EE6310: Image and Video Processing Spring 2023

Gray Scale Point Operations

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Gray Scale Point Operations

Definition

- Recall that binary morphological operations are **neighborhood operations** due to $\mathbf{B}$
- As the name suggests, a point operation is performed on a pixel wise basis **without using neighborhood information**
- **Spatial relationships are not modified**
- $\mathbf{J}(i, j) = f[\mathbf{I}(i, j)]; 0 \leq i \leq N - 1, 0 \leq j \leq M - 1$

Gray Scale Point Operations

Linear Point Operations

- Simplest class of point operations: $\mathbf{J}(i, j) = \mathbf{P} \, . \, \mathbf{I}(i, j) + \mathbf{L}$
- **Scale** and **offset** image intensities
- **Affects image histogram**
    - Recall $\mathbf{H_I}(k)$ is the histogram of image $\mathbf{I}$ where $0 \leq k \leq (K-1)$ for a K level image

Gray Scale Point Operations

Linear Point Operations

- If $\mathbf{P} = 1$, $\mathbf{J}(i,j) = \mathbf{I}(i,j) + \mathbf{L}$
  - what happens when $\mathbf{L} > 0$?
  - how about when $\mathbf{L} < 0$?
- If $\mathbf{L} = 0$, $\mathbf{J}(i,j) = \mathbf{P} \,.\, \mathbf{I}(i,j)$
  - what happens when $\mathbf{P} > 1$?
  - how about when $\mathbf{P} < 1$?
- A demo...

