

AM5650 - Course Project

# **COUPLING OF STRUCTURE AND WAKE OSCILLATORS IN VORTEX-INDUCED VIBRATIONS**

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# Vortex - Induced Vibrations



Vortex Shedding

## What are Vortex-Induced Vibrations (VIV) ?

Vortex-induced vibrations (VIV) are vibrations in a structure caused by periodic vortex shedding in a fluid flow around it. This phenomenon occurs when the shedding frequency of vortices matches or is close to the natural frequency of the structure, leading to large-amplitude vibrations.

## Why are Vortex-Induced Vibrations (VIV) Important?

Several kinds of structures subjected to wind or water currents may experience VIV: common posts, chimneys, suspended cables for bridges, power transmission lines in air, and pipes, risers, towing cables, mooring lines in water. In some cases, this has to be taken into account in their design as a potential cause of fatigue damage, such as for offshore structures.



Vortex Induced Vibrations of a Chimney

# Model Considered

## Transverse 2D VIV of 1 dof Structure in Steady Uniform Flow:

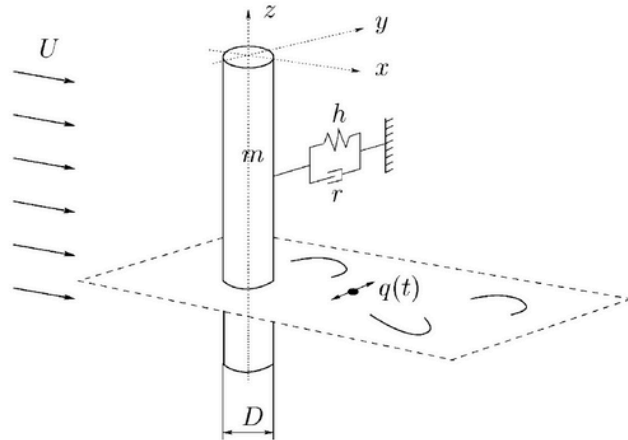


Fig. 1. Model of coupled structure and wake oscillators for 2-D vortex-induced vibrations.

## Non-dimensional Coupled Governing Equations

$$\ddot{y} + \left( 2\xi\delta + \frac{\gamma}{\mu} \right) \dot{y} + \delta^2 y = s, \quad \ddot{q} + \varepsilon(q^2 - 1)\dot{q} + q = f,$$

- The structure oscillation is modeled as a damped linear oscillator with mass and stiffness.
- The wake dynamics are described by a nonlinear van der Pol oscillator.
- Linear coupling terms connect the structure and wake via displacement, velocity, or acceleration.

## Values of Model Parameters

Parameter	Meaning	Value taken
$\delta$	Reduced natural frequency of structure	$\delta = \frac{1}{St \cdot U_r}$
$\mu$	Dimensionless mass ratio	$\mu = \frac{0.05}{M}$
$M$	Mass number, scales effect of wake on structure	$M = 2 \times 10^{-4}$
$\gamma$	Fluid-added damping coefficient	$\gamma = 0.8$
$\xi$	Structural damping ratio	$\xi = 3.1 \times 10^{-3}$
$\varepsilon$	van der Pol nonlinearity parameter	$\varepsilon = 0.3$
$s$	Forcing from wake on structure	$s = Mq$
$f$	Forcing from structure on wake	$f = A\ddot{y}$ or $A\dot{y}$ or $Ay$
$A$	Scaling of coupling force	$A = 12$

Reduced velocity measures how fast the flow is compared to the natural oscillation speed of the structure.

$$U_r = \frac{2\pi U}{\Omega_s D}$$

**Lock-in:** when the vortex shedding frequency synchronizes with the structure's natural frequency, leading to large oscillations. Can be considered as resonance

