Numerical Assignment #1

1. Equation for the deflection curve of a beam is given as:

$y = \frac{w_0}{120\ E \cdot I \cdot L}\left(-x^5 + 2\ L^2\ x^3 - L^4\ x\right)$. Use the bisection method to determine the point of maximum deflection.

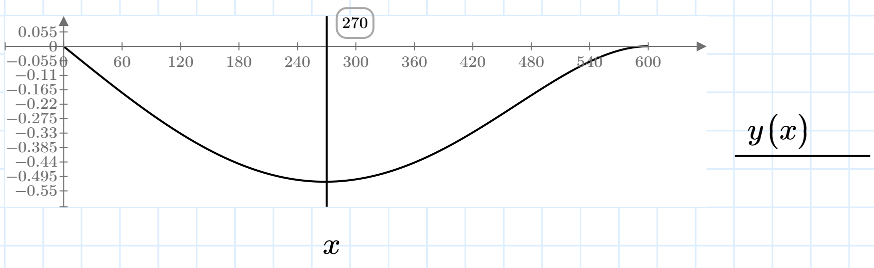
$$y = \frac{w_0}{120\ E \cdot I \cdot L}\left(-x^5 + 2\ L^2\ x^3 - L^4\ x\right) \xrightarrow[simplify]{substitute,\ L=600\ cm,\ E=50000\ \frac{kN}{cm^2},\ I=30000\ cm^4,\ w_0= \quad float,4}$$

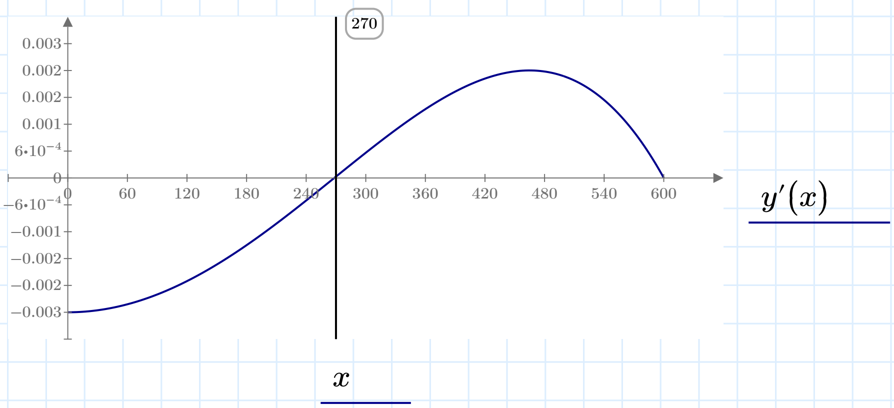
$$y(x) := \frac{-0.2315 \cdot 10^{-13} \cdot x^5}{cm^4} + \left(\frac{0.1667 \cdot 10^{-7} \cdot x^3}{cm^2} - 0.003 \cdot x\right)$$ where x is in cm too...

$$y(x) := -0.2315 \cdot 10^{-13} \cdot x^5 + \left(0.1667 \cdot 10^{-7} \cdot x^3 - 0.003 \cdot x\right)$$

$$y'(x) \rightarrow -\left(0.11575 \cdot 10^{-12}\right) \cdot x^4 + 0.5001 \cdot 10^{-7} \cdot x^2 - 0.003$$

$$x := 0..600$$





So, to solve y'(x)=0 numerically, guess two points where y'(a)*y'(b)<0

let ... $a := 100$ $b := 400$ check if above condition is true for a and b...

$$y'(a) \cdot y'(b) = -5.119 \cdot 10^{-6}$$

Iteration #1). $c1 := \dfrac{a+b}{2} = 250$

condition: if y'(a)*y'(c1)<0, update bounds from (a,b) to (a,c1)
else if, y'(a)*y'(c1)>0, update bounds from (a,b) to (c1, b)

$y'(a) \cdot y'(c1) = 8.201 \cdot 10^{-7}$ so, update bounds from (a,b) to (c1, b)

Iteration #2). $c2 := \dfrac{c1+b}{2} = 325$ Relative error: $\left|\dfrac{c2-c1}{c2}\right| \cdot 100 = 23.077$

$y'(c1) \cdot y'(c2) = -3.236 \cdot 10^{-7}$ so, update bounds from (c1, b) to (c1, c2)

Iteration #3). $c3 := \dfrac{c1+c2}{2} = 287.5$ Relative error: $\left|\dfrac{c3-c2}{c3}\right| \cdot 100 = 13.043$

$y'(c1) \cdot y'(c3) = -1.119 \cdot 10^{-7}$ so, update bounds from (c1, c2) to (c1, c3)

