LDA

Example: Compute the Linear Discriminant Analysis for the given 2-D dataset with two classes X_1 and X_2.

$$X_1 = (x_1, x_2) = \{(4,1), (2,4), (2,3), (3,6), (4,4)\}$$
$$X_2 = (x_1, x_2) = \{(9,10), (6,8), (9,5), (8,7), (10,8)\}$$

Step 1: Compute mean of class1 (μ_1) and class2 (μ_2).

$$\mu_1 = \left\{\frac{4+2+2+3+4}{5}, \frac{1+4+3+6+4}{5}\right\} = \{3, 3.6\}$$

$$\mu_2 = \left\{\frac{9+6+9+8+10}{5}, \frac{10+8+5+7+8}{5}\right\} = \{8.4, 7.6\}$$

Step 2: Calculate the covariance!

$$S_1 = \sum (x-\mu_1)(x-\mu_1)^T$$

$$(x-\mu_1) = \begin{bmatrix} 1 & -1 & -1 & 0 & 1 \\ -2.6 & 0.4 & -0.6 & 2.4 & 0.4 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -2.6 \end{bmatrix}\begin{bmatrix} 1 & -2.6 \end{bmatrix} = \begin{bmatrix} 1 & -2.6 \\ -2.6 & 6.76 \end{bmatrix} \quad -(1)$$

$$\begin{bmatrix} -1 \\ 0.4 \end{bmatrix}\begin{bmatrix} -1 & 0.4 \end{bmatrix} = \begin{bmatrix} 1 & -0.4 \\ -0.4 & 0.16 \end{bmatrix} \quad -(2)$$

$$\begin{bmatrix} -1 \\ -0.6 \end{bmatrix}\begin{bmatrix} -1 & -0.6 \end{bmatrix} = \begin{bmatrix} 1 & 0.6 \\ 0.6 & 0.36 \end{bmatrix} \quad -(3)$$

$$\begin{bmatrix} 0 \\ 2.4 \end{bmatrix}\begin{bmatrix} 0 & 2.4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 5.76 \end{bmatrix} \quad \rightarrow (4)$$

$$\begin{bmatrix} 1 \\ 0.4 \end{bmatrix}\begin{bmatrix} 1 & 0.4 \end{bmatrix} = \begin{bmatrix} 1 & 0.4 \\ 0.4 & 0.16 \end{bmatrix} \quad \rightarrow (5)$$

To get S_1, we need to add (1), (2), (3), (4), (5) and take Avg.

$$S_1 = \frac{1}{5}\begin{bmatrix} 4 & -2 \\ -2 & 13.2 \end{bmatrix} = \begin{bmatrix} 0.8 & -0.4 \\ -0.4 & 2.6 \end{bmatrix}$$

Similarly, we will calculate S_2,

$$S_2 = \begin{bmatrix} 1.84 & -0.04 \\ -0.04 & 2.64 \end{bmatrix}$$

Step 3: Calculate S_W (within class scatter Matrix)

$$S_W = S_1 + S_2 = \begin{bmatrix} 2.64 & -0.44 \\ -0.44 & 5.28 \end{bmatrix}$$

Step 4: Calculate S_B (Between class scatter Matrix).

$$S_B = (\mu_1 - \mu_2)(\mu_1 - \mu_2)^T$$

$$= \begin{bmatrix} -5.4 \\ -4 \end{bmatrix} \begin{bmatrix} -5.4 & -4 \end{bmatrix} = \begin{bmatrix} 29.16 & 21.6 \\ 21.60 & 16 \end{bmatrix}$$

Step 5: Find LDA Projection Vector.

$$S_W^{-1} S_B V = \lambda V$$

$$|S_W^{-1} S_B - \lambda I| = 0$$

$$\Rightarrow \left| \begin{bmatrix} 0.384 & 0.032 \\ 0.082 & 0.192 \end{bmatrix} \begin{bmatrix} 29.16 & 21.6 \\ 21.60 & 16 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\Rightarrow \begin{vmatrix} 11.89 - \lambda & 8.81 \\ 5.08 & 3.79 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (11.89 - \lambda)(3.79 - \lambda) - 44.75 = 0$$

$$\Rightarrow 45.06 - 11.89\lambda + \lambda^2 - 3.79\lambda - 44.75 = 0$$

$$\Rightarrow \lambda^2 - 15.68\lambda + 0.31 = 0$$

$$\lambda_1 = 15.66, \quad \lambda_2 = 0.019$$

So, for $\lambda = 15.66$

$$S_W^{-1} S_B V = \lambda V$$

$$\begin{bmatrix} 11.89 & 8.81 \\ 5.08 & 3.79 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 15.66 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0.91 \\ 0.39 \end{bmatrix} \text{ (Ans.)}$$

$$S_W^{-1} = \frac{1}{(2.64 \times 5.28) - (-0.44) \times (-0.44)} \cdot \begin{bmatrix} 5.28 & 0.44 \\ 0.44 & 2.64 \end{bmatrix}$$

$$= \frac{1}{13.7456} \begin{bmatrix} 5.28 & 0.44 \\ 0.44 & 2.64 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5.28}{13.7456} & \frac{0.44}{13.7456} \\ \frac{0.44}{13.7456} & \frac{2.64}{13.7456} \end{bmatrix}$$

$$= \begin{bmatrix} 0.384 & 0.032 \\ 0.082 & 0.192 \end{bmatrix}$$