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Lab 9 → laplace Transform.

Q1 a) $x(t) = \frac{1}{\sqrt{3}}[\sin(\sqrt{3}t) + 2\cos(\sqrt{3}t)]e^{-t}u(t+1)$.

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt.$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{3}}\left[\sin(\sqrt{3}t) + 2\cos(\sqrt{3}t)\right] e^{-t}\, u(t+1)\, e^{-st}\, dt$$

$$= \int_{-1}^{\infty} \frac{1}{\sqrt{3}}\left[\frac{e^{j\sqrt{3}t} - e^{-j\sqrt{3}t}}{2j} + \cancel{2}\left(\frac{e^{j\sqrt{3}t} + e^{-j\sqrt{3}t}}{\cancel{2}}\right)\right] \cdot e^{-t} e^{-st}\, dt$$

$$= \int_{-1}^{\infty} \frac{1}{\sqrt{3}}\left[\frac{e^{j\sqrt{3}t} - e^{-j\sqrt{3}t} + 2je^{j\sqrt{3}t} + 2je^{-j\sqrt{3}t}}{2j}\right] e^{-t(s+1)}\, dt.$$

$$= \int_{-1}^{\infty} \frac{1}{\sqrt{3}}\left[\frac{e^{j\sqrt{3}t}(1+2j) + e^{-j\sqrt{3}t}(2j-1)}{2j}\right] e^{-t(s+1)}\, dt.$$

$$= \frac{1}{2\sqrt{3}j}\left[(1+2j)\int_{-1}^{\infty} e^{t(\sqrt{3}j - s - 1)}\, dt + (2j-1)\int_{-1}^{\infty} e^{-t(\sqrt{3}j + s + 1)}\, dt\right.$$

$$= \frac{1}{2\sqrt{3}j}\left[\frac{(1+2j)\left(e^{(1+s-\sqrt{3}j)}\right)}{(\sqrt{3}j - s - 1)} + \frac{(2j-1)}{(\sqrt{3}j + s + 1)}\left(e^{-(\sqrt{3}j + s + 1)}\right)\right].$$

After solving this

$$\frac{1}{\sqrt{3}}\left(\frac{2(s+1) + \sqrt{3}}{(s+1)^2 + 3}\right)$$

c) $x(t) = u(t) - 2u(t-1) + u(t-3)$

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$$X(s) = \int_{-\infty}^{\infty} \left[u(t) - 2u(t-1) + u(t-3)\right] e^{-st}\,dt$$

$$= \int_{0}^{\infty} e^{-st}\,dt - 2\int_{1}^{\infty} e^{-st}\,dt + \int_{3}^{\infty} e^{-st}\,dt$$

$$= \left.\frac{e^{-st}}{-s}\right|_{0}^{\infty} - 2\left.\frac{e^{-st}}{(-s)}\right|_{1}^{\infty} + \left.\frac{e^{-st}}{(-s)}\right|_{3}^{\infty}$$

$$= \frac{1}{s}(1-0) - 2\left(\frac{e^{-s}}{s} - 0\right) + \left(\frac{e^{-3s}}{s} - 0\right)$$

$$\boxed{X(s) = \frac{1 - 2e^{-s} + e^{-3s}}{s}}$$

d) $x(t) = e^{-1(t-1)}\, u(t-1)$

$$X(s) = \int_{-\infty}^{1} e^{t-1} u(t-1) e^{-st}\,dt + \int_{1}^{\infty} e^{1-t} u(t-1) e^{-st}\,dt$$

$$= 0 + \int_{1}^{\infty} e^{1-t} \cdot e^{-st}\,dt = \int_{1}^{\infty} e^{1-t-st}\,dt = \int_{1}^{\infty} e^{1-t(1+s)}\,dt$$

$$= e\left[\frac{e^{-t(1+s)}}{-(1+s)}\right]\Bigg|_{1}^{\infty}$$

$$= \frac{e}{1+s}\left[e^{-(1+s)} + 0\right]$$

$$\boxed{X(s) = \frac{e^{-s}}{1+s}}$$