5. A centrifugal pump is used to raise water against a static head of 18.0 m. The suction and delivery pipes, both 0.15 m in diameter, have, respectively, friction head losses amounting to 2.25 and 7.5 times the dynamic head. The impeller, which rotates at 1450 rev/min, is 0.25 m diameter with eight vanes, radius ratio 0.45, inclined backwards at β'2= 60°. The axial width of the impeller is designed to give constant radial velocity at all radii and is 20 mm at the impeller exit. Assuming a hydraulic efficiency of 0.82 and an overall efficiency of 0.72, determine.

i. the volume flow rate;
ii. the slip factor using Busemann's method;
iii. the impeller vane inlet angle required for zero incidence angle;
iv. the power required to drive the pump.

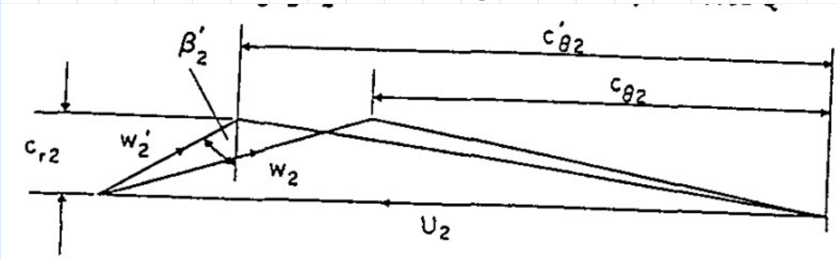
$b_2 := 20\ mm$ $\quad D_2 := 0.25\ m$ $\quad \omega := 1450\ rpm$ $\quad H_S := 18\ m$

$\eta_h := 0.82$ $\quad \beta_2' := 60\ deg$ $\quad d := 0.15\ m$ $\quad H_L(c) := (2.25 + 7.5)\dfrac{c^2}{2\ g} \rightarrow \dfrac{4.875 \cdot c^2}{g}$

i. To calculate the volume flow rate... $\quad \rho := 998\ \dfrac{kg}{m^3}$

$$c := \frac{4\ Q}{\pi \cdot d^2} \xrightarrow{float,4} \frac{56.59 \cdot Q}{m^2} \qquad H := H_S + H_L(c) + \frac{c^2}{2\ g} \xrightarrow{float,4} \frac{17210.0 \cdot Q^2}{g \cdot m^4} + 18.0 \cdot m$$

Velocity Triangle>>



$$U_2 := \omega \cdot \frac{D_2}{2} = 18.98\ \frac{m}{s}$$

$$substitute, H_i = \frac{U_2 \cdot c_{\theta 2}}{g}$$

$$\eta_h = \frac{H}{H_i} \xrightarrow{float,4} 0.82 = \frac{0.9483 \cdot g \cdot s}{c_{\theta 2}} + \frac{906.7 \cdot Q^2 \cdot s}{c_{\theta 2} \cdot m^5}$$

$$\phi_2(Q) := \frac{Q}{\pi \cdot D_2 \cdot b_2 \cdot U_2} \xrightarrow{float,4} \frac{0.003354 \cdot Q \cdot s}{m^2 \cdot mm}$$

$$\eta_h = \frac{g \cdot H}{U_2^{\ 2}\ (0.818 - 1.732\ \phi_2)} \xrightarrow{float,4} 0.82 = \frac{0.002776 \cdot g \cdot s^2 \cdot \left(\dfrac{17210.0 \cdot Q^2}{g \cdot m^4} + 18.0 \cdot m\right)}{m^2 \cdot \left(-1.0 \cdot \dfrac{0.005809 \cdot s}{m^2 \cdot mm} + 0.818\right)}$$

Guess Values

$$Q := 0.1\ \frac{m^3}{s}$$

Constraints

$$0.82 = \frac{0.002776 \cdot g \cdot s^2 \cdot \left(\dfrac{17210.0 \cdot Q^2}{g \cdot m^4} + 18.0 \cdot m\right)}{m^2 \cdot \left(-1.0 \cdot \dfrac{0.005809 \cdot Q \cdot s}{m^2 \cdot mm} + 0.818\right)}$$

Solver

$$\text{Find}(Q) = 0.029\ \frac{m^3}{s}$$

So... $Q := 0.029\ \dfrac{m^3}{s}$

$\phi_2 := \phi_2(Q)$

ii. The Busemann slip factor is then found by : (*Equation from Dixon...)

$$\sigma_B := \frac{A - \phi_2 \cdot \tan\left(\beta_2'\right)}{1 - \phi_2 \cdot \tan\left(\beta_2'\right)} \xrightarrow[float\,,4]{substitute\,, A = 0.818} \frac{1871.0 \cdot mm}{m \cdot \tan\left(60.0 \cdot deg\right) - 10280.0 \cdot mm} + 1.0 = 0.781$$

iii. The impeller vane inlet angle, β_1' for zero flow incidence is obtained from...

$$cot\left(\beta_1'\right) = \frac{c_{r1}}{U_1} = \frac{c_{r2}}{U_2}\left(\frac{r_2}{r_1}\right) = \phi_2 \cdot \frac{r_2}{r_1}$$

$$\cot\left(\beta_1'\right) = \phi_2 \cdot \left(\frac{r_2}{r_1}\right) \xrightarrow[float\,,4]{substitute\,, r_1 = 0.45\ r_2} \cot\left(\beta_1'\right) = \frac{0.0002161 \cdot m}{mm}$$

$$\cot\left(\beta_1'\right) = \frac{0.000216147 \cdot m}{mm} \xrightarrow{solve\,, \beta_1'} \operatorname{acot}\left(\frac{0.000216147 \cdot m}{mm}\right) = 77.803\ deg$$

So, $\beta_1' := 77.803\ deg$

iv. The power required is then...

$$W'_P := \frac{\rho \cdot Q \cdot g \cdot H}{\eta_h} \xrightarrow{float\,,4} \frac{35.3 \cdot g \cdot kg \cdot \left(\frac{17210.0 \cdot Q^2}{g \cdot m^4} + 18.0 \cdot m\right)}{s} = 6.742\ kW$$

So, $W'_P := 6.742\ kW$