Process Dynamics & Control

Process Dynamics & Control (CH 61016)

Class Teacher:

1. Prof Amar Nath Samanta
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Class TA

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Class Timing

Wednesday	:	10 AM to 11 AM
Thursday	:	9 AM to 10 AM
Friday	:	11 AM to 1 PM

Attendance

Biometric method

Institute Deregistration Rule

Syllabus of PDC (my part only)

- Development Process Models
- Development of Control Relevant Models
- Review of linear Dynamics
- Development of controller and observer using state space models
- Non-linear system analysis
  - Phase Plane
  - Bifurcation
  - Stability analysis
- Feedback linearization of nonlinear systems

Process Dynamics & Control

Books:

1. Process Dynamics & Control -- Seborg, Edgar, Mellichamp & Doyle

2. Process Control -- W Bequette

3. Process Dynamics, Modelling & Control – B A Ogunnaike & W H Ray

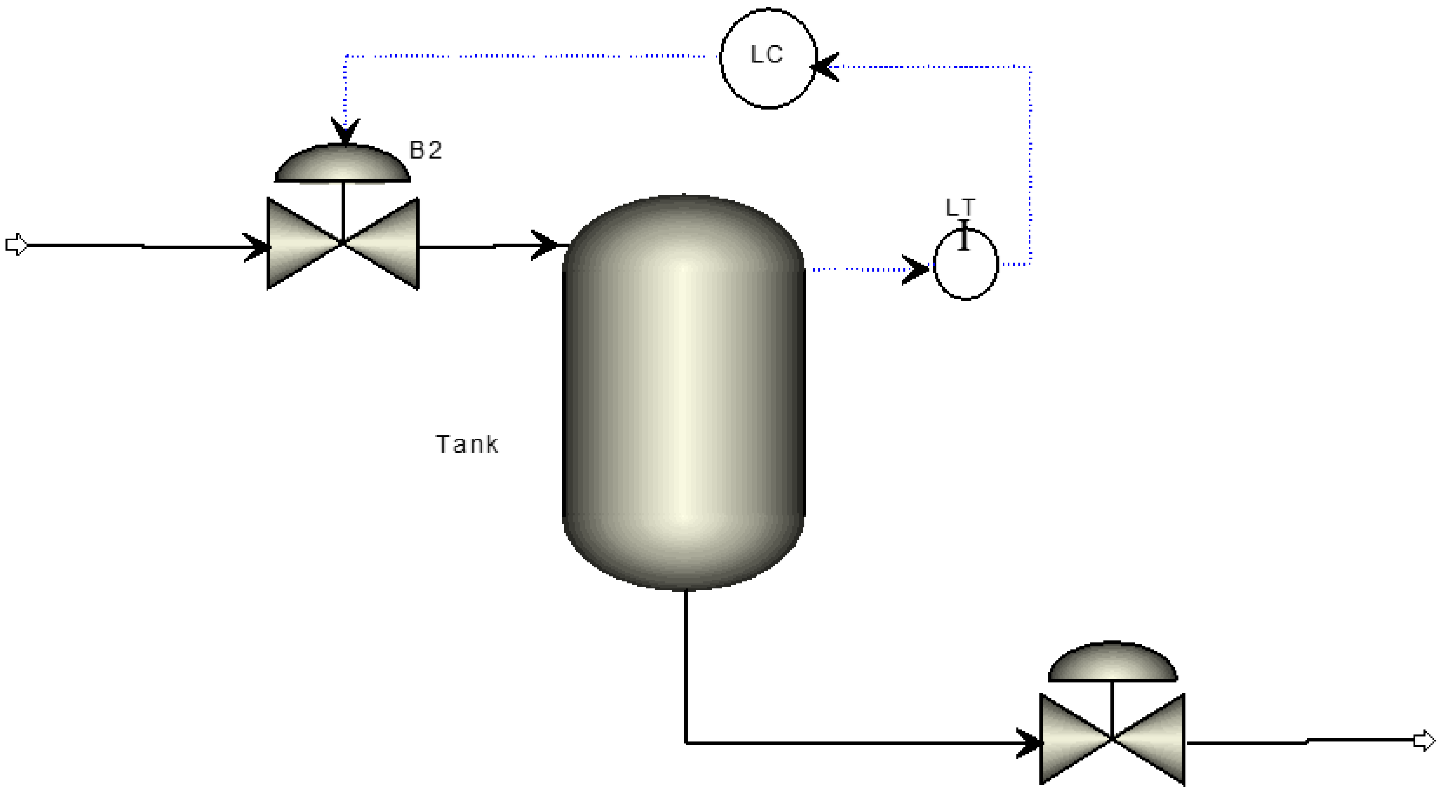
4. Modern Control Engineering -- K Ogata