Iteration #4). $c4 := \dfrac{c1+c3}{2} = 268.75$ Relative error: $\left|\dfrac{c4-c3}{c4}\right| \cdot 100 = 6.977$

$$root = c4 \xrightarrow{float, 4} root = 268.8$$

So, approximate root for the equation $y'(x) = 0$ must be 268.75 with an approximate error of 6.977%

Finally, lets compare against the real answer...

$$\text{clear}(x) \quad y'(x) = 0 \xrightarrow[\text{}]{\substack{solve, x \\ float, 4 \\ assume, x>0}} \begin{bmatrix} 268.3 \\ 600.1 \end{bmatrix}$$

* 600 is a trivial answer because the beam is fixed support there, and so the maximum deflection is at x=268.3 whereas our approximation was x=268.8, pretty close!

$\text{clear}(y)$ $\text{clear}(a, b$

2. $\dfrac{2\ k2 \cdot d^{5 \div 2}}{5} + \dfrac{1}{2}\ k1 \cdot d^2 - m \cdot g \cdot (d-h) = 0$, solve for d given the rest using the secant method...

$$substitute, k1 = 40000\ \frac{g}{s^2}$$

$$substitute, k2 = 40\ \frac{g}{s^2 \cdot m^{0.5}}$$

$$substitute, m = 95\ g$$

$$substitute, g = 9.81\ \frac{m}{s^2}$$

$$\frac{2\ k2 \cdot d^{5 \div 2}}{5} + \frac{1}{2}\ k1 \cdot d^2 - m \cdot g \cdot (d-h) = 0 \xrightarrow{substitute,\ h = 0.43\ m} \frac{16.0 \cdot g \cdot d^{2.5}}{s^2 \cdot m^{0.5}} + \frac{400.7385 \cdot g \cdot m^2}{s^2} +$$

So, if we assume that d is in meters and f(d) is in millijoules, we can drop the rest of the units...

$$f(d) := 16.0 \cdot d^{2.5} + 20000.0 \cdot d^2 - 931.95 \cdot d + 400.7385$$

To solve for the root (f(d)=0) using the secant method, I need to guess two points

$f(-2) = 8.226 \cdot 10^4 + 90.51\mathrm{i}$ hmm... looks like f(d) is not real for d<0

let $x0 := 1$ $x1 := 2$

$x2 := x1 - f(x1) \cdot \dfrac{x1 - x0}{f(x1) - f(x0)} = 0.671$ Error: $\left|\dfrac{x2 - x1}{x2}\right| \cdot 100 = 198.265$

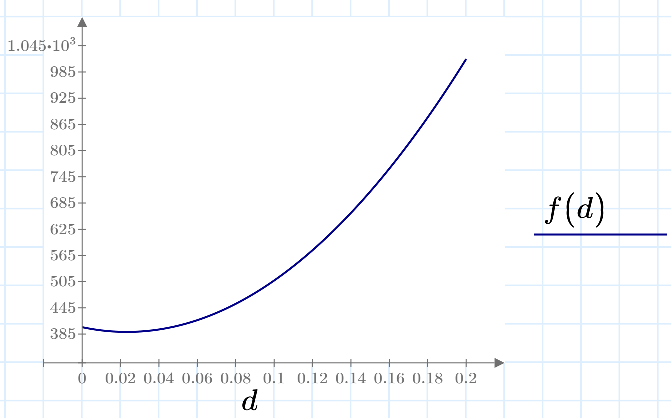
$x3 := x2 - f(x2) \cdot \dfrac{x2 - x1}{f(x2) - f(x1)} = 0.504$ Error: $\left|\dfrac{x3 - x2}{x3}\right| \cdot 100 = 33.163$

$x4 := x3 - f(x3) \cdot \dfrac{x3 - x2}{f(x3) - f(x2)} = 0.282$ Error: $\left|\dfrac{x4 - x3}{x4}\right| \cdot 100 = 78.722$

$x5 := x4 - f(x4) \cdot \dfrac{x4 - x3}{f(x4) - f(x3)} = 0.165$ Error: $\left|\dfrac{x5 - x4}{x5}\right| \cdot 100 = 70.79$

$x6 := x5 - f(x5) \cdot \dfrac{x5 - x4}{f(x5) - f(x4)} = 0.066$ Error: $\left|\dfrac{x6 - x5}{x6}\right| \cdot 100 = 149.507$

Looks like it's not converging, and if we plot f(d), we see that it doesn't even have any real roots!



$$f(d) = 0 \xrightarrow[float,\ 4]{solve,\ d} \begin{bmatrix} 0.02331 + 0.1396\mathrm{i} \\ 0.02331 - 0.1396\mathrm{i} \end{bmatrix}$$

It only has complex roots!

