# Good Looking Mathematics

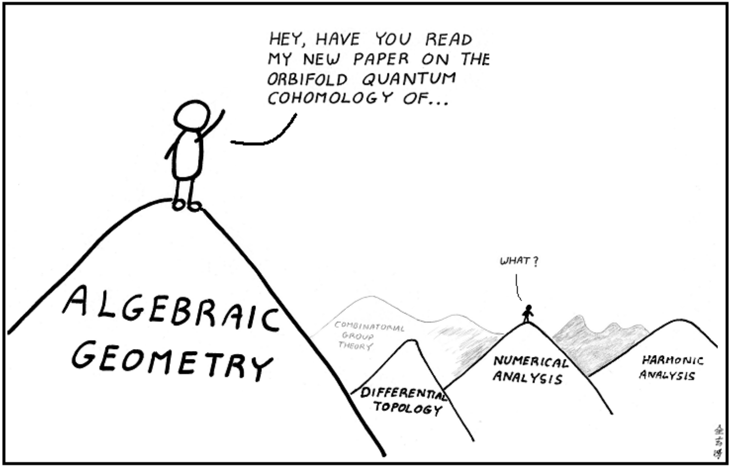
## A Discussion of Mathematical Models

Patrick Rhomberg

The University of Iowa
Applied Mathematics and Computational Science
Computer Science, Computational Epidemiology Group

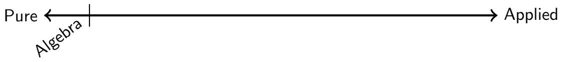
April 9, 2013

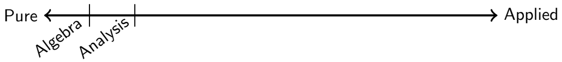
# Abstruse Goose: 211

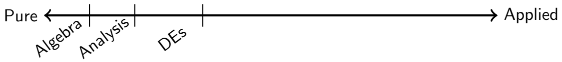


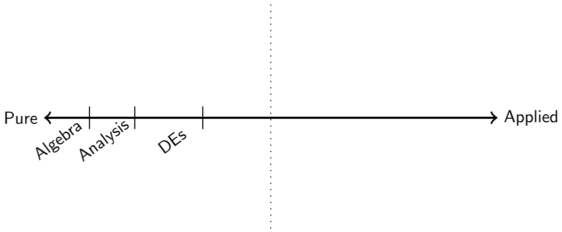

The Landscape of Modern Mathematics

Pure $\longleftrightarrow$ Applied

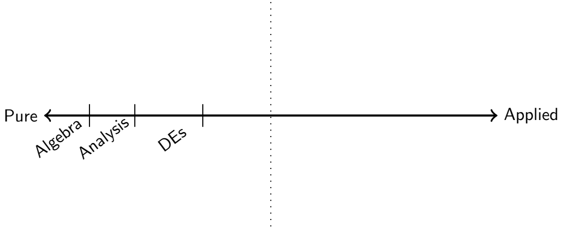

Pure
Applied
Algebra


Pure
Applied
Algebra
Analysis


Pure
Algebra
Analysis
DEs
Applied

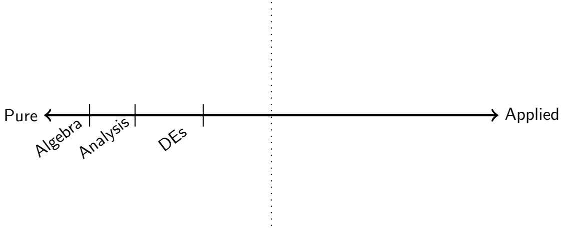

Pure
Algebra
Analysis
DEs
Applied

Mathematics
(Mental pursuits)



Mathematics
(Mental pursuits)

Physical Science
(Tangible per-suits)



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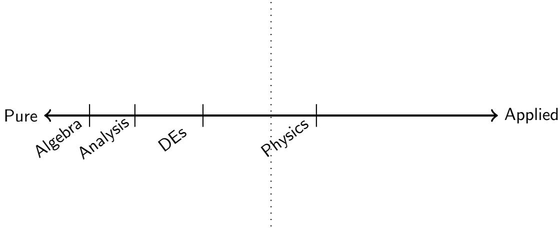

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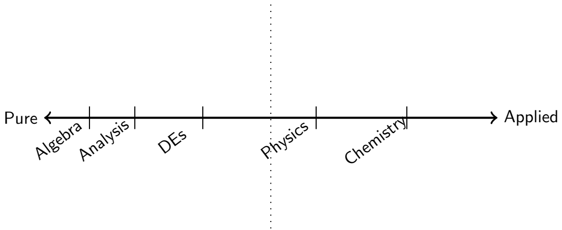

Mathematics
(Mental pursuits)

