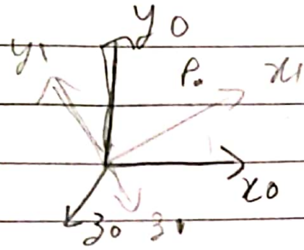
18110131

Assignment - 2

1.) 

Rotation ~~of~~ from 0 to 1 → R_0^1
1 to 0 → R_1^0

We ~~can~~ have $R_0^1 = \begin{bmatrix} x_1 \cdot x_0 & y_1 \cdot x_0 & z_1 \cdot x_0 \\ x_1 \cdot y_0 & y_1 \cdot y_0 & z_1 \cdot y_0 \\ x_1 \cdot z_0 & y_1 \cdot z_0 & z_1 \cdot z_0 \end{bmatrix}$

x_1, y_1, z_1 are unit vectors of frame 1 &
x_0, y_0, z_0 are unit vectors of frame 0

Column matrices: $C_1 = [x_1 x_0 \quad x_1 y_0 \quad x_1 z_0]^T$
$C_2 = [y_1 x_0 \quad y_1 y_0 \quad y_1 z_0]^T$
$C_3 = [z_1 x_0 \quad z_1 y_0 \quad z_1 z_0]^T$

$\therefore C_1 \cdot C_2 = x_1 \cdot x_0 \cdot y_1 \cdot x_0 + x_1 \cdot y_0 \cdot y_1 y_0 + x_1 z_0 y_1 z_0$

we know that x_1 & y_1 are $\perp$ $x_1 \cdot y_1 = 0$

Using commutative property & distributive ~~associative~~ w.g

$C_1 \cdot C_2 = 0$

Similarly, $C_2 \cdot C_3 = 0$ & $C_1 \cdot C_3 = 0$

$\therefore$ All the columns are mutually ~~⊥ to ea~~ orthogonal.

2.) ~~For~~ We have $P_0 = R_0^1 P_1$ - ①

& $R_1^0 = \begin{bmatrix} x_0 \cdot x_1 & y_0 \cdot x_1 & z_0 \cdot x_1 \\ x_0 \cdot y_1 & y_0 \cdot y_1 & z_0 \cdot y_1 \\ x_0 \cdot z_1 & y_0 \cdot z_1 & z_0 \cdot z_1 \end{bmatrix}$ We know that → $x_1 \cdot x_0 = x_0 \cdot x_1$ (commutative)

$\therefore$ We get,

$R_1^0 = (R_0^1)^T$ - ② ($\because$)

Also, $P_1 = R_1^0 P_0$

Take R_1^0 inverse both sides

$\therefore R_1^{0^{-1}} P_1 = P_0$ - ②

From above equation ① & ② we get

$R_0^1 P_1 = R_1^{0^{-1}} P_1 \Rightarrow R_0^1 = R_1^{0^{-1}}$

Scanned with CamScanner

Multiply R_1^0 both sides

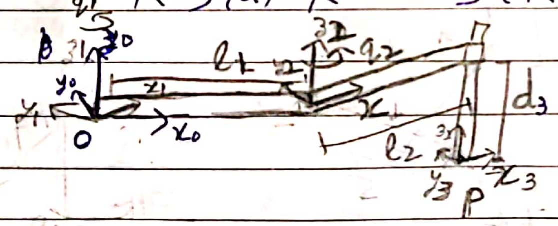
$\therefore \; R_1^0 \, R_0^1 = I$

Using (3) $\rightarrow$ $R_1^0 (R_1^0)^T = I$; Take determinant $(\det(A) = |A|)$

$|I| = 1 = |R_1^0 (R_1^0)^T| = |R_1^0||R_1^{0T}| = |R_1^0|^2 \Rightarrow \underline{|R_1^0| = 1}$

5. To show: $R\,S(a)\,R^T = S(Ra)$; where R is a rotation matrix & a is a vector

$$R\,S(a)\,R^T = R\,(a \times R^T)$$
$$= Ra \times RR^T$$
$$= Ra \times I \quad (\because RR^T = I)$$
$$= (Ra) \times (I)$$
$$= S(Ra)\,I$$
$$R\,S(a)\,R^T = S(Ra) \quad (\because S(d)\,I = S(d))$$

6.) 

