Time-Interleaved Analog-To-Digital Converters: Status and Future Directions

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***Abstract*— We discuss time-interleaved analog-to-digital converters (ADCs) as a prime example of merging analog and digital signal processing. A time-interleaved ADC (TI-ADC) consists of M parallel channel ADCs that alternately take samples from the input signal, where the sampling rate can be increased by the number of channels compared to a single channel. We recall the advantages of time interleaving and investigate the problems involved. In particular, we explain the error behavior of mismatches among the channels, which distort the output signal and reduce the system performance significantly, and provide a concise framework for dealing with them. Based on this analysis, we review the principle possibilities of calibrating TI-ADCs, where we point out the necessities and advantages of digital enhancement. To this end, we discuss open issues of channel mismatch identification as well as channel mismatch correction.**

I. Introduction

Since analog-to-digital converters (ADCs) ultimately limit the performance of today's communication systems, high-speed, high-resolution, and power-aware ADCs are required in order to comply with new communication standards. This also leads to an increased demand for high-speed and high-resolution sampling systems in the measurement industry [1]. Present ADC technologies work on their limits and cannot be properly pushed further, since the downscaling of IC technologies to deep sub-micron technologies makes their design even more difficult. However, the increased component density of digital circuits allows for using additional chip area with small additional costs [2].

One possibility to overcome these performance limits is to use parallelism, i.e., to split the information of the analog input signal into several parallel channels, to convert them independently, and finally to recombine them into one digital output signal. In theory, which was introduced by Papoulis' Generalized Sampling Expansion (GSE) [3], there are many ways to split the information of the input signal. In practice, only a few parallel multi-channel sampling structures [4] have been further analyzed [5]–[7], where the time-interleaved structure is among the most promising ones for the future.

The idea of a time-interleaved ADC (TI-ADC) is that each channel in a system of M parallel channels alternately takes one sample, whereas the sampling frequency of one channel does not need to fulfill the Nyquist Criterion [8]. However, when in the digital domain all samples merge into one sequence we obtain an overall sampling frequency that fulfills the Nyquist criterion. Thus, sampling with an ideal TI-ADC with M channels is equivalent to sampling with an ideal ADC with an M times higher sampling rate. The channels of a TI-ADC can be realized in different converter technologies to achieve for example high-rate and low-power ADCs [9] or high-rate and high-resolution ADCs [10].

The typical structure of a TI-ADC is shown in Fig. 1. We see the analog input signal $x_a(t)$, the M time-interleaved parallel channels,

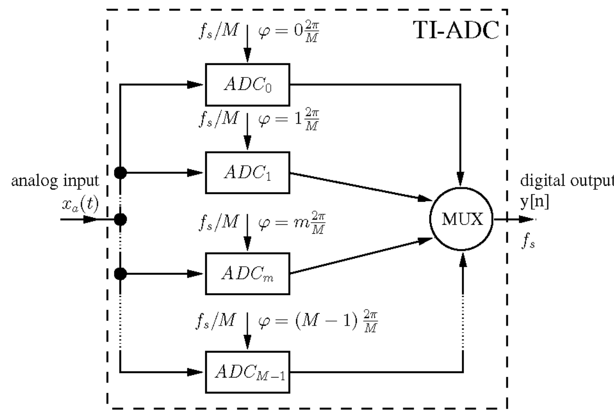

Fig. 1. Time-interleaved ADC (TI-ADC) with M channels. Each channel alternately takes samples at a rate $\frac{f_s}{M}$ from the input signal $x_a(t)$. At the multiplexer (MUX), the samples from the M parallel channels are merged into one output channel running at an M times higher rate f_s.

and the multiplexer (MUX) to recombine the digital outputs of the channels. The conversion rate of the overall system is increased by the number of channels M. It should be noticed that each channel has to deal with the entire input signal $x_a(t)$, and, therefore, the sample-and-holds in each channel have to resolve the full input signal bandwidth.

