Information Theory, Quiz 3 (April 12, 2023)

Serial Number: ______________ **Roll Number:** ________________________

Important

- Any malpractice will lead to instant fail grade.
- This is a question paper and answer sheet. Please provide only final answers in the spaces provided. Do not include derivations or proofs. You will be given a 8-page booklet for rough work.
- **Write your roll number and "serial number" in this sheet.**
- **No breaks during the exam.** No calculators, books, notes or electronic devices allowed.

All the best! Listen to prog metal:
♪And Plague Flowers the Kaleidoscope ♫

1. Consider the message sequence with alphabet $\{A, B\}$

$$B\ B\ B\ A\ A\ B\ B\ B\ A\ B\ B\ B\ B\ B\ A\ B$$

(a) (1 mark) List all the phrases (separated by commas) in this sequence (including the empty phrase), in the same order as they appear in the message.
`Answer:`

(b) (2 marks) Suppose we extend the message sequence be appending one more phrase. List all the phrases that can be the parents of this new phrase.
`Answer:`

(c) (2 mark) Suppose we extend the message sequence by appending one more phrase, and the parent of this new phrase is B. What will be this new phrase?
`Answer:`

2. (4 marks) Consider the two-pass LZ78 encoder for a sequence with alphabet $\{A, B\}$. Say the encoder represents the suffix A by 0, and B by 1. Suppose the number of phrases c is 7, and the codeword generated by LZ78 is:

$$0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 1$$

Build the dictionary for decoding this sequence.

Phrase	Index	Index of Parent	Suffix (A or B)

3. (2 marks) Let the alphabet $\mathcal{X} = \{0, 1, 2, 3, 4\}$, and let pmf $p_k = P[X = k]$ be as follows:

$$p_0 = 0.4, p_1 = 0.3, p_2 = 0.2, p_3 = 0.05, p_4 = 0.05.$$

Find the expected length of the Fano code for this random variable.
`Answer:`

4. (2 marks) Suppose a random variable has alphabet $\mathcal{X} = \{0, 1, 2, 3, 4\}$, and it is encoded using a prefix-free code with codeword lengths

$$\ell_0 = 3, \ell_1 = 3, \ell_2 = 4, \ell_3 = 4, \ell_4 = 2.$$

We do not know the pmf of this random variable. What is the relation between the entropy of the random variable $H(X)$ and the expected length L of the codeword when this prefix-free code is used? The relation must be as tight as possible.
`Answer:`