Point P w.r.t 3,

$$P_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For 1 to 0: $R_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

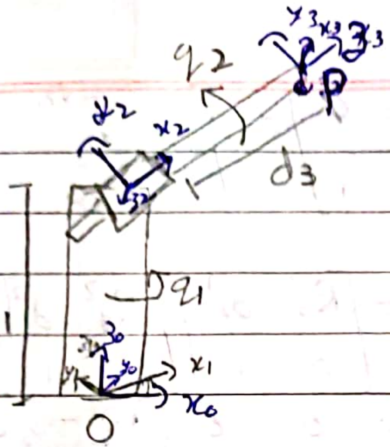
For 2 to 1: $R_1^2 = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$

For 3 to 2: $R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ -d_3 \end{bmatrix}$

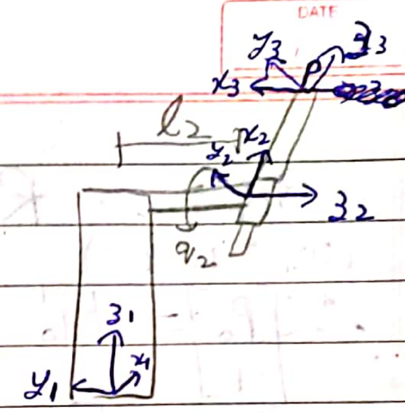
$P_0 = O_3$ in Q11

$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_0^1 & d_0^1 \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} R_2^3 & d_2^3 \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

8.)



Side View $\longrightarrow$



Stanford Type RRP   End Point P : $P_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

For 1 to 0 : $R_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

For 2 to 1 : $R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\frac{\pi}{2} & -s\frac{\pi}{2} \\ 0 & s\pi/2 & c\pi/2 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $d_1^2 = \begin{bmatrix} 0 \\ -l_2 \\ l_1 \end{bmatrix}$

~~For 3 to~~ $R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

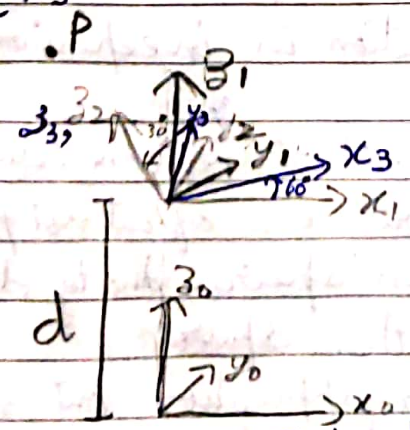
For 3 to 2 : $R_2^3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ~~$d_2^3 = \begin{bmatrix} d_3 \\ 0 \\ d_3 \end{bmatrix}$~~ $d_2^3 = \begin{bmatrix} d_3 \\ 0 \\ 0 \end{bmatrix}$

(Rotate $\frac{\pi}{2}$ about y_2)

$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} cq_1 cq_2 d_3 + l_2 sq_1 \\ sq_1 cq_2 d_3 - l_2 cq_1 \\ sq_2 d_3 + l_1 \\ 1 \end{bmatrix}$$

9.)



Obstacle P w.r.t. Frame 3 (drone)

$P_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$

$d = 10\,m$

1 to 0 : $R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$

2 to 1 : (about x_1 30°) $R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c30 & -s30 \\ 0 & s30 & c30 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix}$ $d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

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$3 \text{ to } 2$: $R_2^3 = \begin{bmatrix} c60 & -s60 & 0 \\ s60 & c60 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ d_2^3

(about z_2 $60°$)

$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ \sqrt{3}/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 \\ 1 \end{bmatrix}$$

$$\therefore \cancel{P_0 = \ldots} \quad P_0 = \begin{bmatrix} 0 \\ -3/2 \\ 10 + 3\sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

10.) Types of Gear Boxes:

1.) Spur Gear Train:

→ Simplest Gear with straight cuts

→ Easy to manufacture & assemble

→ ~~Large~~ Noisy & weaker

→ Used for gear reduction in inexpensive motor-gearbox assemble

2.) Helical Gear: (& Herringbone (dual helical))

→ Similar to spur but axis of cut not parralled to axis of gear

→ More expensive than spur due to difficulties in mfg

→ Quiter & stronger than spur

→ Similar to spur gear application but non parralel shafts are there so spur can't be used, so helical.

3.) Planetary gear:

→ Combination of 3 types of gears — Sun, planet & ring

→ Provides 2 degree of freedom, high reduction in ~~spa~~ small space

→ Difficult assembly, complex design ⇒ Expensive
→ Due to its advantages in space - used in Wheel hub drives AGVs, conveyer belts etc.