Gray Scale Point Operations

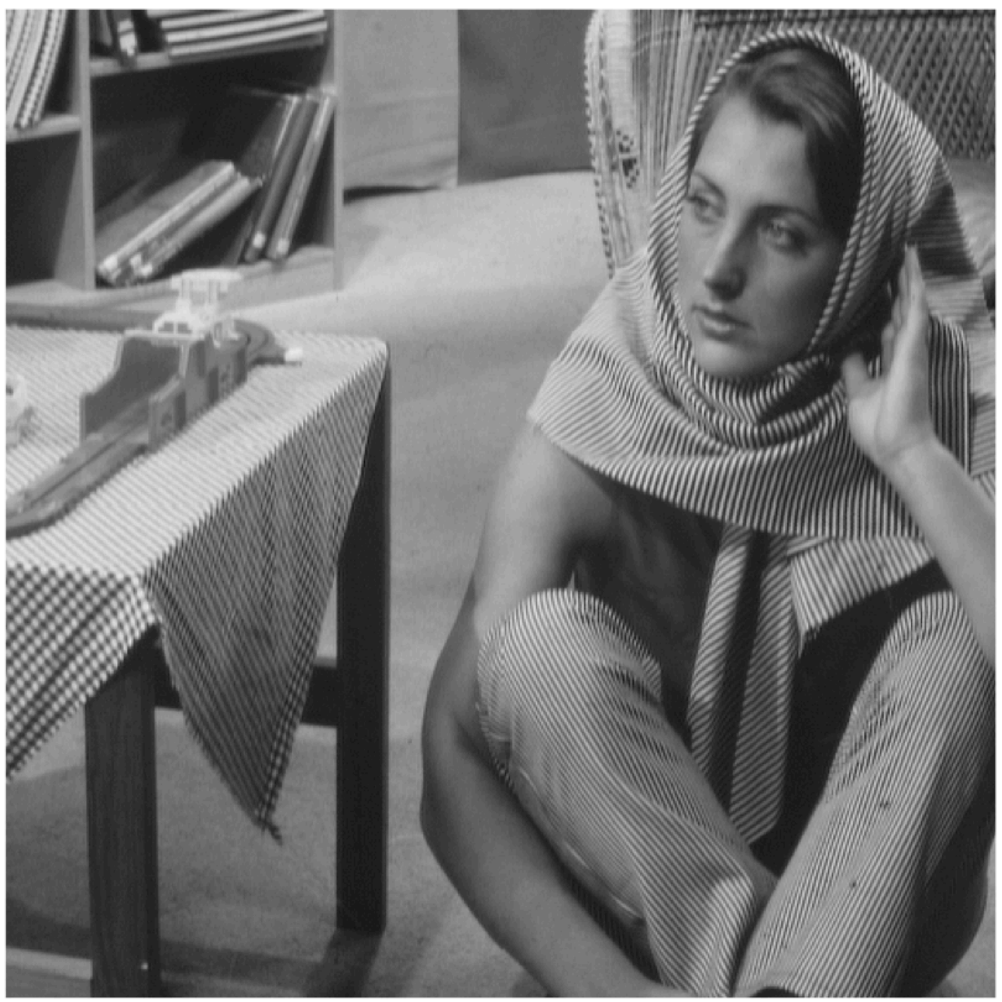
Linear Point Operations

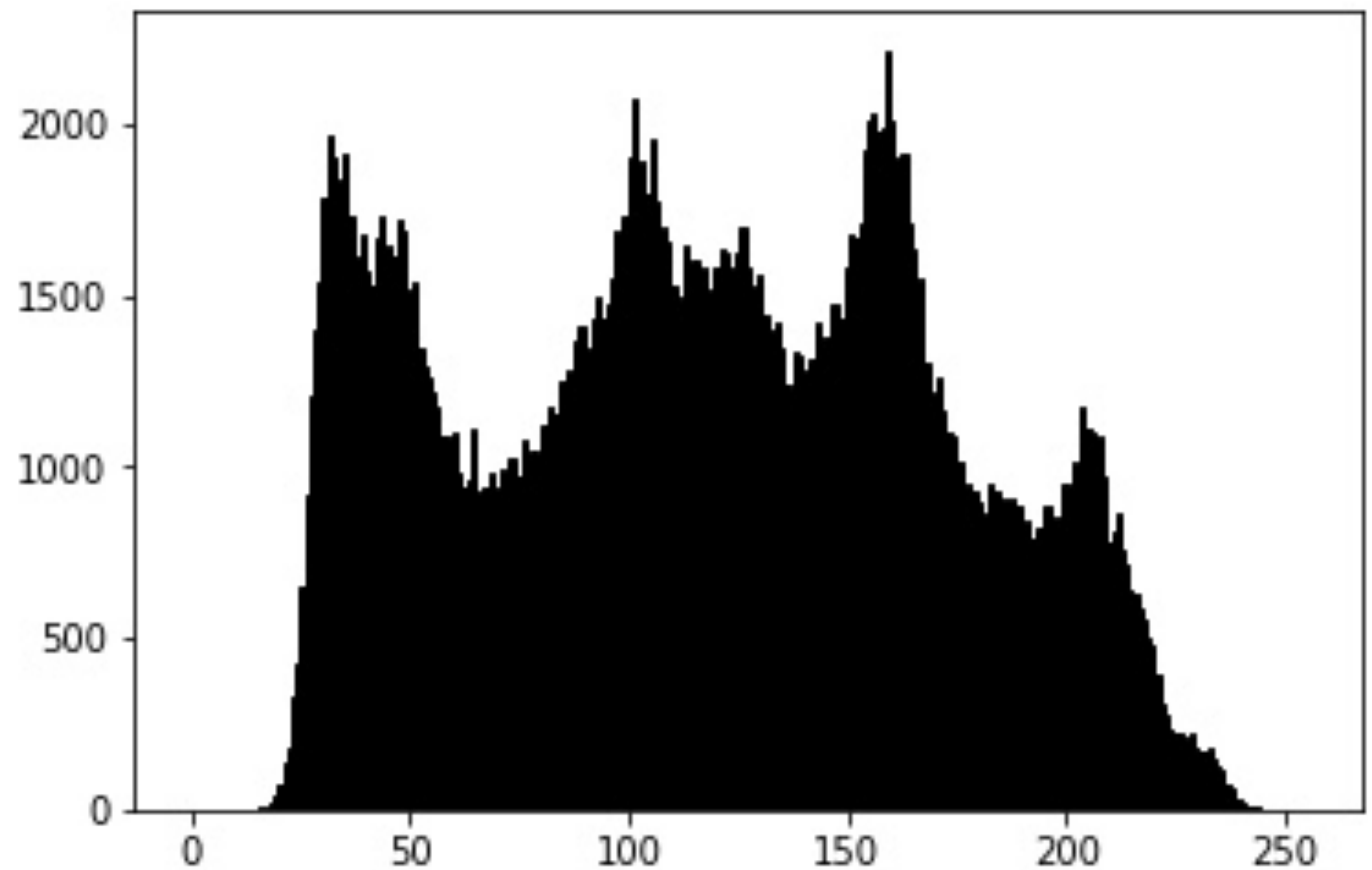
- To summarize:
  - If $\mathbf{P} = 1$, $\mathbf{J}(i, j) = \mathbf{I}(i, j) + \mathbf{L}$, $\mathbf{H}_{\mathbf{J}}(k) = \mathbf{H}_{\mathbf{I}}(k - \mathbf{L})$
  - If $\mathbf{L} = 0$, $\mathbf{J}(i, j) = \mathbf{P} \,.\, \mathbf{I}(i, j)$, $\mathbf{H}_{\mathbf{J}}(k) = \mathbf{H}_{\mathbf{I}}(\text{int}(k/\mathbf{P}))$

