Analog Lab
Experiment 12: Signal strength detector

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1. Finding frequency components of half wave rectified sinusoidal signal

Consider a half wave rectified sinusoidal signal x(t) with-

$$x(t) = \begin{cases} sin(\omega_o t), & \text{if } t \in \left(\frac{2\pi k}{\omega_o}, \frac{2\pi k + \pi}{\omega_o}\right) \text{ and } k \in \mathbb{Z} \\ 0, & \text{otherwise} \end{cases}$$

Now the fourier coefficients (a_k for complex exponentials) of a periodic signal can be written as-

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t} \leftrightarrow a_k = \frac{1}{T_o}\int_{T_o} x(t)e^{-jk\omega_o t}\, dt$$

where the integral is performed over one time period. In this case, we can take $T_0 = \frac{2\pi}{\omega_o}$ and the limits of integral from 0 to $\frac{2\pi}{\omega_o}$. Substituting the above we get-

$$a_k = \frac{\omega_o}{2\pi}\int_0^{\frac{2\pi}{\omega_o}} x(t)e^{-jk\omega_o t}\, dt$$

$$a_k = \frac{\omega_o}{2\pi}\int_0^{\frac{\pi}{\omega_o}} A sin(\omega_o t)e^{-jk\omega_o t}\, dt$$

$$a_k = \frac{A\omega_o}{2\pi}\int_0^{\frac{\pi}{\omega_o}} \frac{e^{j(\omega_o - k\omega_o)t} - e^{-j(\omega_o + k\omega_o)t}}{2j}\, dt$$

$$a_k = \frac{A\omega_o}{4\pi}\left(\frac{1 - e^{j(1-k)\pi}}{\omega_o(1-k)} + \frac{1 - e^{-j(1+k)\pi}}{\omega_o(1+k)}\right) \text{ for } k \neq 1$$

and when k = 1 we have $a_1 = \frac{A}{4j}$.

Now relation between a_k and sinusoidal coefficients can be derived as follows-

$$a_k = A_k e^{j\theta_k}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t} = \sum_{k=-\infty}^{\infty} A_k e^{j(k\omega_o t + \theta_k)}$$

And since x(t) is real we can use the symmetry properties of fourier transform and re-write the above equation as follows-

$$x(t) = a_o + \sum_{k=1}^{\infty} 2A_k cos(k\omega_o t + \theta_k)$$

So the frequency components are $2A_k$ (with sign due to θ_k) for the k^{th} harmonic.

DC component is $a_0 = \frac{A}{\pi}$

For 1^{st} harmonic- $a_1 = \frac{A}{4j} \rightarrow A_1 = \frac{A}{4}$ and $\theta_1 = -90^o$

So the coefficient is $\frac{A}{2}$

For 2^{nd} harmonic- $a_2 = \frac{-A}{3\pi} \rightarrow A_2 = \frac{A}{3\pi}$ and $\theta_2 = 180^o$

So the coefficient is $\frac{-2A}{3\pi}$

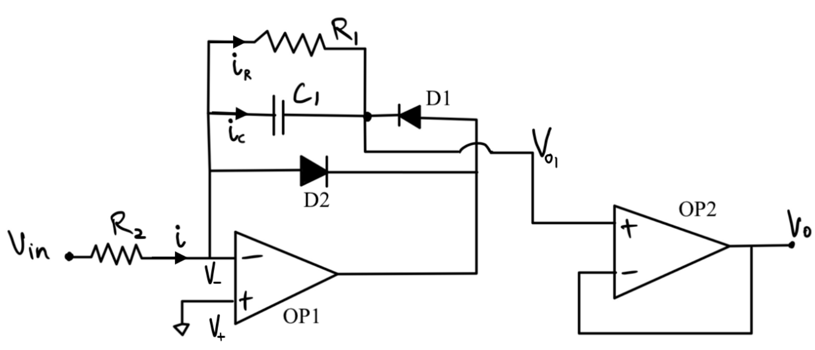
For 3^{rd} harmonic- $a_3 = 0 \rightarrow A_3 = 0$ and $\theta_3 = 0^o$

So the coefficient is 0

2. To implement half-wave rectifier with first-order filter with

Input Frequency: 0.1kHz to 5kHz
Choose RC to reject lowest input frequency
LF347 Op-Amp with +5V/−5V power supply
Diode: 1N4148

Here is the circuit for a half wave rectifier that will be used in a signal strength detector. We can see that OP1 is in negative feedback and so we can use the virtual short condition $V_- = 0V$.