From a theoretical point of view, we can increase the sampling rate of a TI-ADC by the number of channels that work in parallel in the system. Ideally, the sampling rate would linearly scale with the number of channels; however, channel mismatches ultimately limit the performance of TI-ADCs. On the one hand, the downscaling of the IC technologies complicates the matching of the components, but, on the other hand, the increased component density allows for including additional digital components with small additional costs. Therefore, we can add digital circuits to overcome the problems of analog converter circuits [11]. TI-ADCs constitute a prime example of such merging technologies, where the technology can only be properly pushed further, when we consider digitally enhanced analog circuits.

II. Channel Mismatches

Each channel ADC in a TI-ADC has technology dependent errors (e.g., integral nonlinearity errors, clock jitter) like a single-channel ADC, but due to component mismatches among the channels, additional errors, called mismatch errors, are introduced [12]. This is illustrated in Fig. 2, where we see a TI-ADC with channel mismatches and without channel mismatches for a sinusoidal input signal. For

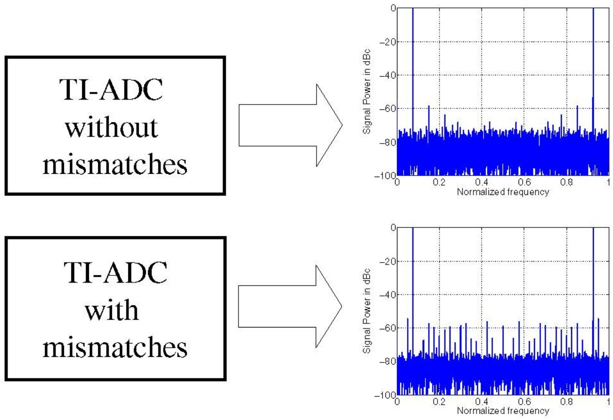

Fig. 2. If we had no mismatches we would see an output spectrum like for a single-channel ADC. As soon as we have mismatches, we obtain additional spectral components, which significantly reduce the TI-ADC performance.

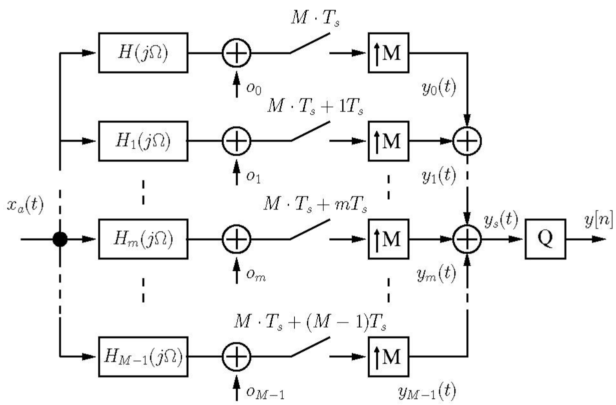

Fig. 3. Linear mismatch model of a time-interleaved ADC with M channels and additional offsets in each channel.

matched channels, we obtain an output spectrum like for a single channel ADC. We see the input signal, harmonics related to the input signal, i.e., integral nonlinearity (INL) errors, and a noise floor determined by the quantization noise, differential nonlinearity (DNL) errors, and jitter effects. In contrast, we see for the TI-ADC with channel mismatches additional spectral components in the output spectrum, which degrade the system performance.

To model channel mismatches we can use the simplified model shown in Fig. 3. The input signal goes to M parallel linear filters given by

$$H_m(j\Omega) = A_m(\Omega)\, e^{-j\phi_m(\Omega)} \tag{1}$$

and is then sampled in a time-interleaved manner. Additionally, we add M offsets o_m in each channel, which are sampled in the same way as well. The signals $y_m(t)$ (sampled input signals and offsets) of all channels are merged into one output stream $y_s(t)$, which becomes after quantization the digital output signal $y[n]$. To determine the output signal $y_s(t)$, we separate the input signal part and the offset part and neglect the quantization process to simplify the model.