Gray Scale Point Operations

Linear Point Operations: Full Scale Contrast Stretch

- The **most common** linear point operation





Notice how the entire range of [0, 255] is not occupied

Gray Scale Point Operations

Linear Point Operations: Full Scale Contrast Stretch

- Let A and B be the minimum and maximum gray values of a K level image $\mathbf{I}$
- Apply point operation to $\mathbf{I}$: $\mathbf{J}(i, j) = \mathbf{P} \,.\, \mathbf{I}(i, j) + \mathbf{L}$
- Pick $\mathbf{P}$ and $\mathbf{L}$ such that
  - $\mathbf{P}A + \mathbf{L} = 0$
  - $\mathbf{P}B + \mathbf{L} = (K - 1)$
- Solving for $\mathbf{P}$ and $\mathbf{L}$:
  - $\mathbf{P} = \dfrac{K-1}{B-A}$
  - $\mathbf{L} = -A\dfrac{K-1}{B-A}$
- Another demo...

Gray Scale Point Operations

Non-linear Point Operations

- Let's consider non-linear point functions:
  - $\mathbf{J}(i,j) = f[\mathbf{I}(i,j)]; 0 \leq i \leq (N-1); 0 \leq j \leq (M-1)$
- A very large class of functions!
- A few **common** ones:
  - $\mathbf{J}(i,j) = [\mathbf{I}(i,j)]^2$
  - $\mathbf{J}(i,j) = \sqrt{[\mathbf{I}(i,j)]}$
  - $\mathbf{J}(i,j) = \log(1 + [\mathbf{I}(i,j)])$
  - $\mathbf{J}(i,j) = \exp([\mathbf{I}(i,j)])$

Gray Scale Point Operations

Log Range Compression

- Scenario: A small group of **very bright pixels** obscure rich information present in **less bright pixels** that are **less visible**
- Example: Astronomical images of galaxies

Gray Scale Point Operations

Log Range Compression

- Apply log range compression: $\mathbf{J}(i,j) = \log(1 + [\mathbf{I}(i,j)])$
- Bright intensities compressed heavily causing **faint details** to appear
- Apply **FSCS** to use full range


Gray Scale Point Operations

Histogram Flattening

- An image with a **flat** histogram
    - makes rich use of available grayscale
    - *might* be an image with smooth changes in intensities
    - *might* be an image with lots of texture
- Nonlinear point operations can be applied to an image to obtain an **approximately** flat histogram
    - How is this different from FSCS?