Applying KVL across R_2 -

$$i = \frac{V_{in} - V_-}{R_2} = \frac{V_{in}}{R_2}$$

So when V_{in} is positive (current i is positive), current must flow through D2 (as it cannot flow through D1) and hence V_o of OP1 will become $-V_D$ and all current i will flow through D2. When V_{in} is negative (current i is negative), current must flow into V_{in} and hence will flow through D1. In that case, applying KCL at V_- of OP1-

$$\frac{0-V_{in}}{R_2} + \frac{0-V_{o1}}{R_1} + \left(0 - V_{o1}\right)sC_1 = 0$$

$$V_{o1} = \left(\frac{-V_{in}R_1}{R_2}\right)\left(\frac{1}{1+sC_1R_1}\right)$$

So essentially as you can see, V_{in} passes through the circuit to V_{o1} with a gain of low pass filter to remove the higher frequency components (only when V_{in} is negative).

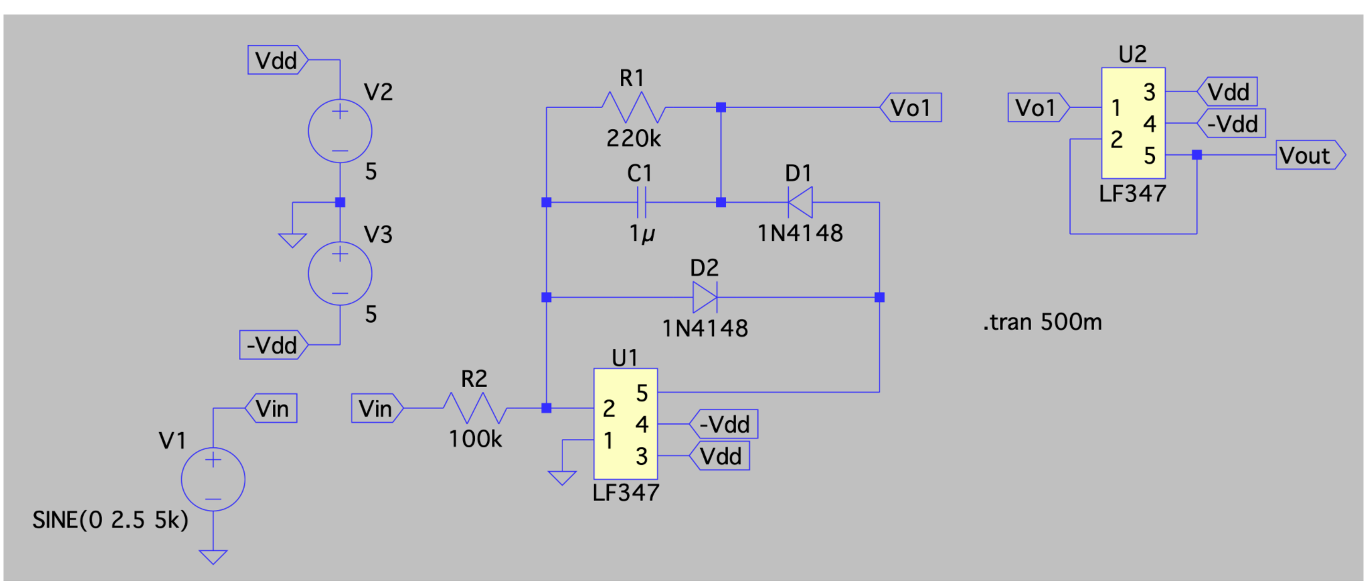
Since we are designing a signal strength detector that outputs a DC voltage (whose magnitude depends on the peak of input signal) we need to reject all the AC components, i.e. cutoff frequency of the parallel RC must be much lower than the minimum input frequency (100Hz). So-

$$\frac{10}{2\pi R_1C_1} < f_{min} \quad \Rightarrow \quad R_1C_1 > 15.916\, ms$$

