INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Department of Chemical Engineering

End-semester (Autumn) Examination 2023-2024

Subject: Advanced Mathematical Techniques in Chemical Engineering (CH61015)

Remarks:

1. This question paper contains two parts: **Part A** and **Part B**. Attempt both parts.
2. Unless otherwise stated, usual mathematical notations apply.
3. Time = 3 h; maximum marks = 50; total number of printed pages = 2.

Part A: Linear algebra

1. Determine whether the following functions belong to the space of square integrable functions. Subsequently, normalise the appropriate ones.

(a) $f(x) = \dfrac{1}{\sqrt{x}}$ on [0,1]

(b) $g(x) = \dfrac{1}{x^{0.25}}$ on [0,1]

. . . 5 marks

2. Consider the following initial value problem:

$$\frac{dx}{dt} = -2x - y$$

$$\frac{dy}{dt} = x - 2y$$

with the initial condition vector as $[1 \quad 0]^T$.

(a) Solve the above system of equations.

(b) Draw the $x - y$ phase portrait.

(c) Draw $x - t$ and $y - t$ solution projections.

. . . 10 marks

3. Solve the following problem using Frobenius method.

$$\frac{d^2y}{dx^2} + y = 0$$

. . . 10 marks

Part B: Differential equations

4. Find the steady state temperature distribution in a semi-circular plate of radius a insulated on both faces. The governing equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}=0$$

At $r=a$, $T=T_o$ for any θ. At $\theta=0$ and π, $T=0$. This means the temperature is maintained zero on boundary diameter. Use suitable physical boundary conditions.

...7 marks

5. Solve the following problem:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{\partial^2 u}{\partial z^2}=0$$

At $r=1, u=0$, at $z=0, u=0$, at $z=1, u=1$. Use suitable physical boundary conditions.

...8 marks

6. Solve completely using Green's function method:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+x \qquad (1)$$

At $t=0$, $u=1$. At $x=0$, $\frac{\partial u}{\partial x}=0$. At $x=1$, $\frac{\partial u}{\partial x}+u=0$.

...10 marks

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Class Test 4

Advanced Mathematical Techniques in Chemical Engineering (CH 61015)

Full Marks:20

Completely solve the following PDE:

1. Consider $\frac{d^2u}{dx^2}+\frac{du}{dx}=x$ subject to at x=0, u=0 and at x=1, u=1.

(i) Find the solution of Causal Green's function. (8)

(ii) Find the Adjoint operator. (7)

(iii) Find the solution expression of adjoint green's fn operator (5)

Class Test 2

Advanced Mathematical Techniques in Chemical Engineering (CH 61015)

Full Marks:20

Completely solve the following PDE:

1. $$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

Subject to at t=0, u=0 and $\frac{\partial u}{\partial t} = u_{01}$; at x=0, $\frac{\partial u}{\partial x} = 0$; at x=1, u=2

Class Test 01

Advanced Mathematical Methods in Chemical Engineering

(CH 61015) FM:20

4. Check whether the PDES are linear, nonlinear, homogeneous, non-homogeneous: (5)

(vi) $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+t$ → Linear nonhomogenous

(vii) $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+ut$ → Linear homogenous

(viii) $u\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+\sin x$ → Non linear Non homogenous

(ix) $\left(\frac{\partial u}{\partial t}\right)^2=\frac{\partial^2 u}{\partial x^2}$ → Non linear, homogenous

(x) $\left(\frac{\partial u}{\partial t}\right)=\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial x}+\frac{\partial^2 u}{\partial y^2}-u\frac{\partial u}{\partial y}-xy$ → Non linear, non homogenous

5. Consider the ODE 5

$y''+\lambda y=0$ subject to

$at\ x=0, \frac{dy}{dx}+2y=0$

$at\ x=1, y=0$

Find the eigenvalues, λ_n

Find the eigenfunctions, y_n

$T = X(x)\,Y(y)$

$\frac{1}{Y}\frac{d^2T}{dy^2} = -\frac{1}{X}\frac{d^2T}{dx^2} = -\alpha^2$

6. Solve completely for $T(x,y)$: 10

$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0$

$at\ x=0, T=T_1;\ at\ x=a, \frac{\partial T}{\partial x}=0$

$at\ y=0, T=T_\infty;\ at\ y=b, -k\frac{\partial T}{\partial y}=h(T-T_\infty)$

$\theta = \frac{T-T_\infty}{T_1-T_\infty}$

$T = T_\infty + (T_1-T_\infty)\sum_{n=1} \frac{\sin \alpha y}{\sin h\alpha}\left(\frac{1-\cos\alpha}{\alpha}\right) \sin h\alpha(1-x)$

$(T_1-T_\infty)\frac{\partial T}{\partial x} + \frac{h}{k}(T_1-T_\infty)T = 0$

Linear Transformⁿ :-

$t(v_1+v_3) = t(v_1)+t(v_3)$

$t(\alpha v_1) = \alpha t(v_1)$

INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Department of Chemical Engineering

Mid-semester (Autumn) Examination 2023-2024

Subject: Advanced Mathematical Techniques in Chemical Engineering (CH61015)

Remarks:

1. This question paper contains two parts: **Part A** and **Part B**. Attempt both parts.
2. Write all the answers of a part together.
3. Unless otherwise stated, usual mathematical notations apply.
4. Time = 2 h; maximum marks = 30; total number of printed pages = 2.

Part A: Partial Differential Equations

1. If $L = \dfrac{d^2}{dx^2} + x\dfrac{d}{dx}$ and

at $x = 0, \dfrac{du}{dx} = 0,$

at $x = 1, \dfrac{du}{dx} + u = 0,$

Find the adjoint operator and the boundary conditions for the adjoint problem.

. . . 7 marks

2. The one dimensional, transient heat conduction problem takes the following form after non-dimensionalization.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

At $t = 0, u = 2x$; at $x = 0, \dfrac{\partial u}{\partial x} = 0$; $at x = 1, \dfrac{\partial u}{\partial x} + 2u = 0.$

Solve completely for $u(x, t)$.

. . . 8 marks

Part B: Linear Algebra

3. Consider $(V, \otimes)$ such that

$$V = \left\{ \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} : x_{ij} \in \{0, 1\} \right\}$$

Determine the number of elements present in V for which $\otimes = +$ is a binary operation on V.

. . . 5 marks

4. Consider $(V, F, \otimes, \odot)$ with $V = \left\{ \sum_{i=0}^{2} a_i x^i : a_i \in \mathbb{R} \right\}$, $F = (\mathbb{R}, \otimes, \odot)$, $\otimes = +$, and $\odot = \cdot$. If this algebraic structure makes a linear vector space then verify whether $f(x)$, $g(x)$ and $h(x)$ constitute a basis. Linear independence test, if necessary, must be done by the Wronksian determinant.

$$f(x) = x^2 - 2x + 5$$

$$g(x) = 2x^2 - 3x$$

$$h(x) = x + 3$$

. . . 5 marks

5. $(V, \mathbb{R}, +, \cdot)$ is a linear vector space with $V = \{[x \quad y \quad z]^T : x, y, z \in \mathbb{R}\}$ and the following equations with all real coefficients and b_i's defined on the space.

$$x + ay + (a^2 - bc)z = b_1$$

$$x + by + (b^2 - ac)z = b_2$$

$$x + cy + (c^2 - ab)z = b_3$$

Determine the dimension of this linear vector space.

. . . 5 marks

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