

INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Department of Chemical Engineering

Mid-semester (Autumn) Examination 2022-2023

Subject: Advanced Mathematical Techniques in Chemical Engineering (CH61015)

Remarks:

1. This question paper contains two parts: **Part A** and **Part B**. Attempt both parts.
 2. Write all the answers of a part together.
 3. Unless otherwise stated, usual mathematical notations apply.
 4. Time = 2 h; maximum marks = 60; total number of printed pages = 2.
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Part A: Differential equations

1. The one dimensional transient heat transfer problem is given as:

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$$

At $t = 0, T = T_0$; at $x = 0, k \frac{\partial T}{\partial x} + h(T - T_\infty) = 0$; at $x = L, \frac{\partial T}{\partial x} = 0$. Find the temperature profile $T(x, t)$ completely.

... 10 marks

2. If $L = 5 \frac{d^2}{dx^2} + x \frac{d}{dx} + 1$, at $x = 0, \frac{du}{dx} + u = 0$ and at $x = 1, \frac{du}{dx} - 3u = 0$ then find adjoint operator and boundary conditions for the adjoint problem.

... 10 marks

3. Consider the ODE $y'' + \lambda y = 0$ subject to the boundary conditions, at $x = 0, \frac{dy}{dx} + \beta_1 y = 0$ and at $x = 1, \frac{dy}{dx} + \beta_2 y = 0$ where β_1, β_2 are constants. Find the eigenvalues and eigenfunctions.

... 10 marks

Part B: Linear algebra

4. If V is a non-empty set such that $V = \{(x, y) : x, y \in \mathbb{R}\}$ and the following operations are defined on V :

$$\oslash : (a_1, b_1) + (a_2, b_2) = (3b_1 + 3b_2, -a_1 - a_2)$$

$$\odot : k(a_1, b_1) = (3kb_1, -ka_1)$$

$$\forall (a_i, b_i) \in V, k \in \mathbb{R}.$$

- (a) Verify whether \otimes and \odot are binary operations on V .
- (b) Verify whether \otimes is distributive over \odot .
- (c) Verify whether \odot is distributive over \otimes .

... 4+3+3 = 10 marks

5. For the following two systems of equations in unknowns x_i 's and y_i 's, determine whether the corresponding range spaces are identical.

System 1

$$2x_1 + 6x_2 + 9x_3 + 7x_4 = p_1$$

$$x_1 + 3x_2 + 3x_3 + 2x_4 = p_2$$

$$-x_1 - 3x_2 + 3x_3 + 4x_4 = p_3$$

System 2

$$3y_1 + y_2 + 10y_3 + 14y_4 = q_1$$

$$y_1 + 2y_3 + 3y_4 = q_2$$

$$y_2 + 4y_3 + 5y_4 = q_3$$

... 10 marks

6. Determine a basis for the solution space of the following simultaneous equations:

$$\frac{dy}{dx} - 2y = 0$$

$$\frac{dy}{dx} - \frac{y}{x} = 0$$

... 10 marks
