Roll No. ....

Total No. of Questions : 09]

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Paper ID [A0459]

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B.Tech. (Sem. - $3^{rd}/4^{th}$)

MATHEMATICS - III (CS - 203/204)

Time : 03 Hours **Maximum Marks : 60**

Instruction to Candidates:

1) Section - A is **Compulsory**.
2) Attempt any **Four** questions from Section - B.
3) Attempt any **Two** questions from Section - C.

Section - A

Q1) *($10 \times 2 = 20$)*

a) Discuss the relationships between limit, continuity and differentiation of a function.

b) Define fundamental theorem of Integral Calculus.

c) Distinguish between Trapezoidal and Simpson's $\frac{1}{3}$rd rule of numerical integration.

d) Define analytic function and Cauchy-Riemann equations.

e) Poles and residues of analytic functions.

f) Define residue theorem of functions of complex variables.

g) Explain the conditions for the existance of Laplace transform of a function.

h) Define conformal mapping.

i) Discuss the categorization of Laplace, wave and heat equations.

j) State Rolle's theorem.

Section - B

(4 × 5 = 20)

Q2) Determine the analytic function $f(z)=u+iv$ if

$$u-v=\frac{\text{Cos}\, x+\text{Sin}\, x-e^{-y}}{2(\text{Cos}\, x-\text{Cosh}\, y)}$$

and $f\left(\frac{\pi}{2}\right)=0.$

Q3) Evaluate

$$\int_C \frac{z-3}{z^2+2z+5}dz$$

where C is the circle $|z+i+1|=2.$

Q4) An impulse I (kg - sec) is applied to a mass m attached to a spring having a spring constant k. The system is damped with damping constant μ. The differential equation governing the phenomena is

$$m\frac{d^2x}{dt^2}+kx+\mu\frac{dx}{dt}=\text{I}\delta(x).$$

where $\delta(x)$ is unit impulse function. Derive expression for displacement $x(t)$ of the mass, assuming initial conditions $x(0)=x'(0)=0.$

Q5) Find the area included between the curves $y^2(2a-x)=x^3$ and its asymptote.

Q6) Employ Taylor's method to obtain approximate solution of y at $x = 0.2$ for the differential equation $\frac{dy}{dx}=2y+3e^x,\ y(0)=0.$

Section - C

(2 × 10 = 20)

Q7) Using Runge-Kutta method of order 4, compute y (0.2) and y (0.4) from

$$10\frac{dy}{dx}=x^2+y^2,\ \ y(0)=1.$$

taking h = 0.1.

Q8) A tightly stretched string of length l with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $v_0 \,\text{Sin}^3\left(\frac{\pi x}{l}\right)$. Find the displacement $y\ (x,t)$, using separation of variable technique.

Q9) Show that the transformation

$$w=z+\frac{a^2}{z}$$

transforms circles with origin at the centre in the z-plane into coaxial concentric, confocal ellipses in the w - plane.

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