Name :- Purval Madhukar Bhude

Roll no. S20230010193

MAHATMA GANDHI MISSION

Q1 i) $x(t) = 4\cos 4t \sin 3t$

$= 2[\sin(7t) - \sin t]$

$= 2\left[\frac{e^{j7t} - e^{-7tj}}{2j} - \frac{e^{jt} - e^{-jt}}{2j}\right]$

$= \frac{e^{7jt}}{j} - \frac{e^{-7jt}}{j} - \frac{e^{jt}}{j} + \frac{e^{-jt}}{j}$

$a_7 = \frac{1}{j}$, $a_{-7} = \frac{-1}{j}$ $a_1 = \frac{-1}{j}$, $a_{-1} = \frac{1}{j}$

ii) $x(t) = \cos(12t) - \sin(6t)$

$x(t) = \left(\frac{e^{12jt} + e^{-12jt}}{2}\right) - \left(\frac{e^{6jt} - e^{-6jt}}{2j}\right)$

$\therefore$ $a_{12} = 1/2$ $a_{-12} = +1/2$ $a_6 = -1/2j$, $a_{-6} = 1/2j$

iii) $x(t) = \frac{1}{2}(10 - \sin(6\pi t))$.

$= \frac{1}{2}\left[10 - \frac{(e^{6\pi tj} - e^{-6\pi tj})}{2j}\right)$

$= 5 - \frac{e^{6\pi tj}}{4j} + \frac{e^{-6\pi jt}}{4j}$

$a_0 = 5$ $a_6 = -1/4j$, $a_{-6} = 1/4j$

iv) $x[n] = \{ \ldots\ 0, 1, 2, 3, \underset{\uparrow}{0}, 1, 2, 3, 0, 1, 2, \ldots \}$

$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n)\, e^{-jk\omega_0 n}$ $N = 4$

$\omega_0 = \pi/2$

$= \frac{1}{4} \sum_{n=0}^{3} x(n)\, e^{-jk\pi/2\, n}$

$= \frac{1}{4}\left(x(0) e^{0} + x(1) e^{-jk\pi/2} + x(2) e^{-jk\pi} + x(3) e^{-jk3\pi/2}\right.$

$= \frac{1}{4}\left[0 \cdot 1 + 1\, e^{-jk\pi/2} + 2 \cdot e^{-jk\pi} + 3 e^{-jk(3\pi/2)}\right.$

for $k=0$,

$$a_k = \frac{1}{4}[1+2+3] = 3/2$$

for $k=1$,

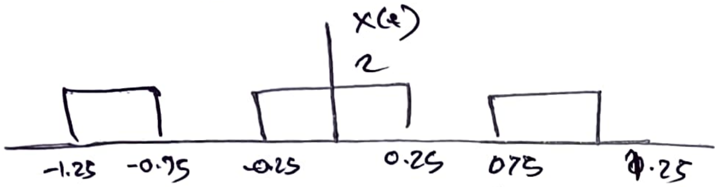
$$a_1 = \frac{1}{4}\left(e^{-j\pi/2} + 2e^{-j\pi} + 3e^{-3\pi/2 j}\right]$$

$$= \frac{1}{4}\left(-j + 2(-1) + 3(j)\right]$$

$$= \frac{j-1}{2}$$

and so on.

(5)



$T_0 = 1$ $\qquad \omega = 2\pi$

$$C_k = \int_{-1/2}^{1/2} x(t)\, e^{-j2\pi kt}$$

$$= \int_{-1/2}^{1/2} 2\, e^{-j2\pi kt}\, dt$$

$$= \frac{-2\, e^{-j2\pi kt}}{j\pi k}\Bigg|_{-1/2}^{1/2}$$

$$= \cancel{-2\left[e^{-j\pi k} - e^{j\pi k}\right]} \Rightarrow \cancel{2\left[e^{j\pi k} - \frac{1}{e^{j\pi k}}\right]}$$

$$= 2\cancel{\left[\frac{e^{2j\pi k} - 1}{e^{j\pi k}}\right]}$$

$$\cancel{\text{for } k=0 \quad C_k = 2\left[\frac{1-1}{1}\right] = 0.}$$

$$a_k = \frac{2}{\pi k}\left\{\frac{e^{j\frac{\pi}{2}k} - e^{-j\frac{\pi}{2}k}}{2j}\right]$$

$$a_k = \frac{2}{\pi k}\sin\left(\frac{\pi}{2}k\right)$$

$$a_k = \begin{cases} 2/\pi k, & k = \text{odd} \\ 0, & k = \text{even.} \end{cases}$$

