Department of Electrical Engineering
IIT Hyderabad

EE 6340/3801
Wireless Communications

Channel

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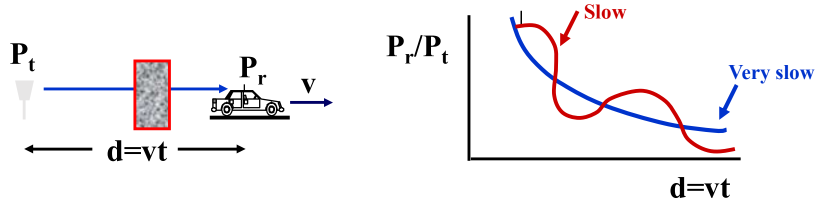
Lecture 3 Outline

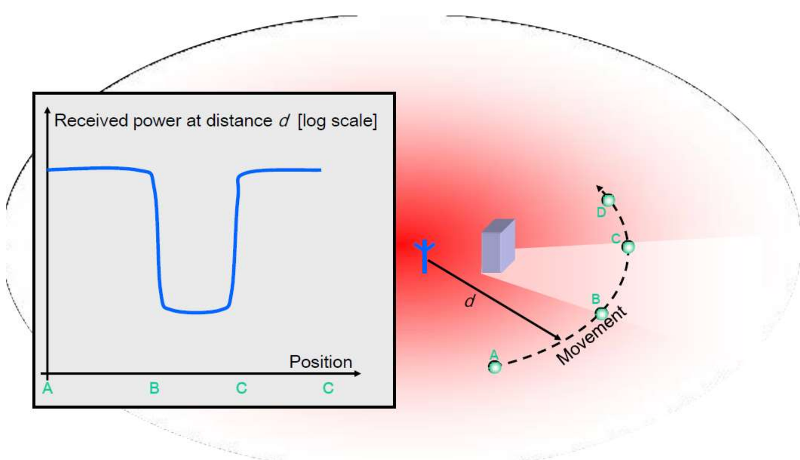
- Announcements
  - 1st HW posted, due next Monday 12 pm.
- Review of Last Lecture
  - Wireless Channel
  - TX and RX Signal Models
  - Path Loss Models
    - Free-space and 2-Ray Models
    - Simplified Path Loss Model
    - General Ray Tracing
    - Empirical Models
    - mmWave Models
- Narrowband Channel Model

Propagation Characteristics

- Path Loss (includes average shadowing)
- Shadowing (due to obstructions)
- Multipath Fading



Shadowing (Large Scale Fading)