Software: Knowledge of MATLAB & Simulink desirable

Why study Process dynamics

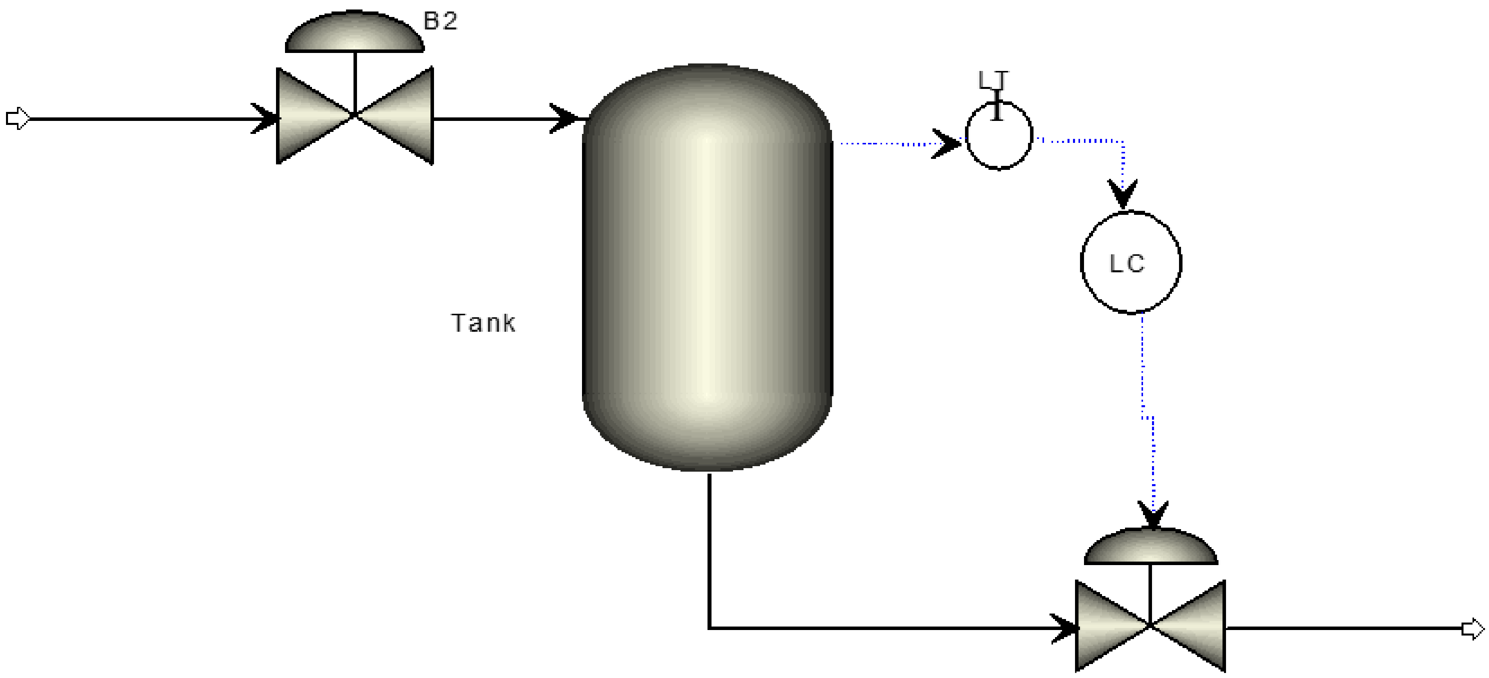
Let us consider a very simple example of tank level control problem

Option 1:



Tank Level problem

Option 2:



Which configuration is better from the control view point and why?

Benchmark Systems to be used for the course

- Tank Level System
  - 3 tank in series
  - Quadruple Tank
- Continuous Stirred Tank Heater
- Continuous Stirred Tank Reactor system
  - Isothermal Reactor (Van de Vusse reaction)
  - Non-isothermal Reactor
  - Polymerization Reactor
- Distillation Column

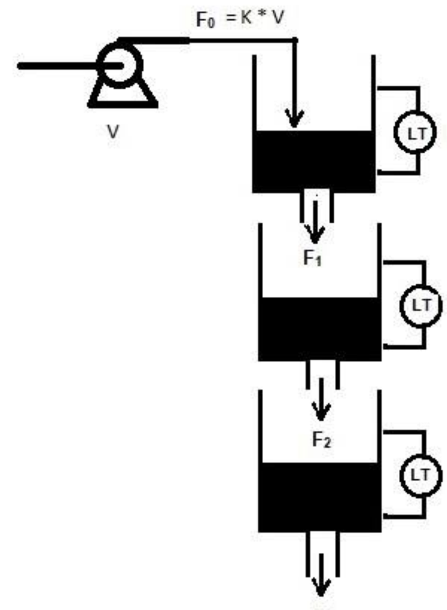
MATLAB / SIMULINK model will be developed based on nonlinear model

Mathematical Model of tank level system

$$\frac{dh_1}{dt} = \frac{F_0}{A_1} - \frac{a_1}{A_1}\sqrt{2gh_1}$$

$$\frac{dh_2}{dt} = \frac{a_1}{A_2}\sqrt{2gh_1} - \frac{a_2}{A_2}\sqrt{2gh_2}$$

$$\frac{dh_3}{dt} = \frac{a_2}{A_3}\sqrt{2gh_2} - \frac{a_3}{A_3}\sqrt{2gh_3}$$



Control objective:

Control the 3rd tank level by manipulating pump voltage V, where $F_0 = K\,V$

Add q_2 input flow on the tanks 2 and set q_2 as manipulated variable.

