Probabilistic Versus Quantum Algorithms

1. Introduction
Algorithms can be broadly classified based on how they handle uncertainty and computation. Two important classes are **probabilistic algorithms** and **quantum algorithms**. Both aim to solve problems more efficiently than classical deterministic algorithms, but they do so using different principles.

2. Probabilistic Algorithms
Probabilistic algorithms use randomness to make decisions during computation. They do not always guarantee a correct answer but can give the correct result with high probability.

- **Example**: The **Monte Carlo algorithm** estimates π by random sampling.
- **Working**: Random inputs are generated, and outcomes are analyzed statistically to produce an approximate solution.
- **Time Complexity**: Often faster than deterministic algorithms, especially for problems like primality testing or optimization.
- **Advantage**: Simpler and faster for certain problems where exact computation is expensive.
- **Limitation**: There is always a small chance of error, which may require repeated runs to increase accuracy.

3. Quantum Algorithms
Quantum algorithms leverage the principles of **quantum mechanics**: superposition, entanglement, and interference. They can explore many possible solutions simultaneously, achieving exponential speedups for certain problems.

- **Example**: **Shor's Algorithm** for factoring large integers.
- **Working**: Quantum superposition allows simultaneous evaluation of a function for many inputs. Quantum Fourier Transform (QFT) helps find periodicity efficiently.
- **Time Complexity**: Solves problems in **polynomial time** that would take **exponential time** classically.
- **Advantage**: Can solve specific hard problems like integer factorization and unstructured search much faster than classical or probabilistic algorithms.
- **Limitation**: Requires a quantum computer; practical implementation is still in early stages.

4. Comparison

Feature	**Probabilistic Algorithms**	**Quantum Algorithms**
Principle	Uses randomness	Uses quantum superposition & entanglement
Accuracy	High probability, not always exact	Exact or high-probability solutions
Example Problem	Primality testing, Monte Carlo	Shor's Algorithm, Grover's Algorithm
Speed	Faster than classical in some cases	Exponentially faster for specific problems
Hardware Requirement	Classical computer	Quantum computer

5. Applications

- **Probabilistic Algorithms**: Simulation, randomized optimization, cryptography, approximate counting.
- **Quantum Algorithms**: Breaking RSA encryption, unstructured search, quantum chemistry simulations, optimization problems.

Phase Kick-Back – Explanation

Phase Kick-Back is a **quantum phenomenon** where the **phase of a qubit** in a **control register** is modified (or "kicked back") as a result of applying a **controlled unitary operation** on a target qubit.

- Instead of changing the **state of the target qubit**, the **control qubit's phase** is altered.
- This is extremely useful in algorithms that encode information in the **phase** rather than the amplitude of a qubit.

2. How it Works

Suppose we have:

- **Control qubit:** $|x\rangle$
- **Target qubit:** $|y\rangle$
- **Unitary operation U** such that $U|y\rangle = e^{i\phi}|y\rangle$

A **controlled-U operation** applies Uonly if the control qubit is $|1\rangle$:

$$C_U|x\rangle|y\rangle = |x\rangle U^x|y\rangle$$

- If the target qubit is prepared in a special state, e.g., $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$, then applying controlled-U can **transfer the phase** from the target qubit to the control qubit:

$$C_U|x\rangle|-\rangle = e^{ix\phi}|x\rangle|-\rangle$$

3. Example

- Let target qubit = $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$
- Controlled-Z gate $CZ$is applied:

$$CZ|x\rangle|y\rangle = \begin{cases} -|x\rangle|y\rangle & \text{if } x = y = 1 \\ |x\rangle|y\rangle & \text{otherwise} \end{cases}$$

- Applying $CZ$when target is $|-\rangle$→ control qubit picks up **phase −1** if it is $|1\rangle$.
- Target remains unchanged: $|-\rangle$

This is **phase kick-back in action**.

4. Importance in Quantum Algorithms

Phase kick-back is crucial because it allows **encoding function values into the phase** of a qubit without measuring it.

1. **Deutsch–Jozsa Algorithm**:
    - Oracle flips the phase of the input qubit according to $f(x)$

- Uses phase kick-back to encode f(x) as a **phase** in the superposition

2. **Simon's Algorithm**:
   - Encodes hidden XOR function into phases of input qubits
3. **Shor's Algorithm**:
   - Quantum Fourier Transform and modular exponentiation use phase kick-back to find **periods efficiently**

Deutsch–Jozsa Algorithm – Explanation

- Developed by **David Deutsch and Richard Jozsa (1992–1993)**.
- Solves a **black-box problem** (oracle problem) **deterministically** using a **quantum computer**.
- Shows that quantum computers can solve some problems in **one query**, whereas classical computers may need **exponentially many queries**.

2. Problem Definition

- A function $f:\{0,1\}^n \to \{0,1\}$implemented as a **black-box oracle**.

Promise:

- $f$is either:
  1. **Constant:** $f(x) = 0$or $f(x) = 1$for all x
  2. **Balanced:** $f(x) = 0$for exactly half of the inputs, $f(x) = 1$for the other half

Goal: Determine whether $f$is **constant** or **balanced**.

Classical solution:

- In the worst case, need $2^{n-1} + 1$queries to guarantee correctness.

Quantum solution:

- Deutsch–Jozsa algorithm solves it **with just 1 query**.

3. Steps of Deutsch–Jozsa Algorithm

Step 1: Initialize Quantum Registers

- Use **n qubits for input** and 1 qubit for output.
- Initialize input qubits to $|0\rangle^{\otimes n}$and output qubit to $|1\rangle$:

$$|0\rangle^{\otimes n}|1\rangle$$

Step 2: Apply Hadamard Transform

- Apply **Hadamard gate (H)** to all input qubits and output qubit:

$$|\psi_1\rangle = \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle \cdot \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

- Creates **superposition of all input states**.

Step 3: Apply Oracle U_f

- Oracle $U_f$maps:

$$U_f|x\rangle|y\rangle = |x\rangle|y \oplus f(x)\rangle$$

- After applying oracle, the quantum state becomes:

$$\frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} (-1)^{f(x)}|x\rangle \cdot \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

- The output qubit is no longer important; **phase of input qubits encodes f(x)**.

Step 4: Apply Hadamard Transform Again

- Apply **Hadamard** to all input qubits again.
- This step performs **interference**, combining amplitudes constructively or destructively.

Step 5: Measure Input Qubits

- Measure the input qubits:
  - **All zeros** $|0\rangle^{\otimes n} \to f$is **constant**
  - **Any non-zero state** $\to f$is **balanced**

5. Quantum Advantage

- **Classical:** Worst case $2^{n-1} + 1$queries
- **Quantum:** 1 query, deterministic result
- Demonstrates **quantum parallelism and interference**.

Simon's Algorithm – Explanation

Simon's Algorithm is a **quantum algorithm** developed by **Daniel Simon in 1994**.

- It solves **Simon's Problem**, which is designed to show that **quantum computers can be exponentially faster than classical computers** for certain problems.
- Simon's Algorithm was one of the earliest examples showing an **exponential speedup of quantum algorithms over classical ones**.

2. Simon's Problem

Given a **black-box (oracle) function** $f:\{0,1\}^n \to \{0,1\}^n$with the property that:

1. There exists a secret string $s \in \{0,1\}^n$such that:

$$f(x) = f(y) \iff y = x \oplus s$$

- Here, $\oplus$is **bitwise XOR**.
- s is **unknown**.

2. If $s = 0^n$, then $f$is **1-to-1** (injective).
3. Otherwise, $f$is **2-to-1**, meaning every output value has **exactly two inputs** mapping to it: $x$and $x \oplus s$.

Goal: Find the secret string s.

- **Classical computers** require $O(2^{n/2})$queries to the oracle.
- **Simon's quantum algorithm** solves it in **O(n) queries**, exponentially faster.

3. Steps of Simon's Algorithm

Step 1: Initialize Quantum Registers

- Use **two quantum registers**:
    1. Input register: $n$qubits (for x)
    2. Output register: $n$qubits (for $f(x)$)
- Initialize both registers to $|0\rangle^{\otimes n}$.

Step 2: Apply Hadamard Transform

- Apply **Hadamard gates (H)** to all qubits in the **input register**, creating a **superposition of all possible inputs**:

$$\frac{1}{\sqrt{2^n}}\sum_{x=0}^{2^n-1} |x\rangle|0\rangle$$

Step 3: Query the Oracle

- Apply the **oracle** $U_f$to map $|x\rangle|0\rangle \rightarrow |x\rangle|f(x)\rangle$
- Now, the quantum state encodes **all input-output pairs** in superposition.

Step 4: Measure Output Register

- Measure the **output register**.
- Measurement collapses the output to a specific value $f(x_0)$, leaving the **input register in a superposition of two states**:

$$|x_0\rangle + |x_0 \oplus s\rangle$$

- This is key: the input register now contains **information about the secret string** s.

Step 5: Apply Hadamard to Input Register

- Apply **Hadamard transform** to the **input register** again.
- This produces a superposition of all strings $y \in \{0,1\}^n$such that:

$$y \cdot s = 0 \text{ (mod 2)}$$

- Here, $y \cdot s$is the **dot product modulo 2**.

Step 6: Measure Input Register

- Measure the **input register**, giving a random string $y$such that $y \cdot s = 0$.
- Repeat Steps 1–6 **O(n) times** to collect **n independent equations**.