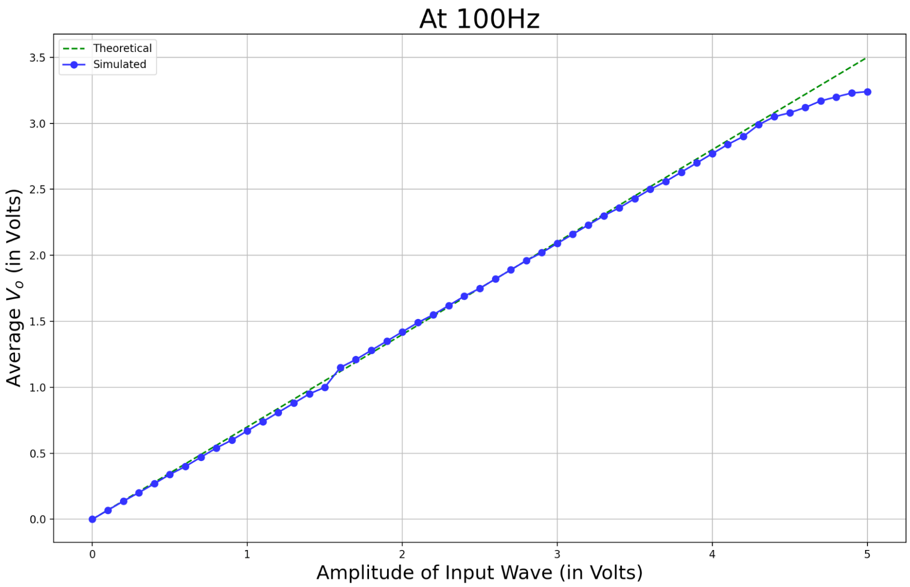
Since this is a first order filter it is not good enough to filter out all higher order frequencies, so we can overdesign the filter and compensate for the voltage gain by reducing R_2 . Also it is ideal to keep small capacitance (less time to charge) and large resistance (less charge loss).

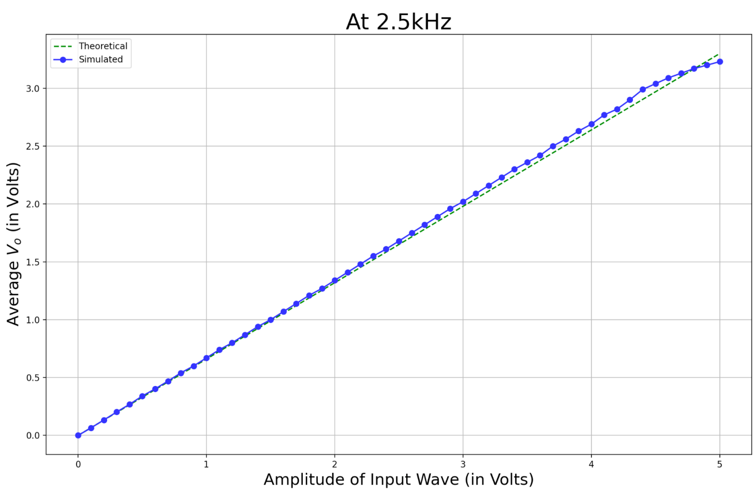
For this experiment I took C_1 as $1\mu F$, R_1 as $220k\Omega$ and R_2 as $100k\Omega$.

