Rheology of complex fluids

1/8/23

viscosity → resistance to flow → even though it might not be the correct parameter for some materials (solids).

soft matter:— anything that can deform (or flow)

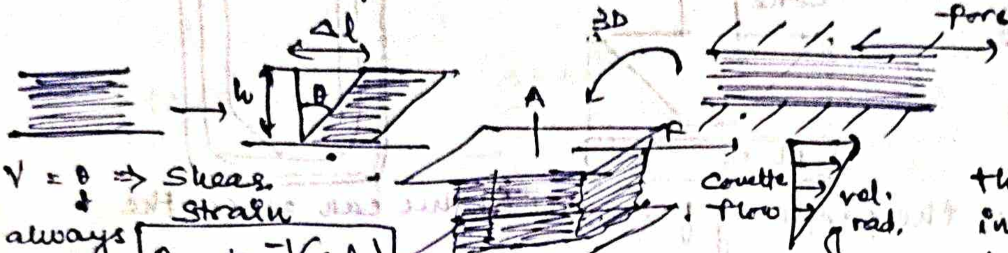
newtonian fluid → $\sigma = \eta \dfrac{\partial v_x}{\partial y}$; $\dot{\gamma} = \dfrac{\partial \gamma}{\partial t}$

uniform viscosity (μ) ; strain rate.

$$\boxed{\text{prove that } \dot{\gamma} = \frac{\partial v_x}{\partial y}}$$

shear strain = γ

strain rate = $\dot{\gamma}$

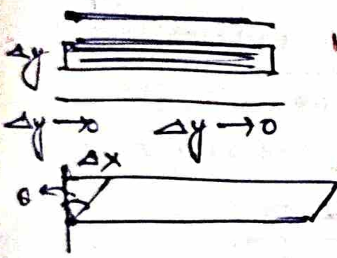


$\gamma = \theta$ ⇒ Shear strain & always an angle

$$\boxed{\theta = \tan^{-1}\left(\frac{\Delta l}{h}\right)}$$

$\sigma = \dfrac{F_{tangential}}{\text{Area of plate}}$

If we assume that there is no slip cond^n the liq. molecules in contact with lower plate don't move and upper plate move so we have a vel. gradient.



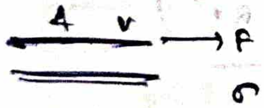
We don't want our h to be $h >> \Delta l$

We want it to be comparable with Δl.

So, we look at Local strain.

Local strain → $\gamma = \theta = \lim_{\Delta y \to 0} \dfrac{\Delta x}{\Delta y}$; $\tan\theta \approx \theta$ ($\theta \approx \tan\theta$)

$\boxed{\dfrac{\partial v_x}{\partial x}}$ → elongation flow or stretching → Eg → stretching of soft matter, 2 particles moving apart in opposite x direction.



$\sigma = \eta \dfrac{\partial v_x}{\partial y} \sim \dfrac{\Delta v_x}{h} = \eta \dot{\theta}$

Newton's Law of viscosity applicable on not just Newtonian fluids but others as well. Thus, does not guarantee that the fluid it is applied on is a Newtonian fluid

$\sigma = \dfrac{F}{A}$ → $\eta = \dfrac{F/A}{\dot{\theta}} = \dfrac{F/A}{u/h}$

(for finding viscosity)

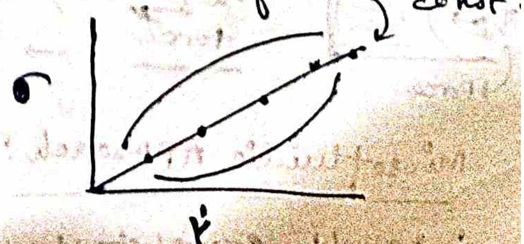
→ viscosity is measured using a rheometer

→ while measuring viscosity we can control only one parameter of the system.

→ for more precision the gap should be less

→ if strain is to be found stress can be controlled and vice versa.

Newtonian fluids → μ const.



→ If expt. needs to be carried out for more time, dimensions of the plates should be very large (impractical) → so rheometer is designed with circular plates.

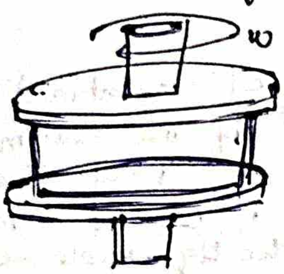
PIV (Particle Image Velocimetry)

use of fluorescent particles to get the vel. profile using particle local $\frac{\partial v_x}{\partial y}$. more precision in μ

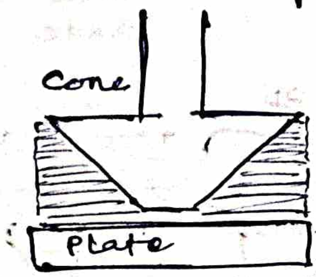
Rheometry

(structure property rel"ship of soft matter)

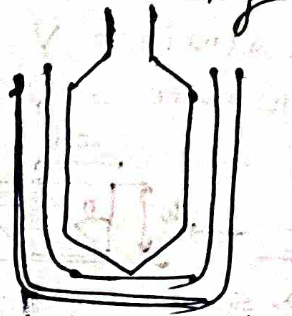
Parallel Plate Rheometry



Cone and Plate Rheometry



Cup and Bob Rheometry



We can measure the property from which we can access the structures

Reynolds no. = $\frac{\text{Inertial force}}{\text{viscous force}} = \frac{\rho v d}{\mu}$

Re depends on surface-to-vol. ratio

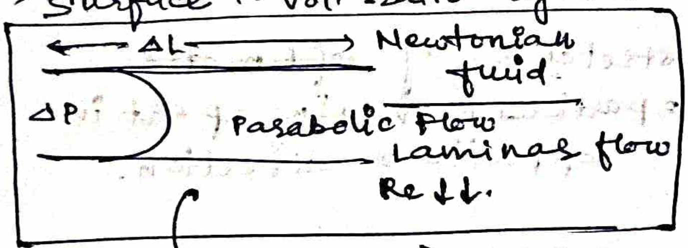
applied by wall

kinetic Δ

in microchannels, the flow is always laminar

Re no. is very low due to increased viscous forces.

surface to vol. ratio high↑



Hagen Poiseuille flow is parabolic :-

$$v_x = \frac{\Delta P}{4 L \mu} R^2 \left(1 - \frac{r^2}{R^2}\right)$$

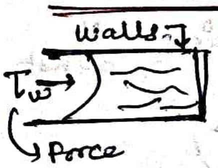
$$v_x = v_0 \left(1 - \frac{r^2}{R^2}\right)$$

$$\frac{\partial v_x}{\partial r} = \frac{2v_0}{R}$$

$v_0 = f(\Delta P, L, \mu)$

$$\left.\frac{\partial v_x}{\partial r}\right|_{r=R} = \frac{2v_0}{R}$$

Wall shear stress → stress applied by fluid on the walls of the pipe / whatever vessel it is flowing through.



$\underbrace{T_w (2\pi r L)}_{\text{force}} = \Delta P \, \pi R^2$

$\rightarrow T_w = \frac{\Delta P R}{2L}$

Microfluidic Approach :- We are approximating the viscosity of the fluid by saying that the wall μ is roughly equal to the μ of bulk.

Viscosity = $\frac{\text{stress}}{\text{strain rate}}$ (at that pt.)

$\eta = \dfrac{\frac{(\Delta P) R}{2L}}{\frac{2}{R}\left(\frac{\Delta P}{L\eta}\right) R^2}$ ✗ → everything will cancel $\eta = \eta$ → So we need to know v_0 independently to find viscosity μ.

$v_0 = \dfrac{\Delta m}{t} = \underbrace{\dot{m}}_{\text{mass flow rate}} = \rho A v_{avg}$ ⟹ we'll find v_{avg}

$v_{avg} = \dfrac{\dot{m}}{\rho A}$ ⟹ $v_{avg} = \dfrac{v_0}{2}$

$v_0 = 2 v_{avg}$

So $\boxed{v_0 = \dfrac{2\dot{m}}{\rho A}}$

$\tau_{\text{fluid at wall}} = \dfrac{\frac{\Delta P R}{2L}}{\left(\frac{4\dot{m}}{\rho A}\right) \times \frac{2}{R}} = \dfrac{\Delta P R^2 \rho A}{8 L \dot{m}}$ (A = Area of cross section)

$= \dfrac{\Delta P R^2 \rho A}{8 L \dot{m}}$

Constitutive eq^n → Eq^n which governs fluid behaviour. Any constitutive eq^n relates 2 terms. (Stress & strain, strain rate or higher derivatives of strain)

$\sigma = f(\gamma, \dot{\gamma}, \ddot{\gamma} ..)$

$\boxed{\sigma = \eta \dot{\gamma}}$ $\boxed{\sigma = G \gamma}$ ← any constant (elastic & viscous particle)

viscoelastic → complex fluids

Weissenberg effect → rod climbing effect

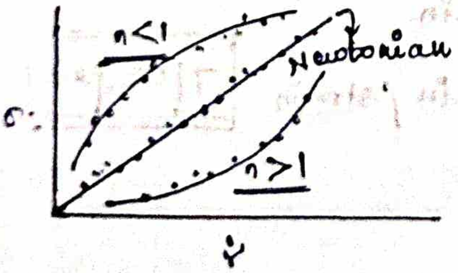
Power Law fluid ⟹ $\boxed{\sigma = \mu (\dot{\gamma})^n}$

3 | 08 | 23

When $n = 1$ ⟹ Newtonian fluid.

$n < 1$ → shear thining fluid.

$n > 1$ → shear thickening fluid



$\dot{\theta} = \dot{\gamma} = \dfrac{\partial v_x}{\partial y}$

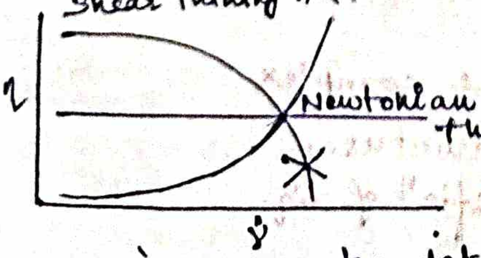
$\dot{\theta} = \lim_{\Delta t \to 0} \dfrac{\theta_{t+\Delta t} - \theta_t}{\Delta t}$

$\theta_t = \theta_{t+\Delta t}$

σ v/s $\dot{\gamma}$ graph is not very initutitive.

*. Shear Thining means when force applied $\mu \downarrow$ (viscosity ↓)

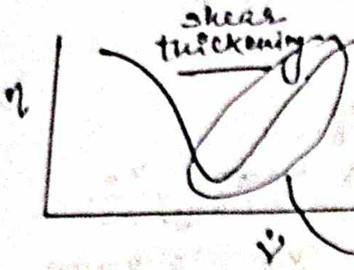
Shear Thining $n < 1$



$\dot{\gamma}$	σ	$\frac{\sigma}{\dot{\gamma}} = \eta$

we don't need extra data. We can use the data to build σ v/s $\dot{\gamma}$ to build η v/s $\dot{\gamma}$ graph.

what happens if $\dot{\gamma} \to 0$
we do not know, high precision device needed for $\dot{\gamma}$ as less than 10^{-3}



Shear Thickening fluid, $n > 1$ → pure liq. can never show shear thickening. → always a dispersion is needed (emulsifier) polymer

↓

shear thickening happens within a shear rate window → They get so close to each other that they form a strong cluster → Hydrocluster formation Theory.

sudden ↑ in viscosity

↓

first it breaks & then suddenly starts thickening.

if very slow shear rate particles do not come together.

So shear rate needs to be very high.

Complex fluids → complex microstructures (somewhere in b/w macro level and molecular level)

oil in water emulsion :→

What happens is eventually the oil molecules coalesce to form a single cluster.
↓ phase sepn

To sustain this emulsion, we need to add emulsifier (surface active agents) to entrap the molecules

3D Network → Gel (Eg. - curd)
oriented protein molecules. Eg. - cheese.
air trapped inside → foam → gas entrapped inside 3-D. network of liq.

Rheology — Science of flow.
Newton's Law of viscosity : stress ∝ strain rate
Hooke's Law of Elasticity : stress ∝ strain.

7	08	23

only one of the parameters (stress / strain / strain rate) can be controlled.

$\sigma = f(\gamma, \dot{\gamma}, \ldots$ material properties$)$
↓
flow curve
constitutive eqn

Complex fluids → constitutive eqn are complex
↑
complex microstructures.

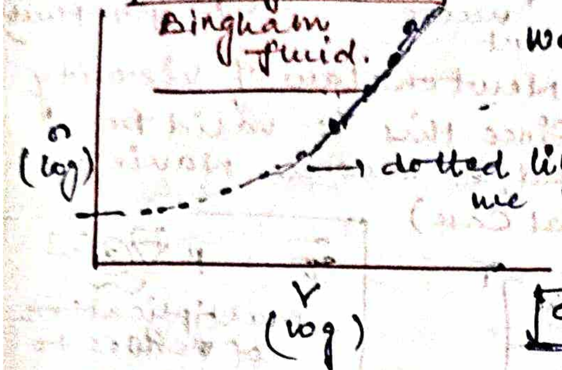
Cushion → Solid foam → Solidificn of liq.

Liquid crystal → complex fluids → atleast a combo. of 2 polymers or fluids.

gel, emulsion, foam, conc^c dispersion (toothpaste)

Constitutive eq^n :-

$$\boxed{\sigma = \sigma_y + k\dot{\gamma}}$$



$\dot{\gamma} \to 0$ (we cannot make rate of change 0)

We are controlling the shear rate (rate of change of θ is being controlled)

↓ Torque is being applied to maintain the θ
↓ software then gives stress

→ dotted line (bcoz in expts we never get a continuous line. we get discrete pt lines)

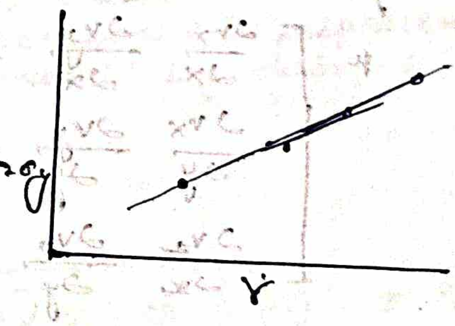
$\boxed{\text{cause} \to \text{effect}}$

$k \to$ material property

If the rate of deformation is very very small, then also there will be stress (residual stress), we need to overcome it to make the fluid / material flow.

$$(\sigma - \sigma_y) = (k \cdot \dot{\gamma})$$

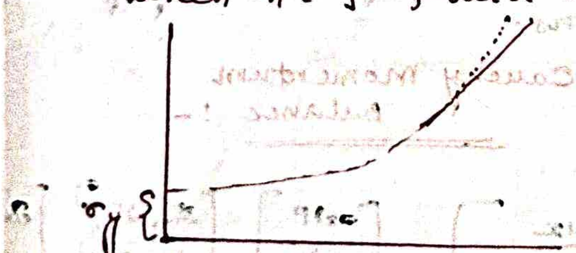
$\dot{\gamma}$	σ	$\sigma - \sigma_y$



Herschel Berkley fluid

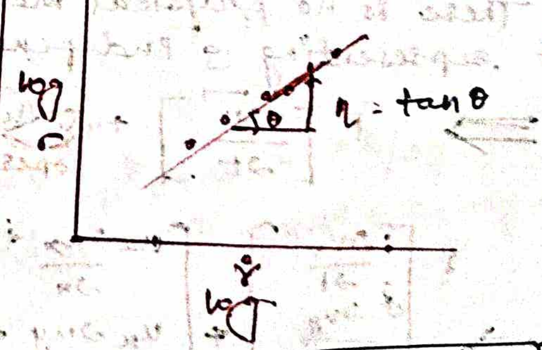
$$\sigma = \sigma_y + k(\dot{\gamma})^n$$

when $n = 1$ fluid is Bingham.



Power Law fluid :-

$$\sigma = k(\dot{\gamma})^n$$



8 | 08 | 23

Visco Elasticity (Elastic + viscous)

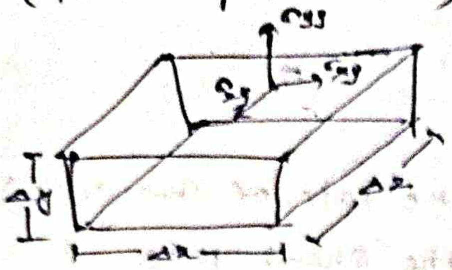
Stress → Tensor

$$x: \frac{\partial u_x}{\partial t} + u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + u_z\frac{\partial u_x}{\partial z} = \frac{1}{\rho}\left(-\frac{\partial P}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right) + g_x$$

$$y: \frac{\partial u_y}{\partial t} + u_x\frac{\partial u_y}{\partial x} + u_y\frac{\partial u_y}{\partial y} + u_z\frac{\partial u_y}{\partial z} = \frac{1}{\rho}\left(-\frac{\partial P}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right) + g_y$$

$$z: \frac{\partial u_z}{\partial t} + u_x\frac{\partial u_z}{\partial x} + u_y\frac{\partial u_z}{\partial y} + u_z\frac{\partial u_z}{\partial z} = \frac{1}{\rho}\left(-\frac{\partial P}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right) + g_z$$

We want to define stress at a pt. but that is not possible. So, mathematically, we say that stress is defined on a very small cube which is almost the size of a pt. and nine vectors just represent this cube (Tensor representation) n dimensional property.



$$x: \frac{\partial u_x}{\partial t} + u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + u_z\frac{\partial u_z}{\partial z} = \frac{1}{\rho}\left(-\frac{\partial P}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right) + g_x \rightarrow$$ Cauchy Momentum Balance for fluids (for gen. fluid)

$$\boxed{\overline{\overline{\tau}} = \mu \vec{\nabla} \otimes \vec{v}}$$

Tensor, dyadic product, vector

$$\begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix}_{3\times 1} [v_x \ v_y \ v_z]^{1\times 3} = 3\times 3$$

When we plug this Newton's law of viscosity in above eq^n. Since this is valid for a Newtonian fluid, it becomes Navier Stokes Eq^n (special case)

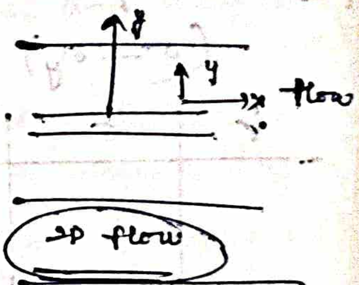
$\overline{\overline{\tau}} = \mu \vec{\nabla} \otimes \vec{v}$ multiplication of vectors to give tensor → Dyadic product

$$\begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix} = \mu \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{bmatrix}^T$$

(Newton's Law of viscosity)

$$\downarrow$$

$$\mu \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_y}{\partial x} & \frac{\partial v_z}{\partial x} \\ \frac{\partial v_x}{\partial y} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{bmatrix}$$



$$\boxed{\tau_{yx} = \frac{\partial v_x}{\partial y}}$$

$\tau_{xz} \rightarrow$ (means force in x dir^n on z plate.) Area vector

RHS → shear force + pressure force + Body force.