Without offsets the sampled output, i.e., linear mismatches only, can be written as

$$y_s^l(t) = \sum_{m=0}^{M-1} (x_a(t) * h_m(t)) \sum_{n=-\infty}^{\infty} \delta\left(t - (m+nM)T_s\right), \tag{2}$$

where T_s is the sampling period. The Fourier transform of (2) can

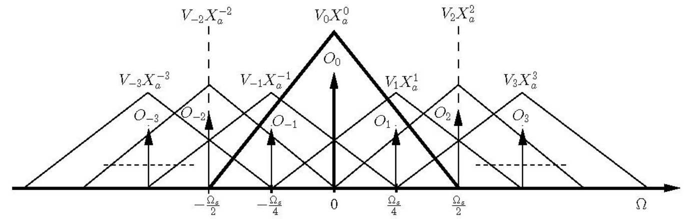

Fig. 4. Spectrum of a TI-ADC. Each additional spectral component is a shifted (by $k\frac{\Omega}{M}$) copy of the input spectrum $X_a(j\Omega)$, which is weighted by the complex transfer function $V_k(j\Omega)$. Furthermore, we have Dirac delta pulses at $k\frac{\Omega}{M}$, which are weighted by O_k. For this figure we have used the abbreviation $V_kX_a^k$ for $V_k\left(j\Omega - jk\frac{\Omega_s}{M}\right) X_a\left(j\Omega - jk\frac{\Omega_s}{M}\right)$.

be calculated by developing the Dirac delta distribution in (2) into a continuous Fourier series, which leads us to

$$Y_s^l(j\Omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} V_k\left(j\Omega - jk\frac{\Omega_s}{M}\right) X_a\left(j\Omega - jk\frac{\Omega_s}{M}\right), \tag{3}$$

where

$$V_k(j\Omega) = \frac{1}{M} \sum_{m=0}^{M-1} H_m(j\Omega)\, e^{-jkm\frac{2\pi}{M}}. \tag{4}$$

Without an input signal the sampled output, i.e., offset mismatches only, is

$$y_s^o(t) = \sum_{m=0}^{M-1} o_m \sum_{n=-\infty}^{\infty} \delta\left(t - (m+nM)T_s\right), \tag{5}$$

where the Fourier transform of (5) gives

$$Y_s^o(j\Omega) = \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} O_k \delta\left(\Omega - k\frac{\Omega_s}{M}\right), \tag{6}$$

where

$$O_k = \frac{1}{M} \sum_{m=0}^{M-1} o_m e^{-jkm\frac{2\pi}{M}}. \tag{7}$$

The final output is the linear combination of (3) and (6), that is

$$Y_s(j\Omega) = Y_s^l(j\Omega) + Y_s^o(j\Omega). \tag{8}$$

From (2) to (8) we can recognize frequency domain characteristics of TI-ADCs. The input signal spectrum $X_a(j\Omega)$ is shifted by $k\frac{\Omega_s}{M}$ and weighted by the corresponding mismatch transfer function $V_k(j\Omega)$, which is illustrated in Fig. 4. The mismatch transfer function $V_k(j\Omega)$ is the discrete Fourier transform (DFT) of the frequency responses $H_m(j\Omega)$ of the channels. For matched channels, all shifted spectral components become zero, since the mismatch transfer function is zero for all $k \neq 0, \pm M, \pm 2M, \ldots$ The output distortions caused by offset mismatches do not depend on the input signal $x_a(t)$. We therefore obtain a fixed output signal pattern at $k\frac{\Omega_s}{M}$ that is weighted by the factor O_k, which is the DFT among the offsets of all channels. If all offsets are identical we will have no mismatches but we could still have an overall offset error O_0.

For a general analysis of dynamic and static nonlinearity mismatches we can use nonlinear hybrid filter banks, which unify and simplify the treatment of channel mismatches [13].