Which one is difficult to control?

Data for Simulation in Matlab/Simulink

A_1, A_3 = 28 cm^2 A_2 = 32 cm^2 V^s = 3 V

a_1 = 0.06725 cm^2 a_2 = 0.05683 cm^2
a_3 = 0.07089 cm^2 K=3.14

- Transfer Function model

Define : $x_1 = h_1 - h_1^s$; $x_2 = h_2 - h_2^s$; $x_3 = h_3 - h_3^s$; $u = V - V^s$

Linearizing around steady state value h_1^s

$$\frac{dh_1}{dt} = \frac{K}{A_1}V - \frac{a_1}{A_1}\sqrt{2g}\left[\sqrt{h_1^s} + \frac{1}{2\sqrt{h_1^s}}(h_1 - h_1^s)\right]$$

Subtracting above from steady state equation,

$$\frac{dx_1}{dt} = \frac{K}{A_1}u - \frac{a_1\sqrt{g}}{A_1\sqrt{2h_1^s}}x_1 \quad \Rightarrow \text{Transfer function } \frac{x_1(s)}{u(s)} = \frac{K_{p_1}}{\tau_{p_1}s+1}$$

Where, $K_{p_1} = \frac{K}{a_1}\sqrt{\frac{2h_1^s}{g}}$ and $\tau_{p_1} = \frac{A_1}{a_1}\sqrt{\frac{2h_1^s}{g}}$

Similarly,

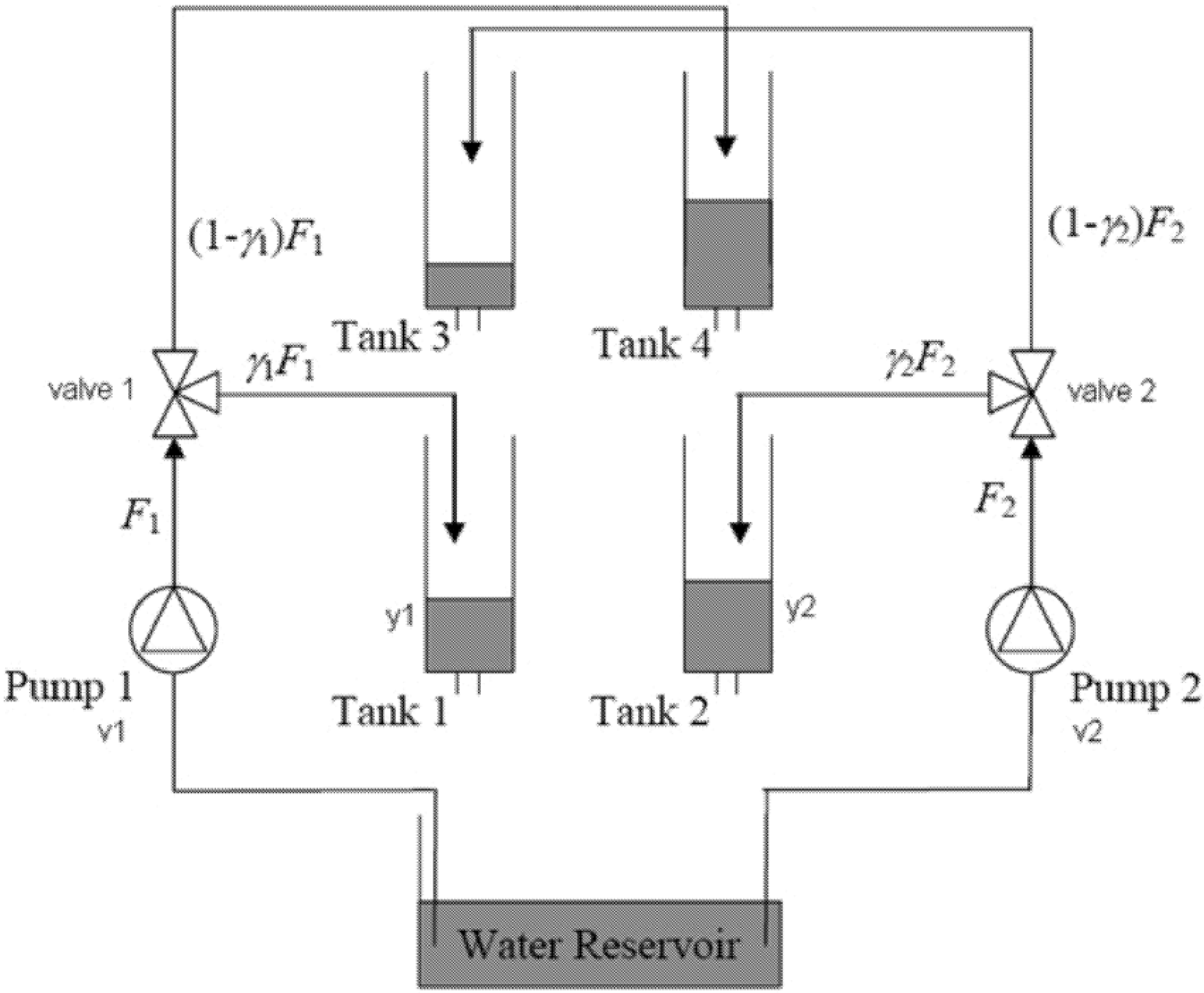
$$\frac{dx_2}{dt} = \frac{a_1}{A_2}\sqrt{\frac{g}{2h_1^s}}\,x_1 - \frac{a_2}{A_2}\sqrt{\frac{g}{2h_2^s}}\,x_2 \text{ and Transfer function } \frac{x_2(s)}{x_1(s)} = \frac{K_{p_2}}{\tau_{p_2}s+1}$$

