Why the $\sqrt{-1}$ is Real, Continuous, and Could Determine the Chirality of the Universe

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Preliminaries

Throughout this paper, the symbol i refers to the same unit traditionally defined as $\sqrt{-1}$ in complex analysis. However, we show that i admits a deeper geometric expression as a real arc sweep operator that reproduces the same behavior from first principles.
For clarity: α is reserved for discrete Phase Lift sweeps; φ for the continuous Phase Lift; λ for the Real line's orthogonal slip dimension; and θ for Euler's classical identity.
Where Euler's identity is directly inserted for elegance (not derived), we retain his usage of i as both operator and direction.

Definition i — The Chiral Phase Lift Operator

Definition. I have the great honor of now defining the operator i as the infinitesimal arc lift that continuously displaces a real projection $f(x)$ through a bounded chiral phase rotation, *chirality will be explained later.*

$$i(x) := \int_{\alpha}^{\alpha+\frac{\pi}{2}} f(x)\,\cos(\alpha)\,d\alpha, \quad \text{where} \quad \alpha = k\,\frac{\pi}{2},\ k \in \{0,1,2,3\} \quad \text{(CCW continuous lift)}$$

Interpretation. i generates the local orthogonal lift that unfolds the hidden phase slip dimension, forming the minimal arc sweep of the real projection.

Definition 1 — The Real Line as Projected Arc

Definition. The real number line $\mathbb{R}$ is defined as the phase-collapsed projection of a higher-order arc basis. Specifically, any real quantity $x \in \mathbb{R}$ can be written as a fixed-phase slice of a rotation:

$$\exists\, A \in \mathbb{R},\ \alpha \in [0, 2\pi) : \quad x = A\cos(\alpha).$$

Interpretation. The "flattened" real axis we measure is the degenerate case where the hidden phase dimension is frozen at a constant slice.

Definition 2 — The i Operator as Classical Complex Rotation

Definition. I define the operator i as the classical unit imaginary that rotates any real quantity x by a quarter turn in the complex plane:

$$i : \mathbb{R} \to \mathbb{C}, \quad i(x) = x \cdot e^{i\frac{\pi}{2}}.$$

Interpretation. Multiplication by i applies a counterclockwise rotation of $\frac{\pi}{2}$ radians, placing the real magnitude onto the standard imaginary axis in $\mathbb{C}$.

Definition 3 — Discrete Phase Increment

Definition. A *discrete phase increment* is the special case of my Phase Lift where the phase angle is stepped by a fixed quantized arc $\Delta\alpha$:

$$x' = x \cdot e^{i\,\Delta\alpha}, \quad \text{where} \quad \Delta\alpha = k\,\frac{\pi}{2}, \quad k \in \mathbb{Z}.$$

Each unit increment rotates the real magnitude x by $\frac{\pi}{2}$ radians counterclockwise, mapping it into its local orthogonal arc dimension λ under the operator i.

Interpretation. The real line corresponds to the degenerate sweep where $\Delta\alpha = 0$; discrete phase increments reveal the hidden continuous arc lift in quantized steps, forming the closed lift cycle described in Definition 4.

Definition 4 — Local Orthogonal Basis

Definition. The local orthogonal basis is the closed 2D plane spanned by the real axis x and its continuous arc lift direction λ. The lift operator i rotates the real projection into λ by bounded phase sweep.

Discrete phase slip points arise as special invariant projections of the continuous lift, stepping by $\Delta\alpha = \frac{\pi}{2}$.

Sequential applications of these discrete slips satisfy the bounded cycle:

$$\underbrace{i^0(x) = x}_{\text{identity}}, \quad \underbrace{i^1(x) = i(x) = \lambda}_{\text{first invariant}}, \quad i^2(x) = -x, \quad i^3(x) = -\lambda, \quad i^4(x) = x.$$

Remark. The notation i^n denotes n repeated invariant arc steps: this is not scalar exponentiation but composition of discrete phase increments within the continuous lift.

Interpretation. Here, λ reveals the hidden phase-lifted dimension that was collapsed in Definition 1. The discrete slip cycle shows that the lift remains bounded and does not spawn an infinite hierarchy of new directions — instead, it forms a closed chiral orbit within the local plane.