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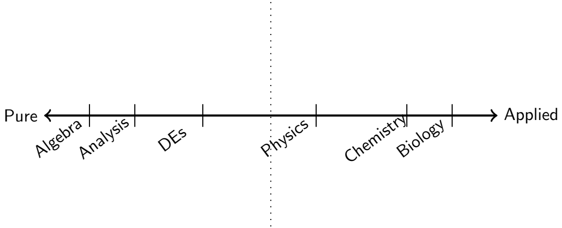
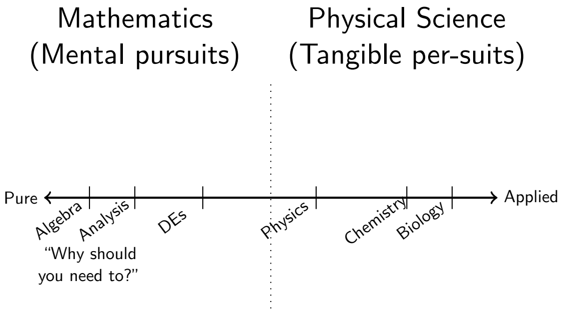
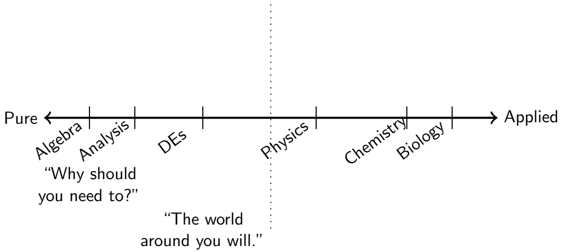



Mathematics
(Mental pursuits)
Physical Science
(Tangible per-suits)
Pure
Applied
Algebra
Analysis
DEs
Physics
Chemistry
Biology
"Why should you need to?"

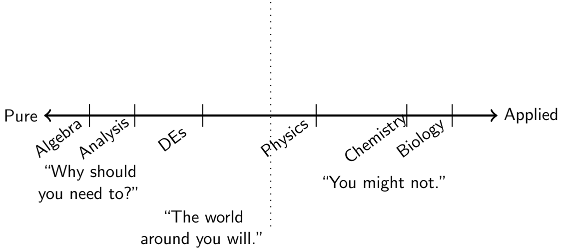
Mathematics
(Mental pursuits)

Physical Science
(Tangible per-suits)

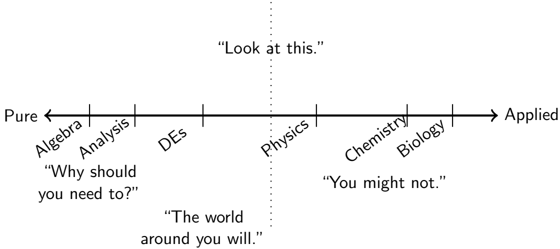


Mathematics
(Mental pursuits)

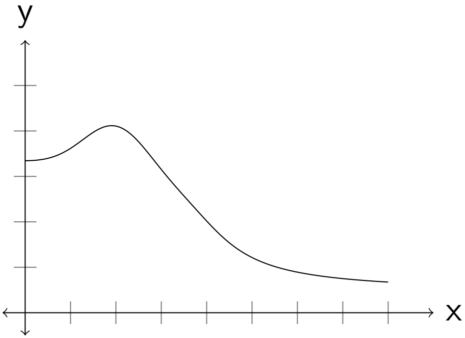
Physical Science
(Tangible per-suits)



Mathematics (Mental pursuits) | Physical Science (Tangible per-suits)



# Table of Contents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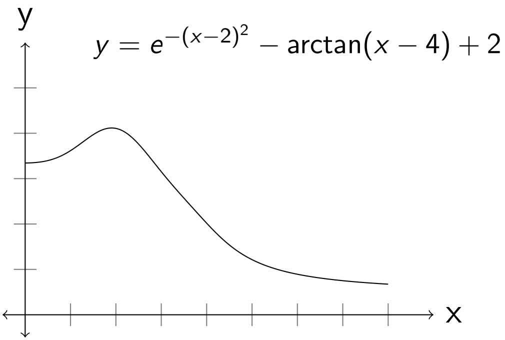
x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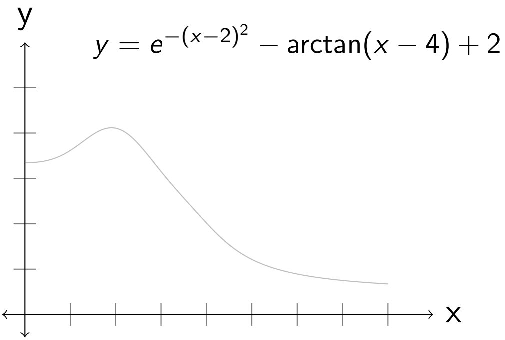
x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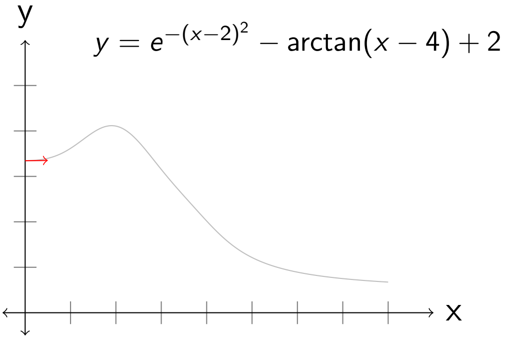
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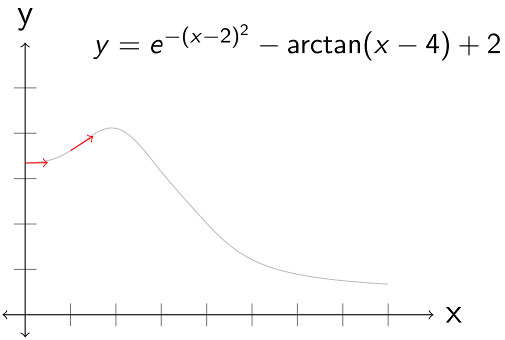
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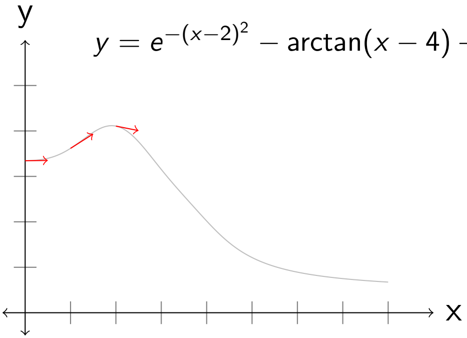


