INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
Department of Chemical Engineering
End-semester (Autumn) Examination 2017-2018
Subject: Advanced Mathematical Techniques in Chemical Engineering (CH61015)

Remarks:
1. This question paper contains two parts: **Part A** and **Part B**. Attempt both the parts.
2. Write all the answers of a part together.
3. Unless otherwise stated, usual mathematical notations apply.
4. Time = 3 h; maximum marks = 100; total number of printed pages = 2.

Part A: Differential Equations

1. Temperature distribution in a homogeneous solid sphere in non-dimensional form is given as

$$\frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial t}\right)$$

At $t = 0$, $T = k(1 - r)$, where k is a constant. At $r = 1$ for all t, $T = 0$. Obtain the temperature distribution.

. . . 10 marks

2. Steady state temperature distribution of thermally conducting solid bounded by concentric spheres of radii a and b, such that $T = f_1(\theta)$ at $r = a$ and $T = f_2(\theta)$ at $r = b$. Find the temperature distribution in the solid (assuming ϕ symmetry). The governing equation is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{1}{sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial T}{\partial \theta}\right) = 0$$

. . . 10 marks

3. Find the steady state temperature distribution in a semi-circular plate of radius a insulated on both faces. The governing equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} = 0$$

At $r = a$, $T = T_o$ for any θ. At $\theta = 0$ and π, $T = 0$. This means the temperature is maintained zero on boundary diameter.

. . . 10 marks

4. Solve the following equation completely using Green's function method.

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + x$$

At $t = 0$, $T = T_o$. At $x = 0$, $\dfrac{\partial T}{\partial x} = 0$. At $x = 1$, $T = T_1$.

. . . 20 marks

Part B: Linear Algebra

5. Consider the following system:

$$\frac{d}{dt}\underline{\mathrm{X}} = \underline{\underline{A}}\,\underline{\mathrm{X}}$$

(a) Given that

$$\underline{\underline{A}} = \begin{bmatrix} -2 & 5 \\ 5 & -2 \end{bmatrix}$$

Sketch the phase portrait and comment upon the stability of the system.

(b) Carry out the similarity transformation of $\underline{\underline{A}}$ to obtain a new phase portrait with straight line solutions along $x-y$ axes.

. . . 10 marks

6. Solve the following equation using *similarity transformation.*

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

Given the initial conditions $x(0) = \frac{dx}{dt}(0) = 1.$

. . . 15 marks

7. Find the general solution of the following equation.

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 6y = 0$$

. . . 15 marks

8. Consider the nonautonomous differential equation given below.

$$\frac{dx}{dt} = \begin{cases} x-4 & t<5 \\ 2-x & t \geq 5 \end{cases}$$

(a) Find a solution of this equation satisfying $x(0) = 4$. Describe the qualitative behavior of this solution.

(b) Find a solution of this equation satisfying $x(0) = 3$. Describe the qualitative behavior of this solution.

(c) Draw the phase portrait indicating the qualitative behavior of the solutions of this system as $t \to \infty$.

. . . 10 marks

***