Communication Systems Lab
Assignment 3

Name- Pushkal Mishra
Roll- EE20BTECH11042

Downsampling and Decimation-

Downsampling by a factor of M is the process of sampling every M^{th} time instant in a discrete time signal, essentially reducing the number of samples. Corresponding mathematical equation is as follows-

$$x_d[n] = x[Mn]$$

In the frequency domain, we are convolving the input signal with a periodic impulse train. The issue comes when the bandwidth of the input signal is greater than half the period of the impulse train which leads to aliasing. To prevent this, we can pass the input signal first through an anti-aliasing filter to remove high frequency components and then perform downsampling. This process is known as decimation. Calculation of cutoff frequency of LPF can be performed as follows-

$$2B \leq \frac{2\pi}{M} \quad \Rightarrow \quad \omega_c = \frac{\pi}{M}$$

In the given question, the input signal has 80 samples and the filter has 51 samples so after convolution there will be 130 samples which does not match the number of samples of the input signal. So we extract 80 samples having the highest energy, i.e. discard the first and last 25 samples ((51 – 1) / 2) and then send this signal to the downsampler.

Upsampling and Interpolation-

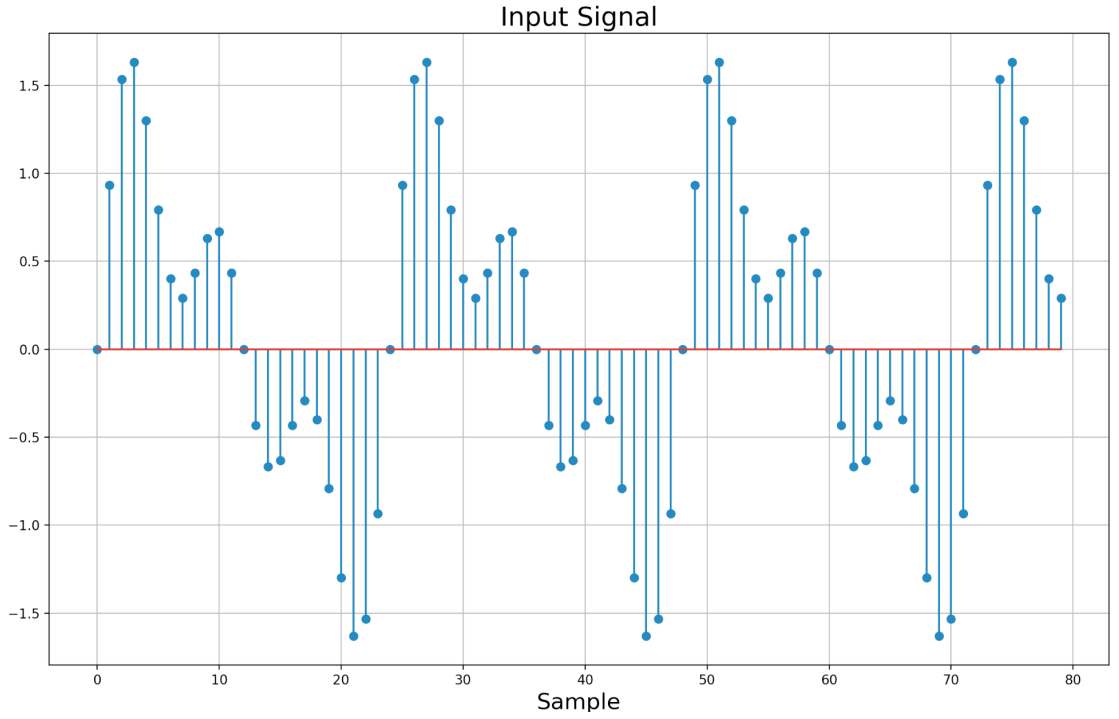
Upsampling by a factor of L is the process of inserting L – 1 zeros between every consecutive sample in a discrete time signal, essentially increasing the length of the signal. Corresponding mathematical equation is as follows-

