Test Solutions

1. The statement is false. For all $n \geq 2$, we have that: $3m+5n \geq 3m+10 \geq 13$ So the only possibility could be that $n=1$ would satisfy the equation $3m+5n=12$ for some m, but letting $n=1$ into the equation, we get: $3m+5=12$, which becomes $3m=7$. There is no natural number m to satisfy this equation. Therefore, the statement has been proved as false.

2. This statement is true. Let the five consecutive integers be defined as $n-2, n-1, n, n+1, n+2$; we can add them together as follows: $(n-2)+(n-1)+n+(n+1)+(n+2)=5n$ Since 5n is divisible by 5, this proves the following statement.

3. The statement is true. Now, we can rewrite $n^2 + n + 1$, as $n*(n + 1) + 1$. If n is odd, then n + 1 is even. In either case, $n * (n + 1)$ is even because the product of an even number and an odd number is always even. Then, we can write $n * (n + 1) + 1$ as $2k+1$, That's strange. Thus, the result is proved.

4. Division theorem: Any integers a and b, if $b > 0$, exist uniquely of the integers q and r in such a manner that $a = bq + r$, where $0 \leq r < b$. Putting $b = 4$ and $n = q$ we get: $a = 4n + r$, $0 \leq r < 4$, & for $r = 0$ or 2 we get $a = 4n$/ $a = 4n + 2$ which is the even natural number. And for $r = 1$ or 3, it should be such that $a=4n + 1$ or $a = 4n + 3$ is the odd natural number. Since a is any odd natural number satisfying the antecedent, we have to have that it is one of the following forms: $a = 4n + 1$ or $a = 4n + 3$. Hence, the result is proved.

5. Recall that the division algorithm states that for any integers a, b where $b > 0$, there are unique integers q and r so that $a = bq + r$ where $0 \leq r < b$. So we let $b = 3$. Then the statement is $a = 3q + r$ where $0 \leq r < 3$. We can write this out fully, letting $n = a$ that: $n = 3q$ or, $n = 3q +1$ or $n = 3q + 2$. We are now able to put the following in such forms. Let us take 'n', n + 2, and 'n' + 4 as follows: 'n' is either, 3q or, 3q + 1 or 3q + 2; 'n' + 2: is either 3q + 2, 3q+ 3 or 3q + 4;'n' + 4: is either, 3q + 4 or, 3q + 5 or 3q + 6; Now we realize in every form that is in those three parts there does exist at least a particular element from which 1 is divisible by 3 since if 'n' it will be: $3|3q$ if n+ 2, $3|(3q + 3)$ or else if 'n'+ 4 $3|\ 3q + 6$. So it is proved.

6. We prove this by contradiction, which is to assume there exists $n > 3$ such that *n+2, n+4 & n* are prime. But from the proof of number 5, we have just discovered/proved that one of *n+2, n+4 & n* can be divided by 3. And since $n > 3$, 3 is not one of the primes. So one of *n+2, n+4, & n* is not *prime*. Hence we have proved the result.

7. Let the addition, $2+2^2+2^3+...+2^n$, be indicated by S. Multiplied by 2, I have that $2S=2^3+2^2+...+2^{n-1}$. From this expression $2S$-S, I have that $S=2^{n+1}-2$, which was to be demonstrated or proven.

8. According to my assumption, for given any $0 < \in$, there will exist n, an integer, for such that all $n \leq m$, the situation $|-L+a_m| < \in$ remains true. The statement that Ma_n approaches ML as n reaches ∞ is equal to saying that for any positive $\in_{1,}$ there will exist n, an integer, for all $m \leq n$, $\in_1 > |-ML + Ma_m|$. This can be restated as $|M|\ |-L+a_m| < \in_1$. This statement will be true if we set $\in/M = \in_1$ & identify an n. Because one can always do this, and hence we have a valid result.

9. Let $A_n=(0,1/n)$: We know A_n is a subset of A_1 because (0,1/n) is also a subset of (0,1). Assume x belongs to (0,1). We always can identify a natural # m for that x>1/m. But, this shows that x isn't member of A_m. Thus, x can't belong in intersection of A_n for n– which is any natural number. Because we always can establish an m, it accepts the intersection of A_n is null. Hence we established a valid result.

10. Let $[0, 1/n] = A_n$: We can show this set as $0 \cup B_n$, where $(0,1/n) = B_n$. Thus, the intersection of A_n for n in natural numbers can therefore be presented as $0 \cup (\cap B_n)$. Since we have proved from number 9 that the intersection of all B_n is the null set, we determine that $\{0\} = 0 \cup \varnothing$. Thus, we have proved the result.