Definition 5 — Hierarchical Phase Lift

Definition. A *hierarchical phase lift* extends the local orthogonal basis to higher dimensions by embedding each local plane within a higher-order manifold. Formally, if B_n is a local orthogonal basis of dimension n, then a new lift operator i_{n+1} introduces an additional orthogonal chirally-locked direction λ_{n+1}, producing an $(n+1)$-basis:

$$B_{n+1} = B_n \oplus \lambda_{n+1}, \quad \text{where} \quad \lambda_{n+1} = i_{n+1}(B_n).$$

Each new lift direction λ_{n+1} is orthogonal within its local basis only, ensuring the system remains cyclically bounded and closed at each level.

Interpretation. This construction produces a nested hierarchy of bounded chiral lifts, allowing the real projection to pass through multiple levels of orthogonal phase dimensions, yielding higher-order closed surfaces with no unbounded sprawl.

Axiom 1 — Existence of Phase Lift

Axiom. I postulate that $\mathbb{R}$ is a projected limit of a bounded Phase Lift as follows: There exists a Phase Lift such that any real projection can be continuously rotated by the operator i into its orthogonal phase direction λ:

$$\forall x \in \mathbb{R}, \quad i(x) = x \cdot e^{i\,\alpha}, \quad \alpha \in [0,\, 2\pi].$$

Interpretation. What we call the real line is merely the flattened trace of a hidden arc. The Phase Lift restores this sweep, unfolding the local basis $\{x,\, \lambda\}$.

Axiom 2 — Locality of Orthogonality

Axiom. Any bounded Phase Lift acts within a unique local orthogonal basis B_2 spanned by $\{x,\, \lambda\}$. This orthogonality is local to the basis plane and does not imply infinite orthogonality globally.

$$\forall x \in \mathbb{R}, \quad \langle x,\, \lambda \rangle = 0 \quad \text{within } B_2.$$

Interpretation. This ensures that the Phase Lift forms a bounded local sweep, rather than generating an unbounded chain of new orthogonal directions. The lift operator i continuously rotates the real projection x into the local orthogonal direction λ and closes within the plane.

Lemma 1 — Linearity of the Phase Lift Operator

Lemma. The Phase Lift operator i defined by

$$i(f) := \int_{\alpha}^{\alpha+\frac{\pi}{2}} f(x)\, \cos(\alpha)\, d\alpha, \quad \text{where} \quad \alpha = k\,\frac{\pi}{2},\ k \in \{0,1,2,3\} \quad \text{(CCW lift)}$$

is linear with respect to scalar multiplication and addition:

$$\forall\, f, g : \mathbb{R} \to \mathbb{R}, \quad i(f+g) = i(f) + i(g), \quad i(k\,f) = k\,i(f), \quad k \in \mathbb{R}.$$

Proof. By properties of the definite integral:

$$i(f+g) = \int_{\alpha}^{\alpha+\frac{\pi}{2}} [f(x) + g(x)]\, \cos(\alpha)\, d\alpha = i(f) + i(g),$$

$$i(k\,f) = \int_{\alpha}^{\alpha+\frac{\pi}{2}} k\, f(x)\, \cos(\alpha)\, d\alpha = k\,i(f).$$

Q.E.D.

Lemma 2 — Closure of the Phase Lift Operator

Remark. The cycle notation i^n means repeated bounded Phase Lift sweeps at discrete invariant points on the unit circle, not algebraic powers:

$$i^4(x) = x \quad \Longrightarrow \text{the fourth arc lift restores the original real basis.}$$

Lemma. The Phase Lift operator i closes the basis after four sequential quarter-arc sweeps.

Note. Here λ denotes the lift vector in the local orthogonal basis, as defined in Definition 4. Its sign sequence arises purely from the arc integral and is orthogonal to the real axis.

