HW 2

III.

a)
$$\left.\begin{aligned} x(t) &= x_1 t + x_0 \\ y(t) &= y_1 t + y_0 \\ z(t) &= z_1 t + z_0 \end{aligned}\right\} \quad \vec{v}(t) = \langle x(t), y(t), z(t) \rangle$$

b) Given a point, $\vec{p} = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$, its affine coordinates are $\vec{p}' = \begin{bmatrix} x(t)/w \\ y(t)/w \\ z(t)/w \end{bmatrix}$, where $w = z(t)$.

Therefore, $\vec{p}' = \begin{bmatrix} x(t)/z(t) \\ y(t)/z(t) \\ 1 \end{bmatrix}$.

$$x(t) = \frac{x_0 + x_1 t}{z_0 + z_1 t} \qquad y(t) = \frac{y_0 + y_1 t}{z_0 + z_1 t} \qquad z(t) = \frac{z_0 + z_1 t}{z_0 + z_1 t} = 1$$

c) $\frac{x(t)}{1} = \frac{x_0 + x_1 t}{z_0 + z_1 t}$

$x(t) \cdot (z_0 + z_1 t) = x_0 + x_1 t$

$x(t) z_0 + x(t) z_1 t = x_0 + x_1 t$

$x(t) z_1 t - x_1 t = x_0 - x(t) z_0$

$t(x(t) z_1 - x_1) = x_0 - x(t) z_0$

$t = \frac{x_0 - x(t) z_0}{x(t) z_1 - x_1}$

$y(t) = \frac{y_0 + y_1 t}{z_0 + z_1 t}$

$$y(t) = \frac{y_0 + y_1 \cdot \left(\frac{x_0 - x(t) z_0}{x(t) z_1 - x_1}\right)}{z_0 + z_1 \left(\frac{x_0 - x(t) z_0}{x(t) z_1 - x_1}\right)}$$

$$y(t) = \frac{\left(y_0 + y_1 \cdot \frac{x_0 - x(t) z_0}{x(t) z_1 - x_1}\right) x(t) z_1 - x_1}{\left(z_0 + z_1 \cdot \frac{x_0 - x(t) z_0}{x(t) z_1 - x_1}\right) \cdot x(t) z_1 - x_1}$$

$$y(t) = \frac{y_0 \cdot (x(t) z_1 - x_1) + y_1 (x_0 - x(t) \cdot z_0)}{z_0 \cdot (x(t) z_1 - x_1) + z_1 (x_0 - x(t) z_0)}$$

$$y(t) = \frac{y_0 \cdot x(t) z_1 - y_0 x_1 + y_1 \cdot x_0 - y_1 \cdot x(t) \cdot z_0}{z_0 x(t) z_1 - z_0 x_1 + z_1 x_0 - z_1 x(t) z_0}$$

$$y(t) = \frac{x(t) \cdot (y_0 z_1 - y_1 z_0) - y_0 x_1 + y_1 x_0}{x(t) \cdot (z_0 z_1 - z_0 z_1) - z_0 x_1 + z_1 x_0}$$

$$y(t) \cdot (x(t) \cdot (z_0 z_1 - z_0 z_1) - z_0 x_1 + z_1 x_0) + x(t)(y_1 z_0 - y_0 z_1) + y_0 x_1 - y_1 x_0 = 0$$

d) $y(t) \cdot (x(t) \cdot (z_0 z_1 - z_0 z_1) - z_0 x_1 + z_1 x_0 + x(t)(y_1 z_0 - y_0 z_1) + y_0 x_1 - y_1 x_0 = 0$

Since $(z_0 z_1 - z_0 z_1) = 0$, we have

$y(t) \cdot (x(t) \cdot (0) - z_0 x_1 + z_1 x_0) + x(t)(y_1 z_0 - y_0 z_1) + y_0 x_1 - y_1 x_0 = 0$

$y(t)(z_1 x_0 - z_0 x_1) + x(t)(y_1 z_0 - y_0 z_1) - y_0 x_1 - y_1 x_0 = 0$

$\underbrace{x(t)}_{a}(y_1 z_0 - y_0 z_1) + \underbrace{y(t)}_{b}(z_1 x_0 - z_0 x_1) + \overbrace{y_0 x_1 - y_1 x_0}^{c} = 0$

For the given line to be a point,

$x(t) = y(t) \; \forall t$, which occurs when the direction vector(s), $\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$, are parallel — or the same.

For the above to occur,

$$\frac{x_1}{z_1} = \frac{y_1}{z_1}$$

$$x_1 = y_1$$

Therefore, the line collapses to a point, $\vec{p}$