# Results in the Paper

To a leading order the solution of the governing equation is considered of the form:

$$y(t) = y_o \cos(\omega t), \quad q(t) = q_o \cos(\omega t - \varphi),$$

## Appearance of Lock - In Phenomenon

- Defined as the deviation of the wake and structure common frequency from 1. Out of lock-in, the coupled system is synchronized onto the vortex shedding angular frequency 1.
- Hysteretic behaviour is observed as the deviation of angular frequency from 1 does not follow the same trend when  $U_r$  is increased and decreased

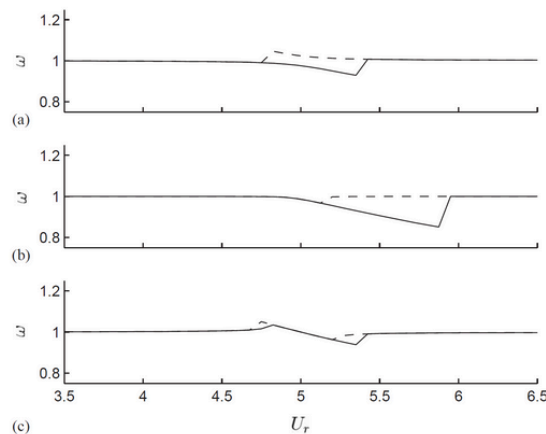
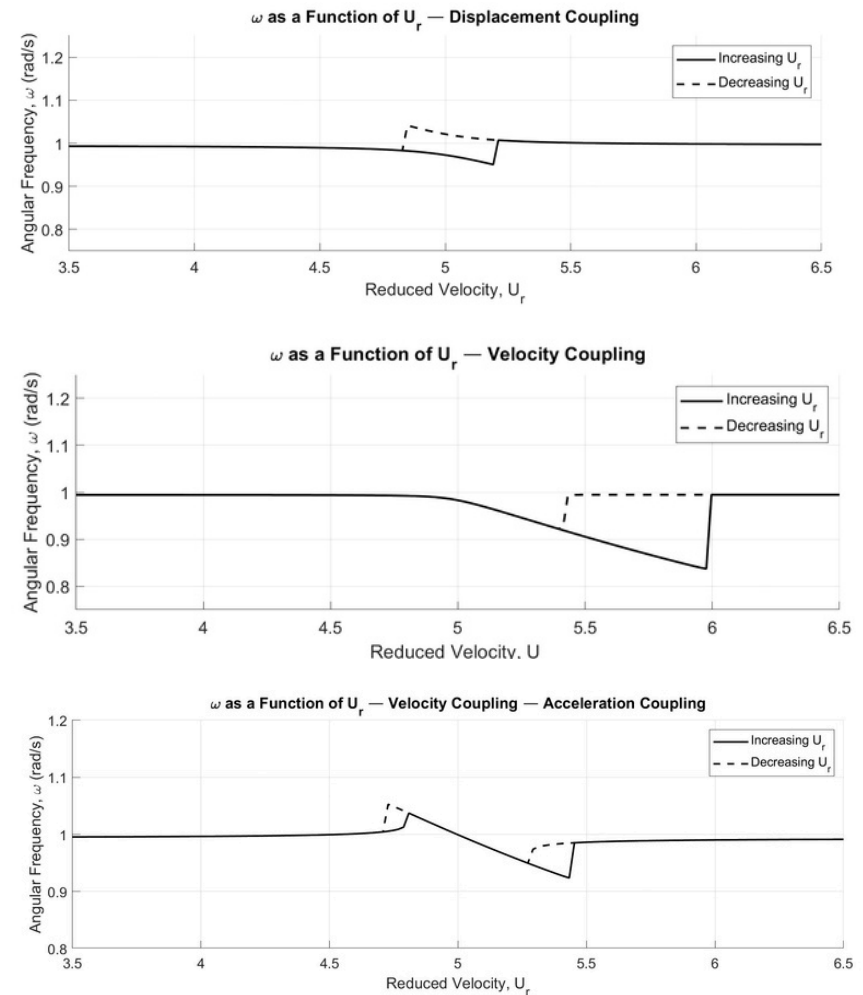
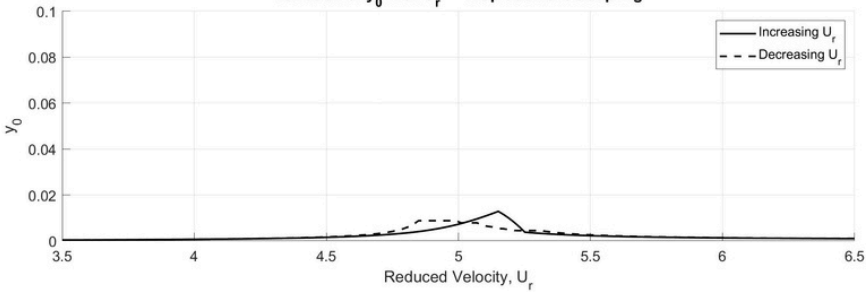