* There is no physical meaning of Tensor. It is just a way of representing 9 independent relations.

$$\Rightarrow \rho\left[\frac{\partial \vec{u}}{\partial t}\right] + u \nabla \vec{u}$$ → scalar operator

Cauchy Momentum Balance :-

$$\Rightarrow \rho \begin{bmatrix} \hat{i}\frac{\partial v_x}{\partial t} \\ \hat{j}\frac{\partial v_y}{\partial t} \\ \hat{k}\frac{\partial v_z}{\partial t} \end{bmatrix}_{3\times 1} + \begin{bmatrix} u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + u_z\frac{\partial u_x}{\partial z} \\ u_x\frac{\partial u_y}{\partial x} + u_y\frac{\partial u_y}{\partial y} + u_z\frac{\partial u_y}{\partial z} \\ u_x\frac{\partial u_z}{\partial x} + u_y\frac{\partial u_z}{\partial y} + u_z\frac{\partial u_z}{\partial z} \end{bmatrix}_{3\times 1} = \begin{bmatrix} -\frac{\partial P}{\partial x} \\ -\frac{\partial P}{\partial y} \\ -\frac{\partial P}{\partial z} \end{bmatrix} + \begin{bmatrix} \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \cdots \\ \vdots \\ \vdots \end{bmatrix} + \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}$$

$$\underbrace{u_x\frac{\partial}{\partial x} + u_y\frac{\partial}{\partial y} + u_z\frac{\partial}{\partial z}}$$

$$(\vec{u}\,\vec{\nabla}) \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \Longrightarrow (u_x\hat{i} + u_y\hat{j} + u_z\hat{k})\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$$

$$\boxed{\rho\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u} = -\nabla p + \vec{\nabla}\cdot(\overline{\tau}) + (\vec{g})}$$

* Product of a vector and tensor is ~~a tensor~~

$$\boxed{\rho\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u} = -\nabla p + \underbrace{\nabla\cdot\overline{\tau}}_{\text{divergence vector}} + \vec{g}}$$

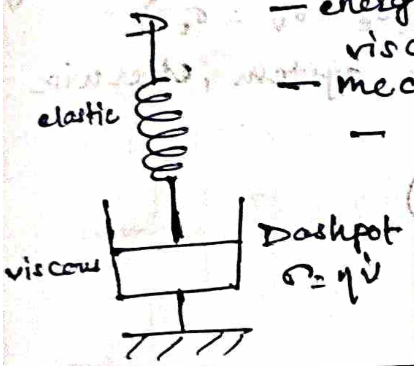
$\overline{\tau} \rightarrow f(\gamma, \dot{\gamma}, \gamma..)$

$\boxed{\text{Shear rate} \uparrow \ \text{viscosity} \downarrow .}$

$\sigma = \sigma_y + K(\dot{\gamma})^n \rightarrow HB$
$\sigma = \sigma_y + K\dot{\gamma} \rightarrow$ Bingham } give some idea of the viscosity & flow of fluids.

But we need other viscoelastic models to get a better idea.

Maxwell Model :– more interest in the microstructure (complexity)



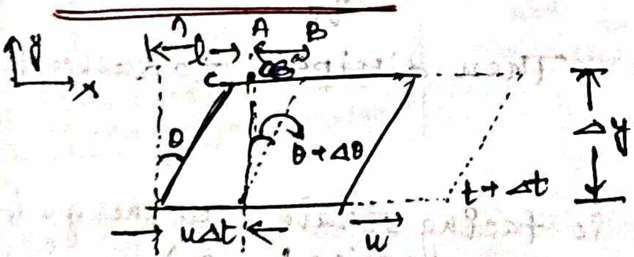
– energy transport to the spring gets dissipated to viscous fluid.
– mechanical fluid.
– elastic component is represented by spring and viscous comp. by dashpot.

10 | 08 | 23

$$\boxed{\frac{\partial}{\partial t}(\rho\vec{V}) + \nabla(\rho\vec{V}\vec{V}) = \rho\vec{g} + \nabla\cdot\sigma_{ij}}$$

Shear flow :–

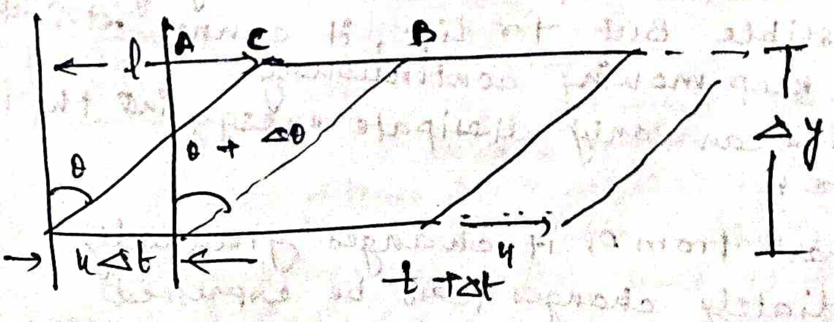
Shear rate $= \dot{\gamma}_{xy} = \dfrac{\partial u_x}{\partial y}$



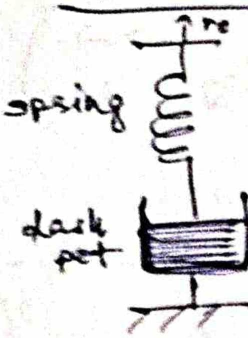
$\gamma_t = \theta_t = \dfrac{l}{\Delta y}$

$\gamma_{t+\Delta t} = \theta_{t+\Delta t} = AB/\Delta y = \dfrac{AC+CB}{\Delta y}$

$= \dfrac{(l - u\Delta t) + u\Delta t + \frac{\partial u}{\partial y}\Delta y\,\Delta t}{\Delta y}$



Mechanical Models :-



→ not a real device, just a representation

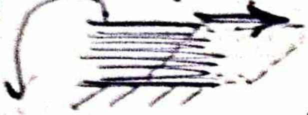
elastic nature $\boxed{\sigma_0 = G_0 \gamma_0}$ → elasticity.

G_0 → some kind of modulus

viscous model → $\boxed{\sigma = \eta_0 \dot{\gamma}_v}$

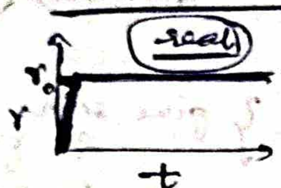
η_0 → viscosity

Why series ? Not a parallel combination ?



Step Strain

(real)



$\eta_0, \gamma_0, \sigma, G_0$

in real expt. what are the parameters we can control? → strain we are applying

$\boxed{\gamma_0 = \gamma_e + \gamma_v}$ → strain is divided b/w elastic and viscous.

rel^n b/w σ_e & σ_v → They are same ($\sigma_e = \sigma_v = \sigma_0$)

there should be uniform throughout the system, otherwise the spring will break.

$$0 = \dot{\gamma}_e + \dot{\gamma}_v \quad \text{(in case of no strain)}$$

$$0 = \frac{\dot{\sigma}_e}{G_0} + \frac{\sigma_v}{\eta_0} \Rightarrow \boxed{\frac{\dot{\sigma}}{G_0} + \frac{\sigma}{\eta_0} = 0}$$

$$\boxed{\frac{1}{G_0}\frac{d\sigma}{dt} + \frac{\sigma}{\eta_0} = 0}$$

$$\Rightarrow \frac{d\sigma}{dt} = \frac{-G_0 \sigma}{\eta_0} \Rightarrow \ln \sigma = -\frac{G_0 t}{\eta_0} \rightarrow \sigma = Ce^{-\frac{G_0 t}{\eta_0}}$$

at $t=0$ Whole Energy → spring. Then dissipates to dashpot.

$t = 0, \gamma = \gamma_0$

$\boxed{\gamma_0 = \gamma_{e_{t=0}}}$ → only spring is facing strain (all energy is stored in the spring, $t = 0$)

for spring material or elastics step strain or staying at deform for some time is possible. But for liq., it cannot stop at deform^n, it needs to keep moving continuously.

Viscous nature of fluids can only dissipate energy in the form of heat it cannot store it.

Dashpot → γ_v will start from 0, it changes gradually.
γ_e immediately changes, can be expressed immediately.

$t=0,\ \gamma=\gamma_0,\ \gamma_e=\gamma_0,\ \gamma_0=0$

$$\int_{\sigma_0}^{\sigma}\frac{d\sigma}{\sigma}=-\int_0^t \frac{G_0\,dt}{\eta_0} \rightarrow \ln\left(\frac{\sigma}{\sigma_0}\right)=-\frac{G_0 t}{\eta_0}$$

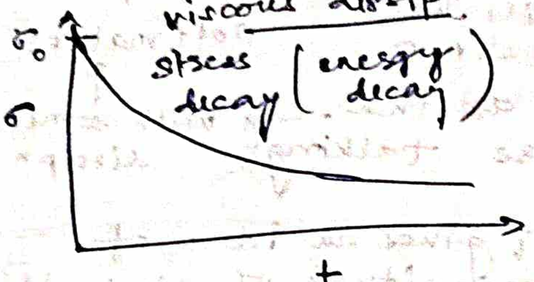
$$\Rightarrow \sigma=\sigma_0 e^{-(G_0/\eta_0)t} \rightarrow \sigma=\sigma_0 e^{-(t/\tau)}$$

$\tau=\frac{\eta_0}{G_0}$ = timescale $\boxed{\sigma_0=G_0\gamma_0}$

(Dashpot resists deform^n)

$\sigma(t)=G_0\gamma_0 e^{(-t/\tau)}$

considers viscous part like inductor (resists change in strain)



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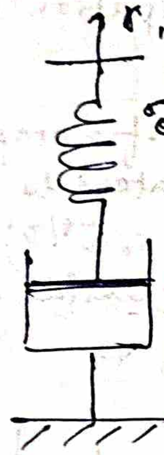
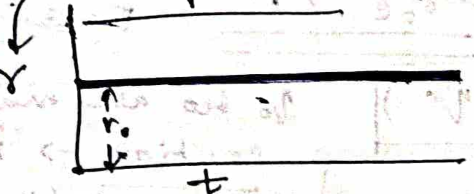




IC $t=0 \rightarrow \sigma=\sigma_e=G$

$\gamma=\gamma_e=\gamma_0$ $\sigma_0=G\gamma_0$

$t>0$

$\gamma_0=\gamma_e+\gamma_v$

$\dot{\gamma}=\dot{\sigma}_{\infty}+\sigma_0=0$

not modulus $\leftarrow G_0$ $\quad \eta_0 \rightarrow$ not the actual viscosity

some kind of model parameters (material property)

Special expt. where we are keeping $\dot{\gamma}=0$

$\dot{\gamma}=at$

$\dot{\gamma}=\frac{1}{G_0}\left(\dot{\sigma}+\frac{\sigma}{\tau}\right)$

MaxWell Model

obj :- To get something measurable out of model parameters

η_0, G_0 are model parameters.

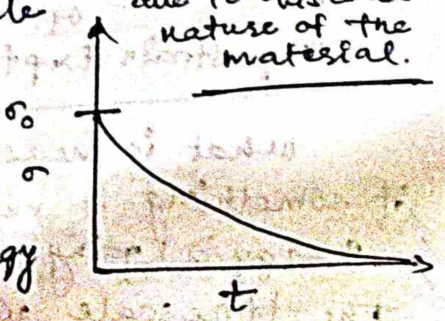
$$\boxed{\sigma_{(t)}=G_0\gamma_0 e^{-t/\left(\frac{\eta_0}{G_0}\right)}}$$

$\frac{\eta_0}{G_0}=\tau$ = relaxation timescale

$$\boxed{\sigma=\underbrace{G_0\gamma_0}_{\sigma_0} e^{-t/\tau}}$$

σ_0 : stress is decaying.

initially :- σ_0 = stress stored in the spring.

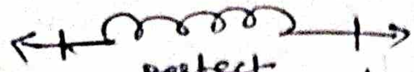


Relaxation ↓ ~~energy~~ dissip^n due to viscous nature of the material.

whenever we are putting step strain, energy is getting stored in spring but at the same time we have viscous component. So, what is happening is viscous dissipation of energy

Energy ~ Stress → Energy is decaying → Stress is decaying
Either it remains const. or it dissipates (decays) → (takes some other form of energy which is not helpful for us)

σ → Total output → Total stress experienced by the system.



perfect spring

some energy will get stored and it will remain forever
So relx^n timescale → ∞.

Ex. Water & similar fluids have relx^n timescale → ~0.

Somewhere in between are the material about which we are talking (soft matter with some viscous dissip^n)

elastic part or (spring) stores energy
viscous part dissipates energy.

Ex. τ = lose for 10 s.
σ as compared to σ_0
$\sigma = \sigma_0 e^{-10/\tau}$

τ gives an idea of magnitude of dissipation of Energy.

$\boxed{\eta_{actual} = f(G_0, \eta_0)}$

η_0 G_0 are material specific property

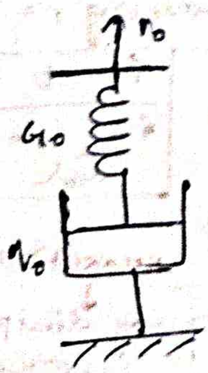
Strain energy → uses it (dissip^n) → converts to a new form

Caution → Not all materials behave like this ($\sigma = \sigma_0 e^{-t/\tau}$) stress decaying exponentially.

Step Strain Test

17/08/23

Input → Const. strain (unitless) (b/w 0 to 1 or %)
output → Stress.



$\sigma = \sigma_0 \exp(-t/\tau)$ (σ_0 as γ_0)
τ = relx^n time scale

"Higher relx^n time scale signifies relaxation process is slow"

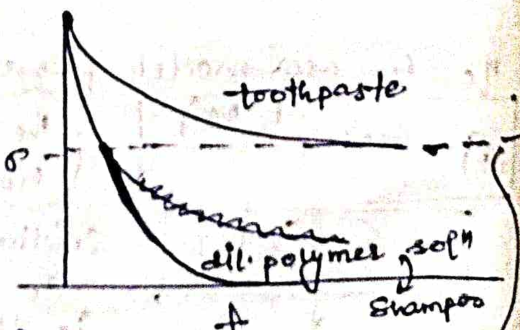
$\frac{\eta_0}{G_0} = \tau$

$\boxed{\sigma = f(\tau)}$

Step strain Expt : – $\boxed{\sigma \text{ v/s } \tau(t)}$

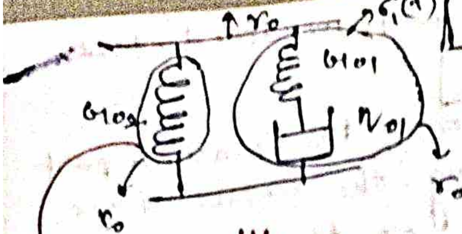
What is measurable? – τ

If something is very viscous, lots of energy is going to dissipate but the timescale will vary (depends)



only this much dissip^n is possible
The rest of the energy is going to remain for quite longer period of time (maybe ∞)

$\boxed{\sigma_2 = \gamma_0 G_{02}}$ **Tooth paste Model :-**

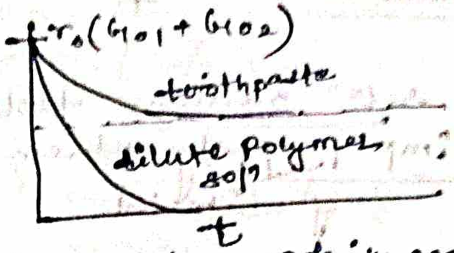


These will always be energy stored in this spring

$$\sigma_1(t) = \sigma_{10} \exp(-t/\tau)$$

$$\boxed{\sigma(t) = \sigma_0 G_{02} + \gamma_0 G_{01} \exp(-t/\tau)}$$

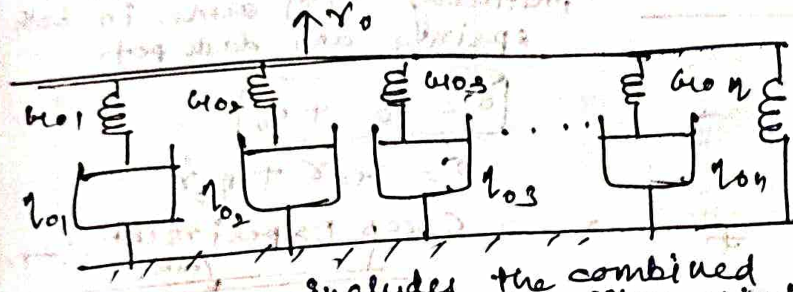
$t = 0,\ \sigma_0 = \gamma_0 (G_{01} + G_{02})$



$$\sigma(t) = \sigma_0 + \sigma_{01} \exp\left(\frac{-t}{\tau_1}\right) + \sigma_{02} \exp\left(\frac{-t}{\tau_2}\right) + \sigma_{03} \exp\left(\frac{-t}{\tau_3}\right)$$

Relaxn time distribn :- multiple relxn time scales work in real fluids instead of one τ, there are a no. of τs which cumulatively decide the decay behaviour of the fluid.

$$\sigma(t) = \sum_{i=1}^{n} \sigma_i t$$



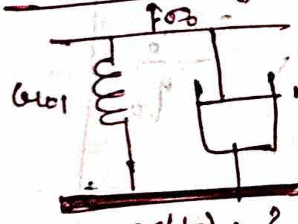
$\sigma_i(t) = \sigma_{i0} \exp\left(\frac{-t}{\tau_i}\right)$ $\tau_i = \frac{\eta_{0i}}{G_{0i}}$

$$\boxed{\sigma(t) = \sum_{i=1}^{n} \sigma_{i0} \exp\left(-\frac{t}{\tau_i}\right)}$$

$+ \sigma_\infty$ → energy that stays in the system

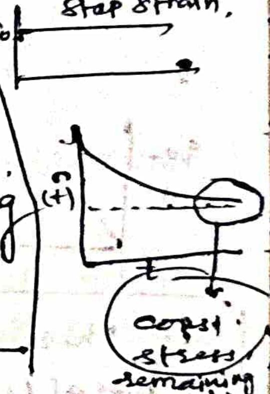
includes the combined effect of multiple substances