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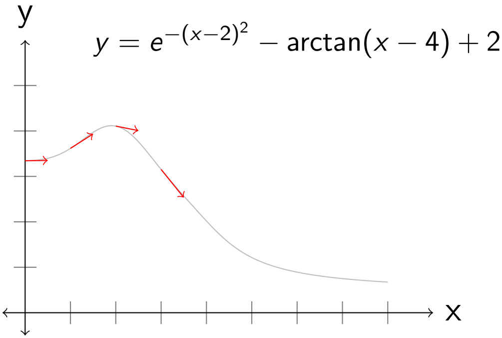
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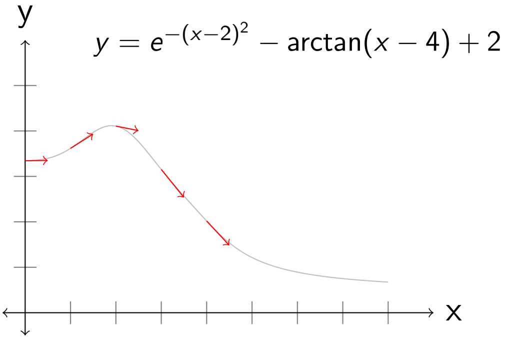
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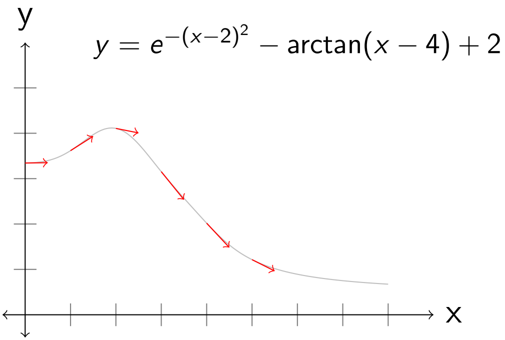
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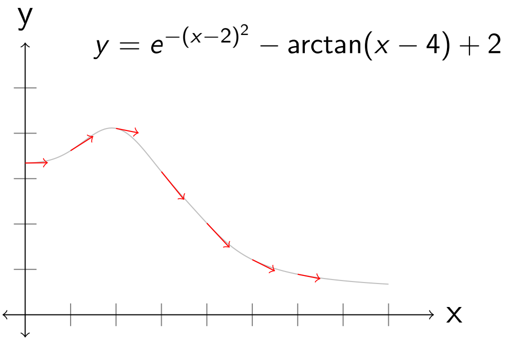
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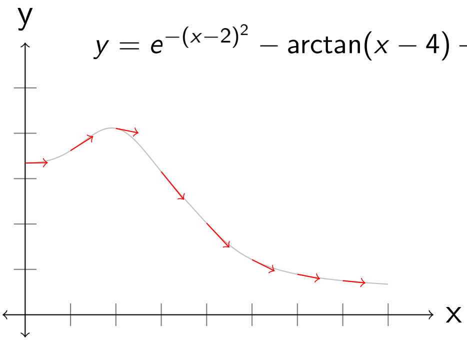

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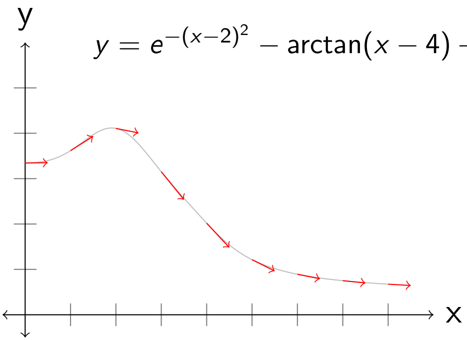

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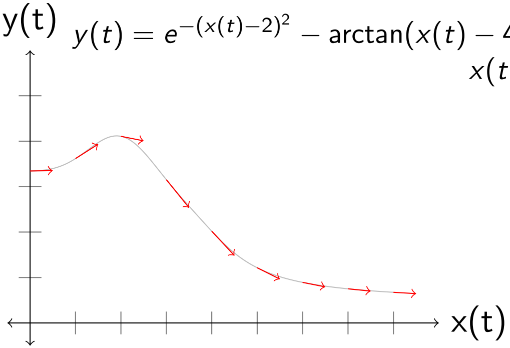
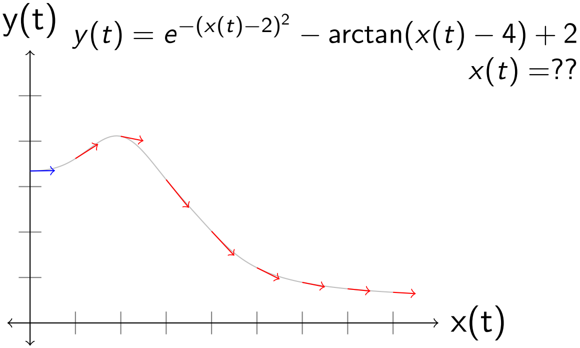
$$x(t) = ??$$