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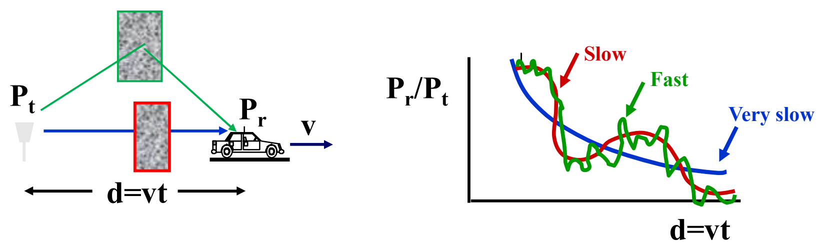
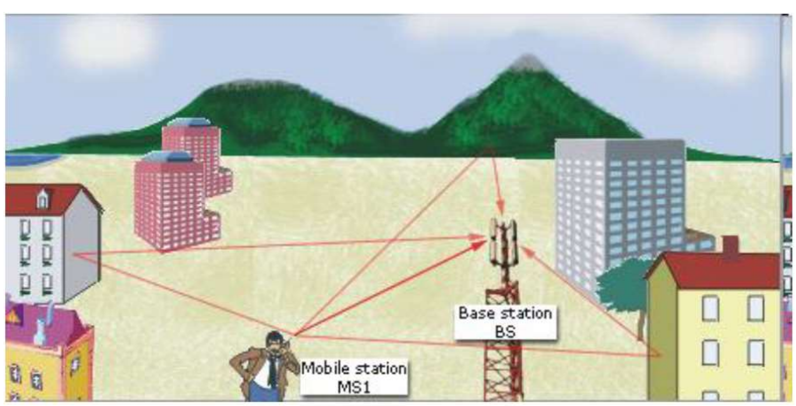
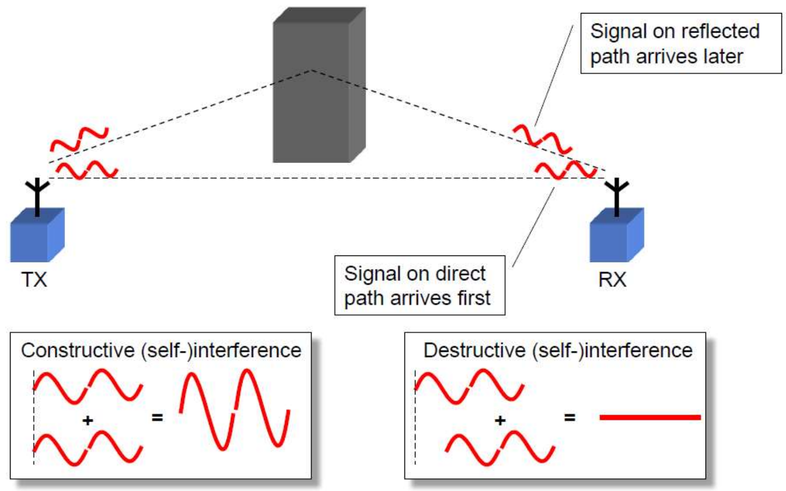


Multipath Fading



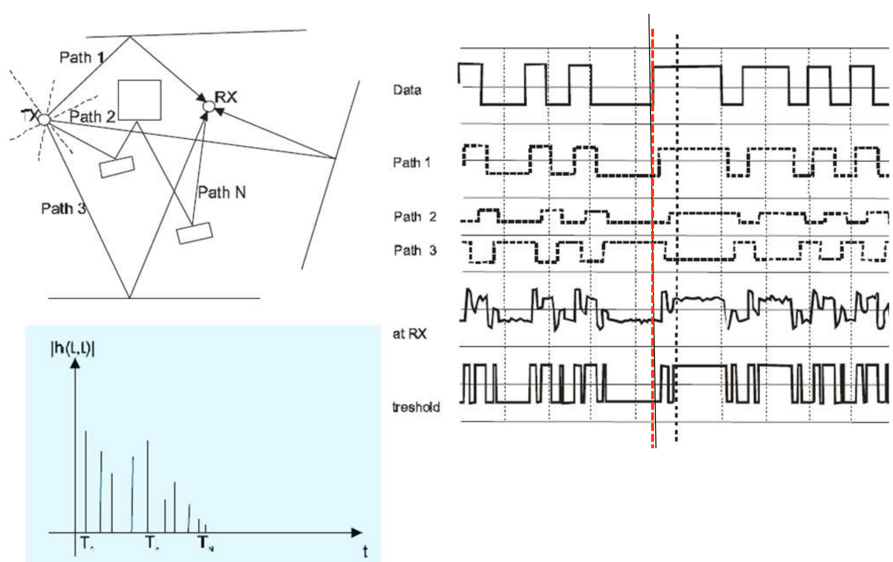
Effects of Multipath: 2

Fading (small scale)



Effects of Multipath: 2

Intersymbol Interference



Friis' Formula

- The received power for a nonisotropic TX antenna is given by

$$P_{RX}(d) = G_{TX} A_{RX} \frac{P_{TX}}{4\pi d^2}$$

- The effective Area is related to G_{RX} as

$$A_{RX} = \frac{\lambda^2}{4\pi} G_{RX}$$

- The Friis Formula is given by

$$\boxed{P_{RX}(d) = P_{TX} G_{TX} G_{RX} \left(\frac{\lambda}{4\pi d} \right)^2}$$

