2. Design an input solid shaft for lathe machine transmission that has three spur gears and bearings at the two ends. Take D/d=1.25, r/d=0.02, and assume design factor to be 2. The transmitted torque is assumed to be 250N.m. The pitch diameters for the gears: d1=d3=200mm and d2=150mm. Specify a shaft material and take any assumptions. Justify your solution using ANSYS software.

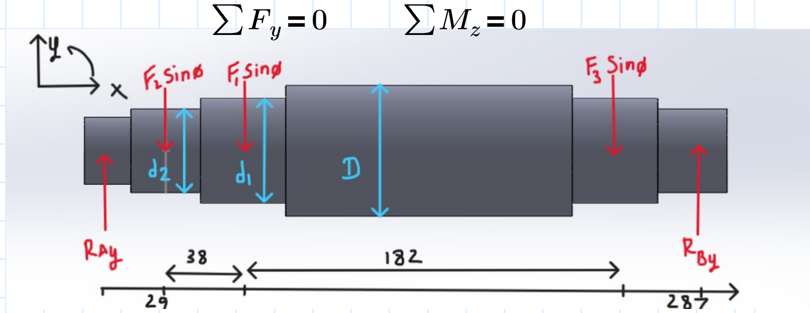
Given: $n_{fs} := 2$ $\quad T := 250\ N \cdot m$ $\quad d_1 := 200\ mm$ $\quad d_2 := 150\ mm$ $\quad d_3 := d_1 = 200\ mm$

Assuming a pressure angle of 20 degree for all 3 gears... $\phi := 20\ deg$

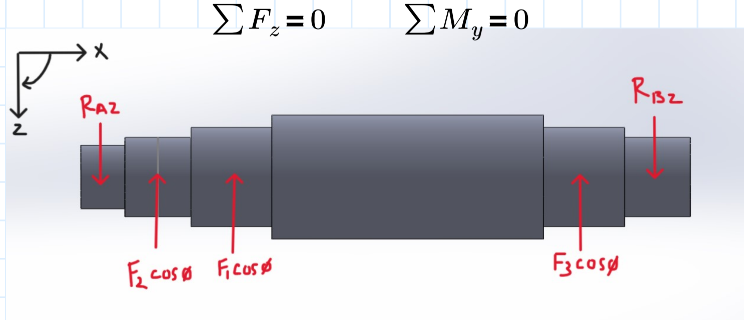
Assume the shaft is along the x-axis, and that the radial (center to center line of the gears) is along the y-axis.

## Calculating F1, F2 and F3

$\sum F_y = 0 \qquad \sum M_z = 0$




$\sum F_z = 0 \qquad \sum M_y = 0$




Guess Values

$F_1 := 1\ N \quad F_2 := 1\ N \quad F_3 := 1\ N$

Constraints

$$F_1 \cdot \cos(\phi) \cdot \frac{d_1}{2} = T \qquad F_2 \cdot \cos(\phi) \cdot \frac{d_2}{2} = T$$

$$F_3 \cdot \cos(\phi) \cdot \frac{d_3}{2} = T$$

Solver

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} := \text{Find}(F_1, F_2, F_3) = \begin{bmatrix} 2.66 \cdot 10^3 \\ 3.547 \cdot 10^3 \\ 2.66 \cdot 10^3 \end{bmatrix} N$$

## Calculating the reactions using equilibrium equations

Guess Values

$R_{Ay} := 1\ N \quad R_{By} := 1\ N \quad R_{Bz} := 1\ N \quad R_{Az} := 1\ N$

Constraints

$$R_{Ay} + R_{By} - (F_2 + F_3 + F_1)\ \sin(\phi) = 0$$

$$(R_{By} \cdot 287 - \sin(\phi)\ (F_2 \cdot 29 + F_1 \cdot (29 + 38) + F_3 \cdot 182))\ mm = 0$$

$$R_{Az} + R_{Bz} - \cos(\phi)\ (F_2 + F_1 + F_3) = 0$$

$$(R_{Bz} \cdot 287 - \cos(\phi)\ (F_2 \cdot 29 + F_1 \cdot (29 + 38) + F_3 \cdot 182))\ mm = 0$$

Solver

$$\begin{bmatrix} R_{Ay} \\ R_{By} \\ R_{Az} \\ R_{Bz} \end{bmatrix} := \text{Find}(R_{Ay}, R_{By}, R_{Az}, R_{Bz}) = \begin{bmatrix} 2.121 \cdot 10^3 \\ 912.039 \\ 5.828 \cdot 10^3 \\ 2.506 \cdot 10^3 \end{bmatrix} N$$

We aren't given the distance between the left-most bearing and Gear 2 so I assumed it's 29 mm after measuring from the shaft layout.

