MA 203 PROJECT PRESENTATION

MATHEMATICAL MODELING OF GLUCOSE CONTROL IN DIABETIC PATIENTS

GROUP – 19

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ABSTRACT

Our project focuses on enhancing glucose control in diabetic patients using the Bergman minimal model. Diabetes, an endocrine disorder, results in poor blood glucose regulation, often made worse by obesity. Current treatments lack predictive control, potentially leading to hypoglycemic episodes. Recent biomedical advancements offer continuous glucose monitoring and insulin pumps for finer control. Our main goal is to develop an efficient numerical solution to solve the Ordinary Differential Equations (ODEs) within the Bergman model, addressing the dynamic interplay between glucose and insulin in diabetes management.

PROBLEM STATEMENT

Diabetes, a chronic endocrine disorder, presents a significant challenge in maintaining optimal blood glucose levels. While treatments such as exercise, dieting, oral medications, and insulin injections are available, they often lack predictive control, potentially causing hypoglycemic episodes. Recent advances in biomedical technology offer continuous glucose monitoring devices and insulin pumps, but efficient integration is needed. The main problem is the absence of a numerical solution for effectively addressing the dynamics of glucose and insulin within the human body, specifically within the Bergman minimal model. Our project aims to develop this numerical approach to optimize glucose control and improve the management of diabetes.

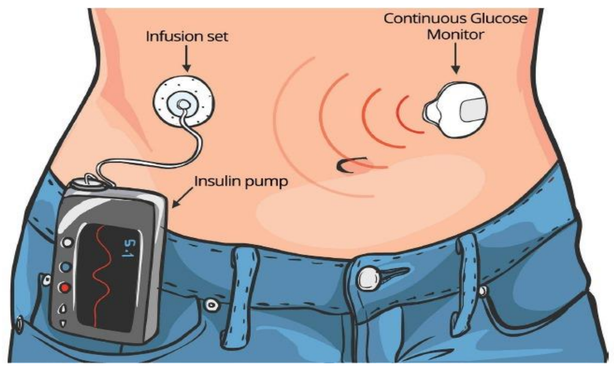


Figure 1 Insulin Pump

MATHEMATICAL MODEL

Our mathematical model starts with some simple mass balance equations to help us keep track of how glucose and insulin change as time goes by. Imagine glucose being split into different parts, each responsible for maintaining a certain baseline glucose level (known as Gb). In our model, we pay special attention to processes like how glucose is taken up by tissues in the body, how the liver deals with it, and how the liver itself produces glucose. These processes all work together to establish what we call the Net Hepatic Glucose Balance (NHGB), and insulin has a role in influencing this balance. We have equations that describe how these processes are interconnected.

Additionally, our model takes into account the changes in active insulin (referred to as I2) over time.

We introduce a time delay because insulin takes a bit of time to move through the tiny blood vessels called capillaries. This means that when there's an excess of insulin, it gets stored away in a sort of reserve, and when there's not enough, it comes back into circulation. We've also included a process for clearing insulin from the system. All of these elements combined make our model more detailed and accurate in representing how glucose and insulin behave.

These equations lead to differential equations for glucose (dG/dt) and insulin (dI2/dt) dynamics. Insulin sensitivity (X) is also derived from these equations.

EQUATIONS

The equations used are:

$$\frac{dG}{dt} = -(p_1 + X) \cdot G + p_1 \cdot G_b$$

Here p1 is glucose clearance rate. This equation describes the rate of change in glucose concentration as influenced by insulin concentration (X) and glucose basal production (Gb).

$$\frac{dI}{dt} = p_6 \cdot (G - p_5)^{+} t - p_4 \cdot (I - I_b)$$

where $(\dots)^{+}$ will be 0 if $(\dots) < 0$.

The rate of insulin concentration change (dI/dt) is influenced by glucose levels relative to a threshold (p5). When glucose is below the threshold, insulin decreases due to clearance (p4), and when it's above, insulin increases proportionally with a rate constant (p6) based on the deviation from the threshold. This accounts for insulin secretion and its impact on glucose regulation.