$$\frac{dx_3}{dt} = \frac{a_2}{A_3}\sqrt{\frac{g}{2h_2^s}}\,x_2 - \frac{a_3}{A_3}\sqrt{\frac{g}{2h_3^s}}\,x_3 \text{ and Transfer function } \frac{x_3(s)}{x_2(s)} = \frac{K_{p_3}}{\tau_{p_3}s+1}$$

Where, $K_{p_j} = \frac{a_{j-1}}{a_j}\sqrt{\frac{h_j^s}{h_{j-1}^s}}$ and $\tau_{p_j} = \frac{A_j}{a_j}\sqrt{\frac{2h_j^s}{g}}$ for j = 2,3

So, Process Transfer function $G(s) = \frac{x_3(s)}{u(s)} = \frac{K_{p_1}K_{p_2}K_{p_3}}{(\tau_{p_1}s+1)(\tau_{p_2}s+1)(\tau_{p_3}s+1)}$

Quadruple Tank problem



Quadruple Tank problem

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2$$

$$\frac{dh_2}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1$$

Control Objective

Both the levels of tank1 and tank2 should be controlled by manipulating voltages to the pumps.

Data for simulation:

A_1, A_3 = 28 cm^2

A_2, A_4 = 32 cm^2

a_1, a_3 = 0.071 cm^2

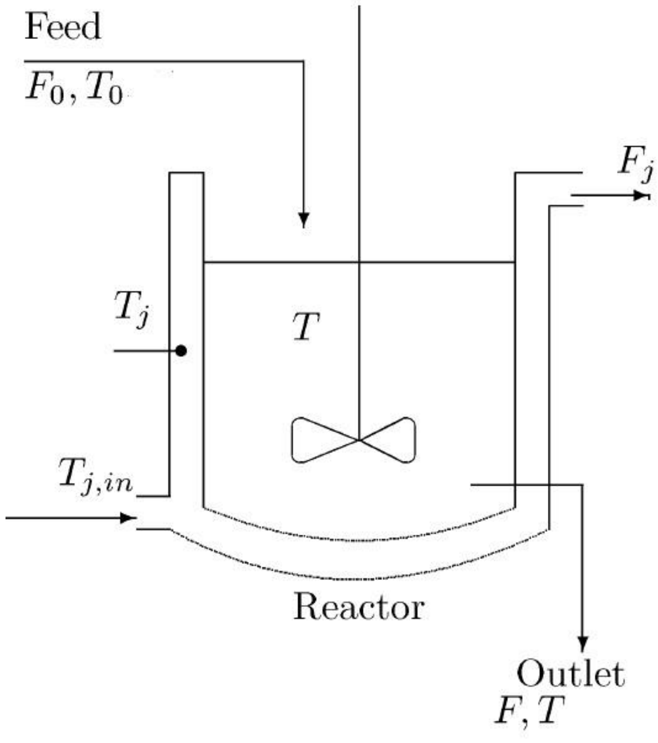
a_2, a_4 = 0.057 cm^2

k_1, k_2 = 3.33, 3.35

v_1, v_2 = 3.0, 3.0

γ_1, γ_2 = 0.7, 0.6

Jacketed heated stirred tank



Assumptions:

Constant hold-up in tank and jacket
Constant heat capacities and densities
Incompressible flow

$$\frac{dV}{dt} = F_0 - F = 0$$

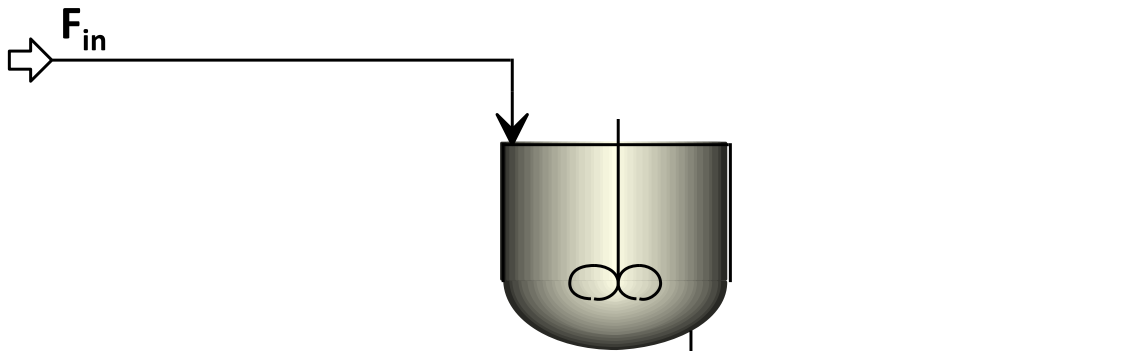
$$\frac{dT}{dt} = \frac{F}{V}(T_0 - T) + \frac{UA}{\rho C_P V}(T_j - T)$$

$$\frac{dT_j}{dt} = \frac{F_j}{V_j}(T_{ji} - T_j) - \frac{UA}{\rho_j C_{Pj} V_j}(T_j - T)$$

Parameter and Steady State Values

F = 30 l/min; F_j^s = 50 l/min; T_0 = 15^{0}C; T_{Jin} = 93^{0}C; V = 300 l; V_j = 30 l;
ρC_p = 1 Kcal/K l; ρC_{pj} = 1.384 ; UA = 100 Kcal/min K; T^s = 60^{0}C ; T_J = 73.5 ^{0}C

Continuous Stirred Tank Reactor



$$A \xrightarrow{k_1} B \xrightarrow{k_2} C$$

$$A \xrightarrow{k_3} 2D$$

Assumption:

1. **Exit Condition = Reactor Condition**
2. **Isothermal Reaction**
3. **Constant volume, i.e, F_{in} = F_{out}=F**
4. **Only reactant A in feed is consumed, i.e, other reactant is in large excess. No product in the feed**

Continuous Stirred Tank Reactor

Constant volume : dV/dt = 0

Dynamic model

- $\frac{dC_A}{dt} = \frac{F}{V}\left(C_{Af} - C_A\right) - k_1 C_A - k_3 C_A^2$
- $\frac{dC_B}{dt} = -\frac{F}{V} C_B + k_1 C_A - k_2 C_B$
- $\frac{dC_C}{dt} = -\frac{F}{V} C_C + k_2 C_B$
- $\frac{dC_D}{dt} = -\frac{F}{V} C_D + \frac{1}{2} k_3 C_A^2$

