Turbulence

$u = \bar{u} + u' \quad v = \bar{v} + v' \quad w = \bar{w} + w' \quad T = \bar{T} + T' \quad P = \bar{P} + P'$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\rightarrow \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} + \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0$$

Now integrating the terms over time we get :- $\frac{1}{t}\int_0^t \frac{\partial u}{\partial x}$

$$= \frac{1}{t}\int_0^t \frac{\partial(\bar{u} + u')}{\partial x} \, 0 = \frac{\partial \bar{u}}{\partial x}.$$

$$\Rightarrow \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0, \quad \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0$$

Turbulent BL :-

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial x} + \nu\frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\partial(\overline{v'u'})}{\partial y}$$

$$\bar{u}\frac{\partial \bar{T}}{\partial x} + \bar{v}\frac{\partial \bar{T}}{\partial y} = \alpha\frac{\partial^2 \bar{T}}{\partial y^2} - \frac{\partial(\overline{v'T'})}{\partial y}$$

additional terms - (stress)
Reynolds stress

$$-\rho\overline{u'v'} = \rho\epsilon_m\frac{\partial \bar{u}}{\partial y} \text{ (Eddy shear stress)}$$

$$-\rho c_p\overline{v'T'} = \rho c_p \epsilon_H\frac{\partial \bar{T}}{\partial y} \text{ (Eddy heat flux)}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial y}\left[\mu\frac{\partial \bar{u}}{\partial y} - \rho\overline{u'v'}\right]$$

$$\bar{u}\frac{\partial \bar{T}}{\partial x} + \bar{v}\frac{\partial \bar{T}}{\partial y} = \frac{1}{\rho c_p}\left[k\frac{\partial \bar{T}}{\partial y} - \rho c_p\overline{v'T'}\right]$$

τ_{app} = apparent shear stress $= \mu\frac{\partial \bar{u}}{\partial y} - \rho\overline{u'v'} = \rho(\nu + \epsilon_M)\frac{\partial \bar{u}}{\partial y}$

JULY

Week	27	28	29	30	31
Mon	1	8	15	22	29
Tue	2	9	16	23	30
Wed	3	10	17	24	31
Thu	4	11	18	25	
Fri	5	12	19	26	
Sat	6	13	20	27	
Sun	7	14	21	28	

AUGUST

Week	31	32	33	34	35
Mon		5	12	19	26
Tue		6	13	20	27
Wed		7	14	21	28
Thu	1	8	15	22	29
Fri	2	9	16	23	30
Sat	3	10	17	24	31
Sun	4	11	18	25	

SEPTEMBER

Week	35	36	37	38	39
Mon	30	2	9	16	23
Tue		3	10	17	24
Wed		4	11	18	25
Thu		5	12	19	26
Fri		6	13	20	27
Sat		7	14	21	28
Sun	1	8	15	22	29

OCTOBER

Week	40	41	42	43	44
Mon		7	14	21	28
Tue	1	8	15	22	29
Wed	2	9	16	23	30
Thu	3	10	17	24	31
Fri	4	11	18	25	
Sat	5	12	19	26	
Sun	6	13	20	27	

NOVEMBER

Week	44	45	46	47	48
Mon		4	11	18	25
Tue		5	12	19	26
Wed		6	13	20	27
Thu		7	14	21	28
Fri	1	8	15	22	29
Sat	2	9	16	23	30
Sun	3	10	17	24	

DECEMBER

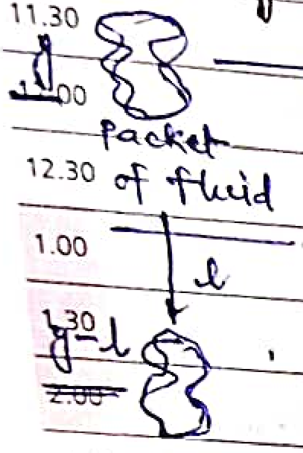
Week	48	49	50	51	52
Mon	30	2	9	16	23
Tue	31	3	10	17	24
Wed		4	11	18	25
Thu		5	12	19	26
Fri		6	13	20	27
Sat		7	14	21	28
Sun	1	8	15	22	29

$$-q''_{app} = k\frac{\partial \bar{T}}{\partial y} - \rho c_p \overline{v'T'} = \rho c_p(\alpha + \epsilon_H)\frac{\partial \bar{T}}{\partial y}$$ = apparent heat flux

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{d\bar{P}}{dx} + \frac{\partial}{\partial y}\left[(\nu + \epsilon_M)\frac{\partial \bar{u}}{\partial y}\right]$$

$$\bar{u}\frac{\partial \bar{T}}{\partial x} + \bar{v}\frac{\partial \bar{T}}{\partial y} = \frac{\partial}{\partial y}\left[(\alpha + \epsilon_H)\frac{\partial \bar{T}}{\partial y}\right]$$

Mixing Length model :-



$\longrightarrow u$ $\quad O(\bar{u}(y)) - O(\bar{u}(y-l)) \simeq u'$

$$O(u') = l\frac{\partial \bar{u}}{\partial y} = O(v')$$

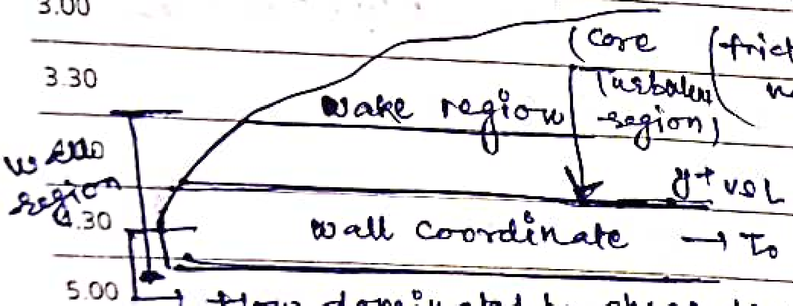
Prandtl mixing length

$$-\overline{u'v'} = l^2\left(\frac{\partial \bar{u}}{\partial y}\right)^2$$

forced assump^n — order of change of pos^n is a f^n of $\bar{u}$

$$\epsilon_M = -\overline{u'v'} = l^2\left|\frac{\partial \bar{u}}{\partial y}\right|^2 \qquad l \propto y \rightarrow l = ky$$

k: Von Karman's const



(frictional vel.) $u^+ = \frac{\bar{u}}{u^*}$, $\quad u^* = \left(\frac{\tau_0}{\rho}\right)^{1/2}$, $\quad v^+ = \frac{\bar{v}}{u^*}$

wall coordinate $\longrightarrow \tau_0$ is const $\tau_0 \neq f(y)$ — wall coordinate

$$y^+ = y\left(\frac{u^*}{\nu}\right)$$

flow dominated by shear stress

laminar shear layer that grows until it breaks down (unstable)

$$x^+ = x\left(\frac{u^*}{\nu}\right)$$

shear stress const. (No effect of inertia).

- in wall coordinates
- inertia $\simeq 0$ $\quad \frac{dP}{dx} = 0$.

$$\frac{\partial}{\partial y}\left[(\nu + \epsilon_M)\frac{\partial \bar{u}}{\partial y}\right] = 0 \rightarrow (\nu + \epsilon_M)\frac{\partial \bar{u}}{\partial y} = \text{const.} = \frac{\tau_0}{\rho}$$

$$(\nu+\epsilon_M)\frac{\partial(u^+u^*)}{\partial\left(\frac{y^+\nu}{u^*}\right)}=\frac{\tau_0}{\rho} \rightarrow (\nu+\epsilon_m)\frac{u^{*2}}{\nu}\frac{\partial u^+}{\partial y^+}=\frac{\tau_0}{\rho}$$

$$\Rightarrow \left(1+\frac{\epsilon_m}{\nu}\right)\frac{\partial u^+}{\partial y^+}=1 \Rightarrow \left(1+\frac{\epsilon_m}{\nu}\right)\frac{\partial u^+}{\partial y^+}=1$$

Case 1: Viscous Sublayer :- $\nu >> \epsilon_m$

$$\Rightarrow \frac{\partial u^+}{\partial y^+}=1 \rightarrow \int_0^{u^+}\partial u^+=\int_0^{y^+_{vsL}}\partial y^+ \rightarrow u^+=y^+_{vsL} \quad \text{when } 0<y<y^+_{vsL}$$

Case 2 :- $\epsilon_m >> \nu$

$$\Rightarrow \frac{\epsilon_m}{\nu}\frac{\partial u^+}{\partial y^+}=1 \Rightarrow \frac{\partial u^+}{\partial y^+}=\frac{\nu}{\epsilon_m}$$

$$\epsilon_m = l^2\left|\frac{\partial\bar{u}}{\partial y}\right| = K^2y^2\left|\frac{\partial\bar{u}}{\partial y}\right|$$

$$\epsilon_m = K^2(y^+)^2\frac{\nu^2}{(u^*)^2}\left(\frac{u^{*2}}{\nu}\right)\frac{\partial u^+}{\partial y^+}$$

$$\epsilon_m = K^2y^{+2}\,\nu\frac{\partial u^+}{\partial y^+}$$

$$\Rightarrow (Ky^+)^2\left|\frac{\partial u^+}{\partial y^+}\right|^2=1 \rightarrow \frac{\partial u^+}{\partial y^+}=\frac{1}{Ky^+}$$

$$\rightarrow \int_0^{u^+}\partial u^+=\int\frac{1}{Ky^+}\partial y^+=\int_0^{y^+_{vsL}}\frac{1}{Ky^+}\partial y^+ + \int_{y^+_{vsL}}^{y^+}\frac{1}{Ky^+}\partial y^+$$

$$\Rightarrow u^+=\frac{1}{K}\ln y^+ + y^+_{vsL}-\frac{1}{K}\ln y^+_{vsL} \rightarrow u^+=A\ln y^++B.$$

8.00 Assume a vel. profile :- $u^+ = f(y^+)$

8.30 $$\frac{\bar{u}}{u^*} = f\left(\bar{y}\left(\frac{u^*}{\nu}\right)\right)$$ at the edge of BL :- $\bar{y} = \delta$, $\bar{u} = u_\infty$

9.00

9.30 $$\frac{u_\infty}{(\tau_0/\rho)^{1/2}} = f\left(\frac{\delta(\tau_0/\rho)^{1/2}}{\nu}\right) \quad \text{—①}$$

10.00

10.30 $$\frac{d}{dx}\int u(u_\infty - u)\,dy = \cancel{\frac{1}{\rho}\frac{dP_\infty}{dx}}^{\,0} + \cancel{\frac{du_\infty}{dx}\int u\,dy}^{\,0} + \nu\left(\frac{\partial u}{\partial y}\right)_{y=0} \rightarrow \frac{\tau_0}{\rho}$$

11.00

11.30 $$\frac{d}{dx}\int_0^\delta u(u_\infty - u)\,dy = \frac{\tau_0}{\rho} \quad \text{—②}$$

12.00

12.30 2 eqns. 2 unknowns.