Gray Scale Point Operations

Histogram Flattening

- Defined **normalized histogram** to be
  - $p_{\mathbf{I}}(k) = \frac{1}{MN}\mathbf{H}_{\mathbf{I}}(k); 0 \leq k \leq (K-1)$
  - Satisfies $\sum_{k=0}^{K-1} p_{\mathbf{I}}(k) = 1$ (like a PMF)
  - The **cumulative histogram** is $P_{\mathbf{I}}(r) = \sum_{k=0}^{r} p_{\mathbf{I}}(k); r = 0,1,\ldots,(K-1)$, a non-decreasing function
  - $P_{\mathbf{I}}(K-1) = 1$
- Probabilistic interpretation:
  - $P_{\mathbf{I}}(r) = Pr\{I(i,j) \leq r\}$
  - $p_{\mathbf{I}}(r) = P_{\mathbf{I}}(r) - P_{\mathbf{I}}(r-1); r = 0,1,\ldots,(K-1)$

Gray Scale Point Operations

Histogram Flattening: Continuous Case

- If $p(x)$ and$P(x)$ are **continuous**, they can be regarded as probability density and cumulative distribution respectively
- $p(x) = \dfrac{dP(x)}{dx}$
- $P^{-1}(x)$ exists of defined by **convention**
- Problem: Transform $\mathbf{I}$ with $p(x), P(x)$ into image $\mathbf{K}$ with flat histogram
- Solution: $\mathbf{J}(i, j) = P(\mathbf{I}(i, j))$ will have a **flattened histogram** with range $[0,1]$
- $\mathbf{K} =$ FSCS($\mathbf{J}$) solves the problem

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Histogram Flattening: Continuous Case

- Recall: $\mathbf{J} = P(\mathbf{I})$ will have a **flattened histogram** with range $[0,1]$
- This means the following:
  - Let $\mathbf{Q}(x) = Pr\{\mathbf{J} \leq x\}$, then,
  - $\mathbf{Q}(x) = Pr\{P(\mathbf{I}) \leq x\} = Pr\{\mathbf{I} \leq P^{-1}(x)\}$
  - $\implies \mathbf{Q}(x) = P[P^{-1}(x)] = x$
  - $\therefore q(x) = \dfrac{dP(x)}{dx} = 1; 0 \leq x \leq 1$

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Histogram Flattening: Discrete Case

- To approximately flatten the histogram of image $\mathbf{I}$: define the cumulative histogram image $\mathbf{J} = P(\mathbf{I})$
- By definition, $P(\mathbf{I}),\ 0 \leq \mathbf{J} \leq 1$
- This means that the elements of $\mathbf{J}$ are approximately linearly distributed between 0 and 1
- As before, $\mathbf{K} = \text{FSCS}(\mathbf{J})$ yields the histogram flattened image

