CSci 4270 and 6270
Computational Vision,
Spring Semester, 2025
Lecture 03 Practice Exercises

Overview

Please see the lecture 01 and 02 practice problems for an overview.

Problems

1. For **unit vectors** $\hat{x}$ and $\hat{y}$ in N dimensional space, the *square Euclidean distance* between the vectors is $\|\hat{x} - \hat{y}\|^2$ while the *cosine distance* is $\cos\theta = \hat{x} \cdot \hat{y}$. Give a derivation showing that

$$\|\hat{x} - \hat{y}\|^2 = 2(1 - \cos\theta).$$

2. If $\hat{\mathbf{x}}$ is a unit vector and $\mathbf{y}$ is any vector, show that

$$(\mathbf{y} \cdot \hat{\mathbf{x}})\hat{\mathbf{x}} \qquad \text{and} \qquad \mathbf{y} - (\mathbf{y} \cdot \hat{\mathbf{x}})\hat{\mathbf{x}}$$

   are orthogonal.

3. Given a set of points $\mathbf{x}_i$ in N dimensions, how do we know that the direction of maximum and minimum variation in the data are orthogonal to each other?

4. Under what conditions are the spectral decomposition of a matrix and the singular value decomposition of a matrix equal?

5. Suppose you are given set of N vectors stored in a two dimensional NumPy array. In particular, each column should be thought of as a different vector, so that M, the number of rows, is the length of each vector. Write code to normalize the array so that each column is a unit vector. No for loops are necessary.