Mid Sem Solution 2023

1. **OMB :** $F = P + R$ $\quad\cdots(1)$

Overall solute bal : $F C_0 = R C_R$ $\quad\cdots(2)$

Since, S.S. CF system acts like a CSTR,

$$C_0 = C_R$$

$$\therefore \quad v_w = K \ln \frac{C_g}{C_R} \quad \cdots(3)$$

$P = v_w \rho A$; $\quad v_w$ = volumetric permeate flux

ρ = Density of permeate

A = membrane area

$$P = \rho A K \ln \frac{C_g}{C_R} \quad \cdots(3a)$$

(1) & (3a) $\Rightarrow$ $F = \rho A K \ln \frac{C_g}{C_R} + R$

(2) $\Rightarrow$ $F = \rho A K \ln\left(\frac{C_g R}{F C_0}\right) + R$

$$\frac{P}{F} = \text{Recovery of product} = \frac{\rho A K \ln\left(\frac{C_g R}{F C_0}\right)}{\rho A K \ln\left(\frac{C_g R}{F C_0}\right) + R}$$

$$\frac{d(P/F)}{dR} = 0$$

$$\Rightarrow \boxed{R_{opt} = e \frac{F C_0}{C_g} = 2.7 \frac{F C_0}{C_g}}$$

$$R_{opt} = 2.7 \times \frac{50 \times 1}{5} = 27 \text{ kg/h}$$

$$P = F - R = 50 - 27 = 23 \text{ kg/h}$$

$$\boxed{\left(\frac{P}{F}\right)_{OPT} = \frac{23}{50} = 0.46}$$

(ii) $C_R = \frac{F C_0}{R} = \frac{50 \times 1}{27} = 1.85 \ kg/m^3$

$P_{opt} = \rho A K \ \ln\left(\frac{C_g R}{F C_0}\right)$ ⟶ $= 1$ under optimum condition

$\therefore P_{opt} = \rho A K$

$P_{op} = 23 \ kg/h$; $\rho = 1000 \ kg/m^3$; $A = LW = 0.3L \ m^2$

$K = \frac{23}{1000 \times 3600 \times 0.3L}$ $m^3/m^2 s$

$= \frac{2.1 \times 10^{-5}}{L} \ m/s.$

Laminar flow:

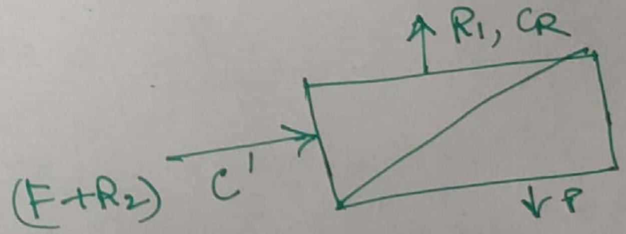
$Sh = \frac{k d_e}{D} = 1.85 \left(Re\, Sc \frac{d_e}{L}\right)^{1/3}$

$K = \frac{1.7 \times 1.85 \times 10^{-6}}{L^{1/3}}$

$\therefore \frac{2.1 \times 10^{-5}}{L} = \frac{1.7 \times 1.85 \times 10^{-6}}{L^{1/3}}$

$\Rightarrow \boxed{L = 17.25 \ m}$

(iii)



$F + R_2 = R_1 + P \Rightarrow 50 + R_2 = R_1 + 23$

$\Rightarrow R_1 = 27 + R_2 \quad \cdots (1)$

$(F + R_2) C' = R_1 C_R$

$(50 + R_2) 1.5 = R_1 \, 1.85$

$\Rightarrow 50 = 1.35 R_1 \Rightarrow R_1 = 37 \ kg/h$

$R_2 = 10 \ kg/h$

$\boxed{R_c = \frac{R_2}{R_1} = \frac{10}{37} = 0.27}$

#2

$$v_w = K \ln \frac{C_m - C_p}{C_0 - C_p} = L_p(\Delta P - \Delta \pi)$$

$$K \ln \frac{C_m - C_p}{C_0 - C_p} = L_p\left[\Delta P - a(C_m - C_p)\right]$$

$$v_w C_p = B(C_m - C_p)$$

$$v_w = B\left(\frac{C_m - C_p}{C_p}\right)$$

$$L_p(\Delta P - a(C_m - C_p)) = B\left(\frac{C_m - C_p}{C_p}\right)$$

$$\Rightarrow 1 - \alpha(C_m - C_p) = \beta \frac{C_m}{C_p} - \beta \;; \qquad \alpha = a/\Delta P, \quad \beta = \frac{B}{L_p \Delta P}$$