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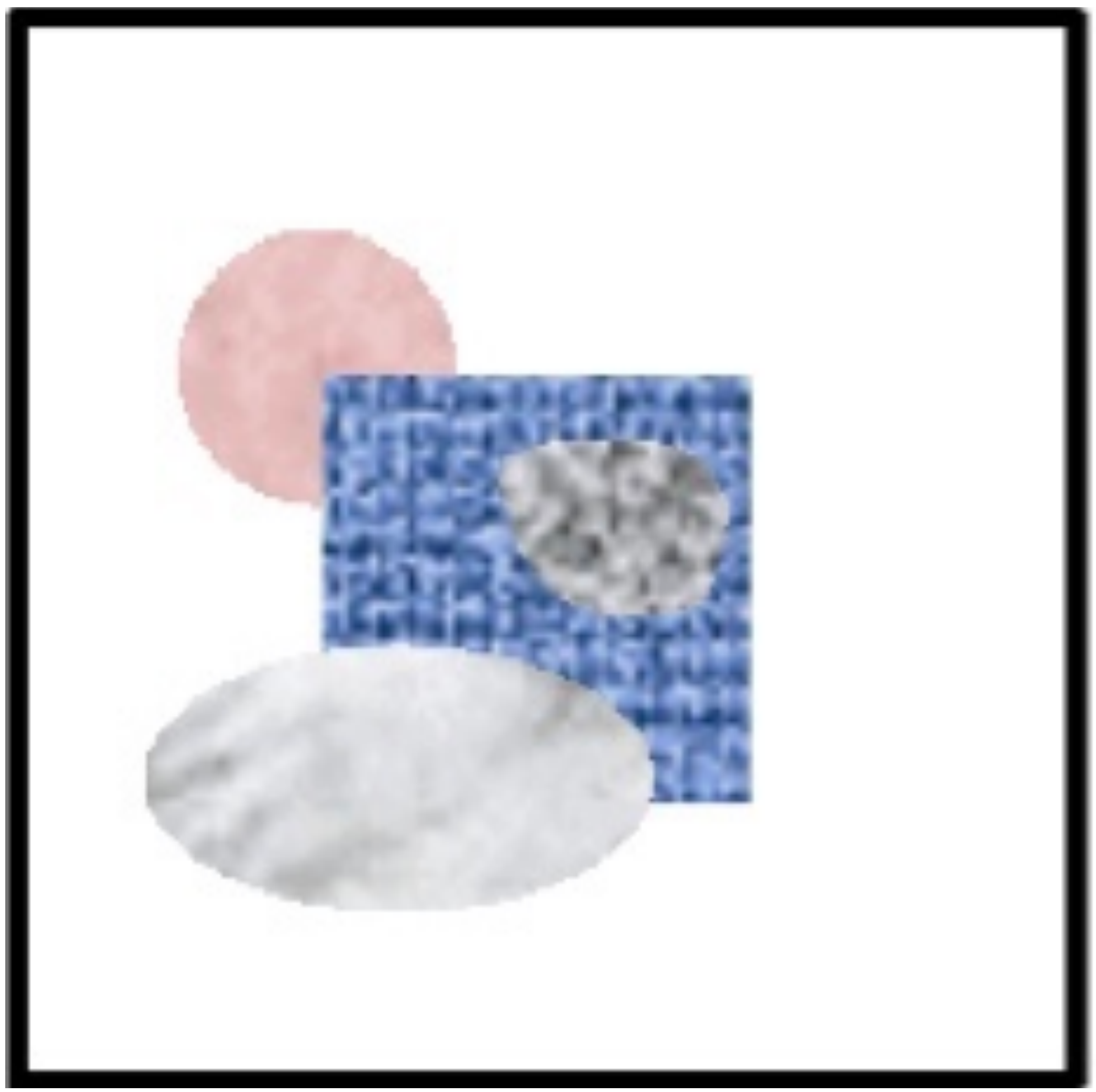
Histogram Flattening: Discrete Case