Step 7: Solve Linear System

- Solve the **linear system of equations modulo 2** to find the secret string s.

✅ Done! Simon's Algorithm finds s**exponentially faster than classical methods**.

4. Example (n=2)

Suppose $n = 2$, and the secret string $s = 10$.

- Oracle maps:

$$f(00) = f(10) = 01, f(01) = f(11) = 11$$

Step 1: Input register in superposition:

$$|00\rangle + |01\rangle + |10\rangle + |11\rangle$$

Step 2: Apply oracle → entangled state:

$$|00\rangle|01\rangle + |01\rangle|11\rangle + |10\rangle|01\rangle + |11\rangle|11\rangle$$

Step 3: Measure output → e.g., $f(x) = 01$→ input register collapses to:

$$|00\rangle + |10\rangle$$

Step 4: Hadamard on input → get a string $y$such that $y \cdot s = 0$

- Repeat and solve → find $s = 10$

5. Quantum Advantage

- **Classical approach:** Need $O(2^{n/2})$oracle queries.
- **Quantum approach:** Only $O(n)$queries needed → **exponential speedup**.
- Shows that **quantum computers can outperform classical computers** for some problems.

Shor's Algorithm – Detailed Explanation

Shor's Algorithm is a **quantum algorithm** developed by **Peter Shor in 1994** for **integer factorization**. It can factor a large number Nin **polynomial time**, which is exponentially faster than the best-known classical factoring algorithms (like trial division or the general number field sieve).

- Factoring large numbers is the **basis of RSA encryption**.
- On a quantum computer, Shor's Algorithm can break RSA encryption efficiently.
- Classical computers struggle because factoring large numbers requires **exponential time**, whereas Shor's Algorithm does it in **O((log N)^3)** operations.

2. Mathematical Foundation

Shor's Algorithm is based on the idea that **factoring can be reduced to period finding**:

1. Suppose $N$is the number we want to factor.
2. Pick a number $a < N$such that gcd(a, N) = 1.
3. Consider the function:

$$f(x) = a^x \text{mod } N$$

This function is **periodic**, meaning there exists a smallest positive integer $r$such that:

$$a^r \equiv 1 \text{ (mod } N)$$

- Finding this period $r$is **the hardest part classically**, but quantum computers can do it efficiently using **quantum Fourier transform (QFT)**.
- Once the period $r$is known, the **factors of** N**can be computed using gcd(a^(r/2) ± 1, N))**, provided $r$is even.

3. Detailed Steps

Step 1: Random Selection of a

- Choose a random number $a$less than N.
- If gcd(a, N) ≠ 1, then we already have a factor.

- Otherwise, proceed to find the period.

Step 2: Quantum Period Finding

- Construct a quantum superposition of all possible values of x.
- Compute $f(x) = a^x \text{mod } N$for all x**simultaneously** using quantum parallelism.
- Apply **quantum Fourier transform (QFT)** to find the period r.

Step 3: Classical Post-processing

- Check if $r$is even. If not, choose another a.
- If $r$is even, compute:

$$\text{factor}_1 = \gcd(a^{r/2} - 1, N) \text{factor}_2 = \gcd(a^{r/2} + 1, N)$$

- These usually give **non-trivial factors of N.**

4. Example: Factor N = 15

Let's see a small example:

- Pick $a = 2$(gcd(2,15) = 1).
- Compute $f(x) = 2^x \text{mod } 15$:

x	**0 1 2 3 4**
2^x mod 15	1 2 4 8 1

- Period $r = 4$(smallest x where 2^x ≡ 1 mod 15)
- $r$is even → compute gcd(2^(4/2) ± 1, 15) = gcd(4-1, 15), gcd(4+1, 15) = gcd(3,15), gcd(5,15)
- Factors = **3 and 5**

5. Quantum Advantage

- **Classical Approach:** Finding period $r$is hard. For large N, classical algorithms take **exponential time**.
- **Quantum Approach:** Quantum superposition and interference allow the algorithm to **find the period efficiently**, in **polynomial time**.

6. Applications and Importance

1. **Breaking RSA Encryption:**
   - RSA security depends on factoring large semiprime numbers.
   - Shor's Algorithm can factor them efficiently, threatening RSA.
2. **Cryptography:**
   - The development of quantum-resistant algorithms is crucial because Shor's Algorithm can break current public-key cryptography.
3. **Number Theory and Quantum Computing:**
   - Shor's Algorithm demonstrates that quantum computing can solve certain **mathematical problems exponentially faster** than classical computing.

Factoring Integers

Factoring integers is the process of breaking a number into smaller numbers (factors) that, when multiplied together, give the original number. It is a fundamental problem in **number theory** and has major applications in **cryptography**, especially in RSA encryption.

2. Classical Factoring Methods
Classical algorithms try to find factors using deterministic or probabilistic approaches:

- **Trial Division**: Check divisibility by all numbers up to √N. Simple but **slow for large numbers**.
- **Fermat's Method**: Express N as a difference of squares, useful when factors are close together.
- **Pollard's Rho Algorithm**: A probabilistic method that can find non-trivial factors faster than trial division.
- **Time Complexity**: For very large numbers, classical methods take **exponential time**, making them inefficient for cryptography.

3. Quantum Factoring (Shor's Algorithm)
Quantum algorithms can factor integers **efficiently** using quantum mechanics.

- **Shor's Algorithm**: Developed by Peter Shor (1994), it factors a number N in **polynomial time**.
- **Key Idea**: Reduce factoring to **period finding**. For a number a<N such that gcd(a, N)=1, consider f(x) = a^x mod N. The smallest period r such that a^r ≡ 1 mod N is used to compute factors.
- **Steps**:
  1. Randomly choose a number a < N. If gcd(a, N) ≠ 1, a factor is found.
  2. Use **quantum superposition** and **Quantum Fourier Transform (QFT)** to find the period r.
  3. Compute gcd(a^(r/2) ± 1, N) to get non-trivial factors.
- **Advantage**: Exponentially faster than classical algorithms for large integers.

4. Example (N = 15)

- Pick a = 2, gcd(2,15) = 1.
- f(x) = 2^x mod 15 → sequence: 1, 2, 4, 8, 1 …
- Period r = 4 → factors = gcd(2^(4/2) ± 1, 15) = gcd(3,15), gcd(5,15) = 3 and 5.

Grover's Algorithm – Explanation

Grover's Algorithm, developed by **Lov Grover in 1996**, is a **quantum search algorithm** that can find a specific item in an **unsorted database** of size Nin roughly $\mathbf{O}(\sqrt{\mathbf{N}})$operations.

- Classical search in an unsorted database requires $O(N)$steps.
- Grover's Algorithm achieves a **quadratic speedup**, which is significant for large N.

Applications:

- Searching large unsorted databases
- Breaking symmetric cryptography (e.g., brute-forcing keys)
- Solving combinatorial problems

2. Problem Setup

Given:

- An **unsorted database** of Nelements.
- A **black-box (oracle) function** $f(x)$that outputs:

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is the target item} \\ 0 & \text{otherwise} \end{cases}$$

Goal: Find $x$such that $f(x) = 1$.

3. Steps of Grover's Algorithm

Step 1: Initialize Quantum State

- Use n qubits to represent $N = 2^n$items.
- Initialize all qubits to $|0\rangle$.

$$|0\rangle^{\otimes n}$$

- Apply **Hadamard transform** to create **equal superposition** of all states:

$$|\psi_0\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle$$

Step 2: Oracle (Phase Inversion)

- Apply **oracle** Owhich flips the **phase of the target state** $|x_t\rangle$by multiplying it by -1:

$$O|x\rangle = \begin{cases} -|x\rangle & x = x_t \\ |x\rangle & x \neq x_t \end{cases}$$

- This marks the correct item in the superposition.

Step 3: Amplify Target (Grover Diffusion Operator)

- Apply the **Grover Diffusion Operator** $D$to amplify the probability amplitude of the target state:

$$D = 2|\psi_0\rangle\langle\psi_0| - I$$

- Intuition: Inverts the amplitudes about the **average amplitude**, increasing the target's probability.

Step 4: Repeat Steps 2–3

- Repeat the **Oracle + Diffusion** operation approximately:

$$R = \frac{\pi}{4}\sqrt{N} \text{ times}$$

- After these iterations, the **target state's probability** is very close to 1.

Step 5: Measure

- Measure the qubits.
- The result will most likely be the **target element** x_t.

4. Example: Search in a 4-Item Database

Suppose database: [00,01,10,11]and the target is $x_t = 10$.

Step 1: Initialize superposition:

$$|\psi_0\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

Step 2: Oracle flips target phase:

$$|\psi_1\rangle = \frac{1}{2}(|00\rangle + |01\rangle - |10\rangle + |11\rangle)$$

Step 3: Apply Grover diffusion:

- Reflect about average amplitude → probability of $|10\rangle$increases.

Step 4: Repeat (~1 iteration for N=4)

Step 5: Measure → result: $|10\rangle$with high probability.

5. Quantum Advantage

- **Classical search:** O(N) queries to find the item.
- **Grover's search:** O(√N) queries.
- Quadratic speedup may seem modest compared to Shor's Algorithm (exponential speedup), but it is significant for **large unstructured datasets**.