1.00 Wall coordinates are not independent of experiment.

1.30

2.00

2.30

3.00

3.30

Temp. Profile :-

q_0'' : wall heat flux does not depend on y sufficiently close to the wall

wake region

CTR → core Turbulent Region

conduct^n sublayer CSL

$$\frac{\partial}{\partial y}\left[(\alpha+\epsilon_H)\frac{\partial \bar{T}}{\partial y}\right] = 0$$

$$(\alpha+\epsilon_H)\frac{\partial \bar{T}}{\partial y} = \text{const.} = \frac{-q_0''}{\rho c_p}$$

$$\frac{\rho c_p u^*\,\partial(T_0-\bar{T})}{q_0''} = \frac{\partial y}{\left(\frac{\alpha}{\nu}+\frac{\epsilon_H}{\nu}\right)}, \quad y^+ = y\left(\frac{u^*}{\nu}\right)$$

$$\frac{\rho c_p u^*\,d(T_0-\bar{T})}{q_0''} = \frac{dy^+}{\left[\frac{1}{Pr}+\left(\frac{\epsilon_H}{\epsilon_m}\right)\left(\frac{\epsilon_m}{\nu}\right)\right]}$$

$$\left\{T^+(x^+,y^+) = \frac{\rho c_p u^*(T_0-\bar{T})}{q_0''}\right\}$$

$$dT^+ = \frac{dy^+}{\frac{1}{Pr}+\frac{1}{Pr_t}\left(\frac{\epsilon_m}{\nu}\right)}$$

$$\int dT^+ = \int_0^{y^+_{CSL}} \frac{dy^+}{\frac{1}{Pr}+\underbrace{\frac{1}{Pr_t}\left(\frac{\epsilon_m}{\nu}\right)}_{\text{negligible term.}}} + \int_{y^+_{CSL}}^{y^+} \frac{dy^+}{\underbrace{\frac{1}{Pr}}_{\text{negligible term}}+\frac{1}{Pr_t}\left(\frac{\epsilon_m}{\nu}\right)}$$

$$T^+ = Pr\,y^+ \qquad 0 < y^+ < y^+_{CSL}$$

$$T^+ - T^+_{CSL} = Pr_t \int \frac{dy^+}{(\epsilon_m/\nu)}$$

$$\boxed{\frac{\epsilon_m}{\nu}\frac{du^+}{dy^+} = 1 \rightarrow \frac{\epsilon_m}{\nu} = \frac{dy^+}{du^+} = Ky^+}$$

$\Rightarrow T^+ - T^+_{csL} = Pr_t \int \frac{dy^+}{\kappa y^+} \rightarrow \boxed{T^+ = Pr\, y^+_{csL} + \frac{Pr_t}{\kappa} \ln\left(\frac{y^+}{y^+_{csL}}\right)}$

for $y^+_{csL} < y^+ < y^+$

Summary :—

Vel. Distribⁿ :— $u^+ = y^+ \quad 0 < y^+ < y^+_{vsL}$

$u^+ = \frac{1}{\kappa} \ln y^+ + B \quad y^+_{vsL} < y^+ < y^+ \quad \kappa \cong 0.4 \quad B = 5.5$

Temp. Distribⁿ :— $T^+ = Pr\, y^+ \quad 0 < y^+ < y^+_{csL}$ $\quad \boxed{0.5 \le Pr < 5}$

$T^+ = Pr\, y^+_{csL} + \frac{Pr_t}{\kappa} \ln\left(\frac{y^+}{y^+_{csL}}\right)$ $\quad \boxed{y^+_{csL} \cong 13.2 \quad Pr_t \cong 0.9 \quad \kappa \cong 0.41}$

$T^+ = 2.195 \ln y^+ + 13.2\, Pr - 5.66$

We got some T distribution. But T^+ depends on some expression that involves q_0'' which is unknown (even u^* is not known)

$T^+ = \frac{\rho c_p u^* (T_0 - \bar{T})}{q_0''} \quad u^* = \left(\frac{\tau_0}{\rho}\right)^{1/2}$

manipulating the expressions we have got for finding the expression valid at the edge of wake region

$\frac{\rho c_p u^* (T_0 - T_\infty)}{q_0''} = Pr\, y^+_{csL} + \frac{Pr_t}{\kappa} \ln\left(\frac{\delta u^*}{\nu}\right) - \frac{Pr_t}{\kappa} \ln(y^+_{csL})$ — (1)

Now q_0'' app not const at the edge but we have assumed.

$u^+ = \frac{\bar{u}}{u^*} \rightarrow \frac{u_\infty}{u^*} = \frac{1}{\kappa} \ln\left(\frac{\delta u^*}{\nu}\right) + B$ — (2)

Use (1) & (2) to derive a correlⁿ with condⁿ $\frac{u_\infty}{u^*} = \left(\frac{2}{c_{fx}}\right)^{1/2}$

$c_{f,x} = \frac{\tau_0}{\frac{1}{2}\rho u_\infty^2} \longrightarrow \tau_0 = \frac{c_{fx}\, \rho u_\infty^2}{2}$

$\left(\frac{u_\infty}{u^*}\right)^2 = \frac{u_\infty^2}{\tau_0/\rho} = \frac{2}{c_{f,x}}$

Desired heat transfer result :-

$$St_x = \frac{h}{\rho C_p u_\infty} = \frac{\frac{1}{2} C_{f,x}}{Pr_t + \left(\frac{1}{2}C_{f,x}\right)^{1/2}\left[Pr\, y^+_{CSL} - B\, Pr_t - \left(\frac{Pr_t}{K}\right)\ln y^+_{CSL}\right]}$$

Stanton no.

$$\boxed{St_x = \frac{\frac{1}{2} C_{fx}}{0.9 + \left(\frac{1}{2}C_{f,x}\right)^{1/2}\left(13.2 Pr - 10.25\right)}}$$

Denominator in St_x is not very sensitive to changes in Re

$0.5 \le Pr \le 5 \to$ denominator is of order $O(1)$ and Pr - dependent

$$St_x \propto \frac{1}{2}C_{f,x} \Rightarrow \boxed{St_x\, Pr^{2/3} = \frac{1}{2}C_{f,x}}$$ Colburn analogy

Same expression using Von - Karman analysis $(Pr_t = 1)$

$$\boxed{St_x = \frac{\frac{1}{2}C_{f,x}}{1 + 5\left(\frac{1}{2}C_{f,x}\right)^{1/2}\left\{Pr - 1 + \ln\left[1 + \frac{5}{6}(Pr - 1)\right]\right\}}}$$

Using $\Rightarrow$ $\boxed{St_x = \frac{1}{2}C_{f,x}}$ $\to$ Reynold's analogy

$$\frac{\tau_0}{\rho} = (\nu + \epsilon_m)\frac{\partial \bar{u}}{\partial y} \to \frac{\partial \bar{u}}{\partial y}(\mu + \rho\epsilon_m) = \tau_0 \quad (1)$$

$$\frac{-q_0''}{\rho c_p} = (\alpha + \epsilon_H)\frac{\partial \bar{T}}{\partial y} \Rightarrow \frac{\partial \bar{T}}{\partial y}(K + \rho C_p \epsilon_H) = -q_0'' \quad (2)$$

(1) $\div$ (2) :- $$\frac{\tau_0}{-q_0''} = \left(\frac{\mu + \rho\epsilon_m}{K + \rho c_p \epsilon_H}\right)\frac{d\bar{u}}{d\bar{T}} \quad (3)$$

$Pr = \frac{\mu C_p}{K}$ let's take $Pr = 1$, $\mu C_p = K$

for $Pr_t \simeq 1$, $\epsilon_H = \epsilon_m$

Imposing above (2) condns in (3)

$$\frac{\tau_o}{q_o''} = \frac{(\mu + \rho\epsilon_M)}{c_p(\mu + \rho\epsilon_M)} \cdot \frac{d\bar{u}}{d\bar{T}} = \frac{1}{c_p}\frac{d\bar{u}}{d\bar{T}} \qquad \left\{ \frac{q_o''}{T_o - T_\infty} = h \right\}$$

$$\rightarrow \frac{\tau_o}{q_o''} = \frac{1}{c_p}\frac{u_\infty}{(T_\infty - T_o)} \Rightarrow \frac{\frac{1}{2}\rho u_\infty^2 c_{f,x}}{h\cancel{\Delta T}} = \frac{1}{c_p}\frac{u_\infty}{\cancel{\Delta T}}$$

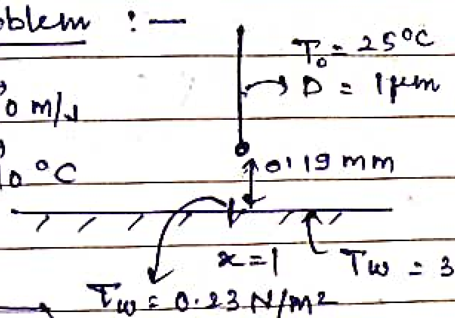
$$\Rightarrow \boxed{\frac{h}{u_\infty \rho c_p} = \frac{1}{2} c_{f,x} = St_x} \qquad Pr = \frac{\mu c_p}{k} \qquad Pr_t = \frac{\epsilon_M}{\epsilon_H}$$

Replace molecules with eddies then only this analogy can be used

$\boxed{Pr_t = f(\text{system})}$
$\boxed{Pr = f(\text{property of material})}$

if T changes, Pr won't change (material independent of T) and Pr_t will change.