$$\Rightarrow C_m = \frac{C_p + \alpha C_p^2 + \beta C_p}{\beta + \alpha C_p}$$

$$K \ln\left(\frac{C_m - C_p}{C_0 - C_p}\right) = L_p \Delta P\left[1 - \alpha(C_m - C_p)\right]$$

$$\Rightarrow \ln\left(\frac{C_m - C_p}{C_0 - C_p}\right) = \gamma\left[1 - \alpha(C_m - C_p)\right], \qquad \gamma = \frac{L_p \Delta P}{K}$$

$$\Rightarrow \boxed{\ln\left[\frac{C_p}{(\beta + \alpha C_p)(C_0 - C_p)}\right] = \gamma\left[\frac{\beta}{\beta + \alpha C_p}\right]} \quad \text{— (1)}$$

$$\alpha = \frac{a}{\Delta P} = \frac{84000}{45 \times 10^5} = 0.02$$

$$\beta = \frac{B}{L_p \Delta P} = \frac{1.5 \times 10^{-6}}{5 \times 10^{-12} \times 45 \times 10^5} = 0.07$$

$$Re = \frac{\rho u_0 d_e}{\mu} = 3 \times 10^4 \;; \qquad Sc = 666.7$$

$$Sh = 0.023 (Re)^{0.8} Sc^{0.33}$$

$$\Rightarrow K = 1.12 \times 10^{-4} \text{ m/s.}$$

$$\gamma = \frac{L_p \Delta P}{K} = 0.2$$

(1) $\Rightarrow$ Trial & error $\Rightarrow$ $C_p \sim 3.5 \text{ kg/m}^3$

$C_m = 28.5 \text{ kg/m}^3$

$$\boxed{\begin{aligned} R_o &= 0.86 \\ R_r &= 0.88 \\ v_w &= 1.2 \times 10^{-5} \text{ m}^3/\text{m}^2\text{-s} \end{aligned}}$$

3. $Q = 50\ L/h = 1.39 \times 10^{-5}\ m^3/s$

No. of fibers = n

$$1.39 \times 10^{-5} = n \cdot \frac{\pi d^2}{4} u_0$$

$$\Rightarrow u_0 = \frac{31.48}{n}, \quad m/s$$

$$Sh = 1.62 \left(Re\ Sc\ \frac{d}{L}\right)^{1/3}$$

$$\Rightarrow K = \frac{1.23 \times 10^{-5}}{n^{1/3}}$$

$$v_w = K \ln C_0/C_0 = \frac{1.23 \times 10^{-5}}{n^{1/3}} \ln 10$$

$$P = 30\ L/h = 8.33 \times 10^{-6}\ m^3/s$$

$$P = v_w(n\pi dL)$$

$$v_w = \frac{0.015}{n}$$

$$\frac{1.23 \times 10^{-5} \ln 10}{n^{1/3}} = \frac{0.015}{n}$$

$$\Rightarrow \boxed{\begin{aligned} n &= 12{,}188.6 \\ &\simeq 12189 \end{aligned}}$$

4. $$D\frac{dc}{dy} + v_w(c - c_p) = 0$$

$$\Rightarrow D_0(1+k_1c)\frac{dc}{dy} + v_w(c - c_p) = 0$$

$$\Rightarrow \int_{c_m}^{c_0}\left(\frac{1+k_1c}{c-c_p}\right)dc = -\frac{v_w\delta}{D_0}$$

$$\Rightarrow -\frac{v_w}{k} = \int_{c_m}^{c_0}\frac{dc}{c-c_p} + k_1\int_{c_m}^{c_0}\frac{c\,dc}{c-c_p}$$

$$-\frac{v_w}{k} = \ln\left[\frac{R_o(1-R_r)}{R_r(1-R_o)}\right] + k_1c_0(1-R_o)\left[\frac{R_o-R_r}{(1-R_o)(1-R_r)} + \ln\left\{\frac{R_o(1-R_r)}{R_r(1-R_o)}\right\}\right]$$

$$\boxed{\frac{v_w}{k} = \ln\left[\frac{R_r}{R_o}\,\frac{1-R_o}{1-R_r}\right] + k_1c_0(1-R_o)\left[\frac{R_r-R_o}{(1-R_o)(1-R_r)} + \ln\left(\frac{R_r}{R_o}\,\frac{1-R_o}{1-R_r}\right)\right]}$$