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Quantum Algorithms for Linear Algebra

Quantum Algorithms for Linear Algebra are quantum computing techniques designed to perform **linear algebra operations** more efficiently than classical methods. They exploit **superposition, entanglement, and quantum interference** to handle large matrices and vectors in high-dimensional Hilbert spaces, offering potential speedups in computation.

Quantum Representation of Vectors and Matrices: Classical vectors $x \in \mathbb{R}^n$and matrices $A \in \mathbb{R}^{n\times n}$are encoded as quantum states $|x\rangle$and operators acting on quantum states. This enables **parallel processing** of multiple elements simultaneously.

- **Quantum Phase Estimation (QPE)**
- **Quantum Singular Value Estimation (QSVE)**

Steps of Quantum Algorithms for Linear Algebra

1. **Encode Data into Quantum States:** Map classical vectors and matrices into quantum states using amplitude encoding or other quantum encoding methods.
2. **Apply Quantum Operations:** Use quantum circuits for matrix multiplication, eigenvalue estimation, or singular value decomposition.
3. **Compute Solution or Transformation:** For linear systems, obtain quantum states representing solutions. For PCA, extract eigenvalues or principal components using QSVE.
4. **Measurement and Extraction:** Measure quantum states to extract classical information, such as approximate solutions, eigenvalues, or singular vectors.
5. **Optional Optimization or Post-processing:** Perform classical or hybrid quantum-classical

optimization for tasks like regression or dimensionality reduction.

Advantages of Quantum Linear Algebra Algorithms

- **Exponential Speedup:** For sparse and well-conditioned matrices, quantum algorithms can outperform classical methods significantly.
- **High-Dimensional Processing:** Can handle vectors and matrices too large for classical computers.
- **Efficient Computation:** Quantum parallelism allows simultaneous evaluation of many elements or operations.

Regression and Clustering in QML

Regression and clustering are key tasks in machine learning. Quantum Machine Learning (QML) leverages **quantum computing principles**—superposition, entanglement, and interference—to perform these tasks more efficiently, especially for high-dimensional or complex datasets.

Quantum Regression: Quantum regression extends classical regression techniques (like linear or ridge regression) using quantum algorithms.

Quantum Feature Encoding: Classical input data $x \in \mathbb{R}^n$is mapped to quantum states $|\phi(x)\rangle$in high-dimensional Hilbert space.

Steps of Quantum Regression

1. **Encode Data into Quantum States:** Map classical features to quantum states $|\phi(x)\rangle$.
2. **Construct Quantum Circuit:** Represent the regression model using quantum gates.
3. **Solve Linear System:** Use HHL or related algorithms to compute regression coefficients.
4. **Prediction:** Encode new data points and compute outputs via quantum measurements.

Quantum Clustering: Quantum clustering algorithms group data points based on similarity using quantum computing.

Quantum k-Means: Classical k-means is enhanced using **quantum distance estimation**. Quantum states represent data points, and distances are calculated efficiently using quantum inner products.

Quantum Hierarchical Clustering: Quantum circuits estimate similarity matrices, enabling faster clustering of large datasets.

Steps of Quantum Clustering

1. **Encode Data into Quantum States:** Represent data points as quantum states.
2. **Compute Similarity/Distance:** Use quantum inner products or kernel evaluations to compute distances or similarities.
3. **Assign Clusters:** Group points based on minimum distance or similarity in quantum feature space.
4. **Iterate and Optimize:** Update cluster centers iteratively using quantum circuits until convergence.

Advantages of QML Regression and Clustering

- **Exponential Feature Space:** Can process data in dimensions impossible for classical algorithms.
- **Efficient Computation:** Quantum inner products and linear algebra reduce computational complexity.
- **Better Pattern Recognition:** Captures complex correlations in high-dimensional datasets efficiently.
- **Potential Quantum Advantage:** Faster convergence and scalability for large datasets.

Nearest Neighbour Search in QML

Nearest Neighbour Search (NNS) is a fundamental task in machine learning used to find the most similar data points to a given query. Quantum Machine Learning (QML) enhances classical nearest neighbour methods by exploiting **superposition, entanglement, and interference**, enabling faster search in high-dimensional datasets

Quantum k-Nearest Neighbours (QkNN): Extends classical k-NN by using quantum circuits to evaluate distances to all training points **in parallel** and select the k closest points efficiently.

Quantum Representation of Data: Classical data vectors $x \in \mathbb{R}^n$are encoded into quantum states $|\phi(x)\rangle$, which allows simultaneous processing of multiple points using quantum superposition.

Steps of Quantum Nearest Neighbour Search

1. **Encode Data into Quantum States:** Map each classical data point to a quantum state $|\phi(x)\rangle$.
2. **Construct Quantum Circuit for Distance Calculation:** Build a circuit to compute the inner product or similarity between the query and all training points.
3. **Compute Quantum Distance/Similarity:** Evaluate distances or similarities efficiently in parallel using quantum operations.
4. **Amplitude Amplification and Selection:** Apply quantum amplitude amplification to identify the nearest neighbours with high probability.
5. **Classification or Retrieval:**
    - **For Classification:** Use majority voting of nearest neighbours' labels.
    - **For Retrieval/Search:** Return the nearest neighbour(s) as the query result.

Advantages of Quantum Nearest Neighbour Search

- **Exponential Feature Space:** Can handle high-dimensional datasets efficiently.
- **Parallel Distance Computation:** Evaluates distances to multiple points simultaneously.
- **Faster Search:** Quantum amplitude amplification reduces the number of steps compared to classical search.
- **Scalability:** Effective for very large datasets where classical NNS becomes slow.

Classification in Quantum Machine Learning

Classification is a fundamental task in machine learning where the goal is to assign labels to data points based on their features. Quantum Machine Learning (QML) enhances classical classification methods by leveraging **superposition, entanglement,**

and quantum interference to process high-dimensional datasets efficiently.

Quantum Feature Encoding:
Classical data vectors $x \in \mathbb{R}^n$ are mapped to quantum states $| \phi(x) \rangle$, creating a **high-dimensional Hilbert space** where complex patterns and correlations can be captured naturally.

Quantum Kernel Methods:
Quantum classifiers often use kernels to compute similarity between data points.

Decision Function:
Quantum classifiers construct a decision function similar to classical SVMs.

Steps of Quantum Classification

1. **Encode Data into Quantum States:** Map classical data points into quantum states using a quantum feature map.
2. **Construct Quantum Circuit or Kernel:** Build circuits to compute quantum kernels or implement parameterized quantum neural networks.
3. **Train Classifier:** Optimize weights (α_i for QSVM) or quantum gate parameters to separate classes effectively.
4. **Decision Function Evaluation:** Compute the output for new data points using the trained quantum classifier.
5. **Classify New Data:** Assign labels based on the decision function or measurement outcomes from quantum circuits.

Advantages of Quantum Classification

- **Exponential Feature Space:** Can handle complex datasets and high-dimensional feature spaces.
- **Efficient Kernel Evaluation:** Inner products in quantum space are computed faster than classically.
- **Parallel Processing:** Superposition allows simultaneous evaluation of multiple inputs.
- **Potential Quantum Advantage:** May provide speedup and improved accuracy for certain datasets.

Quantum Boosting (QBoost) – Detailed Explanation

Quantum Boosting, often referred to as **QBoost**, is a **quantum version of the classical boosting algorithm** used in machine learning. It combines **weak classifiers** into a **strong classifier** with the help of **quantum computing**, specifically **quantum optimization techniques** like **Quantum Annealing** or **Variational Quantum Algorithms**.

Boosting itself is a classical technique that improves prediction accuracy by combining multiple “weak” learners (models that perform slightly better than random guessing) into a **strong learner**. Quantum Boosting leverages **quantum hardware** to **optimize the weights of these weak learners** efficiently, potentially handling **larger and more complex datasets** than classical boosting.

1. Key Concepts

1. **Weak Classifiers:**
   - Small models that perform slightly better than random guessing.
   - Examples: decision stumps or small decision trees.
2. **Strong Classifier:**
   - A weighted combination of weak classifiers, designed to reduce overall error.
3. **Quantum Optimization:**
   - QSVM uses **quantum computing** to optimize the weights assigned to each weak classifier.
   - Quantum techniques like **Quantum Annealing** or **Variational Quantum Circuits (VQC)** can find an optimal combination faster than classical methods in certain cases.

3. Steps of Quantum Boosting (QBoost)

Step 1: Prepare Weak Classifiers

- Train multiple weak classifiers $h_i(x)$ on the dataset.
- Weak classifiers can be simple models like **decision stumps** or **shallow trees**.

Step 2: Map Weight Optimization to Quantum Problem

- Assign a **binary variable** w_i for each weak classifier.
- Encode the problem as a **Hamiltonian** or **cost function** suitable for a quantum computer:

$$\hat{H} = \sum_{i,j} C_{ij} w_i w_j + \sum_{i} C_i w_i$$

The coefficients C_i, C_{ij} are derived from **classifier errors and correlations**.

Step 3: Solve Optimization Using Quantum Hardware

- Use a **quantum annealer** (like D-Wave) or **variational quantum circuit** to **minimize the Hamiltonian**.
- The quantum computer finds the optimal combination of weights w_i that minimize training error.

Step 4: Construct Strong Classifier

- Combine selected weak classifiers using the **optimized weights**:

$$H(x) = \text{sign}(\sum_{i} w_i h_i(x))$$

- This gives a **strong quantum-boosted classifier** with improved accuracy.