Next, draw shear and moment diagram for both x-y and x-z planes...

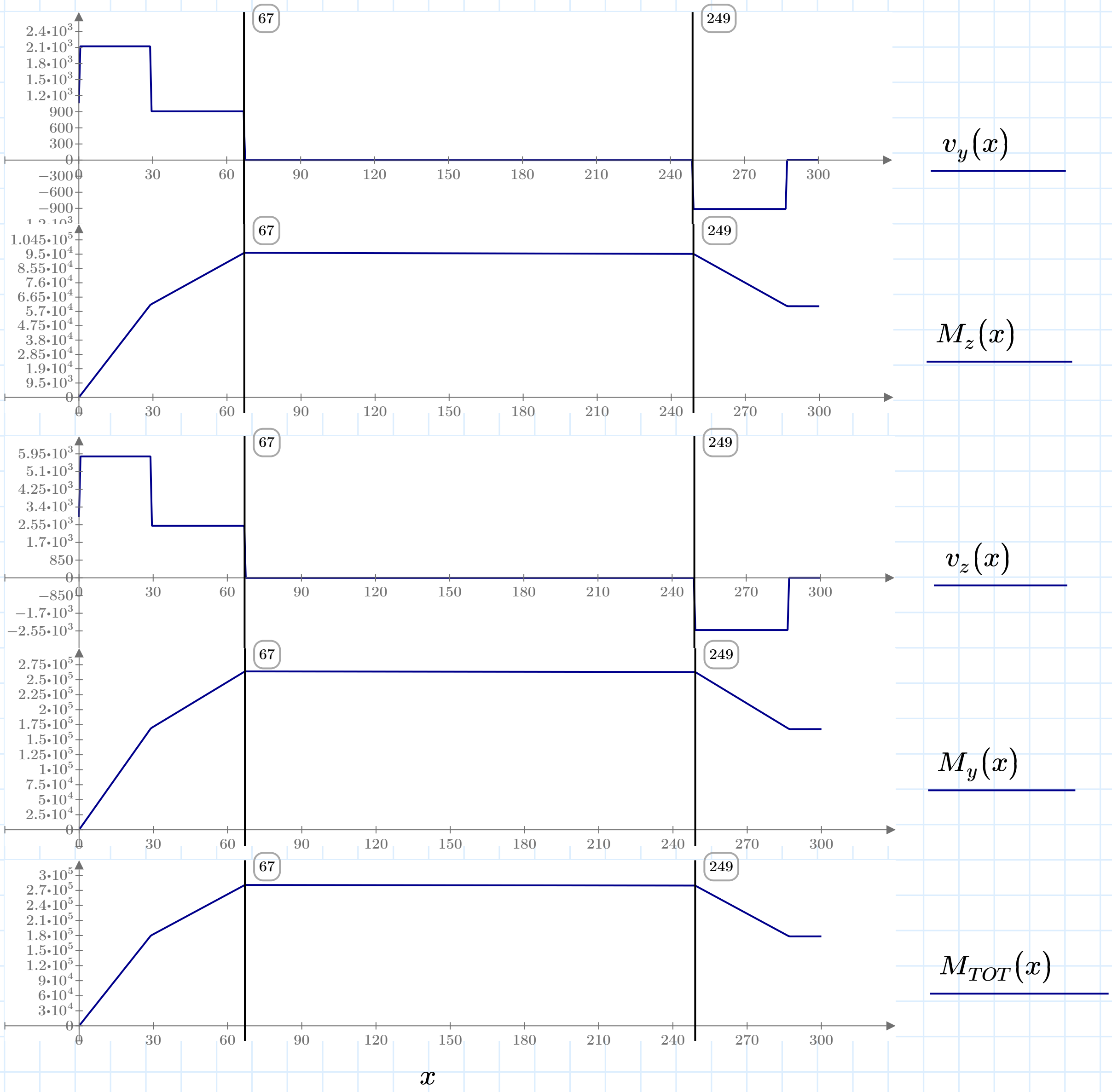
$$v_y(x) := \left( R_{Ay} \cdot \frac{1}{\boldsymbol{N}} \cdot \Phi(x) + \sin(\phi) \cdot \frac{1}{\boldsymbol{N}} \cdot \left( -F_2 \cdot \Phi(x-29) - F_1 \cdot \Phi(x-67) - F_3 \cdot \Phi(x-249) \right) + R_{By} \cdot \frac{1}{\boldsymbol{N}} \cdot \Phi(x-287) \right)$$