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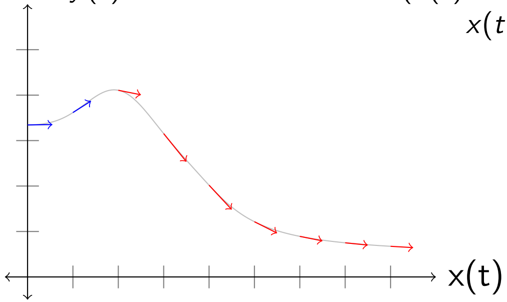

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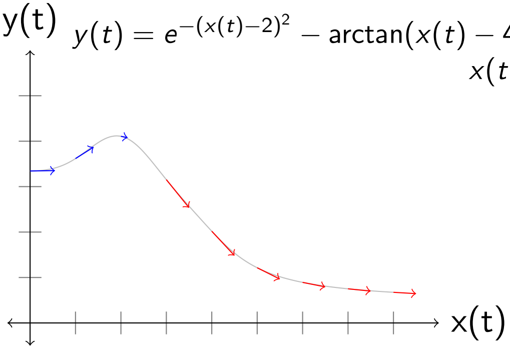

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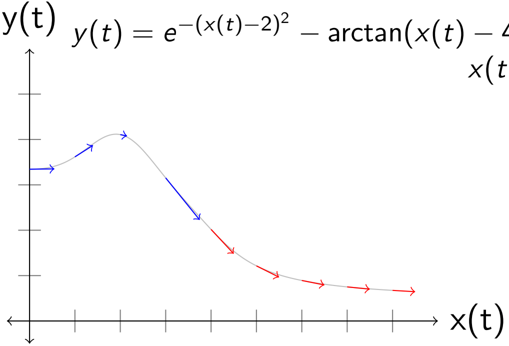

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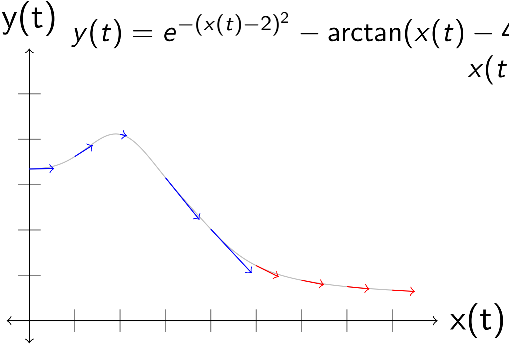

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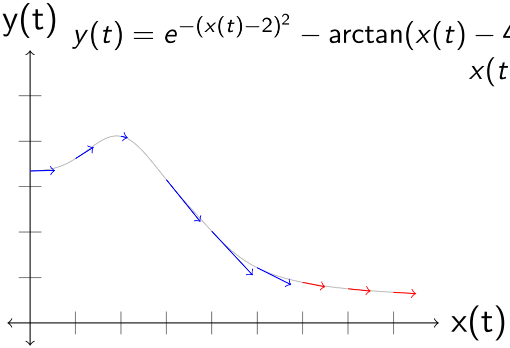

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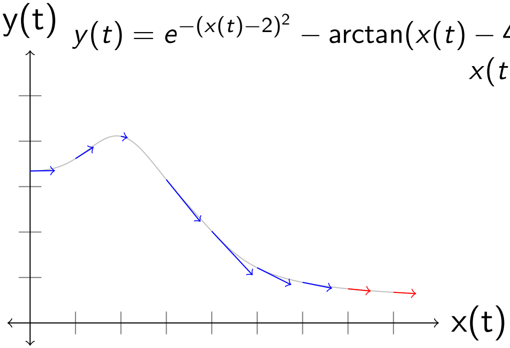

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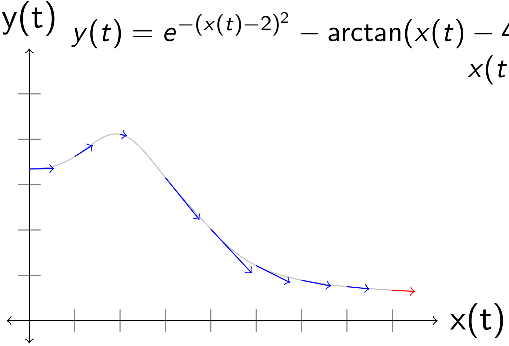