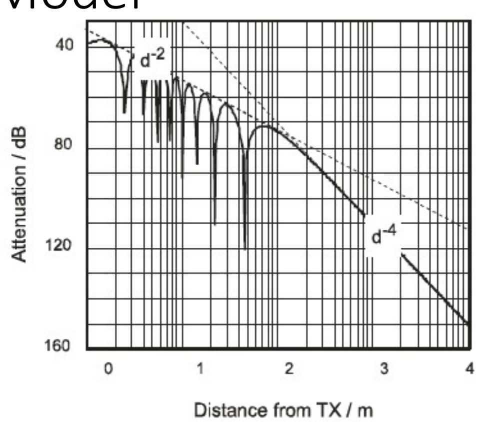
$$r(t) = Re\{v(t) e^{j(2fct+\emptyset 0)}\} = Re\left\{\left[\frac{\lambda\sqrt{G} e^{-j2\pi d/\lambda}}{4\pi d} u(t) + n(t)\right] e^{j(2fct+\emptyset 0)}\right\}$$

Limitations
- >30 MHz (VHF & higher)
- Far field of the antenna:

the far field requires
$d >> \lambda, La$
Where La Is the largest dimension of the antenna
- Received Signal

Simplified Path Loss Model

- Typically exponent varies based on surroundings with d ϵ [1.5, 8]
- Used when path loss dominated by reflections.
- Most important parameter is the path loss exponent γ, determined empirically.



$$P_r = P_t K \left[\frac{d_0}{d} \right]^{\gamma}, \qquad 2 \le \gamma \le 8$$

Empirical Channel Models

(not covered in lecture/HW/exams)

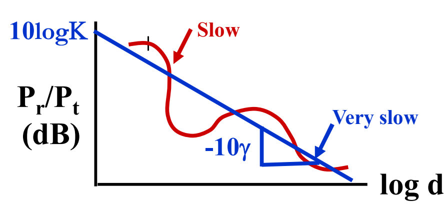
- Early cellular empirical models:
  - Empirical path loss models for early cellular systems were based on extensive measurements.
  - Okumura model: empirically based (site/freq specific), uses graphs
  - Hata model: Analytical approximation to Okumura
  - Cost 231 Model: extends Hata to higher freq. (2 GHz)
  - Multi-slope model
  - Walfish/Bertoni: extends Cost 231 to include diffraction
- Current cellular models (LTE and 5G):
  - Detailed path loss models for UE (3GPP TS 36.101) and base stations (3GPP TS 36.104) for different multipath delay spreads, user speeds and MIMO antenna correlations.
  - The 5G model includes higher frequencies (up to 100 GHz).
- WiFi channel models: TGn and TGac
  - Indoor and outdoor path loss models with MIMO (4x4 & greater), 40 MHz channels (& greater), and different multipath delay spread.

Commonly used in cellular and WiFi system simulations

Combined Path Loss and Shadowing

- Linear Model: ψ lognormal

$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma}\psi$$



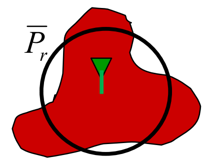
- dB Model

$$\frac{P_r}{P_t}(dB) = \underbrace{10\log_{10}K}_{K_{dB}} - 10\gamma\log_{10}\left(\frac{d}{d_0}\right) - \psi_{dB},\ \ \psi_{dB} \sim N(\mu_\psi, \sigma_\psi^2)$$