Kelvin - Voigt Model :- Stress ($\sigma(t)$) decays when we apply step strain.



$$\sigma(t) = \left(\sum_{i=1}^{n} \sigma_{0i} \exp\left(\frac{-t}{\tau_i}\right)\right)$$

$+ \sigma_\infty$ (some const. stress remaining to represent we use a perfect string)

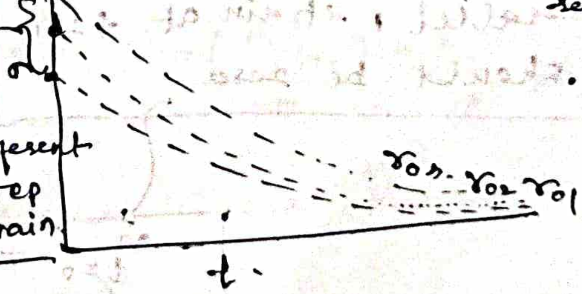


$\boxed{21 | 08 | 23}$

Single Maxwell Element :-

$$\sigma = \gamma_0 G_0 \exp\left(\frac{-t}{\tau}\right)$$

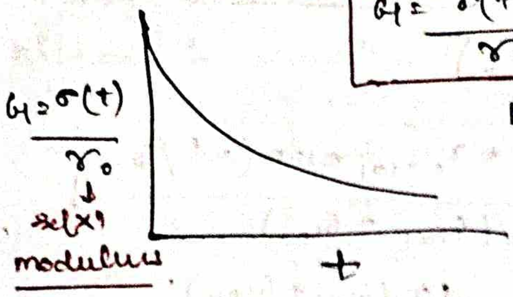
γ_0 changing in the graph beside.



$$\frac{\sigma(t)}{\gamma_0} = \exp\left(\frac{-t}{\tau}\right)$$

t	σ/γ_{01}	σ/γ_{02}	σ/γ_{03}

→ if we plot σ/γ_0 we get a master curve (all the single curves we are getting ($\gamma_{01}, \gamma_{02}, \gamma_{03}$) collapse into one) for the representation of material property



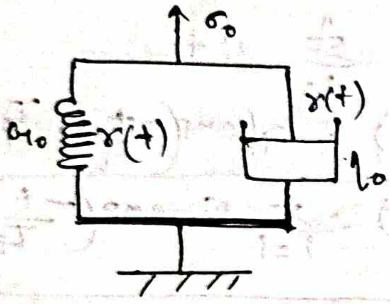
$\boxed{G_1 = \frac{\sigma(t)}{\gamma_0} = \text{relx}^n \text{ modulus}} \rightarrow$ Normalized stress

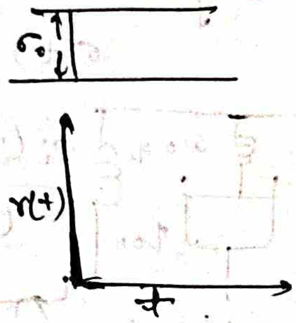
We generally eliminate the plateau part while constructing the curve as we want to focus on the exponential part $\boxed{G_1(t) = G_{10}\exp\left(\frac{-t}{\tau_i}\right)}$ (generalized maxwell modelli)

relx^n modulus tells about the nature of the material. It is an imp. property which decays with time. during stress relx^n exp. of material.

Kelvin Voight Model :—

Charging a resistor with a capacitor



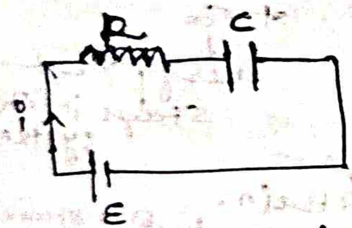


initially $\gamma(t)$ same in both spring and dash pot

$\boxed{\sigma = \sigma_e + \sigma_v}$

$$\sigma = G_{10}\gamma + \eta_0\dot{\gamma}$$

Creep Experiment :— (aim: get some property)



$$E = iR + \frac{q}{C}$$

$$E = \frac{dq}{dt}R + \frac{q}{C}$$

$$q = q_0\left(1 - \exp\left(\frac{-t}{RC}\right)\right)$$

$$E = \left(\frac{dq}{dt}\right)R + \frac{q}{C}$$

$$\boxed{\frac{d\gamma}{dt} + \frac{\gamma}{\left(\frac{\eta_0}{G_0}\right)} = \frac{\sigma_0}{\eta_0}}$$

$$\frac{d\gamma}{dt} + \frac{\gamma}{\tau_0} = \frac{\sigma_0}{\eta_0}$$

$$\gamma\exp\left(\frac{t}{\tau_0}\right)\Big|_0^t = \int_0^t \frac{\sigma_0}{\eta_0}\exp\left(\frac{t}{\tau_0}\right)dt$$

$$\gamma(t)\exp\left(\frac{t}{\tau_0}\right) - \gamma_{t=0} = \frac{\tau_0\sigma}{\eta_0}\exp\left(\frac{t}{\tau_0}\right)$$

$$\tau_0 = \frac{\eta_0}{G_{10}}$$

If there is a viscous element in parallel, strain at $t = 0$ should be zero.

$$\boxed{\cancel{-\gamma_{t=0}} + \gamma(t)\exp\left(\frac{t}{\tau_0}\right) = \frac{\sigma_0}{G_{10}}\exp\left(\frac{t}{\tau_0}\right)}$$

$$\gamma(t)\exp\left(\frac{t}{\tau_0}\right) = \frac{\sigma_0}{G_{10}}\left[\exp\left(\frac{t}{\tau_0}\right) - 1\right]$$

$$\boxed{\gamma(t) = \frac{\sigma_0}{G_{10}}\left(1 - \exp\left(\frac{-t}{\tau_0}\right)\right)}$$

Retardation Time Scale

$$\tau_0 = \frac{\eta_0}{G_{10}}$$

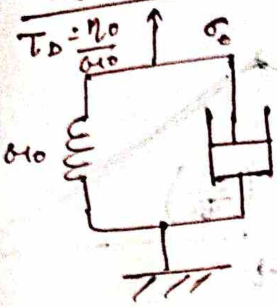
Strain at $t=0$ is σ_0 / G_0 for maxwell model.

* Strain takes some time to reach its max. value bcoz of the viscous nature of liq. (Energy dissip^n occurs over a time scale). T_D → dissip^n → retard^n time scale.

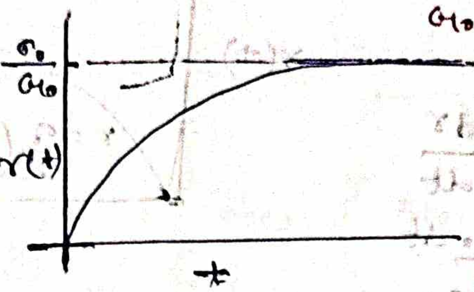
22/08/23

KV Model:—

$T_D = \frac{\eta_0}{G_0}$



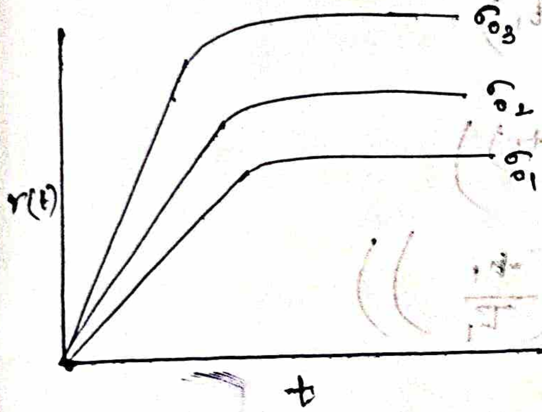
(Creep Test) $\gamma(t) = \frac{\sigma_0}{G_0}\left(1 - \exp\left(\frac{-t}{T_D}\right)\right)$



→ if the spring is not there, no delay in response.

→ But here the process is creepy (lagging) bcoz of the spring

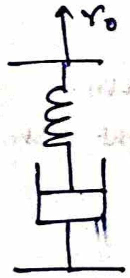
→ There is a retardation in strain evolution



$$\frac{\gamma(t)}{\sigma_0} = \underbrace{\frac{1}{G_0}\left(1 - \exp\left(\frac{-t}{T_D}\right)\right)}$$

$$\boxed{J(t) = \frac{1}{G_0}\left(1 - \exp\left(\frac{-t}{T_D}\right)\right)}$$

Creep Compliance

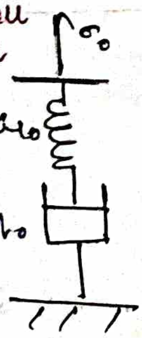


$$G(t) = \frac{\sigma(t)}{\gamma_0} = G_0 \exp\left(\frac{-t}{T}\right)$$

Why J and G are not reciprocals?
They are correlated but not straightforward reciprocals of each other.

⟹ We cannot do creep test with Maxwell model bcoz we will not get something useful out of it

const stress in maxwell model



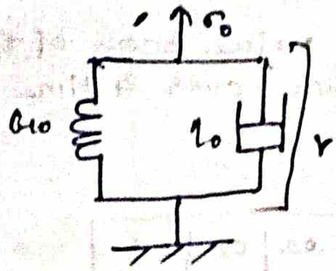
$$\dot{\gamma} = \dot{\gamma}_e + \dot{\gamma}_v = \frac{d\gamma_e}{dt} + \dot{\gamma}_v = \frac{d(\sigma_0/G_0)}{dt} + \frac{\sigma_0}{\eta_0}$$

$$\frac{d\gamma}{dt} = \cancel{\frac{\dot{\sigma}}{G_0}}^{\,0} + \frac{\sigma}{\eta_0}$$

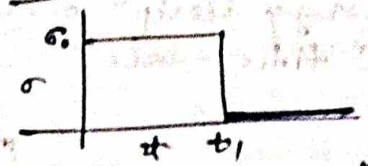
Linear stress is applied. Linear evolution of stress const^d

$$\frac{d\gamma}{dt} = \frac{\dot{\sigma}}{G_0} + \frac{\sigma}{T\, G_0}$$

Trivial sol^n	Non Trivial sol^n
η_0 & G_0 const	$\eta_0(t)$ & $G_0(t)$ f^n (t)



Input profile :-



Creep Recovery Test

$\gamma(t)$ for $t > t_1$

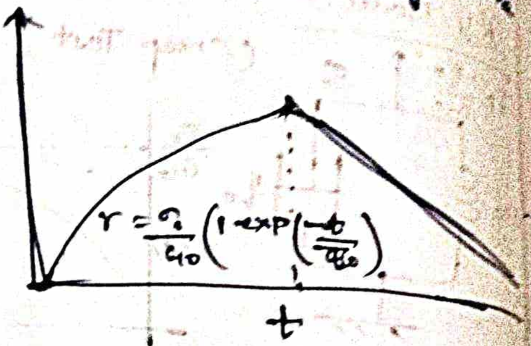
Plot strain $\gamma(t)$ as a fn of time

$$\sigma = G_{10}\gamma + \eta_0\dot{\gamma}$$

$t > t_1$

$$0 = G_{10}\gamma + \eta_0\dot{\gamma}$$

$$0 = G_{10}\gamma + \eta_0\frac{d\gamma}{dt}$$



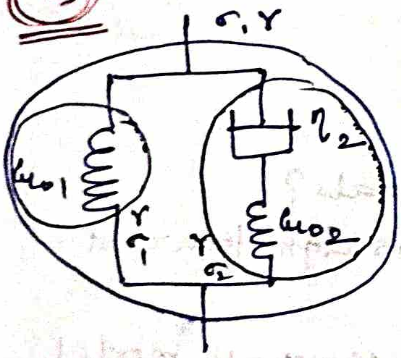
$$\rightarrow \int_{\gamma_{t_1}}^{\gamma(t)} \frac{d\gamma}{\gamma} = -\int_{t_1}^{t} \frac{G_{10}\,dt}{\eta_0}$$

$$\rightarrow \ln\left(\frac{\gamma}{\gamma_{t_1}}\right) = \frac{-G_{10}}{\eta_0}(t-t_1)$$

$$\rightarrow \gamma = \gamma_{t_1}\exp\left(\frac{-G_{10}(t-t_1)}{\eta_0}\right)$$

$$\therefore \gamma(t_1) = \frac{\sigma_0}{G_{10}}\left(1-\exp\left(\frac{-t_1}{\tau_1}\right)\right)$$

8.



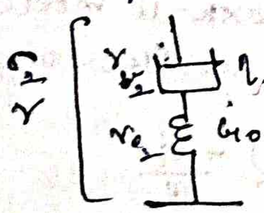
Decoupling the system :-

γ experienced by both the arms

σ_1 by left arm & σ_2 by right arm

$$\sigma = \sigma_1 + \sigma_2 \quad \text{—— (a)}$$

$$\sigma_1 = G_{01}\gamma \quad \text{—— (b)}$$



$$\dot{\gamma} = \dot{\gamma}_{e_2} + \dot{\gamma}_{v_2}$$

$$\dot{\gamma} = \frac{\dot{\sigma}_2}{G_{02}} + \frac{\sigma_2}{\eta_2} \quad \text{—— (c)}$$

$f(\sigma, \gamma, \dot{\sigma}, \dot{\gamma}, G_{0_1}, G_{0_2}, \eta)$:-

$$\sigma = G_{01}\gamma + \sigma_2 \rightarrow \sigma_2 = \sigma - G_{01}\gamma$$

$$\dot{\sigma}_2 = \dot{\sigma} - G_{01}\dot{\gamma}$$

$$\dot{\gamma} = \frac{\dot{\sigma} - G_{01}\dot{\gamma}}{G_{02}} + \frac{\sigma_2}{\eta_2} \rightarrow \dot{\gamma} = \frac{\dot{\sigma} - G_{01}\dot{\gamma}}{G_{02}} + \frac{\sigma - G_{01}\gamma}{\eta_2}$$

$\Rightarrow$ Constitutive eqn

$$\boxed{\frac{\sigma}{\eta_2} + \frac{\dot{\sigma}}{G_{02}} = \dot{\gamma}\left(1 + \frac{G_{01}}{G_{02}}\right) + \frac{G_{01}\gamma}{\eta_2}}$$

after stress relax^n test (const strain applied)

$\boxed{\text{or step strain test}} \to \dot{\gamma} = 0$

$$\Rightarrow \frac{\sigma}{\eta_2} + \frac{\dot{\sigma}}{G_{02}} = \cancel{\dot{\gamma}}^{0}\left(1 + \frac{G_{01}}{G_{02}}\right) + \frac{G_{01}\gamma_0}{\eta_2}$$

$$\Rightarrow \frac{\sigma}{\eta_2} + \frac{\dot{\sigma}}{G_{02}} = \frac{G_{01}\gamma_0}{\eta_2} \quad (\gamma = \gamma_0)$$

$$\Rightarrow \frac{d\sigma}{dt} + \frac{G_{02}\sigma}{\eta_2} = \frac{G_{01}G_{02}\gamma_0}{\eta_2}$$

$$\Rightarrow \frac{d\sigma}{dt} = \frac{G_{02}}{\eta_2}\left(G_{01}\gamma_0 - \sigma\right)$$

$$\to \int_{\sigma_0}^{\sigma} \frac{d\sigma}{G_{01}\gamma_0 - \sigma} = \int_0^t \frac{-G_{02}}{\eta_2}\,t$$

at $t = 0$ $= (G_{01} + G_{02})\gamma_0$

Spring only.

$$\to \ln\left(\frac{G_{01}\gamma_0 - \sigma}{G_{01}\gamma_0 - \sigma_0}\right) = \frac{-G_{02}t}{\eta_2}$$

$$\to G_{01}\gamma_0 - \sigma = e^{-(G_{02}t/\eta_2)}\left(G_{01}\gamma_0 - \sigma_0\right)$$

$$\cancel{\sigma = G_{01}\gamma_0\left(1 - e^{(G_{02}t/\eta_2)}\right)}$$

$$\to \sigma = G_{01}\gamma_0 - \left(\cancel{G_{01}\gamma_0} - \cancel{G_{01}\gamma_0} - G_{02}\gamma_0\right)e^{(-G_{02}t/\eta_2)}$$

$$= \gamma_0\left(G_{01} + G_{02}\,e^{(-G_{02}t/\eta_2)}\right)$$

$$\boxed{\sigma(t) = (\sigma_0 - G_{01}\gamma_0)\exp\left(\frac{-G_{02}t}{\eta_2}\right) + G_{01}\gamma_0}$$

Ultimately no energy remains in the system. It gets dissipated into some other form.

Oscillatory Rheology :—

24-8-2023

$G(t) = \frac{\sigma(t)}{\gamma_0}$ Relxⁿ modulus (Relaxⁿ time Scale)

$G(t)$ → Normalised Stress

$J(t) = \frac{\gamma(t)}{\sigma_0}$ (Retardation Time Scale)

$J(t)$ → Creep compliance

Oscillate in a controlled manner as we want to obtain some useful informⁿ

$\gamma = \gamma_0 \sin(\omega t)$

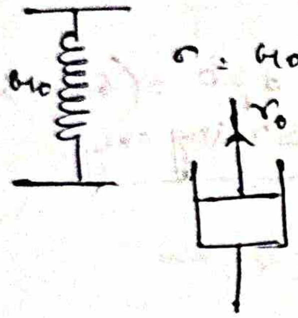
ω → angular frequency

$\boxed{f = \frac{\omega}{2\pi}}$

f = no. of cycles per second / per unit time (Hz)

ω = radian per sec.

1 cycle — Amp Extreme — mean — Amp Extreme



$\sigma = G_0 \gamma_0 \sin(\omega t)$ → in phase

$\gamma_0 \sin \omega t$

$\sigma = \eta_0 \dot{\gamma}_0$
$= \eta_0 \gamma_0 \omega \cos(\omega t)$
$= \eta_0 \gamma_0 \omega \sin(\omega t + \frac{\pi}{2})$ → out of phase $\phi = \pi/2$

in phase with elastic part

out of phase with viscous part (stress & strain out of phase)

elastic: σ, γ are in phase means $\phi = 0$ ϕ → phase angle

viscoelastic - $\boxed{\sigma = \sigma_0 \sin(\omega t + \phi)}$ $0 < \phi < \pi/2$

$\gamma = \gamma_0 \sin \omega t$

$\sigma = \sigma_0 \sin(\omega t + \phi)$

$\sigma = \sigma_0 (\cos\phi) \sin \omega t + \sigma_0 (\sin\phi) \cos \omega t)$

$\sigma_0\cos\phi$ → in phase elastic nature

$\sigma_0\sin\phi$ → out of phase viscous nature

const. — time dependent

models are valid only for linear regime (small defⁿmns) if stress / strain are changed very rapidly or are very large it fails.