In the literature, two kinds of linear mismatches, i.e., gain mismatches and timing mismatches, have been treated extensively. The gain of an ADC is often defined as the magnitude response for a DC input signal, which in our notation corresponds to

$$g_m = H_m(j0) = A_m(j0). \tag{9}$$

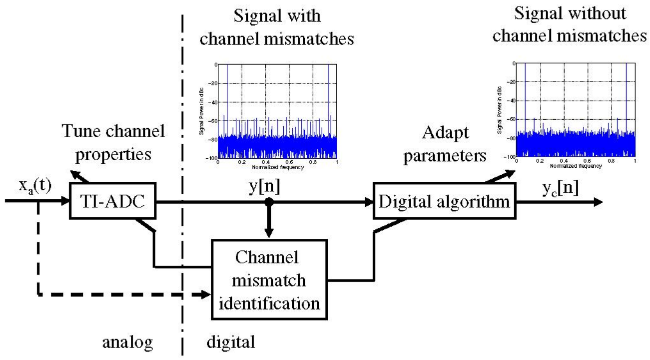

Fig. 5. Possibilities to calibrate a TI-ADC.

Thus, the gain mismatch is the deviation of the gains g_m from the average gain of all channels. In practical TI-ADC designs, however, we have to deal with magnitude mismatches. To compensate for them, a first solution is to use some kind of average magnitude response for each channel over the frequency band of interest instead of DC gains.

The timing mismatch is the deviation from the averaged linear-phase responses of the channels normalized by the frequency. To see this, we split the phase responses into a linear and a nonlinear part, i.e.,

$$\phi_m(\Omega) = t_m\Omega + \Phi_m(\Omega), \tag{10}$$

where $t_m\Omega$ is the linear-phase response over the frequency band of interest. The timing mismatch Δt_m is the deviation from the averaged and frequency normalized linear-phase response, i.e.,

$$\Delta t_m = t_m - \frac{1}{M}\sum_{m=0}^{M-1} t_m. \tag{11}$$

It should be noticed that we can treat aperture-delay mismatches with this model as well. The aperture-delay mismatch is the deviation from the ideal sampling instant caused by time-shifted clock signals. This delay can be represented by an equivalent time-shift (linear phase shift) of the input signal in each channel which can be accomplished by the linear filters $H_m(j\Omega)$.

III. Calibration of Channel Mismatches

Avoiding mismatches is the main concern in designing fast TI-ADCs [9]. Unfortunately, shrinking IC technologies and increasing clock rates make component matching even more difficult. Furthermore, the matching is influenced by time-variant parameters such as temperature or component aging. Therefore, calibration methods for TI-ADCs have been proposed, which tune the component matching, e.g., [14], [15], or digitally correct the distorted output signal, e.g., [16]–[20].

In Fig. 5 the principal calibration methods are illustrated. We see a TI-ADC driven by a sinusoidal input signal $x_a(t)$. At the output of the TI-ADC ($y[n]$) we see the sampled spectrum of the sinusoidal input signal and distortions caused by channel mismatches. In order to reduce these distortions we have to identify the significant mismatch parameters. This task is much easier to realize when we know the input signal $x_a(t)$, which is indicated by a dashed arrow from the identification box to the input signal. In many cases, however, we have no or just little statistical knowledge, e.g., bandlimitation or modulation technique, about the input signal. This little knowledge makes it much more difficult to obtain reliable estimates for the mismatch parameters. If the mismatch parameters are known, we can tune the matching on the analog side or we can reconstruct the distorted signal on the digital side. It is also possible to combine both approaches [21].

Digital calibration is attractive in many ways. The digital calibration is, like the principal topology of a TI-ADC, independent from the used channel converter technology. Hence, we can apply the same digital calibration method to TI-ADCs with different channel converter technologies, since the TI-ADC environment and the production process do not directly influence the functionality of the calibration method.

