INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Department of Chemical Engineering

End-semester (Autumn) Examination 2022-2023

Subject: Advanced Mathematical Techniques in Chemical Engineering (CH61015)

Remarks:

1. This question paper contains two parts: **Part A** and **Part B**. Attempt both parts.
2. Unless otherwise stated, usual mathematical notations apply.
3. Time = 3 h; maximum marks = 100; total number of printed pages = 2.

Part A: Linear algebra

1. For the following set of simultaneous equations, verify whether the system has only real solutions for t, θ, $x_1(0), x_2(0) \in \mathbb{R}$. θ may be treated as a parameter independent of x_i and t. You may solve only for x_1 and deduce the conclusions from there.

$$\frac{dx_1}{dt} = (cos\theta)x_1 - (sin\theta)x_2 \tag{1}$$

$$\frac{dx_2}{dt} = (sin\theta)x_1 + (cos\theta)x_2 \tag{2}$$

. . . 20 marks

2. Determine the dimension and basis for the range space of the following set of equations using Fredholm's alternative theorem.

$$ix_1 + 2x_2 - 3ix_3 = 2 \tag{3}$$

$$5ix_1 + 10x_2 - 15ix_3 = 9 \tag{4}$$

$$2ix_1 + 4x_2 - 6ix_3 = 5 \tag{5}$$

. . . 15 marks

3. The function $H_n : \mathbb{R} \to \mathbb{R}$ defined as

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}(e^{-x^2}) \tag{6}$$

yields a set of Hermite polynomials. Sketch the first three polynomials and verify if the polynomials form an orthogonal set in [-1 , 1] by considering $n = 0, 1, 2$. You must test all inner products.

. . . 15 marks

Part B: Differential equations

4. Solve completely:

$$\frac{\partial u}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} \tag{7}$$

At $t = 0$, $u = f(r, \theta)$. At $r = 1$, $u = 0$. Use the suitable physical boundary conditions on the rest of the boundaries.

. . . 10 marks

5. Solve completely:

$$\frac{\partial u}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) \tag{8}$$

At $r = 1, \dfrac{\partial u}{\partial r} + 3u = 0$ and at $t = 0, u = 1$.
Use the suitable physical boundary conditions on the rest of the boundaries.

. . . 10 marks

6. Solve completely:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{1}{sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial u}{\partial \theta}\right) + \frac{1}{r^2}\frac{1}{sin^2\theta}\frac{\partial^2 u}{\partial \phi^2} = 0 \tag{9}$$

The boundary condition at $r = 1, u = f(\theta, \phi)$. Use the suitable physical boundary conditions on rest of the boundaries.

. . . 10 marks

7. Solve completely using Green's function method:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + t \tag{10}$$

At $t = 0$, $u = 1$. At $x = 0$, $\dfrac{\partial u}{\partial x} = 0$. At $x = 1$, $u = 2$.

. . . 20 marks

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