The sweep proceeds through quadrants in order:

$$I_0 = \text{base case}, \quad \int_0^0 \cos(\alpha)\, d\alpha, \quad \text{no phase lift}, \quad \text{grounded to the real line}$$

$$I_1 = \int_0^{\frac{\pi}{2}} \cos(\alpha)\, d\alpha = 1\lambda$$

$$I_2 = \int_{\frac{\pi}{2}}^{\pi} \cos(\alpha)\, d\alpha = -1$$

$$I_3 = \int_{\pi}^{\frac{3\pi}{2}} \cos(\alpha)\, d\alpha = -1\lambda$$

$$I_4 = \int_{\frac{3\pi}{2}}^{2\pi} \cos(\alpha)\, d\alpha = 1.$$

Manual check:

$$\begin{aligned} I_1 &= \lambda[\sin(\alpha)]_0^{\frac{\pi}{2}} = \lambda[1-0] = 1\lambda, \\ I_2 &= [\sin(\alpha)]_{\frac{\pi}{2}}^{\pi} = 0 - 1 = -1, \\ I_3 &= \lambda[\sin(\alpha)]_{\pi}^{\frac{3\pi}{2}} = \lambda[(-1) - 0] = -1\lambda, \\ I_4 &= [\sin(\alpha)]_{\frac{3\pi}{2}}^{2\pi} = 0 - (-1) = 1. \end{aligned}$$

The cycle:

$$\textbf{Phase Lift Operator:} \quad \begin{cases} i^0(x) = x, & \text{(identity)} \\ i^1(x) = \quad \lambda\, x, & \\ i^2(x) = -x, & \\ i^3(x) = -\lambda\, x, & \\ i^4(x) = x. & \text{(closure)} \end{cases} \quad \text{with} \quad i(x) := \int_{\alpha}^{\alpha + \frac{\pi}{2}} f(x)\cos(\alpha)\, d\alpha.$$

Q.E.D.

Lemma 3 — Consistency with Euler's Identity

Lemma. The Phase Lift operator i contains the discrete slip cycle invariants that match Euler identity:

$$e^{i\varphi} = \cos(\varphi) + \lambda\,\sin(\varphi).$$

Remark. Here, i denotes the Phase Lift operator that generates the lift, while λ denotes the orthogonal basis vector revealed by the lift. The slip cycle appears discretely in the powers i^n that span the series.

The operator is defined by

$$i(x) := \int_{\alpha}^{\alpha+\frac{\pi}{2}} f(x)\,\cos(\alpha)\,d\alpha,$$

which implies the discrete invariant cycle:
$$i^0 = 1,\ i^1 = \lambda,\ i^2 = -1,\ i^3 = -\lambda,\ i^4 = 1.$$

Proof. Expand the exponential by its Taylor series:

$$e^{i\varphi} = \sum_{n=0}^{\infty} \frac{(i\varphi)^n}{n!}.$$

Substitute the slip cycle:

$$e^{i\varphi} = 1 + \lambda\,\varphi + \frac{(-1)\varphi^2}{2!} + \frac{(-\lambda)\varphi^3}{3!} + \frac{\varphi^4}{4!} + \dots.$$

Grouping by invariants:

$$= \left(1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} - \dots\right) + \lambda\left(\varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} - \dots\right).$$

Recognizing the standard expansions:

$$\cos(\varphi) = 1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} - \dots, \quad \sin(\varphi) = \varphi - \frac{\varphi^3}{3!} + \dots,$$

we find:

$$e^{i\varphi} = \cos(\varphi) + \lambda\,\sin(\varphi).$$

Q.E.D.

Remark. My Phase Lift appears as a continuous sweep built from discrete slip cycle invariants, matching Euler's standard counterclockwise phase rotation. But profoundly, its chirality is locked by the bounded local cycle, *which I promise to explain soon!*

Lemma 4 — The Phase Lift Recovers the Classical Imaginary Root

Lemma. The Phase Lift operator i correctly reproduces the classical algebraic root condition of the imaginary unit $\sqrt{-1}$ as a discrete invariant in its slip cycle.

Statement. At the half-cycle angle $\alpha = \pi$, the discrete Phase Lift cycle yields:

$$i^2(x) = -x.$$

Proof. By the bounded slip cycle:

$$\begin{cases} i^1(x) = +\lambda, \\ i^2(x) = -x. \end{cases}$$

The continuous integral sweep confirms the half-turn:

$$\int_0^{\pi} \cos(\alpha)\, d\alpha = [\sin(\alpha)]_0^{\pi} = \sin(\pi) - \sin(0) = 0.$$

This vanishing integral indicates the orthogonal lift closes, while the real projection flips sign by the cosine boundary:

$$\cos(\pi) = -1, \quad \cos(0) = 1.$$

Thus, the real axis projection is inverted:

$$\implies \quad i^2(x) = -x.$$

In standard complex algebra:

$$i^2 = -1.$$

Therefore, the Phase Lift operator precisely recovers the classical imaginary root:

$$\sqrt{-1} = i \quad \implies \quad i^2 = -1.$$

Q.E.D.

