Name :- Purval Madhukar Bhude
Roll no. S20230010193
Section :- 4

MAHATMA GANDHI MISSION

Q) $x(t) = e^{-|2t|} u(t)$.

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \int_{0}^{\infty} e^{-2t} u(t)\, e^{-j\omega t}\, dt.$$

$$= \int_{0}^{\infty} e^{-2t} e^{-j\omega t}\, dt$$

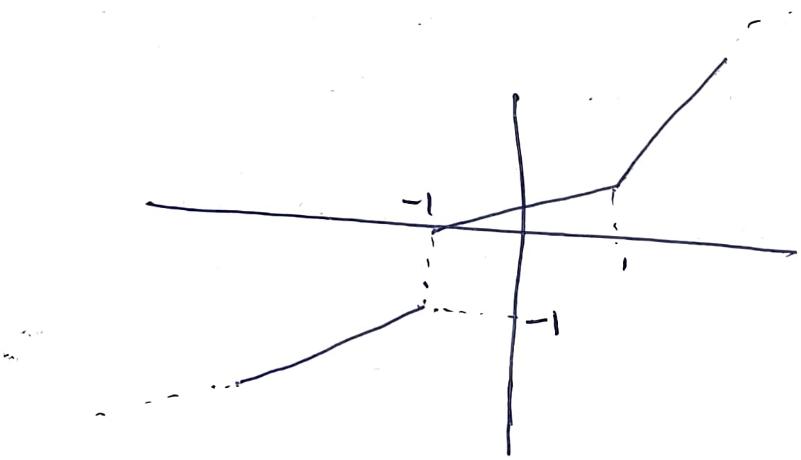
$$= \int_{0}^{\infty} e^{-t(2+j\omega)}\, dt$$

$$= \left[\frac{e^{-(2+j\omega)t}}{-(2+j\omega)} \right]_{0}^{\infty}$$

$$= -\frac{1}{2+j\omega} [0-1]$$

$$\boxed{X(j\omega) = \frac{1}{2+j\omega}}$$

b) $$x(t) = \begin{cases} t & , \ t < -1 \\ t+1 & , \ -1 \leq t < 1 \\ 2t & , \ t \geq 1 \end{cases}$$



$$x(j\omega) = \underbrace{\int_{-\infty}^{-1} t\, e^{-j\omega t}\, dt}_{I_1} + \underbrace{\int_{-1}^{1} (t+1)\, e^{-j\omega t}\, dt}_{I_2} + \underbrace{\int_{1}^{\infty} 2t\, e^{-j\omega t}\, dt}_{I_3}$$

$$I_1 = \int_{-\infty}^{1} t e^{-j\omega t} dt = t \int e^{-j\omega t} dt - \int \frac{e^{-j\omega t}}{-j\omega} dt$$

$$= \left[\frac{t e^{-j\omega t}}{(-j\omega)} + \frac{e^{-j\omega t}}{\omega^2} \right]_{-\infty}^{-1} \quad \text{—} (1) \quad (j^2 = -1).$$

$$= \frac{(-1)\, e^{j\omega}}{+j\omega} + \frac{e^{j\omega}}{\omega^2} - (0+0)$$

$$= e^{j\omega} \left[\frac{1}{j\omega} + \frac{1}{\omega^2} \right]$$

$$I_2 = \int_{-1}^{1} (t+1)\, e^{-j\omega t} dt = \int_{-1}^{1} t e^{-j\omega t} + e^{-j\omega t} dt$$

$$= \left[\frac{t e^{-j\omega t}}{(-j\omega)} + \frac{e^{-j\omega t}}{\omega^2} \right]_{-1}^{1} + \left[\frac{e^{-j\omega t}}{-j\omega} \right]_{-1}^{1} \quad \text{— from (1)}$$

$$= \left[\frac{e^{-j\omega}}{-j\omega} + \frac{e^{-j\omega}}{\omega^2} \right] - \left[\frac{e^{+j\omega}}{j\omega} + \frac{e^{j\omega}}{\omega^2} \right]$$

$$+ \left[\frac{e^{-j\omega}}{-j\omega} \right] - \left[\frac{e^{j\omega}}{-j\omega} \right]$$

$$= e^{-j\omega} \left(\frac{1}{-j\omega} + \frac{1}{\omega^2} \right) - e^{j\omega} \left[\frac{1}{j\omega} + \frac{1}{\omega^2} \right] + \frac{e^{-j\omega} - e^{+j\omega}}{-j\omega}$$

$$= e^{-j\omega} \left(\frac{-1}{j\omega} + \frac{1}{\omega^2} \right) - I_1 + \frac{e^{-j\omega} - e^{+j\omega}}{-j\omega}$$

$$I_3 = 2\int_1^{\infty} t\, e^{-j\omega t}\, dt$$

$$= 2\left[\frac{t\, e^{-j\omega t}}{-j\omega} + \frac{e^{-j\omega t}}{\omega^2}\right]_1^{\infty} \quad \text{-- from (I).}$$