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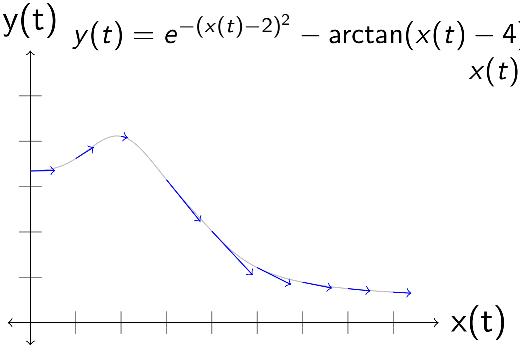
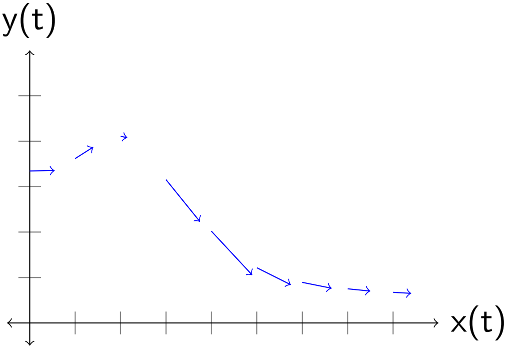
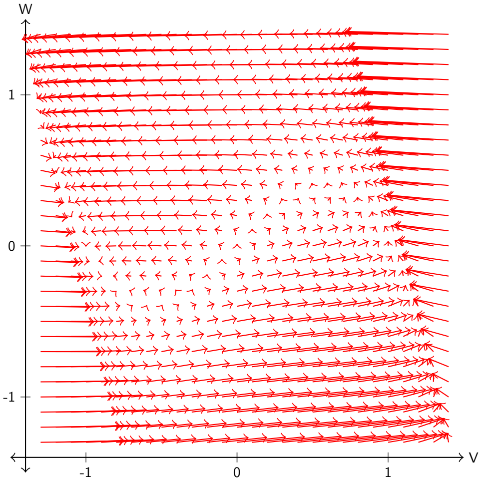
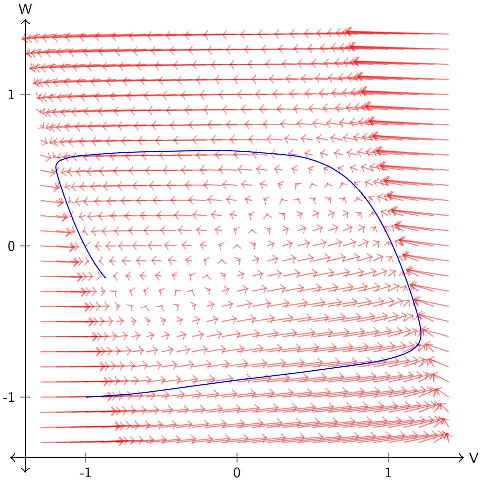
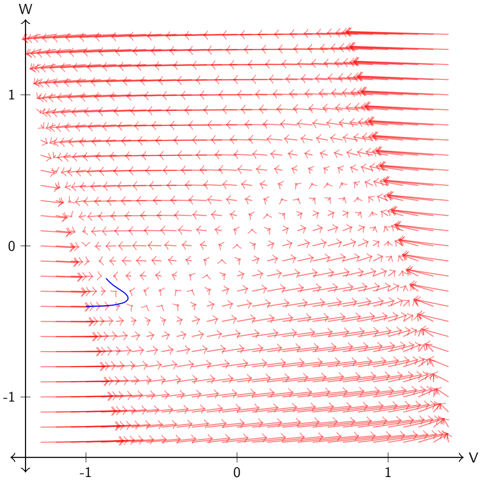
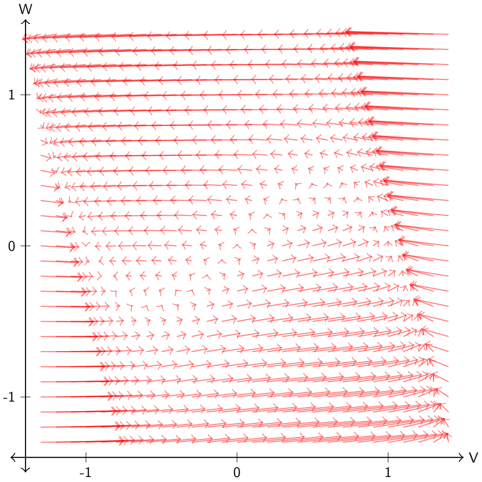


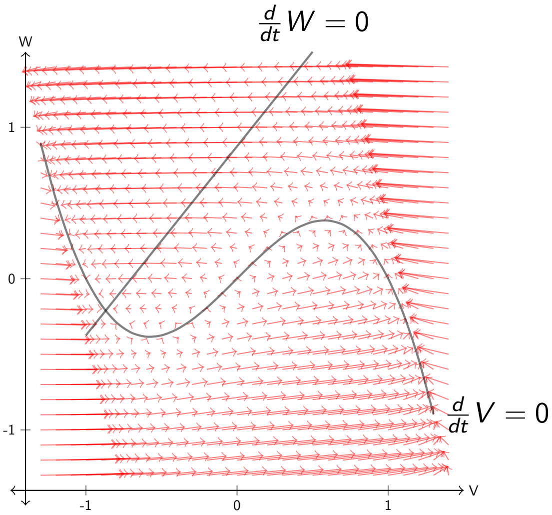

y(t)
x(t)