ALL LAWS are linear (dependence is linear). So Models fail in a non-linear system.

Water has no elasticity

Models :—

$G' = \frac{\sigma_0 \cos\phi}{\gamma_0}$ (Storage or Elastic Modulus)

$G'' = \frac{\sigma_0 \sin\phi}{\gamma_0}$ (Loss or viscous modulus)

Loss Tangent $= \frac{\text{Storage}}{\text{Loss}} = \frac{G''}{G'} = \tan\phi$

$\boxed{G^* = G' + iG''}$ G^* → complex modulus (No physical meaning)

$G^* = \frac{\Delta\sigma}{\Delta\gamma} = \frac{\sigma(t+\Delta t) - \sigma(t)}{\gamma(t+\Delta t) - \gamma(t)}$ stress & strain complex i/p $\gamma = \gamma_0 \sin(\omega t) = \gamma_0 e^{i\omega t}$ $\quad \sigma = \sigma_0 e^{i(\omega t + \phi)}$

input

$$G^* = \frac{\sigma_0 e^{i(\omega(t+\Delta t)+\phi)} - \sigma_0 e^{i(\omega t + \phi)}}{\gamma_0 e^{i\omega(t+\Delta t)} - \gamma_0 e^{i\omega t}}$$

$$G^* = \frac{\sigma_0 e^{i\phi}\left(e^{i\omega(t+\Delta t)} - e^{i\omega t}\right)}{\gamma_0\left(e^{i\omega(t+\Delta t)} - e^{i\omega t}\right)} = \frac{\sigma_0 e^{i\phi}}{\gamma_0}$$

$$= \frac{\sigma_0(\cos\phi + i\sin\phi)}{\gamma_0}$$

$$G^* = \underbrace{\left(\frac{\sigma_0}{\gamma_0}\cos\phi\right)}_{G'} + \underbrace{\left(i\left(\frac{\sigma_0}{\gamma_0}\sin\phi\right)\right)}_{G''}$$

What do we measure during expt? how to get G' & G''?

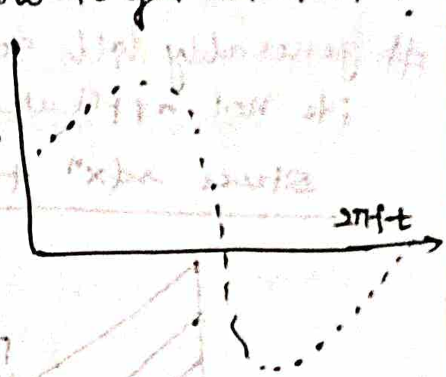
t	$\gamma = 1\sin(2\pi f t)$
0	⋮
0.5	⋮
⋮	⋮
20pts.	⋮

$f = 0.1\,Hz$

Amp := 1 rad.

(in response we get stress.)

Fit sinusoidal f^n



$G' = \dfrac{\sigma_0\cos\phi}{\gamma_0}$ $\quad \sigma = \sigma_0\sin(\omega t + \phi)$

$\boxed{\text{by fitting we get } \phi \,\&\, \sigma_0}$

$G'' = \dfrac{\sigma_0\sin\phi}{\gamma_0}$

$\boxed{29\,|\,08\,|\,23}$

$G^* = \dfrac{d\sigma}{d\gamma} = G' + iG''$ (only for mathematical convenience)

The previous models don't give complete picture. relx^n time scale is very high for both viscous & elastic dominated material.

$\tau = \dfrac{\eta_0}{G_0}$ → Just model parameters → See it as a whole
(not exactly viscosity and other modulus)

Oscillatory Rheology kind of gives us a complete picture

Linear regime:—

The deform^{n}s are very small & stress / strain are additive in nature

Upto what stress value is it possible to get $J(t)$?
↳ There is a cutoff after which non linear regime starts.

How do we find out the linear regime for a material?

Linear regime?

Linear regime ← Strain Amplitude Sweep Test

$\gamma = \gamma_0 \sin(2\pi f t)$

let ~~say~~ f is fixed.

$\gamma_0 = 0.1 \rightarrow \gamma = \gamma_1$
$\gamma_0 = 0.2 \rightarrow \gamma = \gamma_2$
$\vdots \qquad \vdots$

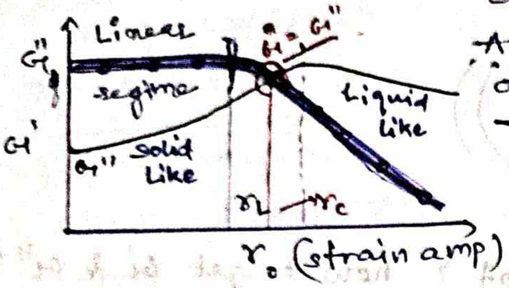
So, the variable of interest which we are controlling is γ_0.

γ_0 is very small (deform^{n}s very small) if nothing is happening to material why change in G'!

And how small the deform^{n}s are, depends on the material.

The pt. of sudden decrease in G' (or elastic modulus) → breaking pt.

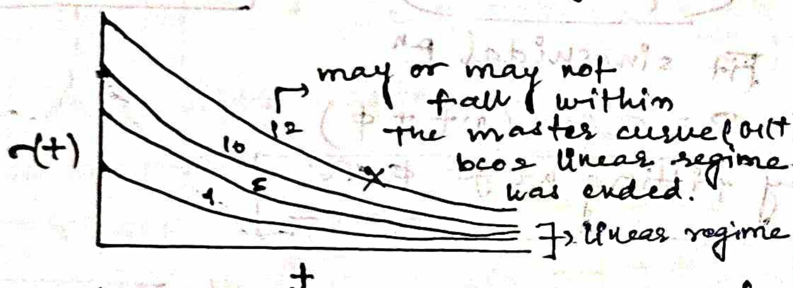
in this expt., at each γ_0 the avg. G' is taken & plotted for each cycle.



generally till 90% of G', linear regime is retained but its not applicable for every material.

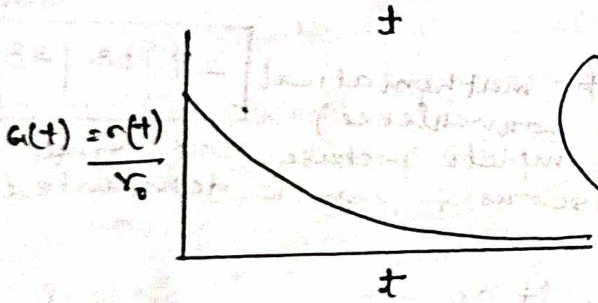
Stress relx^n test :-

What is the cutoff of linear regime from above test. (γ_L)



may or may not fall within the master curve ($G(t)$ curve) bcoz linear regime was ended.

} linear regime



After breaking pt., the material becomes ~~from~~ more liq. like. Earlier it was more solid like.

Going from elastic dominated regime to viscosity dominated regime ($G'' > G'$)

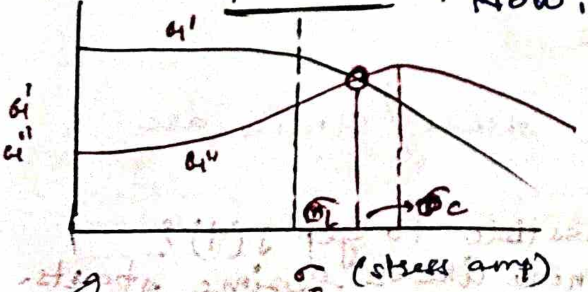
rate of loss of Energy ↑ with γ_0 upto a limit.

To be proved in future :- rate of loss of energy $= f(G'')$

ϕ should change near the breaking pt. bcoz if the material is same ϕ does not change. (should not change)

Stress - Amplitude Sweep Test :—

Now, our input → $\boxed{\sigma_0 \sin \omega t}$



for 1 cycle

We don't need to do seperate tests. We can just put Strain Amplitude instead of only strain. (same for stress) and if we know our γ_L we can get our σ_L. So for ~~xxx~~ every γ_0 we can get σ_0.

We are only interested in amplitude

Frequency → $\gamma = \gamma_0 \sin(2\pi f t)$

⇒ The timescale over which viscous dissipⁿ of energy take place is → Relxⁿ Time Scale → take energy and try to take a new form (configuration)

Relaxation Time scale → (belongs to material)

$\boxed{T_{exp} = \frac{1}{f}}$ experimental Time Scale ↓ we can control.

$f = 0.1$ Hz
↳ 10s ~~minutes~~ to complete 1 cycle

Every cycle is identical to previous cycle if material remains intact.

Lets compare τ and Texp :—

$\tau <<$ Texp → Liquid like. → at very low frequency (relatively more liq. like)

$\tau >>$ Texp → Solid like → at high frequency.

Frequency Sweep Test :—

$\gamma = \gamma_0 \sin(2\pi f t)$ γ_0 → avg. value in linear regime

Linear regime itself is dependent on frequency

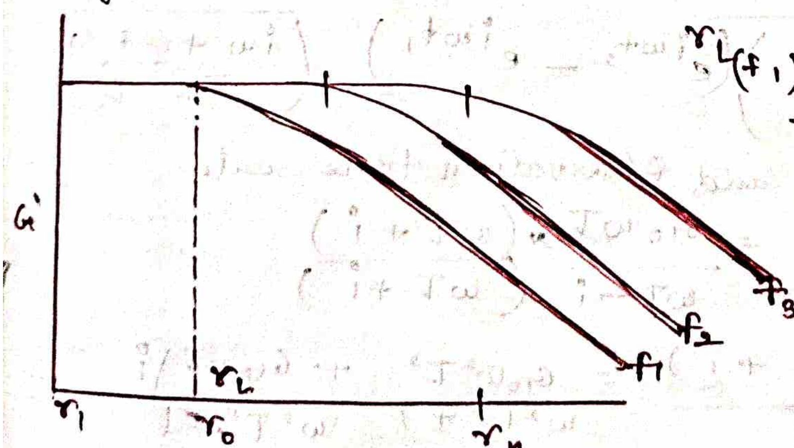
Dependence of γ_0 on frequency :—

Strain-Amplitude Sweep Test :—

$\gamma = \gamma_0 \sin(2\pi f_1 t)$ $\gamma_1 < \gamma_0 < \gamma_n$ → f_1 ∨ f_2

$\gamma_2 = \gamma_0 \sin(2\pi f_2 t)$ $\gamma_1 < \gamma_0$ ~~$<$~~ γ_n

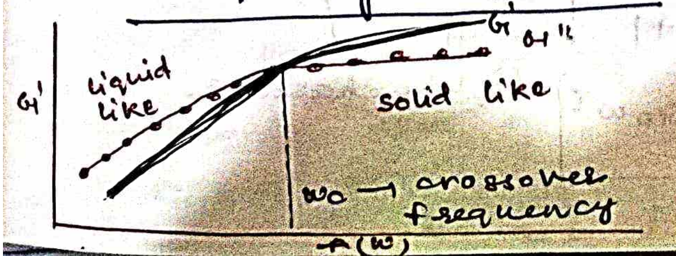
higher the freq. more will be the elasticity



$\gamma_{L(f_1)} < \gamma_{L(f_2)} < \gamma_{L(f_3)}$
$f_1 < f_2 < f_3$

Linear regime is lowest for higher freq.

Frequency sweep Test :—



if viscous character is dominated at lower freq., then G'' ↑ (will be higher)

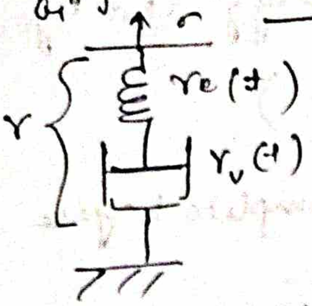
high freq. → more elastic character

w_c is representative of relaxation time scale because relaxation is energy dissipⁿ.

↑ w, elasticity ↑

Mathematical Analysis of Freq. Sweep Test

G', G'' $f(\omega)$ — Maxwell model :-



input :- $\sigma = \sigma_0 \sin \omega t$

$\sigma = \sigma_0 e^{i\omega t}$ (mathematical)

if at any instance of time $\gamma = \gamma_e + \gamma_v$

$$\gamma = \frac{\sigma}{G_0} + \gamma_v \rightarrow \dot{\gamma} = \frac{\dot{\sigma}}{G_0} + \frac{\sigma}{\eta_0}$$

$$\rightarrow \frac{d\gamma}{dt} = \frac{\dot{\sigma}}{G_0} + \frac{\sigma}{\eta_0} \rightarrow \frac{d\gamma}{dt} = \frac{i\sigma_0 \omega e^{i\omega t}}{G_0} + \frac{\sigma_0 e^{i\omega t}}{\eta_0}$$

$$\Rightarrow \frac{d\gamma}{dt} = \frac{\sigma_0 e^{i\omega t}}{G_0}\left(i\omega + \frac{1}{(\eta_0/G_0)}\right) = \frac{\sigma_0 e^{i\omega t}}{G_0}\left(i\omega + \frac{1}{\tau}\right)$$

$$\Rightarrow \frac{d\gamma}{dt} = \frac{\sigma_0 e^{i\omega t}}{G_0}\left(i\omega + \frac{1}{\tau}\right)$$

$$G^* = \frac{\int_{t_1}^{t_2} d\sigma}{\int_{t_1}^{t_2} d\gamma} = \frac{\sigma(t_2) - \sigma(t_1)}{\gamma(t_2) - \gamma(t_1)}$$

$$\int_{\gamma(t_1)}^{\gamma(t_2)} d\gamma = \int_{t_1}^{t_2} \frac{\sigma_0}{G_0} e^{i\omega t}\left(i\omega + \frac{1}{\tau}\right) dt = \left(\frac{\sigma_0}{G_0}\left(i\omega + \frac{1}{\tau}\right)\right) \int_{t_1}^{t_2} e^{i\omega t} dt$$

$$= \frac{\sigma_0}{G_0}\left(i\omega + \frac{1}{\tau}\right) \frac{1}{i\omega} \left(e^{i\omega t}\right)_{t_1}^{t_2}$$

$$= \frac{\sigma_0}{G_0}\left(1 + \frac{1}{\tau i\omega}\right)\left(e^{i\omega t_2} - e^{i\omega t_1}\right)$$

31|08|23

$$G^* = \frac{\sigma_0 e^{i\omega t_2} - \sigma_0 e^{i\omega t_1}}{\frac{\sigma_0}{G_0}\left(\frac{1}{i\omega}\right)\left(i\omega + \frac{1}{\tau}\right)\left(e^{i\omega t_2} - e^{i\omega t_1}\right)} = \frac{G_0 i\omega}{\left(i\omega + \frac{1}{\tau}\right)}$$

Experimental Time Scale and Observation time scale.

$$G^* = \frac{G_0 \tau \omega i}{1 + i\omega\tau} = \frac{G_0 \tau \omega i}{-i^2 + i\omega\tau} = \frac{G_0 \omega\tau}{\omega\tau - i} \times \frac{(\omega\tau + i)}{(\omega\tau + i)}$$

$$= \frac{G_0 \omega\tau(\omega\tau + i)}{\omega^2\tau^2 + 1} = \frac{G_0 \omega^2\tau^2}{\omega^2\tau^2 + 1} + \left(\frac{G_0 \omega\tau}{\omega^2\tau^2 + 1}\right) i$$

$$G' = \frac{G_0 \omega^2 \tau^2}{\omega^2\tau^2 + 1} \qquad G'' = \frac{G_0 \omega\tau}{\omega^2\tau^2 + 1} \qquad \boxed{\omega\tau <<< 1}$$

$\Downarrow$

liq like character

$$\rightarrow \boxed{G' \sim G_0 \omega^2 \tau^2} \qquad \boxed{G'' \sim G_0 \omega\tau}$$

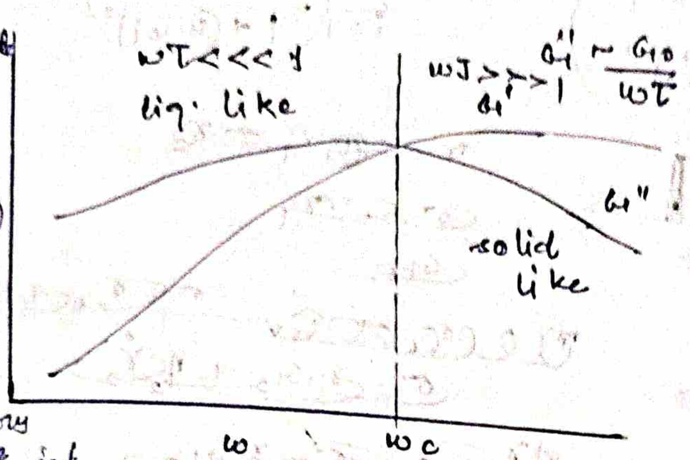
Initially $G' > G''$ and it ↑ rapidly. So both intersect and then $G'' > G'$

$\omega\tau >>> 1$ → solid like (elastic nature dominance)

$G' \simeq G_0$ $\quad G'' \sim \frac{G_0}{\omega\tau}$

log(scale) log

G', G'' → Just gives an idea of viscous nature of material



after removing strain, some elasticity is regained but not completely

→ freq. being varied.

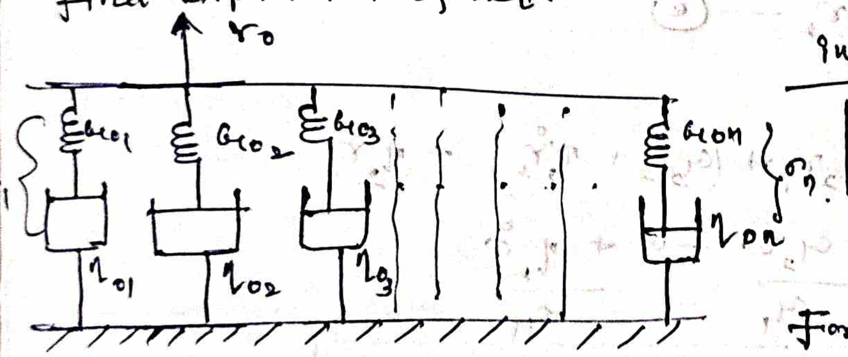
for freq sweep test, acquisition time needs to be changed because of variation of f.

Generalized Maxwell model is needed as it will not work for every system.