Problem :—



$\vec{u_\infty} = 10$ m/s

$\vec{T_\infty} = 10°C$

ⓐ Wall heat flux at the location of the measurement

$$y^+ = y\left(\frac{u^*}{\nu}\right) \qquad y^+_{CSL} = 13.2 \text{ [Given]}$$

$y^+ < 200$ {wall coordinate}

$$y^+ = y\left(\frac{(\tau_o/\rho)^{1/2}}{\nu}\right) = \frac{0.19 \times 0.45}{1000 \times 16.7\times10^{-6}}$$

$\rho = 1.13\ \frac{kg}{m^3}$
$k = 0.026\ \frac{W}{mK}$
$Pr = 0.7$
$C_p = 10^3\ J/kg\cdot K$

$$u^* = \left(\frac{\tau_o}{\rho}\right)^{1/2} = \left(\frac{0.23}{1.13}\right)^{1/2} = 0.45 \qquad \boxed{y^+ = 5.12 < y^+_{CSL}}$$

So, wire tip is inside CSL

$$q_o'' = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = -0.026 \times \left(\frac{30-25}{0.19\times10^{-3}}\right) = 684.21\ \frac{W}{m^2}$$

ⓑ Compare with Colburn rel^n ($St\,Pr^{2/3} = \frac{1}{2}C_{f,x}$) and find the % difference relative to direct measurement.

JULY					
Week	27	28	29	30	31
Mon	1	8	15	22	29
Tue	2	9	16	23	30
Wed	3	10	17	24	31
Thu	4	11	18	25	
Fri	5	12	19	26	
Sat	6	13	20	27	
Sun	7	14	21	28	

AUGUST					
Week	31	32	33	34	35
Mon		5	12	19	26
Tue		6	13	20	27
Wed		7	14	21	28
Thu	1	8	15	22	29
Fri	2	9	16	23	30
Sat	3	10	17	24	31
Sun	4	11	18	25	

SEPTEMBER					
Week	35	36	37	38	39
Mon	30	2	9	16	23
Tue		3	10	17	24
Wed		4	11	18	25
Thu		5	12	19	26
Fri		6	13	20	27
Sat		7	14	21	28
Sun	1	8	15	22	29

OCTOBER					
Week	40	41	42	43	44
Mon		7	14	21	28
Tue	1	8	15	22	29
Wed	2	9	16	23	30
Thu	3	10	17	24	31
Fri	4	11	18	25	
Sat	5	12	19	26	
Sun	6	13	20	27	

NOVEMBER					
Week	44	45	46	47	48
Mon		4	11	18	25
Tue		5	12	19	26
Wed		6	13	20	27
Thu		7	14	21	28
Fri	1	8	15	22	29
Sat	2	9	16	23	30
Sun	3	10	17	24	

DECEMBER					
Week	48	49	50	51	52
Mon	30	2	9	16	23
Tue	31	3	10	17	24
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Fri		6	13	20	27
Sat		7	14	21	28
Sun	1	8	15	22	29

Direct: $St = \dfrac{h}{\rho c_p u_\infty}$ $\left\{ h = \dfrac{q_0''}{\Delta T} \right\}$ $\Rightarrow St = \dfrac{q_0''}{\rho c_p u_\infty \Delta T}$

$St = \dfrac{684.21}{1.13 \times 10^3 \times 10 \times (\underset{30}{T} - \underset{10}{T_\infty})} = 3.027 \times 10^{-3}.$

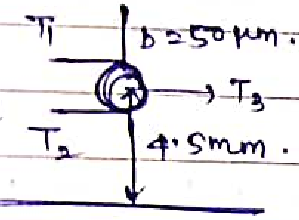
Colburn soln :-

$St\, Pr^{2/3} = \frac{1}{2} C_{f,x} \rightarrow St = \dfrac{\tau_0}{\frac{1}{2}\rho u_\infty^2} \times \dfrac{1}{Pr^{2/3}} = \dfrac{0.23}{1.13 \times 10^2 \times (0.7)^{2/3}}$

$= 2.53 \times 10^{-3}.$

$\%\ \text{difference} = \dfrac{3.027 - 2.53}{3.027} = 16.42\%$

(c)



Expected $T = ?$

$y^+ = y\left(\dfrac{u^*}{\nu}\right) = \dfrac{4.5}{1000} \times \dfrac{0.45}{16.7 \times 10^{-6}} = 121.257$

Core turbulent region ($\because y^+ > 13.2$)

$T^+ = Pr\, y^+_{csL} + \dfrac{Pr_t}{K} \ln\left(\dfrac{y^+}{y^+_{csL}}\right) = (0.7 \times 13.2) + \dfrac{0.9}{0.41} \ln\left(\dfrac{121.257}{13.2}\right)$

Von Karman const $= 0.41$

$T^+ = 9.24 + 4.868 = 14.1$

$T^+ = \dfrac{(T_\infty - \bar{T})\, c_p u^* \rho}{q_w''} \Rightarrow 14.1 = (30 - \bar{T}) \dfrac{\times 10^3 \times 1.13 \times 0.45}{684.21}$

$\rightarrow \bar{T} = 30 - 18.97 = 11°C$

$\bar{T}$ must always be less than T_w. for viscous Temp. $\bar{T} > T_w$ but it generally does not happen.

(d) Sensor size induced uncertainty in the mean temperature measurement at this distance from the wall.

$T^+ = f(y^+) \rightarrow \dfrac{dT^+}{dy^+} = f'(y^+)$

$T^+ = 2.195 \ln y^+ + 3.58 \rightarrow \dfrac{dT^+}{dy^+} = \dfrac{2.195}{y^+} \rightarrow \dfrac{\Delta T^+}{d^+} = \dfrac{2.195}{121.257}$

$\rightarrow \dfrac{\Delta T^+}{d^+} = 0.0181 \rightarrow \dfrac{\Delta T^+}{\left(\dfrac{d u^*}{\nu}\right)} = 0.0181 \rightarrow \Delta T^+ = 0.0181 \left(\dfrac{50 \times 10^{-6} \times 0.45^{6}}{16.7 \times 10^{-6}}\right)$

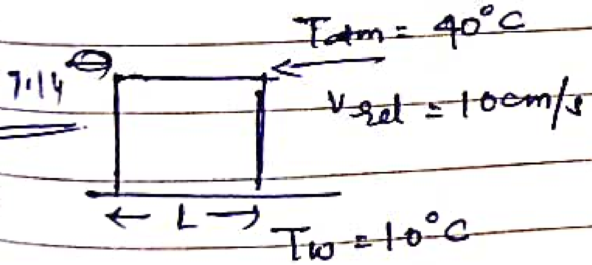
$\Rightarrow \Delta T^+ = 0.0181 \times (1.35)$, $\Delta T^+ = 0.024.$

$\dfrac{|\Delta \bar{T}| \rho c_p u^*}{q_w''} = \Delta T^+ \rightarrow |\Delta \bar{T}| = \dfrac{\Delta T^+ \times q_w''}{\rho c_p u^*} = 0.0323°C$

$\dfrac{\Delta \bar{T}}{T_3} \times 100 = \dfrac{0.0323}{11} \times 100 = 0.3\%$

(e) u_∞ changed to 50 m/s

* initially when wire is located at 0.19 mm above the wall
→ Not expected change in measured Temperature because y^+ is within CSL. So only dependent on sys. property.
* By Thermocouple located 4.5 mm above the wall → beyond y^+_{CSL} ΔT will change bcoz Pr_t will change



$\rho_{air} = 1.205 \times 10^{-3}\ g/cm^3$

$\nu_{air} = 0.15\ cm^2/s$

$F_{D,air} = F_{D,water}$

$\tau_{w,a} \cancel{A} = \tau_{w,w} \cancel{A}$

$\Rightarrow \tau_a = \tau_w \Rightarrow 0.037 \rho_a u_a^2 \left(\dfrac{\nu_a}{u_a L}\right)^{1/5} = 0.037 \rho_w u_w^2 \left(\dfrac{\nu_w}{\nu_w L}\right)^{1/5}$

$\Rightarrow \dfrac{U_{air}}{U_{water}} = \left(\dfrac{\rho_w}{\rho_a}\right)^{5/9} \left(\dfrac{\nu_w}{\nu_a}\right)^{1/9} = \left[\dfrac{1\,kg}{(10\,cm)^3} \times \dfrac{cm^3}{1.205 \times 10^{-3} g}\right]^{5/9} \left(\dfrac{0.015}{0.15}\right)^{1/9}$

$Re = \frac{\rho v d}{\mu}$ $\frac{v x}{\nu} = Re_x$

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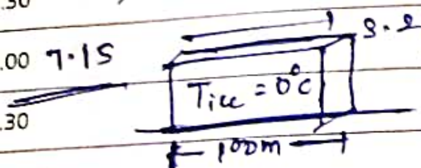
8.00 $u_{water} = 100 m/s$

8.30 $\Rightarrow u_{air} = 11.7 \, km/h$.

