Information Theory, Quiz 1 (Jan 19, 2023)

Serial Number: _____ **Roll Number:** _____

Important

- Any malpractice will lead to instant fail grade, irrespective of your performance in past/future quizzes.
- This is a question paper and answer sheet. Please provide only final answers in the spaces provided. Do not include derivations or proofs. You will be given a 8-page booklet for rough work.
- **Write your roll number and "serial number" in this question/answer sheet.** The serial number is available in the attendance list with the TA.
- No breaks during the exam. No books, notes, laptops, calculators, mobile devices etc. are allowed.

Instructions: Choose **ALL the correct options** for the following questions. If none of the options are correct, you must write "NONE OF THE ABOVE" as your answer.

Example. If the correct options to a question are both (a) and (b), then the only correct answer to this question is: "(a) and (b)". In this case the answers "(a)", "(b)", or "None of the above" will all be incorrect and will be awarded no marks.

1. (1 mark) Choose all the correct answers.
   (a) If $\mathcal{X}$ is a finite set then $H(X)$ is finite.
   (b) $H(X) \geq 0$.

   Answer:

2. (1 mark) Which of the following statements are true for all jointly distributed random variables X, Y, Z ?
   (a) $H(Y|X = x) \leq H(Y)$ for all $x \in \mathcal{X}$.
   (b) $H(Y|X, Z) \leq H(Y|X)$.

   Answer:

3. (1 mark) Choose all the correct answers.
   (a) $D\left(p_{X,Y,Z} \| p_X p_Y p_Z\right) = H(X) + H(Y) + H(Z) - H(X, Y, Z)$.
   (b) $D\left(p_{X,Y|Z} \,\|\, p_{X|Z} p_{Y|Z} \,|\, p_Z\right) = I(X; Y|Z)$.

   Answer:

4. (1 mark) Which of the following detection rules provide the smallest value of probability of error P_e ?
   (a) $g(y) = \arg\max_{x \in \mathcal{X}} p_{X|Y}(x|y)$.
   (b) $g(y) = \arg\max_{x \in \mathcal{X}} p_{X,Y}(x, y)$.

   Answer:

Instructions: Provide only the final answers for the below questions.

5. (1 mark) Write the chain rule expansion of the joint entropy $H(X_1, \ldots, X_n)$.

   Answer:

6. (1 mark) Write the chain rule expansion of $D(p_{X,Y} \| q_{X,Y})$.

   Answer:

7. (1 mark) Write the Fano's inequality for the case $|\mathcal{X}| = 2$, i.e., write the inequality using the fact $|\mathcal{X}| - 1 = 1$.

   Answer:

8. (2 marks) Let $\mathcal{X} = \{1, 2, 3, 4\}$ and let $P[X = k] = p_k$, for $k = 1, 2, 3, 4$. Let $h_2(\alpha) = \alpha \log\left(\frac{1}{\alpha}\right) + (1 - \alpha) \log\left(\frac{1}{1-\alpha}\right)$ be the binary entropy function.

   Write $H(X)$ in terms of $h_2\left(p_1 + p_2\right), h_2\left(\frac{p_1}{p_1+p_2}\right)$ and $h_2\left(\frac{p_3}{p_3+p_4}\right)$.

   Answer:

9. (2 marks) If X is distributed as follows, find the value of $H(X)$.

$$P[X = k] = \frac{1}{2^k}, \text{ for } k = 1, 2, 3, \ldots$$

   Answer:

10. (1 mark) With X distributed as in the previous question, what is the value of $H(X \,|\, X \geq 3)$?

    Answer: