5. Consider the cross section of a long rectangular bar as shown in Fig. 1 below. Internal energy is generated in the bar at a constant rate $\dot{q}$ per unit volume. q_1'' and q_2'' are given constant heat fluxes out of and into the bar at $x = a$ and $x = b$, respectively. The surfaces at $x = 0$ and $y = 0$ are perfectly insulated. The thermal conductivity of the material of the bar is constant. Find the relationship between q_1'', q_2'' and $\dot{q}$ so that the temperature distribution $T(x, y)$ can attain steady state. **[5]**

(Please Turn Over)

6. Consider a long solid cylinder of circular cross section with a radius r_0. The surface of the cylinder at $r = r_0$ is held at an arbitrary temperature $f(\phi)$. There are no internal energy sources or sinks, and the thermo-physical properties of the material of the cylinder can be assumed to be constant. Determine the steady state temperature distribution $T(r, \phi)$ in the cylinder using **separation of variables**. **[7]**

7. Consider a plane wall of thickness L as shown in Fig. 2 below. This is initially kept at a temperature $T_i(x)$. The internal energy is generated in this wall at a rate of $\dot{q}(x,t)$ per unit volume for times $t \geq 0$. Also, heat is dissipated by convection from the surfaces at $x = 0$ and $x = L$ into a surrounding medium whose temperature T_∞ varies with time. The thermo-physical properties may be assumed to be constant and the heat transfer coefficients h_1 and h_2 are very large. Determine the unsteady-state temperature distribution $T(x, t)$ in the wall using **method of integral transforms**. **[7]**

8. Consider steady state heat conduction in a long square slab $(2L \times 2L)$ as shown in Fig. 3 below. The internal energy is generated in the slab at a constant rate of $\dot{q}$ per unit volume. All four sides are maintained at temperature T_∞. The thermal conductivity of the material of the slab is constant.
(a) Write down the governing energy equation and the boundary conditions for the system in non-dimensional forms. **[2]**
(b) Using **central difference approximation**, write down the finite-difference forms of the governing equation and the boundary conditions. `**[1+2]**
(c) How will you handle the corner points? **[1]**

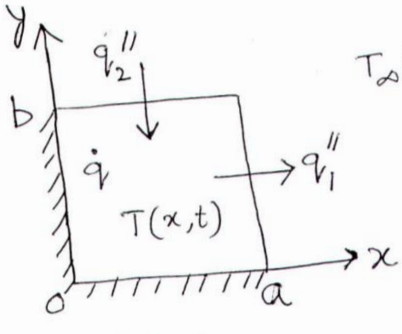

Fig. 1

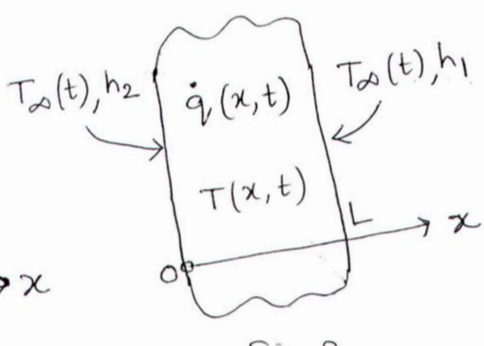

Fig. 2

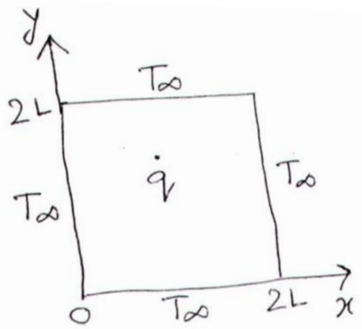

Fig. 3

5. Consider a rectangular fin with thickness (*b*) and length (*L*) as shown in Fig. 1. The width (*W*) of the fin is very large compared to its length (*W* >> *L*). The fin has to dissipate heat to the surroundings with heat transfer coefficient (*h*) and temperature (T_∞). The temperature at the fin base is (T_b) and the fin has an adiabatic tip. The profile area of the rectangular fin (A_P) is defined as $A_p = bL$ and thus there may be several shapes (various combinations of *b* and *L*) of the fin for the same profile area (A_P). For a given profile area, find the optimum thickness and length of the fin which removes maximum amount of heat per unit mass of the fin. **[7]**

<u>Given:</u> The following function $f(x)$ has a maximum at $x = 1.4192$, where $\pi = 3.14$.

$$f(x) = \pi \frac{\tanh(x)}{x^{1/3}}$$

6. Consider a long solid cylinder of circular cross section with radius r_0 as shown in Fig. 2. The surface of the cylinder is held at an arbitrary temperature $f(\phi)$. There is no internal heat generation in the cylinder and thermo-physical properties of the cylinder may be assumed to be constant. Determine the steady-state temperature distribution $(T(r,\phi))$ in the cylinder using **Separation of Variables**. **[8]**

7. Consider a semi-infinite rectangular strip as shown in Fig. 3. The surface at $x = 0$ is perfectly insulated. The initial $(t = 0)$ temperature distribution in the strip is given as $T_i(x, y)$. For timcs $t \geq 0$, the surface at $x = L$ is kept at a temperature $T_1(y,t)$ and the surface at $y = 0$ is kept at a temperature $T_2(x,t)$. Both $T_i(x, y)$ and $T_1(y,t)$ vanish as $y \to \infty$. There is no internal heat generation in the strip and thermo-physical properties may be assumed to be constant. Determine the unsteady-state temperature distribution $T(x, y,t)$ in the above semi-infinite rectangular strip for $t \geq 0$ using **Fourier Transforms.** **[10]**

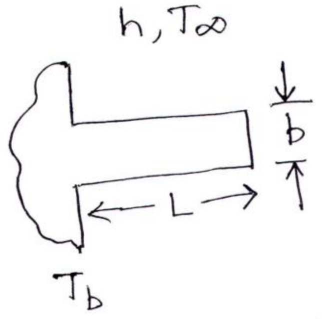

Fig. 1

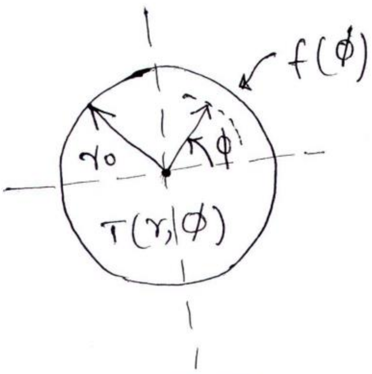

Fig. 2

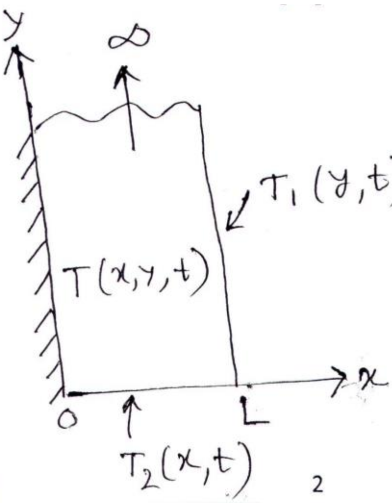

Fig. 3

5. Consider the heat conduction in a rectangular bar as shown in cross-section in Fig. 1. Internal energy is generated in this bar at a constant rate $\dot{q}$ per unit volume (W/m^3). The boundary conditions are shown on the figure itself. There is no temperature gradient in z-direction and the thermo-physical properties of the material of the bar may be considered as constant. Determine the steady-state temperature distribution $T(x,y)$ in the bar by

(a) Method of **separation of variables** [6]

(b) Method of **finite Fourier transforms** [6]

6. Consider a solid sphere of radius r_0. The surface of the sphere is maintained at some arbitrary temperature distribution $f(\theta)$. There are no internal energy sources or sinks in the sphere and the thermo-physical properties of the material of the sphere may be considered as constant. Find the steady-state two dimensional temperature distribution $T(r,\theta)$ in the sphere using **Fourier-Legendre series.** [7]

7. Consider a long solid cylinder of semi-circular cross-section as shown in Fig. 2. The cylindrical surface at $r = r_0$ is maintained at some arbitrary temperature distribution $f(\phi)$. The planar surfaces at $\phi = 0$ and $\phi = \pi$ are both maintained at constant temperature T_0. There are no internal energy sources or sinks in the cylinder and the thermo-physical properties of the material of the cylinder may be considered as constant. Find the steady-state two dimensional temperature distribution $T(r,\phi)$ in the cylinder using **Hankel transforms.** [6]

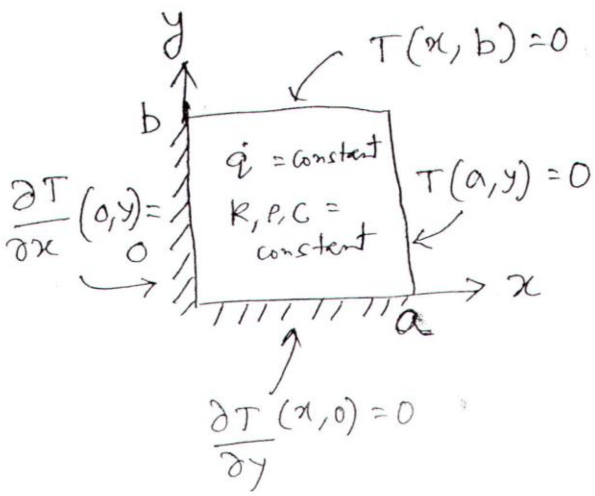

Fig. 1

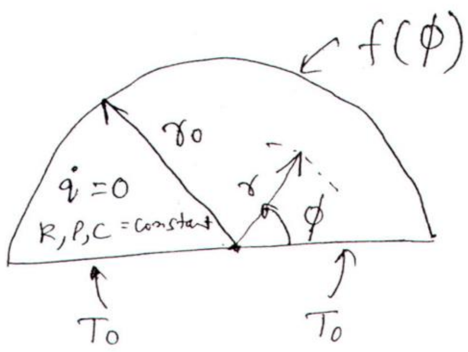

Fig. 2

2

5. Consider the cross-section of a long rectangular bar ($0 < x < a$, $0 < y < b$) made from a material with constant thermo-physical properties. Internal energy is generated at a constant rate Q per unit volume. The surfaces at ($x = 0, y$) and ($x, y = 0$) are insulated. A constant heat flux Q_1 leaves the surface at ($x = a, y$) and the surface at ($x, y = b$) receives a constant heat flux Q_2.

   (a) Determine the relationship among Q, Q_1, and Q_2 at steady-state.

   (b) Does the steady-state problem have a unique solution for $T(x, y)$? Justify. **[2+2 = 4]**

6. Consider a long solid cylinder of circular cross-section (Figure - B1). The surface of the cylinder is held at an arbitrary temperature $f(\phi)$. There is no internal energy sources or sinks. Assuming constant thermo-physical properties, obtain an expression for the steady-state temperature distribution $T(r, \phi)$ in cylinder using **Separation of Variables Method.** **[6]**

   __Given:__ In *cylindrical* coordinate system, the Laplacian of temperature T is

$$\nabla^2 T = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

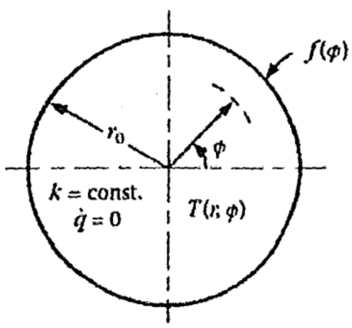

Figure – B1

7. A one-dimensional slab of thickness $2L$ (extending from $x = -L$ to $x = L$) is initially at a uniform temperature T_0. For times $t \geq 0$, internal energy is generated in the slab at a rate

$$Q = Q_0\{1 + \beta(T - T_0)\}$$

where Q_0 and β are given constants, while the surfaces at $x = L$ and $x = -L$ are maintained at the initial temperature T_0. Assuming constant thermo-physical properties, obtain an expression for the unsteady-state temperature distribution $T(x, t)$ in the slab for $t > 0$ using **Finite Fourier Transform Method.** [6]