Exercise 14.5. Symplectic Euler integration of the Lotka–Volterra model.
with x explicit and y implicit

In [ ]:
```
import numpy as np
from matplotlib import pyplot as plt
from scipy.integrate import odeint

alpha =2/3
beta = 4/3
gamma = 1
kronicha_delta = 1
x0=1
y0=1
dt=0.05
total_t = int(20/dt)

def calcualte_invariant(x,y):
    return kronicha_delta*x - gamma*np.log(x)+ beta*y - alpha*np.log(y)
print(total_t)

def model(y, t):
    x, y = y
    dxdt = alpha*x - beta*x*y
    dydt = -gamma*y + kronicha_delta*x*y
    return [dxdt, dydt]


x= np.zeros((total_t,1))
y= np.zeros((total_t,1))
x[0]=x0
y[0]=y0
invarient =np.zeros((total_t,1))
invarient[0]= calcualte_invariant(x[0],y[0])
t_list= np.linspace(0,20, total_t)

for i in range(1,total_t):
    y_inter = y[i-1]/(1-(dt*(kronicha_delta*x[i-1]-gamma)))

    x[i] = x[i-1] + (alpha*x[i-1] - beta*x[i-1]*y_inter)*dt
    y[i] = y[i-1] + (kronicha_delta*x[i-1]*y_inter- gamma*y[i-1])*dt
    invarient[i]= calcualte_invariant(x[i],y[i])

x_initials = np.linspace(0.5, 2, 3)
y_initials = np.linspace(0.5, 2, 3)

plt.plot(t_list,x, label='Prey x')
plt.plot(t_list,y, label='predators y')
plt.legend(fontsize ='small')
plt.xlabel('t',fontsize ='small')
plt.ylabel('N(t)',fontsize ='small')
# plt.title('Numerical solutions x(t) and y(t) of the Lotka-Volterra model' \'obtained using the symplectic Euler int
plt.title('Numerical solutions x(t) and y(t) of the Lotka-Volterra model\n'
          'obtained using the symplectic Euler integration scheme (equations (14.12),\n'
          'x implicit and y explicit', fontsize='small')
plt.xticks(fontsize='10')
plt.yticks(fontsize='10')
