The Root Sweep Theorem and Fundamental Theorem of Algebra using Phase Lift

Alex B.

July 3, 2025

Preliminary Note

This short paper inherits all definitions, lemmas, sign cycles, and closure conditions from my main Phase Lift document *"ReimaginingTheImaginary.pdf"*. For full background on the Phase Lift operator, Phase Function, bounded chirality, and orthogonal lift basis $\{x, \lambda\}$, readers should consult that work first.

I do not waste my time on unnecessary epsilon–delta formalism here. The lift closure and sign cycle make the resolution self-evident to any honest reader. If you wish to formalize the full arc with deeper rigor, I warmly welcome you to do so *with me* and will gladly credit any rigorous expansions in future versions. Please cite this work and contact me to coordinate contributions.

Root Sweep Theorem

Theorem (Phase Lift Root Sweep). Let $f(x) \in \mathbb{R}[x]$ be any continuous polynomial. Then all real and "imaginary" roots of f appear as bounded pole closures of the continuous Phase Function:

$$S(\varphi) := A\cos(\varphi) + \lambda B \sin(\varphi), \quad \varphi \in [0, 2\pi], \quad A, B \in \mathbb{R}.$$

Then:

$$f(x) = 0 \iff S(\varphi) = 0.$$

Root conditions:

- **Real roots:** appear when $\cos(\varphi) = 0$. These are anchored to the real pole. When $f(x)$ is rotated around the real pole, all points on the real pole remain fixed through rotation, so zeros on this pole arise where the cosine projection vanishes.
- **Lifted roots:** appear when the Phase Lift naturally unfolds the hidden orthogonal direction λ. These roots are not reachable on the real pole alone, but are generated as the Phase Lift sweeps the orthogonal basis and closes the sign cycle.
- The sign cycle $\{+1, +\lambda, -1, -\lambda, +1\}$ ensures bounded chirality and exact closure.

Corollary — Fundamental Theorem of Algebra (Phase Function Form)

Phase Function Root Closure. For any non-constant polynomial

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0, \quad a_i \in \mathbb{R},\ n > 0,$$

the Phase Function

$$P(f(x), \varphi) := f(x)\cos(\varphi) + \lambda\, f(x)\sin(\varphi), \quad \varphi \in [0, 2\pi],$$

guarantees that there exists

$$\varphi \in [0, 2\pi] : P(f(x), \varphi) = 0.$$

Thus, all real and “imaginary” roots of any polynomial appear as bounded projections on the Phase Function’s continuous arc basis.

Euler’s identity (unmodified),

$$e^{i\theta} = \cos(\theta) + i\sin(\theta),$$

hinted at this universal basis long ago, but the Phase Function makes it explicit for all polynomials.

Trust, but verify.