$\boxed{14|9|23}$

Generalized Maxwell model :-

Find expression of relaxn modulus for GMM. (G_{0i}, τ_{0i})



Initially :-

$$\boxed{\sigma_0 = G_{01}\gamma_0 + G_{02}\gamma_0 + \cdots G_{0n}\gamma_n}$$

~~$\sigma_t = \sigma_1 + \sigma_2 + \cdots + \sigma_n$~~

$\sigma_t = \sigma_1 + \sigma_2 \cdots \sigma_n$

For ① st :-

$\gamma_0 = \gamma_1 + \gamma_2$. $\quad \rightarrow \dot{\gamma}_1 + \dot{\gamma}_2 = \dot{\gamma}_0$

$\Rightarrow \frac{\sigma_1}{G_{01}} = \gamma_1 \quad \dot{\gamma}_2 = \frac{\sigma_1}{\eta_{01}} \quad \rightarrow \dot{\gamma}_0 = \frac{\dot{\sigma}_1}{G_{01}} + \frac{\sigma_1}{\eta_{01}}$

→ Similarly

$$\rightarrow 0 = \frac{1}{G_{01}}\frac{d\sigma_1}{dt} + \frac{\sigma_1}{\eta_{01}} \rightarrow \frac{d\sigma_1}{dt} = \frac{-G_{01}\sigma_1}{\eta_{01}}$$

$$\rightarrow \ln\left(\frac{\sigma_1}{\sigma_{10}}\right) = \exp\left(\frac{-G_{01}t}{\eta_{01}}\right) \rightarrow \sigma_1 = \sigma_{10}\exp\left(\frac{-G_{01}t}{\eta_{01}}\right)$$

Similarly for $\sigma_2 = \sigma_{20}\exp\left(-\frac{G_{02}t}{\eta_{02}}\right)$

$$\sigma_n = \sigma_{n0}\exp\left(\frac{-G_{0n}t}{\eta_{0n}}\right)$$

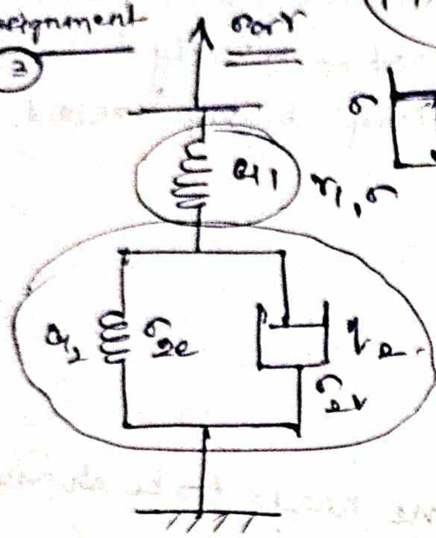
So $\sigma_t = \sigma_1 + \sigma_2 + \dots \sigma_n$

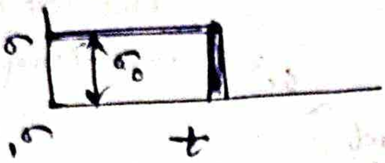
$$G(t) = \sum_{i=1}^{n} G_{0i} \exp\left(\frac{-t}{T_i}\right) \quad \text{where } T_i = \frac{\eta_{0i}}{G_{0i}}$$

To find G' & G'' → Oscillatory strain Test

$$G' = \sum_{i=1}^{n} \left(\frac{G_{0i} (T_{0i}\,\omega)^2}{1 + (\omega T_{0i})^2} \right) \qquad G'' = \sum_{i=1}^{n} \left(\frac{G_{0i}\, T_{0i}\, \omega}{1 + (\omega T_{0i})^2} \right)$$

Assignment (3)





$\cancel{\gamma = \gamma_1 + \gamma_2}$

$$\sigma = G_1 \gamma_1 \quad \text{—(a)}$$

$$\sigma = G_2 \gamma_2 + \eta_2 \dot{\gamma}_2 \quad \text{—(b)}$$

$$\gamma = \gamma_1 + \gamma_2 \quad \text{—(c)}$$

$$G_2 \gamma = G_2 \gamma_1 + G_2 \gamma_2 \quad \text{—(d)}$$

$$\eta_2 \dot{\gamma} = \eta_2 \dot{\gamma}_1 + \eta_2 \dot{\gamma}_2 \quad \text{—(e)}$$

eq^n (d) + (e) :—

$$G_2 \gamma + \eta_2 \dot{\gamma} = G_2 \gamma_1 + (G_2 \gamma_2 + \eta_2 \dot{\gamma}_2) + \eta_2 \dot{\gamma}_1$$

$$\Rightarrow G_2 \gamma + \eta_2 \dot{\gamma} = \frac{G_2 \sigma}{G_1} + \sigma + \frac{\eta_2 \dot{\sigma}}{G_1}$$

$$\Rightarrow \boxed{\sigma \left(1 + \frac{G_2}{G_1} \right) + \left(\frac{\eta_2}{G_1} \right) \dot{\sigma} = G_1 \gamma + \eta_2 \dot{\gamma}}$$

Boltzmann Superposition Principle :-

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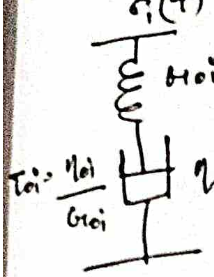
$\sigma_i(t),\ \gamma_i(t)$



$$\dot\gamma = \frac{\dot\sigma}{G_{oi}} + \frac{\sigma}{\eta_o}$$

$$\frac{d\sigma}{dt} + \frac{\sigma}{\tau_o} = G_o \frac{d\gamma}{dt}$$

$$\Rightarrow \sigma e^{t/\tau_o}\Big|_{-\infty}^{t} = \int_{-\infty}^{t} G_{o} e^{t'/\tau_o}\dot\gamma\, dt'$$

$$\rightarrow \sigma e^{t/\tau_o} - \cancel{(\sigma) e^{-\infty/\tau_o}} = \int_{-\infty}^{t} G_{o} e^{t'/\tau_o}\dot\gamma\, dt$$

$\infty \times 0$

finite (we dont deal with ∞ quantities)

$$\rightarrow \sigma e^{t/\tau_o} = \int_{-\infty}^{t} G_{o} e^{t'/\tau_o}\dot\gamma(t')\, dt'$$

$$\rightarrow \sigma = \int_{-\infty}^{t} G_o e^{t'/\tau_o} e^{-t/\tau_o}\underbrace{\dot\gamma(t')}_{f^n \text{ of } t'} dt'$$

$$\rightarrow \sigma = \int_{-\infty}^{t} G_o e^{(t'-t)/\tau_o}\dot\gamma\, dt'$$

$$= \int_{-\infty}^{t} G_o e^{-(t-t')/\tau_o}\dot\gamma\, dt'$$

The above term will only survive if $\dot\gamma$ is being applied otherwise the whole term will become zero.

$$\sigma_i(t) = \int_{-\infty}^{t} G_{o} e^{-(t-t')}\dot\gamma(t')\,dt' \Rightarrow \sum_{i=1}^{n}\sigma_i = \int_{-\infty}^{t}\underbrace{\left(\sum G_{oi} e^{-\frac{(t-t')}{\tau_{oi}}}\right)}_{\text{relx}^n \text{ modulus of generalised maxwell model.}}\dot\gamma\, dt'$$

$$\sigma(t) = \int_{-\infty}^{t}\underbrace{G(t-t')}_{f^n \text{ of } (t-t')}\dot\gamma(t')\,dt' \rightarrow \text{All maxwell models in parallel.}$$

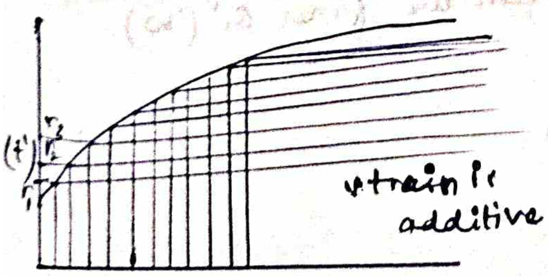
$$\gamma(t) = \int_{-\infty}^{t} J(t-t')\dot\sigma(t')\,dt' \rightarrow \text{all KV models in series.}$$

$$\boxed{t = \int_{0}^{t} G(t)\ J(t-t')\,dt'}$$

extra strain needs to be taken into account

$$\gamma_2 = \gamma_1 + d\gamma \qquad \gamma_3 = \gamma_2 + d\gamma$$

$$\sigma(t) = \gamma_1\phi(t-t_1) + \gamma_2\phi(t-t_2) + \gamma_3\phi(t-t_3) + \ldots\ldots$$

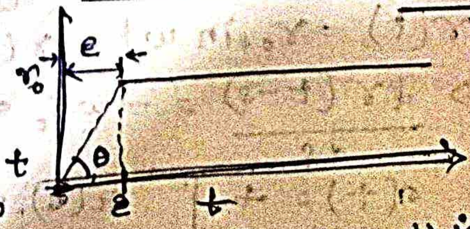


$$\sigma(t) = \int_{-\infty}^{t} G(t-t')\underbrace{\left(\frac{d\gamma}{dt'}\,dt'\right)}_{\text{net} \uparrow \text{ in strain}}$$

Linear strain field = sum of ∞ step strains.

step strain test :- γ_o

$\varepsilon \to 0$



as $\dot\gamma \overset{o}{=}$ const with t

$$\sigma(t) = \int_{0}^{\varepsilon \to 0} G(t-t')\dot\gamma(t')\,dt' + \int_{\varepsilon\to 0}^{t}\cancel{G_1 \dot\gamma dt'}^{\,0} \Leftarrow \sigma(t) = \int_{-\infty}^{t} G(t-t')\dot\gamma(t')\,dt'$$

$$\sigma(t) = \lim_{\varepsilon\to 0} \frac{\int_0^{\varepsilon} G(t-t')\gamma_0\,dt'}{\varepsilon} = \lim_{\varepsilon\to 0} \frac{G(t-\varepsilon)\gamma_0}{(1)} = G(t)\gamma_0$$

$$\boxed{\sigma(t) = \gamma_0 G(t)} \Rightarrow \sigma(t) = \gamma_0 G_0 \exp\left(\frac{-t}{\tau}\right)$$

rel^n b/w η and $G(t)$:—

Const. strain rate experiment :—

$\dot{\gamma} = \text{const.} = \dot{\gamma}_0$

$\boxed{\eta = \frac{\sigma}{\dot{\gamma}_0}}$ Newton's Law of viscosity → applicable on all gen. fluids.

Newtonian fluid → η const

$$\sigma(t) = \int_{-\infty}^{t} G(t-t')\dot{\gamma}_0\,dt' \quad (\text{By BSP})$$

$$\Rightarrow \eta = \frac{\sigma(t)}{\dot{\gamma}_0} = \int_{-\infty}^{t} G(t-t')\,dt' \qquad \begin{pmatrix} t-t' = s \\ -dt' = ds \end{pmatrix}$$

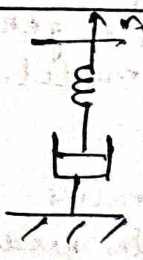
$$\Rightarrow \eta = -\int_{\infty}^{0} G(s)\,ds = \int_0^{\infty} G(s)\,ds \rightarrow \text{time}$$

speed of decay

$\Rightarrow$ rate of dissip^n of energy slow → viscosity high.

relax^n process

η for a single Maxwell model :—



$$\eta = \int_0^{\infty} G_0 e^{-s/\tau_0}\,ds = \left[-G_0\tau_0 e^{-s/\tau_0}\right]_0^{\infty} = G_0\tau_0$$

$\tau_0 = \eta_0 / G_0$

$\boxed{\eta = \eta_0 = G_0\tau_0}$

viscosity of dashpot = viscosity of material [only for 1 sys.]
{single maxwell model}

$G'(\omega) \longrightarrow G(t)$

if we know of $G(t)$ of a material can we find $G'(\omega)$.

$$\sigma(t) = \int_{-\infty}^{t} G(t-t')\dot{\gamma}(t')\,dt'$$

$$t - t' = s \rightarrow -dt' = ds$$

$$\sigma(t) = (-)\int_0^{\infty} G(s)\left(\frac{d\gamma(t-s)}{ds}\right)ds$$

$\gamma(t) = \gamma_0 \sin\omega t \qquad \gamma(t-s) = \gamma_0 \sin(\omega(t-s))$

$$\Rightarrow \frac{d\gamma(t-s)}{ds} = -\omega\gamma_0 \cos(\omega(t-s))$$

$$\sigma(t) = +\int_0^{\infty} G(s)\,\omega\gamma_0 \cos(\omega(t-s))\,ds$$

$$\sigma(t) = r_0 \omega \int_0^{\infty} G(s)\left(\cos\omega t \cos\omega s + \sin\omega t \sin\omega s\right) ds$$

$$\sigma(t) = r_0\omega\underbrace{\left(\int_0^{\infty} G(s)\cos\omega s\, ds\right)}\cos\omega t + r_0\omega\underbrace{\left(\int_0^{\infty} G(s)\sin\omega s\, ds\right)}\sin\omega t$$

$$\Rightarrow G' = r_0\omega \int_0^{\infty} G(s)\sin\omega s\, ds$$

$$\Rightarrow G'' = r_0\omega \int_0^{\infty} G(s)\cos\omega s\, ds.$$

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BSP :-

$$\sigma(t) = \int_{-\infty}^{t} G(t-s)\, \dot{r}\, ds \quad \dot{r} = \frac{\partial r(s)}{\partial s}$$

$$r(t) = \int_{-\infty}^{t} J(t-s)\, \dot{\sigma}\, ds \quad \dot{\sigma} = \frac{\partial \sigma(s)}{\partial s}$$

$r = r_0 \sin\omega t$

$-s = z,\ ds = -dz$

$$-\int_{\infty}^{0} G(z)\frac{dr(t-z)}{-dz}\, dz \Rightarrow \int_0^{\infty} G(z)\frac{dr(t-z)}{dz}\, dz$$

$$\boxed{\sigma(t) = -\int_0^{\infty} G(z)\, \frac{dr(t-z)}{dz}}$$

$r = r_0 \sin\omega t$

$$r(t-z) = r_0 \sin\omega(t-z)$$

$$dr(t-z) = r_0\left((\sin\omega t)\cos\omega z - (\cos\omega t)\sin\omega z\right)$$

$$\frac{dr(t-z)}{dz} = -\left(r_0\, \omega \sin\omega t \sin\omega z + r_0\omega\cos\omega t\cos\omega z\right)$$

$$\Rightarrow \sigma(t) = \omega_0 r\int_0^{\infty} G(z)\left(\sin\omega z \sin\omega t + \cos\omega t\cos\omega z\right) dz$$

$$= \omega_0 r\int_0^{\infty} G(z)\sin\omega z\sin\omega t\, dz + \omega_0 r\int_0^{\infty} G(z)\cos\omega t\cos\omega z\, dz$$

$$G' = \frac{\sigma_0}{r_0} = \frac{r_0\omega\int_0^{\infty} G(z)\sin\omega z\sin\omega t\, dz}{r_0 \sin\omega t} = \frac{r_0\omega\left(\int_0^{\infty} G(z)\sin\omega z\, dz\right)\sin\omega t}{r_0 \sin\omega t}$$

$$\Rightarrow r = r_0 \sin\omega t$$

$$\sigma = \sigma_0 \sin(\omega t + \delta) = \sigma_0 \sin\omega t\cos\delta + \sigma_0\cos\omega t\sin\delta$$

$$G' = \frac{\sigma_0\cos\delta}{r_0} \qquad G'' = \frac{\sigma_0\sin\delta}{r_0}$$

$G'(\omega) = \omega\left(\int_0^\infty G(z)\sin\omega z\,dz\right)$ $G''(\omega) = \omega\left(\int_0^\infty G(z)\cos\omega z\,dz\right)$

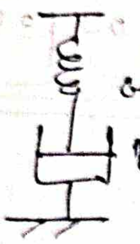
$\frac{\sigma(t)}{\dot{\gamma}}$ $\left(\eta = \int_0^\infty G(s)\,ds\right)$ $\dot{\gamma} \to 0$ → slow/sluggish relx?

const. shear rate for finding viscosity.

$$\boxed{\sigma(t) = \int_{-\infty}^{t} G(t-s)\,\dot{\gamma}(s)\,ds}$$

B.S.P valid only for linear regime → very small defm.

disturbing the sample minimally



G_0 η_0

$$G'(\omega) = \omega \int_0^\infty G_0 e^{-z/\tau} \sin\omega z\,dz$$

$$= -\omega G_0 e^{-z/\tau} \frac{\cos\omega z}{\omega}\Big|_0^\infty - \frac{1}{\tau}\int e^{\omega} G_0 \frac{\cos\omega z}{\omega}\,dz$$

$$G(s) = G_0 e^{-s/\tau}$$

12 | 09 | 23

$$G' = \omega \int_0^\infty G(s)\sin\omega s\,ds$$

$$I = \frac{G'}{G_0\omega} = \int_0^\infty e^{-s/\tau} \sin\omega s\,ds$$

$\tau = \frac{\eta_0}{G_0}$

$$\frac{G'}{G_0\omega} = I = \frac{e^{-s/\tau}\cos\omega s}{\omega}\Big|_\infty^0 \left(\frac{-1}{\tau\omega}\right)\int_0^\infty e^{-s/\tau}\cos\omega s\,ds$$

$$I = \frac{1}{\omega} - \frac{1}{\tau\omega}\underbrace{\left(\int_0^\infty e^{-s/\tau}\cos(\omega s)\,ds\right)}_{I_1} \quad \text{—— (1)}$$

$$I_1 = \frac{e^{-s/\tau}\cancel{\sin\omega s}^{\,0}}{\omega}\Big|_0^\infty + \frac{1}{\tau\omega}\int_0^\infty e^{-s/\tau}\sin\omega s\,ds$$

$$I_1 = \frac{1}{\tau\omega}\int_0^\infty e^{-s/\tau}\sin\omega s\,ds$$

$$I_1 = \frac{I}{\tau\omega}$$

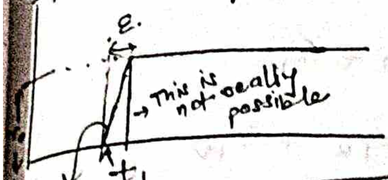
(1):- $I = \frac{1}{\omega} - \frac{1}{\tau\omega} \times \frac{I}{\tau\omega} \Rightarrow I + \frac{I}{\tau^2\omega^2} = \frac{1}{\omega} \rightarrow I\left(\frac{1+\omega^2\tau^2}{\tau^2\omega^2}\right) = \frac{1}{\omega}$

$\rightarrow I = \frac{\omega\tau^2}{1+\tau^2\omega^2} \rightarrow \frac{G'}{G_0\omega} = \frac{\omega\tau^2}{1+\tau^2\omega^2} \rightarrow G' = \frac{G_0\omega^2\tau^2}{1+\tau^2\omega^2}$

Similarly we can find our G''.

