KKT条件.

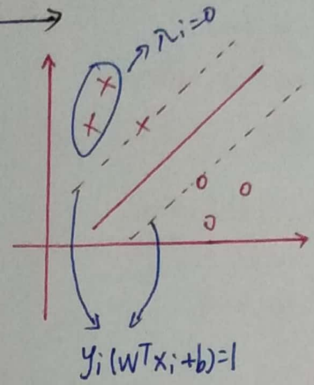
$$\begin{cases} 对\ \frac{\partial \mathcal{L}}{\partial w} = 0 \quad \frac{\partial \mathcal{L}}{\partial b} = 0 \quad \frac{\partial \mathcal{L}}{\partial \lambda} = 0 \\ \boxed{\lambda_i (1 - y_i (w^T x + b)) = 0} \rightarrow \text{slackness complementary} \\ \lambda_i \geq 0 \\ 1 - y_i (w^T x_i + b) \leq 0 \end{cases}$$

w^*: 是数据data线性组合



w^*, b^*

$$\boxed{w^* = \sum_{i=0}^{N} \lambda_i y_i x_i} \quad \cancel{b^*}$$



$\exists (x_k, y_k)$, s.t. $1 - y_k (w^T x_k + b) = 0$

$y_k (w^T x_k + b) = 1$

$y_k = \pm 1 \quad y_k^2 (w^T x_k + b) = y_k$

$$\boxed{b^* = y_k - w^T x_k = y_k - \sum_{i=0}^{N} \lambda_i y_i x_i^T x_k}$$

$f(x) = \mathrm{sign}(w^{*T} x + b^*)$