μ_ψ=0 when average shadowing incorporated into K and γ, else μ_ψ>0

Outage Probability

- Path loss only: circular "cells"; Path loss+shadowing: amoeba-shaped cells



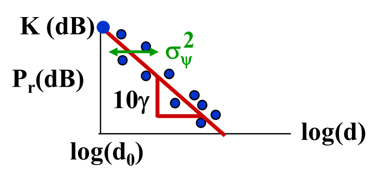
- Outage probability: probability received power falls below given minimum:

$$P_{out} = \mathrm{p}(P_r < P_{min})$$

- For log-normal shadowing model

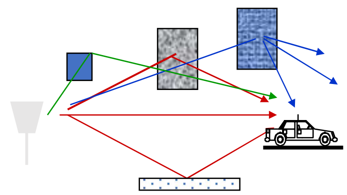
$$P_{out} = 1 - \mathrm{Q}\left(\frac{P_{min} - (P_t + 10log_{10}K - 10\gamma log_{10}(d/d_0))}{\sigma_{\psi_{dB}}}\right)$$

Model Parameters from Empirical Measurements



- Fit model to data
- Path loss (K,γ), d_0 known:
  - "Best fit" line through dB data
  - K obtained from measurements at d_0.
    - Or can solve for (K,γ) simultaneously (least squares fit)
  - Exponent is MMSE estimate based on data
  - Captures mean due to shadowing
- Shadowing variance
  - Variance of data relative to path loss model (straight line) with MMSE estimate for γ

Statistical Multipath Model



- Random # of multipath components, each with
  - Random amplitude
  - Random phase
  - Random Doppler shift
  - Random delay
- Random components change with time
- Leads to time-varying channel impulse response
- The movement of surrounding objects (e.g. vehicles) will also cause the time variation of the signals.

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FADING: WHAT IS FADING?

- **Path loss and shadowing is caused by large objects that are distant from MS.**
  - Even the MS is moving, the change in the relative position between MS and those distant large objects is small.
  - Therefore, the impairments caused by those large distant objects change very slow with respect to (w.r.t.) time and position.
  - Shadowing is also referred to as large scale fading.
- **Small scale fading is caused by the effects of objects that are close to MS.**
  - The movement of MS w.r.t. nearby small objects will dramatically change the reflection or diffractions of propagated signals.
  - The signal at receiver (sum of the signals from all multiple paths) will change rapidly with the movement of MS.

Small scale fading: rapid fluctuation of the received signals over short distance.

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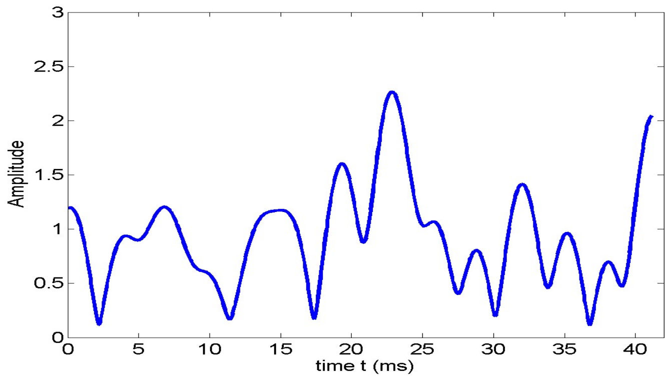
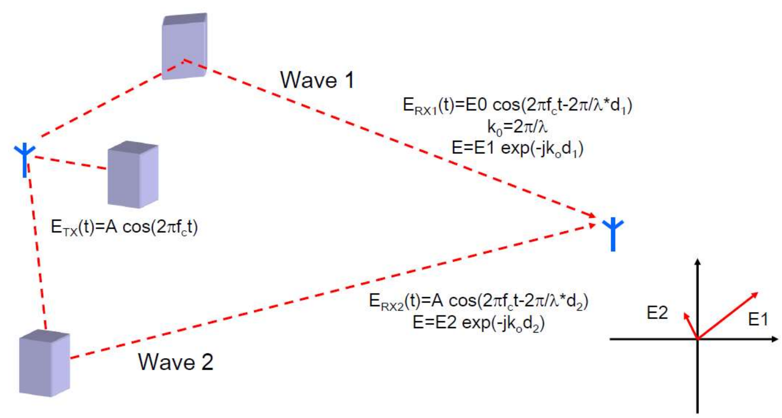
FADING: AN EXAMPLE