Step 5: Classify New Data

- For a new input x, evaluate the strong classifier $H(x)$.
- The quantum-computed weights ensure that the strong classifier generalizes better than any individual weak classifier.

4. Advantages of Quantum Boosting

1. **Efficient Weight Optimization:**
2. **Better Accuracy:**
3. **Handles High-Dimensional Data:**
4. **Potential Quantum Advantage:**

Quantum Support Vector Machines (QSVM)

Quantum Support Vector Machines (QSVMs) are a **quantum machine learning algorithm** designed for **classification tasks**. They extend the concept of traditional SVMs into the **quantum computing domain**, by leveraging **superposition, entanglement, and quantum interference** to process high-dimensional datasets efficiently.

QSVMs use these properties to encode classical data into **quantum states**, compute **quantum kernels** efficiently, and classify data in a **high-dimensional Hilbert space**—which would be computationally infeasible for classical algorithms.

Quantum Feature Map:

- Classical data vectors $x \in \mathbb{R}^n$ are mapped to quantum states $|\phi(x)\rangle$.
- Quantum feature maps allow **complex data correlations** to be captured naturally.

Steps of Quantum Support Vector Machines (QSVM)

1. **Encode Data into Quantum States:**
   - Map each classical data point x to a quantum state $|\phi(x)\rangle$ using a quantum feature map.
2. **Prepare Quantum Circuit for Kernel Evaluation:**
   - Construct a quantum circuit to compute the **overlap (inner product)** between quantum states:

$$K(x_i, x_j) = |\langle\phi(x_i)|\phi(x_j)\rangle|^2$$

3. **Compute Quantum Kernel Matrix:**
   - Evaluate the kernel for all training data pairs to form the **kernel matrix**, capturing similarities.
4. **Determine Support Vector Weights:**
   - Solve for α_i and bias b using an **optimization process** (e.g., quadratic programming).
   - Points with non-zero α_i are **support vectors**.
5. **Construct Decision Function:**
   - Define the classifier using quantum kernels and support vector weights:

$$f(x) = \text{sign}\left(\sum_i \alpha_i y_i K(x_i, x) + b\right)$$

6. **Classify New Data:**
   - Encode new data points into quantum states, compute kernels with support vectors, and apply the decision function to classify.
7. **Optional Fine-Tuning:**
   - Adjust quantum feature maps or parameters to improve classification accuracy.

Advantages of Quantum Computation:

- **Exponential Feature Space**
- **Efficient Kernel Evaluation**
- **Potential Quantum Advantage**

Quantum Neural Networks (QNNs)

Quantum Neural Networks (QNNs) are a **quantum version of classical neural networks**, designed to leverage **quantum computing principles**—such as superposition, entanglement, and interference—to process information in ways that classical neural networks cannot.

- They aim to **enhance machine learning** by allowing operations in **high-dimensional quantum Hilbert spaces**, enabling **faster computation** and **better representation of complex data**.
- QNNs are part of the broader field called **Quantum Machine Learning (QML)**.

Key Concepts

- **Qubits:**
  Unlike classical bits (0 or 1), qubits exist in superposition: $|\psi\rangle=\alpha|0\rangle+\beta|1\rangle$
- **Quantum Gates:**
  Gates act like weights/activations in QNNs. Examples: Pauli (X, Y, Z), Hadamard (H), Rotation (Rx, Ry, Rz), CNOT.
- **Superposition & Entanglement:**
  Superposition processes multiple states simultaneously. Entanglement captures complex correlations between features.
- **Quantum Circuits:**
  QNNs are built as **Parameterized Quantum Circuits (PQC)** with trainable parameters optimized using hybrid quantum-classical methods.

2. Structure of a Quantum Neural Network

1. **Input Encoding (Quantum Feature Map):**
   - Classical data x is encoded into a quantum state $|\phi(x)\rangle$.
   - Techniques: **angle encoding, amplitude encoding, basis encoding**.
2. **Parameterized Quantum Layers:**
   - Quantum gates with **trainable parameters** act like neurons.
   - Layers may include **entangling gates (CNOT, CZ)** to capture correlations.
3. **Measurement Layer:**
   - Qubits are **measured** to extract classical outputs.
   - Measurement probabilities correspond to **predictions or activations**.
4. **Training (Parameter Optimization):**
   - Loss function is defined classically, e.g., **cross-entropy** or **mean squared error**.
   - **Parameters are updated** using gradient-based optimization (classical or hybrid).
   - Techniques include **Quantum Gradient Descent** or **Parameter Shift Rule**.

3. Working of QNNs – Step by Step

1. **Encode Input Data:**

   - Transform classical input into quantum states using **quantum feature maps**.

2. **Apply Quantum Layers:**
   - Pass qubits through **parameterized quantum gates** (rotation + entangling layers).
   - Quantum interference combines features in high-dimensional space.

3. **Measure Output:**
   - Perform quantum measurement on qubits to get probabilities.
   - Map these probabilities to **class labels or regression values**.

4. **Compute Loss Function:**
   - Compare predicted output with actual labels using a classical loss function.

5. **Update Parameters:**
   - Adjust quantum gate parameters to minimize loss (via classical optimizer or hybrid approach).

6. **Iterate Until Convergence:**
   - Repeat the above steps for several epochs until **training converges**.

4. Advantages of QNNs

1. **High-Dimensional Representations:**
2. **Parallelism:**
3. **Capturing Correlations:**
4. **Potential Quantum Advantage:**

5. Limitations

1. **Quantum Hardware Constraints:**
2. **Hybrid Approach Needed:**
3. **Training Complexity:**

Variational Quantum Algorithms (VQAs)

Variational Quantum Algorithms (VQAs) are a class of hybrid quantum-classical algorithms designed to solve **optimization and machine learning problems** using quantum computers. They combine **quantum circuits** with classical optimization to leverage the advantages of quantum computing while overcoming current hardware limitations.

Parameterized Quantum Circuits:
VQAs use quantum circuits with **tunable parameters** (rotation angles, gate parameters) that act like weights in classical models. The goal is to adjust these parameters to minimize or maximize a cost function.

Hybrid Quantum-Classical Loop:

- Quantum circuits evaluate a **cost function** (e.g., energy of a system, prediction error).
- Classical optimizers (gradient descent, Nelder-Mead, or Adam) update the parameters iteratively.
- This loop continues until the cost function converges to an optimal value.

- **Cost Function Evaluation:**
  The quantum circuit prepares a state $|\psi(\theta)\rangle$, and the cost function is measured as the expectation value of a Hamiltonian or operator:

$$C(\theta) = \langle\psi(\theta) \mid H \mid \psi(\theta)\rangle$$

Here, θrepresents the set of circuit parameters.

Steps of Variational Quantum Algorithms

1. **Initialize Parameters:**
   Start with random or heuristic values for circuit parameters θ.
2. **Prepare Parameterized Quantum Circuit:**
   Construct a quantum circuit with gates dependent on $\theta$to encode the problem.
3. **Measure Cost Function:**
   Execute the circuit and measure the expectation value of the operator corresponding to the problem.
4. **Classical Optimization:**
   Use a classical optimizer to update $\theta$to minimize (or maximize) the cost function.
5. **Iterate Until Convergence:**
   Repeat the quantum measurement and classical optimization loop until the cost function reaches an acceptable minimum.
6. **Extract Solution:**
   Use the final optimized quantum state $|\psi(\theta^*)\rangle$to derive the solution to the problem.

Advantages of Variational Quantum Algorithms

- **Hybrid Approach**
- **Flexibility**
- **Scalable**

Applications

- **Quantum Chemistry**
- **Optimization Problems:** Portfolio optimization, logistics, and combinatorial optimization.
- **Material Science:** Simulating physical systems and discovering new materials.
- **Image Recognition:** Identify visually similar images quickly.
- **Recommendation Systems:** Find similar items or users in large datasets.
- **Anomaly Detection:** Detect unusual patterns by comparing with nearest neighbours.
- **Quantum Machine Learning:** Linear regression, classification, and clustering on large datasets
- **Optimization Problems:** Portfolio optimization, resource allocation, and network optimization
- **Data Analysis:** Quantum PCA and dimensionality reduction for big data
- **Finance:** Predicting stock prices, detecting fraud, portfolio optimization

- **Healthcare:** Disease risk prediction, patient clustering for treatment plans
- **Image and Signal Processing:** Classifying complex patterns in images or signals.

U5

Classical Error Correction: The Error Model and

In any communication or computing system, transmitted data can get corrupted due to noise, interference, or hardware faults. To ensure reliable communication, **error correction techniques** are used. These techniques involve **detecting and correcting errors** using redundant information. Classical error correction forms the basis for **quantum error correction**, which is essential in quantum information processing.

2. Error Model
An **error model** describes the types of errors that can occur and their probabilities.

- **Binary Symmetric Channel (BSC):** Each transmitted bit has a probability $p$of flipping (0 → 1 or 1 → 0), and a probability $1 - p$of remaining unchanged.

Mathematically:

$$P(\text{received bit} = b) = 1 - p, P(\text{received bit} \neq b) = p$$

- **Types of errors:**
  - **Single-bit errors:** Only one bit in a codeword is corrupted.
  - **Multiple-bit errors:** More than one bit is corrupted.
  - **Burst errors:** A sequence of consecutive bits is corrupted.

3. Encoding for Error Correction
To detect and correct errors, **redundant bits** are added to the original data bits to form **codewords**. This process is called **encoding**.