9.00 7.15



$h_{sf} = 333.4 \, kJ/kg$

$T_{air} = 40°C$ air properties: $\nu = 0.15 \, cm^2/s$ $k = 2.5 \times 10^{-4} \frac{W}{cm \cdot K}$ $Pr = 0.7$

10.00

10.30 air BL is turbulent. $Re_L = 3.24 \frac{m}{s} \cdot 100 m \cdot \frac{s}{0.15 cm^2} \cong 2.16 \times 10^7$

11.00

11.30 $\therefore \overline{Nu_L} = 0.037 \, Pr^{1/3} Re_L^{4/5} = 0.037 (0.72)^{1/2} (2.16 \times 10^7)^{4/5}$

12.00 $= 2.44 \times 10^4$.

12.30 corresponding L-averaged heat flux into the top surface

1.00 of the iceberg is $\overline{q_L''} = \overline{h_L} \Delta T = \overline{Nu_L} \frac{k}{L} \Delta T$

1.30 $\overline{q_L''} = \frac{2.44 \times 10^4 \times 2.5 \times 10^{-4} \times 40°C}{100}$

2.00

2.30 $= 244 \, W/m^2$

3.00 $\frac{dH}{dt} = \frac{\overline{q_L''}}{\rho h_{sf}} = \frac{244 W}{m^2} \times \frac{(0.1 m)^3}{1 kg} \times \frac{kg}{333.4 kJ} = 0.73 \times 10^{-6} \, m/s$

3.30

4.00 $= 2.6 \, mm/hr$.

4.30

Natural convection

Incompressible flow $\neq$ ρ changing

Volumetric strain rate (rep. deform^n of Vol.) $\rightarrow \{D\forall/Dt\}$

$\frac{D\forall}{Dt} = 0$ $\rightarrow$ ρ = const. is a special case of incompressible flow.

$\boxed{\frac{1}{\forall}\frac{D\forall}{Dt} = \vec{\nabla}\cdot V}$ { linear deform^n of a fluid element }

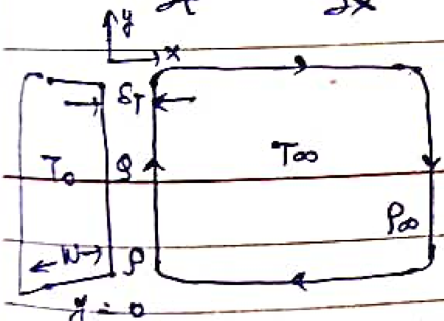
$m = \rho \times V \rightarrow \ln m = \ln \rho + \ln \forall$

$\Rightarrow \frac{1}{m}\frac{Dm}{Dt} = \frac{1}{\rho}\frac{D\rho}{Dt} + \frac{1}{\forall}\frac{D\forall}{Dt}$ (for incompressible fluid)

$\frac{1}{m}\frac{Dm}{Dt} \to 0$ conservn of mass. $\frac{1}{\forall}\frac{D\forall}{Dt} \to 0$

$\Rightarrow \frac{1}{\rho}\frac{D\rho}{Dt} = 0 \Rightarrow \frac{D\rho}{Dt} = 0$ $\rightarrow$ doesn't mean ρ is const.

$\Rightarrow \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} = 0$



$T_o > T_\infty$

Assump^n :- 2D incompressible flow.

continuity eq^n :- $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ —(i)

m^m bal. :- X $\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$

$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial P}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \cancel{\frac{\partial^2 v}{\partial y^2}}\right) - \rho g$

$\frac{\partial P}{\partial y} \to \frac{dP_\infty}{dy}$; $\frac{\partial^2 v}{\partial y^2} \to 0$

Everything is happening close to the wall.

$\hookrightarrow dP/dy = dP_\infty/dy = -\rho_\infty g$.

$\Rightarrow \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}\right) = \mu\frac{\partial^2 v}{\partial x^2} + g(\rho_\infty - \rho)$

$\rho = \rho_\infty + \frac{\partial \rho}{\partial t}(T - T_\infty)$ + higher order terms neglected.

8.00 $\rho - \rho_\infty = \frac{\partial \rho}{\partial t}\Big|_{T_\infty}(T - T_\infty)$

Vol. expansion coeff. $\beta = \frac{1}{V}\frac{\partial V}{\partial T}\Big|_T$

$V = \frac{1}{\rho}$ $\quad \beta = \frac{1}{\rho}\frac{\partial(1/\rho)}{\partial T} = -\frac{1}{\rho}\frac{\partial \rho}{\partial t}$

9.00 $\rho - \rho_\infty = -\beta_\infty \rho_\infty (T - T_\infty)$

9.30 $\rightarrow \rho_\infty - \rho = \beta_\infty \rho_\infty (T - T_\infty)$

$\beta_\infty = -\frac{1}{\rho_\infty}\frac{\partial \rho}{\partial t}\Big|_{T_\infty} \rightarrow -\beta_\infty \rho_\infty = \frac{\partial \rho}{\partial t}\Big|_{T_\infty}$

10.30 $\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \mu\frac{\partial^2 v}{\partial x^2} + g\beta_\infty \rho_\infty (T - T_\infty)$ —— (2)

11.30 $Q = \text{Area} \times h_{0-1}\,\Delta T$ $\quad f^n$ of δ_T $\{h \sim k/\delta_T\}$

12.00 Heat eq^n :— $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left[\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}}\right]$ 0

1.00 $\rightarrow u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2}$ —— (3)

2.00 Boussinesq approximn :— considering all others const. and taking ρ as a f^n of T only

3.00

3.30 **Scaling analysis :—**

4.00 $x \sim \delta_T$ $\quad y \sim H$ $\quad u \sim u$ $\quad v \sim v_c$ (characteristic vel.)

4.30 Using (1) :— $\frac{u}{\delta_T} \sim \frac{v_c}{H} \rightarrow \boxed{u \sim \frac{v_c \delta_T}{H}}$ $\left\{\text{continuity eq}^n\ \frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}\right\}$

5.30 Energy bal. Eq^n Using (2). $\Big\} \rightarrow u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2} \Rightarrow u\frac{\Delta T}{\delta_T} \sim v\frac{\Delta T}{H} \sim \alpha\frac{\Delta T}{\delta_T^2}$

$\frac{v_c \cancel{\delta_T}}{H}\cdot\frac{\Delta T}{\cancel{\delta_T}} \Rightarrow \frac{v_c \Delta T}{H} \sim \frac{\alpha \Delta T}{\delta_T^2} \Rightarrow \boxed{v_c \sim \frac{\alpha H}{\delta_T^2}}$

m^2 bal. Eq^n :— Using (2) $\quad u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 v}{\partial x^2} + g\beta(T - T_\infty)$

$$\frac{u V_c}{\delta_T} \sim \frac{V_c \cdot V_c}{H} \sim \frac{\nu V_c}{\delta_T^2} \sim g\beta\Delta T$$

$$\Rightarrow \underbrace{\frac{V_c^2}{H}}_{\text{inertial term}} \sim \underbrace{\frac{\nu V_c}{\delta_T^2}}_{\text{viscous/friction term}} \sim \underbrace{g\beta\Delta T}_{\text{body force or gravitational force (buoyancy)}}$$

Buoyance is not negligible since without it there would be no flow

$$\frac{\text{Friction}}{\text{Buoyance}} = \frac{\frac{\nu V_c}{\delta_T^2}}{g\beta\Delta T} \Rightarrow \frac{\nu \alpha H}{(\delta_T)^4 g\beta\Delta T} = \frac{\nu\alpha}{g\beta\Delta T H^3}\left(\frac{H}{\delta_T}\right)^4 = Ra_H^{-1}\left(\frac{H}{\delta_T}\right)^4$$

$$\frac{V_c^2}{H} \sim Ra_H^{-1}\left(\frac{H}{\delta_T}\right)^4 \sim 1$$

$$\frac{\text{Inertia}}{\text{Buoyancy}} = \frac{\frac{V_c^2}{H}}{g\beta\Delta T} = \frac{\alpha^2 H^2}{g\beta\Delta T \delta_T^4 H} = \frac{\alpha^2}{g\beta\Delta T H^3}\left(\frac{H^4}{\delta_T^4}\right) = \left(\frac{\alpha\nu}{g\beta\Delta T H^3}\right)\left(\frac{\alpha}{\nu}\right)\left(\frac{H}{\delta_T}\right)^4$$

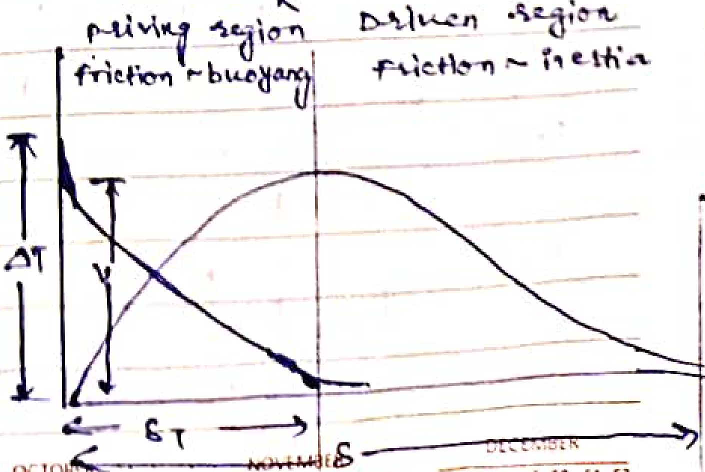
$$= \frac{1}{Ra_H}\frac{1}{Pr}\left(\frac{H}{\delta_T}\right)^4 = Ra_H^{-1} Pr^{-1}\left(\frac{H}{\delta_T}\right)^4$$

$$Ra_H^{-1} Pr^{-1}\left(\frac{H}{\delta_T}\right)^4 \sim Ra_H^{-1}\left(\frac{H}{\delta_T}\right)^4 \sim 1$$