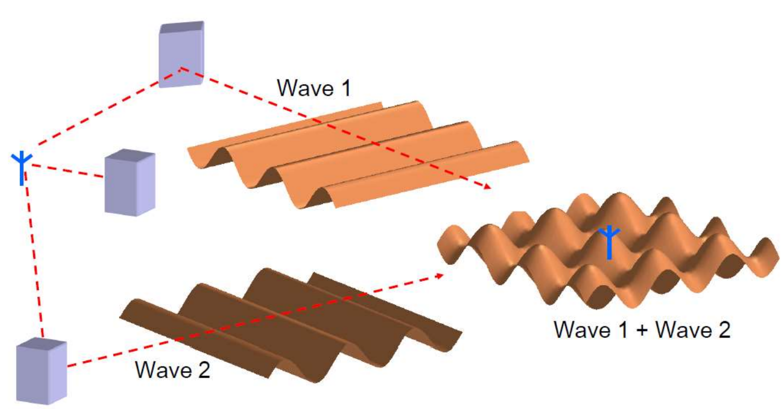
- **The rate of variation depends on two factors:**
  - Relative movement speed between Tx and Rx
  - Speed of surrounding objects

Narrowband Channels

Two Waves: Small Scale Fading



Two Waves: Small Scale Fading



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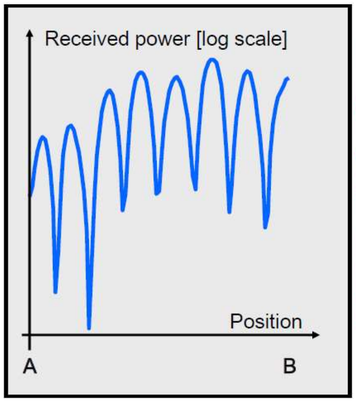
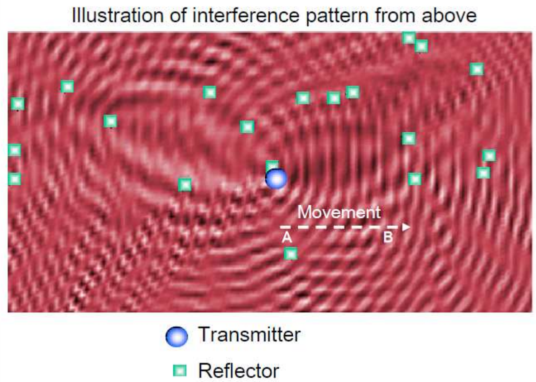
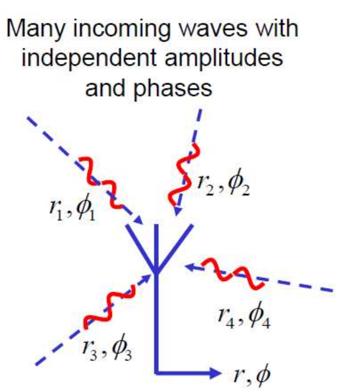
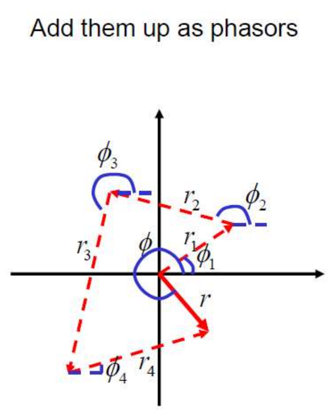




Many Waves: Small Scale Fading





$$r\exp(j\phi) = r_1\exp(j\phi_1) + r_2\exp(j\phi_2) + r_3\exp(j\phi_3) + r_4\exp(j\phi_4)$$