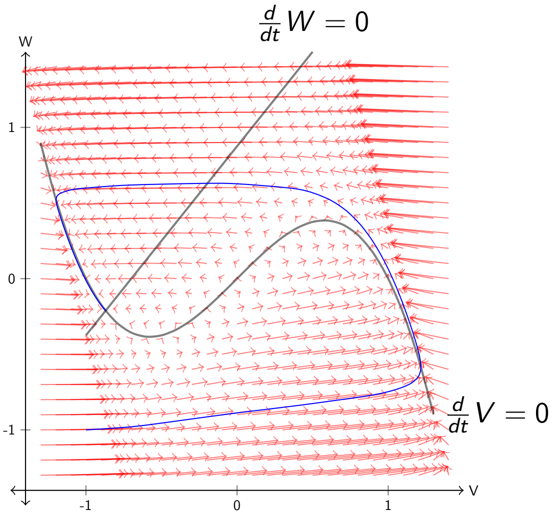

W
1
0
-1
-1
0
1
V


W
1
0
-1
-1
0
1
V


W
1
0
-1
-1
0
1
V


W
1
0
-1
-1
0
1
V


$\frac{d}{dt}W = 0$
W
1
0
-1
-1
0
1
V
$\frac{d}{dt}V = 0$


$\frac{d}{dt}W = 0$
W
1
0
-1
-1
0
1
V
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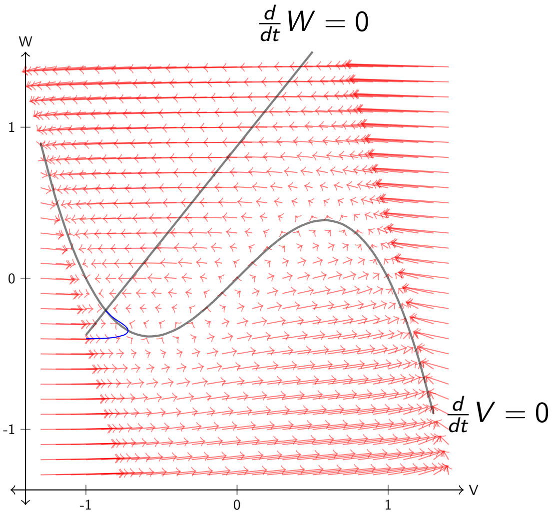
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W
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0
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-1
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1
V
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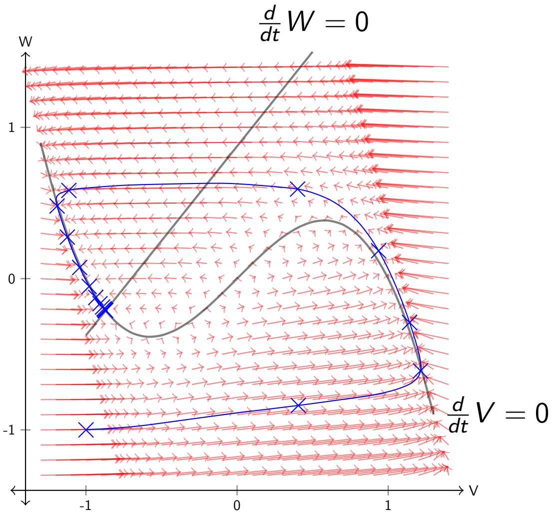
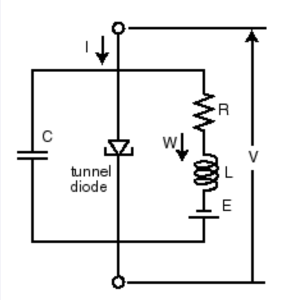

$\frac{d}{dt}W = 0$
W
1
0
-1
-1
0
1
V
$\frac{d}{dt}V = 0$

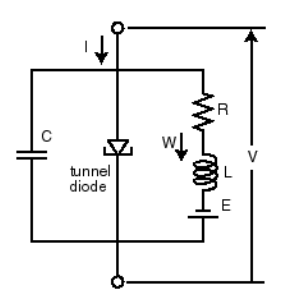
# A Tale of Two Models

## The Nagumo Tunnel-Diode Nerve Model



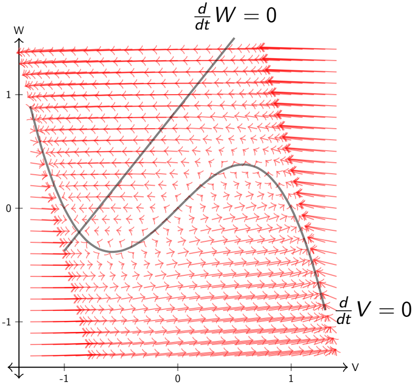
# A Tale of Two Models

## The Nagumo Tunnel-Diode Nerve Model




## The FitzHugh Equations

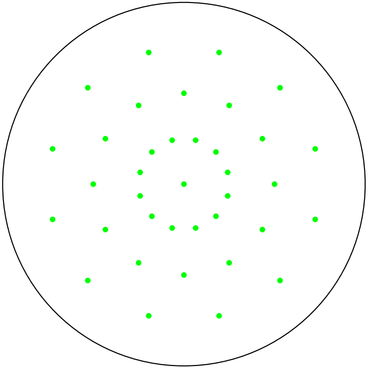
$$
\begin{aligned}
\frac{d}{dt}V &= V - V^3 - W - I \\
\frac{d}{dt}W &= 0.08(V + 0.7 - 0.8W)
\end{aligned}
$$