Fig. 5. Angular frequency  $\omega$  as a function of reduced velocity  $U_r$ : (a) displacement model; (b) velocity model; (c) acceleration model. —, increasing  $U_r$ ; - -, decreasing  $U_r$ .

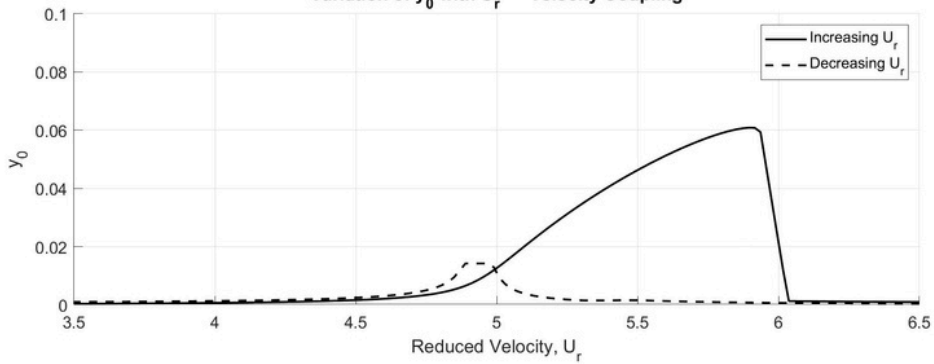


## Effect of Lock - In on $y_0$

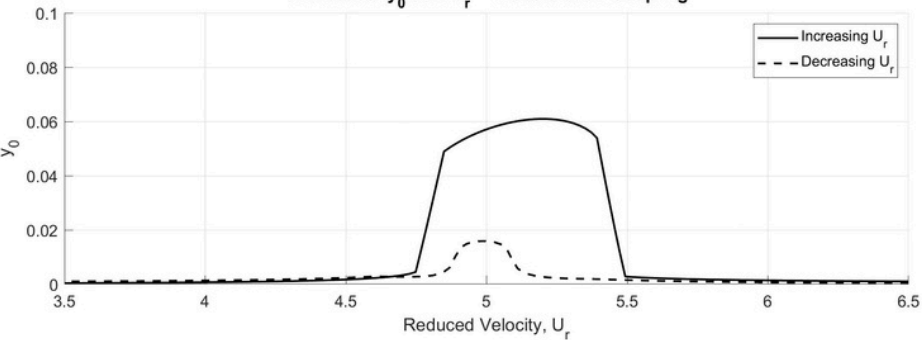
Variation of  $y_0$  with  $U_r$  — Displacement Coupling