plt.show()

matrix = np.full((total_t,1), [invarient[0]])

plt.plot(t_list, invarient, label = 'Numerical solution' )
plt.plot(t_list,matrix, linestyle ='dashed', color='black')
plt.xlabel('t',fontsize ='small')
plt.ylabel('I(t)',fontsize ='small')
plt.title('I (x, y) versus time; x implicit and y explicit',fontsize ='small')
plt.ylim(0,4)
plt.legend(fontsize ='small')
plt.xticks(fontsize='10')
plt.yticks(fontsize='10')
plt.show()

plt.plot(x,y)
plt.plot(x[0],y[0], marker ='s', label ='intial point')
plt.plot(x[-1],y[-1], marker ='o', label ='end point')

plt.xlabel('Preys x', fontsize ='small')
plt.ylabel('Predactors y', fontsize ='small')
for x0 in x_initials:
    for y0 in y_initials:
        initial_conditions = [x0, y0]
        solution = odeint(model, initial_conditions, t_list)
        plt.plot(solution[:, 0], solution[:, 1], color = 'gray', linewidth=0.5)
plt.title('Trajectory in phase space;x implicit and y explicit',fontsize ='small')
plt.legend(fontsize='small')
plt.xticks(fontsize='10')
plt.yticks(fontsize='10')
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plt.show()
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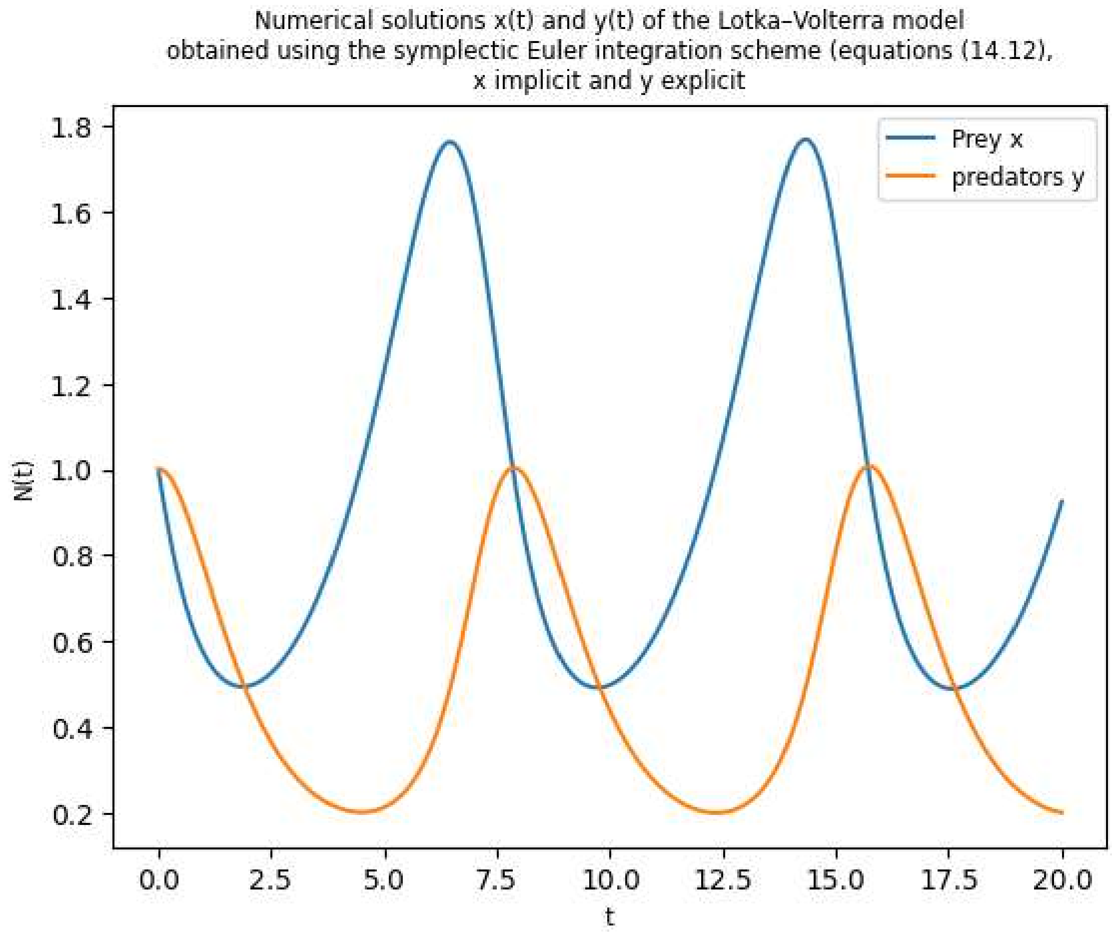
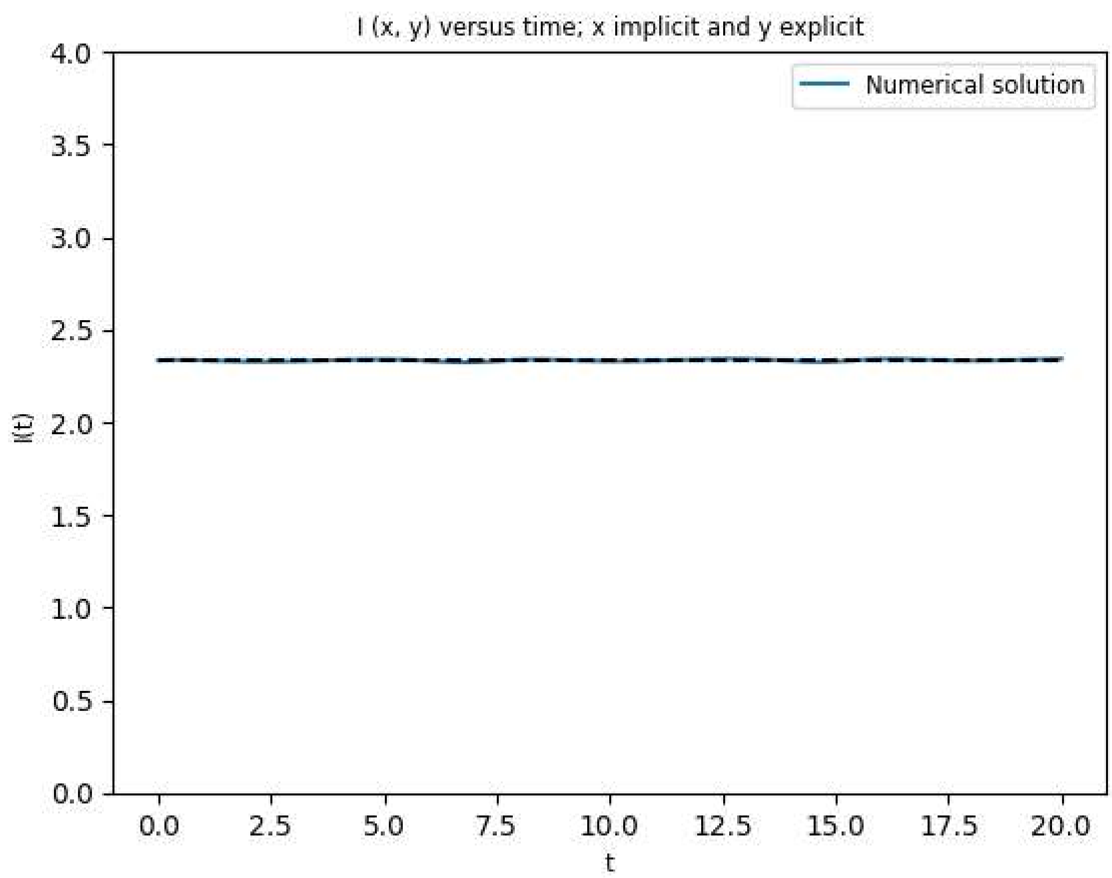
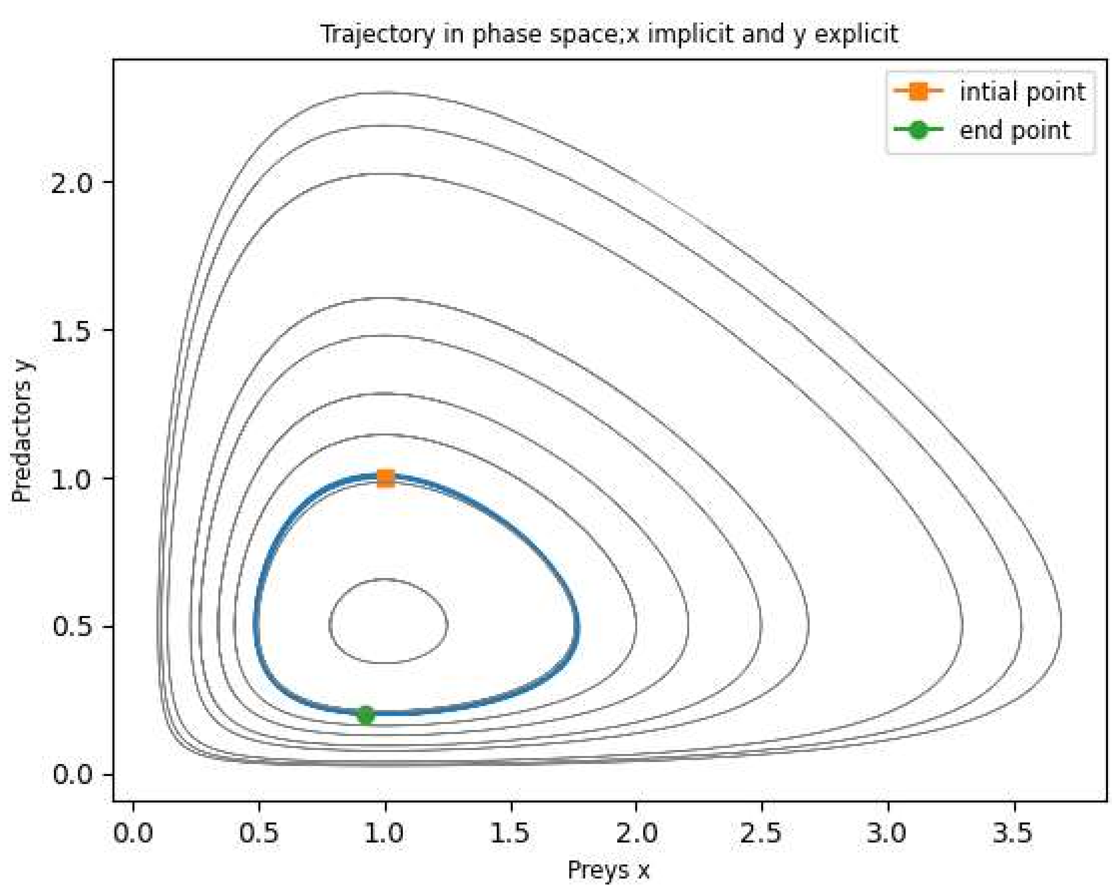
400







In [ ]:
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gamma = 1
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total_t = int(20/dt)

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y= np.zeros((total_t,1))
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invarient =np.zeros((total_t,1))
invarient[0]= calcualte_invariant(x[0],y[0])