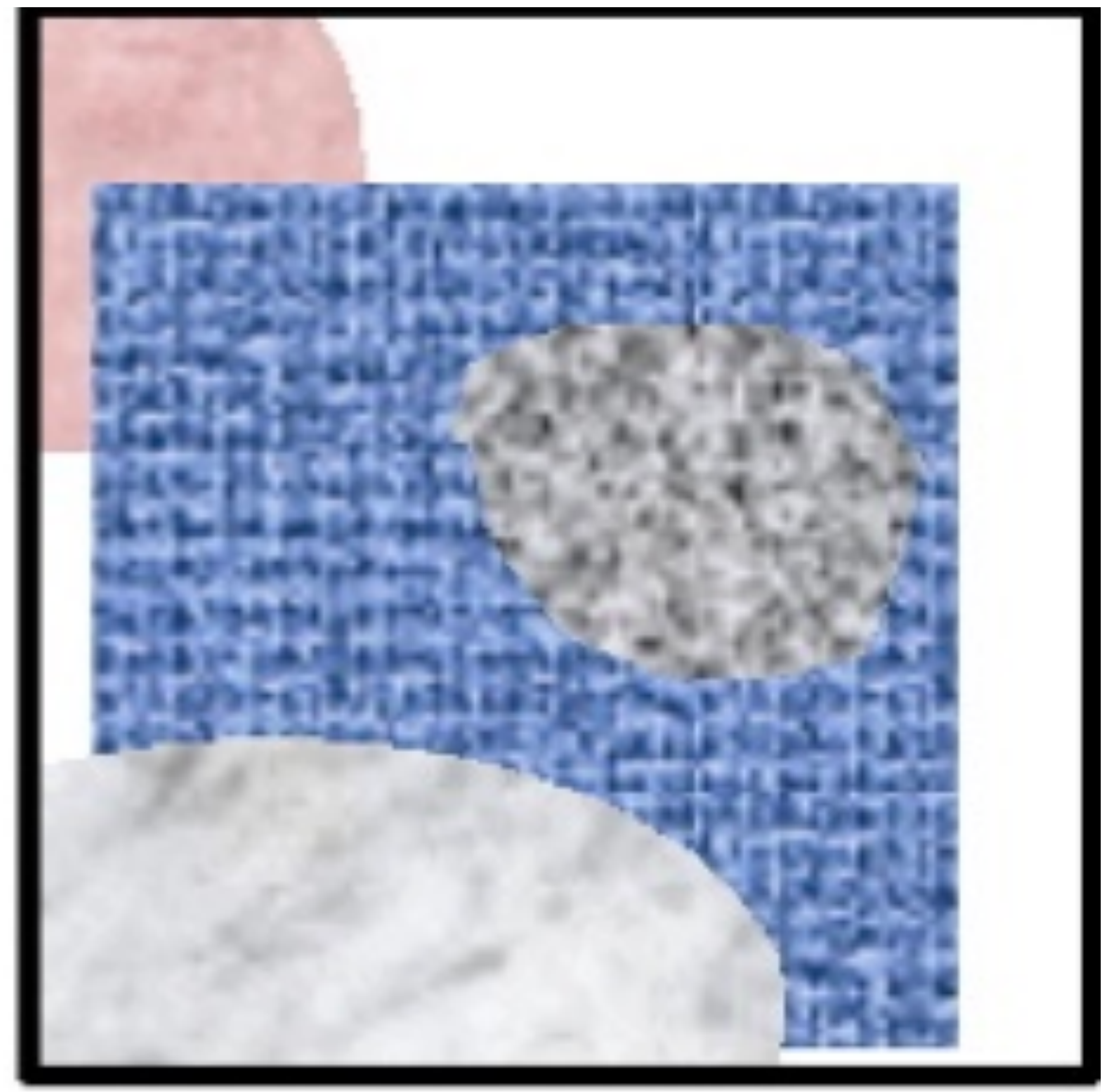
- Observations
  - The height of $H_{\mathbf{I}}(k)$ cannot be reduced
  - Flattening only spreads the histogram - **more flat**
  - Spaces are characteristics of flattened histograms