Q2 ① $x(t) = 4\cos(4t)\sin(3t)$

$T = 2\pi$

$x_N(t) = \sum_{k=-N}^{N} a_k e^{jk(\frac{2\pi}{T})t}$

$$x_N(t) = a_{-7}e^{j-7t} + a_{-1}e^{j(-1)t} + a_1 e^{j(1)t} + a_7 e^{j7t}$$

$$= -\frac{1}{j}e^{-7jt} + \frac{1}{j}e^{7jt} + \frac{1}{j}e^{-jt} - \frac{1}{j}e^{jt}$$

$$= 2\left(\frac{e^{7jt}-e^{-7jt}}{2j}\right) - 2\left(\frac{e^{jt}-e^{-jt}}{2j}\right)$$

$$= 2(\sin 7t) - 2\sin t$$

$$= 4\cos(4t)\sin(3t)$$

$$e(t) = x(t) - x_N(t)$$

$$= 0.$$

② $x(t) = \cos(12t) - \sin(6t).$

$T = \pi/3$

$$x_N(t) = \frac{1}{2}e^{-j7.2t} + \frac{1}{2j}e^{-3.6jt} - \frac{1}{2j}e^{3.6jt} + \frac{1}{2}e^{j7.2t}$$

$$= \frac{1}{2}\left(e^{j7.2t} + e^{-j7.2t}\right) - \frac{1}{2j}\left(e^{3.6jt} - e^{-3.6jt}\right)$$

$$= \cos(7.2t) - \sin(3.6t)$$

$$e(t) = x(t) - x_N(t)$$

$$e(t) = \cos(12t) - \sin(6t) - \cos(72t) + \sin(36t)$$

③ $x(t) = \frac{1}{2}(10 - \sin(6\pi t)]$

$\tau = 1/3$

$$x_N(t) = \frac{1}{4j} e^{j(-6)(\frac{2\pi}{3})t} + 5 - \frac{1}{4j} e^{j\cdot(6)(\frac{2\pi}{3})t}$$

$$= 5 - \frac{1}{2}\left(\frac{e^{4\pi jt} - e^{-4\pi jt}}{2j}\right)$$

$$= 5 - \frac{1}{2}\sin(4\pi t)$$

$$e(t) = x(t) - x_N(t)$$

$$= \cancel{5} - \frac{\sin(6\pi t)}{2} - \cancel{5} + \frac{1}{2}\sin(4\pi t)$$

$$\boxed{e(t) = \frac{\sin(4\pi t) - \sin(6\pi t)}{2}}$$

Q3 $T_0 = 4$ $\omega_0 = 2\pi/4 = \pi/2$

a) $$a_k = \begin{cases} 0 & k = 0 \\ (j)^k \frac{k \sin k\pi/4}{k\pi} & \text{otherwise} \end{cases}$$

$$x(t) = \sum_{-\infty}^{\infty} a_k e^{j\frac{\pi}{2}kt}$$

$$= \sum_{-\infty}^{-1} (j)^k \frac{\sin k\pi/4}{k\pi} e^{j\pi/2 kt} + 0 + \sum_{1}^{\infty} (j)^k \frac{\sin k\pi/4}{k\pi} e^{j\frac{\pi}{2}kt}$$

b) $$a_k = (-1)^k, \frac{\sin k\pi/8}{2k\pi}, \quad a_0 = 1/16$$

$$x(t) = \sum_{-\infty}^{\infty} a_k e^{j\pi/2 kt}$$

$$= \sum_{-\infty}^{-1} (-1)^k, \frac{\sin k\pi/8}{2k\pi} e^{j\frac{\pi}{2}kt} + \frac{1}{16} + \sum_{1}^{\infty} (-1)^k \frac{\sin k\pi/8}{2k\pi} e^{j\frac{\pi}{2}kt}$$