Data for the CSTR

V = 1 l

F = 78 l/h

C_{Af} = 10 mol/l

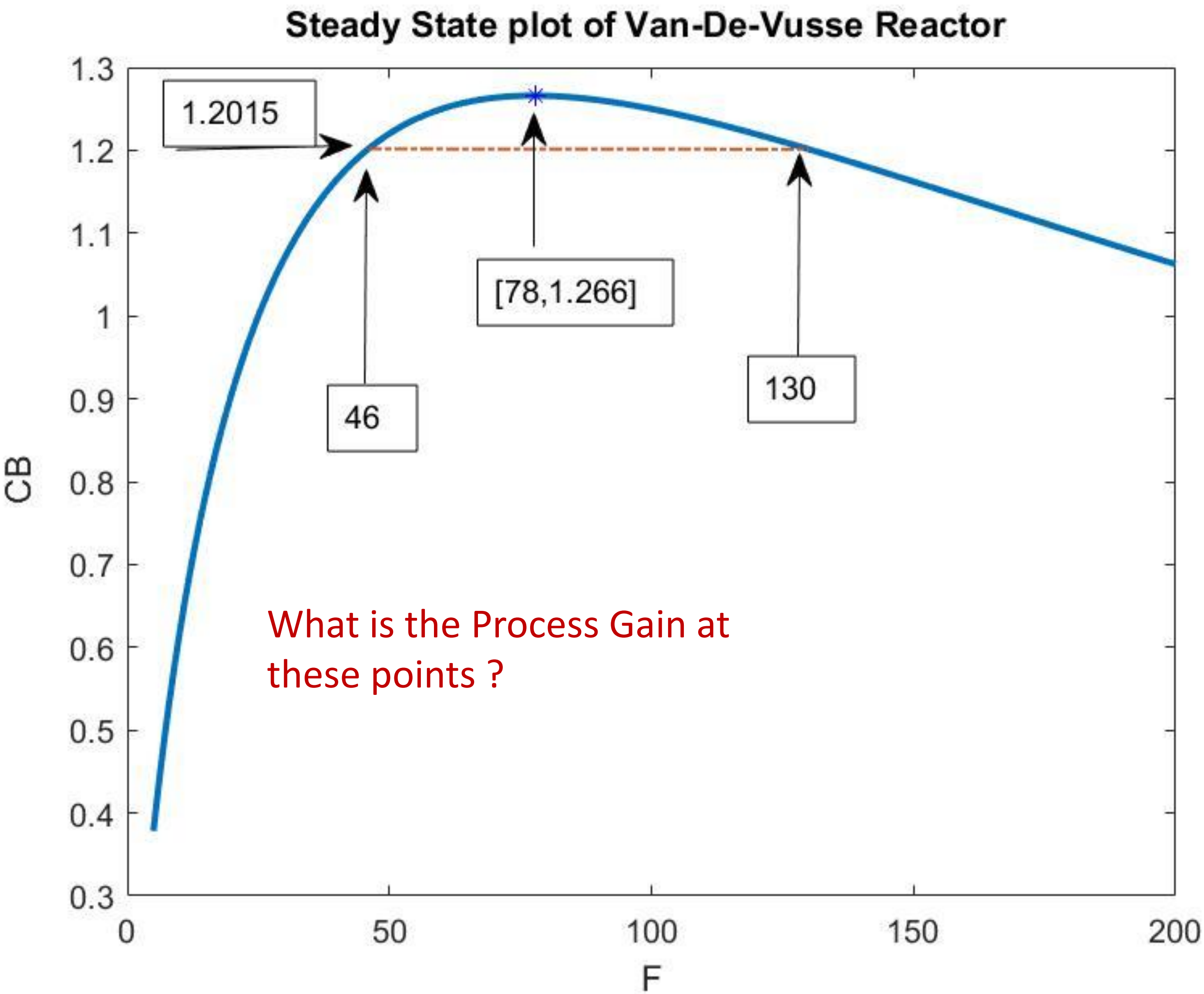
kl = 50 h-1

k2 = 100 h-1

k3 = 10 l.mol-1.h-1

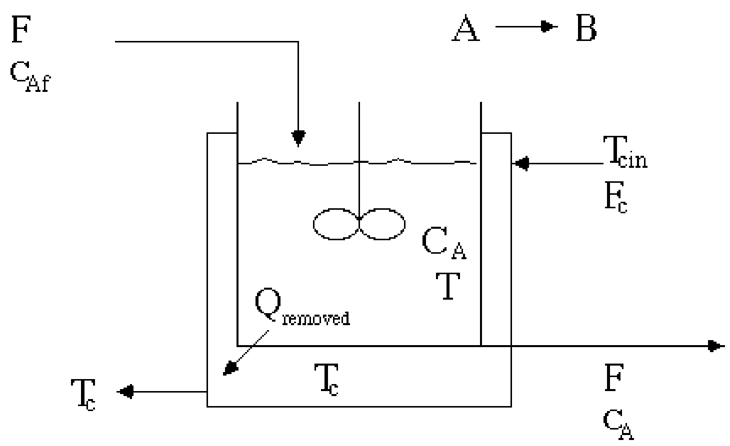
Since our objective is to control C_B, the reduced order model should be

- $\frac{dC_A}{dt} = \frac{F}{V}\left(C_{Af} - C_A\right) - k_1 C_A - k_3 C_A^2 = f_1(C_A, C_B, F)$
- $\frac{dC_B}{dt} = -\frac{F}{V} C_B + k_1 C_A - k_2 C_B = f_2(C_A, C_B, F)$



What is the Process Gain at these points ?