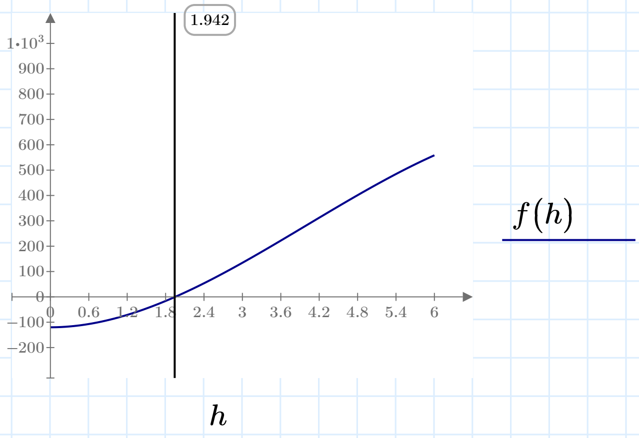
clear $(f, x1, x0, x3, x4, x2)$

3. $V = \pi \cdot h^2 \cdot \frac{(3\ R - h)}{3}$ Given: R=4m and V=40m^3, find h using the Regula Falsi method (Initial guess are 0 and R)

$$V = \pi \cdot h^2 \cdot \frac{(3\ R - h)}{3} \xrightarrow[\text{substitute}, V = 40\ m^3]{\text{substitute}, R = 4\ m} 40 \cdot m^3 = \frac{h^2 \cdot \pi \cdot (12 \cdot m - h)}{3}$$

Assuming h is in meters and dropping units...

$$40 = \frac{h^2 \cdot \pi \cdot (12 - h)}{3} \quad >> \quad f(h) := h^2 \cdot \pi \cdot (12 - h) - 120$$



$let \quad a_0 := 0 \quad b_0 := 4$

The regula-falsi equation: $c(k) = \frac{a_k \cdot f(b_k) - b_k \cdot f(a_k)}{f(b_k) - f(a_k)}$ Error: $\left| \frac{c_k - c_{k-1}}{c_k} \right| \cdot 100$

#1 $k := 0$

$$c(k) = \frac{a_k \cdot f(b_k) - b_k \cdot f(a_k)}{f(b_k) - f(a_k)} \xrightarrow{\text{explicit}, k, a, b} c(0) = \frac{[0]_0 \cdot f([4]_0) - [4]_0 \cdot f([0]_0)}{f([4]_0) - f([0]_0)}$$

$$c_0 := \frac{[0]_0 \cdot f([4]_0) - [4]_0 \cdot f([0]_0)}{f([4]_0) - f([0]_0)} \rightarrow \frac{15}{4 \cdot \pi} \qquad f(a_0) \cdot f(c_0) = 8.595 \cdot 10^3$$

$$a_1 := c_0 \qquad b_1 := b_0$$

since $f(a_0) \cdot f(c_0) > 0$ means the root is NOT between a_0 and b_0 , update the bounds to (c_0, b_0)

#2 $k := 1$

$$c_1 := \frac{a_k \cdot f\left(b_k\right) - b_k \cdot f\left(a_k\right)}{f\left(b_k\right) - f\left(a_k\right)} \rightarrow \frac{-\left(\pi \cdot \frac{\frac{3375}{4 \cdot \pi} - 2700}{4 \cdot \pi}\right) + (450 - 960 \cdot \pi)}{\pi \cdot \left(\frac{-\frac{3375}{4 \cdot \pi} + 2700}{16 \cdot \pi} - 128 \cdot \pi\right)} = 1.762$$

$f\left(a_1\right) f\left(c_1\right) = 1.444 \cdot 10^3$

Error: $\left|\frac{c_1 - c_0}{c_1}\right| \cdot 100 = 32.251$

$a_2 := c_1$ $b_2 := b_1$

#3 $k := 2$

$$c_2 := \frac{a_k \cdot f\left(b_k\right) - b_k \cdot f\left(a_k\right)}{f\left(b_k\right) - f\left(a_k\right)} = 1.911$$

$f\left(a_2\right) \cdot f\left(c_2\right) = 85.401$

$a_3 := c_2$ $b_3 := b_2$

Error: $\left|\frac{c_2 - c_1}{c_2}\right| \cdot 100 = 7.808$

#4 $k := 3$

$$c_3 := \frac{a_k \cdot f\left(b_k\right) - b_k \cdot f\left(a_k\right)}{f\left(b_k\right) - f\left(a_k\right)} = 1.942$$

Error: $\left|\frac{c_3 - c_2}{c_3}\right| \cdot 100 = 1.592$

So, final root= 1.942 and it's indeed the point where the graph crosses the x-axis from the previous plot.