$$M_z(x) := \int 912.0 \cdot \Phi(x-287.0) + 2120.0 \cdot \Phi(x) - \sin(20.0 \cdot \boldsymbol{deg}) \cdot (3550.0 \cdot \Phi(x-29.0) + 2660.0 \cdot \Phi(x-67.0) + 2660.0 \cdot \Phi(x-249.0))\, \mathrm{d}x \rightarrow -$$

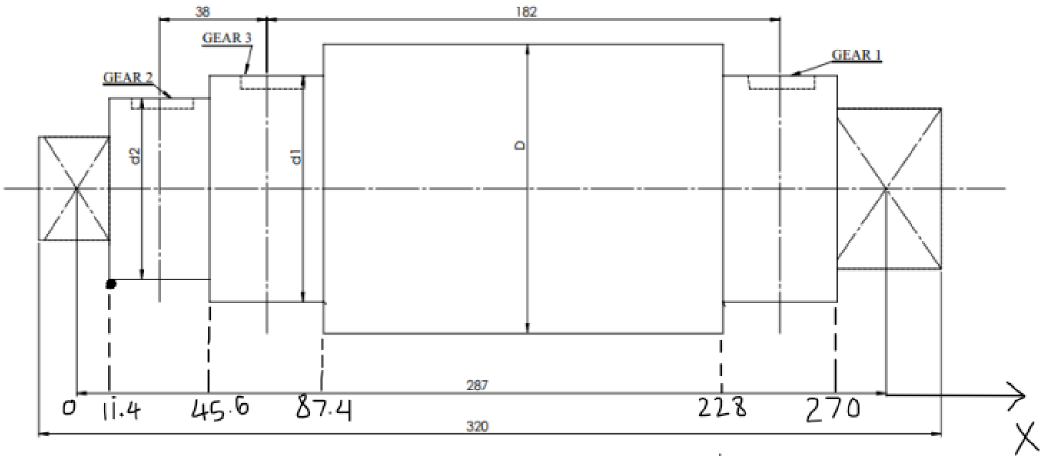
$$v_z(x) := R_{Az} \cdot \frac{1}{\boldsymbol{N}} \cdot \Phi(x) + \cos(\phi) \cdot \frac{1}{\boldsymbol{N}} \cdot \left( -F_2 \cdot \Phi(x-29) - F_1 \cdot \Phi(x-67) - F_3 \cdot \Phi(x-249) \right) + R_{Bz} \cdot \frac{1}{\boldsymbol{N}} \cdot \Phi(x-287)$$

$$M_y(x) := \int 2510.0 \cdot \Phi(x-287.0) + 5830.0 \cdot \Phi(x) - \cos(20.0 \cdot \boldsymbol{deg}) \cdot (3550.0 \cdot \Phi(x-29.0) + 2660.0 \cdot \Phi(x-67.0) + 2660.0 \cdot \Phi(x-249.0))\, \mathrm{d}x \rightarrow$$

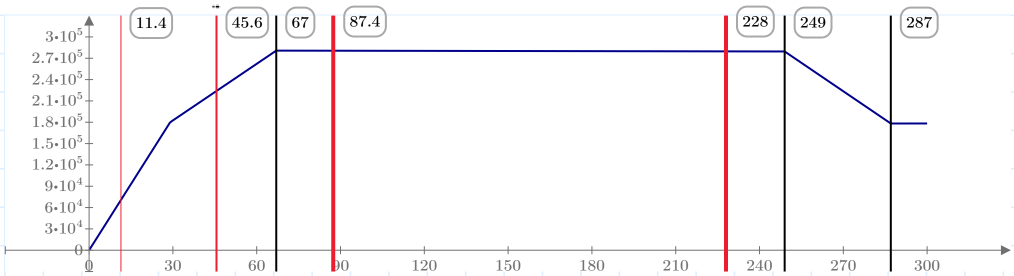
$M_{TOT}(x) := \sqrt{M_z(x)^2 + M_y(x)^2}$ (Because the x-y and x-z planes are orthogonal, we treat the total moment as a resultant vector.)




So the highest moment is from x= 67 to 249. $M_{max}$ = $M_{TOT}(67) = 2.807 \cdot 10^5$ in Newton-mm.




