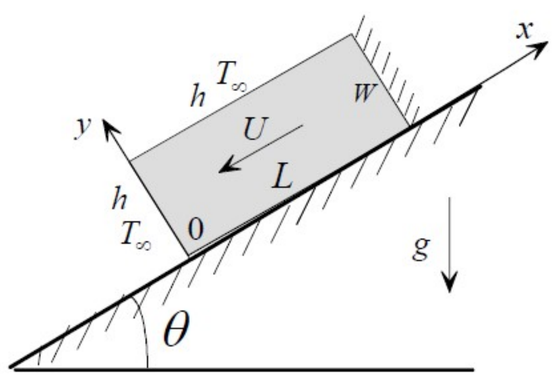
**Indian Institute of Technology Kharagpur**

**Department of Chemical Engineering**

**Advanced Heat Transfer CH61014 (Spring 2024)**

**Assignment 1**

1. A rectangular plate of length $L$ and height $H$ slides down an inclined surface with a velocity $U$. Sliding friction result in surface heat flux $q''$. The front and top sides of the plate exchange heat by convection. The heat transfer coefficient is h and the ambient temperature is $T_\infty$ Neglect heat loss from the back side and assume that no frictional heat is conducted through the inclined surface. Write the two-dimensional steady state heat equation and boundary conditions.




2. A slab, which extends from $x = -L$ to $x = L$, is initially at a uniform temperature $T_0$. For times $t \geq 0$, internal energy is generated in the slab at an exponential decay rate per unit volume according to

$$\dot{q} = \dot{q}_o e^{-\beta t}$$

where $\dot{q}_o$ and $\beta$ are two given positive constants, while the surfaces at $x = +L$ are kept at the initial temperature $T_0$. Assuming constant thermos-physical properties, formulate the problem for the unsteady-state temperature distribution $T(x, t)$ in the slab for times $t > 0$.

3. A sphere (radius $\boldsymbol{r_0}$) of homogeneous material is initially at a uniform temperature $\boldsymbol{T_i}$**.** At time $\boldsymbol{t = 0}$, the sphere is immersed in a stream of moving fluid at some different temperature $\boldsymbol{T_\infty}$. The external surface of the sphere exchanges heat by convection and the heat transfer coefficient is $h$. Assume that the heat flux on the surface is uniform. Formulate one-dimensional transient heat conduction problem in dimensionless form.

4. Derive the non-Fourier heat conduction equation by considering the dual phase lagging constitutive relation between heat flux and temperature gradient instead of using classical Fourier's law.

   **Note:** You can start with the following relation for energy balance:

$$-\nabla \cdot \mathbf{q}'' + \dot{q} = \rho c \frac{\partial T}{\partial t}$$

5. Consider two-dimensional ($T$(r, $z$)) steady state heat conduction in a solid cylinder of radius R and length L. At $z = 0$, the cylinder is insulated and at $z = L$ the cylinder receives uniform flux $q$". The cylindrical surface is maintained at a constant temperature $T_0$. The thermal conductivity varies with temperature according to $k = k_0(1 + \beta_1 T + \beta_2 T^2)$.

   (i) Formulate the steady state heat conduction problem (write energy balance equation and Boundary Conditions).

   (ii) Use Kirchhoff transformation method to convert the nonlinear problem to linear problem.

   (iii) Solve the problem in (ii) for steady-state temperature distribution $T(r, z)$.

6. A 2-D rectangular region $0 \le x \le a$, $0 \le y \le b$ is initially at a uniform temperature $T_0$. For times $t > 0$, the boundaries at $x = 0$ and $y = 0$ are kept at zero temperature and the boundaries at $x = a$ and $y = b$ dissipate heat by convection into an environment at zero temperature. The heat transfer coefficients $h$ are the same for both of these boundaries. Using Separation of Variables, obtain an expression for the temperature distribution $T(x, y, t)$.

7. Consider a semi-circular section of a tube with inside radius $R_i$ and outside radius $R_o$. Heat is exchanged by convection along the inside and outside cylindrical surfaces. The inside temperature and heat transfer coefficient are $T_i$ and $h_i$, respectively. The outside temperature and heat transfer coefficient are $T_o$ and $h_o$, respectively. The two plane surfaces are maintained at uniform temperature $T_b$.

   a) Write the two dimensional steady-state heat equation and boundary conditions.

   b) Solve the steady-state heat conduction problem using Separation of Variables.