Variation of  $y_0$  with  $U_r$  — Velocity Coupling

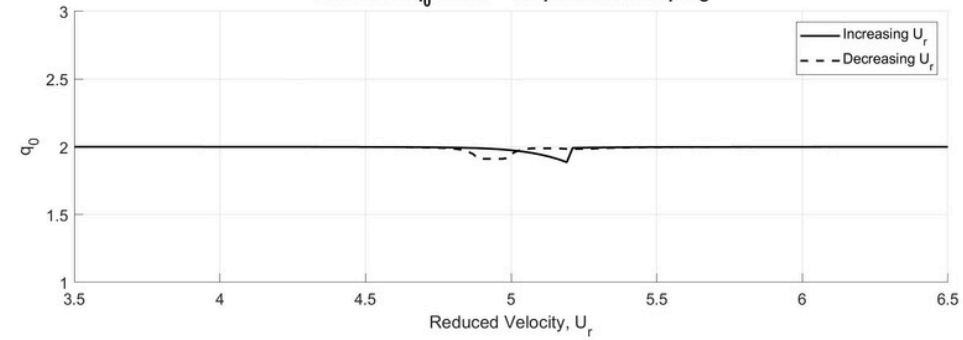


Variation of  $y_0$  with  $U_r$  — Acceleration Coupling

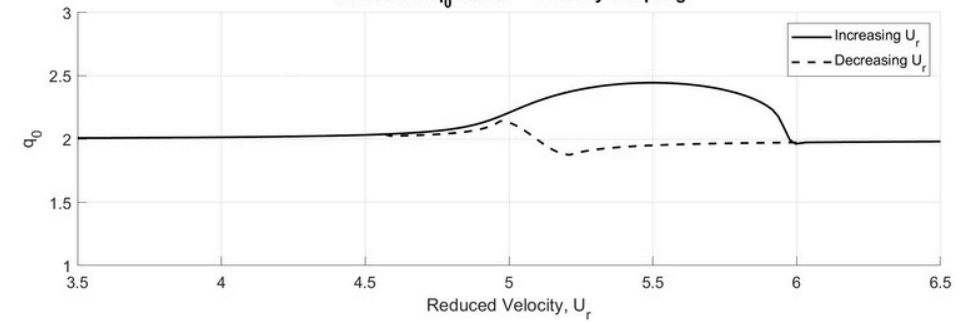


## Effect of Lock - In on $q_0$

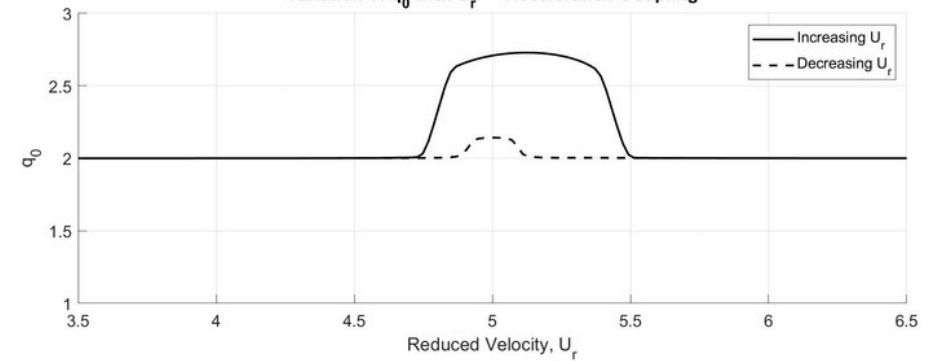
Variation of  $q_0$  with  $U$  — Displacement Coupling



