14.4

$$x_{n+1} = x_n + (\alpha x_{n+1} - \beta x_{n+1} y_{n+1}) \Delta t$$

$$x_{n+1} - \alpha x_{n+1} \Delta t + \beta x_{n+1} y_{n+1} \Delta t = x_n$$

$$x_{n+1} (1 - \alpha \Delta t + \beta y_{n+1} \Delta t) = x_n$$

$$x_{n+1} = \frac{x_n}{1 - \alpha \Delta t + \beta y_{n+1} \Delta t}$$

$$y_{n+1} = y_n + (\delta x_{n+1} y_{n+1} - \gamma y_{n+1}) \Delta t$$

$$y_{n+1} = y_n + \delta x_{n+1} y_{n+1} \Delta t - \gamma y_{n+1} \Delta t$$

$$y_{n+1} - \delta x_{n+1} y_{n+1} \Delta t + \gamma y_{n+1} \Delta t = y_n$$

$$y_{n+1} (1 - \delta x_{n+1} \Delta t + \gamma \Delta t) = y_n$$

$$y_{n+1} = \frac{y_n}{(1 - \delta x_{n+1} \Delta t + \gamma \Delta t)}$$

Sub x_{n+1} in y_{n+1}

$$y_{n+1} = \frac{y_n}{1 - \delta \left(\frac{x_n}{1 - \alpha \Delta t + \beta y_{n+1} \Delta t} \right) \Delta t + \gamma \Delta t}$$

$$1 - \alpha \Delta t + \beta y_{n+1} \Delta t - \delta x_n \Delta t + \gamma \Delta t [1 - \alpha \Delta t + \beta y_{n+1} \Delta t]$$

$$1 - \alpha \Delta t + \beta y_{n+1} \Delta t - \delta x_n \Delta t + \gamma \Delta t - \alpha \Delta t^2 \gamma + \gamma \beta y_{n+1} \Delta t^2$$

$$\Rightarrow y_{n+1} - \alpha \Delta t\, y_{n+1} + \beta (y_{n+1})^2 \Delta t - \delta x_n y_{n+1} \Delta t + \gamma \Delta t\, y_{n+1} - \alpha \Delta t^2 \gamma\, y_{n+1} + \gamma \beta (y_{n+1})^2 \Delta t^2$$

$$= y_n [1 - \alpha \Delta t + \beta y_{n+1} \Delta t]$$

$$= y_n - \alpha \Delta t\, y_n + \beta y_{n+1} y_n \Delta t$$

$$(y_{n+1})^2 [\beta \Delta t + \gamma \beta \Delta t^2] + y_{n+1} [1 - \alpha \Delta t - \delta x_n \Delta t + \gamma \Delta t - \alpha \Delta t^2 \gamma - \beta y_n \Delta t]$$

$$- y_n - \alpha \Delta t\, y_n = 0$$

Sol $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$a_y = [\beta \Delta t + \gamma \beta \Delta t^2]$$

$$b_y = [1 - \alpha \Delta t - \delta x_n \Delta t + \gamma \Delta t - \alpha \Delta t^2 \gamma - \beta y_n \Delta t]$$

$$c_y = - y_n - \alpha \Delta t\, y_n$$

To find a_x b_x c_x Sub y_{n+1} in x_{n+1}

$$x_{n+1} = \frac{x_n}{1 - \alpha \Delta t + \beta y_{n+1} \Delta t} \qquad y_{n+1} = \frac{y_n}{(1 - \delta x_{n+1} \Delta t + \gamma \Delta t)}$$

$$x_{n+1} = \frac{x_n}{1 - \alpha \Delta t + \beta \dfrac{y_n}{1 - \Delta t \delta x_{n+1} + \gamma \Delta t} \Delta t}$$

$$\frac{1 - \Delta t \delta x_{n+1} + \gamma \Delta t - \alpha \Delta t + \beta y_n \Delta t - \alpha \Delta t + \alpha \Delta t^2 \delta x_{n+1} - \alpha \Delta t^2 \gamma}{1 - \Delta t \delta x_{n+1} + \gamma \Delta t}$$

$$x_{n+1} - \Delta t \delta (x_{n+1})^2 + \gamma \Delta t x_{n+1} - \alpha \Delta t x_{n+1} + \beta y_n x_{n+1} \Delta t - \alpha \Delta t x_{n+1} + \alpha \Delta t^2 \delta (x_{n+1})^2 - \alpha \Delta t^2 \gamma x_{n+1} = x_n - \Delta t \delta x_{n+1} x_n + \gamma \Delta t x_n$$

$$(x_{n+1})^2 \left[\alpha \Delta t^2 \delta - \Delta t \delta\right] + \left[1 + \gamma \Delta t - \alpha \Delta t + \beta y_n \Delta t - \alpha \Delta t - \alpha \Delta t^2 \gamma + \Delta t \delta x_n\right] - x_n - \gamma \Delta t x_n = 0$$

Multiply by '-1'

$$a = \Delta t \delta - \alpha \Delta t^2 \delta$$

$$b = -\left[1 - \gamma \Delta t - 2\alpha \Delta t + \beta y_n \Delta t - \Delta t^2 \alpha \gamma + \Delta t \delta x_n\right]$$

$$c = x_n + \gamma \Delta t x_n$$