$$= 2\left[(0+0) - \left(\frac{e^{-j\omega}}{-j\omega} + \frac{e^{-j\omega}}{\omega^2}\right)\right] = -2\, e^{-j\omega}\left(\frac{1}{-j\omega} + \frac{1}{\omega^2}\right)$$

$$\therefore X(j\omega) = I_1 + I_2 + I_3$$

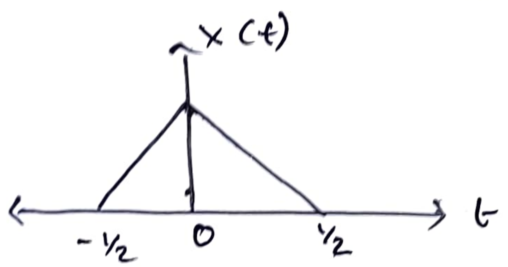
$$= \cancel{I_1} + e^{-j\omega}\left(-\frac{1}{j\omega} + \frac{1}{\omega^2}\right) - \cancel{I_1} - \frac{e^{-j\omega} - e^{j\omega}}{j\omega}$$

$$- 2\, e^{-j\omega}\left(\frac{1}{-j\omega} + \frac{1}{\omega^2}\right)$$

$$= -e^{-j\omega}\left(\frac{1}{-j\omega} + \frac{1}{\omega^2}\right) - \frac{e^{-j\omega}}{j\omega} + \frac{e^{j\omega}}{j\omega}$$

$$= \frac{e^{j\omega}}{j\omega} - \frac{e^{-j\omega}}{\omega^2}$$

(c)



$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \underbrace{\int_{-1/2}^{0} (1+2t)\, e^{-j\omega t}\, dt}_{I} + \int_{0}^{1/2} (1-2t)\, e^{-j\omega t}\, dt.$$

$$I = \int_{-1/2}^{0} (1+2t)\, e^{-j\omega t}\, dt$$

Put $t = -k$

If $t = -1/2$, $k = 1/2$

$t = 0$, $k = 0$

$$= \int_{0}^{1/2} (1-2\cancel{k})\, e^{j\omega k}\, dk$$

change $k \to t$

$$= \int_{0}^{1/2} (1-2t)\, e^{j\omega t}\, dt.$$

$$\therefore X(j\omega) = \int_{0}^{1/2} (1-2t)\, e^{j\omega t}\, dt + \int_{0}^{1/2} (1-2t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{1/2} (1-2t)\left(e^{j\omega t} + e^{-j\omega t}\right) dt.$$

$$= \int_{0}^{1/2} (1-2t)\,(2\cos\omega t)\, dt.$$

$$= 2\left[\int_{0}^{1/2} \cos\omega t\, dt - \underbrace{2\int_{0}^{1/2} t\cos\omega t\, dt}_{\to I'}\right]$$

$$\cancel{= 2\left[\left(\frac{\sin\omega t}{\omega}\right)_0^{1/2} - 2\left(\right.\right.}$$

$$I' = \int t\cos\omega t\, dt = t\,\frac{\sin\omega t}{\omega} - \int \frac{\sin\omega t}{\omega}\, dt$$

$$= \frac{t}{\omega}\sin\omega t + \frac{\cos\omega t}{\omega^2}$$

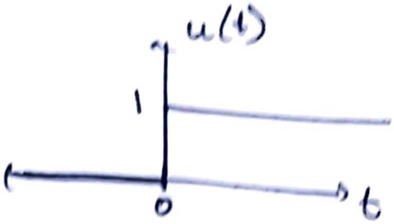
$$X(j\omega) = 2\left[\left(\frac{\sin\omega t}{\omega}\right)_0^{1/2} - 2\left(\frac{t}{\omega}\sin\omega t + \frac{\cos\omega t}{\omega^2}\right)_0^{1/2}\right]$$

$$= 2\left[\left(\frac{\sin\omega/2}{\omega} - 0\right) - 2\left(\frac{\sin(\omega/2)}{2\omega} + \frac{\cos(\omega/2)}{\omega^2} - 0 - \frac{1}{\omega^2}\right)\right]$$

$$= 2\left[\cancel{\frac{\sin\omega/2}{\omega}} - \cancel{\frac{\sin(\omega/2)}{\omega}} - \frac{2\cos(\omega/2)}{\omega^2} + \frac{2}{\omega^2}\right]$$

$$\boxed{X(j\omega) = \frac{4}{\omega^2}\left[1 - 2\cos(\omega/2)\right]}$$

d) $u(t)$



$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} u(t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-j\omega t}\, dt$$

$$\boxed{\text{Property : } \int x(\tau)\, d\tau = \frac{1}{j\omega} X(j\omega) + \pi X(0)\, \delta(\omega)}$$

$$\therefore \quad = \left. \frac{e^{-j\omega t}}{-j\omega} \right|_{0}^{\infty}$$

$$= \frac{1}{-j\omega}(0-1) = \frac{1}{j\omega}$$

$$\therefore x(t) \longleftrightarrow \frac{1}{j\omega} + \pi X(0)\, \delta(\omega)$$

$$\Rightarrow \quad \frac{1}{j\omega} + \pi\, \delta(\omega)$$