Gray Scale Neighborhood Operations

Image Zoom



Original Image



Zoomed Image

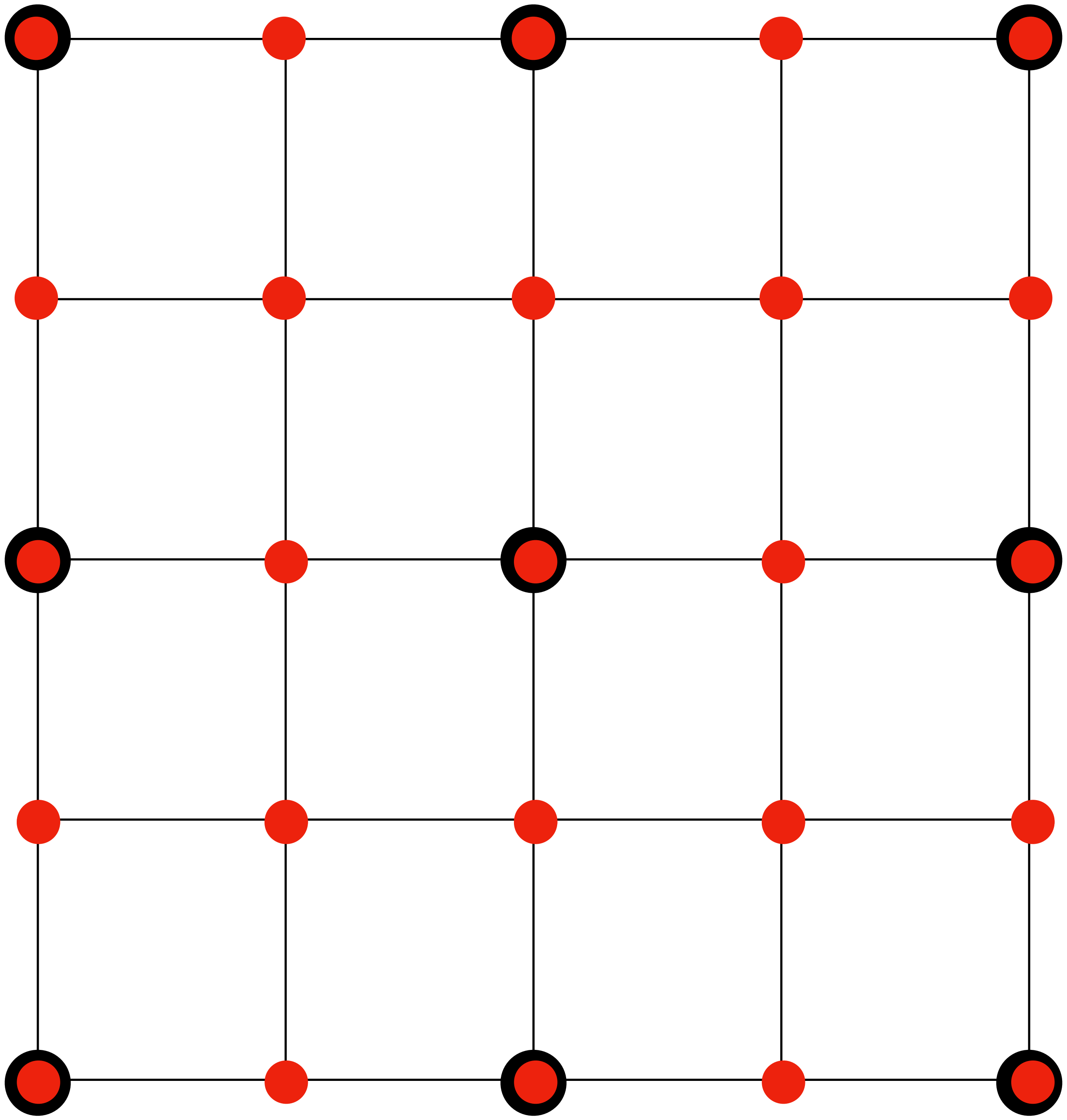
Gray Scale Neighborhood Operations

Image Zoom

- Zooming **magnifies** an image by **resampling** it
- Visualization:
  - Let $N \times N$ be the size of the original image
  - Let $M \times M$ be the size of the zoomed image ($M > N$)
  - Now fit larger image in space occupied by smaller image
  - Compute pixel values at the new grid points using nearest neighbor interpolation

Gray Scale Neighborhood Operations

Image Zoom



Gray Scale Neighborhood Operations

Image Zoom: Bilinear Interpolation

- Use four nearest neighbors for interpolation according to: $J(i, j) = A + Bi + Cj + Dij$
- How to find A, B, C, D?
  - Use four nearest neighbors of the location (i, j)
  - Smoothness assumption: $I(m, n) = A + Bm + Cn + Dmn$
  - Solve: $$\begin{bmatrix} 1 & i_0 & j_0 & i_0j_0 \\ 1 & i_1 & j_1 & i_1j_1 \\ 1 & i_2 & j_2 & i_2j_2 \\ 1 & i_3 & j_3 & i_3j_3 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} I(i_0, j_0) \\ I(i_1, j_1) \\ I(i_2, j_2) \\ I(i_3, j_3) \end{bmatrix}$$