Narrowband Model

$$r(t) = Re\{v(t)e^{j(2fct+\emptyset 0)}\}$$
$$= Re\left\{\left[\frac{\lambda\sqrt{G}e^{-j2\pi d/\lambda}}{4\pi d}u(t) + n(t)\right]e^{j(2fc \quad \emptyset 0)}\right\}$$

- Assume delay spread $\max_{m,n}|\tau_n(t)\text{-}\tau_m(t)|<<1/B$
- Then $u(t\text{-}\tau) \approx u(t)$.
- Received signal given by

$$r(t) = \Re\left\{u(t)e^{j(2fc \quad \emptyset 0)}\left[\sum_{n=0}^{N(t)} \alpha_n(t)e^{-j\varphi_n(t)}\right] + n(t)e^{j(2fct \quad \emptyset 0)}\right\}$$

- No signal distortion (spreading in time)
- Multipath affects complex scale factor in brackets.
- Characterize scale factor by setting $u(t)=e^{j\phi_0}$*: or 1*

$$s(t) = \Re\{e^{j2\pi f_c t}\} = \cos 2\pi f_c t,$$

NarrowBand Channel Model

- The received signal

$$r(t) = \Re\left\{\left[\sum_{n=0}^{N(t)} \alpha_n(t)e^{-j\phi_n(t)}\right] e^{j2\pi f_c t}\right\}$$

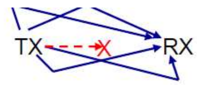
$$= r_I(t)\cos 2\pi f_c t + r_Q(t)\sin 2\pi f_c t,$$

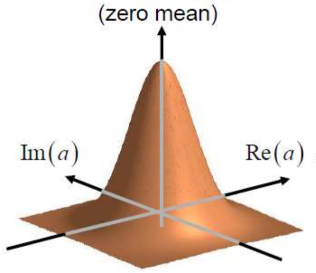
$$r_I(t) = \sum_{n=1}^{N(t)} \alpha_n(t)\cos\phi_n(t),$$

$$r_Q(t) = \sum_{n=1}^{N(t)} \alpha_n(t)\sin\phi_n(t).$$

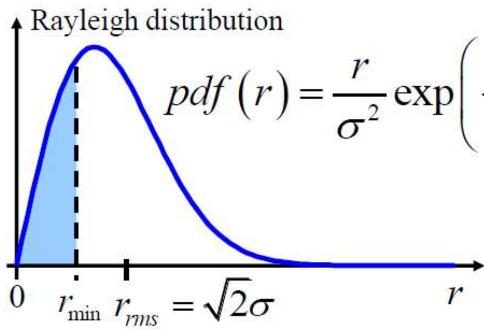
For large $N(t)$, $r_I(t)$ and $r_Q(t)$ jointly Gaussian by CLT

Rayleigh Fading Channel





Rayleigh Fading



$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

Fading Margin

$$M = \frac{r_{rms}^2}{r_{\min}^2}$$

$$M_{|dB} = 10\log_{10}\left(\frac{r_{rms}^2}{r_{\min}^2}\right)$$

$$\Pr(r < r_{\min}) = \int_0^{r_{\min}} pdf(r)\,dr = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right)$$

Rayleigh Fading-Fading Margin

- How many dB fading margin, against Rayleigh fading, do we need to obtain an outage probability of 1 % ?