Doubt clearing. We have applied strain we want to find out stress

General form of BSP :—



$$\sigma(t) = \int_{-\infty}^{t} G(t-s)\frac{\partial \gamma}{\partial s}\,\partial s \qquad \dot\gamma = \frac{\gamma_0}{\varepsilon}$$

$$= \int_{-\infty}^{t_1} G(t-s)(0) + \lim_{\varepsilon\to0}\int_{t_1}^{t_1+\varepsilon} G(t-s)\frac{\gamma_0}{\varepsilon} + \int_{t_1+\varepsilon}^{t} (\ )\,0$$

Do the same for KV model

$$= \lim_{\varepsilon\to 0}\int_{t_1}^{t_1+\varepsilon} G(t-s)\frac{\gamma_0}{\varepsilon}$$

$$\sigma(t) = \lim_{\varepsilon\to0}\frac{\int_{t_1}^{t_1+\varepsilon} G(t-s)\gamma_0\,ds}{\varepsilon} = \lim_{\varepsilon\to0}\frac{\gamma_0 G(t-t_1-\varepsilon)}{1} = \gamma_0 G(t-t_1)$$

$$\sigma(t) = \gamma_0 G(t-t_1) = \gamma_0 G_0 e^{(t-t_1/\tau)}$$

Multimaxwell Model Derivation :—

$$\eta_0^{\dot\gamma\to0} = \int_0^\infty G(s)\,ds \qquad G(s) = \sum G_{0i}\, e^{-s/\tau_i}$$

Zero shear μ

BSP

$$\Rightarrow \eta_0 = \int_0^\infty G_{0i}e^{-s/\tau_i}\,ds = \sum G_{0i}\tau_i e^{-s/\tau_i}\Big|_\infty^0$$

$$= \sum G_{0i}\tau_i(1-0)$$

$$= \sum G_{0i}\tau_i$$

Assignment Qs.

Assignment 2

(3) relaxn modulus :- $G(t) = G_1 e^{-s/\tau_1} + G_2 e^{-s/\tau_2}$

$$\sigma(t) = \int_{-\infty}^{t} G(t-s)\frac{d\gamma}{ds}ds$$

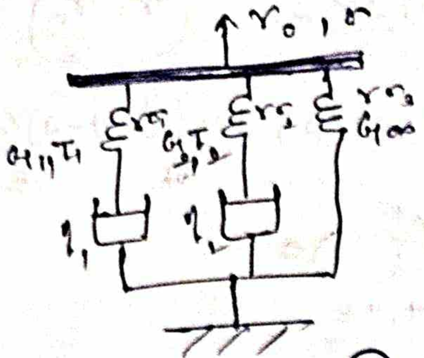
$$= \int_0^{t_1}\left(G_1 e^{-(t-s)/\tau_1} + G_2 e^{-(t-s)/\tau_2}\right)k\,ds - \int_{t_1}^{2t_1}\left(G_1 e^{-(t-s)/\tau_1} + G_2 e^{-(t-s)/\tau_2}\right)k\,ds$$

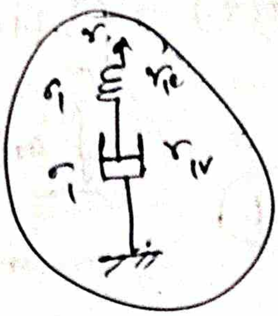
$$= k\left(\tau_1 G_1 e^{-(t-s)/\tau_1}\Big|_0^{t_1} + \tau_2 G_2 e^{-(t-s)/\tau_2}\Big|_0^{t_1}\right)$$

$$- k\left(\tau_1 G_1 e^{-(t-s)/\tau_1}\Big|_{t_1}^{2t_1} + \tau_2 G_2 \exp\left(-\frac{(t-s)}{\tau_2}\right)\Big|_{t_1}^{2t_1}\right)$$

Assignment (9) :-

(1) $\sigma(t) = G_1 e^{-t/\tau_1} + G_2 e^{-t/\tau_2} + G_\infty$





$\gamma_0 = \gamma_{1e} + \gamma_{1v}$

$\dot{\gamma}_0 = \dot{\gamma}_{1e} + \dot{\gamma}_{1v}$

(1) $\dot{\gamma}_0 = \frac{\dot{\sigma}_1}{G_1} + \frac{\sigma_1}{\eta_1}$

(2) $\dot{\gamma}_0 = \frac{\dot{\sigma}_2}{G_2} + \frac{\sigma_2}{\eta_2}$. (3) $\gamma_0 = \frac{\sigma_3}{G_\infty}$.

$\Rightarrow \sigma_3 = \gamma_0 G_\infty$

$\sigma_1 + \sigma_2 + \sigma_3 = \sigma$

$\Rightarrow \sigma_1 + \sigma_2 + \gamma G_\infty = \sigma$

(4) $\Rightarrow \underline{\sigma_1 + \sigma_2 = \sigma - \gamma G_\infty}$

$\dot{\sigma}_1 + \dot{\sigma}_2 + \dot{\sigma}_3 = \dot{\sigma} \Rightarrow \underline{\dot{\sigma}_1 + \dot{\sigma}_2 = \dot{\sigma} - \dot{\gamma} G_\infty}$ (5)

$\Rightarrow$

$$\dot{\gamma} = \frac{\dot{\sigma} - \dot{\gamma} G_\infty - \dot{\sigma}_2}{G_1} + \frac{\sigma - \gamma G_\infty - \sigma_2}{\eta_1}$$

$$\Rightarrow \left(\dot{\gamma} - \frac{\sigma_2}{\eta_2}\right) G_2 = \dot{\sigma}_2$$

$$\Rightarrow \dot{\gamma} = \frac{\dot{\sigma} - \dot{\gamma} G_\infty - \dot{\gamma} G_2 - \frac{\sigma_2 G_2}{\eta_2}}{G_1} + \frac{\sigma - \gamma G_\infty - \sigma_2}{\eta_1}$$

eqn (4)$/\eta_1$ + eqn (5)$/G_1$

$$\frac{\sigma}{\eta_1} + \frac{\dot{\sigma}}{G_1} = \left(\frac{\sigma_1}{\eta_1} + \frac{\dot{\sigma}_1}{G_1}\right) + \frac{\sigma_2}{\eta_1} + \frac{\dot{\sigma}_2}{G_1} + \frac{\gamma G_\infty}{\eta_1} + \frac{\dot{\gamma} G_\infty}{G_1}$$

$$\Rightarrow \frac{\sigma}{\eta_1} + \frac{\dot{\sigma}}{G_1} = \dot{\gamma} + \left(\frac{\sigma_2}{\eta_1} + \frac{\dot{\sigma}_2}{G_1}\right) + \frac{\gamma G_\infty}{\eta_1} + \frac{\dot{\gamma} G_\infty}{G_1} \quad (6)$$

$$\Rightarrow \underbrace{\frac{\sigma}{\eta_1}+\frac{\dot\sigma}{G_1}-\left(\dot\gamma+\frac{\gamma G_\infty}{\eta_1}+\frac{\dot\gamma G_\infty}{G_1}\right)}=\frac{\sigma_2}{\eta_1}+\frac{\dot\sigma_2}{G_1}$$

$$P=\frac{\sigma_2}{\eta_1}+\frac{\dot\sigma_2}{G_1} \longleftarrow (4)$$

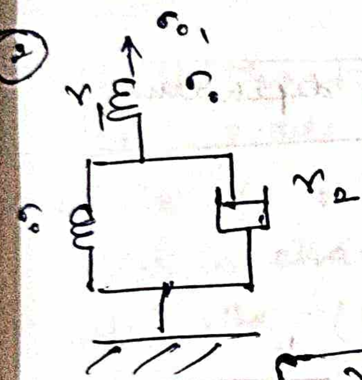
$$\Rightarrow (4)\times G_1-(2)\times G_2$$

$$\Rightarrow G_1P-G_2\dot\gamma=\frac{G_1\sigma_2}{\eta_1}+\cancel{\dot\sigma_2}-\cancel{\dot\sigma_2}-\frac{G_2\sigma_2}{\eta_2}$$

$$\Rightarrow \sigma_2=\frac{PG_1-\dot\gamma G_2}{\frac{G_1}{\eta_1}-\frac{G_2}{\eta_2}}$$

~~σε = ε PG1σ~~

(2)



$$\gamma=\gamma_1+\gamma_2$$

$$\sigma_0=G_1\gamma_1 \qquad \sigma_0=G_2\gamma_2+\eta_2\dot\gamma_2$$

$$\frac{d\gamma_2}{dt}+\frac{G_2\gamma_2}{\eta_2}=\frac{\sigma_0}{\eta_2}$$

$$\boxed{\begin{aligned}\gamma&=\frac{\sigma_0}{G_2}+\gamma_2\\ \gamma(t)&=\frac{\sigma_0}{G_1}+\gamma_2(t)\end{aligned}}$$

$$\gamma_2\exp\left(\frac{G_2t}{\eta_2}\right)\Big|_0^t=\frac{\sigma_0}{\eta_2}\int_0^t\exp\left(\frac{G_2t}{\eta_2}\right)dt$$

$$\Rightarrow \gamma_2(t)\exp\left(\frac{G_2t}{\eta_2}\right)-\gamma_2(t=0)$$

$$=\frac{\sigma_0}{\eta_2}\frac{\eta_2}{G_2}\exp\left(\frac{G_2t}{\eta_2}\right)\Big|_0^t$$

$$=\frac{\sigma_0}{G_2}\left(\exp\left(\frac{G_2t}{\eta_2}\right)-1\right)$$

$$\Rightarrow \gamma_2(t)\exp\left(\frac{G_2t}{\eta_2}\right)-0=\frac{\sigma_0}{G_2}\left(\exp\left(\frac{G_2t}{\eta_2}\right)-1\right)$$

$\because$ at $t=0,\ \gamma=\gamma_1=\frac{\sigma_0}{G_1}$

$$\gamma(t)=\frac{\sigma_0}{G_1}+\text{KV part}=\frac{\sigma_0}{G_1}+\frac{\sigma_0}{G_2}\left(1-\exp\left(\frac{-G_2t}{\eta_2}\right)\right)$$

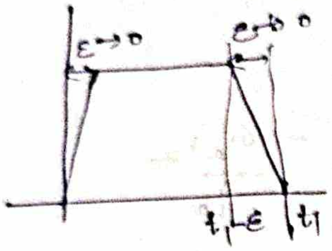
For $(t>t_1)\rightarrow$ initial will be t_1

$$\frac{\gamma(t)}{\sigma_{10}} = \frac{1}{G_1} + \frac{1}{G_2}\left(1 - \exp\left(-\frac{G_2 t}{\eta_2}\right)\right)$$

$\boxed{14/09/23}$

Assignment (2)

(5) $G(t) = G_0 e^{-t/\tau}$

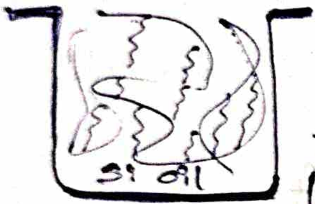


$$\sigma(t) = \lim_{\varepsilon \to 0} \frac{\gamma \int_0^{\varepsilon} G_0 e^{-(t-s)/\tau}\, ds}{\varepsilon} - \lim_{\varepsilon \to 0} \frac{\gamma_0 \int_{t_1-\varepsilon}^{t_1} G(t-s)}{\varepsilon}$$

$$\sigma(t) = \gamma_0 \left[\lim_{\varepsilon \to 0} G(t-\varepsilon) - \lim_{\varepsilon \to 0} \frac{0 - G(t - t_1 + \varepsilon)}{1} \right]$$

$$= \gamma_0 \left[G(t) - G(t - t_1) \right]$$

$$= \gamma_0 G_0 \left[e^{-t/\tau} - e^{-(t-t_1)/\tau} \right]$$



10 : 1
V/V

(cross linker) (liq → Gel)

$\boxed{\text{Arrhenius dependence}}$

Chemical gel →	Si + crosslinker
Physical gel (clay) →	no crosslinking

TEM → Transmission Electron Microscope.

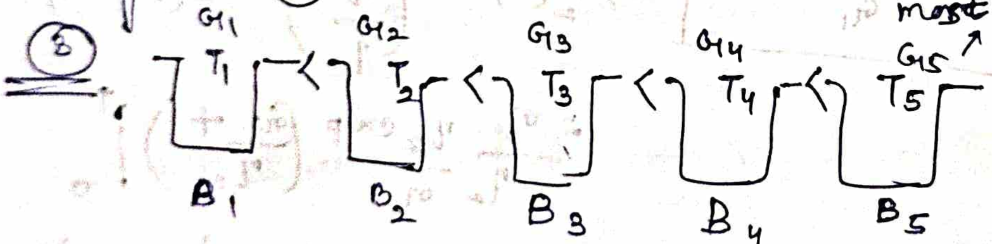
KE = eV_{app}.

(particle nature) ↔ Wave Nature → De Broglie

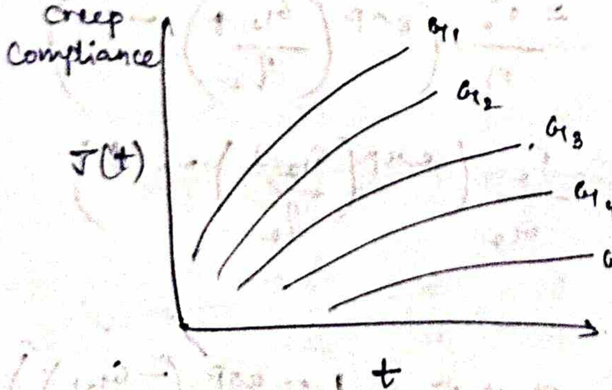
$P = \frac{h}{\lambda}$ $\quad \lambda = \frac{h}{\sqrt{2meV_{app}}}$

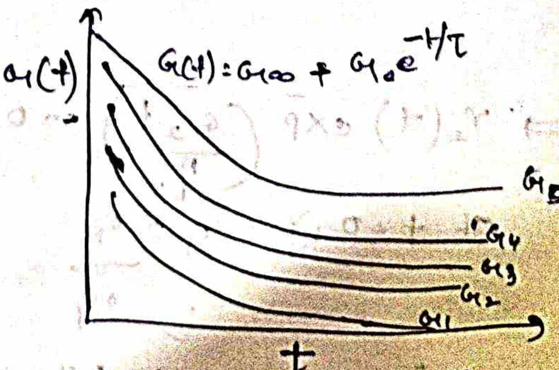
Assignment (3)

(5)



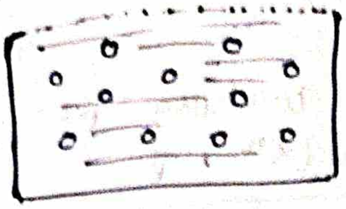
most elastic
↓
decay is slow



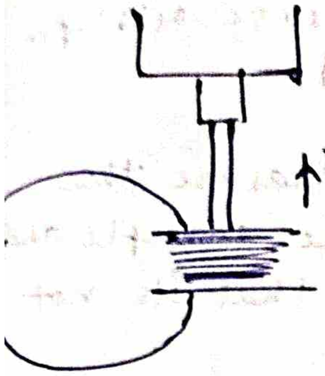


Slower relx^n → more elasticity.
↓
higher relx^n time scale

Project 1 → Electro – Rheology (ER)
(ER fluids)

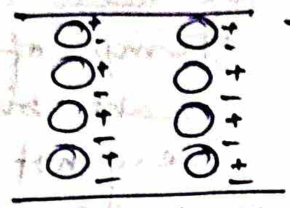


- Dipolar dielectric particles in oil → stir it
dispersion obtained.
- no settling because of increased viscosity



homogenous dispersion → dielectric particles become dipole in the presence of electric field. (polarised)

$\uparrow \vec{E}$



→ formation of chains

Like toothpaste, there ~~st~~ is a certain threshold of ~~strain~~ stress that needs to be crossed for the particles to flow. Particles align in chain like fashion and the material becomes chain of microstructures with elastic character. solid like. (on applicⁿ of $\vec{E}$) → ($\mu\uparrow$)

Transition of liq. like to solid like on applicⁿ of $\vec{E}$

$\boxed{\text{Yield stress after } \vec{E} > \text{Yield stress before } \vec{E}}$

Using $\vec{E}$, we can tune the flowability of a material (how much they can flow)

Project 2 → sinusuidal strain profile to be put in practical sense.

input → $\gamma_0 \sin(2\pi f t)$

$\gamma_0 = 0.1$ (fix)

$f = 0.1 Hz$ (fix)

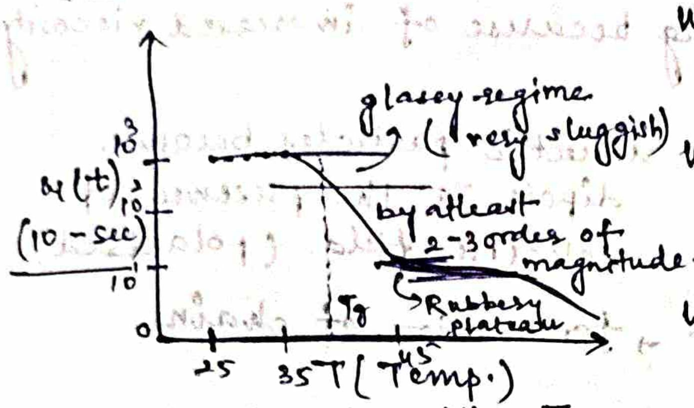
change γ_0 with every iteration (new idea)

t	γ
	0.1
	0.2
	⋮
	10

Polymer Rheology

what is a polymer? → collection of monomers connected with each other.

Time-Temperature Superposition :-



Tg → glass transition Temp.

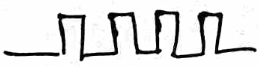
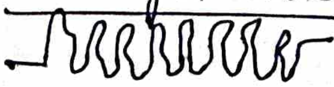
We are not looking for complete relx^n,

We only measure $G(t)$ for $10\,s \rightarrow G_{10}(t)$

We need to measure the Temp. of the sample and make sure that its not very hot.

Generally, the T is set to be 25 °C.
We apply strain (const.) for sometime and then remove the step strain. We measure the relx^n modulus.
When we go to another T we ignore that again apply step strain for like 10 s and measure the relx^n modulus
The graph is log-log scale.
Big polymer molecules - many connected monomers. At the start of the experiment, due to application of T vibrational & translational energy ↑. Thus, as a result entanglement ↓. Beyond a certain T, the molecules can flow freely. Initially the decrease in entanglement is very slow but at some point there is jump by atleast 2-3 order of magnitude. That is Tg → Glass Transition T.
Then we get a weak plateau or rubbery plateau (Cross linked polymer). Then cross links will break or if there are entanglements they will diminish.

