Department of Electrical Engineering
IIT Hyderabad

EE 6340/3801
Wireless Communications

Channel

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Lecture 5 Outline

- Announcements
  - TA: **ANNU SURAJPU,** ee21resch01010@iith.ac.in
- Review of Last Lecture
  - Doppler
  - Introduction to Wideband Channel
- Today's lecture
  - Wideband Channel
  - Scattering Function
  - Multipath Intensity Profile
  - Doppler Power Spectrum

Narrowband Model

$$r(t) = Re\{v(t)e^{j(2fct+\emptyset 0)}\}$$
$$= Re\left\{\left[\frac{\lambda\sqrt{G}e^{-j2\pi d/\lambda}}{4\pi d}u(t) + n(t)\right]e^{j(2f \quad \emptyset 0)}\right\}$$

- Assume delay spread $\max_{m,n}|\tau_n(t)-\tau_m(t)|<<1/B$
- Then $u(t-\tau) \approx u(t)$.
- Received signal given by

$$r(t) = \Re\left\{u(t)e^{j(2fc \quad \emptyset 0)}\left[\sum_{n=0}^{N(t)} \alpha_n(t)e^{-j\varphi_n(t)}\right] + n(t)e^{j(2fct+\emptyset 0)}\right\}$$

- No signal distortion (spreading in time)
- Multipath affects complex scale factor in brackets.
- Characterize scale factor by setting $u(t)=e^{j\phi_0}$: *or 1*

$$s(t) = \Re\{e^{j2\pi f_c t}\} = \cos 2\pi f_c t,$$

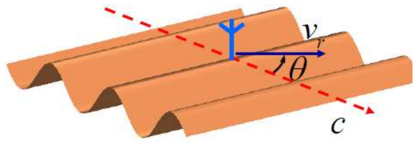
NarrowBand Channel Model

- The received signal

$$r(t) = \Re\left\{\left[\sum_{n=0}^{N(t)} \alpha_n(t)e^{-j\phi_n(t)}\right]e^{j2\pi f_c t}\right\}$$
$$= r_I(t)\cos 2\pi f_c t + r_Q(t)\sin 2\pi f_c t,$$
$$r_I(t) = \sum_{n=1}^{N(t)} \alpha_n(t)\cos\phi_n(t),$$
$$r_Q(t) = \sum_{n=1}^{N(t)} \alpha_n(t)\sin\phi_n(t).$$

For large $N(t)$, $r_I(t)$ and $r_Q(t)$ jointly Gaussian by CLT

Doppler



Receiving antenna moves with speed v_r at an angle ϑ relative to the propagation direction of the incoming wave, which has frequency f_c.

Frequency of received signal:

$$f = f_c + \upsilon$$

where the Doppler shift is

$$\upsilon = -f_c \frac{v_r}{c} \cos(\theta)$$

The maximal Doppler shift is

$$\upsilon_{\max} = f_c \frac{v}{c}$$

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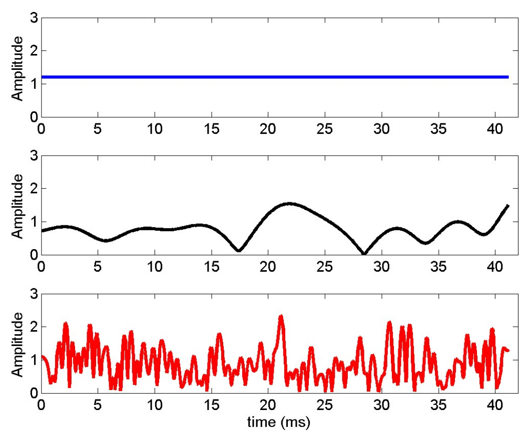
FADING: DOPPLER

- **At given frequency**
  - $v \uparrow \rightarrow f_D \uparrow \rightarrow$ channel changes more rapidly