## The FitzHugh Equations

$$\frac{d}{dt}V = V - V^3 - W - I$$
$$\frac{d}{dt}W = .08(V + .7 - .8W)$$

# The Spread of Infection: The Idea



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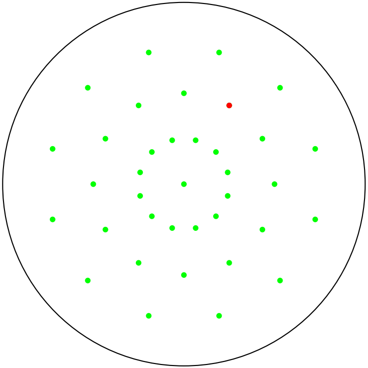

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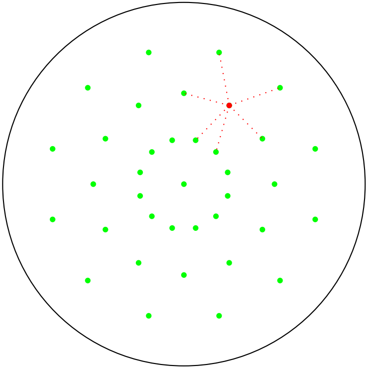

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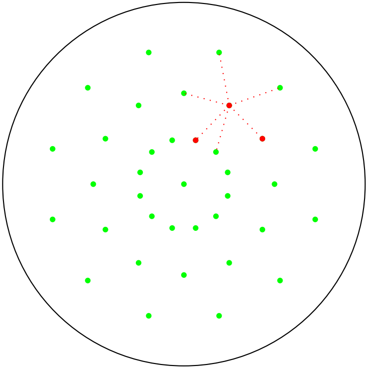

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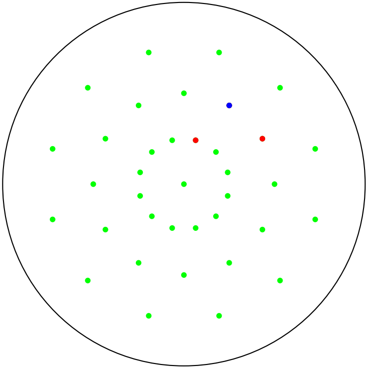

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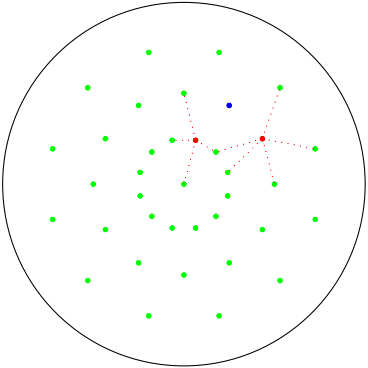
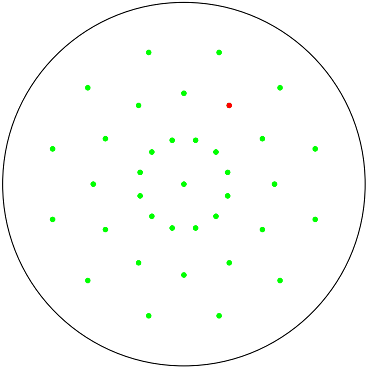


# The Spread of Infection: The Model



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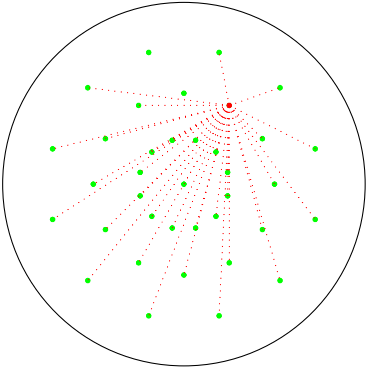

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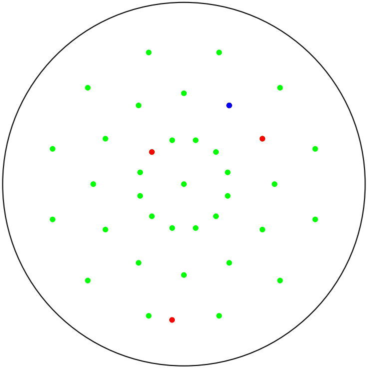

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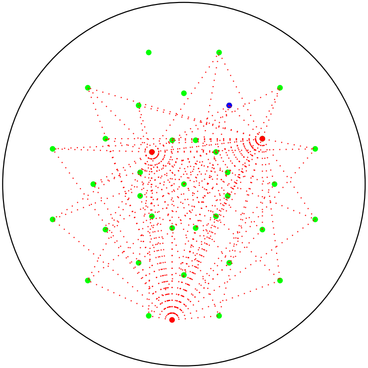

# Compartmental Epidemiology Model

# Compartmental Epidemiology Model

Susceptible

# Compartmental Epidemiology Model

# Compartmental Epidemiology Model

S

Infected

# Compartmental Epidemiology Model

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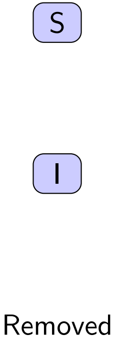
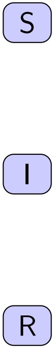


