

INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Department of Chemical Engineering

End-semester (Autumn) Examination 2022-2023

Subject: Advanced Mathematical Techniques in Chemical Engineering (CH61015)

Remarks:

1. This question paper contains two parts: **Part A** and **Part B**. Attempt both parts.
 2. Unless otherwise stated, usual mathematical notations apply.
 3. Time = 3 h; maximum marks = 100; total number of printed pages = 2.
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Part A: Linear algebra

1. For the following set of simultaneous equations, verify whether the system has only real solutions for t , θ , $x_1(0)$, $x_2(0) \in \mathbb{R}$. θ may be treated as a parameter independent of x_i and t . You may solve only for x_1 and deduce the conclusions from there.

$$\frac{dx_1}{dt} = (\cos\theta)x_1 - (\sin\theta)x_2 \quad (1)$$

$$\frac{dx_2}{dt} = (\sin\theta)x_1 + (\cos\theta)x_2 \quad (2)$$

... 20 marks

2. Determine the dimension and basis for the range space of the following set of equations using Fredholm's alternative theorem.

$$ix_1 + 2x_2 - 3ix_3 = 2 \quad (3)$$

$$5ix_1 + 10x_2 - 15ix_3 = 9 \quad (4)$$

$$2ix_1 + 4x_2 - 6ix_3 = 5 \quad (5)$$

... 15 marks

3. The function $H_n : \mathbb{R} \rightarrow \mathbb{R}$ defined as

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} (e^{-x^2}) \quad (6)$$

yields a set of Hermite polynomials. Sketch the first three polynomials and verify if the polynomials form an orthogonal set in $[-1, 1]$ by considering $n = 0, 1, 2$. You must test all inner products.

... 15 marks

Part B: Differential equations

4. Solve completely:

$$\frac{\partial u}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} \quad (7)$$

At $t = 0$, $u = f(r, \theta)$. At $r = 1$, $u = 0$. Use the suitable physical boundary conditions on the rest of the boundaries.

... 10 marks

5. Solve completely:

$$\frac{\partial u}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u}{\partial r} \right) \quad (8)$$

At $r = 1$, $\frac{\partial u}{\partial r} + 3u = 0$ and at $t = 0$, $u = 1$.

Use the suitable physical boundary conditions on the rest of the boundaries.

... 10 marks

6. Solve completely:

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial u}{\partial \theta} \right) + \frac{1}{r^2} \frac{1}{\sin^2 \theta} \frac{\partial^2 u}{\partial \phi^2} = 0 \quad (9)$$

The boundary condition at $r = 1$, $u = f(\theta, \phi)$. Use the suitable physical boundary conditions on rest of the boundaries.

... 10 marks

7. Solve completely using Green's function method:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + t \quad (10)$$

At $t = 0$, $u = 1$. At $x = 0$, $\frac{\partial u}{\partial x} = 0$. At $x = 1$, $u = 2$.

... 20 marks