High Pr. fluids :- $\{Pr >> 1\}$

$$\delta_T \sim H Ra_H^{-1/4} \qquad h \sim \frac{k}{\delta_T} \sim \frac{k}{H}(Ra_H)^{1/4} \qquad Nu = \frac{hH}{k} \sim Ra_H^{1/4}$$

$$\boxed{V_c \sim \frac{\alpha H}{\delta_T^2} \sim \frac{\alpha Ra_H^{1/2}}{H}}$$



$$\frac{V_c^2}{H} \sim \frac{\nu V_c}{\delta^2} \Rightarrow \frac{V_c}{H} \sim \frac{\nu}{\delta^2}$$

$$\Rightarrow \frac{\nu H}{V_c} \sim \delta^2$$

$$\Rightarrow \delta^2 \sim \frac{\nu H^2}{\alpha Ra_H^{1/2}}$$

$\Rightarrow \boxed{\delta \sim H\, Pr^{1/2}\, Ra_H^{-1/4}}$

$\dfrac{\delta_T}{\delta} \sim \dfrac{H\, Ra_H^{-1/4}}{H\, Pr^{1/2}\, Ra_H^{-1/4}} \rightarrow \dfrac{\delta_T}{\delta} \sim Pr^{-1/2}$ δ_T is smaller than δ

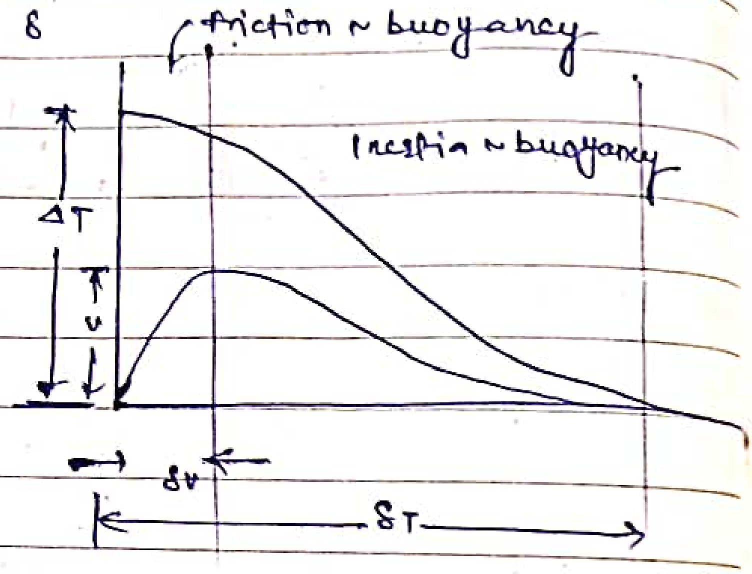
Low Pr fluids :- $Pr \ll 1$

$\delta_T \sim H (Ra_H\, Pr)^{-1/4}$

$V_c \sim \dfrac{\alpha}{H} (Ra_H\, Pr)^{1/2}$

$Nu = \dfrac{hH}{k} \sim (Ra_H\, Pr)^{1/4}$

$Bo_H = Ra_H\, Pr = \dfrac{g\beta \Delta T H^3}{\alpha^2}$



$\dfrac{\nu V_c}{\delta_v^2} \sim g\beta\Delta T \Rightarrow \delta_v^2 \sim \dfrac{\nu V_c}{g\beta\Delta T} \Rightarrow \delta_v^2 \sim \dfrac{\nu\alpha}{g\beta\Delta T H}(Ra_H Pr)^{1/2}$

$\rightarrow \delta_v^2 \sim \underbrace{\dfrac{\alpha\nu}{g\beta\Delta T H^3}}_{(Ra_H)} H^2 (Ra_H\, Pr)^{1/2} \rightarrow \delta_v^2 \sim H^2 \dfrac{Pr^{1/2}}{Ra_H^{1/2}}$

Gr_H = grashof No. $= \dfrac{Ra_H}{Pr} = \dfrac{g\beta\Delta T H^3}{\nu^2}$

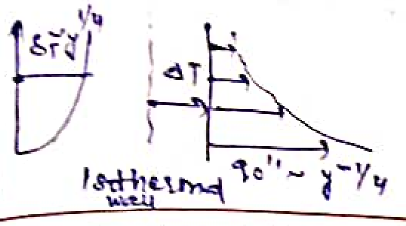
$\Rightarrow \delta_v^2 \sim H^2 Gr_H^{-1/2} \Rightarrow \delta_v \sim H\, Gr_H^{-1/4}$

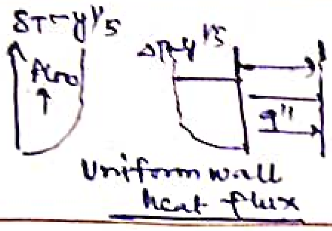
$\dfrac{\delta_v}{\delta_T} \sim \dfrac{H (Ra_H Pr)^{1/4}}{H} \left(\dfrac{Pr^{1/4}}{Ra_H^{1/4}}\right) \Rightarrow \dfrac{\delta_v}{\delta_T} \sim Pr^{1/2} < 1$

$Ra_H^{1/4} \sim \left(\dfrac{H}{\delta_T}\right)$ $Pr \gg 1$ $\Rightarrow$ measure of slenderness of BL region occupied by the buoyancy induced flow.

$Bo^{1/4} \sim \left(\dfrac{H}{\delta_T}\right)$ $Pr \ll 1$

$Gr_H^{1/4} \sim \left(\dfrac{H}{\delta_v}\right)$ $Pr \ll 1$





Uniform Wall heat flux:-

$T = f(y)$ $\quad q_0'' = \text{const.}$ $\quad q_0'' = h\Delta T$ $\quad h \sim \dfrac{k}{\delta_T}$ $\Rightarrow q_0'' \sim \dfrac{k}{\delta_T}(\Delta T)$

$$\boxed{\Delta T \sim \frac{q_0'' \delta_T}{k}}$$

High Pr :- ($Pr \gg 1$) $\qquad$ Low Pr :- ($Pr \ll 1$)

$$\delta_T \sim H\left(\frac{g\beta\Delta T H^3}{\alpha\nu}\right)^{-1/4} \qquad \boxed{\delta_T \sim H\left(Ra_H^* Pr\right)^{-1/5}}$$

$$\Rightarrow \delta_T^4 \sim H^4 \frac{\nu\alpha k}{g\beta q_0'' \delta_T H^3}$$

$$\Rightarrow \delta_T^5 \sim H^5\left[\frac{\nu\alpha k}{g\beta q_0'' H^4}\right]$$

$\dfrac{1}{Ra_H^*}$ → based on heat flux q''

$$\Rightarrow \delta_T \sim H\left(Ra_H^*\right)^{-1/5}$$

$$Nu \sim \frac{H}{\delta_T} \sim \left(Ra_H^*\right)^{1/5}$$

How to decide which one is dominating? Natural or forced?

Boussinesq approxim^n :- $u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} = \nu\dfrac{\partial^2 v}{\partial x^2} + g\beta_\infty(T - T_\infty)$

$u^* = \dfrac{u}{V_c}$; $v^* = \dfrac{v}{V_c}$; $x^* = \dfrac{x}{H}$; $y^* = \dfrac{y}{H}$

$\theta = \dfrac{T - T_\infty}{T_0 - T_\infty}$; $u = u^* V_c$; $v = v^* V_c$; $x = x^* H$; $y = y^* H$

$$\frac{V_c^2}{H} u^* \frac{\partial v^*}{\partial x^*} + \frac{V_c^2}{H} v^* \frac{\partial v^*}{\partial y^*} = \frac{\nu V_c}{H^2}\frac{\partial^2 v_c^*}{\partial x^{*2}} + g\beta_\infty \theta (T_0 - T_\infty)$$

$$\Rightarrow u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*} = \frac{\nu}{V_c H}\frac{\partial^2 v_c^*}{\partial x^{*2}} + \underbrace{\frac{H}{V_c^2} g\beta_\infty\theta(T_0 - T_\infty)}_{\text{Effect of Buoyancy dictates the presence of Natural convection}}$$

const. coeff of θ :- $\frac{H^2 \nu^2 g\beta_\infty (T_0 - T_\infty) H^3}{H^2 \cdot H^3 \, V_c^2 \; \nu^2} = \frac{Gr_H}{Re^2} \sim Ri$ (Richardson no.)

$Ri >> 1 \rightarrow$ Natural conv. $\quad Ri \sim 1$ mixed conv.

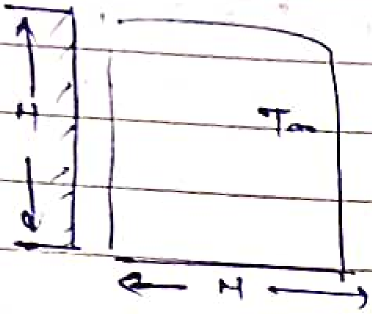
$Ri << 1 \rightarrow$ forced conv.

$(\delta_T)_{NC} \sim y\, Ra_y^{-1/4} \; (Pr > 1) \quad (\delta_T)_{FC} \sim y\, Re_y^{-1/2} Pr^{-1/3} \; (Pr > 1)$

$(\delta_T)_{NC} < (\delta_T)_{FC}$ Natural conv.

$(\delta_T)_{NC} > (\delta_T)_{FC}$ forced conv.