3.1 Parity Check Codes

- Add a single parity bit to make the total number of 1s even (even parity) or odd (odd parity).
- Can detect **single-bit errors** but cannot correct them.

Example: For data bits d_1, d_2, d_3, parity bit $p$is:

$$p = d_1 \oplus d_2 \oplus d_3$$

where $\oplus$denotes the XOR operation.

3.2 Repetition Codes

- Each bit is repeated $n$times.
- Example: 0 → 000, 1 → 111 (3-bit repetition).
- Error correction is done using **majority voting**.

3.3 Hamming Codes

- Multiple parity bits are added at positions $2^0, 2^1, 2^2, ...$in the codeword.
- Can detect and **correct single-bit errors** and detect double-bit errors.

4. Working Principle

1. Original data → **Encoding** (add redundancy).
2. Transmit through **noisy channel** → errors may occur.
3. At the receiver, **syndrome or parity check** is used to detect errors.
4. Correct errors (if possible) → recover original data.

Equation for received codeword R:$R = C \oplus E$

where C= transmitted codeword, E= error vector.

Error Recovery

In any communication or storage system, errors in data are inevitable due to noise, interference, or hardware faults. Detecting errors is only the first step; the system must **recover the original data** to maintain reliability. **Error recovery** refers to the techniques used to **correct detected errors** and ensure that the transmitted or stored information is accurately reconstructed. Error recovery is a key component of **classical error control** and also forms the foundation for **quantum error correction**.

2. Goals of Error Recovery

- **Correct corrupted data:** Restore the original message accurately.
- **Maintain system reliability:** Ensure smooth operation of communication or computation systems.
- **Optimize resources:** Reduce retransmission and bandwidth usage where possible.

3. Methods of Error Recovery

3.1 Automatic Repeat Request (ARQ)

- ARQ is a **feedback-based recovery method**. The receiver detects errors using parity checks, checksums, or cyclic redundancy check (CRC).
- If an error is detected, the receiver **requests retransmission** of the affected data.
- Common ARQ protocols:
  - **Stop-and-Wait ARQ:** Sender transmits one frame at a time and waits for acknowledgment. Efficient for low-error channels.
  - **Go-Back-N ARQ:** Sender continues sending a number of frames; if an error is detected in a frame, all subsequent frames are resent. Suitable for high-speed links.
  - **Selective Repeat ARQ:** Only erroneous frames are retransmitted, improving efficiency over Go-Back-N.

Advantages: Reliable and simple.
Disadvantages: Increased latency due to retransmission; requires feedback channel.

3.2 Forward Error Correction (FEC)

- FEC is a **proactive method** where redundancy is added at the transmitter. The receiver can **detect and correct errors without retransmission**.
- Uses **mathematical coding techniques** such as:
  - **Repetition Codes:** Each bit is repeated multiple times; majority voting corrects errors.
  - **Hamming Codes:** Multiple parity bits allow detection and correction of single-bit errors.

- **Cyclic Codes (CRC):** Can detect burst errors efficiently.

- FEC is widely used in **satellite communications, deep-space communication, and streaming applications**, where retransmission is costly or impossible.

Advantages: Reduces the need for retransmission; useful in high-latency systems.
Disadvantages: Requires extra bandwidth for redundant bits; complexity increases with code strength.

4. Mathematical Representation
Let the transmitted codeword be $C$and the received codeword be R, with an error vector E: $R = C \oplus E$

The recovery process identifies Eusing **syndromes or parity checks**. The original codeword is then recovered: $C = R \oplus E$

Where $\oplus$denotes XOR operation.

For example, in a Hamming (7,4) code, 3 parity bits are added to 4 data bits. The **syndrome** at the receiver indicates which bit, if any, is in error.

The Classical Three-Bit Code
The Classical Three-Bit Code is one of the **simplest error-correcting codes** used in classical information theory. It is a type of **repetition code**, where each bit of the original data is repeated three times to allow **detection and correction of single-bit errors**. This code is a foundational example in **classical error correction** and provides insight into more advanced coding schemes.

2. Encoding Procedure

- Each original bit b(0 or 1) is encoded as **three identical bits**.
- Encoding rules:
  - $0 \rightarrow 000$
  - $1 \rightarrow 111$
- The resulting **codeword** contains redundancy that helps detect and correct errors during transmission.

Example:
If the original message is 101, the encoded codeword becomes:

$$101 \rightarrow 111\ 000\ 111$$

3. Error Detection and Correction

- Suppose a codeword is transmitted through a **noisy channel**, and a single bit flips.
- At the receiver, **majority voting** is used to determine the original bit:
  - If two or more bits are 0 → decoded as 0
  - If two or more bits are 1 → decoded as 1

Example:

- Transmitted codeword: 111
- Received codeword: 101 (error in second bit)
- Majority voting: two 1s → decoded as 1 (correct recovery)

5. Advantages and Disadvantages

Advantages:

- Simple and easy to implement.
- Corrects single-bit errors reliably.

Disadvantages:

- Inefficient: triples the number of bits transmitted (high redundancy).
- Cannot correct multiple-bit errors.

Classical Error Correction: Fault Tolerance

In classical information processing, **faults or errors** in transmitted or stored data can occur due to noise, interference, or hardware faults. **Fault tolerance** is the ability of a system to **continue correct operation even when such errors occur**. Classical fault tolerance ensures **reliable communication, computation, and data integrity** by combining **error detection, error correction, and redundancy**.

2. Objectives of Fault Tolerance

- **Reliability:** Ensure the system works correctly despite errors.
- **Data Integrity:** Protect information from corruption or loss.
- **Continuous Operation:** Maintain system functionality even under component failures.
- **Error Recovery:** Detect and correct errors automatically to avoid manual intervention.

3. Principles of Fault Tolerance

- **Redundancy:** Add extra information or components (e.g., repeated bits, parity bits, or additional hardware).
- **Error Detection and Correction:** Use classical codes like **parity codes, Hamming codes, or repetition codes**.
- **Replication:** Duplicate critical components or processes to mask failures.
- **Majority Voting:** When multiple copies of data or processes exist, the **most frequent result** is assumed correct.

4. Techniques in Classical Error Correction for Fault Tolerance

4.1 Repetition Codes

- Each bit is repeated multiple times
- **Majority voting** at the receiver corrects single-bit errors.
- Error correction capability $t = \left\lfloor \frac{n-1}{2} \right\rfloor$ where n= number of repeated bits.

4.2 Hamming Codes

- Add multiple **parity bits** at positions 2^0, 2^1, 2^2, … to detect and correct single-bit errors.
- Number of parity bits required:

$$2^r \geq m + r + 1$$

where m= data bits, r= parity bits.

4.3 Triple Modular Redundancy (TMR)

- Three identical systems perform the same computation.
- The **majority vote** decides the correct output:

$$O = \text{majority}(O_1, O_2, O_3)$$

- This approach **tolerates a single faulty module** and ensures correct operation.

Quantum Information: Quantum Teleportation

Quantum teleportation is a **protocol in quantum information theory** that allows the **transfer of an unknown quantum state** from one location to another **without physically sending the particle itself**. It uses **entanglement, classical communication, and quantum measurement** to achieve this. Quantum teleportation is a key concept in **quantum communication, quantum computing, and quantum networks**.

2. Principles Used in Quantum Teleportation

1. **Quantum Entanglement:** Two qubits share a correlated state such that the state of one qubit depends on the other, even when separated by large distances.
2. **Superposition:** A qubit can exist in a combination of $|0\rangle$ and $|1\rangle$ states.
3. **Classical Communication:** Two classical bits are sent from sender to receiver to complete the teleportation process.
4. **Quantum Measurement:** Measurement of entangled qubits collapses their states, allowing the receiver to reconstruct the original quantum state.

Steps of Quantum Teleportation

1. **Initial Setup**
   - Alice has qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ (unknown state).
   - Alice and Bob share a Bell state ($\Phi^+ = (|00\rangle + |11\rangle)/\sqrt{2}$).
2. **Entangling Alice's Qubits**
   - Alice applies CNOT between her unknown qubit and her part of Bell pair.
   - Then applies a Hadamard gate on the unknown qubit.
3. **Measurement**
   - Alice measures her two qubits in the computational basis.
   - She gets one of four possible results (00, 01, 10, 11).
   - She sends these **two classical bits** to Bob.
4. **Bob's Correction**

- Depending on Alice's result, Bob applies a correction gate on his qubit:
  - If result = 00 → apply I (do nothing)
  - If result = 01 → apply X
  - If result = 10 → apply Z
  - If result = 11 → apply XZ
- After correction, Bob's qubit becomes **$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$** (the original state).

6. Applications

- **Quantum Communication**
- **Quantum Computing**
- **Quantum Networks**

Quantum Dense Coding
Quantum dense coding is a **quantum communication protocol** that allows **sending two classical bits of information by transmitting only one qubit**, using the principle of **quantum entanglement**. It is an important concept in **quantum information theory**, demonstrating the advantage of quantum resources over classical systems.

2. Principles Used in Quantum Dense Coding

1. **Quantum Entanglement:** Two parties, Alice (sender) and Bob (receiver), share an **entangled qubit pair**.
2. **Unitary Operations:** Alice applies one of four unitary operations (I, X, Z, XZ) to encode 2 classical bits into her qubit.
3. **Quantum Measurement:** Bob performs a **Bell-state measurement** on the received qubit and his half of the entangled pair to decode the information.

