Chapter 4 Part 3

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Using Deviances to Compare Models

- A **deviance** is a way in which to measure how the observed data deviates from the model predictions; it will be defined more later in this chapter, but it is similar to sum of squared errors (unexplained variability in the response) in LLSR regression.
- Because we want models that minimize deviance, we calculate the **drop-in-deviance** when adding age to the model with no covariates (the **null model**).
- The deviances for the null model and the model with age can be found in the model output.
- Lets go indentify this in our output.

Using Deviances to Compare Models

- A residual deviance for the model with age is reported as 2337.1 with 1498 df.
- The output also includes the deviance and degrees of freedom for the null model (2362.5 with 1499 df).
- The drop-in-deviance is 25.4 (2362.5 - 2337.1) with a difference of only 1 df, so that the addition of one extra term (age) reduced unexplained variability by 25.4.

Using Deviances to Compare Models

- If the null model were true, we would expect the drop-in-deviance to follow a χ^2 distribution with 1 df. - The p-value for comparing the null model to the model with age is found by determining the probability that the value for a χ^2 random variable with one degree of freedom exceeds 25.4, which is essentially 0.
- Once again, we can conclude that we have statistically significant evidence ($\chi^2_{\text{df}=1} = 25.4$, $p < .001$) that average household size decreases as age of the head of household increases.
- Lets conduct this test in R
- Lets use the χ^2 function in R to get this p value

Formalizing this Test

- We are testing:

$$\text{Null (reduced) Model} : \log(\lambda) = \beta_0 \text{ or } \beta_1 = 0$$
$$\text{Larger (full) Model} : \log(\lambda) = \beta_0 + \beta_1 \text{age or } \beta_1 \neq 0$$

Nested Models

- In order to use the drop-in-deviance test, the models being compared must be **nested**
  - All the terms in the smaller model must appear in the larger model.
- Here the smaller model is the null model with the single term β_0 and the larger model has β_0 and β_1, so the two models are indeed nested.
- For nested models, we can compare the models' residual deviances to determine whether the larger model provides a significant improvement.

Comparing Two Approaches

Drop-in-deviance test to compare models

- Compute the deviance for each model, then calculate: drop-in-deviance = residual deviance for reduced model – residual deviance for the larger model.
- When the reduced model is true, the drop-in-deviance $\sim \chi^2_d$ where d= the difference in the degrees of freedom associated with the two models (that is, the difference in the number of terms/coefficients).
- A large drop-in-deviance favors the larger model.

Wald test for a single coefficient

- Wald-type statistic = estimated coefficient / standard error
- When the true coefficient is 0, for sufficiently large n, the test statistic $\sim$ N(0,1).
- If the magnitude of the test statistic is large, there is evidence that the true coefficient is not 0.

Comparing Methods

- The drop-in-deviance and the Wald-type tests usually provide consistent results
  - However, if there is a discrepancy, the drop-in-deviance is preferred.
- Not only does the drop-in-deviance test perform better in more cases, but it's also more flexible.
- If two models differ by one term, then the drop-in-deviance test essentially tests if a single coefficient is 0 like the Wald test does, while if two models differ by more than one term, the Wald test is no longer appropriate.

Second Order Model

- The Wald-type test and drop-in-deviance test both suggest that a linear term in age is useful.
- Our exploratory data analysis suggested that a quadratic model might be more appropriate.
- A quadratic model would allow us to see if there exists an age where the number in the house is, on average, a maximum.
- Lets fit this in R

Assesing the New Model

- We can assess the importance of the quadratic term in two ways.
  - First, the p-value for the Wald-type statistic for age^2 is statistically significant ($Z = -11.058$, $p < 0.001$).
  - Another approach is to perform a drop-in-deviance test.
- Lets do both in R

The Test

H_0: $\log(\lambda) = \beta_0 + \beta_1 \text{age}$ (reduced model)

H_A : $\log(\lambda) = \beta_0 + \beta_1 \text{age} + \beta_2 \text{age}^2$ (larger model)

The Test Results

- The first order model has a residual deviance of 2337.1 with 1498 df and the second order model, the quadratic model, has a residual deviance of 2200.9 with 1497 df.
- The drop-in-deviance by adding the quadratic term to the linear model is 2337.1 - 2200.9 $=$ 136.2 which can be compared to a χ^2 distribution with one degree of freedom.
- The p-value is essentially 0, so the observed drop of 136.2 again provides significant support for including the quadratic term.

Fitted Model

- We now have an equation in age which yields the estimated log(mean number in the house).

$$\log(\text{mean numHouse}) = -0.333 + 0.071\text{age} - 0.00071\text{age}^2$$

- These are found using the method of maximum likelihood estimation
- Lets try this in R