4.) **Bevel gears:**
→ Conical shaped spur gears used for 90° rotation of transmission
→ Requires strong gearbox to contain it & have stable transmission
→ Used in differential drives, used to rotate axis by 90°

5.) **Worm Gears:**
→ Worm gears also transmit rotation at 90°
→ They are non-backdrivable - so prevents back force
→ ~~Large space requirements~~, costly to manufacture.
→ Used in shutter mechanism (warehouse), Automotive steering

6.) **Lead Screw:**
→ Turn rotation to linear motion
→ Precise motion, resistance to back drive
→ Low efficiency, manufacturing
→ Used in CNCs, 3d printers, Lathes etc.

Drone application require smooth motion of motors & very high RPM and minimal load. Gear boxes will increase the load, decrease efficiency of motors & output due to gear friction. Also, BLDC motors are generally used with drones whose RPM can be manipulated easily. Not much torque requirements for drones so no gearbox needed.

* Note: Jacobian derivation used is slightly different since, frame selection is different from Text book

11) RRP SCARA Jacobian:

$$\begin{bmatrix} v_0^n \\ \omega_0^n \end{bmatrix} = J_0^n \dot{q}$$

$$\dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ -\dot{d}_3 \end{bmatrix}$$ In this, Frame 0→1, 1→2 are revolute & 2→3 is prismatic

$$\therefore J_0^n = \begin{bmatrix} J_1 & J_2 & J_3 \end{bmatrix} = \begin{bmatrix} z_1 \times (O_3 - O_1) & z_2 \times (O_3 - O_2) & z_3 \\ z_1 & z_2 & 0 \end{bmatrix}$$

Here, O_1 = Origin of frame $1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

O_2 = Origin of frame $2 =$

$$\begin{bmatrix} O_2 \\ 1 \end{bmatrix} = H_0^1 H_1^2 \begin{bmatrix} O_{3\times1} \\ 1 \end{bmatrix} = \begin{bmatrix} l_1 \cos q_1 \\ l_1 \sin q_1 \\ 0 \\ 1 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad z_1 = R_0^1 z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$z_2 = R_0^1 R_1^2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad z_3 = R_0^1 R_1^2 R_2^3 z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} O_3 \\ 1 \end{bmatrix} = \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore O_3 = \begin{bmatrix} l_2 c_{q_1} c_{q_2} - l_2 s_{q_1} s_{q_2} + l_1 c_{q_1} \\ l_2 s_{q_1} c_{q_2} + l_2 s_{q_2} c_{q_1} + l_1 s_{q_1} \\ -d_3 \end{bmatrix}$$

$A \times B = S(A) B$

$$\therefore z_1 \times (O_3 - O_1) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} l_2 c_{q_1} c_{q_2} - l_2 s_{q_1} s_{q_2} + l_1 c_{q_1} \\ l_2 s_{q_1} c_{q_2} + l_2 s_{q_2} c_{q_1} + l_1 s_{q_1} \\ -d_3 \end{bmatrix}$$

$$z_1 \times (O_3 - O_1) = \begin{bmatrix} -(l_2 s_{q_1} c_{q_2} + l_2 s_{q_2} c_{q_1} + l_1 s_{q_1}) \\ l_2 c_{q_1} c_{q_2} - l_2 s_{q_1} s_{q_2} + l_1 c_{q_1} \\ 0 \end{bmatrix}$$

$$z_2 \times (O_3 - O_2) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -(l_2 sq_1 cq_2 + l_2 sq_2 cq_1) \\ l_2 cq_1 cq_2 - l_2 sq_1 sq_2 \\ 0 \end{bmatrix}$$

$$\therefore J_0^3 = \begin{bmatrix} -(l_2 sq_1 cq_2 + l_2 sq_2 cq_1 + l_1 sq_1) & -(l_2 sq_1 cq_2 + l_2 sq_2 cq_1) & 0 \\ l_2 cq_1 cq_2 - l_2 sq_1 sq_2 + l_1 cq_1 & l_2 cq_1 cq_2 - l_2 sq_1 sq_2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}_{6\times 3}$$