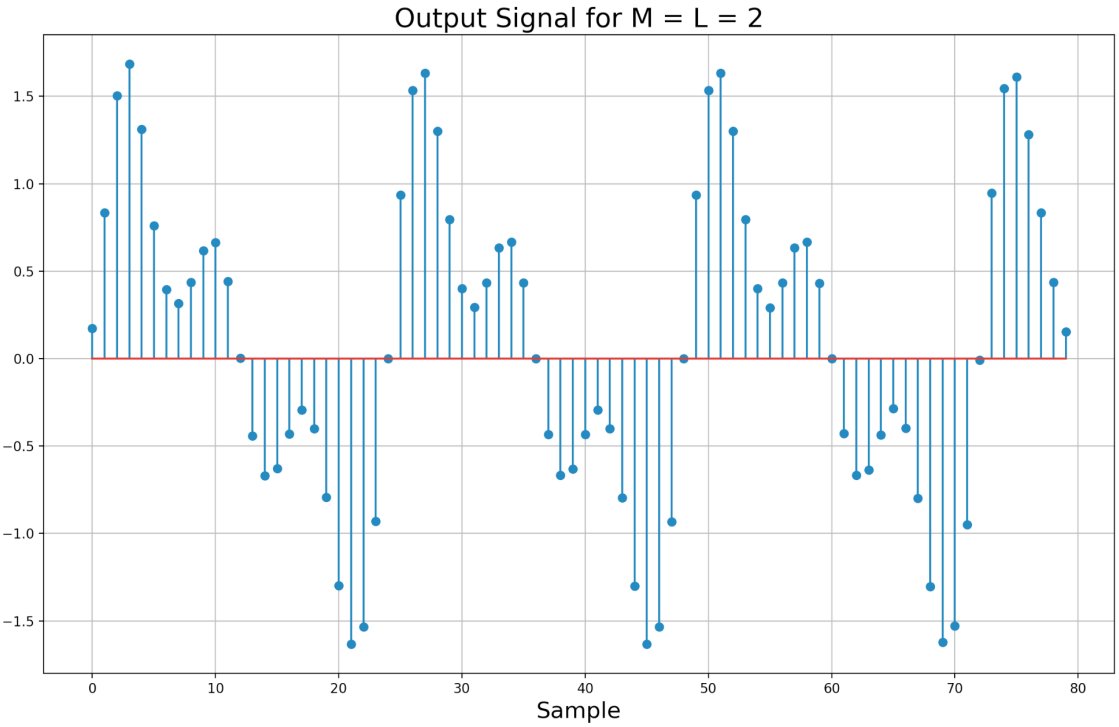
$$x_u[n] = \begin{cases} x_d\left[\frac{n}{L}\right] & \text{if n is a multiple of L} \\ 0 & \text{otherwise} \end{cases}$$

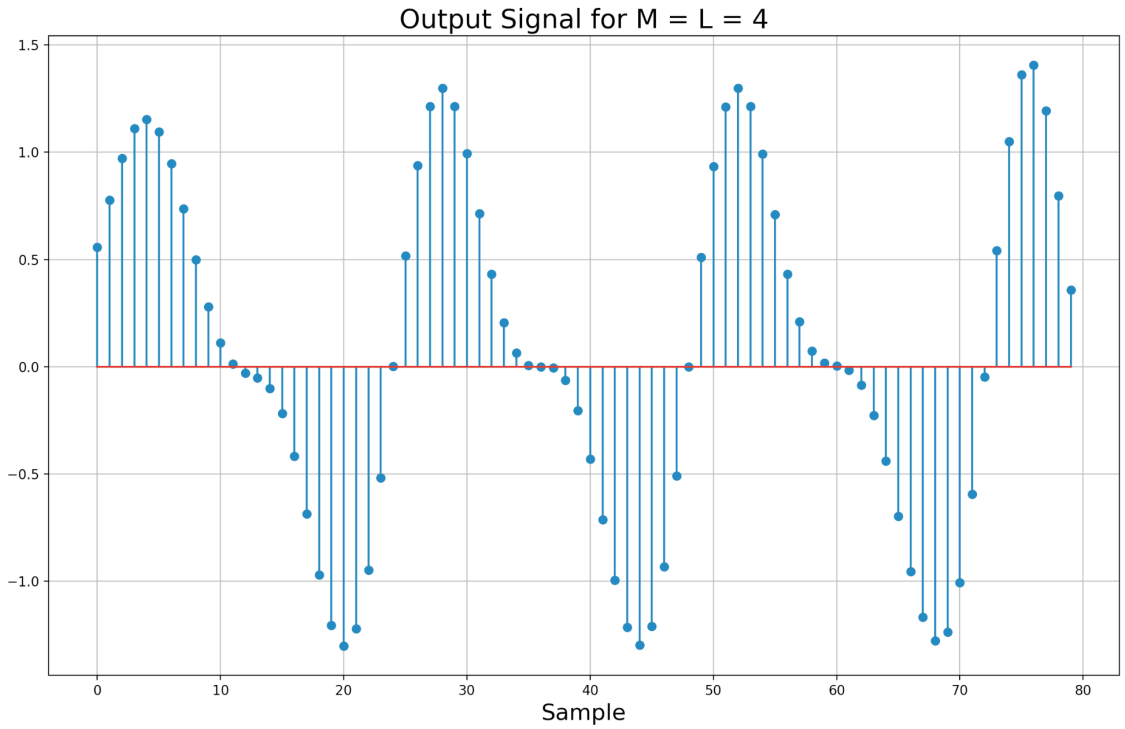
During upsampling in the frequency domain multiple copies of the signal are replicated at periodic intervals. So to choose the band of interest, we pass this signal through a low pass filter with cutoff frequency $\omega_c = \frac{\pi}{L}$.

Here the gain of the filter is set to L (= M) because during downsampling the input signal is scaled with a factor of $\frac{1}{M}$ in the frequency domain. This process is called interpolation.

Again the similar issue of length of signal rises up, in the question after upsampling the length of signal is 80 samples but after convolution the length becomes 130 samples. So we choose the samples with highest energy, i.e. discard the first and last 25 samples.







Average error in case of- M = 2 is 0.000061

M = 4 is 0.003171

Also observe that as the value of M increases, so does the error in the reconstructed signal increases which happens due to loss of information.