Test2
Solution

$$v_w = K \ln \frac{C_w - C_f}{C_0 - C_f}$$

$$\Rightarrow \frac{C_w - C_f}{C_0 - C_f} = e^{v_w/K}$$

$$\Rightarrow \frac{C_w/C_f - 1}{C_0/C_f - 1} = e^{v_w/K}$$

$$\Rightarrow \frac{\frac{1}{1-R_r} - 1}{\frac{1}{1-R_0} - 1} = e^{v_w/K}$$

$$\Rightarrow \frac{R_r}{1-R_r} \cdot \frac{1-R_0}{R_0} = e^{v_w/K} = z$$

$$\Rightarrow \frac{R_r}{1-R_r} = \frac{R_0 z}{1-R_0}$$

$$\Rightarrow \frac{1-R_r}{R_r} = \frac{1-R_0}{R_0 z}$$

$$\Rightarrow \frac{1}{R_r} = 1 + \frac{1-R_0}{R_0 z} = \frac{1 + R_0 z - R_0}{R_0 z}$$

$$\Rightarrow R_r = \frac{R_0 z}{1 + R_0 z - R_0}$$

Case1: $v_w/K = 1 \Rightarrow z = e = 2.7$

$$R_r = \frac{2.7 R_0}{1 + 1.7 R_0}$$

$$0.5 < R_r < 0.8$$

$$0.5 < \frac{2.7 R_0}{1 + 1.7 R_0} < 0.8$$

$$\Rightarrow 0.5 + 0.85 R_0 < 2.7 R_0 < 0.8 + 1.36 R_0$$

$$0.5 + 0.85 R_0 < 2.7 R_0 \Rightarrow 1.85 R_0 > 0.5 \Rightarrow R_0 > 0.27$$

$$2.7 R_0 < 0.8 + 1.36 R_0 \Rightarrow 1.34 R_0 < 0.8 \Rightarrow R_0 < 0.597 \approx 0.6$$

$$\boxed{0.27 < R_0 < 0.6}$$

Case2: $v_w/K = 0.5 \Rightarrow z = e^{0.5} = 1.64$

$$0.5 < \frac{1.64 R_0}{1 + 0.64 R_0} < 0.8 \Rightarrow 0.5 + 0.32 R_0 < 1.64 R_0 < 0.8 + 0.512 R_0$$

$$0.5 + 0.32 R_0 < 1.64 R_0 \Rightarrow 1.32 R_0 > 0.5 \Rightarrow R_0 > 0.38$$

$$1.64 R_0 < 0.8 + 0.512 R_0 \Rightarrow 1.128 R_0 < 0.8 \Rightarrow R_0 < 0.71$$

$$\boxed{0.38 < R_0 < 0.71}$$

Case3: $v_w/K = 2 ; \quad z = e^2 = 7.29$

$$0.5 < \frac{7.29 R_0}{1 + 6.29 R_0} < 0.8$$

$$\Rightarrow 0.5 + 3.145 R_0 < 7.29 R_0 < 0.8 + 5 R_0$$

$$\Rightarrow 0.5 + 3.145 R_0 < 7.29 R_0 \Rightarrow R_0 > 0.12 ;$$

$$7.29 R_0 < 0.8 + 5 R_0 \Rightarrow R_0 < 0.35$$

$$\boxed{0.12 < R_0 < 0.35}$$