

Assignment_6

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Q1: Formulate and solve the binary integer programming (BIP) model for this problem using library lpSolve or equivalent in R

Formulating an LP function. We set the function to maximization because we are trying to find the longest route.

```
library(lpSolveAPI)
lproute <- make.lp(0,12)
lp.control(lproute, sense="max")

## $anti.degen
## [1] "fixedvars" "stalling"
##
## $basis.crash
## [1] "none"
##
## $bb.depthlimit
## [1] -50
##
## $bb.floorfirst
## [1] "automatic"
##
## $bb.rule
## [1] "pseudononint" "greedy"          "dynamic"          "rcostfixing"
##
## $break.at.first
## [1] FALSE
##
## $break.at.value
## [1] 1e+30
##
## $epsilon
##      epsb      epsd      epsel      epsint  epsperturb  epspivot
##      1e-10      1e-09      1e-12      1e-07      1e-05      2e-07
##
## $improve
## [1] "dualfeas" "thetagap"
##
## $infinite
## [1] 1e+30
##
## $maxpivot
```

```
## [1] 250
##
## $mip.gap
## absolute relative
##      1e-11      1e-11
##
## $negrange
## [1] -1e+06
##
## $obj.in.basis
## [1] TRUE
##
## $pivoting
## [1] "devex"      "adaptive"
##
## $presolve
## [1] "none"
##
## $scalelimit
## [1] 5
##
## $scaling
## [1] "geometric"    "equilibrate" "integers"
##
## $sense
## [1] "maximize"
##
## $simplextype
## [1] "dual"    "primal"
##
## $timeout
## [1] 0
##
## $verbose
## [1] "neutral"
```

Setting the objective function

```
t_value <- c(5, 3, 4, 2, 3, 1, 4, 6, 2, 5, 4, 7)
set.objfn(lproute, 1*t_value)
set.type(lproute, 1:12, "binary")
```

Setting the constraints

```
add.constraint(lproute,c(1,1),"=",1,indices = c(1,2))
add.constraint(lproute,c(1,-1,-1),"=",0,indices = c(1,3,4))
add.constraint(lproute,c(1,-1),"=",0,indices = c(2,5))
add.constraint(lproute,c(1,-1,-1),"=",0,indices = c(3,6,7))
add.constraint(lproute,c(1,1,-1,-1),"=",0,indices = c(4,5,8,9))
add.constraint(lproute,c(1, -1),"=",0,indices = c(6,10))
add.constraint(lproute,c(1,1,-1),"=",0,indices = c(7,8,11))
```

```
add.constraint(lproute,c(1,-1),"=",0,indices = c(9,12))
add.constraint(lproute,c(1,1,1),"=",1,indices = c(10,11,12))
```

Solving the LP Problem

```
solve(lproute)

## [1] 0

get.objective(lproute)

## [1] 17

get.variables(lproute)

## [1] 1 0 0 1 0 0 0 1 0 0 1 0

get.constraints(lproute)

## [1] 1 0 0 0 0 0 0 0 1
```

Mapping input to the output

```
arc <- c("X12", "X13", "X24", "X25", "X35", "X46", "X47", "X57", "X58",
        "X69", "X79", "X89")
vars<-get.variables(lproute)
output<-data.frame(arc,vars)
output

##      arc vars
## 1  X12     1
## 2  X13     0
## 3  X24     0
## 4  X25     1
## 5  X35     0
## 6  X46     0
## 7  X47     0
## 8  X57     1
## 9  X58     0
## 10 X69     0
## 11 X79     1
## 12 X89     0
```

Formulate and solve the binary integer programming (BIP) model for this problem using library lpSolve or equivalent in R. From the variables, the longest route will be X12 - X24 - X57 - X79. The maximum objective function is 17

Q2: 1) Determine the maximum return on the portfolio. What is the optimal number of shares to buy for each of the stocks? What is the corresponding dollar amount invested in each stock? 2) Compare the solution in which there is no integer restriction on the number of shares invested. By how much (in percentage terms) do the integer restrictions alter the value of the optimal objective function? By how much (in percentage terms) do they alter the optimal investment quantities?

