Debasis Sarkar

Indian Institute of Technology Kharagpur

Department of Chemical Engineering

Advanced Heat Transfer CH61014 (Spring 2024)

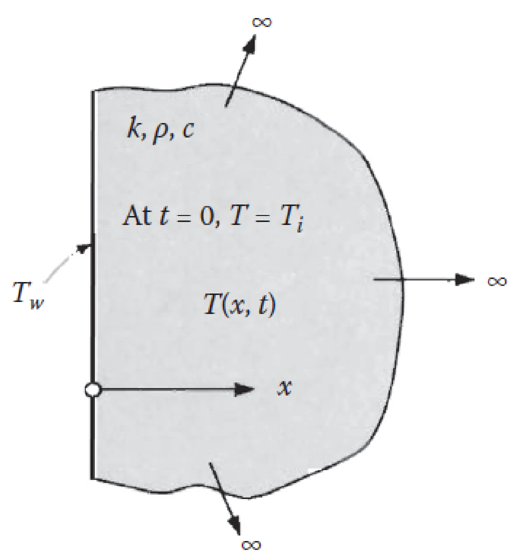
Assignment 2

1. A slab, extending from $x = 0$ to $x = L$ and of infinite extent in the y and z directions, is initially at a uniform temperature T_i. For times $t \geq 0$, a constant heat flux q'', is applied to the surface at $x = L$, while the surface at $x = 0$ is kept perfectly insulated. Assume that the thermo-physical properties of the slab are constant. Obtain an expression for the unsteady-state temperature distribution $T(x, t)$ in the slab for $t > 0$ using
   a) Separation of Variables
   b) Integral Transform Technique

2. A long solid cylinder of constant thermo-physical properties and radius r_0 is initially at a uniform temperature T_i. For times $t \geq 0$, a constant heat flux q'' is applied to the peripheral surface at $r = r_0$. Obtain an expression for the unsteady-state temperature distribution $T(r, t)$ in the cylinder for $t > 0$ using
   a) Separation of Variables
   b) Integral Transform Technique

3. A solid sphere, $0 \leq r \leq r_0$, of constant thermo-physical properties is initially at a uniform temperature T_i. For times $t \geq 0$, the sphere is heated by applying a constant heat flux q'' to its surface at $r = r_0$. Obtain an expression for the unsteady-state temperature distribution $T(r, t)$ in the sphere for $t > 0$ using
   a) Separation of Variables
   b) Integral Transform Technique

4. Consider a semi-infinite solid, $0 \le x < \infty$, initially at a uniform temperature T_i. The surface temperature at $x = 0$ is changed to and kept at a constant temperature T_w for times $t \ge 0$. Assume constant thermo-physical properties (k, ρ, c). Obtain an expression for the unsteady-state temperature distribution $T(x, t)$ in the slab using
    a) Finite Fourier Transform
    b) Similarity Method



5. A slab of thickness L is initially at zero temperature. For times $t > 0$, the boundary surface at $x = 0$ is kept at zero temperature, while the surface at $x = L$ is subjected to a time-varying temperature $f(t)$ defined by

$$f(t) = \begin{cases} bt & \text{for} \quad 0 < t < \tau_1 \\ 0 & \text{for} \quad t > \tau_1 \end{cases}$$

Assume constant thermo-physical properties (k, ρ, c). Obtain an expression for the unsteady-state temperature distribution $T(x, t)$ in the slab using Duhamel's Method.

6. A slab of thickness L is initially at zero temperature. For times $t > 0$, the boundary surface at $x = L$ is kept insulated, while the surface at $x = 0$ is subjected to a time-dependent heat flux $f(t)$ of the functional form:

$$-k \left.\frac{\partial T}{\partial x}\right|_{x=0} = f(t) \equiv \begin{cases} t & \text{for} \quad 0 < t < \tau_1 \\ 0 & \text{for} \quad t > \tau_1 \end{cases}$$

Assume constant thermo-physical properties (k, ρ, c). Using Duhamel's theorem, develop an expression for the temperature distribution $T(x, t)$ in the slab for times:
(i) $t < \tau_1$ and for (ii) $t > \tau_1$.

7. Consider a large solid, $x \geq 0$, initially at the fusion temperature T_f. At $t = 0$, the temperature of the boundary surface at $x = 0$ is raised to T_0 ($> T_f$) and maintained at that constant temperature for times $t > 0$. Assuming constant thermo-physical properties for the liquid phase, and neglecting any convective motion in the melt, obtain exact expressions for both the temperature distribution in the liquid phase and the solid–liquid interface location as a function of time.