Assignment

Q.1. For the manufacture of Swiss cheese, Lactobaccilus casei is cultivated under anaerobic conditions as a starter culture. The culture produces lactic acid as a by-product of energy metabolism. The organism has the following characteristics: $Y_{X/S}$ = 0.23 Kg/Kg, Ks = 0.15 Kg/m^3, μ_{max} = 0.35 h^{-1}, and m_s = 0.135 kg/kg h A stirred-tank reactor is operated in fed batch mode at quasi-steady state with a feed flow rate of 4 m^3/h and feed substrate concentration of 80 kg/m^3. After 6 h, the liquid volume is 40 m^3.

(a) What was the initial culture volume?

(b) What is the concentration of the substrate at the quasi-steady state?

Solution:

(a) Given F = 4 m^3/ h, V = 40 m^3, and t = 6 h.

For a fed batch operation at quasi-steady state (variable volume fed batch), the volume of the reactor at time t and the initial culture volume V_0 can be given as-

$$V = V_0 + Ft$$

$$V_0 = V - Ft = (40) - (4)(6) = 16 \text{ m}^3$$

(b) Given F = 4 m^3/h, V = 40 m^3, Ks = 0.15 kg/m^3, μ_{max} = 0.35 h^{-1}.

We know that the dilution rate is

$$D = \frac{F}{V} = \frac{4}{40} = 0.10 \ \text{h}^{-1}$$

At quasi-steady state,

$$\mu = D = \frac{\mu_{\max} S}{K_s + S}$$

So, the substrate concentration at quasi-steady state can be given as

$$S = \frac{K_s D}{\mu_{\max} - D}$$

Putting the given values in the above equation, we get

$$S = \frac{0.15(0.10)}{0.35 - 0.10} = \ 0.06 \text{kg/m}^3$$

Q. 2. The growth of a microorganism on glucose is described by the Monod model. During the growth of the cell, the glucose concentration is 10 g/L, and the μ_{max} and Ks values are 0.5 h^{-1} and 0.1 g/L, respectively. Find the time required to triple the biomass concentration.

Solution:

Given μ_{max} = 0.5 h^{-1}, Ks = 0.1 g/L, and S = 10 g/L.

The Monod equation for cell growth is

$$\mu = \frac{\mu_{\max} S}{K_S + S} = \frac{0.5 \times 10}{0.1 + 10} = 0.495\,\mathrm{h}^{-1}$$

If cell death is negligible as compared to growth ($\mu_g = \mu_{net}$),

$$\frac{dX}{dt} = \mu X$$

Rearranging and integrating the above equation, we get

$$\int_{X_0}^{X} \frac{dX}{X} = \int_0^t \mu\, dt$$

$$\ln\left(\frac{X}{X_0}\right) = \mu t$$

$$t = \frac{\ln X/X_0}{\mu}$$

Time required to triple cell mass ($X = 3X_0$) is

$$t_{\text{triple}} = \frac{\ln 3}{\mu} = \frac{\ln 3}{0.495}$$

$$t_{\text{triple}} = \frac{\ln 3}{\mu} = 2.22\,\mathrm{h}$$

Q.3. Pseudomonas sp. has a minimum doubling time of 2.4 h when grown on acetate in a chemostat that follows the Monod model.

Given K_S = 1.3 g/L, $Y_{X/S}$ = 0.46 g cell / g acetate, S_0 = 38 g / L.

(a) Find out steady-state S and X when D = 1/2 D_{max}.

(b) Find the cell mass productivity at 0.8 D_{max}.

(c) Determine the value of $D_{washout}$.

Solution: It is known that

$$t_{d_{\min}} = \frac{\ln(2)}{\mu_{\max}}$$

$$\mu_{\max} = \frac{\ln(2)}{t_{d_{\min}}} = 0.288\,\mathrm{h}^{-1}$$

$$D_{\max} = \mu_{\max}\left(1 - \sqrt{\frac{K_S}{K_S + S_0}}\right) = 0.288\left(1 - \sqrt{\frac{1.3}{1.3 + 38}}\right)$$

$$= 0.288(1 - 0.1818) = 0.235\,\text{h}^{-1}$$

(a)

$$D = \frac{1}{2} D_{\max} = 0.1178$$

$$S = \frac{K_S D}{\mu_{\max} - D} = \frac{1.3(0.1178)}{0.288 - 0.1178} = 0.899\,\frac{\text{g}}{\text{L}}$$

$$X = Y_{X/S}(S_0 - S) = 0.46\,(38 - 0.899) = 17\frac{g}{L}$$

(b) 0.8Dmax = 0.8 X 0.235 h^{-1} = 0.188 h^{-1}

Cell mass productivity is XD.

$$X = Y_{X/S}\,(S_0 - S)$$

$$S = \frac{1.3(0.188)}{0.28 - 0.188} = 2.44\,\frac{g}{L}$$

$$X = 0.46(38 - 2.4) = 16.35\,\frac{\text{g}}{\text{L}}$$

Cell mass productivity (at $0.8D_{max}$) is

$$XD = 16.35 \times 0.188 = 3.07\,\frac{\text{g}}{\text{L h}}$$

(c)

$$D_{\text{washout}} = \frac{\mu_{\max} S_0}{K_S + S_0} = \frac{0.288 \times 38}{1.3 + 38} = 0.278\,\text{h}^{-1}$$