Problem 1 :- $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial x^2} + g\beta\Delta T$



$x \sim H \quad y \sim H \quad u \sim u \quad v \sim u_c$

$\frac{V_c^2}{H} \sim \frac{u V_c}{H} \sim \frac{\nu V_c}{H^2} \sim g\beta\Delta T$

$\frac{u}{H} \sim \frac{V_c}{H} \rightarrow \frac{V_c^2}{H} \sim \frac{\nu V_c}{H^2} \sim g\beta\Delta T$

fr. ~ Buoyancy

$\frac{\nu V_c}{H^2} \sim g\beta\Delta T \rightarrow V_c \sim \frac{g\beta\Delta T H^2}{\nu} \rightarrow \theta \sim \frac{g\beta\Delta T H^3}{\nu^2}$

inertia ~ friction

$\frac{V_c^{\cancel{2}}}{\cancel{H}} \sim \frac{\nu \cancel{V_c}}{H^{\cancel{2}}} \rightarrow V_c \sim \frac{\nu}{H} \rightarrow \frac{g\beta\Delta T H^3}{(\nu\alpha)}\left(\frac{\alpha}{\nu}\right) \sim 1$

$\rightarrow \frac{Ra_H}{Pr} \sim 1 \rightarrow Gr_H \sim 1$

cond^n ~ convec^n

$\sim \frac{\alpha\Delta T}{H^2} \sim \frac{V_c \Delta T}{H} \rightarrow V_c \sim \frac{\alpha}{H}$

Problem 2 :- $\frac{t_1}{t_2}$ ratio to be calculated.

$Pr >> 1$

$\left(\frac{g\beta}{\alpha\nu}\right)_{air} = 125 \left(\frac{g\beta}{\alpha\nu}\right)_{cd} = 4910$

k_{air} 0.025 $\quad k_{cd} =$ 0.58

$H = 5L \quad Nu_{vertical} = \frac{Hh}{k} \sim Ra_H^{1/4} \quad Nu_{horizontal} = \frac{Lh}{k} \sim Ra_L^{1/4}$

$[(T_{cd} - T_s) = (T_s - T_{air})]$

$\dot{q} = mc_p \frac{dT}{dt} = m_{cd} c_{cd} \frac{dT}{dt}$ (rate of cooling down)

$q = hA(T_{cd} - T_{air})$ (heat dissipated in air)

8.00 $hA(T_{cd} - T_{air}) = (mc)_{cd} \frac{dT}{dt} \rightarrow t \sim \frac{(mc)_{cd}}{hA}$.

8.30

9.00 $\Rightarrow \frac{t_1}{t_2} \approx \frac{h_2}{h_1}$ $\boxed{\frac{1}{h} = \frac{1}{h_{cd}} + \frac{1}{h_{air}}}$ → overall heat transfer coeff.

9.30

10.00 where, $h_{cd} \sim \frac{1}{H}(kRa_H^{1/4})_{cd}$ & $h_{air} \sim \frac{1}{H}(kRa_H^{1/4})_{air}$

10.30

11.00 $\frac{h_{cd}}{h_{air}} \sim \frac{(k(Ra_H)^{1/4})_{cd}}{(k(Ra_H)^{1/4})_{air}} = \frac{\left[k\left(\frac{g\beta\Delta T H^3}{\alpha\nu}\right)^{1/4}\right]_{cd}}{\left[k\left(\frac{g\beta\Delta T H^3}{\alpha\nu}\right)^{1/4}\right]_{air}} = \frac{\left[k\left(\frac{g\beta}{\alpha\nu}\right)^{1/4}\right]_{cd}}{\left[k\left(\frac{g\beta}{\alpha\nu}\right)^{1/4}\right]_{air}}$

11.30

12.00

12.30 $\frac{h_{cd}}{h_{air}} = \frac{(kC^{1/4})_{cd}}{(kC^{1/4})_{air}} = \frac{0.58\,(4910)^{1/4}}{0.025\,(125)^{1/4}} = 58.1$

1.00

$\{C = g\beta/\alpha\nu\}$ $h_{cd} \gg h_{air}$

$h \sim h_{air}$ side $\sim \frac{1}{H}(kRa_H^{1/4})_{air} = (\text{const})\, H^{-1/4}$

$\frac{t_1}{t_2} = \frac{h_2}{h_1} = \left(\frac{H_1}{H_2}\right)^{1/4} = 5^{1/4} \approx 1.5$

cooling the cold drink by placing it vertically will take 50% more time compared to horizontal pos.

Problem 3 :-

NS :- $\cancel{u\frac{\partial v}{\partial x}} + \cancel{v\frac{\partial v}{\partial y}} = \cancel{-\frac{\partial p}{\partial y}} + \nu\frac{\partial^2 v}{\partial x^2} + g\beta_\infty(T - T_\infty)$

FDF

$\nu\frac{\partial^2 v}{\partial x^2} + g\beta_\infty(T - T_\infty) = 0$

$(T_0 - T_\infty) \gg T - T_\infty$ → max. temp. diff. possible is significantly high

$$\frac{\partial v}{\partial x} = -\frac{g\beta_\infty \Delta T}{\nu} x + c_1 = -K_1 x + c_1$$

$$v(y) = -\frac{K_1 x^2}{2} + c_1 x + c_2$$

BCs :- at $x = 0$, $\frac{\partial v}{\partial x} = 0$ (origin) $\to c_1 = 0$

at $x = \frac{D}{2}$, $v = 0 \to c_2 = -\frac{K_1 D^2}{8}$

$$v(y) = \frac{g\beta_\infty (T_0 - T_\infty) D^2}{8\nu}\left[1 - \left\{\frac{x}{(D/2)}\right\}^2\right]$$

$\dot{m}$ = discharge rate $= \rho \times (2 \times W \times h) \times \int_0^{D/2} v(x)\,dx$

$$= \frac{2\rho W g\beta_\infty (T_0 - T_\infty) D^2}{8\nu}\left[x - \frac{x^3}{3(D/2)^2}\right]_0^{D/2}$$

$$= \frac{\cancel{2}\rho W g\beta_\infty (T_0 - T_\infty) D^2}{\cancel{8}_4 \nu}\left[\frac{D}{2} - \frac{D}{6}\right]$$

$$= \frac{\rho W g\beta_\infty (T_0 - T_\infty)}{\nu} \frac{\cancel{2}D^3}{2\cancel{4} \times 6}$$

$$\Rightarrow \frac{\rho W g\beta_\infty (T_0 - T_\infty) D^3}{12\nu}$$

$\dot{m} C_p (T_0 - T_\infty) = q''_{0-H}$ = Total heat transfer

$$\Rightarrow q''_{0-H} = \frac{g\beta_\infty \rho \cancel{W} D^3 C_p (T_0 - T_\infty)^2}{12\nu} \times \frac{1}{2 \times H \times \cancel{W}} = \frac{g\beta_\infty \rho D^3 C_p (T_0 - T_\infty)}{24\nu H}$$

$$\frac{q}{D \times H \times W}$$

$$Nu = \frac{Ra_D}{24} = \frac{1}{24}\left(\frac{g\beta \Delta T D^3}{\nu \alpha}\right)$$

$$Nu = \frac{h_{0-H} \times H}{k} = \frac{Ra_D}{24}$$

$$\frac{q''_0}{\Delta T}$$

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Case 1 :- $Pr >> 1$ $\quad y_T$ = Transition length $\quad y_T << H$ For PDF

$\delta_T \sim H\, Ra_H^{-1/4}$

$D/2 \sim y_T\, Ra_{y_T}^{-1/4}$

$\cancel{\Rightarrow \left(\frac{y_T}{D}\right)^4 \sim \left(\frac{1}{2} Ra_{y_T}^{1/4}\right)^4}$

$\cancel{\Rightarrow y_T\, Ra_y}$

8.00 $\rightarrow \left(\frac{y_T}{D}\right) \sim \frac{Ra_{y_T}^{-1/4}}{2} \rightarrow \left(\frac{y_T}{D}\right)^4 \sim \frac{1}{2^4} Ra_{y_T}$.

8.30

9.00 $\frac{Ra_D}{Ra_{y_T}} \sim \frac{D^3}{y_T^3} \rightarrow Ra_{y_T} = \left(\frac{y_T}{D}\right)^3 Ra_D$

9.30

10.00 $\cancel{\left(\frac{y_T}{D}\right)^4 \sim \frac{1}{2^4} Ra_{y_T} \Rightarrow Ra_{y_T} \sim \left(\frac{y_T}{D}\right)^4 \cdot 2^4 \rightarrow \frac{y_T}{D}}$

10.30

11.00 $\left(\frac{y_T}{D}\right)^4 \sim \frac{1}{2^4}\left(\frac{y_T}{D}\right)^3 Ra_D \rightarrow \frac{y_T}{D} \sim \frac{1}{2^4} Ra_D$

11.30

12.00 $\frac{D \times Ra_D}{2^4} << H \rightarrow y_T << H \rightarrow Ra_D << 2^4\left(\frac{H}{D}\right)$

12.30

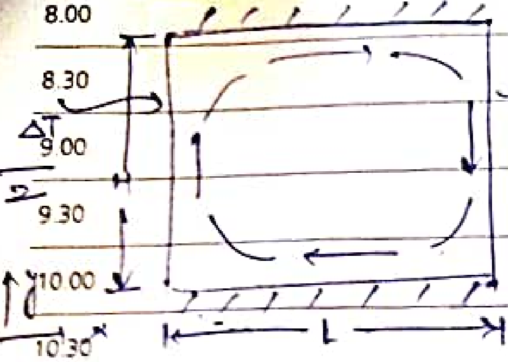
1.00 $Ra_D^{1/4} << 2\left(\frac{H}{D}\right)^{1/4}$

1.30

2.00 **Case 2 :-** $Pr << 1 \rightarrow Bo_D^{1/4} << 2\left(\frac{H}{D}\right)^{1/4}$.