t_list= np.linspace(0,20, total_t)

for i in range(1,total_t):
    x_inter = x[i-1]/(1-dt*(alpha- beta*y[i-1]))

    x[i]= x[i-1] + (alpha*x_inter - beta* x_inter*y[i-1])*dt
    y[i] = y[i-1] + (kronicha_delta*x_inter*y[i-1]-gamma*y[i-1])*dt

    invarient[i]= calcualte_invariant(x[i],y[i])

x_initials = np.linspace(0.5, 2, 3)
y_initials = np.linspace(0.5, 2, 3)

plt.plot(t_list,x, label='Prey x')
plt.plot(t_list,y, label='predators y')
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for x0 in x_initials:
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        solution = odeint(model, initial_conditions, t_list)
        plt.plot(solution[:, 0], solution[:, 1], color = 'gray', linewidth=0.5)
plt.title('Trajectory in phase space;x implicit and y explicit',fontsize ='small')
plt.legend(fontsize='small')
plt.xticks(fontsize='10')
plt.yticks(fontsize='10')

plt.show()
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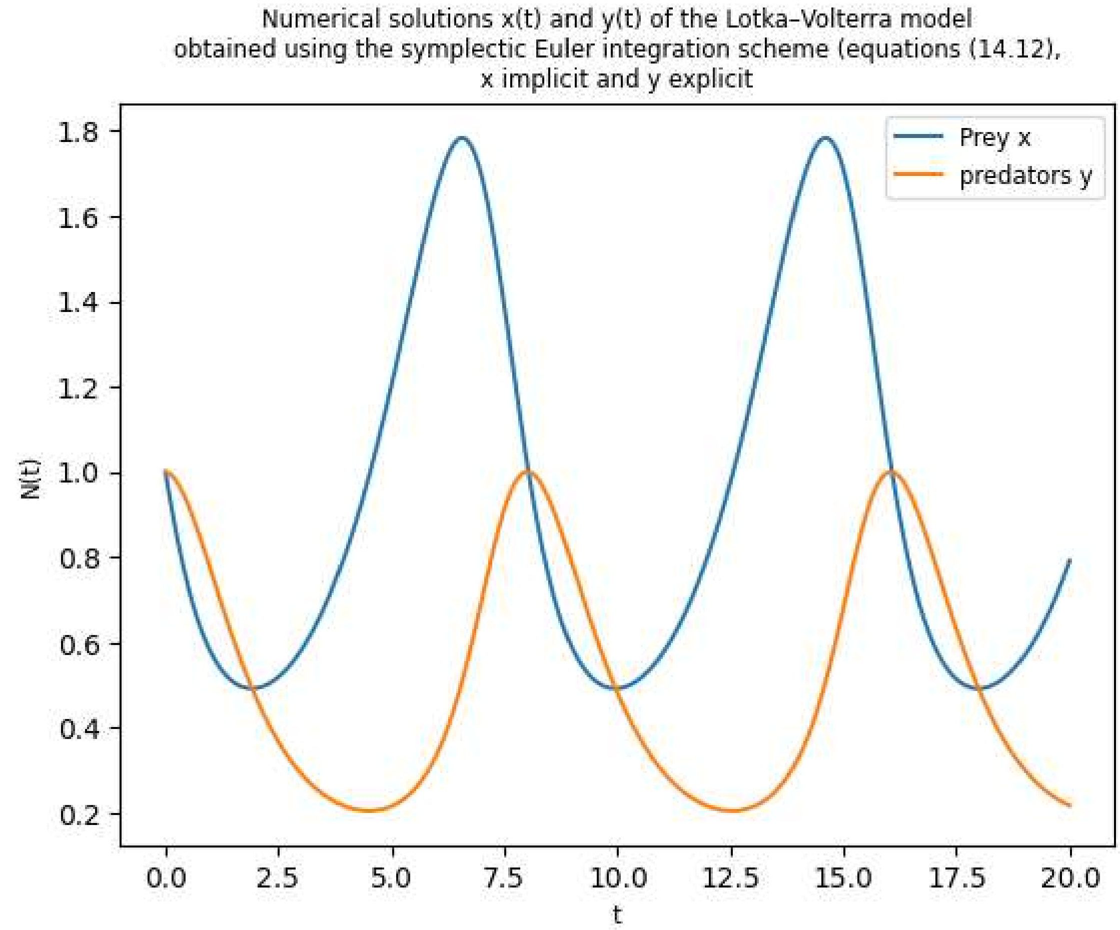
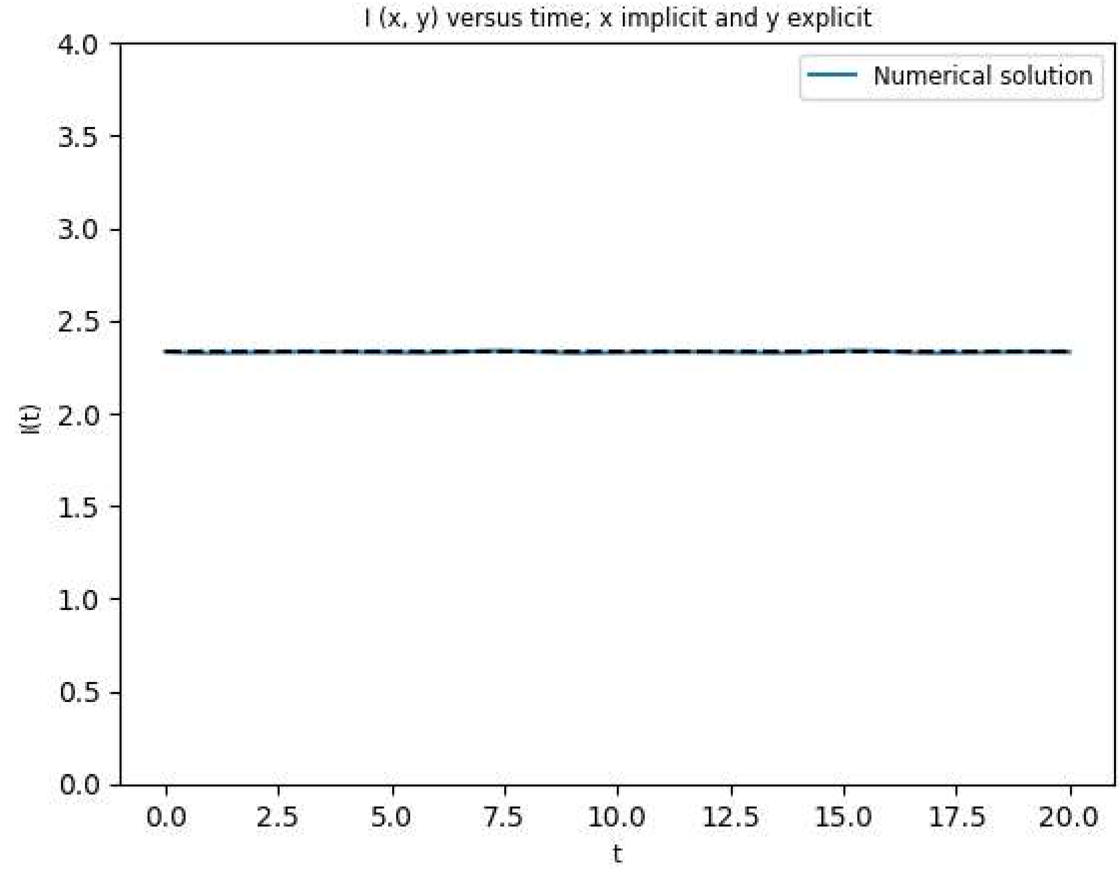
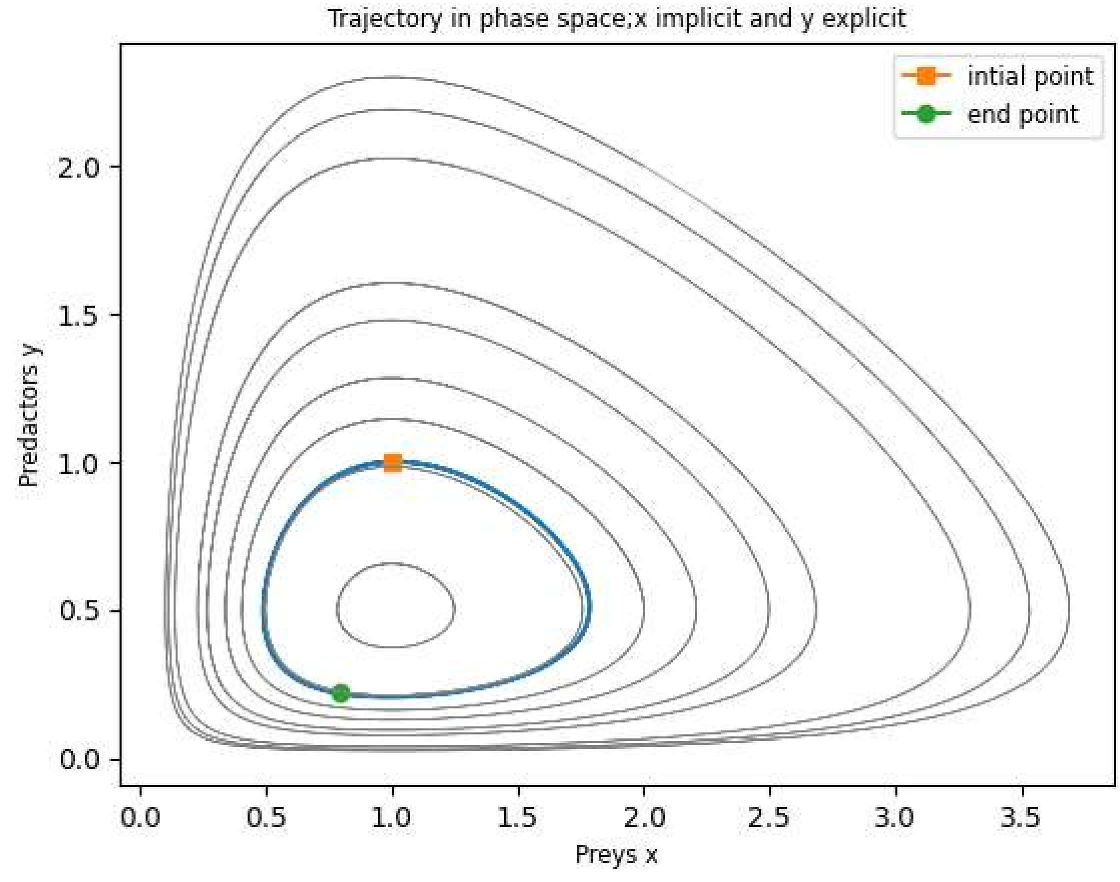

Numerical solutions x(t) and y(t) of the Lotka–Volterra model
obtained using the symplectic Euler integration scheme (equations (14.12),
x implicit and y explicit
Prey x
predators y
N(t)
t
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0


I (x, y) versus time; x implicit and y explicit
Numerical solution
I(t)
t
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0


Trajectory in phase space;x implicit and y explicit
intial point
end point
Predactors y
Preys x
0.0
0.5
1.0
1.5
2.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5