A. Correction Methods

Although the correction of gain and offset mismatches in the digital domain is quite simple, since we only have to add at most one adder and one multiplier to the signal path of each channel ADC, the correction of timing mismatches (linear-phase mismatches) is much more difficult. In fact, it is a sub-problem of the non-uniform sampling problem. For TI-ADCs the problem simplifies to periodically non-uniform sampled signals, i.e., the time shifts Δt_m exhibit a periodicity, where the time shifts Δt_m are small compared to the sampling period T_s. However, under the constraint of an on-chip implementation the problem becomes difficult again.

For the timing-mismatch problem accurate solutions have been found in [19], [20], [22], [23], although only for some of them [19], [20] the implementation on a TI-ADC chip is maintainable. However, for changing timing mismatches the used reconstruction method has to be easily adaptable. Thus, an open question is to find reconstruction methods where the needed coefficients can be simply derived from the estimated timing mismatches. A first solution to this problem can be found in [21], [24]. In [24] the authors show that by using fractional delay filters they only have to redesign one coefficient in each channel to adapt to changed timing mismatches and in [21] a method was introduced which reorders the channel sequence in order to achieve a spectrally shaped output signal. Unfortunately, both methods need some amount of additional oversampling.

Therefore, the goal of timing mismatch correction is to find an accurate, power-aware method, which only needs a slight oversampling and which can be easily adapted to changing timing mismatches. If these problems are solved, magnitude and nonlinear-phase mismatches (bandwidth mismatches) will limit the effective resolution of TI-ADCs and will therefore have to be corrected for a further improvement [18].

B. Identification Methods

The identification of mismatch parameters is the most critical component in the channel mismatch compensation process of TI-ADCs. If the identified parameters are wrong even the best correction method cannot improve the TI-ADC performance.

For the identification of the channel mismatches with special input signals we can find accurate solutions [18], [25], [26]. Nevertheless, identification with special input signals is suitable for measurement applications with calibration cycles but not suitable for communications systems, where in general we have no extra duty cycles for the calibration. There, the identification has to be done during the normal operation of the TI-ADC with the only restriction of bandlimited input signals. For single communication protocols, we can assume particular signal statistics, however, the trend towards software defined radio (SDR) does not allow such assumptions.

For gain and offset mismatch identification we can find some methods [17], [27], [28]. In many cases, we obtain good offset and gain mismatch estimates with a simple comparison of the averaged output values and the averaged output power among all channels. Unfortunately, such methods are vulnerable to an input signal correlation with the switching sequence of the channels, i.e., we only get a limited number of different samples from each channel for the estimation process.

The most challenging problem is the on-line identification of timing mismatches. The identification should be accurate and its implementation should be cost-efficient. Although we can find identification methods in the literature [17], [29], they are either imprecise, limited in the number of channels, or have an enormous computational complexity.

Hence, we need stable on-line identification methods for offset, gain, and timing mismatches, where an accurate and implementation efficient timing mismatch identification is the foremost challenge for the future. After having solved these problems, however, we will also have to deal with the identification of bandwidth mismatches and nonlinearity mismatches.

IV. Conclusion

We have presented a unified framework for dealing with linear channel mismatches and offset mismatches. In particular, we can use this framework to describe gain and timing mismatches as a special case of linear mismatches. We have discussed correction and identification of channel mismatches and we have pointed out the most challenging problems for the near future. Since the major problems of offset, gain, and timing mismatch correction have been recently solved, the main concern for the future will be finding stable and reliable on-line mismatch identification methods. However, to achieve very high-resolution TI-ADCs more errors, e.g., bandwidth mismatches, INL mismatches, and DNL mismatches, have to be considered and calibrated as well. For example, for a 100 MHz signal bandwidth, a 750 MHz ADC bandwidth, and an analog device matching of 1%, the bandwidth mismatch will limit the resolution to some 12-13 bits. The research on these mismatches has just started.

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