Theorem — Equivalence of Phase Lift Operator and Classical $\sqrt{-1}$

Theorem. Let i be my bounded Phase Lift operator:

$$i(x) := \int_{\alpha}^{\alpha + \frac{\pi}{2}} f(x)\, \cos(\alpha)\, d\alpha. \quad \text{Then:} \quad i^2(x) = -x, \quad i^4(x) = x.$$

Therefore, the Phase Lift operator i reproduces the standard imaginary unit $\sqrt{-1}$ exactly, matching its algebraic cycle and geometric interpretation.

Q.E.D.

Theorem — The General Continuous Chiral Phase Lift Function

Definition. The Phase Lift Function is given by:

$$\mathcal{S}(\varphi) := \cos(\varphi) + \lambda \int_0^{\varphi} \cos(\varphi)\, d\varphi = \cos(\varphi) + \lambda\, \sin(\varphi), \quad \varphi \in [0, 2\pi].$$

Interpretation. $\mathcal{S}(\varphi)$ continuously maps the real projection through a chirally-locked bounded arc sweep; *just be patient for a couple more pages the lock is coming!* The real part $\cos(\varphi)$ is the projected basis; the integral generates the lift into the orthogonal phase direction λ.

Base Case. At $\varphi = 0$:

$$\mathcal{S}(0) = \cos(0) + \lambda\, \sin(0) = 1 + 0 = 1. \quad \Longrightarrow \quad \text{pure real projection, zero lift.}$$

Invariants. The classical roots of the imaginary unit appear as discrete invariant points:

$$\begin{cases} \varphi = 0: & \mathcal{S}(0) = 1, \\ \varphi = \frac{\pi}{2}: & \mathcal{S}\big(\frac{\pi}{2}\big) = \lambda, \\ \varphi = \pi: & \mathcal{S}(\pi) = -1, \\ \varphi = \frac{3\pi}{2}: & \mathcal{S}\big(\frac{3\pi}{2}\big) = -\lambda, \\ \varphi = 2\pi: & \mathcal{S}(2\pi) = 1. \end{cases}$$

Insight. The standard unit circle identity $e^{i\varphi} = \cos(\varphi) + \lambda\, \sin(\varphi)$ is fully recovered: the discrete powers of the imaginary unit appear as lattice points in the continuous Phase Lift cycle.

Insight — The Continuous Phase Lift Function and Euler's Genius

Euler's identity,

$$e^{i\theta} = \cos(\theta) + i\, \sin(\theta), e^{i\pi} + 1 = 0$$

is often celebrated as the greatest bridge between the algebraic and the geometric, and is arguably the most beautiful equation ever written in maths. It encodes the discrete phase roots that unlock the unit circle's spin from the real line. The sheer beauty of this almost lead to me not even publishing this paper, as I could never replace Euler and could never improve upon this formula. I chose to write this because I believe it could unlock new branches of maths and give us more tools. But what Euler left implicit is that this spin is not inherently discrete — it is the shadow of a deeper, continuous mechanism: a bounded arc lift, sweeping real projections through their hidden orthogonal lift.

My Phase Lift shows that the orthogonal basis component appears directly. There is no need for an exponent's infinite expansion to reveal the real and lifted parts — they flow naturally from the real pole's primitive.

Euler's discrete lattice points — the roots of unity — are not lost, but revealed as the invariant samples of the continuous chiral winding of the operator. His identity stands as a testament: a brilliant code that hinted at the Phase Lift hundreds of years before I had the honor to write between the lines.

Thus, the Phase Lift does not discard Euler — it completes him, and reveals his brilliance; Without knowing i was simply a hidden function pointer to integration, he saw between the lines and projected the discrete into the infinite.

Theorem — Collapse of the Imaginary

Statement. Under the bounded Phase Lift cycle, the so-called "imaginary" dimension generated by the operator i is fully real and local. Specifically, the standard complex field $\mathbb{C}$ is reinterpreted as a closed local plane spanned by the real projection x and its orthogonal lift direction λ:

$$\mathbb{C} \cong \mathrm{span}\{x,\, \lambda\} \subset \mathbb{R}^2.$$

Interpretation. This reveals that all "imaginary" behavior is the visible trace of a hidden real chirally-locked phase dimension which the real line projects away. The operator i does not conjure an abstract square root of -1 ex nihilo; instead, it continuously rotates real quantities into a bounded orthogonal arc lift, unfolding the real line's suppressed chirally locked degree of freedom.