Non-Isothermal Jacketed CSTR

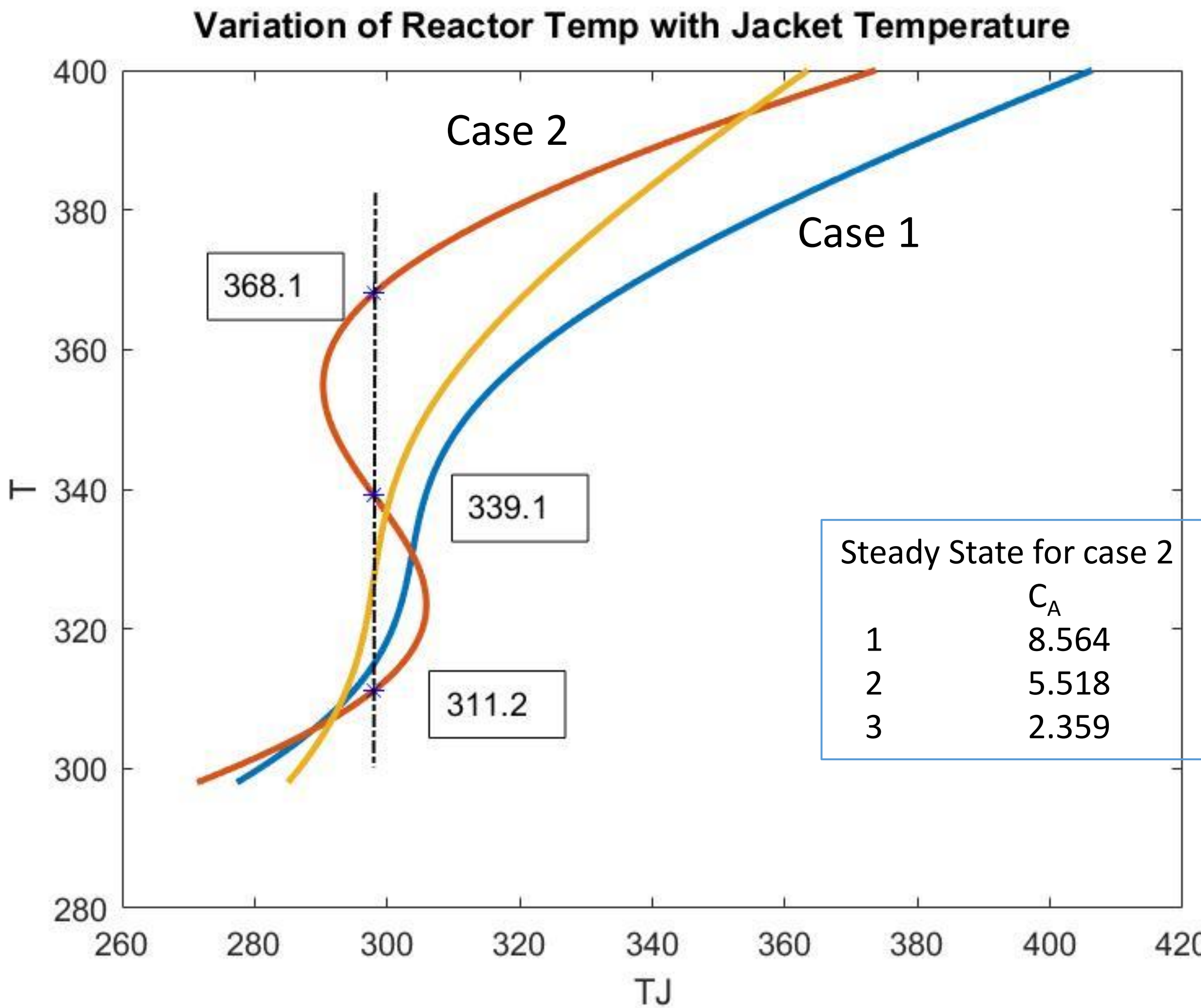


$$\frac{dC_A}{dt} = \frac{F}{V}\left(C_{A_f} - C_A\right) - r$$

$$\frac{dT}{dt} = \frac{F}{V}\left(T_f - T\right) + \left(\frac{-\Delta H}{\rho C_p}\right)\mathrm{r} - \frac{\mathrm{UA}}{\mathrm{V}\rho C_p}(\mathrm{T} - \mathrm{T_C})$$

$$r = k_0 e^{-\frac{E}{RT}} C_A$$

parameter	case 1	case 2	case 3
F/V, hr-1	1	1	1
k_0, hr-1	14,825*3600	9,703*3600	18,194*3600
$(-\Delta H)$, kcal/kgmol	5215	5960	8195
E, kcal/kgmol	11,843	11,843	11,843
ρcp, kcal/(m^3 ^oC)	500	500	500
T_f, ^oC	25	25	25
C_{Af}, kgmol/m3	10	10	10
UA/V, kcal/(m^3 ^oC hr)	250	150	750
T_C, ^oC	25	25	25



Steady State for case 2

	C_A	T
1	8.564	311.2
2	5.518	339.1
3	2.359	368.1

Styrene Polymerization Reactor

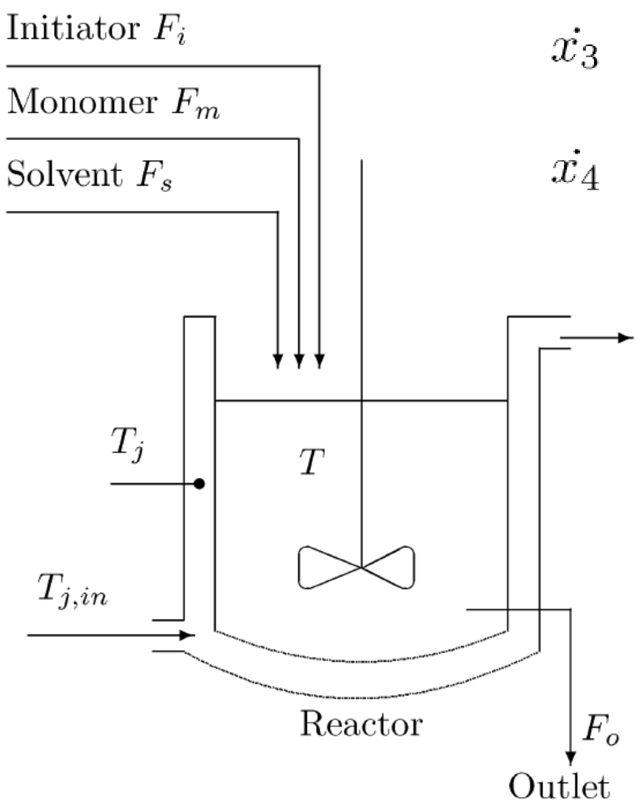
x_1 Initiator Conc
x_2 Monomer Conc
x_3 Reactor Temp
x_4 Jacket Temp.

