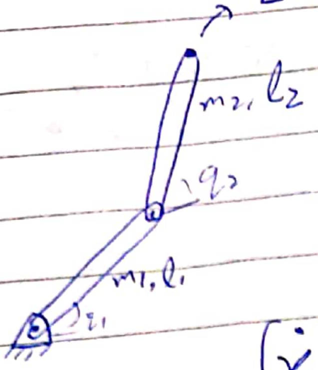
~~Assignment~~

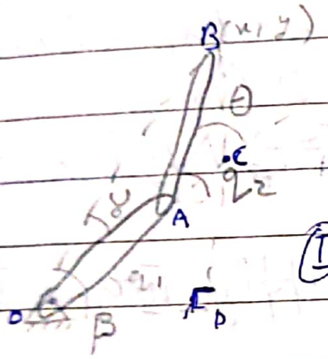
Mini Project

Task - 1



$$\therefore \quad x = l_1 \cos q_1 + l_2 \cos q_2 \quad -(0)$$
$$y = l_1 \sin q_1 + l_2 \sin q_2$$
$$\therefore \quad \dot{x} = -l_1 \sin q_1 \dot{q}_1 - l_2 \sin q_2 \dot{q}_2$$
$$\dot{y} = l_1 \cos q_1 \dot{q}_1 + l_2 \cos q_2 \dot{q}_2$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 \sin q_1 & -l_2 \sin q_2 \\ l_1 \cos q_1 & l_2 \cos q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad -(\mathrm{I})$$



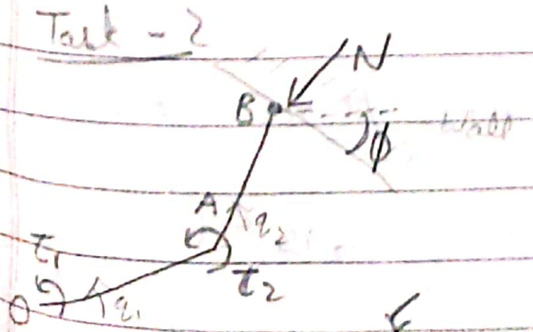
Using Cosine rule on Δ OAB

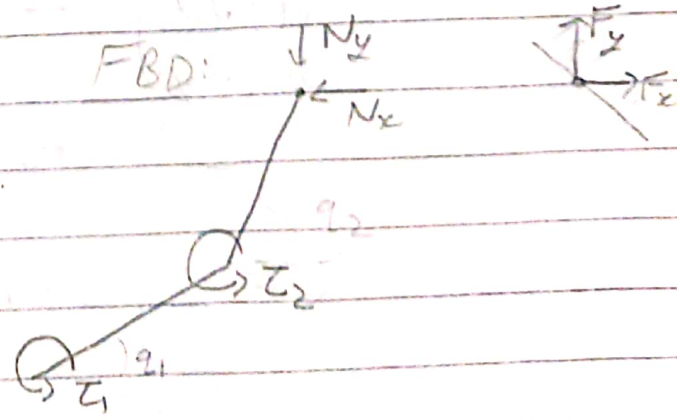
(II)
$$\angle BAC = \theta = \cos^{-1}\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$
$$q_1 = \beta - \gamma = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2 \cos\theta}\right)$$

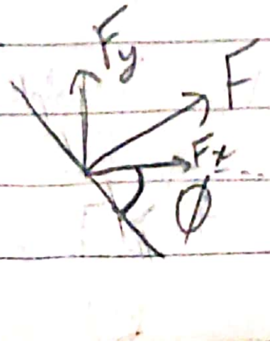
$$q_2 = \theta + q_1$$

2 Motors at O & A → Position Control mode to achieve q_1 & q_2 at each time step.

Task - 2





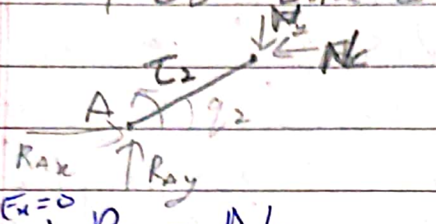


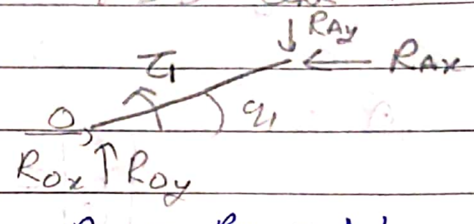
$$\therefore \quad F_y = -F\cos\phi$$
$$F_x = F\sin\phi$$