3. The Dense Coding Protocol

Step 1: Prepare Entangled Pair

- Alice and Bob share a **Bell state**:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Step 2: Alice Encodes Classical Bits

- To send **2 classical bits**, Alice applies one of four unitary operations to her qubit:
  - **00 → I (identity)**
  - **01 → X (bit flip)**
  - **10 → Z (phase flip)**
  - **11 → XZ (bit + phase flip)**

Step 3: Alice Sends Her Qubit to Bob

- After applying the operation, Alice sends **her single qubit** to Bob.

Step 4: Bob Decodes Information

- Bob performs a **Bell-state measurement** on both qubits.
- The measurement outcome corresponds to the **2 classical bits** Alice sent.

5. Advantages

- Dense coding uses **entanglement as a communication resource**.
- It allows **sending more classical information than the number of qubits transmitted**.
- Requires both **quantum entanglement** and **quantum operations** for encoding and decoding.

6. Applications

- **Quantum Communication**
- **Quantum Cryptography**

- **Quantum Networks**

Quantum Key Distribution (QKD)

Quantum Key Distribution is a **secure communication protocol** that allows two parties, typically called Alice (sender) and Bob (receiver), to **generate and share a secret cryptographic key** using **quantum mechanics principles**. The main advantage of QKD is that it can **detect any eavesdropping** attempt due to the **fundamental laws of quantum physics**.

2. Principles Used in QKD

1. **Quantum Superposition:** Qubits can exist in a combination of |0⟩ and |1⟩ states.
2. **Quantum Measurement:** Measuring a qubit **disturbs its state**, allowing detection of eavesdropping.
3. **No-Cloning Theorem:** An unknown quantum state **cannot be copied**, preventing an eavesdropper from duplicating qubits without detection.
4. **Entanglement (optional):** Some QKD protocols, like E91, use entangled pairs to distribute secure keys.

3. The BB84 Protocol (Most Common QKD Protocol)

Step 1: Preparation and Transmission

- Alice prepares a random sequence of qubits using **two bases**:
  - **Rectilinear basis (+):** |0⟩, |1⟩
  - **Diagonal basis (×):** |+⟩ = (|0⟩ + |1⟩)/√2, |−⟩ = (|0⟩ − |1⟩)/√2
- Alice sends these qubits to Bob over a quantum channel.

Step 2: Measurement by Bob

- Bob randomly chooses a basis (+ or ×) to measure each qubit.
- Due to quantum mechanics, **correct measurement only occurs if Alice's and Bob's bases match**.

Step 3: Sifting

- Alice and Bob publicly compare their bases (not the actual bit values).
- They **keep only the bits where the bases matched**, forming the **raw key**.

Step 4: Error Checking and Privacy Amplification

- Alice and Bob check a subset of the raw key for errors to detect **eavesdropping (Eve)**.
- If errors exceed a threshold, the key is discarded.
- Privacy amplification reduces Eve's potential knowledge, generating a **final secure key**.

4. Mathematical Representation

- Qubit in rectilinear basis: |0⟩ or |1⟩
- Qubit in diagonal basis: |+⟩ = (|0⟩ + |1⟩)/√2, |−⟩ = (|0⟩ − |1⟩)/√2
- Probability of correct measurement if bases match: 1
- Probability of correct measurement if bases mismatch: 0.5

5. Advantages

- QKD **guarantees unconditional security** based on quantum mechanics.
- Detects eavesdropping automatically, unlike classical cryptography.

6. Applications

- **Secure Communication**
- **Quantum Networks**
- **Cryptography**

Noise and Error Models in Quantum Systems

Quantum systems are extremely sensitive to **environmental interactions, decoherence, and operational imperfections**. These unwanted interactions introduce **noise and errors**, which can corrupt quantum information. Understanding noise and error models is essential for designing **quantum error correction codes** and achieving **fault-tolerant quantum computation**.

2. Types of Quantum Noise

1. **Decoherence:** Loss of quantum coherence due to interaction with the environment. It causes **superposition states to collapse** into classical mixtures.
2. **Amplitude Damping:** Models energy loss from a qubit to the environment, e.g., a qubit |1⟩ decaying to |0⟩.
3. **Phase Damping (Dephasing):** Only the **phase information** of the qubit is lost, without energy change, affecting superposition.
4. **Depolarizing Noise:** The qubit randomly becomes **completely mixed** with a certain probability, modeling uniform random errors.
5. **Bit-flip and Phase-flip Errors:**
   - **Bit-flip (X error):** |0⟩ ↔ |1⟩
   - **Phase-flip (Z error):** |+⟩ ↔ |−⟩
6. **Combination Errors:** Often, a qubit experiences **simultaneous bit-flip and phase-flip errors (Y error)**.

3. Quantum Error Models
Quantum errors are often represented using **Pauli operators** (I, X, Y, Z):

- **Identity (I):** No error
- **Bit-flip (X):** X|0⟩ = |1⟩, X|1⟩ = |0⟩
- **Phase-flip (Z):** Z|+⟩ = |−⟩, Z|−⟩ = |+⟩
- **Bit-phase-flip (Y):** Y = iXZ, combination of bit-flip and phase-flip

4. Common Quantum Noise Channels

 Bit-Flip Channel: Randomly flips a qubit state from |0⟩|0⟩|0⟩ to |1⟩|1⟩|1⟩ or vice versa, similar to a classical bit error.

 Phase-Flip Channel: Randomly changes the phase of a qubit by flipping the sign of the |1⟩|1⟩|1⟩ state.

 Depolarizing Channel: Replaces the qubit state with a completely mixed state with certain probability, causing loss of information.

 Amplitude Damping Channel: Models energy loss where a qubit relaxes from ∣1⟩|1⟩∣1⟩ to ∣0⟩|0⟩∣0⟩, common in real quantum systems.

Quantum Cryptography and Secure Communication

Quantum cryptography is a branch of **quantum information science** that uses the principles of **quantum mechanics** to achieve **unconditionally secure communication**. Unlike classical cryptography, whose security depends on **computational complexity**, quantum cryptography guarantees security based on **fundamental laws of physics**, such as the **no-cloning theorem** and **measurement disturbance**.

2. Principles of Quantum Cryptography

1. **Quantum Superposition:** Qubits can exist in a combination of states |0⟩ and |1⟩.
2. **Quantum Measurement:** Measuring a quantum state **disturbs it**, revealing any eavesdropping attempt.
3. **No-Cloning Theorem:** Unknown quantum states **cannot be copied**, preventing duplication of qubits by an adversary.
4. **Entanglement:** Pairs of entangled qubits exhibit correlations that can be used for secure key generation and communication.

3. Quantum Key Distribution (QKD)

- QKD is the **most widely used quantum cryptography protocol** for secure communication.
- **BB84 Protocol:**
  - Alice sends qubits prepared in **rectilinear (+) or diagonal (×) bases**.
  - Bob randomly measures in one of the two bases.
  - Bases are compared publicly, and matching results form the **raw key**.
  - Error checking detects **eavesdropping**. Privacy amplification ensures **secure final key**.
- **E91 Protocol:** Uses **entangled qubit pairs** to distribute secure keys.

Security Features:

- Any eavesdropping introduces **detectable errors** in the quantum channel.
- Keys are **information-theoretically secure**, not depending on computational assumptions.

4. Secure Communication using Quantum Cryptography

1. **Key Generation:** QKD generates a **shared secret key** between Alice and Bob.
2. **Encryption:** The shared key is used in **classical symmetric encryption** (e.g., one-time pad) to encrypt messages.
3. **Transmission:** Encrypted message is sent over a **classical channel**.
4. **Decryption:** Receiver uses the shared key to decrypt the message securely.

Advantages:

- **Unconditional security** guaranteed by quantum physics.
- **Eavesdropping detection:** Any attempt to intercept key qubits introduces errors.
- **Resistance to future attacks** (including attacks by quantum computers).

6. Applications of Quantum Cryptography

- **Military and government communication**
- **Banking and finance**
- **Quantum networks and quantum internet**
- **Future-proof security**

U6

Quantum Problem Solving: Heuristic Search

Quantum heuristic search refers to using **quantum algorithms** (based on quantum mechanics principles) to efficiently find approximate or optimal solutions in large and complex search spaces. It is a **quantum version** of classical heuristic search methods that exploit superposition and interference to speed up search processes.

In classical computing, heuristic search algorithms (like A*, hill climbing, or simulated annealing) use heuristics — *educated guesses* — to guide the search toward a solution more quickly. In **quantum computing**, we apply similar ideas but use **quantum parallelism**, where many possible solutions are processed at once in a *superposition of states*.

A common approach is **Grover's Algorithm**, which performs an **unsorted search** in $O(\sqrt{N})$time instead of $O(N)$— a quadratic speed-up over classical search.

Working:

1. **Problem Representation:**
   Encode all possible solutions as quantum states $|\, x\rangle$.
2. **Superposition Initialization:**
   A quantum register is prepared in a superposition of all possible states:$|\, \psi\rangle = \frac{1}{\sqrt{N}}\sum_{x=0}^{N-1} \quad |\, x\rangle$
3. **Oracle Function:**
   A quantum oracle marks the solution state(s) by flipping their phase if they meet the heuristic or target condition.
4. **Amplitude Amplification:**
   Quantum interference amplifies the probability of the correct state(s) and suppresses incorrect ones through Grover iterations.
5. **Measurement:**
   Measuring the final state collapses it to the most probable (i.e., best or near-optimal) solution.