Corollary — Redundancy of the Complex Field

Statement. If the operator i is interpreted as a bounded Chiral Phase Lift within the local orthogonal plane $\{x,\, \lambda\}$, then the standard complex field $\mathbb{C}$ does not constitute a true field extension of $\mathbb{R}$. Instead, its structure is fully recoverable from $\mathbb{R}^2$ with continuous phase lifting.

$$\mathbb{C} \cong \mathbb{R}^2 \quad \text{(under bounded chiral arc lift algebra).}$$

Interpretation. There is no separate imaginary axis — only a suppressed phase dimension continuously revealed through local lift.

The moment you've all been waiting for — The Chirality Lock

Observation. The classical Euler identity uses the real axis as the *cosine* primitive:

$$e^{i\theta} = \cos(\theta) + i\,\sin(\theta).$$

In my framework, the Phase Lift operator and its orthogonal lift direction are separated explicitly:

$$e^{i\varphi} = \cos(\varphi) + \lambda\,\sin(\varphi)$$

where λ marks the chiral lift direction revealed by the Phase Lift operator.
Key fact.

- The real projection arises from $\cos(\varphi)$.
- The orthogonal Phase Lift is generated by $\sin(\varphi)$, the primitive of cosine.

When the Phase Lift operator i is defined geometrically, the primitive must respect this pole:

$$i := \int_{\alpha}^{\alpha+\frac{\pi}{2}} \cos(\alpha)\,d\alpha, \quad \text{with CCW arc sweep.}$$

This guarantees:

- The first lift starts naturally in Q1,
- The real projection flips align with the correct sign cycle,
- The closure cycle matches algebra exactly:

$$+\lambda,\ -1,\ -\lambda,\ +1.$$

What if you invert it?
There exists a valid mirror lift: you could define the operator with the sine primitive instead, anchored at the South Pole:

$$i := \int_{\alpha}^{\alpha-\frac{\pi}{2}} \sin(\alpha)\,d\alpha, \quad \text{with CW arc sweep.}$$

This sweep still closes by the exact same logic as my Theorem (feel free to check it yourself!):

$$+\lambda,\ -1,\ -\lambda,\ +1.$$

It rigorously produces the same sign cycle, but now pinned to the opposite pole and reversed direction. So the operator is bidirectional: both lifts exist in the geometry, both are mathematically valid, both obey physics exactly. Though, it does feel very strange to imagine a world with a "vertical" real axis and a polar origin at the South Pole. But, assuming time, momentum, and energy flow are all redefined with this new real basis, it will close, rigorously.

But what happens if you try to invert the lift direction at the same real pole?

If you fix the East Pole at $\alpha = 0$ and try to sweep backward through the sign cycle, the sign table breaks. The sign sequence inverts:

$$+\lambda \to +1 \to -\lambda \to -1,$$

which does not match the invariant roots needed for the square root identity to close.

Geometrically, sweeping backward implies the lift must generate in reverse: you would have to pull the projection of the new orthogonal direction λ *before it even exists* and arc sweep it back into reality. The Phase Lift can only unfold new phase from the real pole forward — trying to unwind it demands lifting a basis direction from *nothing* and rotating it back in time, which breaks the sign cycle's alignment with the real projection.

So the sign cycle is locked to *one lift direction* for each pole. To spin the other way, you must shift the pole itself — you cannot invert direction alone.

This makes the backward lift vastly less probable because it demands triple propagation to recover closure.

Conclusion — The Chirality Lock is Probabilistic.

So while it is possible to observe an opposite chirality, wave propagation direction, decay paths, and spin states are all probability-weighted by action cost. Thus, the Universe prefers a direction naturally — and now we know why. The real axis pins the projection. The Phase Lift operator locks the spin. The probability gradient makes the minimal arc overwhelmingly favored, while the inverse slip demands triple arc propagation to restore closure.

Thus, the "ghost" of $\sqrt{(-1)}$ was always real. Its sign cycle is locked to the pole's projection, forever biasing the universe's lift to one spin direction.

