

Time period	Number of consultants
8AM – 12PM	4
12PM-4PM	8
4PM-8PM	10
8PM-12AM	6

### Question 1

a) Determine a minimum-cost staffing plan for the center. In your solution, how many consultants will be paid to work full time and how many will be paid to work part time? What is the minimum cost?

$F_i$  = Number of full time computer consultants in 3 shifts.  $i = 1,2,3$ (Each shift is of 8 hours)

$P_j$  = Number of part time computer consultants in 4 shifts.  $j = 1,2,3,4$  (Each shift is of 4 hours)

Pay for full time consultants = \$14/hr

Pay for part time consultants = \$12/hr

$Z_{\min}$  = Minimum Cost

$$Z_{\min} = (F_1 + F_2 + F_3) * 14 * 8 + (P_1 + P_2 + P_3 + P_4) * 12 * 4$$

ST

$$F_1 + P_1 \geq 4$$

$$F_1 + F_2 + P_2 \geq 8$$

$$F_2 + F_3 + P_3 \geq 10$$

$$F_3 + P_4 \geq 6$$

$$F_1 \geq P_1$$

$$F_1 + F_2 \geq P_2$$

$$F_2 + F_3 \geq P_3$$

$$F_3 \geq P_4$$

$$F_i \geq P_j \geq 0$$

$$P_1 = 2, P_2 = 4, P_3 = 5, P_4 = 3$$

and

$$F_1 = 2, F_2 = 2, F_3 = 3$$

$$\begin{aligned} Z_{\min} &= (2 + 2 + 3) * 14 * 8 + (2 + 4 + 5 + 3) * 12 * 4 \\ &= 784 + 672 \\ &= 1456 \end{aligned}$$

Total Full-time workers = 7

Total Part time workers = 14

b) After thinking about this problem for a while, you have decided to recognize meal breaks explicitly in the scheduling of full-time consultants. In particular, full-time consultants are entitled to a one-hour lunch break during their eight-hour shift. In addition, employment rules specify that the lunch break can start after three hours of work or after four hours of work, but those are the only alternatives. Part-time consultants do not receive a meal break. Under these conditions, find a minimum-cost staffing plan. What is the minimum cost?

1 hour break to full time workers and no break to part time workers. Considering this, we need to subtract the pay/hour from the  $Z_{\min}$  function.

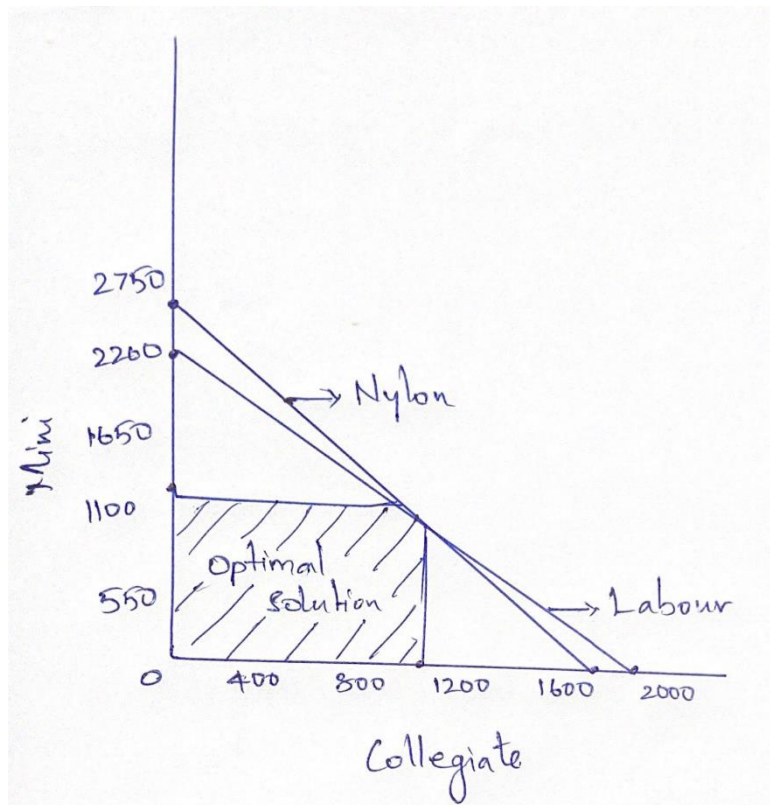
Constraints will remain the same. The new cost function is as below:

$$\begin{aligned} Z_{\min 1} &= (F_1 + F_2 + F_3) * 14 * 8 + (P_1 + P_2 + P_3 + P_4) - (F_1 + F_2 + F_3) * 14 \\ &= 7 * 112 + 14 * 48 - 7 * 14 \\ &= 1358 \end{aligned}$$

$$Z_{\min} - Z_{\min 1} = 1456 - 1358 = 98$$

## Question 2

Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of \$32. Each Mini requires 40 minutes of labor and generates a unit profit of \$24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week. Solve this problem graphically.