For Static Equilibrium (Assume no gravity)

$\Sigma M_O = 0$ & $\Sigma M_A = 0$ $\qquad N_x = F_x$ & $N_y = F_y$

FBD Link 2 $\qquad\qquad$ FBD link 1





$\Sigma F_x = 0$

$\therefore R_{Ax} = N_x \qquad\qquad R_{Ox} = R_{Ax} = N_x$

$R_{Ay} = N_y \qquad\qquad R_{Oy} = R_{Ay} = N_y$

$\Sigma M_A = 0 \qquad\qquad \Sigma M_O = 0$

Eq^n $\quad N_y l_2 \cos q_2 - N_x l_2 \sin q_2 = \tau_2 \qquad \tau_1 = N_y l_1 \cos q_1 - N_x l_1 \sin q_1$

After reaching the wall, apply torques τ_1 & τ_2

Task - 3

Langrange's Eq^n: $\quad \mathcal{L} = K - P$

K → Kinetic E. $\quad P$ → Potential E

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i'$$

Q_i' are generalized forces derived

$$\therefore K = \underbrace{\frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right)\dot{q}_1^2}_{\text{Pure rotation of } l_1} + \underbrace{\frac{1}{2} m_2 v_{c_2}^2}_{\text{Translate } l_2} + \underbrace{\frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2}_{\text{Pure rotation of } l_2}$$

What wa

$$v_{c_2}^2 = (l_1^2 \dot{q}_1^2) + \left(\frac{l_2^2}{4}\dot{q}_2^2\right) + \cancel{2}\, l_1 \frac{l_2}{\cancel{2}} \dot{q}_1 \dot{q}_2 \cos(q_2 - q_1)$$

Including Gravity:

$$V = m_1 g \frac{l_1}{2} \sin q_1 + m_2 g \left(l_1 \sin q_1 + \frac{l_2}{2} \sin q_2 \right)$$

$\therefore$ For q_1:

$$\frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + \frac{m_2 l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1) - \frac{m_2 l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_1 g \frac{l_1}{2} \cos q_1 + m_2 g l_1 \cos q_1 = \tau_1$$

$$\frac{1}{12} m_2 l_2^2 \ddot{q}_2 + \frac{1}{4} m_2 l_2^2 \ddot{q}_2 + \frac{m_2 l_1 l_2}{2} \ddot{q}_1 \cos(q_2 - q_1) + \frac{m_2 l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_2 g \frac{l_2}{2} \cos q_2 = \tau_2$$

In Matrix form:

$$\begin{bmatrix} \frac{1}{3} m_1 l_1^2 + m_2 l_1^2 & \frac{m_2 l_1 l_2}{2} \cos(q_2 - q_1) \\ \frac{m_2 l_1 l_2}{2} \cos(q_2 - q_1) & \frac{1}{12} m_2 l_2^2 + \frac{1}{4} m_2 l_2^2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{m_2 l_1 l_2}{2} (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) \\ \frac{m_2 l_1 l_2}{2} (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$= \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} - \begin{bmatrix} m_1 g \frac{l_1}{2} \cos q_1 + m_2 g l_1 \cos q_1 \\ m_2 g \frac{l_2}{2} \cos q_2 \end{bmatrix}$$

$$A\ddot{q} + B\dot{q} = \bar{\tau} - P \qquad \therefore \ddot{q} = A^{-1}(\tau - P - B\dot{q})$$

Next, we note that (III) is true for any end effector F_x, F_y & not just wall reaction

$$F_x = Rx$$
$$F_y = Ry$$

→ More general $F_x = R_x(x - x_0)$, $F_y = R_y(y - y_0)$

For simplicity, $F_x = Rx \quad F_y = Ry$

From (I)

$$F_x = R(l_1 \cos q_1 + l_2 \cos q_2)$$
$$F_y = R(l_1 \sin q_1 + l_2 \sin q_2)$$

Using (III)

$$\therefore \tau_{2s} = R(l_1 \sin q_1 + l_2 \sin q_2) l_2 \cos q_2 - R(l_1 \cos q_1 + l_2 \cos q_2) l_2 \sin q_2$$

What we want

$$\tau_{1s} = R(l_1 \sin q_1 + l_2 \sin q_2) l_1 \cos q_1 - R(l_1 \cos q_1 + l_2 \cos q_2) l_1 \sin q_1$$

Set Motor Torques as $\tau_1 + \tau_{1s}$ & $\tau_2 + \tau_{2s}$