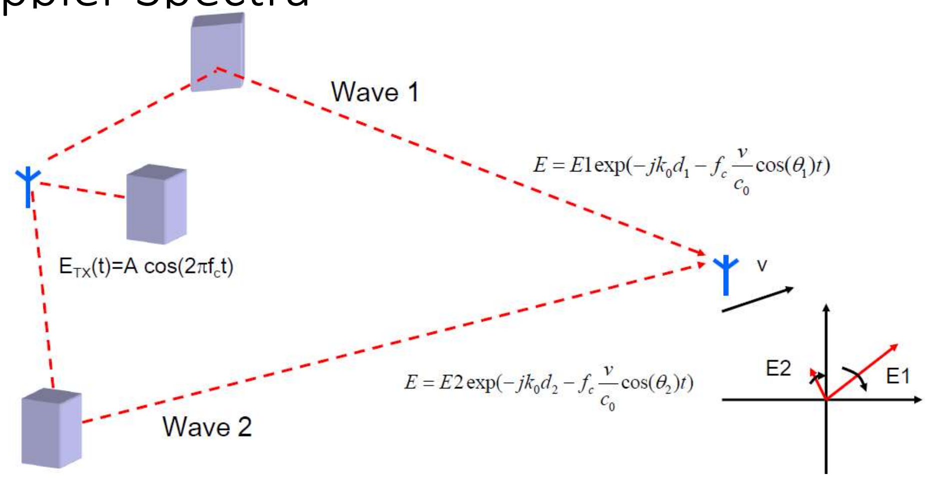


$f_D = 0$Hz

$f_D = 100$ Hz

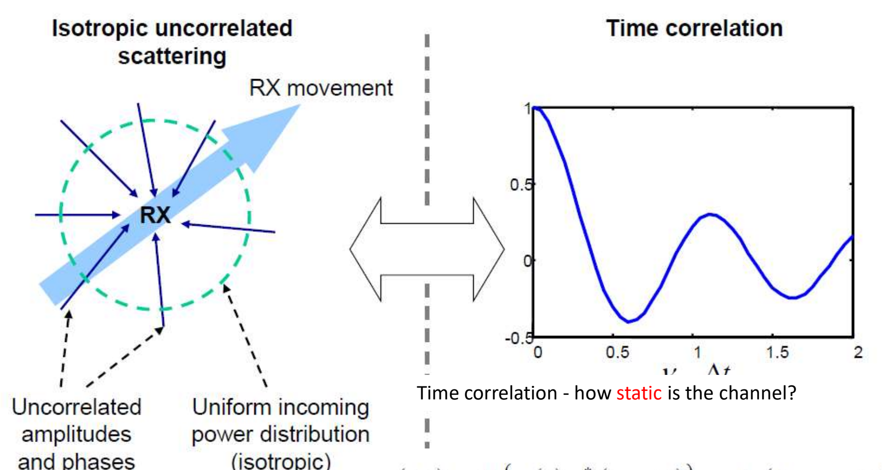
$f_D = 1000$ Hz

Doppler Spectra



The two components have different Doppler shifts!
The Doppler shifts will cause a random frequency modulation

Doppler Spectrum

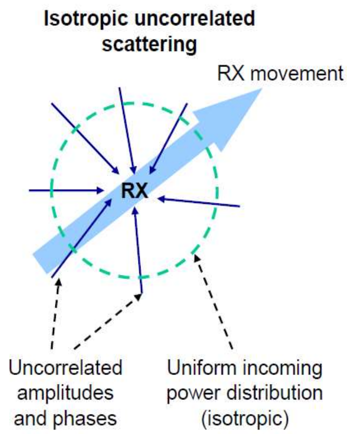


Time correlation - how static is the channel?

$$\rho(\Delta t) = E\{a(t)a^*(t+\Delta t)\} \propto J_0(2\pi v_{\max}\Delta t)$$

Doppler Spectrum

Jakes Model



$$v = -f_c \frac{v_r}{c} \cos(\theta)$$

A MPC arriving in the direction θ has to be multiplied by the pattern $G(\theta)$

$$S(\overline{\theta}) = \overline{\Omega}\left[pdf_\theta(\theta)G(\theta) + pdf_\theta(-\theta)G(-\theta)\right]$$

Change of variable from θ *to* v

$$\left|\frac{d\theta}{dv}\right| = \left|\frac{1}{\frac{dv}{d\theta}}\right| = \frac{1}{|v_{max}\sin\theta|} = \frac{1}{\sqrt{v_{max}^2 - v^2}}$$

Using $pdf(\theta) = 1/2\pi$ *and* $G(\theta)$=1.5

$$S_D(\nu) \propto \frac{1}{\pi\sqrt{\nu_{\max}{}^2 - \nu^2}}$$

Coherence Time

- Coherence time is the time domain dual of Doppler spread
- Used to characterize the time varying nature of the frequency dispersiveness of the channel in the time domain.
- The Maximum Doppler spread and coherence time are inversely proportional to one another

$$T_c \approx \frac{0.4}{v_{max}}$$

- **Slow fading** arises when the T_c>> the delay requirement of the application. The channel can be considered roughly constant over the period of use.
- **Fast fading** occurs when the T_c<< relative to the delay requirement of the application.