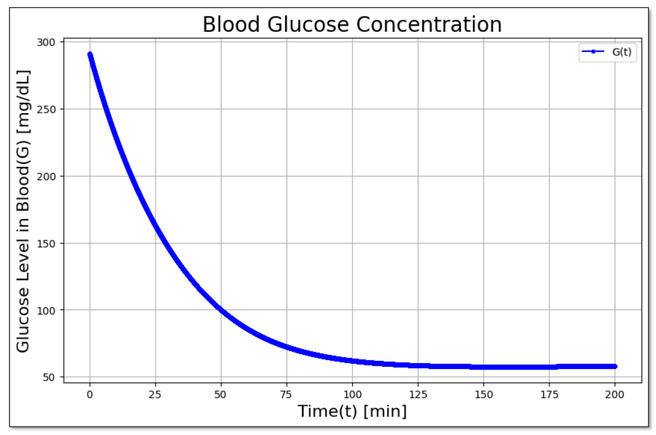
$$\frac{dX}{dt} = -p_2 \cdot X + p_3 \cdot (I - I_b)$$

Here Ib is insulin basal concentration. The rate of change of dynamic insulin sensitivity (dX/dt) represents the patient's insulin sensitivity over time. It accounts for the rate of insulin uptake (p2) and its effect on glucose uptake (p3).

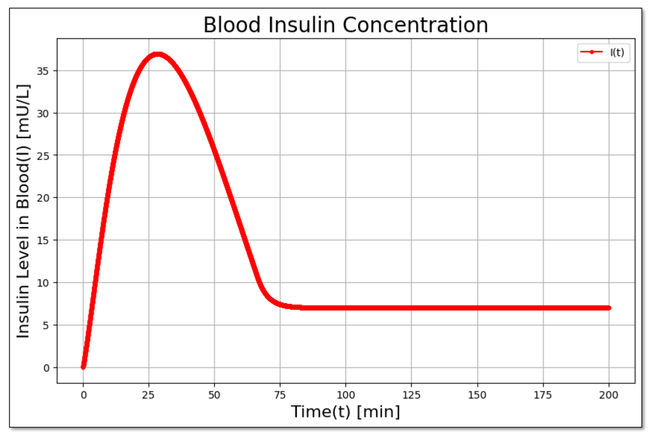
NUMERICAL APPROACH

We will use the Runge-Kutta method, specifically the fourth-order variant, to solve our system of three first-order differential equations. This numerical method iteratively computes solutions and depends on step size choice and method convergence for accuracy. Runge-Kutta methods yield accuracy comparable to Taylor series approaches without needing higher-order derivatives. The method involves increment functions and recurrence relationships, making it efficient for computer calculations. So, to solve the equations, we used the fourth-order Runge-Kutta method.

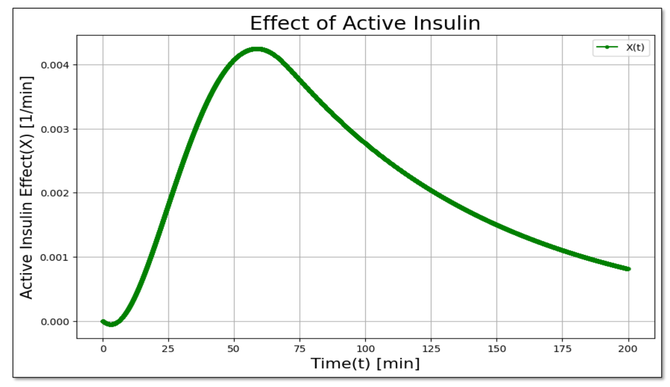
RESULTS



The first graph shows glucose levels over time. Initially, at t=0, diabetic patients have maximum blood glucose. Over time, it decreases due to insulin infusion into fatty tissue. The closed system monitor can set the maximum glucose threshold for insulin infusion.



In the second graph, we track insulin concentration in the blood over time. Starting at t=0 with zero insulin concentration. Continuous monitoring sets an infusion threshold, leading to a rapid increase in insulin concentration upon infusion of insulin into fatty tissue. However, a time lag occurs before the active insulin effect takes place.



The third graph shows the active insulin effect over time, initially negligible at t=0 due to a delay in action. Continuous monitoring prompts insulin infusion in response to high glucose, increasing insulin's glucose-lowering effect. As the active insulin effect reaches the peak of graph, glucose nears normal levels. As glucose decreases, the active insulin effect diminishes alongside insulin concentration.

CONCLUSION

The continuous glucose monitor detects high blood glucose levels and triggers insulin infusion. This sudden infusion raises insulin concentration and subsequently, the active insulin effect, with a lag. As a result, glucose levels decrease. Over time, insulin concentration decreases, leading to a gradual reduction in the active insulin effect. Ultimately, glucose stabilizes at a normal level. This cycle repeats as needed for diabetic patients. In conclusion, the Bergman minimal model, using differential equations, clarifies how insulin pumps in a closed system maintain normal blood glucose levels for diabetic patients.