BONUS Stanford:

$$\begin{bmatrix} v_0^3 \\ \omega_0^3 \end{bmatrix} = J_0^3 \dot{q} \qquad \dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{d}_3 \end{bmatrix}$$

$$J_0^3 = \begin{bmatrix} z_1 \times (O_3 - O_1) & z_2 \times (O_3 - O_2) & z_3 \\ z_1 & z_2 & 0 \end{bmatrix}$$

$$O_1 \rightarrow \begin{bmatrix} O_1 \\ 1 \end{bmatrix} = H_0^1 \begin{bmatrix} O_{3\times1} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = O_0$$

$$O_{3\times1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} O_2 \\ 1 \end{bmatrix} = H_0^1 H_1^2 \begin{bmatrix} O_{3\times1} \\ 1 \end{bmatrix} \Rightarrow O_2 = \begin{bmatrix} l_2 \sin q_1 \\ -l_2 \cos q_1 \\ l_1 \end{bmatrix}$$

$$\begin{bmatrix} O_3 \\ 1 \end{bmatrix} = \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \rightarrow O_3 = \begin{bmatrix} cq_2 cq_1 d_3 + l_2 sq_1 \\ sq_1 cq_2 d_3 - l_2 cq_1 \\ sq_2 d_3 + l_1 \end{bmatrix}$$

Z-Axes: $z_1 = R_0^1 z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$z_2 = R_0^1 R_1^2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin q_1 \\ -\cos q_1 \\ 0 \end{bmatrix}; \quad z_3 = R_0^1 R_1^2 R_2^3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} cq_1 cq_2 \\ sq_1 cq_2 \\ sq_2 \end{bmatrix}$$

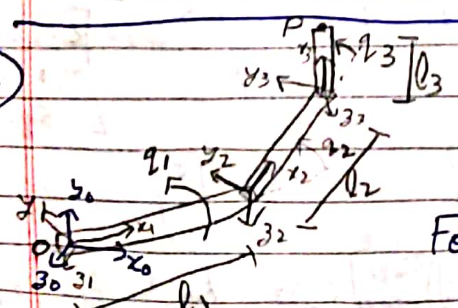
$$z_1 \times (O_3 - O_1) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Cq_1 Cq_2 d_3 + l_2 Sq_1 \\ Sq_1 Cq_2 d_3 - l_2 Cq_1 \\ Sq_2 d_3 + l_1 \end{bmatrix}$$

$$= \begin{bmatrix} -(Sq_1 Cq_2 d_3 - l_2 Cq_1) \\ Cq_1 Cq_2 d_3 + l_2 Sq_1 \\ 0 \end{bmatrix}$$

$$z_2 \times (O_3 - O_2) = \begin{bmatrix} 0 & 0 & -Cq_1 \\ 0 & 0 & -Sq_1 \\ Cq_1 & Sq_1 & 0 \end{bmatrix} \begin{bmatrix} Cq_1 Cq_2 d_3 \\ Sq_1 Cq_2 d_3 \\ Sq_2 d_3 \end{bmatrix}$$

$$= \begin{bmatrix} -Sq_2 Cq_1 d_3 \\ -Sq_1 Sq_2 d_3 \\ C^2q_1 Cq_2 d_3 + S^2 q_1 Cq_2 d_3 \end{bmatrix} = \begin{bmatrix} -Sq_2 Cq_1 d_3 \\ -Sq_2 Sq_1 d_3 \\ Cq_2 d_3 \end{bmatrix}$$

$$\therefore J_0^3 = \begin{bmatrix} -(Sq_1 Cq_2 d_3 - l_2 Cq_1) & -Sq_2 Cq_1 d_3 & Cq_1 Cq_2 \\ Cq_1 Cq_2 d_3 + l_2 Sq_1 & -Sq_2 Sq_1 d_3 & Cq_2 Sq_1 \\ 0 & Cq_2 d_3 & Sq_2 \\ 0 & Sq_1 & 0 \\ 0 & -Cq_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}_{6\times 3}$$

13.) 

RRR: Elbow

End effector: $P \rightarrow P_3 = \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix}$

For 1 to 0: $R_0^1 = \begin{bmatrix} Cq_1 & -Sq_1 & 0 \\ Sq_1 & Cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

2 to 1: $R_1^2 = \begin{bmatrix} Cq_2 & -Sq_2 & 0 \\ Sq_2 & Cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$

3 to 2 : $R_2^3 = \begin{bmatrix} cq_3 & -sq_3 & 0 \\ sq_3 & cq_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$

All 3 are revolutes.