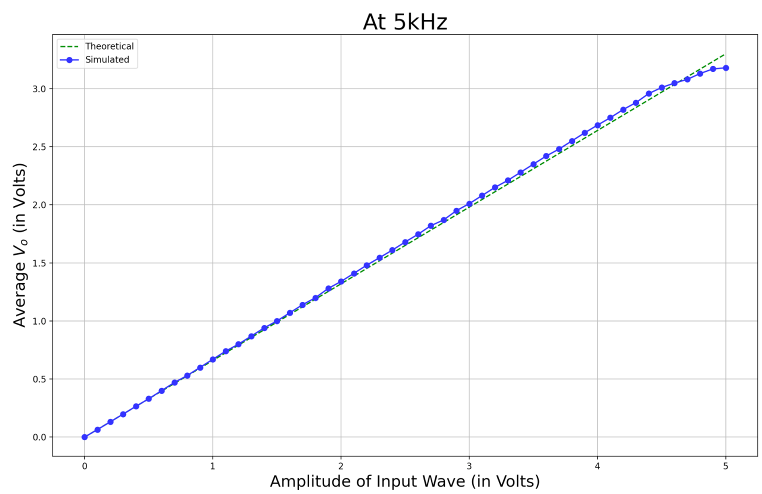
Implemented Circuit in LTSpice

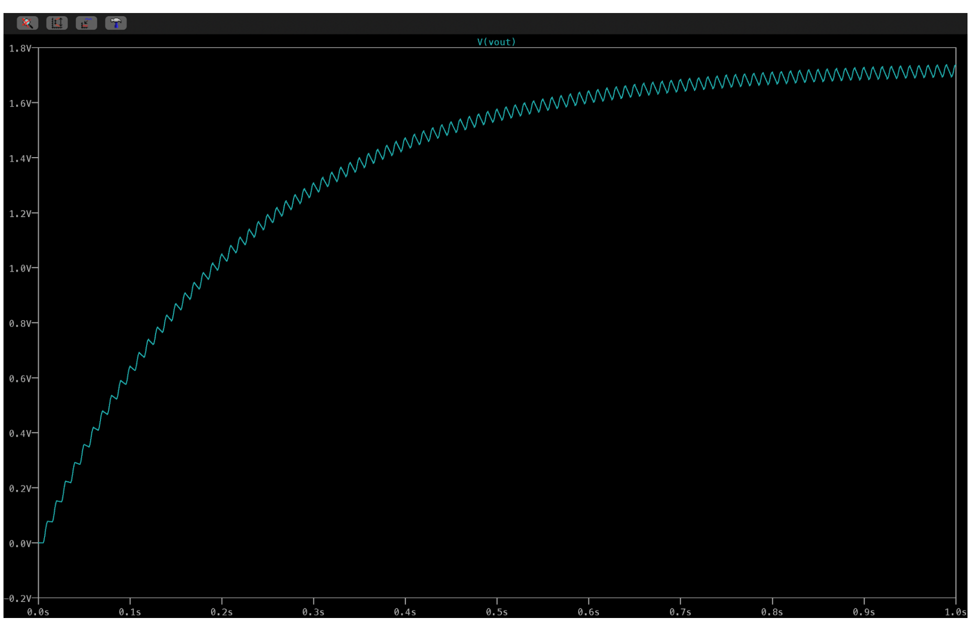


Average V_{out} at steady state vs Amplitude of input voltage



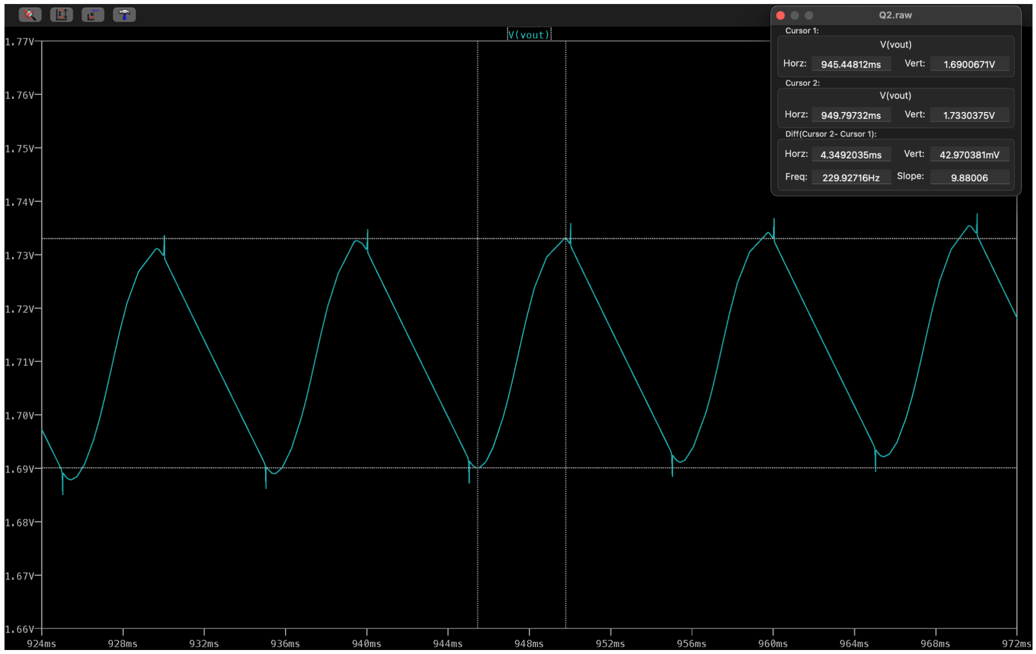


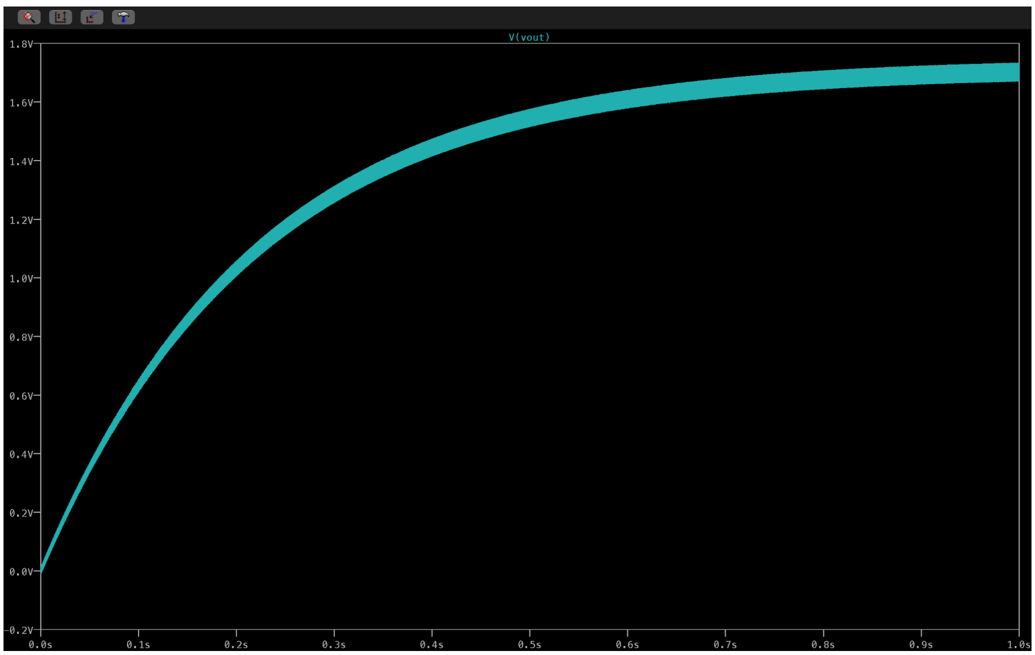




Transient Plot at A = 2.5V and f = 100Hz

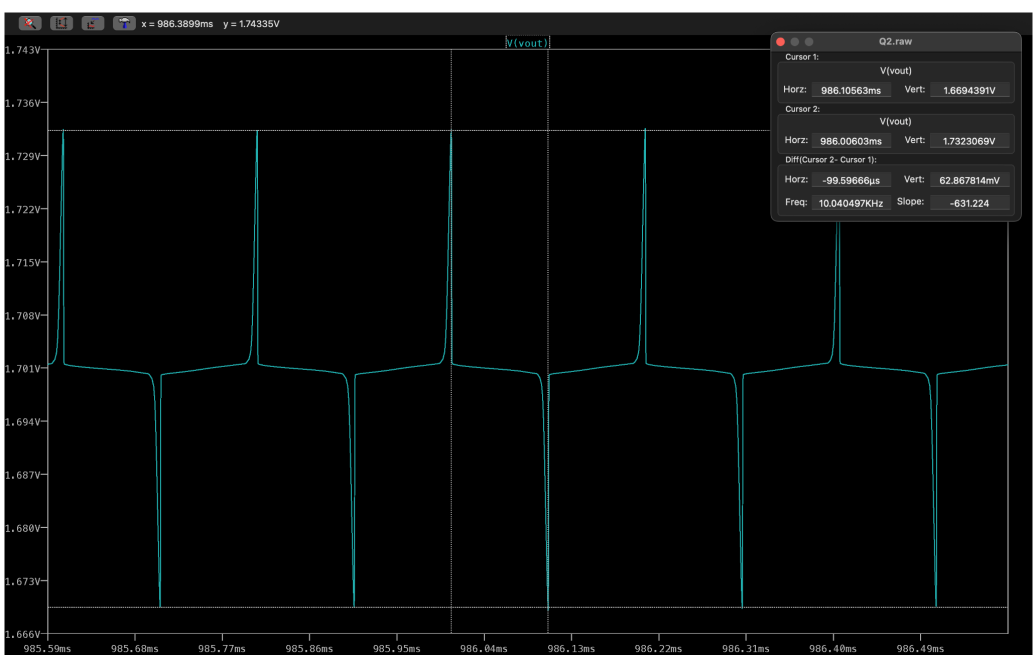
The ripple voltage here is 43mV and average voltage is around 1.71V





Transient Plot at
A = 2.5V and f = 5kHz

The ripple voltage here is 62.9mV and average is around 1.7V



Some observations-

1) The rectifier works close to ideal behavior as seen in the average V_o vs A plots except at higher frequencies.

2) The rectifier works almost the same even after varying input frequencies.

3) There are ripples in output voltages due to unfiltered harmonics.

4) From the transient plots, we can see that it takes around 500ms for the output to settle which is not good enough for commercial systems and this can be improved by reducing the time constant RC of the circuit. One setback that this might cause is the leakage of higher frequency components (can be avoided by using higher order filters).

5) The average voltage for A = 2.5V and frequencies at 0.1kHz and 5kHz is at 1.7V which is good as steady state average output voltage should only depend on the amplitude of input voltage and not frequency.

6) The spikes observed at higher frequencies can be attributed to the non-ideality of the Op-Amps.

7) V_o gets clipped at higher input amplitudes due to Op-Amp saturation voltage at 3.5V.