2.30

Internal natural conv.



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\begin{cases} \rho = \rho_0 - \beta\rho_0 (T - T_0) \\ \quad = \rho_0[1 - \beta(T - T_0)] \end{cases}$$

$$(2) - \frac{\partial}{\partial y}\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\right]$$

$$(3) - \frac{\partial}{\partial x}\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - g[1 - \beta(T - T_0)]\right]$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \quad (4)$$

(2) − (3) :–

$$\frac{\partial}{\partial x}\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] - \frac{\partial}{\partial y}\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right]$$

$$= \nu\left[\frac{\partial}{\partial x}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\partial}{\partial y}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\right] + g\beta\frac{\partial T}{\partial x}$$

Initially :– $y \sim H \quad x \sim \delta_T \quad t \sim t \quad u \sim u \quad v \sim v_c$

at time $t = 0^+$, $v \to 0$

$$\left(\frac{\partial^2 T}{\partial x^2} >> \frac{\partial^2 T}{\partial y^2}\right)$$

Energy bal.

$$\sim \frac{\Delta T}{t} \sim \alpha\frac{\Delta T}{\delta_T^2} \Rightarrow \delta_T^2 \sim \alpha t$$

$$\Rightarrow \delta_T \sim (\alpha t)^{1/2}$$

Thermal inertia cond^n.

$$\boxed{\frac{u}{\delta_T} \sim \frac{v_c}{H} \Rightarrow u \sim \frac{v_c \delta_T}{H}}$$

L.H.S :–

$$\frac{1}{\delta_T}\left[\sim\frac{v_c}{t} \quad \sim\frac{u v_c}{\delta_T} \quad \sim\frac{v_c^2}{H}\right] - \frac{1}{H}\left[\sim\frac{u}{t} \quad \sim\frac{u^2}{\delta_T} \quad \sim\frac{u v_c}{H}\right]$$

$$= \left[\sim\frac{v_c}{\delta_T t} \quad \underbrace{\sim\frac{v_c^2}{H} \quad \sim\frac{v_c^2}{H}}_{\text{neglect smaller than } \frac{v_c}{\delta_T t}}\right] - \underbrace{\left[\sim\frac{v_c\delta_T}{H^2 t} \quad \sim\frac{v_c^2\delta_T}{H^2} \quad \sim\frac{v_c^2\delta_T}{H^2}\right]}_{\text{neglect b/c's smaller than } \frac{v_c}{\delta_T t}}$$

$$\Rightarrow \frac{v_c}{\delta_t t}\left[\sim 1 \quad \sim \frac{v_c t}{H} \quad \sim \frac{v_c t}{H}\right]$$

Inertia $\sim \frac{v_c}{\delta_t t}\left[\frac{\partial^2 v}{\partial x \partial t}\right]$

R.H.S :- $\sim \frac{1}{\delta_T}\left[\frac{\nu v_c}{\delta_T^2} \quad \frac{\nu v_c}{H^2}\right] \sim \frac{1}{H}\left[\sim \frac{\nu}{\delta_T^2} \quad \sim \frac{\nu}{H^2}\right] \sim \frac{g\beta\Delta T}{\delta_T}$

$$\Rightarrow \sim \left[\frac{\nu v_c}{\delta_T^3} \quad \frac{\nu v_c}{H^2 \delta_T}\right] \sim \left[\frac{v_c}{\delta_T H^2} \quad \sim \frac{v_c \delta_T}{H^3}\right] \sim \underbrace{\frac{g\beta\Delta T}{\delta_T}}$$

Buoyancy can't be neglected.

dominating term

viscous $\sim \frac{\nu v_c}{\delta_T^3} \sim \left[\nu \frac{\partial^3 v}{\partial x^3}\right]$ Buoyance :- $\frac{g\beta\Delta T}{\delta_T}$

Ines $\sim \frac{v_c}{\delta_t t} \sim \frac{\nu v_c}{\delta_T^3} \sim \frac{g\beta\Delta T}{\delta_T} \rightarrow \frac{\delta_T^2}{\nu t} \sim 1 \sim \frac{g\beta\Delta T \delta_T^2}{\nu v_c}$

for $Pr >> 1$:- Friction ~ Buoyancy

$1 \sim \frac{g\beta\Delta T \delta_T^2}{\nu v_c} \rightarrow v_c \sim \frac{g\beta\Delta T \delta_T^2}{\nu}$ [initial vel. scale]

energy Eqⁿ :- $\frac{\Delta T}{t} \sim \frac{v_c \Delta T}{H} \sim \frac{\alpha \Delta T}{\delta_T^2}$

As t ↑ convecⁿ. $\frac{\alpha\Delta T}{\delta_{T_f}^2} \sim v_c \frac{\Delta T}{H} \Rightarrow v_c \sim \frac{\alpha H}{\delta_{T_f}^2}$

condⁿ ~ convecⁿ

$\Rightarrow \cancel{t_f \sim \alpha H} \quad \frac{g\beta\Delta T (\delta_{T_f})^2}{\nu} \sim \frac{\alpha H}{(\delta_{T_f})^2}$

$\rightarrow (\delta_{T_f})^4 \sim \left[\frac{\alpha \nu}{g\beta\Delta T H^3}\right] H^4 \rightarrow \delta_{T_f} \sim H Ra_H^{-1/4}$

$t_f \sim \frac{\delta_T^2}{\alpha} \rightarrow t_f \sim \left(\frac{\nu v_c}{g\beta\Delta T \alpha}\right)^{1/2}$

JULY

Week	27	28	29	30	31
Mon	1	8	15	22	29
Tue	2	9	16	23	30
Wed	3	10	17	24	31
Thu	4	11	18	25	
Fri	5	12	19	26	
Sat	6	13	20	27	
Sun	7	14	21	28	

AUGUST

Week	31	32	33	34	35
Mon		5	12	19	26
Tue		6	13	20	27
Wed		7	14	21	28
Thu	1	8	15	22	29
Fri	2	9	16	23	30
Sat	3	10	17	24	31
Sun	4	11	18	25	

SEPTEMBER

Week	35	36	37	38	39
Mon	30	2	9	16	23
Tue		3	10	17	24
Wed		4	11	18	25
Thu		5	12	19	26
Fri		6	13	20	27
Sat		7	14	21	28
Sun	1	8	15	22	29

OCTOBER

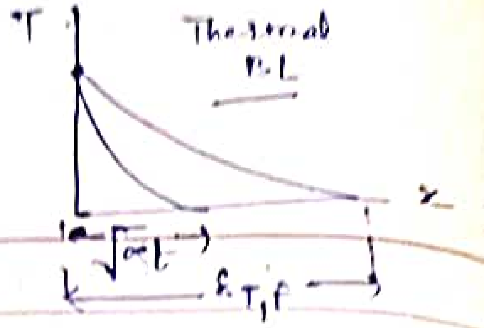
Week	40	41	42	43	44
Mon		7	14	21	28
Tue	1	8	15	22	29
Wed	2	9	16	23	30
Thu	3	10	17	24	31
Fri	4	11	18	25	
Sat	5	12	19	26	
Sun	6	13	20	27	

NOVEMBER

Week	44	45	46	47	48
Mon		4	11	18	25
Tue		5	12	19	26
Wed		6	13	20	27
Thu		7	14	21	28
Fri	1	8	15	22	29
Sat	2	9	16	23	30
Sun	3	10	17	24	

DECEMBER

Week	48	49	50	51	52
Mon	30	2	9	16	23
Tue	31	3	10	17	24
Wed		4	11	18	25
Thu		5	12	19	26
Fri		6	13	20	27
Sat		7	14	21	28
Sun	1	8	15	22	29



8.00 $\boxed{\delta_{T,f} \sim (\alpha t_f)^{1/2} \sim H Ra_H^{-1/4}}$

8.30

9.00 Sidewalls :- ~ inertia ~ Buoyancy friction

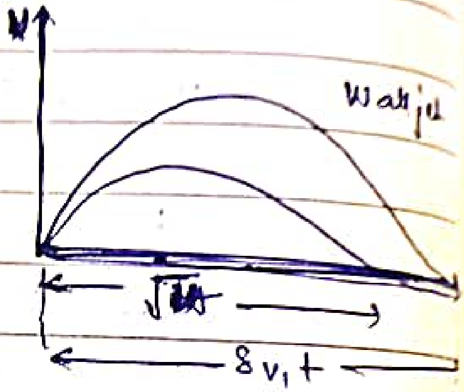
9.30 $\frac{v_c}{\delta_T t} \sim \frac{\nu v_c}{\delta_v^3} \Rightarrow \delta_v^2 \sim \nu t \rightarrow \delta_v \sim (\nu t)^{1/2}$

10.00 $\rightarrow \delta_v^2 \sim \alpha\left(\frac{\nu}{\alpha}\right) t \sim \alpha Pr\, t$

10.30 $\Rightarrow \delta_v^2 \sim Pr\, \delta_T^2$

11.00 $\Rightarrow \left(\frac{\delta_v}{\delta_T}\right) \sim Pr^{1/2}$

11.30



12.00 for distinct vel. BL. :- $\delta_{v,L} < L$

12.30 $\frac{H}{L} < Ra_H^{1/4} Pr^{-1/2}$

1.00

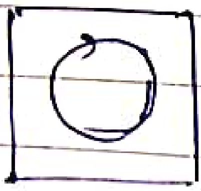
1.30 for distinct Thermal layers $\rightarrow \frac{H}{L} < Ra_H^{1/4}$ $[\delta_{T,f} < L]$