### Question 3

#### a. Define the decision variables

$W_{xy}$ , here  $x$  denotes plants 1,2,3 whereas  $y$  is the sizes l, m, s.

$W_{1l}, W_{1m}, W_{1s}$  – variables of plant 1

$W_{2l}, W_{2m}, W_{2s}$  – variables of plant 2

$W_{3l}, W_{3m}, W_{3s}$  – variables of plant 3

#### b. Formulate a linear programming model for this problem.

$$Z_{\max} = (W_{1l} + W_{2l} + W_{3l}) * 420 + (W_{1m} + W_{2m} + W_{3m}) * 360 + (W_{1s} + W_{2s} + W_{3s}) * 300$$

Subject to:

$$W_{1l} + W_{1m} + W_{1s} \leq 750$$

$$W_{2l} + W_{2m} + W_{2s} \leq 900$$

$$W_{3l} + W_{3m} + W_{3s} \leq 450$$

$$20*W_{1l} + 15*W_{1m} + 12*W_{1s} \leq 13000$$

$$20*W_{2l} + 15*W_{2m} + 12*W_{2s} \leq 12000$$

$$20*W_{3l} + 15*W_{3m} + 12*W_{3s} \leq 5000$$

$$900*(W_{1l} + W_{1m} + W_{1s}) - 750*(W_{2l} + W_{2m} + W_{2s}) = 0$$

$$450*(W_{2l} + W_{2m} + W_{2s}) - 900*(W_{3l} + W_{3m} + W_{3s}) = 0$$

$$450*(W_{1l} + W_{1m} + W_{1s}) - 750*(W_{3l} + W_{3m} + W_{3s}) = 0$$

$$W_{1l} + W_{2l} + W_{3l} \leq 900$$

$$W_{1m} + W_{2m} + W_{3m} \leq 1200$$

$$W_{1s} + W_{2s} + W_{3s} \leq 750$$

$$W_{xy} \geq 0 \text{ here } x = 1, 2, 3 \text{ and } y = l, m, s$$

#### c. Solve the problem using lpsolve, or any other equivalent library in R.

```
library(lpSolveAPI)
```

```
w <- make.lp(0,3,verbose = "neutral")
```

```
w
```

```
add.constraint(w, c(1,1,1), "<=", 750)
```

```
add.constraint(w, c(1,1,1), "<=", 900)
```

```
add.constraint(w, c(1,1,1), "<=", 450)
```

```
add.constraint(w, c(20,15,12), "<=", 13000)
```

```
add.constraint(w, c(20,15,12), "<=", 12000)
```

```
add.constraint(w, c(20,15,12), "<=", 5000)
```

```
add.constraint(w, c(1,1,1), "<=", 900)
```

```
add.constraint(w, c(1,1,1), "<=", 1200)
```

```
add.constraint(w, c(1,1,1), "<=", 750)
```

```
w.col <- c("Plant 1", "Plant 2", "Plant 3")
```

```
w.row <- c("W1l", "W1m", "W1s", "W2l", "W2m", "W2s", "W3l", "W3m", "W3s")
```

```
dimnames(w) <- list(w.row, w.col)
```

w

solve(w)

```
> add.constraint(w, c(1,1,1), "<=", 750 )
> add.constraint(w, c(1,1,1), "<=", 900)
> add.constraint(w, c(1,1,1), "<=", 450)
> add.constraint(w, c(20,15,12), "<=", 13000)
> add.constraint(w, c(20,15,12), "<=", 12000)
> add.constraint(w, c(20,15,12), "<=", 5000)
> add.constraint(w, c(1,1,1), "<=", 900)
> add.constraint(w, c(1,1,1), "<=", 1200)
> add.constraint(w, c(1,1,1), "<=", 750)
>
>
> w.col <- c("Plant 1","Plant 2","Plant 3")
> w.row <- c("w1l","w1m","w1s","w2l", "w2m","w2s","w3l","w3m","w3s")
> dimnames(w) <- list(w.row,w.col)
>
```

> w

Model name:

	Plant 1	Plant 2	Plant 3		
Minimize	0	0	0		
w1l	1	1	1	<=	750
w1m	1	1	1	<=	900
w1s	1	1	1	<=	450
w2l	20	15	12	<=	13000
w2m	20	15	12	<=	12000
w2s	20	15	12	<=	5000
w3l	1	1	1	<=	900
w3m	1	1	1	<=	1200
w3s	1	1	1	<=	750
Kind	Std	Std	Std		
Type	Real	Real	Real		
Upper	Inf	Inf	Inf		
Lower	0	0	0		

> solve(w)

[1] 0