Variation of  $q_0$  with  $U$  — Velocity Coupling



Variation of  $q_0$  with  $U_r$  — Acceleration Coupling



# Considering Wake Induced Damping on Structure

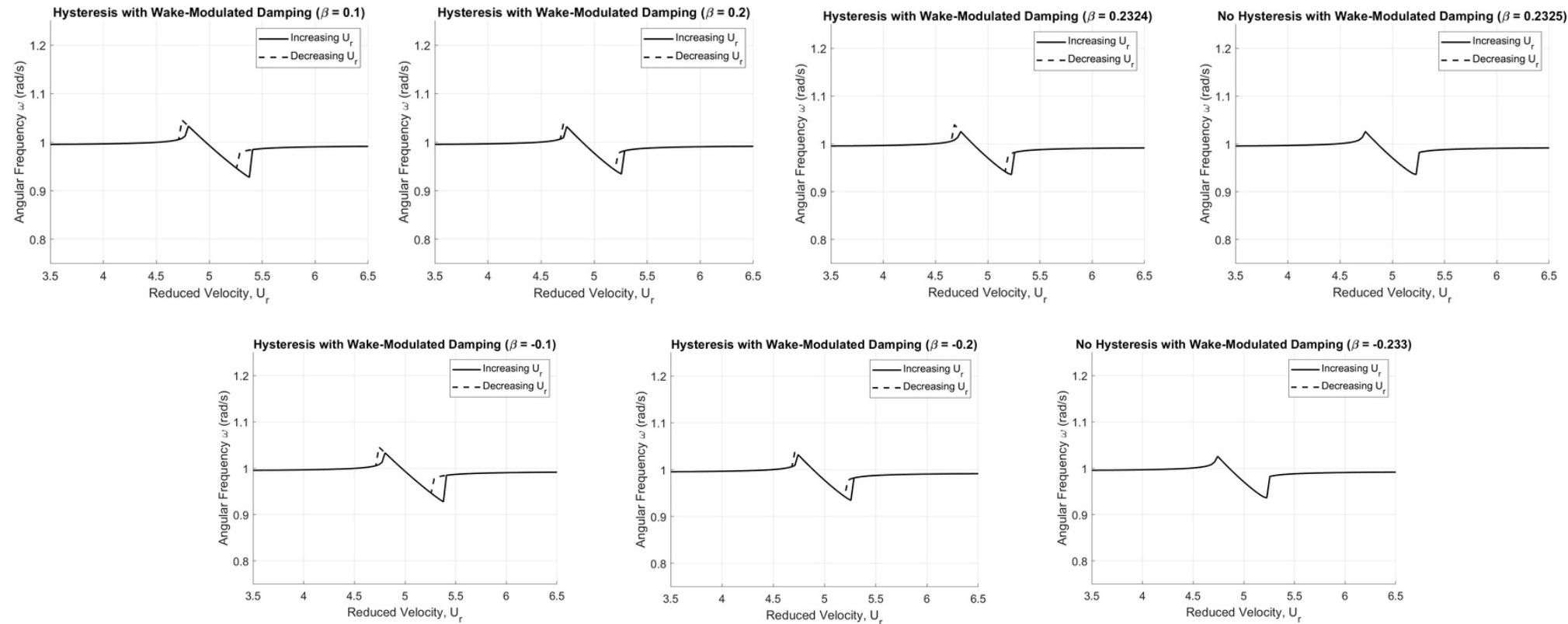
New set of governing equations:

$$\ddot{y} + \left( 2\xi\delta + \frac{\gamma}{\mu} + \beta q(t) \right) \dot{y} + \delta^2 y = Mq(t)$$

$$\ddot{q} + \epsilon (q^2 - 1) \dot{q} + q = A\ddot{y}$$

Including wake-induced damping accounts for the fact that stronger vortex shedding can dissipate more energy from the structure.