2.00

2.30 regimes :-

3.00

3.30 regime 1



diffusing condn dominating over convecn

4.00

4.30 $$\underbrace{u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}}_{\text{convec}^n} = \alpha\underbrace{\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)}_{\text{conduc}^n}$$

convecn << conducn

5.00 $x \sim H \quad y \sim H$

5.30 $\frac{v_c \Delta T}{H} << \alpha \frac{\Delta T}{H^2} \rightarrow v_c << \frac{\alpha}{H}$

6.00

$Pr > 1$:- ~ friction ~ $\frac{\nu v_c}{H^3}$ Buoyancy ~ $\frac{g\beta\Delta T}{H}$

$\frac{\nu v_c}{H^3} \sim \frac{g\beta\Delta T}{H} \rightarrow v_c \sim \frac{g\beta\Delta T H^2}{\nu}$

$\frac{\alpha}{H} >> \frac{g\beta\Delta T H^2}{\nu}$

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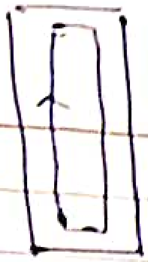
→ $\frac{g\beta \Delta T H^3}{\alpha \nu} << 1 \rightarrow Ra_H^{-1} << 1 \rightarrow Ra_H >> 1$.

regime 2 :-



$\delta_{T,F} < L$

$H Ra_H^{-1/4} < 1$

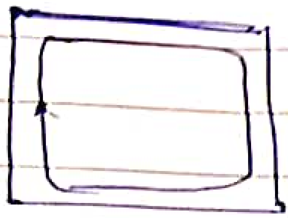
$\frac{H}{L} < Ra_H^{1/4}$

$\delta_{v,t} <$

$H Ra_H^{-1/4} Pr^{1/2}$

$\frac{H}{L} < Pr^{-1/2} Ra_H^{1/4}$

regime 3 :-



$\delta_{T,F} \sim H Ra_H^{-1/4}$

$\rightarrow \frac{\delta_{T,F}}{H} \sim Ra_H^{-1/4}$

significantly high rayleigh no.

regime 4 :-

Counter current heat exchanger
Upper & lower layers of fluid will have different T.

$q''_{conv,\,0-H} \sim \dot{m} c_p \Delta T \sim (v_c \rho \delta_{T,F}) c_p \Delta T$

$v_c \sim \frac{\alpha H}{\delta_{TF}^2} \rightarrow \dot{q}_{conv} \sim \frac{\alpha H}{H Ra_H^{-1/4}} \left(\frac{\rho c_p}{k}\right) \Delta T k^{1/2}$

$\rightarrow \dot{q}_{conv} \sim k \Delta T Ra_H^{1/4}$

$\dot{q}_{cond.} \sim \frac{k \Delta T}{H} \cdot L \rightarrow \dot{q}_c$

$\boxed{\dot{q}_c << \dot{q}_{conv}}$

$\frac{k \Delta T L}{H} << k \Delta T Ra_H^{1/4} \rightarrow \frac{L}{H} << Ra_H^{1/4}$

$\rightarrow \frac{H}{L} >> Ra_H^{1/4}$.

Problem 1

{continuity eqn} — ①

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\underbrace{u\frac{\partial \xi}{\partial x}+v\frac{\partial \xi}{\partial y}}_{\text{inertia}}=\nu\underbrace{\left(\frac{\partial^2 \xi}{\partial x^2}+\frac{\partial^2 \xi}{\partial y^2}\right)}_{\text{friction}}+\underbrace{g\beta\frac{\partial T}{\partial x}}_{\text{buoyancy}}$$ where $\xi=\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}$

$u=-\frac{\partial \psi}{\partial y}$ $\quad v=\frac{\partial \psi}{\partial x}$ $\quad \psi$: stream fn.

$u=-\frac{\partial \psi}{\partial y}\to\frac{\partial u}{\partial y}=-\frac{\partial^2\psi}{\partial y^2}$

$v=\frac{\partial \psi}{\partial x}\to\frac{\partial v}{\partial x}=\frac{\partial^2\psi}{\partial x^2}$

$\xi=-\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}$ $\quad \frac{\partial \xi}{\partial x}=-\frac{\partial^2 u}{\partial y\partial x}+\frac{\partial^2 v}{\partial x^2}$

$\frac{\partial \xi}{\partial y}=\frac{\partial^2 v}{\partial x\partial y}-\frac{\partial^2 u}{\partial y^2}$

LHS $u\frac{\partial \xi}{\partial x}+v\frac{\partial \xi}{\partial y}\Rightarrow$

$$u\left(\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 u}{\partial y\partial x}\right)+v\left(\frac{\partial^2 v}{\partial x\partial y}-\frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial}{\partial x}\left[u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]-\frac{\partial}{\partial y}\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]$$

RHS : $\nu\left[\frac{\partial^2\xi}{\partial x^2}+\frac{\partial^2\xi}{\partial y^2}\right]$

$$=\nu\left[\frac{\partial^3 v}{\partial x^3}-\frac{\partial^3 u}{\partial y\partial x^2}+\frac{\partial^3 v}{\partial x\partial y^2}-\frac{\partial^3 u}{\partial y^3}\right]=\nu\left[\frac{\partial}{\partial x}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\partial}{\partial y}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\right]$$

$x\sim L_x$ $\quad u\sim\frac{\psi}{H}$ $\quad v\sim\frac{\psi}{L_x}$ $\quad y\sim H$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}\quad (L_x\gg H)\quad — ③$$

① $\frac{\psi/H}{L_x}$ $\quad \frac{\psi/L_x}{H}$ $\quad \frac{u}{L_x}\sim\frac{v}{H}\Rightarrow v\sim\frac{uH}{L_x}$

② $u\frac{\Delta T}{L_x}\sim v\frac{\Delta T}{H}\sim\alpha\frac{\Delta T}{H^2}\to u\frac{\Delta T}{L_x}\sim\frac{\alpha\Delta T}{H^2}\to u\sim\frac{\alpha L_x}{H^2}$

$\Rightarrow$ $\boxed{\psi\sim\frac{\alpha L_x}{H}}$

$Pr >> 1$:- friction ~ Buoyancy.

$\nu \frac{\partial^2 \xi}{\partial y^2} \sim g\beta \frac{\partial T}{\partial x} \rightarrow \frac{\nu \xi}{H^2} \sim \frac{g\beta \Delta T}{L_x}$

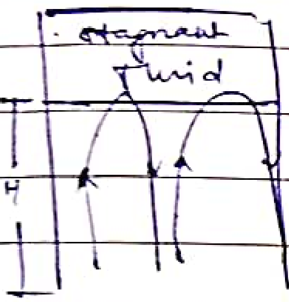
$\rightarrow \frac{\nu \psi}{H^4} \sim \frac{g\beta \Delta T}{L_x}$

$\rightarrow \frac{\alpha L_x \nu}{H^5} \sim \frac{g\beta \Delta T}{L_x} \rightarrow L_x^2 \sim \frac{g\beta \Delta T H^5}{\alpha \nu}$

$\rightarrow L_x \sim Ra_H^{1/2} H$

Problem 2 :- $H >> L$:- $x \sim L$ $y \sim L_y$ $u \sim \frac{\psi}{L_y}$ $v \sim \frac{\psi}{L}$

$H >> L$ $L_y >> L$:- $\xi \sim \frac{\psi}{x^2} \sim \frac{\psi}{L^2}$



$\sim \frac{u \Delta T}{L} \sim \frac{v \Delta T}{L_y} \sim \frac{\alpha \Delta T}{L^2}$

continuity

$\frac{u}{L} \sim \frac{v}{L_y}$

$u \sim \frac{vL}{L_y}$

$\frac{v \Delta T}{L_y} \sim \frac{\alpha \Delta T}{L^2} \rightarrow \frac{\psi}{\cancel{L} L_y} \sim \frac{\alpha}{L^{\cancel{2}}}$

$\rightarrow \boxed{\psi \sim \frac{\alpha L_y}{L}}$

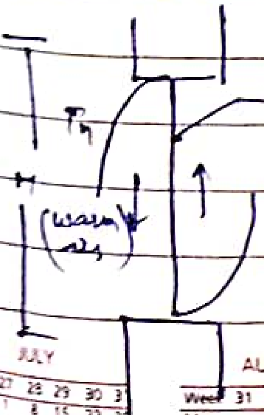
~ friction ~ buoyancy

$\sim \nu \frac{\partial^2 \xi}{\partial x^2} \sim g\beta \frac{\partial T}{\partial x} \rightarrow \frac{\nu \xi}{L_y^2}$ $\frac{\nu \xi}{L^{\cancel{2}}} \sim \frac{g\beta \Delta T}{\cancel{L}} \rightarrow \xi \sim \frac{g\beta \Delta T L}{\nu}$

$\rightarrow \frac{\psi}{L^2} \sim \frac{g\beta \Delta T L}{\nu} \rightarrow \psi \sim \frac{g\beta \Delta T L^3}{\nu \alpha} \cdot \alpha$

$\rightarrow \frac{\cancel{\alpha} L_y}{L} \sim \frac{g\beta \Delta T L^3 \cancel{\alpha}}{\nu \alpha} \rightarrow L_y \sim Ra_H L$

Problem 3 :-



glass sheet

T_w

T_c (cold air)

$Nu \sim 0.517 Ra_H^{1/4}$

$q_0'' = \cancel{Nu} \, h \Delta T$

$q_0'' = \overline{Nu}_L \frac{k \Delta T}{l}$

$q_0'' = 0.517 Ra_H^{1/4} \frac{k (T_h - T_w)}{H}$

$T_h - T_w = \frac{1}{2}(T_h - T_c)$

$q_0'' = 0.517 \left[\frac{g\beta (T_h - T_c) H^3}{2 \alpha \nu}\right]^{1/4} \frac{k}{H} \frac{(T_h - T_c)}{2}$