Minimal Math Example:

Grover's algorithm uses $O(\sqrt{N})$oracle queries to find a solution among Npossibilities.
For example, if $N = 1{,}000{,}000$:

- Classical search: needs ~1,000,000 checks.
- Quantum search: needs only about 1,000 checks.

Advantages:

- **Faster search**
- **Parallel exploration**
- **Better for complex problems**

Disadvantages:

- **Hardware limitations**
- **Decoherence and noise**
- **Probabilistic results**
- **Algorithm design complexity.**

Quantum Tree Search

Quantum Tree Search is a **quantum version of classical tree search algorithms**, where quantum computation is used to explore multiple branches of a search tree simultaneously. It applies **quantum parallelism** and **amplitude amplification** to speed up finding goal nodes in large or complex search trees.

In classical AI, **tree search** explores possible actions and their outcomes in a hierarchical structure (tree) until a goal is found. Quantum Tree Search replaces sequential traversal with **quantum superposition**, allowing simultaneous exploration of many branches. Using **Grover's algorithm** or similar quantum search techniques, it reduces the number of steps required to find the desired node.

Instead of exploring one path at a time, the quantum algorithm encodes all possible paths into quantum states and applies interference to amplify the correct path.

Working:

1. **Tree Representation:**
   Each node and path in the tree is represented as a binary string $|x\rangle$(a quantum state).
2. **Superposition Initialization:**
   A superposition of all possible paths is created:

$|\psi\rangle = \frac{1}{\sqrt{N}}\sum_{x=0}^{N-1} |x\rangle$ where $N$is the number of possible paths.

3. **Oracle Operation:**
   A quantum oracle marks the goal nodes (those that satisfy the target condition) by changing their phase.
4. **Amplitude Amplification:**
   Similar to Grover's algorithm, amplitudes of goal nodes are amplified, increasing their measurement probability.
5. **Measurement:**
   After a few iterations, measuring the quantum state yields the goal node with high probability.

If a classical tree search requires $O(N)$steps to check Nnodes, Quantum Tree Search can do it in $O(\sqrt{N})$steps using amplitude amplification.

For example:
If there are 10,000 nodes,

Classical: 10,000 checks   Quantum: ~100 checks

Advantages:

- **Speed-up**
- **Parallel exploration**
- **Useful for AI and optimization**
- **Reduced computation time**

Disadvantages:

- **Hardware challenges**
- **Complex implementation**
- **Probabilistic nature**
- **Limited to specific problems**

Quantum Production System

A **Quantum Production System** is a quantum version of the classical **production system** used in Artificial Intelligence for problem solving. It applies **quantum computation principles** such as superposition and parallelism to execute multiple production rules simultaneously, improving efficiency in rule-based reasoning.

A **production system** in AI consists of three components:

1. **Set of production rules** (IF–THEN statements)
2. **Working memory** (current state information)
3. **Control system** (decides which rule to apply)

In a **quantum production system**, these elements are represented in **quantum states**. Instead of testing one rule at a time, the system can apply **all possible rules in parallel** due to quantum superposition.Quantum interference is then used to strengthen (amplify) correct rule outcomes and weaken incorrect ones.This approach allows **faster reasoning** and **parallel rule evaluation**.

Working:

1. **State Encoding:**
   Each possible state of the system is represented as a quantum state $|s\rangle$.
   The rule base and data are encoded into quantum bits (qubits).
2. **Superposition Initialization:**
   The system starts in a superposition of all possible states:$|\psi\rangle = \frac{1}{\sqrt{N}}\sum_{s=0}^{N-1} |s\rangle$ where $N$is the number of possible states or rules.
3. **Rule Application (Quantum Parallelism):**
   All applicable rules are applied simultaneously to all states through unitary transformations (quantum operations).
4. **Oracle and Interference:**
   An oracle marks the states that satisfy goal conditions, and interference amplifies these correct results.
5. **Measurement:**
   Measuring the final quantum state collapses it into the state (or rule) that leads to the goal.

Minimal Math:

If a classical production system checks Nrules, Quantum Production System can find the correct rule in about $O(\sqrt{N})$steps, providing **quadratic speed-up**.

Example:
For 1,000 rules –

Classical: 1,000 checks Quantum: ~32 checks

Advantages:

- **Parallel rule evaluation**
- **Speed-up**
- **Scalable**
- **Better decision-making**

Disadvantages:

- Complex implementation
- Hardware limitations
- Probabilistic outcomes
- Limited practical uses

Tarrataca's Quantum Production System

Tarrataca's Quantum Production System (QPS) is a **quantum adaptation** of the classical production system model proposed by **Alexandre M. Tarrataca**. It integrates the principles of **quantum computation**—such as **superposition, reversibility, and parallelism**—to simulate intelligent, rule-based reasoning on a quantum computer.

In classical AI, a **production system** consists of:

1. A set of **production rules** (IF–THEN logic),
2. A **working memory** (stores the current state),
3. A **control system** (selects which rule to apply).

Tarrataca extended this idea into the **quantum domain**, where both rules and states are encoded as **quantum states** (qubits). The system operates using **unitary transformations**, ensuring all operations are **reversible**—a key requirement in quantum mechanics.

Unlike classical systems that apply one rule at a time, Tarrataca's QPS applies **all possible rules in parallel** using **quantum superposition** and identifies the correct path to the goal using **amplitude amplification**.

Working:

1. **Initialization:**
   - Encode all possible initial states into a quantum register as a superposition: $|\psi\rangle = \frac{1}{\sqrt{N}}\sum_{s=0}^{N-1} |s\rangle$ where $N$is the number of possible states.
2. **Rule Representation:**
   - Each production rule is represented as a **unitary operator** U_r, which maps input states to output states (maintaining reversibility).
3. **Quantum Inference Cycle:**
   - The system applies all rules simultaneously to all states in superposition.
   - Quantum interference is used to **amplify** states that move toward the goal and **suppress** incorrect ones.
4. **Oracle Function:**
   - An oracle marks goal states (solutions) by flipping their phase.
5. **Measurement:**
   - Measuring the quantum state collapses it into the **goal configuration**, giving the correct sequence of rule applications.

Minimal Math:

If there are Npossible states or rule applications:

- **Classical system:** $O(N)$time
- **Tarrataca's Quantum system:** $O(\sqrt{N})$time

This gives a **quadratic speed-up** similar to Grover's search algorithm.

Advantages:

- **Quantum parallelism**
- **Reversibility**
- **Faster reasoning**
- **Efficient problem-solving**

Disadvantages:

- Complex encoding
- Hardware limitations
- Probabilistic output
- Still theoretical

Quantum AI Application: Introduction to PennyLane

PennyLane is an open-source, **cross-platform Python library** developed by **Xanadu** for **quantum machine learning (QML)**, **quantum computing**, and **hybrid quantum-classical** computations. It allows users to build and train **quantum neural networks** and integrate quantum algorithms with popular deep learning frameworks like **TensorFlow** and **PyTorch**.

PennyLane acts as a **bridge between quantum hardware and classical AI tools**. It enables **differentiable programming** — meaning users can compute gradients of quantum circuits and optimize them using classical optimization techniques. This is especially useful for **quantum machine learning**, **quantum chemistry**, and **quantum optimization**.

The main idea is to treat quantum circuits like layers in a neural network — they take inputs, perform transformations on qubits, and return outputs that can be optimized just like classical models.

Working (Simplified):

1. **Device Setup:**
   You choose a **quantum device** — either a simulator or real hardware (like IBM Q, Rigetti, or Xanadu's own "Strawberry Fields").
   Example: dev = qml.device("default.qubit", wires=2)
2. **Define a Quantum Circuit:**
   Quantum functions (called *QNodes*) define operations on qubits using gates.

```
@qml.qnode(dev)

def circuit(x):

    qml.RX(x, wires=0)

    qml.CNOT(wires=[0, 1])

    return qml.expval(qml.PauliZ(0))
```

3. **Integrate with Classical ML:** The QNode can be combined with classical layers and trained using optimizers from TensorFlow or PyTorch.

4. **Optimization:** PennyLane automatically calculates gradients (using the **parameter-shift rule**) and updates circuit parameters to minimize loss functions, similar to backpropagation in neural networks.

Key Features:

- **Cross-platform:** Works with multiple quantum hardware providers (IBM, Google, Rigetti, etc.).
- **Hybrid computing:** Combines quantum and classical ML seamlessly.
- **Automatic differentiation:** Calculates gradients of quantum circuits automatically.
- **Plugin support:** Works with frameworks like TensorFlow, PyTorch, and JAX.
- **Accessible and flexible:** Designed for researchers and developers in quantum AI.

Applications:

- **Quantum Machine Learning (QML)**
- **Quantum Chemistry**
- **Optimization Problems**
- **Quantum Data Processing**

Quantum AI Application: Quantum Neural Computation

Quantum Neural Computation (QNC) is the study and development of **neural network models that use quantum computing principles** such as **superposition, entanglement, and parallelism** to perform learning and decision-making tasks. It is essentially the **quantum version of artificial neural networks (ANNs)**.

In classical AI, neural networks process information using interconnected neurons that adjust weights during learning. In **Quantum Neural Computation**, the neurons and their connections are represented using **quantum states (qubits)** and **quantum gates**.