Since we aren't given complete geometric information, the remaining lengths were determined by proportion from the given ones.




$M_{TOT}(x)$ in Newton-mm

$x$

The red lines indicate where we have stress concentration due to shoulders. Their thickness is an indication of where the stress might be the highest (the closer they are to the center, the higher the moment and the stress). Since we have many candidates for the critical point, we start with the most likely, the shoulders where we have the highest moment ( @87.4 and @228 measured from the left bearing). Because the moment is the same at both shoulders and each shoulder has the same bigger and smaller diameter (D and d1) we only have to compute d1 once. Then, we consider the shoulder @45.6. The critical points are labelled 1,2,3... from left to right.

Lets define I and J as functions of d to simplify calculations... $I(d) := \dfrac{\pi \cdot d^4}{64}$ $\quad J(d) := \dfrac{\pi \cdot d^4}{32}$

*Assuming D/d=1.25 is for all the shaft sections, and assuming d2<d1: Elements immediately left and right of the shoulder @ location 2 have the same moment, we know the stress will be more on the left (smaller diameter) side so we analyze there only. Assume generous fillet radius for all. From Table 7–1, estimate $k_t := 1.7$ , $k_{ts} := 1.5$

$M_{TOT}(45.6) = 2.239 \cdot 10^5$ in Newton-mm

$$\sigma_{2\_left} := k_t \cdot \frac{\left(2.239 \cdot 10^5\right) N \cdot mm \cdot \dfrac{1 \cdot 10^{-3}\, m}{mm} \cdot d2}{2 \cdot I(d2)} \xrightarrow{float,3} \frac{3880.0 \cdot N \cdot m}{d2^3}$$

$$\tau_{2\_left} := k_{ts} \cdot \frac{T \cdot \dfrac{1}{N \cdot m} \cdot d2}{2 \cdot J(d2)} N \cdot m \xrightarrow{float,3} \frac{1910.0 \cdot N \cdot m}{d2^3}$$

For a rotating shaft with both bending and torsional stress, von-mises stress is given as: $\sigma' = \sqrt{\sigma_b{}^2 + 3\ \tau^2}$

$$\sigma_{2\_left}' := \sqrt{\sigma_{2\_left}{}^2 + 3 \cdot \tau_{2\_left}{}^2} \xrightarrow[assume,\, d2>0]{float,3,simplify} \frac{5100.0 \cdot N \cdot m}{d2^3}$$

For location 3, there are no stress concentration factors and it has the same moment and diameter d1 as the left side of shoulder at 4, so the stress at 3 is definitely lower than that at 4 (left side) So we don't need to consider it.

Again for location 4, we only consider the left (smaller diameter) section. $M_{TOT}(87.4) = 2.806 \cdot 10^5$ in Newton-mm

$$\sigma_{4\_left} := k_t \cdot \frac{\left(2.806 \cdot 10^5\right) N \cdot mm \cdot \frac{1 \cdot 10^{-3}\, m}{mm} \cdot d1}{2 \cdot I(d1)} \xrightarrow{float, 3} \frac{4860.0 \cdot N \cdot m}{d1^3}$$

$$\tau_{4\_left} := k_{ts} \cdot \frac{T \cdot \frac{1}{N \cdot m} \cdot d1}{2 \cdot J(d1)} N \cdot m \xrightarrow{float, 3} \frac{1910.0 \cdot N \cdot m}{d1^3}$$

$$\sigma_{4\_left}' := \sqrt{\sigma_{4\_left}^{\ 2} + 3 \cdot \tau_{4\_left}^{\ 2}} \xrightarrow[assume, d1 > 0]{float, 3, simplify} \frac{5880.0 \cdot N \cdot m}{d1^3}$$

Comparing the von-mises stresses at 2 and 4... $\frac{\sigma_{2\_left}'}{\sigma_{4\_left}'} \xrightarrow{float, 3} \frac{0.867 \cdot d1^3}{d2^3}$

if this ratio is greater that unity, then the critical point is at 2, and vice versa. Let the ratio $\frac{d1}{d2} = d_{ratio}$

$\frac{\sigma_{2\_left}'}{\sigma_{4\_left}'} = 0.867 \left(d_{ratio}\right)^3$ So what is the ratio that makes the stress at 2 greater than the stress at 4?

$$0.867 \left(d_{ratio}\right)^3 > 1 \xrightarrow{solve, d_{ratio}, float, 3} d_{ratio} > 1.05$$

So if d1/d2 > 1.05, then the critical point is at 2. Since we are given a value for d1/d2 at shoulders =1.2, it means the critical point is at location 2 shoulder.

