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B.Tech. (Sem. - 3rd/4th)

MATHEMATICS - III

SUBJECT CODE : CS - 203/204 (2K2 Batch)

Paper ID : [A0457]

[Note : Please fill subject code and paper ID on OMR]

Time : 03 Hours **Maximum Marks : 60**

Instruction to Candidates:

1) Section - A is **Compulsory**.
2) Attempt any **Four** questions from Section - B.
3) Attempt any **Two** questions from Section - C.

Section - A

Q1) *(10 × 2 = 20)*

a) Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 4$ in the interval $1 \le x \le 4$.

b) Find the Laplace transform of $f(t) = \sqrt{t}\, e^{3t}$.

c) Give an example of bilinear transformation.

d) Prove that $\lim_{x \to a}(x-a)\sin\frac{1}{x-a} = 0$.

e) State the Taylor's theorem with Lagrange's form of remainder.

f) Give the statement of Cauchy Riemann equations.

g) Using Picard's method find first approximation of $\frac{dy}{dx} = x + y^2, y(0) = 1$

h) Write the necessary and sufficient conditions for Riemann's integration.

i) Evaluate $\int_0^1 dx \int_0^x e^{\frac{y}{x}} dy$.

j) Evaluate $\oint_C \frac{e^{-z}}{z+1} dz$, where C is the circle $|z| = 2$.

Section - B

(4 × 5 = 20)

Q2) Find the volume of tetrahedron bounded by the co-ordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

Q3) Find the sum of the residues of the function $f(z)=\frac{\sin z}{z\cos z}$ at its poles inside the circle $|z|=2$.

Q4) Using convolution theorem, find the inverse of $\frac{s+2}{(s^2+4s+5)^2}$.

Q5) Using Fourier sine integral, show that

$$\int_0^\infty \frac{1-\cos\pi\lambda}{\lambda}\sin x\lambda\, d\lambda=\begin{cases}\frac{1}{2}\pi, & \text{when } 0<x<\pi\\ 0, & \text{when } x>\pi\end{cases}$$

Q6) Find the entire length of the cardioid $r = a(1 + \cos\theta)$.

Section - C

(2 × 10 = 20)

Q7) Using Runge Kutta method of order 4, find $y(0.1)$ and $y(0.2)$ given that $\frac{dy}{dx}=x^2-y$, $y(0) = 1$. (Take $h = 0.2$)

Q8) Find the Laurent series of $f(z)=\frac{1}{(z-1)(z-2)}$ for the following intervals

(a) $1<|z|<2$ (b) $|z|>2$

Q9) (a) A tightly stretched string with fixed end points $x = 0$ and $x = L$ is initially in a position given by $f(x)$. The initial velocity is zero, where

$$f(x)=\begin{cases}\frac{2K}{L}x & 0\le x\le\frac{L}{2}\\ \frac{2K}{L}(L-x) & \frac{L}{2}<x<L\end{cases}$$

Obtain the displacement at any point x and any time t. (use wave equation).

(b) Find the values of a, b, c, d so that the function $f(x)$ is analytic. where $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$

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