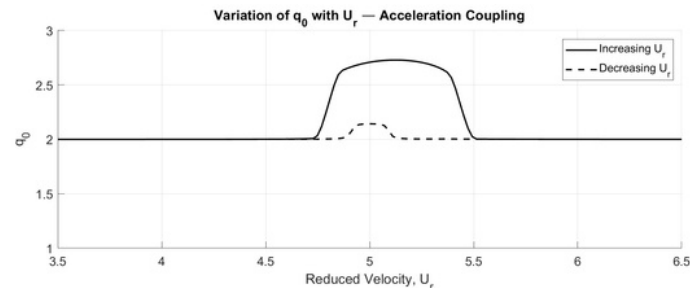
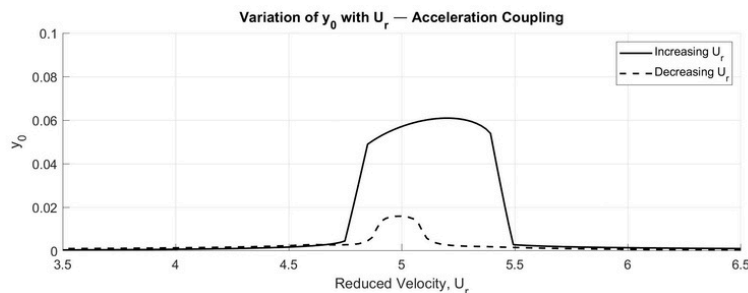
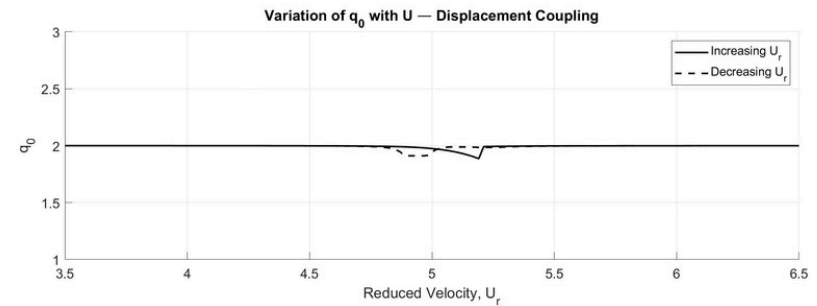
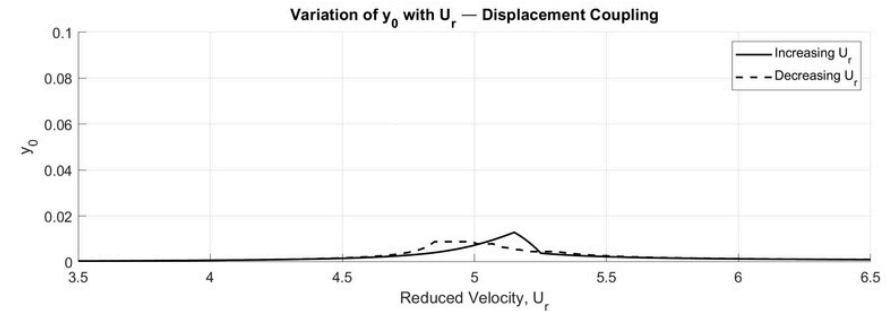
The variation of frequency with reduced velocity was found for various values of beta. The hysteresis in the system was found to vanish when magnitude approaches 0.233



# Discussions

## From the paper it was found that:

- The displacement coupling model fails to capture key features of lock-in: lift magnification and large-amplitude structural response.
- Outside lock-in, the wake oscillator behaves like a self-sustained van der Pol system with fixed amplitude  $q_0 = 2q_0 = 2q_0 = 2$ , while the structure remains nearly stationary.
- The Skop-Griffin parameter (SG) is commonly used to characterize the maximum structural displacement amplitude at lock-in, but velocity coupling fails to predict lift phase and lock-in range at low SG.
- The Skop-Griffin parameter (SG) is commonly used to characterize structural response, but velocity coupling fails to predict lift phase and lock-in range at low SG, and was considered to be the most accurate model for the given system.



## From considering wake induced damping it was found that:

- Including a wake-modulated damping term is physically reasonable, as stronger vortex shedding can act to resist structural motion — effectively behaving like a dynamic damping source
- We observed that for  $\beta \gtrsim -0.23-0.24$ , the system no longer exhibits hysteresis, indicating some sort of bifurcation occurring in the system changing its characteristics.