$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} l_3 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} l_3 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$\Rightarrow \cancel{P_0 = [\, l_1 cq_1 + l_2 cq_1 cq_2 + l_3 cq_1 cq_2 cq_3 - l_3 cq_1}$

$P_0 = \begin{bmatrix} l_1 cq_1 + l_2(cq_1 cq_2 - sq_1 sq_2) + l_3(cq_1 cq_2 cq_3 - cq_1 sq_2 sq_3 - sq_1 sq_2 cq_3 - sq_1 cq_2 sq_3) \\ l_1 sq_1 + l_2(sq_1 cq_2 + sq_2 cq_1) + l_3(sq_1 cq_2 cq_3 - sq_1 sq_2 sq_3 + cq_1 sq_2 cq_3 + cq_1 sq_3 cq_2) \\ 0 \end{bmatrix}$

$O_P = P_0$

For Jacobian:

$\begin{bmatrix} V_0^P \\ \omega_0^P \end{bmatrix} = J_0^P \dot{q} = [J_1 \;\; J_2 \;\; J_3]_{6\times 3} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$

$J_0^P = \begin{bmatrix} z_1 \times (O_P - O_1) & z_2 \times (O_P - O_2) & z_3 \times (O_P - O_3) \\ z_1 & z_2 & z_3 \end{bmatrix}$

$\begin{bmatrix} O_1 \\ 1 \end{bmatrix} = H_0^1 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} O_2 \\ 1 \end{bmatrix} = H_0^1 H_1^2 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} l_1 cq_1 \\ l_1 sq_1 \\ 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} O_3 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} l_1 cq_1 + l_2 cq_1 cq_2 - l_2 sq_1 sq_2 \\ l_1 sq_1 + l_2 sq_1 cq_2 + l_2 sq_2 cq_1 \\ 0 \\ 1 \end{bmatrix}$

Z- Unit Vectors:

$z_1 = R_0^1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $z_2 = R_0^1 R_1^2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$z_3 = R_0^1 R_1^2 R_2^3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\therefore z_1 \times (O_P - O_1) = \begin{bmatrix} 0 & -1 & 0 \\ +1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} P_0 = \begin{bmatrix} -P_{0y} \\ +P_{0x} \\ 0 \end{bmatrix}$

$= \begin{bmatrix} \cancel{l_1 sq_1 + l_2(sq_1 cq_2 + sq_2 cq_1) + l_3(sq_1 cq_2} \\ \\ 0 \end{bmatrix}$

$$z_2 \times (O_P - O_2) = \begin{bmatrix} 0 & -1 & 0 \\ +1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} l_2(C_{q_1}C_{q_2} - S_{q_1}S_{q_2}) + l_3(\qquad) \\ l_2(S_{q_1}C_{q_2} + S_{q_2}C_{q_1}) + l_3(\qquad) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -[l_2(S_{q_1}C_{q_2} + S_{q_2}C_{q_1}) + l_3(S_{q_1}C_{q_2}C_{q_3} - S_{q_1}S_{q_2}S_{q_3} + C_{q_1}S_{q_2}C_{q_3} + C_{q_1}S_{q_3}C_{q_2})] \\ +[l_2(C_{q_1}C_{q_2} - S_{q_1}S_{q_2}) + l_3(C_{q_1}C_{q_2}C_{q_3} - C_{q_1}S_{q_2}S_{q_3} - S_{q_1}S_{q_2}C_{q_3} - S_{q_1}S_{q_3}C_{q_2})] \\ 0 \end{bmatrix}$$

$$= [A_x \quad A_y \quad 0]^T$$

$$z_3 \times (O_P - O_3) = \begin{bmatrix} 0 & -1 & 0 \\ +1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} l_3(C_{q_1}C_{q_2}C_{q_3} - C_{q_1}S_{q_2}S_{q_3} - S_{q_1}S_{q_2}C_{q_3} - S_{q_1}S_{q_3}C_{q_2}) \\ l_3(S_{q_1}C_{q_2}C_{q_3} - S_{q_1}S_{q_2}S_{q_3} + C_{q_1}S_{q_2}C_{q_3} + C_{q_1}S_{q_3}C_{q_2}) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -l_3(S_{q_1}C_{q_2}C_{q_3} - S_{q_1}S_{q_2}S_{q_3} + C_{q_1}S_{q_2}C_{q_3} + C_{q_1}S_{q_3}C_{q_2}) \\ +l_3(C_{q_1}C_{q_2}C_{q_3} - C_{q_1}S_{q_2}S_{q_3} - S_{q_1}S_{q_2}C_{q_3} - S_{q_1}S_{q_3}C_{q_2}) \\ 0 \end{bmatrix}$$

$$= [B_x \quad B_y \quad 0]^T$$

$$\therefore J_0^P = \begin{bmatrix} -P_{oy} & A_x & B_x \\ +P_{ox} & A_y & B_y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$