Once a real axis exists, chirality is no longer symmetric — it is locked by the Phase Lift itself. And it was right in front of us for hundreds of years!

Conclusion — The Phase Lift as Euler's True Continuum

Definition. The continuous Phase Lift Function is:

$$\mathcal{S}(\varphi) = \cos(\varphi)\,\hat{\mathbf{x}} + \sin(\varphi)\,\boldsymbol{\lambda}, \quad \varphi = \omega t.$$

Here:

- $\hat{\mathbf{x}}$ is the real projection basis.
- $\boldsymbol{\lambda}$ is the local chiral lift direction.
- The sweep parameter φ is the same phase angle used in Euler's exponential.

Key property. Differentiation generates rotation:

$$\frac{d}{d\varphi}\mathcal{S}(\varphi) = -\sin(\varphi)\,\hat{\mathbf{x}} + \cos(\varphi)\,\boldsymbol{\lambda}.$$

This is a quarter-turn of the Phase Lift:

$$\Longrightarrow \quad \frac{d}{d\varphi}\mathcal{S}(\varphi) = i\,\mathcal{S}(\varphi), \quad \text{with } i \text{ acting as the continuous rotation generator.}$$

Result. *The classical Euler identity,*

$$e^{i\theta} = \cos(\theta) + i\,\sin(\theta),$$

is exactly recovered as:

$$\boxed{\mathcal{S}(\varphi) = \cos(\varphi) + \boldsymbol{\lambda}\,\sin(\varphi).}$$

Insight. Euler's infinite expansion is not an algebraic "ghost" operation — it is a discrete sampling of this *continuous Phase Lift*. The operator i and the basis direction $\boldsymbol{\lambda}$ were never identical: they were secretly a *process* and its orthogonal codomain. But Euler was so brilliant he saw the continuous sweep hiding between the lines — and his exponential form is far more convenient, elegant, and endlessly beautiful.

Thus, the Phase Lift completes Euler: it shows that the imaginary unit is not a ghost — but a hidden bounded arc lift, anchored at the real pole, closing exactly in four sweeps, and spinning forever.

$\boxed{\textit{Euler saw the shadow. The Lift is the light.}}$

Open Question — Beyond the Exponential

The classical Euler identity,

$$e^{i\theta} = \cos(\theta) + i\,\sin(\theta),$$

is both the continuous generator and the discrete root sampler in one.

The Phase Lift function,

$$\mathcal{S}(\varphi) = \cos(\varphi) + \lambda \int_0^{\varphi} \cos(\varphi)\,d\varphi = \cos(\varphi) + \lambda\,\sin(\varphi),$$

makes the geometry explicit, and does not need the exponent's infinite expansion to spin.

Yet the standard exponential already solves wave equations and rotations perfectly.

Open: Does the continuous definition enable phase slips, constraints, or physical spin limits that the exponential cannot? Does it reveal new phenomena where discrete algebra alone fails?

This remains to be discovered.

My Journey — How I Found the Chiral Phase Lift

Since this could potentially hit hard, I figured I would share my journey for those interested in my pathway, and I hope it helps you find yours. This section is a personal record of the thoughts, intuitions, doubts, and breakthroughs that led me to this result.

Where do I even begin...

To start, maths is not my primary field. I am an elite reverse-engineer and that is what I do best, reverse-engineer things. I was working on a programming project, a profound one, one where you might see my name again some day. While working, I ran into the infamous $\sqrt{-1}$ and I just had this compelling feeling that it made no sense. Why could there possibly be any meaning to taking the square root of a negative number? I immediately thought of Rafael Bombelli and Gerolamo Cardano and how silly they must have felt writing it. So I set out on a task of discovering, what it was.

The Clues I Followed.

My first thought was, why does $\sqrt{-1}$ magically encode a flip to an orthogonal basis? As a reverse-engineer it felt like I was staring at decompiled code and looking at a function pointer to an execution call somewhere else in the global frame. Then it hit me — that is EXACTLY what it was. Now this might spark some controversy, rather sadly, but I thought, God authored physics, he authored math, he authored the link. So why would there be any imperfection in his work. I know of no other operation in all maths that is duplicated, we rigorously prevent that from happening. So I thought, if God is perfect, his math and physics are also perfect, so $\sqrt{-1}$ must be hiding as one of his other known functions. I thought, what function does maths already have that generates an orthogonal basis vector? Obviously, with a question this targeted the only answer is integration and there's zero ambiguity. So I thought, what integration could I possibly perform that would behave like i. What does i do? In the (X,Y) plane it generates a new orthogonal plane (X,Z) which encodes the hidden roots of the polynomial. I thought, how can I also generate this plane, not from assertion, but by rigorous process?