formulating the lp function. Setting the function to maximization

```
library(lpSolveAPI)
lpstock<-make.lp(0,8)
lp.control(lpstock,sense="max")

## $anti.degen
## [1] "fixedvars" "stalling"
##
## $basis.crash
## [1] "none"
##
## $bb.depthlimit
## [1] -50
##
## $bb.floorfirst
## [1] "automatic"
##
## $bb.rule
## [1] "pseudononint" "greedy"          "dynamic"          "rcostfixing"
##
## $break.at.first
## [1] FALSE
##
## $break.at.value
## [1] 1e+30
##
## $epsilon
##      epsb      epsd      epsel      epsint  epsperturb  epspivot
##      1e-10      1e-09      1e-12      1e-07      1e-05      2e-07
##
## $improve
## [1] "dualfeas" "thetagap"
##
## $infinite
## [1] 1e+30
##
## $maxpivot
## [1] 250
##
## $mip.gap
## absolute relative
##      1e-11      1e-11
```

```
##
## $negrange
## [1] -1e+06
##
## $obj.in.basis
## [1] TRUE
##
## $pivoting
## [1] "devex"      "adaptive"
##
## $presolve
## [1] "none"
##
## $scalelimit
## [1] 5
##
## $scaling
## [1] "geometric"    "equilibrate" "integers"
##
## $sense
## [1] "maximize"
##
## $simplextype
## [1] "dual"      "primal"
##
## $timeout
## [1] 0
##
## $verbose
## [1] "neutral"
```

Since we are trying a interger programming we will set the type to integer.

```
set.objfn(lpstock,c(4,6.5,5.9,5.4,5.15,10,8.4,6.25))
set.type(lpstock,c(1:8),type = "integer")
```

Adding the constraints

```
add.constraint(lpstock,c(40,50,80,60,45,60,30,25),"<=",2500000,indices =
c(1:8))
add.constraint(lpstock,1000,">=",0,indices = 1)
add.constraint(lpstock,1000,">=",0,indices = 2)
add.constraint(lpstock,1000,">=",0,indices = 3)
add.constraint(lpstock,1000,">=",0,indices = 4)
add.constraint(lpstock,1000,">=",0,indices = 5)
add.constraint(lpstock,1000,">=",0,indices = 6)
add.constraint(lpstock,1000,">=",0,indices = 7)
add.constraint(lpstock,1000,">=",0,indices = 8)
add.constraint(lpstock,40,">=",100000,indices = 1)
add.constraint(lpstock,50,">=",100000,indices = 2)
add.constraint(lpstock,80,">=",100000,indices = 3)
```

```

add.constraint(lpstock,60,">=",100000,indices = 4)
add.constraint(lpstock,45,">=",100000,indices = 5)
add.constraint(lpstock,60,">=",100000,indices = 6)
add.constraint(lpstock,30,">=",100000,indices = 7)
add.constraint(lpstock,25,">=",100000,indices = 8)
add.constraint(lpstock,c(40,50,80),"<=",1000000,indices = c(1,2,3))
add.constraint(lpstock,c(60,45,60),"<=",1000000,indices = c(4,5,6))
add.constraint(lpstock,c(30,25),"<=",1000000,indices = c(7,8))

```

solving the lp problem

```

solve(lpstock)

## [1] 0

get.objective(lpstock)

## [1] 487145.2

get.variables(lpstock)

## [1] 2500 6000 1250 1667 2223 13332 30000 4000

get.constraints(lpstock)

## [1] 2499975 2500000 6000000 1250000 1667000 2223000 13332000
30000000
## [9] 4000000 100000 300000 100000 100020 100035 799920
900000
## [17] 100000 500000 999975 1000000

```

- 1) Determine the maximum return on the portfolio. What is the optimal number of shares to buy for each of the stocks? What is the corresponding dollar amount invested in each stock? With integer restriction, the maximum return on investment is 487145.2. The optimal number of shares to buy are $S_1 = 2500$, $S_2 = 6000$, $S_3 = 1250$, $H_1 = 1667$, $H_2 = 2223$, $H_3 = 13332$, $C_1 = 30000$ and $C_2 = 4000$ and the corresponding dollars to be invested are $S_1 = 100000$, $S_2 = 300000$, $S_3 = 100000$, $H_1 = 100020$, $H_2 = 100035$, $H_3 = 799920$, $C_1 = 900000$, $C_2 = 100000$

formulating lp problem without integer restrictions

```

lpstock2 <-make.lp(0,8)
lp.control(lpstock2,sense="max")

## $anti.degen
## [1] "fixedvars" "stalling"
##
## $basis.crash
## [1] "none"
##
## $bb.depthlimit
## [1] -50