$$\dot{x_1} = \frac{(F_i\, C_{ia} - F_o\, x_1)}{V} - k_d\, x_1$$

$$\dot{x_2} = \frac{(F_m\, C_{ma} - F_o\, x_2)}{V} - k_p\, x_2\, \mathcal{R}$$

$$\dot{x_3} = \frac{F_o\,(T_a - x_3)}{V} - \frac{\Delta H}{\rho\, C_p}\, k_p\, x_2\, \mathcal{R} - \frac{U\,A}{\rho\, C_p\, V}\,(x_3 - x_4)$$

$$\dot{x_4} = \frac{F_j(T_{j,in} - x_4)}{V_j} + \frac{U\,A}{\rho_j\, C_{pj}\, V_j}\,(x_3 - x_4)$$



$$F_o = F_i + F_m + F_s.$$

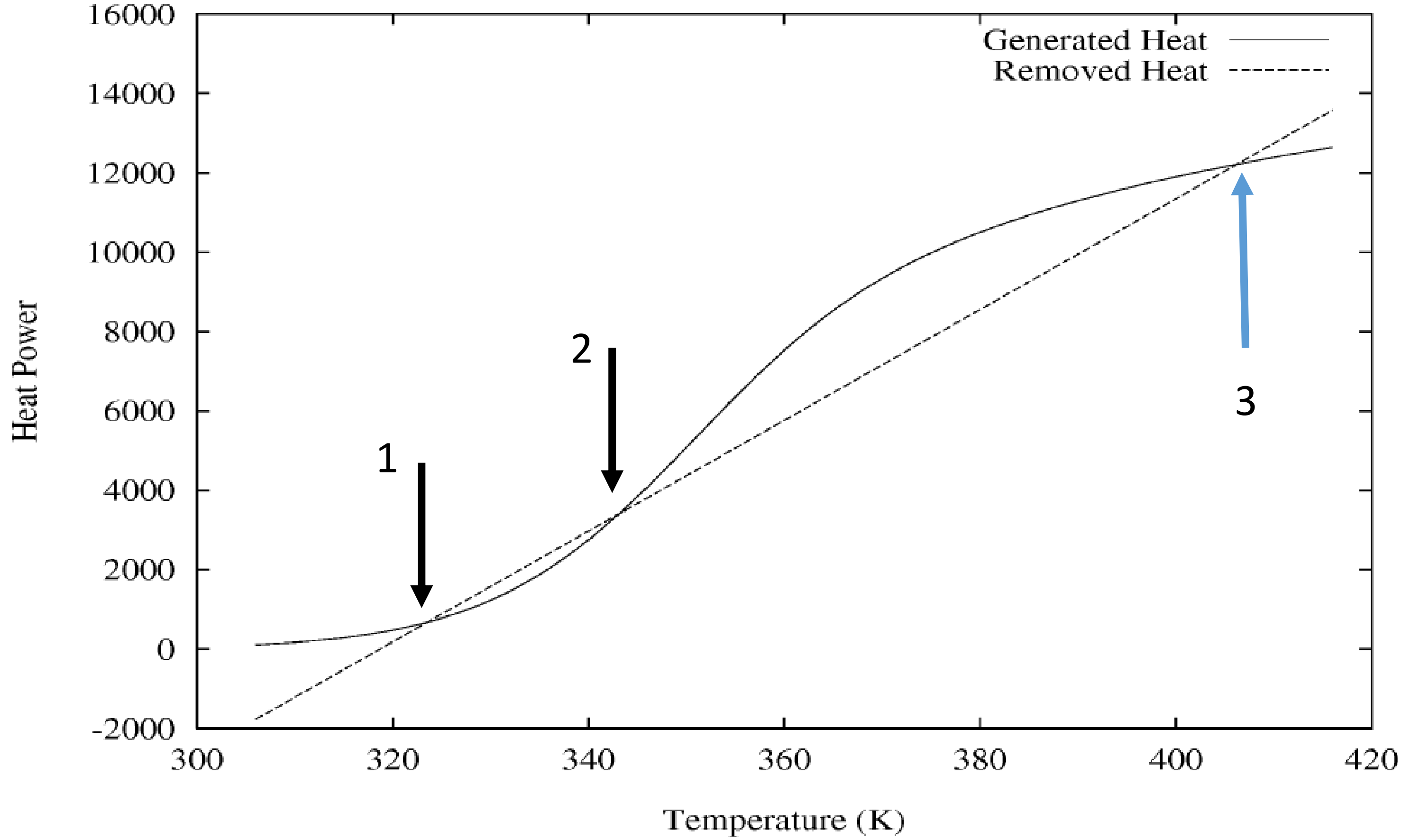
The chain concentration of growing polymer is equal to

$$\mathcal{R} = (2\, f\, k_d\, x_1/k_t)^{0.5}$$

The dissociation, propagation and termination Rate constants follow Arrhenius law :

$$k_i = k_{i0}\, e^{-\frac{E_i}{RT}} \quad \text{for I =d, p or t}$$

Steady State Heat Power vs Reactor Temperature



Phase Portrait