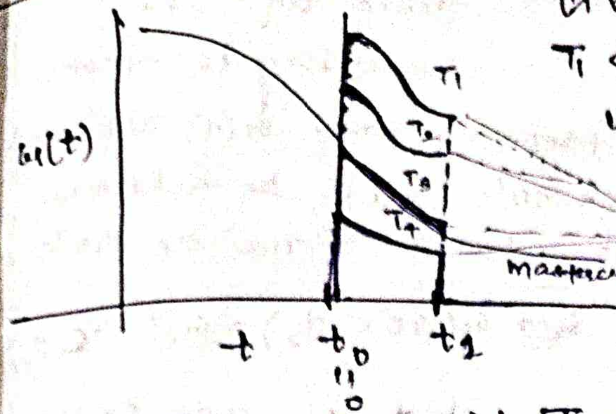
Phase of polymers

Crystalline	Semi Crystalline	Amorphous
		
proper shape and order	order but no proper shape.	no order

More force reqd. to make polymers crystalline or semi crystalline

When these are crystals — proper shape & order is there.

Time Temperature Superposition :—



$T_1 > T_2 > T_3 > T_4$

$T_1 < T_2 < T_3 < T_4$

We took a sample, applied step strain for t_1 time and after making sure that the Temp. has reached T_1 for all molecules macromolecule. We stop (after removing strain for enough time).

T_1

t	G(t)
0.1	⋮
⋮	
10	

$a_1 > 1$ T_2

$a_1 t$	G(t)
0.1	
⋮	
10	

T_3

$a_2 t$	G(t)

T_4

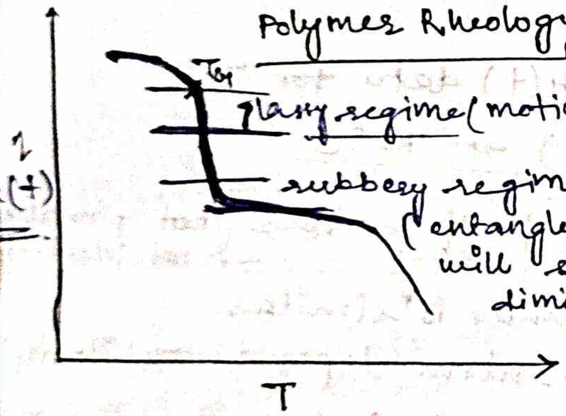
$a_3 t$	G(t)

By finding out the response of the polymer for T_2, T_3, T_4 (short term) we can find the long term response of the polymer at T_1.

At lower T_1 the phenomena which is going to happen we can shift it before.

5	10	23

Polymer Rheology

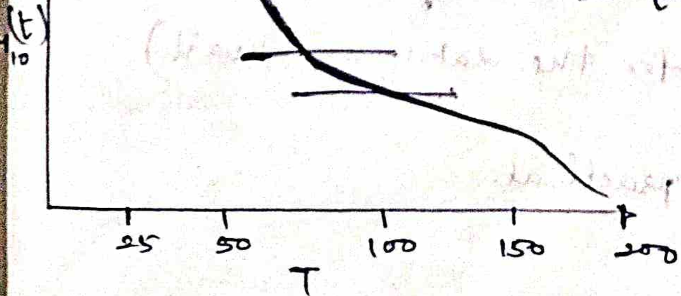


→ range over which entanglements disentangle

* In case of cross linked polymer → cross link will break down (V Temp. Specific)

→ 2 variables → T, G(t)

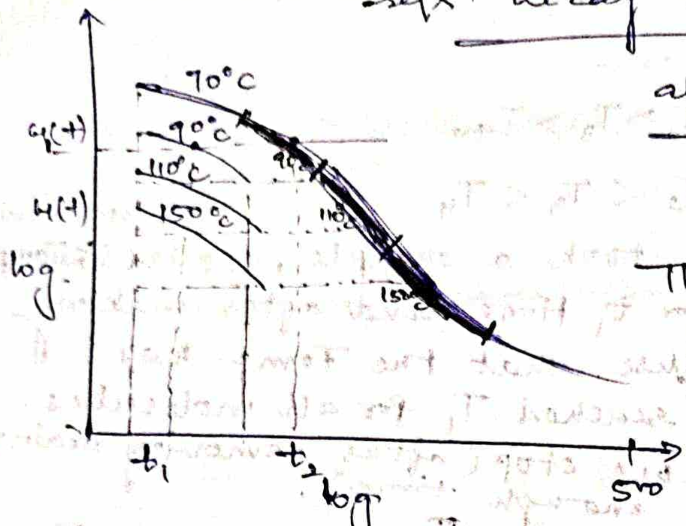
→ We perform the expt. discretely on different T's.



— Effect of Molecular Wt.

— Effect of crosslinking density.

self x" decay for 70°C



at 70°C → mobility ↑
more liq. like
So decay is faster

The property ~~achi~~ $G(t)$ achieved on 90°C will be achieved at a later time on 70°C

$G(70°C, t_2) = G_1(90°C, t_1)$

So Just by knowing the property at t_1 we can known the property at t_2.

By doing short-term Expt at higher T we can estimate the ~~properties~~ long-term properties at low Temp.

Few materials are exceptions which show faster change in lesser Temp.

90°C

t	G(t)

* Horizontal shifting of higher Temp. curve to lower temperature by multiplying with a factor (found by hit & trial method).

100s inform^n → G(t) data for 70°C
at 500s → G(t) at 70°C

Curves have to be self-similar (not parallel, not identical)

only if dynamics is similar

rates changing
mechanism same

only rate of mechanism depends on Temp.

The mechanism is independent of Temp

10 - 10 - 2023

Linear chain polymer → small beads connected by small springs.

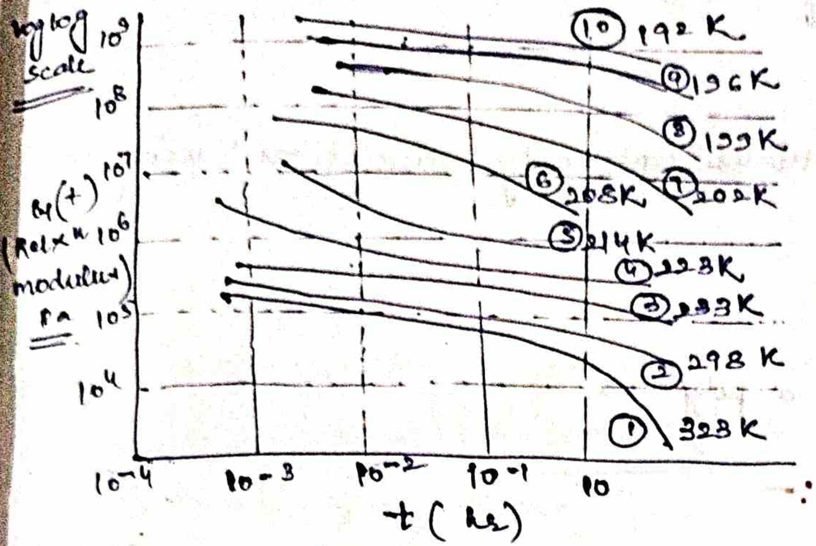
→ our objective is to get macroscopic property.

→ All the T's higher than T_g. (for the data see mail)

Endsem is going to be purely practical.

→ T is a parameter that is going to vary.

→ We are measuring $\underline{\underline{G(t)}}$.



Vertical shifting

⇓

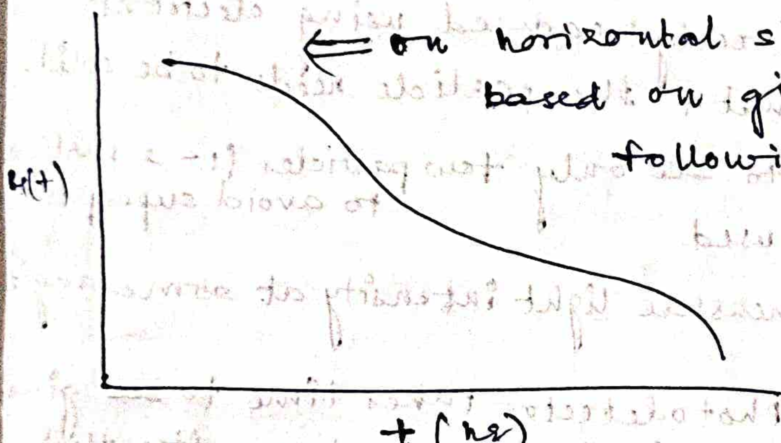
material change occurs slightly.

Horizontal shifting

⇓

rate of change of relaxation to see if we get a single curve

→ If we have to find out if Time-Temp. superpos^n is valid We do look at the $\boxed{\text{Temp. range}}$



⇐ on horizontal shifting of the curves based on given data we get the following master curve.

$$G(t) = \sum_{i=1}^{n} G_{0i} \exp\left(\frac{-t}{\tau_i}\right)$$

$$\tau = f(T)$$

→ Higher is the T, faster is the relx^n process. So τ is a decreasing f^n of Temp.

→ What accounts for change in τ? → shift factors are manifestation of change in τ.

$$G_{T_1}(t) = G_0 \exp\left(\frac{-t}{\tau_1}\right) \quad \underline{(\text{at temp. } T_1)}$$

$$G_{T_2}(t) = G_0 \exp\left(\frac{-t}{\tau_2}\right) \quad \underline{(\text{at temp. } T_2)}$$

$$\tau_2 = \underset{\text{some no.}}{a}\,\tau_1 \Rightarrow G_{T_2}(t) = G_0 \exp\left(\frac{-t}{a\tau_1}\right)$$

Fundamental parameter for us is relx^n time scale

$$\tau \rightarrow a_T \text{ (shift factor)}$$

τ ↓ $f(T)$; a_T ↓ f^n of Temp.

Why we get master curve?
Bcoz only T is getting affected, rest of the process remains same.

~~[illegible]~~

* To predict long term behaviour (property estimation) using short term experimental data.

Light scattering Expt :-

radius of gyration of a polymer

Dynamic Light Scattering :-

Effective particle size depends on the solvent the particle is dissolved in.

→ not valid for conc sol^n, use dil. sol^n

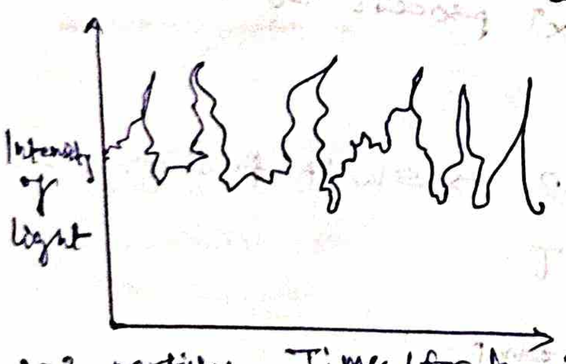
→ Particle size can be directly measured using electronic microscope but for that, the particle needs to be solid.

→ Cross-section small to see only few particles (1-2 particles) to avoid superpos^n

→ monochromatic light used.

→ photodetector used to measure light intensity at some angle

Scattered light :-



2-3 particles

⇒ Photodetector takes time to ~~see~~ give one intensity values after that time only, we get one data point

⇒ The particle goes away after some time, so intensity memory will fade with time, correl^n b/w 2 particles dies out after sometime

⇒ Mobility is hindered in case of more no. of particles (or ↑ conc) so memory of correl^n may remain for some time. (for ↓ conc, mobility ↑, memory fading ↑)

⇒ correl^n b/w intensity pts. decay.

⇒ Incase of bigger particles, relx^n is slower as mobility ↓ for smaller particles, mobility ↑, So correl^n memory decays faster (relx^n faster)

Autocorrel^n f^n is a type of method which can be used to explain a lot of things.

Using DLS, we get an idea → size of particles through mobility correl^n

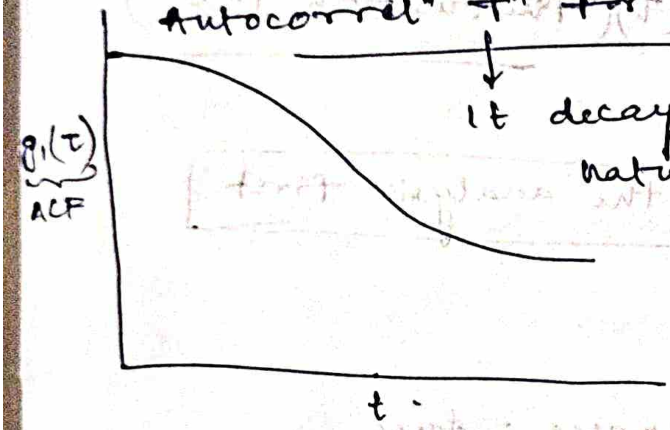
↳ from here, we get to know if the polymer likes the solvent (more soluble & radius of gyration ↑) or does not (rad ↓)

Generic use of autocorrel^n f^n → [checking how much brownian motion is getting affected] & the [correl^n b/w the particles & solvent] and [before-after cond^n of particles for a duration of time.]

correlate 2 data pts. → take the reference & multiply it.

any data changing with time is time series.

Autocorrel^n f^n for Intensity data in our case



It decays due to the electromagnetic nature of waves (smaller particles → decay faster)

Any intensity data decays with time for a fixed point

$$\boxed{ACF = 1 + B\exp(-\Gamma t)}$$

If we have monodispersed particles, then this is valid.

Not valid for polydispersed particles.

Γ → related to particle properties → $D q^2$

D → Diffusivity

$$\boxed{q = \frac{4\pi}{\lambda} \sin\theta} \qquad \boxed{D = \frac{KT}{6\pi\mu R}}$$

ACF will die out.

We can estimate particle size using Intensity data fitting.

2 process –

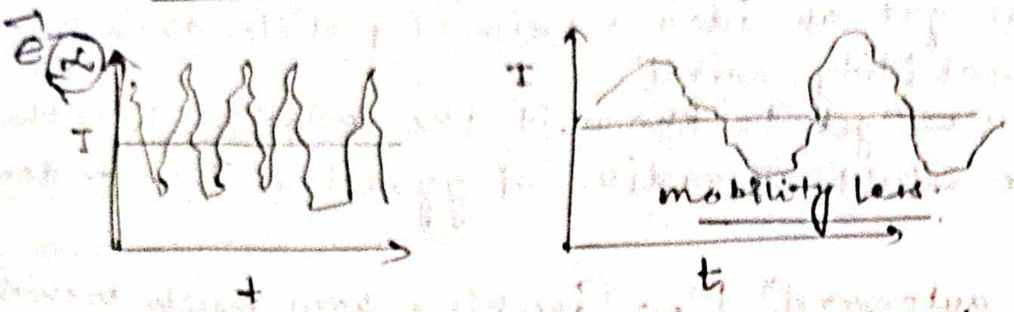
① Hydrodynamic force.

② Thermodynamic force.

Dynamic Light Scattering :-

12/10/22

Bruce J. Berne and Robert Pecora



interaction b/w molecules electron cloud
&
electric field of electromagnetic wave (light)

Whenever you correlate ACF of Intensity fluctuation

$$\frac{\langle I(t), I(t+\tau)\rangle}{\langle I(t)\rangle^2} \quad g^2(\tau) = 1 + A \underbrace{\exp(-\tau t)}_{\text{due to the } \vec{E} \text{ of the particle}} \quad \langle E(t) E(t+\tau)\rangle$$

$g'(\tau)$

$g'(\tau)$ (fluctuation in $\vec{E}$)

Multiexponential Decay

$\mu_2 \rightarrow$ polydispersity index.

* Polymer grafted Nanoparticle

$\boxed{\text{Explain the analysis first}}$

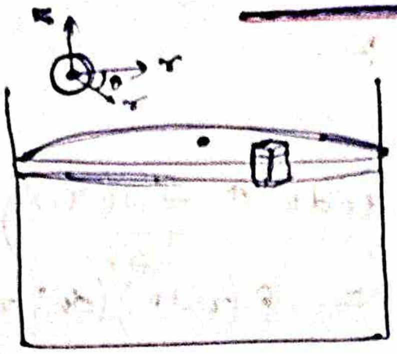
Weissenberg eff

Step 1:- force balance $\rightarrow$ stress × area = force

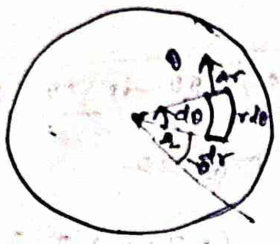
normal, shear

Weissenberg Effect → Vortex form^n
→ rod climbing effect

17-10-23



- differential volume.
- Fluid rotation → clockwise dir^n.
- Dir^n of flow → θ dir^n.
- vel. gradient in r dir^n;
  - → max. at the centre
  - → min (or zero) at the tip (edge)



CV element.

$\sigma_{\theta\theta}$ → acting on the plane exactly $\perp^r$ to θ dir^n

We are only considering normal stresses

Force balance in radial dir^n :-

Eq^n of motion in circular dir^n.

all the forces = centripetal force $\left(\frac{mv^2}{r}\right)$

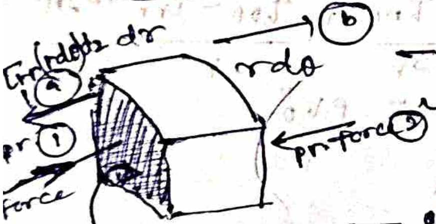
pr. can change ρ. τ can only change shape

$$\boxed{\begin{aligned}\sigma_{11} &= \text{normal stress} + \text{deviatoric stress} \\ \sigma_{11} &= \underset{(\text{pr.})}{-p} + \tau_{11}\ (\text{change in vol.})\end{aligned}}$$

→ both are opp. in nature

→ pr. tries to compress vol. τ_{11} tries ↑ the vol.

(Try finding more differences b/w the 2)



$-p(rd\theta)\,dr$

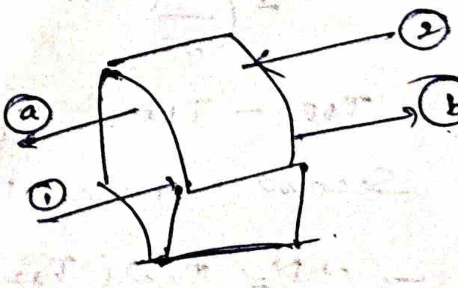
pr. force ①

This is a continuum means we can use Taylor Series. All the f^{n}s are continuous and differentiable f^{n}s.
(smooth f^{n}s)

continuum approxim^n is generally valid till μm.
continuous or not.