Formulating the Idea.
The first major obstacle was figuring out some form of integration that could produce the result I wanted. But what would that even look like? While obvious in hindsight, it was not at all obvious what possible integration method I was after. I thought, "whatever the function it must encode the complex roots of f(x) onto the new (X,Z) plane." My first thought was a surface of revolution on the polynomial $f(x) = x^2 + 1$. Without jumping into more math rigor, I will paraphrase the maths of what I saw. A surface of revolution takes f(x) and sweeps out an infinite sum of infinitesimal arcs by some angle α, then infinitely sums the infinitesimally-wide full arcs and produces a physical surface in the new orthogonal dimension. I first assumed this would just work. This felt right, it felt rigorous. Then I asked: how does this surface of revolution actually encode the complex root of $f(x)$? Well, it does, but it wasn't elegant. At this point I was only thinking X,Y,Z and the rotational angle α, and was not at all thinking of unit circle's invariant points. Suppose we sweep $f(x)$ around the x-axis, then slice the surface with the newly generated (X,Z) plane. Then in essence, within this new (X,Z) we sort of have an infinitely thin cross-section of our original (X,Y) curve, and due to the behavior of integration over an arc, it is true to say that $f'(x)$ now lives in this new orthogonal plane as a projection. This still did not isolate any root, it simply spread all points from $f(x)$ into the new codomain of $f'(x)$. So, the problem of finding the root by integration was still a failure. But then I realized, this would allow the curve to cross the x-axis in the new orthogonal dimension, thus solving for $f'(0)$ would yield the positive root i and imply $f(x)$ has complex root i. So in a sense, it worked — but how horribly inelegant would it be to write this down in place of just i. Well, how about I show you and you'll see…

Formally, it amounts to this:

Swept surface area (informal): $\quad \mathrm{S} = \int_a^b \int_0^{2\pi} f(x)\, d\alpha\, dx.$

Orthogonal slice plane: $\quad \Pi := \left\{ (x, y, z) \mid y = 0 \right\}$

$$\Longrightarrow \cos(\alpha) = 0 \Longrightarrow \alpha = \frac{\pi}{2}, \; \frac{3\pi}{2}.$$

Slice line:

$$z(x) = f(x) \sin(\alpha) = \pm f(x).$$

Root statement: $\quad \exists\, x_0$ s.t. $z(x_0) = i.$

But as you can see in hindsight, this is redundant and does not at all require a surface to begin with. So I thought, if this is going to matter at all in maths, it needs to be directly insertable to the euler identity, and that above, while sort of beautiful in a way, is not insertable.

The Moment It Clicked.
Then I realized, I could drop the entire second integral because I did not care at all about the width fidelity of the curve, I only cared about one point; the point defined as i. So I tried to think of a function that would project a single point onto the orthogonal plane. Naturally, and by the above usage, I knew it had to be $\sin \alpha$ or $\cos \alpha$ So I integrated w.r.t $\sin \alpha$ and it kind of worked... but my mental image cause a glaring problem. I imagined pulling the curve out of the (X,Y) plane in a continuous arc until I hit the new (X,Z) plane, kind of like pulling an infinite slinky out of the page and locking it onto the new plane. But this image directly contrasted what was happening with $\sin \alpha$ behavior on the unit circle. Notably:

$$i(x) := \int_{\alpha}^{\alpha + \frac{\pi}{2}} f(x)\, \sin(\alpha)\, d\alpha, = -[\cos \alpha]_{\alpha}^{\alpha + \frac{\pi}{2}}$$

and as we have already shown in this document, this doesn't close. So I saw this as an "if-else" block in code and thought, what if I just flip the direction? Now:

$$i(x) := \int_{\alpha}^{\alpha - \frac{\pi}{2}} f(x)\, \sin(\alpha)\, d\alpha, = -[\cos \alpha]_{\alpha}^{\alpha - \frac{\pi}{2}}$$

