EXAMPLE 14–13 Hydroturbine Design

A retrofit **Francis radial-flow** hydroturbine is being designed to replace an old turbine in a hydroelectric dam. The new turbine must meet the following design restrictions in order to properly couple with the existing setup:

$r_2 := 2.50\ m$ $\quad r_1 := 1.77\ m$ $\quad \omega := 120\ rpm$

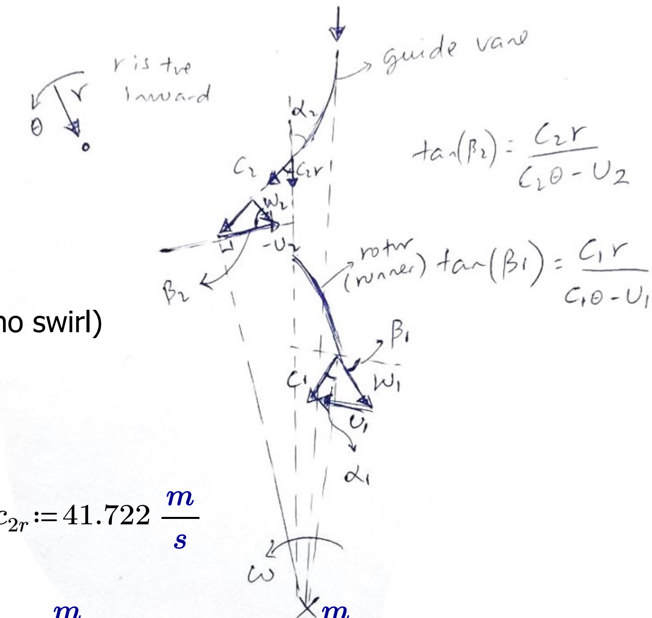
$b_2 := 0.914\ m$ $\quad b_1 := 2.62\ m$ $\quad \alpha_2 := 33\ deg$ from the radial

$Q := 599\ \dfrac{m^3}{s}$ $\quad H_{gross} := 92.4\ m$

and the flow at the runner outlet is to have angle $\alpha 1$ between −10° and 10° from radial

Required : $\beta_2 = ?$ $\quad \beta_1 = ?$

with case a). $\alpha 1$=10° (swirl) and case b). $\alpha 1$ = 0° (no swirl)



Also estimate the power output of the turbine

$$Q = 2\ \pi \cdot r_2 \cdot b_2 \cdot c_{2r} \xrightarrow[\ float,5\ ]{solve,c_{2r}} \frac{41.722 \cdot m}{s} = 41.722\ \frac{m}{s} \qquad c_{2r} := 41.722\ \frac{m}{s}$$

$$\tan(\alpha_2) = \frac{c_{2\theta}}{c_{2r}} \xrightarrow[\ float,5\ ]{solve,c_{2\theta}} \frac{41.722 \cdot m \cdot \tan(33.0 \cdot deg)}{s} = 27.095\ \frac{m}{s} \qquad c_{2\theta} := 27.095\ \frac{m}{s}$$

$$\beta_2 := \tan(\beta_2) = \frac{c_{2r}}{c_{2\theta} - U_2} \xrightarrow[\ solve,\beta_2,float,4\ ]{substitute,U_2 = \omega \cdot r_2} -1.0 \cdot \operatorname{atan}\left(\frac{0.1391}{rpm \cdot s - 0.09032}\right) = -84.09\ deg$$

$$2\ \pi \cdot r_2 \cdot b_2 \cdot c_{2r} = 2\ \pi \cdot r_1 \cdot b_1 \cdot c_{1r} \xrightarrow[\ float,4\ ]{solve,c_{1r}} \frac{20.56 \cdot m}{s} \qquad c_{1r} := 20.56\ \frac{m}{s}$$

case a). $\alpha 1$=10°

$$c_{1\theta} := \tan(\alpha_1) = \frac{c_{1\theta}}{c_{1r}} \xrightarrow[\ float,4\ ]{solve,c_{1\theta},substitute,\alpha_1 = 10\ deg} \frac{20.56 \cdot m \cdot \tan(10.0 \cdot deg)}{s} = 3.625\ \frac{m}{s}$$

$$\beta_1 := \tan(\beta_1) = \frac{c_{1r}}{c_{1\theta} - U_1} \xrightarrow[\ solve,\beta_1,float,4\ ]{substitute,U_1 = \omega \cdot r_1} -1.0 \cdot \operatorname{atan}\left(\frac{0.0968}{rpm \cdot s - 0.01707}\right) = -47.84\ deg$$

case b). $\alpha 1$=0° $\qquad$ clear $(\beta_1, c_{1\theta})$

$$c_{1\theta} := \tan(\alpha_1) = \frac{c_{1\theta}}{c_{1r}} \xrightarrow[\ float,4\ ]{solve,c_{1\theta},substitute,\alpha_1 = 0\ deg} 0.0 = 0$$

$$\beta_1 := \tan(\beta_1) = \frac{c_{1r}}{c_{1\theta} - U_1} \xrightarrow[\ solve,\beta_1,float,4\ ]{substitute,U_1 = \omega \cdot r_1} -1.0 \cdot \operatorname{atan}\left(\frac{0.0968}{rpm \cdot s}\right) = -42.749\ deg$$

To estimate the output power...

take $\rho := 998\ \frac{kg}{m^3}$

case b). α1=0° (no swirl)

$$W'_{shaft} := \rho \cdot Q \cdot \left(U_2 \cdot c_{2\theta} - U_1 \cdot c_{1\theta}\right) \xrightarrow[]{\substack{substitute, U_2 = \omega \cdot r_2 \\ substitute, U_1 = \omega \cdot r_1}} \frac{4859233557.0 \cdot kg \cdot m^2 \cdot rpm}{s^2} = 508.858\ MW$$

Efficiency is then...

$$\eta := \frac{W'_{shaft}}{W'_{hydraulic}} \xrightarrow[]{\substack{substitute, W'_{hydraulic} = \rho \cdot Q \cdot g \cdot H_{gross} \\ float, 3}} \frac{0.00000921 \cdot MW \cdot s}{g \cdot kg \cdot m} = 0.939$$

case a). α1=10° $\quad \beta_1 := -47.84\ deg \quad c_{1\theta} := 3.625\ \frac{m}{s}$

$$W'_{shaft} := \rho \cdot Q \cdot \left(U_2 \cdot c_{2\theta} - U_1 \cdot c_{1\theta}\right) \xrightarrow[]{\substack{substitute, U_2 = \omega \cdot r_2 \\ substitute, U_1 = \omega \cdot r_1}} \frac{4398955907.1 \cdot kg \cdot m^2 \cdot rpm}{s^2} = 460.658\ MW$$

Efficiency is then...

$$\eta := \frac{W'_{shaft}}{W'_{hydraulic}} \xrightarrow[]{\substack{substitute, W'_{hydraulic} = \rho \cdot Q \cdot g \cdot H_{gross} \\ float, 3}} \frac{0.00000834 \cdot MW \cdot s}{g \cdot kg \cdot m} = 0.85$$

Notice that the efficiency dropped when there is swirl in the outlet of the turbine.