# Compartmental Epidemiology Model



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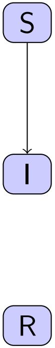

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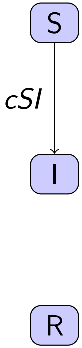

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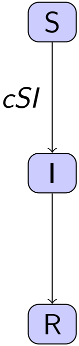

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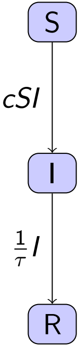

# Compartmental Epidemiology Model

$$\begin{tikzcd} \boxed{S} \arrow[d, "cSI"'] \\ \boxed{I} \arrow[d, "\frac{1}{\tau}I"'] \\ \boxed{R} \end{tikzcd}$$

**The SIR Model**

$$\begin{aligned} \frac{d}{dt}S &= \\ \frac{d}{dt}I &= \\ \frac{d}{dt}R &= \end{aligned}$$

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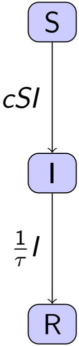


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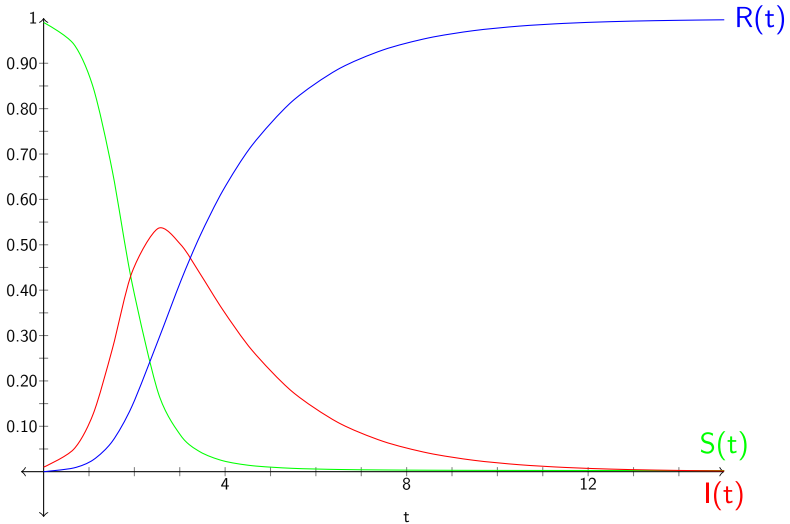
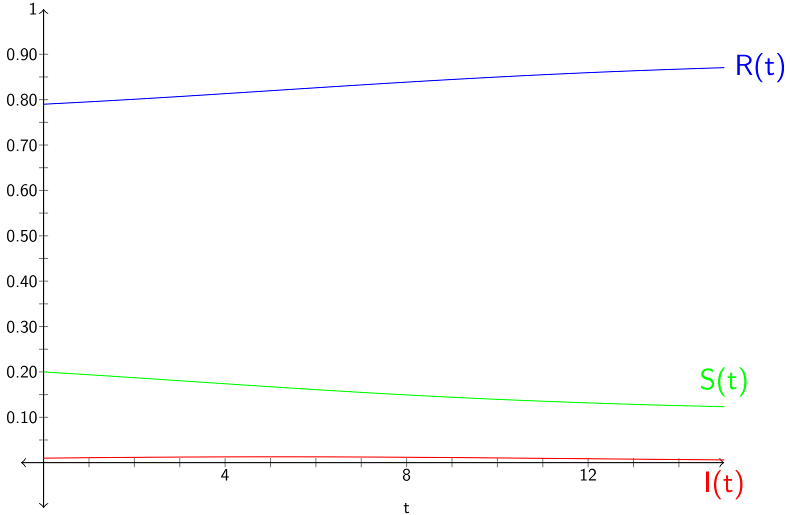
\end{aligned}$$

# An SIR Example: A Pandemic Outbreak



# An SIR Example: Herd Immunity



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$$\begin{aligned}
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### After Some Math Magic:

The Basic Reproductive Number is given $R_0 = c\tau$.

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$$\frac{d}{dt}R = \frac{1}{\tau}I$$

## After Some Math Magic:

The Basic Reproductive Number is given $R_0 = c\tau$.

If $R_0 < \frac{1}{S(0)}$, then $\frac{d}{dt}I(0) < 0$, so we avoid a pandemic.

If $R_0 > \frac{1}{S(0)}$, then $\frac{d}{dt}I(0) > 0$, so we have a bad time.

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- Easily adapted to many different situations.

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- Probabilities assumed to be exponential.
- Focus on the whole, not the individual

## Images used:

- Landscape: http://abstrusegoose.com/211
- Airplane: http://upload.wikimedia.org/wikipedia/commons/thumb/3/34/DieCastModelsWIKI1.jpg/220px-DieCastModelsWIKI1.jpg
- Nagumo Circuit: http://www.scholarpedia.org/article/FitzHugh-Nagumo_model
- Animated Excitation Block: http://www.scholarpedia.org/w/images/4/43/FitzHugh_block.gif
- Fast Depolarization: http://www.scholarpedia.org/w/images/3/30/FitzHugh_accommodation.gif

### YOU!

Thanks for having me!

Thanks for coming!