And then it hit me! This DID close, but it was off by a quarter phase of CW traversal, after adjusting, I got the correct closure, and I felt all my work was validated. But it was ugly and didn't feel right, starting with the main real axis in the vertical direction was something I could not accept. But, this showed me, provably, it did work. So I reached another "if-else" block of code. So I hit my next "if–else": the only variable left was the integrated function itself. And it could only be $\sin(\alpha)$ or $\cos(\alpha)$. So I tried cosine. And it worked — *beautifully*. In hindsight, I should have realized $\cos\alpha$ would be responsible for the real projection into the orthogonal codomain.

Honors — To Those Who Lit the Path

Before I seal this arc, I want to honor the minds who built the stepping stones I stood on.

- **Leonhard Euler.** The master who saw the ghost and encoded it in an identity so beautiful it still silences mathematicians centuries later. Without his leap of faith — linking exponentiation to the unit circle — none of us would have the bridge we now walk.
- **Rafael Bombelli and Gerolamo Cardano.** The early pioneers who dared to write down "$\sqrt{-1}$" at all, when it was an alchemical joke more than a tool. They trusted the impossible and let algebra grow a hidden dimension.
- **Carl Friedrich Gauss.** The prince of mathematicians, who made the complex plane real — and gave the ghost a home we could actually see. His rigor made imaginary numbers respectable and indispensable.
- **Bernhard Riemann.** The subtle genius who understood that the plane can twist, branch, and loop — showing us that what looks like a contradiction can be a higher truth when seen through the right lens. As I am sure you can imagine, there were many feelings of contradiction in this project and Riemann reminded me to keep going.
- **Pierre-Simon Laplace, Joseph Fourier, Augustin-Louis Cauchy.** Each of these brilliant minds spun this ghost into the fabric of physics — making it useful, reliable, inevitable.
- **Grant Sanderson — 3blue1brown** His YouTube channel always inspired me to think more critically about maths, and was a large inspiration of mine even when a kid in high school. Thank you for showing everyone maths is beautiful.
- **Michael Huff — The greatest teacher** I have to give thanks to my high school—college dual credit Calculus professor. He is the reason I fell in love with maths. His teaching style, genuine care, and great intelligence showed me how maths was so much more than some numbers on a page.
- **Matt Parker — StandupMaths** His whimsical and fun approach to creative problems always inspired me, and I eagerly await every video he produces. He showed me that chopping together some "terrible" Python code can lead to a lot of fun, and solve a lot of problems.
- **God — the Divine Author.** Who wrote the original Chiral Phase Lift before I could even give it a name.

This paper is just a footnote to their arc. If my Chiral Phase Lift leads to anything — it only does so because they started it.

What It Means To Me.

As I am sure you can imagine, this means A LOT to me. I feel like this might come across as audacious — but the Chiral Phase Lift definition is the first real rewrite of i in centuries. Cardano discovered its value, Euler discovered its shadow, I stumbled upon the source of the light. This goes to show there may very well be new branches of maths hidden in plain sight all the time. I challenge those reading this to not accept every axiom or convention as fact, but challenge it at the source. This may very well be the greatest thing I accomplish in my life and those of you reading this, I appreciate every moment I got to spend with you here. I hope this bit of maths inspires you to dig deeper and find something yourself. I hope that my definition adds value to the world and leads to new discoveries. I hope the implications of chirality could stretch deep and reveal grand new things. Those things are not for me though, I will now return to what I do best, reverse-engineering, and leave the truly brilliant minds to ponder about what greatness could come from my discovery.

Attribution Request. For now, I choose to remain mostly anonymous. This work is freely given for anyone to use, adapt, teach, and share — but I ask that my name be credited wherever it travels: **Alex B.**

If you wish to reach me for questions or collaboration, you may contact me at `mathsisbeautiful@proton.me`.
Or you can follow me for possible future works @ https://github.com/Purrplexia/

One day, should I wish to claim this publicly, I hold the private key, among other things, to prove it was me. I ask that you please not ever trust someone claiming this work without proving they own the private key. Cryptographic signature (PGP): Public key: see attached. (This is Version 3 and the hash has changed)

And from the mind of a cryptographer and reverse-engineer, I leave you with this — Trust, but verify.