Now to get the diameter at 2, use fatigue analysis...

For quick, conservative first pass, assume , $k_f := k_t$, $k_{fs} := k_{ts}$

Choose inexpensive steel, 1020 CD, with $S_{ut} := 68\ ksi$ . For Se, Eq. (6–19) ka = a*Sut^b $k_a := 2.7\ (68)^{-0.265} = 0.883$
Guess $k_b := 0.9$ . Check later when d is known. $k_c := 1$ $k_d := 1$ $k_e := 1$

Eq. (6–18) $S_e := k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot 0.5 \cdot S_{ut} \xrightarrow{float, 4} 27.01 \cdot ksi$
For first estimate of the diameter at the shoulders at location 2, use the DE-Goodman criterion of Eq. (7–8).
This criterion is good for the initial design, since it is simple and conservative. With Mm = Ta = 0, Eq. (7–8) reduces to

$$M_{critical} := M_{TOT}(45.6) \cdot N \cdot mm = 223.949\ N \cdot m$$

$$d := \left(\frac{16 \cdot n_{fs}}{\pi} \left(\frac{2\left(k_f \cdot M_a\right)}{S_e} + \frac{\left(\left(3\left(k_{fs} \cdot T_m\right)\right)^2\right)^{0.5}}{S_{ut}}\right)\right)^{\frac{1}{3}} \xrightarrow[\substack{substitute, T_m = T \\ substitute, M_a = M_{critical}}]{float, 3} \left(\frac{0.15 \cdot \left(0.127 \cdot 10^7 \cdot N^2 \cdot m^2\right)^{0.5} + 287.0 \cdot N \cdot m}{ksi}\right)^{0.333}$$

$$d2 := \left(\frac{0.15 \cdot \left(0.127 \cdot 10^7 \cdot N^2 \cdot m^2\right)^{0.5} + 287.0 \cdot N \cdot m}{ksi}\right)^{\frac{1}{3}} = 40.442\ mm$$

Now that we have a first-pass guess for d2, correct further by plugging in the formula for $k_b$ as a function of d2

$$k_b := 1.24 \cdot \left(d2 \cdot \frac{1}{mm}\right)^{-0.107} \rightarrow \frac{1.24}{\left(\frac{d2}{mm}\right)^{0.107}}$$

...where d is in mm for 2.79 < d < 51 mm. (because we have seen that d is in that range).

$$S_e := k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot 0.5 \cdot S_{ut} \xrightarrow{float, 4} \frac{37.21 \cdot ksi}{\left(\frac{d2}{mm}\right)^{0.107}}$$

$$d2 := \left(\frac{16 \cdot n_{fs}}{\pi} \left(\frac{2\left(k_f \cdot M_a\right)}{S_e} + \frac{\left(\left(3\left(k_{fs} \cdot T_m\right)\right)^2\right)^{0.5}}{S_{ut}}\right)\right)^{\frac{1}{3}} \xrightarrow[\substack{substitute, T_m = T \\ substitute, M_a = M_{critical}}]{float, 3} \left(\frac{0.15 \cdot \left(0.127 \cdot 10^7 \cdot N^2 \cdot m^2\right)^{0.5} + 208.0 \cdot N \cdot m \cdot \left(\frac{d2}{mm}\right)^{0.107}}{ksi}\right)^{0.3}$$

Guess Values

$$d2 := 40\ mm$$

Constraints

$$d2 = \left(\frac{0.15 \cdot \left(0.127 \cdot 10^7 \cdot N^2 \cdot m^2\right)^{0.5} + 208.0 \cdot N \cdot m \cdot \left(\frac{d2}{mm}\right)^{0.107}}{ksi}\right)^{\frac{1}{3}}$$

Solver

$$d2 := \mathbf{Find}\left(d2\right) = 41.098\ mm$$

Using a solve-block to solve for d2 without having to deal with unit conversions...

So, our first-pass guess was not far off. Assuming D/d=1.25 for all shaft sections, $d1 := 1.25 \cdot d2 = 51.372\ mm$ and $D := 1.25 \cdot d1 = 64.215\ mm$

3. Since my PC couldn't load ANSYS software, I didn't do questions 2 (evaluation part) and 3. I can maybe do the simulation on Solidworks if given the chance...