```

```

##
## $bb.floorfirst
## [1] "automatic"
##
## $bb.rule
## [1] "pseudononint" "greedy"          "dynamic"          "rcostfixing"
##
## $break.at.first
## [1] FALSE
##
## $break.at.value
## [1] 1e+30
##
## $epsilon
##      epsb      epsd      epsel      epsint  epsperturb  epspivot
##      1e-10      1e-09      1e-12      1e-07      1e-05      2e-07
##
## $improve
## [1] "dualfeas" "thetagap"
##
## $infinite
## [1] 1e+30
##
## $maxpivot
## [1] 250
##
## $mip.gap
## absolute relative
##      1e-11      1e-11
##
## $negrange
## [1] -1e+06
##
## $obj.in.basis
## [1] TRUE
##
## $pivoting
## [1] "devex"      "adaptive"
##
## $presolve
## [1] "none"
##
## $scalelimit
## [1] 5
##
## $scaling
## [1] "geometric"  "equilibrate" "integers"
##
## $sense
## [1] "maximize"

```

```

##
## $simplextype
## [1] "dual"    "primal"
##
## $timeout
## [1] 0
##
## $verbose
## [1] "neutral"

set.objfn(lpstock2,c(4,6.5,5.9,5.4,5.15,10,8.4,6.25))

add.constraint(lpstock2,c(40,50,80,60,45,60,30,25),"<=",2500000,indices =
c(1:8))
add.constraint(lpstock2,1000,">=",0,indices = 1)
add.constraint(lpstock2,1000,">=",0,indices = 2)
add.constraint(lpstock2,1000,">=",0,indices = 3)
add.constraint(lpstock2,1000,">=",0,indices = 4)
add.constraint(lpstock2,1000,">=",0,indices = 5)
add.constraint(lpstock2,1000,">=",0,indices = 6)
add.constraint(lpstock2,1000,">=",0,indices = 7)
add.constraint(lpstock2,1000,">=",0,indices = 8)
add.constraint(lpstock2,40,">=",100000,indices = 1)
add.constraint(lpstock2,50,">=",100000,indices = 2)
add.constraint(lpstock2,80,">=",100000,indices = 3)
add.constraint(lpstock2,60,">=",100000,indices = 4)
add.constraint(lpstock2,45,">=",100000,indices = 5)
add.constraint(lpstock2,60,">=",100000,indices = 6)
add.constraint(lpstock2,30,">=",100000,indices = 7)
add.constraint(lpstock2,25,">=",100000,indices = 8)
add.constraint(lpstock2,c(40,50,80),"<=",1000000,indices = c(1,2,3))
add.constraint(lpstock2,c(60,45,60),"<=",1000000,indices = c(4,5,6))
add.constraint(lpstock2,c(30,25),"<=",1000000,indices = c(7,8))

solve(lpstock2)

## [1] 0

get.objective(lpstock2)

## [1] 487152.8

get.variables(lpstock2)

## [1] 2500.000 6000.000 1250.000 1666.667 2222.222 13333.333 30000.000
## [8] 4000.000

get.constraints(lpstock2)

## [1] 2500000 2500000 6000000 1250000 1666667 2222222 13333333
30000000
## [9] 4000000 100000 300000 100000 100000 100000 800000

```


900000
[17] 100000 500000 1000000 1000000

- 2) Compare the solution in which there is no integer restriction on the number of shares invested. By how much (in percentage terms) do the integer restrictions alter the value of the optimal objective function? By how much (in percentage terms) do they alter the optimal investment quantities?

The return on investment is 487152.8 The number of stocks to buy are $S1 = 2500$, $S2 = 6000$, $S3 = 1250$, $H1 = 1666.667$, $H2 = 2222.22$, $H3 = 13333.3$, $C1 = 30000$, $C2 = 4000$ The amounts to be invested are 100000, 300000, 100000, 100000, 100000, 800000, 900000, 100000 respectively.

There is a difference of \$7.6 between the lp problems with and without integer restrictions. The value of the objective function differs by 0.00156% The investment quantities are altered as follows: $S1 = 0\%$, $S2 = 0\%$, $S3 = 0\%$, $H1 = 0.03996\%$ increased, $H2 = 0.03501\%$ decreased, $H3 = 75.01\%$ increased and $C1 = C2 = 0\%$