Q.4. A 1 m^3 mixed flow reactor, at the initial substrate concentration of S0 = 500 g/m3 in the feed, produces 100 g/h of yeast cells in the exit stream at two different flow rates mentioned below:

(i) At 0.5 m^3/h of feed for which the steady state S = 100 g/m^3

(ii) At 1 m^3/h of feed for which the steady state S = 300 g/m^3

The Monod cell growth kinetics follow yeast formation. Compute the following:

(a) The fractional yield of yeast

(b) The kinetic equation for yeast formation

(c) The flow rate for maximum yeast production (Hint: flow rate at D_{max})

(d) The maximum production rate of yeast

Solution:

(a) We know that

Flow rate × Concentration of cells = Rate of cell mass production

Therefore, the concentration of yeast is

$$\frac{100\ \mathrm{g/h}}{0.5\ \mathrm{m^3/h}} = 200\ \mathrm{g/m^3}$$

Case (i): In the case of sterile feed, $X_{SS} = Y_{X/S}(S_0 - S_{SS})$.

$$Y_{X/S} = \frac{200\ \mathrm{g/m^3}}{(500-100)\ \mathrm{g/m^3}} = 0.5$$

(b) From the Monod model under steady state and sterile conditions,

$$\mu = \frac{\mu_{\max} S}{K_S + S} = D$$

Case (i):

$$D = \frac{F}{V} = \frac{0.5\ \mathrm{m^3/h}}{1\ \mathrm{m^3}}\ 0.51/\mathrm{h}$$

So from the Monod equation, we can write

$$0.5 = \frac{\mu_{\max}\ 100}{K_S + 100}$$

Similarly, in Case (ii)

$$1 = \frac{\mu_{\max}\ 300}{K_S + 300}$$

$$\mu_{\max} = 2\frac{1}{\mathrm{h}} \text{ and } K_S = 300\ \mathrm{g/m^3}$$

So, the kinetic equation is:

$$\mu = \frac{2S}{300 + S}$$

(c) We know that

$$D_{\max} = \mu_{\max}\left(1 - \sqrt{\frac{K_S}{(K_S + S_0)}}\right)$$

$$D_{\max} = 0.776\ 1/\mathrm{h} = \frac{F}{V}$$

Flow rate for the maximum yeast production is 0.776 × 1 =0.776 $\mathrm{m^3/h}$.

(d)

$$S = \frac{K_S S}{(\mu_{\max} - D)} = \frac{300 \times 0.776}{(2 - 0.776)} = 190\ \mathrm{g/m^3}$$

$$X = Y_{X/S}(S_0 - S) = 0.5\ (500 - 190) = 155\ \mathrm{g/m^3}$$

Maximum cell mass productivity is 0.776 × 155 = 120.28 g/h $\mathrm{m^3}$

Q. 5. In a chemostat, the steady-state substrate and biomass concentrations are given in the Table. The initial substrate concentration (S0) is 700 mg/L. Find out μ_{max} and Ks, the growth yield coefficient, $Y'_{X/S}$ (growth), and the maintenance coefficient m.

D (h^{-1})	S (mg/L)	X (mg/L)
0.3	45	326
0.25	41	328
0.20	16	340
0.12	8	342
0.08	3.8	344

Solution:

The Monod equation is

$$\mu = \frac{\mu_{max} S}{K_S + S}$$

Under steady-state chemostat operation and sterile feed

$\mu = D$

$$D = \frac{\mu_{max} S}{K_S + S}$$

$$\frac{1}{D} = \frac{K_S}{\mu_{max}} \frac{1}{S} + \frac{1}{\mu_{max}}$$

Plotting 1/D versus 1/S yields the value of K_S and μ_{max}.

$$\mu_{max} = \frac{1}{2.9} = 0.34\ \mathrm{h}^{-1}$$

$$K_S = 37.53 \times \mu_{max} = 12.76\ \frac{\mathrm{mg}}{\mathrm{L}}$$

According to the Pirt model,

$$\frac{1}{Y_{X/S\,(\text{overall})}} = \frac{1}{Y'_{X/S\,(\text{growth})}} + \frac{m}{\mu}$$

$$\frac{1}{Y_{X/S\,(\text{overall})}} = \frac{1}{Y'_{X/S\,(\text{growth})}} + \frac{m}{D}$$

Plotting $1/Y_{X/S}$(overall) versus 1/D of Table 6.9 yields the values of m

and 1 $Y'_{X/S\ /\ (\text{growth})}$.

D (h^{-1})	$1/D$ (h)	S (mg/L)	X (mg/L)	$Y_{X/S(\text{overall})}$	$1/Y_{X/S(\text{overall})}$
0.3	3.33	45	326	0.498	2.008
0.25	4.00	41	328	0.4977	2.009
0.20	5.00	16	340	0.4970	2.012
0.12	8.33	8	342	0.4942	2.023
0.08	12.50	3.8	344	0.4941	2.024

$$Y_{X/S(\text{overall})} = -\frac{dX}{dS} = \frac{X_n - X_0}{S_0 - S_n}$$

For the first sampling point, n = 1

$$Y_{X/S(\text{overall})} = -\frac{dX}{dS} = \frac{X_1 - X_0}{S_0 - S_1} = \frac{326 - 0}{700 - 45} = 0.498$$

From Plot of $1/Y_{X/S(\text{overall})}$ versus 1/D, we get m = 0.0019 h^{-1} (from slope) and $Y_{X/S(\text{growth})}$= 0.50 (from intercept of the Y-axis).

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