Knudsen approxim^n

② $p(rd\theta)\,dz + \left(\frac{\partial(pr\,\partial\theta\,\partial z)}{\partial r}\right) dr$

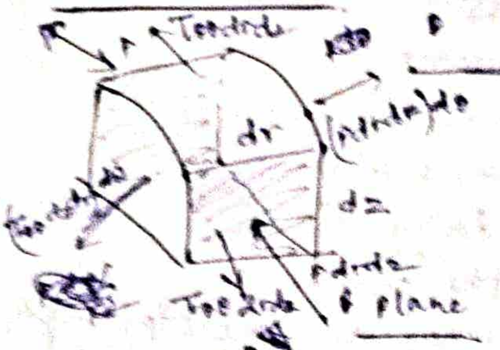


$f_{z+dz} = f_z + \left(\frac{\partial f}{\partial x}\right) dz$

ⓐ $(\tau_{rr})\,rd\theta$

ⓑ $(\tau_{rr})\,rd\theta + \left(\frac{\partial(\tau_{rr}\,rd\theta)}{\partial r}\right) dr$

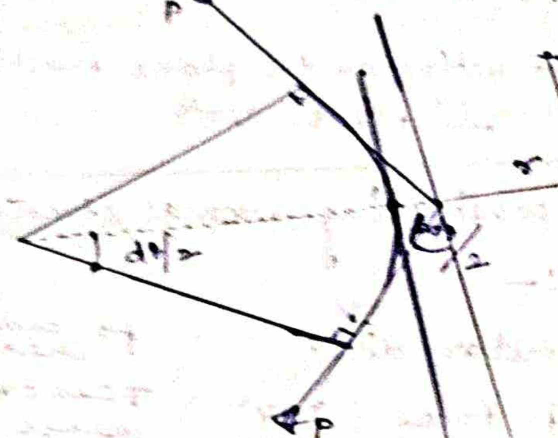
$v_r = v_z = 0$



θ plane balance :-

$$p_{\theta\theta} = -p + \tau_{\theta\theta}$$

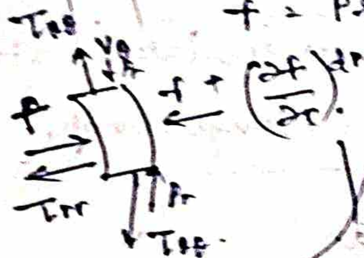
$$-p\, dr\, d\theta + (\tau_{\theta\theta})\, dr\, d\theta - \frac{\partial (r\tau_{rr})}{\partial r} d\theta\, dr + \frac{\partial}{\partial r}(r\, d\theta\, dr) = \rho (r\, d\theta)(dr) \frac{v_\theta^2}{r}$$



$$(p\, dr\, dz)\, d\theta + (\tau_{\theta\theta}\, dr\, dz)\, \frac{d\theta}{2} \times 2$$

$$-\frac{\partial}{\partial r}(\tau_{rr}\, r\, d\theta\, dz)$$

$$\rightarrow -p + \tau_{\theta\theta} - \frac{\partial (r\tau_{rr})}{\partial r} + \frac{\partial (pr)}{\partial r} = \rho v_\theta^2$$



$$f = p\, r\, d\theta\, dz \qquad f + \left(\frac{\partial f}{\partial r}\right) dr$$

$$\rightarrow \tau_{\theta\theta} - \frac{r\,\partial \tau_{rr}}{\partial r} - \tau_{rr} + \frac{r\,\partial p}{\partial r} = \rho v_\theta^2$$

$$\rightarrow -\frac{\partial \tau_{rr}}{\partial r} + \frac{\tau_{\theta\theta} - \tau_{rr}}{r} + \frac{\partial p}{\partial r} = \frac{\rho v_\theta^2}{r}$$

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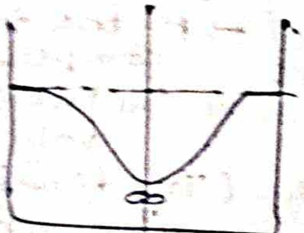
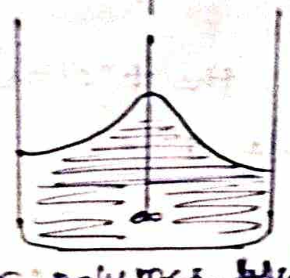


$\rightarrow \theta = 1 \quad r = 2 \quad z = 3$



$$\rightarrow -\frac{\partial \tau_{rr}}{\partial r} + \frac{\tau_{\theta\theta} - \tau_{rr}}{r} + \frac{\partial p}{\partial r} = \frac{\rho v_\theta^2}{r}$$

$$+\frac{\partial \tau_{zz}}{\partial r} - \frac{\partial \tau_{zz}}{\partial r}$$

for polymer melt

$$\rightarrow -\left(\frac{\partial}{\partial r}(\tau_{rr} - \tau_{zz})\right) + \left(-\frac{\partial}{\partial r}(\tau_{zz} - p)\right) - \left(\frac{\tau_{\theta\theta} - \tau_{rr}}{r}\right) = -\frac{\rho v_\theta^2}{r}$$

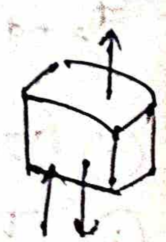
$\tau_{\theta\theta} - \tau_{rr} = N_1$ = first Normal stress difference

Second Normal stress difference $\tau_{rr} - \tau_{zz} = N_2$

$$-\cancel{\frac{\partial N_2}{\partial r}} + \frac{\partial (\tau_{zz} - p)}{\partial r} = \frac{N_1}{r} - \frac{\rho v_\theta^2}{r}$$

$N_2 << N_1$ ignore $\Rightarrow \frac{\partial (\tau_{zz} - p)}{\partial r} = \frac{N_1}{r} - \frac{\rho v_\theta^2}{r}$

$$\rightarrow \boxed{\frac{\partial \sigma_{zz}}{\partial r} = \frac{N_1}{r} - \frac{\rho v_\theta^2}{r}}$$



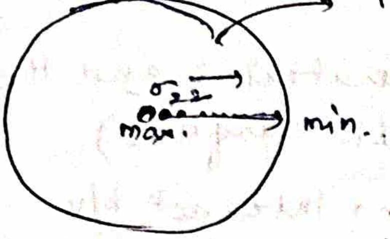
$-p + \tau_{zz}$

σ_{zz}

$\begin{pmatrix} \text{Newtonian fluid} \\ \text{Dilute Sol}^n \end{pmatrix} \Rightarrow N \simeq 0$ $\boxed{\dfrac{\partial \sigma_{22}}{\partial r} = \dfrac{\rho v_\theta^2}{r}}$

*. This ($\sigma_{22}\uparrow$) force is max. near the centre and lowest near the periphery

*. → Therefore we see vortex formation

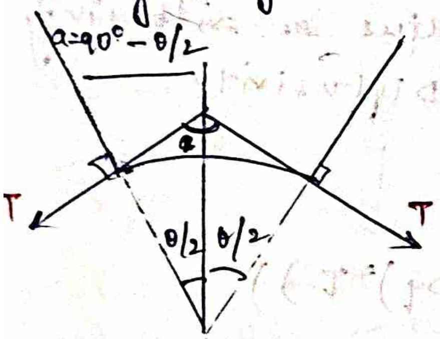


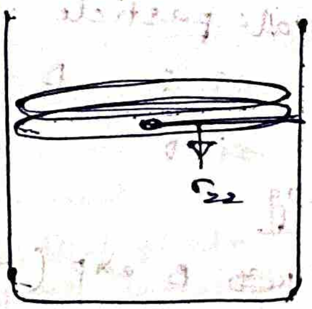
(σ_{22} decreases as we go away from the center in radial direction). This explains the ~~rod~~ vortex formation.

*. if for some fluid $N_1 >> \rho v_\theta^2$ → Then σ_{22} will be an ↑ f^n of r → So fluid will be pushed most at the periphery (pulled most at the center) This explains the rod climbing effect (N_1 is large) → $N_1 = T_{\theta\theta} - T_{rr}$

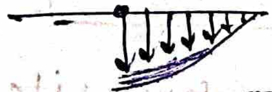
↓

Long polymer solution.





occurs in non linear regime

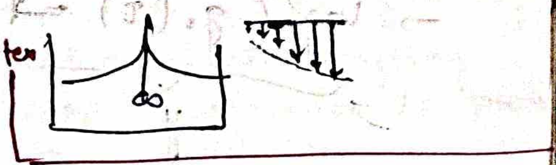


$N_1 = T_{\theta\theta} - T_{rr}$ conc

Long polymer chain solution

N_1 is large

↓



For dilute solution :-

particle dispersion $N_1 \simeq 0$

* Why the rise & drop (form^n of a vortex up or down)?

$T\cos\left(90° - \dfrac{\theta}{2}\right) = T\sin\dfrac{\theta}{2}$

θ is very small → $T\left(\dfrac{\theta}{2}\right)$

T Both sides ⇒ Total $T = T\left(\dfrac{\theta}{2}\right) \times 2 = T\theta$

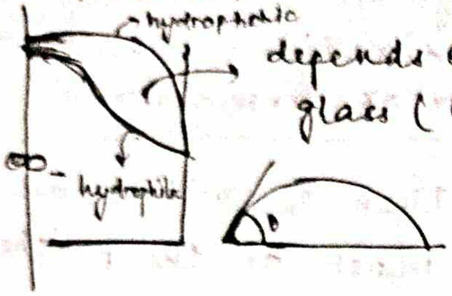
So, Total force → radially inward when the beaker is rotated vigorously.

* must have the property to get stretched (the long polymer chain being rotated). rotation → vigorously → to stretch the polymer chains

In particle dispersion, rod climbing effect cannot be observed because they cannot be stretched.

~~the particle die~~

* Tension in polymer chain → originated due to the alignment of polymer chain along the flow.
* Because of τ_{rr}, $\tau_{\theta\theta}$ gets generated → these generate only bcos of the rotation and stretching.



→ depends on the nature of the material and the glass (hydrophylic / hydrophobic surface)
↓
contact angle → interacⁿ b/w material and glass (surface forces)

t	Ar_c $g_2(\tau)$
⋮	⋮

You are given autocorrelation function

$$g_2(\tau) = 1 + B\exp(-(Dq)^2\tau)$$

Cal. particle radius or diffusivity

$q = \frac{4\pi\lambda}{\sin\theta}$ D = Diffusivity.

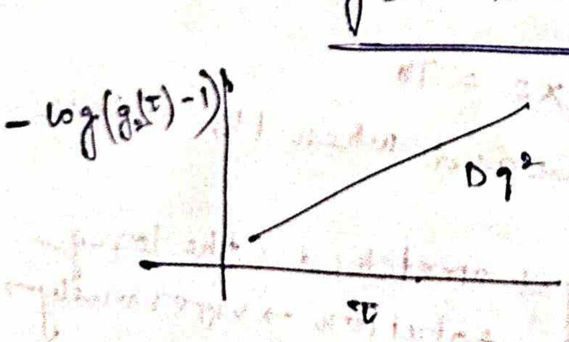
Sir ka favourite → Log

$$\log(g_2(\tau) - 1) = \log(B\exp(-(Dq)^2\tau))$$

$$\to \underbrace{\log(g_2(\tau) - 1)}_{-y} = \log B - (Dq)^2 \underbrace{\tau}_{x}$$

$$\to -\log(g_2(\tau) - 1) = -B_1 + (Dq)^2 \tau$$

$$y = mx + c$$



Theory depends on the assumption that all the particles are following Brownian motion
↓
Concept of DLS can be used.

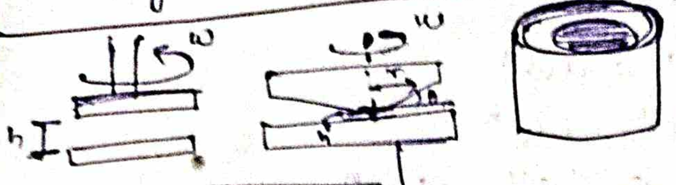
— Time Temp Superposⁿ

— Other types of Superposⁿ → when mechanism is not changing only rate is changing. (self similar curves)

— Purpose of Superposⁿ

→ Is it always guaranteed that on horizontal displacement we get a ~~sing~~ master curve? → Entanglement mechanism should be same otherwise no.

- Time conc superposn
- Any parameter that can slow down the relxn.



h very small

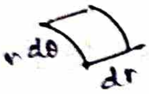
material need to be compressible enough

$$\boxed{\dot{\gamma} = \frac{\omega r}{h}}$$

$$\tau = \eta \frac{\omega r}{h}$$

no way to apply const. stress bcos at each pt. stress is different (Torq. can be const.)

$$T (\text{Torque}) = \vec{r} \times \vec{F}$$



$$\boxed{dF = \tau\,(r d\theta) dr}$$
$$\boxed{\text{stress} \times \text{area}}$$

problem with parallel plate rheometer → can't get const. shear rate (different tangential velocities)

$$\tau (\text{Torque}) = \tau (r\, d\theta) dr \times r \times \sin\left(\frac{\pi}{2}\right)$$

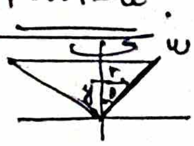
$$dM = \int r\, dF = \int \tau r^2\, d\theta\, dr$$

all the torques in axial dirn (Dirn same) → So we can simply integrate.

$$M = \int_0^R \frac{\eta \omega r^3\, dr (2\pi)}{h}$$

$$M = \left(\frac{\eta \omega \pi}{h} \frac{R^4}{2}\right)$$

stress derivation is not really possible.



$$\tan\theta = \frac{r}{y} \rightarrow y = \frac{r}{\tan\theta}$$

$$v = \omega r$$

$$\dot{\gamma} = \frac{\omega r}{y + h} = \frac{\omega r}{y} = \omega \tan\theta$$

h v.v. small. — same Const.

We can maintain a const shear rate & shear stress in this case

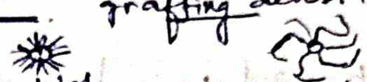
1 bead = 4C chain

Polymer grafted Nanoparticle

Ng (no. of polymers) →

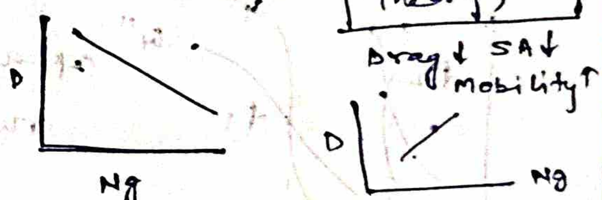
N (no. of beads)

covalent bond b/w polymer & NP — grafting density.



Ng is high	Ng is Low
Brush config	Mushroom config
Steric repulsion (Polymer chains cannot fold)	(polymer chains fold within themselves)
Mobility Low → stokes	$\boxed{\text{decay faster bcos mobility higher (DLS Theory)}}$

Drag ↓ SA ↓ Mobility ↑



if high conc resin → interentanglement diffusivity ↓ as we graft more nanoparticles. (grafting density ↑)

Beyond a certain grafting density the repulsion is much stronger b/w chains.

Mean sq. displacement v/s ~~time~~

dist. 1 min | 5 min | 10 min → PLN

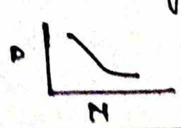
$L^2 = Dt$ on avg is Diffusivity

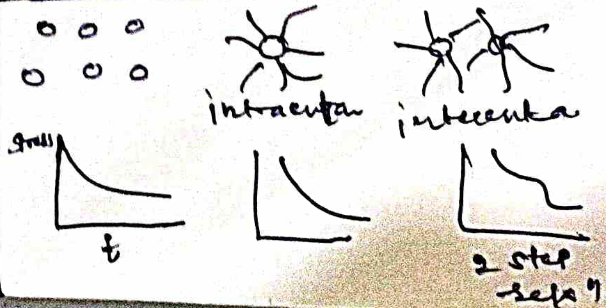
Track the trajectory

Mean disp. → (Square of it) / Time

Then get diffusivity.

Chain length ↑ → interentanglement intermolecular interacn will ↑.





~~initial to gel~~ $g_2^0(\tau) = A + B\exp(-q^2 D \tau)$

~~gel pt~~ $-\ln\left(g_2^0(\tau) - 1\right) = -B_1 + (q^2 D)\tau$

τ	$g_2(\tau) - 1$
10^{-6}	

~~Expt~~ Cyclic frequency Sweep

$\gamma = \gamma_0 \sin(2\pi f t)$

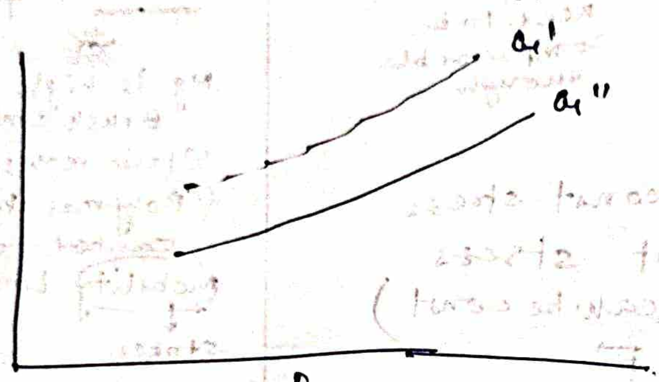
$G' = a\omega^n$

$G'' = b\omega^n$

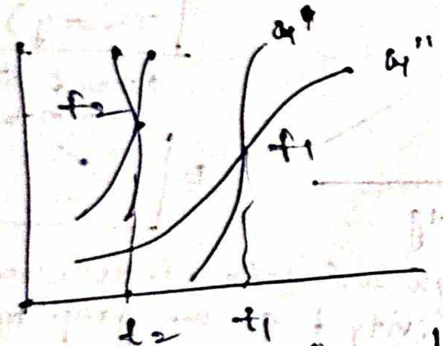
$\tan\delta = \dfrac{G''}{G'} = \dfrac{b\omega^n}{a\omega^n} = \dfrac{b}{a}$

f	G'	G''

for gel :-



t	f	G'	G''	tan δ



apparent $t_{G'} = $ gel pt.

our freq. should not change our gel pt.

$G' \propto G'' \rightarrow$ Method should be independent of frequency.

Both G' and G'' show identical exponent only const. associated is different

f	t
f_1	t_1
f_2	t_2

increasing ↓ … ↑ decreasing

$f_2 > f_1$ $\quad t_2 < t_1$

Gel pt.

The t where tan δ is identical for all the pts. (all the frequencies same), is known as the true gel pt.