$$\Pr(r < r_{\min}) = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) = 1\% = 0.01$$

$$1 - 0.01 = \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) \Rightarrow \ln(0.99) = -\frac{r_{\min}^2}{r_{rms}^2}$$

$$\Rightarrow \frac{r_{\min}^2}{r_{rms}^2} = -\ln(0.99) = 0.01 \quad \Rightarrow M = \frac{r_{rms}^2}{r_{\min}^2} = 1/0.01 = 100$$

$$\boxed{M_{|dB} = 20}$$

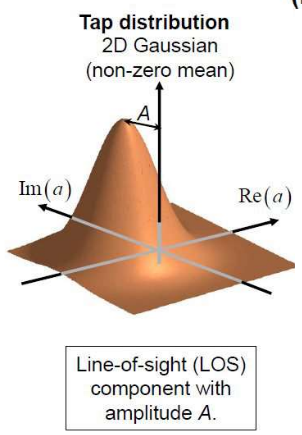
Ricean Fading: One Dominant Factor

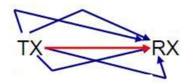
- In case of Line-of-Sight (LOS) one component dominates.
- Assume it is $\mathrm{Re}(r) \in N(A, \sigma^2) \quad \mathrm{Im}(r) \in N(0, \sigma^2)$
- The received amplitude has now a Ricean distribution instead of a Rayleigh
- The ratio between the power of the LOS component and the diffuse components is called Ricean K-factor

$$k = \frac{\text{Power in LOS component}}{\text{Power in random components}} = \frac{A^2}{2\sigma^2}$$

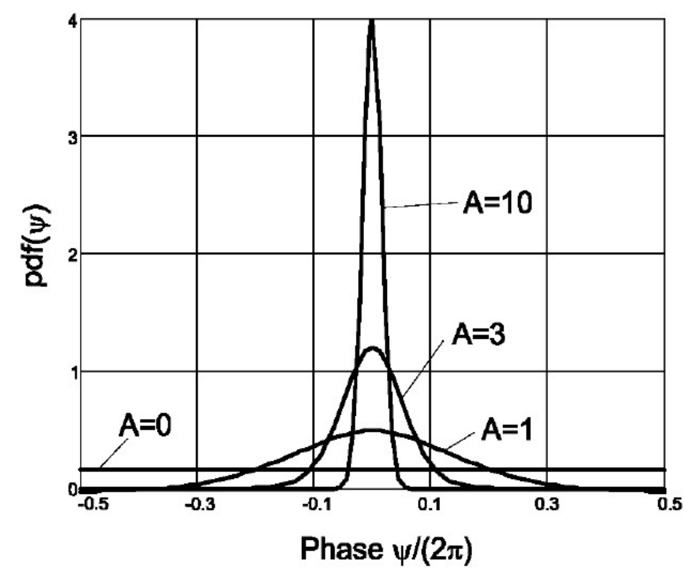
Ricean fading

A dominant component (line of sight)



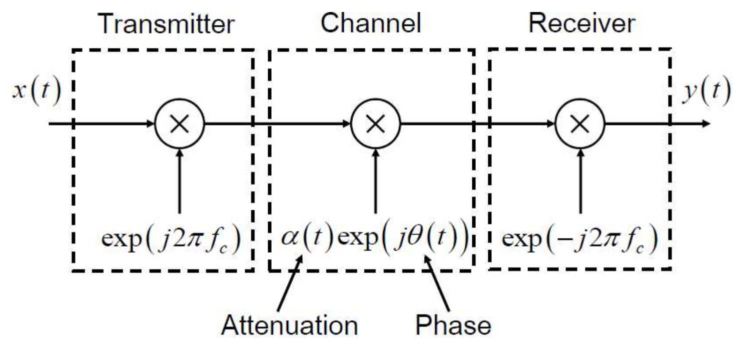


Phase Distribution



The Narowband Multipath Channel without Noise

$$\mathrm{In}: \quad x(t) = \boxed{A(t)}\exp(j\boxed{\phi(t)})$$



$$\text{Out: } y(t) = A(t)\exp(j\phi(t))\,\cancel{\exp(j2\pi f_c t)}\,\alpha(t)\exp(j\theta(t))\,\cancel{\exp(-j2\pi f_c t)}$$
$$= \boxed{A(t)\alpha(t)}\exp(j(\boxed{\phi(t)+\theta(t))})$$