The main goal is to use quantum properties to achieve **faster learning**, **better pattern recognition**, and **enhanced problem-solving** compared to traditional neural networks.

QNC combines two major fields:

1. **Quantum Computing** – which provides computational speed and parallelism.
2. **Neural Networks / AI** – which provides learning and adaptability.

Working (Simplified):

1. **Data Encoding:** Classical data is encoded into quantum states (qubits).

2. **Quantum Processing:** Quantum gates act as “neurons” that process information. These gates manipulate the qubits in **superposition**, allowing multiple inputs to be processed simultaneously.

3. **Learning Mechanism:** Parameters (like weights in a neural net) are optimized using **quantum algorithms** and **interference patterns** to improve accuracy.

4. **Measurement:** The final state of the qubits is measured, collapsing into the most probable output — similar to predicting a class label or result.

Applications:

- **Pattern recognition**,**Optimization problems**
- **Quantum control systems**
- **Financial modeling and prediction**
- **Medical diagnosis**

Quantum Walk – Random Insect

A **Quantum Walk** is the **quantum analog of a classical random walk**, where a particle or “walker” (e.g., a random insect) moves across positions in **superposition**, allowing it to explore multiple paths simultaneously.

In the **Random Insect analogy**, the insect represents a particle performing a random walk on a graph or grid, but in the **quantum version**, it can move in multiple directions at once.

In classical random walks, a walker moves randomly step by step (like an insect wandering randomly).
In a **quantum walk**:

- The walker is in a **superposition of multiple positions**, exploring many paths simultaneously.
- **Quantum interference** ensures that some paths amplify while others cancel out, leading to different probability distributions than classical walks.

Quantum walks are a fundamental tool in **quantum algorithms**, enabling faster search, optimization, and graph traversal.

Working (Simplified):

1. **State Representation:** The position of the walker (insect) is encoded as a **quantum state** $|x\rangle$. The walker’s “coin” state determines its movement direction.

2. **Superposition Initialization:** The walker is placed in a superposition of all possible starting positions.

3. **Quantum Step (Coin + Shift):**
    - **Coin flip:** A quantum coin (Hadamard gate) decides movement in multiple directions.
    - **Shift:** Moves the walker according to the coin’s state, creating superposition across positions.

4. **Interference:**
   Paths interfere constructively or destructively, altering the probability of finding the walker at a certain position.

5. **Measurement:**
   Measuring the system gives the walker's position, representing the outcome of the quantum walk.

Applications:

- **Quantum search algorithms**
- **Optimization**
- **Quantum simulation**
- **Artificial Intelligence**

Quantum Walk – Walk on Graph

A **Quantum Walk on a Graph** is the **quantum version of a classical walk** where a particle (walker) moves across the nodes of a graph in **superposition**, exploring multiple paths simultaneously instead of following a single trajectory.

It is widely used in **quantum algorithms** for searching, optimization, and network analysis.

- In a **classical walk on a graph**, a particle moves randomly from node to node along the edges.
- In a **quantum walk**, the particle exists in a **superposition of multiple nodes**, allowing it to explore many paths at once.
- **Quantum interference** alters the probabilities of landing on certain nodes, creating a different probability distribution from classical walks.

Working (Simplified):

1. **Graph Representation:**

○ Nodes of the graph are represented as quantum states $|v\rangle$.

○ Edges define possible transitions between states.

2. **Superposition Initialization:**

○ The walker is placed in a **superposition of starting nodes**, allowing multiple paths to be explored simultaneously.

3. **Quantum Step (Coin + Shift):**

○ **Coin operation:** Determines direction of movement in superposition (like a quantum coin flip).

○ **Shift operation:** Moves the walker to neighboring nodes based on the coin state.

4. **Interference:**

○ Paths interfere constructively or destructively, amplifying the probability of reaching target nodes.

5. **Measurement:**

○ Observing the walker collapses the state to a particular node, providing the outcome of the quantum walk.

Applications:

- **Quantum search algorithms**
- **Optimization**
- **Network analysis**
- **Quantum computing**

Quantum-Centric Supercomputing: The Next Wave of Computing

Quantum-centric supercomputing combines **classical high-performance computing (HPC)** with **quantum computing** to solve problems that are beyond the reach of either technology alone. It uses **quantum processors** to accelerate critical parts of computation while leveraging classical supercomputers for large-scale tasks.

Classical supercomputers are excellent at massive numerical calculations but struggle with **combinatorial, optimization, or quantum simulation problems**.

Quantum-centric supercomputers integrate quantum processors as **accelerators**, like GPUs in classical computing, for tasks that benefit from **quantum parallelism** and **entanglement**.

This hybrid approach can tackle **complex scientific simulations, optimization problems, and AI workloads** faster and more efficiently than classical-only systems.

Case Studies / Examples:

1. **NASA's Quantum Computing Application:**
   - ○ NASA integrates **D-Wave quantum annealers** with classical HPC to study **aircraft optimization** and **spacecraft trajectory planning**.
   - ○ The quantum processor accelerates the search for optimal solutions in complex parameter spaces.
2. **IBM Quantum + Summit Supercomputer:**
   - ○ IBM uses quantum processors alongside the **Summit supercomputer** for **molecular simulations** and **materials science research**.
   - ○ Quantum modules handle quantum chemistry calculations that classical systems struggle with.
3. **Google Quantum AI & Classical HPC:**
   - ○ Google combines its **Sycamore quantum processors** with classical computing resources to simulate **quantum circuits** and explore **optimization problems**.
   - ○ This hybrid method demonstrates early examples of **quantum advantage** in practical tasks.

Quantum Computing for Data Science

Quantum computing for data science refers to the use of **quantum computing principles**—such as superposition, entanglement, and quantum parallelism—to **process, analyze, and extract insights from large datasets** more efficiently than classical computers.

Data science involves handling **large, complex datasets** and solving **optimization, classification, and prediction problems**.Classical computers can struggle with very large datasets or complex models.

Quantum computing offers new ways to **accelerate data analysis**, perform **faster optimization**, and improve **machine learning algorithms** using **quantum-enhanced models**. Quantum techniques like **Quantum Machine Learning (QML)**, **quantum clustering**, and **quantum principal component analysis (qPCA)** are applied to data science problems.

Working (Simplified):

1. **Data Encoding:**
   - Classical data is mapped into **quantum states (qubits)** for processing.
2. **Quantum Processing:**
   - Quantum gates and circuits manipulate qubits in **superposition**, exploring multiple solutions at once.
   - Quantum interference amplifies correct solutions or important patterns.
3. **Quantum Algorithms for Data Science:**

Quantum Support Vector Machines (QSVM), Quantum k-means,Quantum PCA (qPCA)

4. **Measurement & Output:**
   - Observing qubits collapses them into classical outputs like prediction s, classifications, or optimization results.

Applications:

- **Machine learning & AI**
- **Financial analytics:**
- **Healthcare & genomics:**
- **Big data analysis:**

Challenges and Limitations of Quantum Computing in DS

1. Limited Qubit Availability

- Large data science problems require thousands to millions of logical qubits, which are not yet feasible.

2. Noise and Decoherence

- Qubits are extremely sensitive to environmental noise.
- Decoherence causes loss of quantum information, reducing accuracy of computations.

3. Data Encoding (Quantum Data Loading)

- Classical data must be encoded into quantum states (amplitude, angle, basis encoding).
- Data loading is costly and often removes theoretical speed advantages.

4. Error Rates and Low Fidelity

- Quantum gates and measurements have high error rates.
- Accumulated errors limit the depth of quantum circuits.

5. Lack of Scalable Quantum Algorithms

- Only a few quantum algorithms (e.g., Grover, Shor) show proven advantage.
- Many data science tasks lack practical quantum equivalents.

6. Hardware Constraints

- Quantum hardware requires extreme conditions (cryogenic temperatures, vacuum).
- Hardware stability and availability remain limited.

7. Integration with Classical Systems

- Data science workflows are mostly classical.
- Efficient hybrid quantum–classical integration is still immature.

8. Skill and Tooling Gap

- Requires expertise in quantum physics, linear algebra, and programming.
- Limited mature libraries and debugging tools compared to classical ML frameworks

2□ Limitations of Error Model Encoding in Quantum Computing

1. Complex Noise Characteristics

- Quantum noise is non-uniform and hardware-dependent.
- Accurately modeling real-world noise is difficult.

2. Incomplete Noise Models

- Standard error models (bit-flip, phase-flip, depolarizing) are idealized.
- Real quantum devices exhibit correlated and time-varying errors.

3. Qubit-Specific Error Variability

- Different qubits have different error rates.
- Encoding a single error model for all qubits leads to inaccuracies.

4. Gate-Dependent Errors

- Each quantum gate introduces different error probabilities.
- Modeling errors for every gate increases computational overhead.

5. Measurement Errors

- Readout errors distort final results.
- Correctly encoding measurement noise is challenging but crucial.

6. Error Propagation in Circuits

- Errors accumulate and propagate through quantum gates.
- Small inaccuracies grow rapidly in deep circuits.

7. High Cost of Error Mitigation

- Error mitigation techniques increase circuit depth and execution time.
- Not scalable for large data science problems.

8. Lack of Universal Error Correction

- Full quantum error correction requires many physical qubits per logical qubit.
- Current hardware cannot support large-scale error-corrected systems.