Wideband Channels

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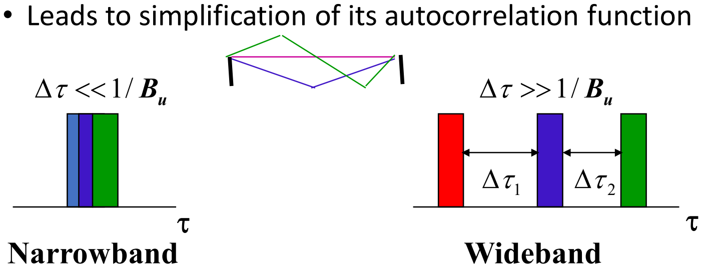
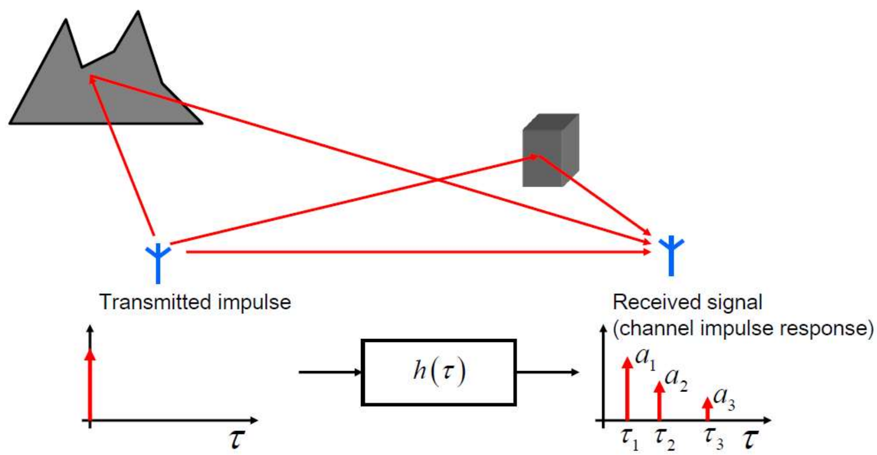
Wideband Channels

- Individual multipath components resolvable
- True when time difference between components exceeds signal bandwidth
- Requires statistical characterization of h(τ,t)
  - Assume CLT, stationarity and uncorrelated scattering
  - Leads to simplification of its autocorrelation function



Delay dispersion: A simple case



$$h(\tau) = a_1\delta(\tau - \tau_1) + a_2\delta(\tau - \tau_2) + a_3\delta(\tau - \tau_3)$$

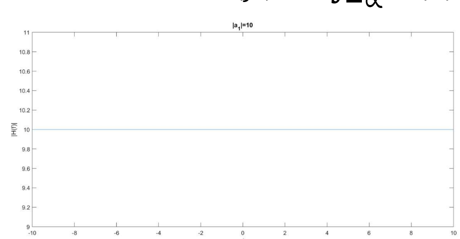
Effect of No Delay Dispersion: Frequency Flat Fading

- Single Path:

$$h(\tau) = a_1\delta(\tau - \tau_1)$$

Taking Fourier Transform, we have

$$H(f) = \int_{-\propto}^{\propto} h(\tau) e^{-j2\pi f\tau}\, d\tau$$



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Effect of Delay Dispersion: Frequency Selective Fading

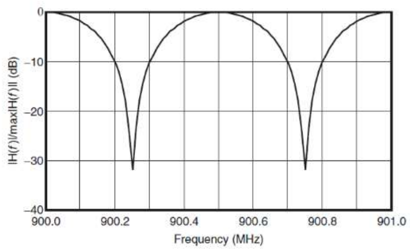
- Two Path:

$$h(\tau) = a_1\delta(\tau-\tau_1) + a_2\delta(\tau-\tau_2)$$

Taking Fourier Transform, we have $H(f) = \int_{-\infty}^{\infty} h(\tau)e^{-j2\pi f\tau}\,d\tau$

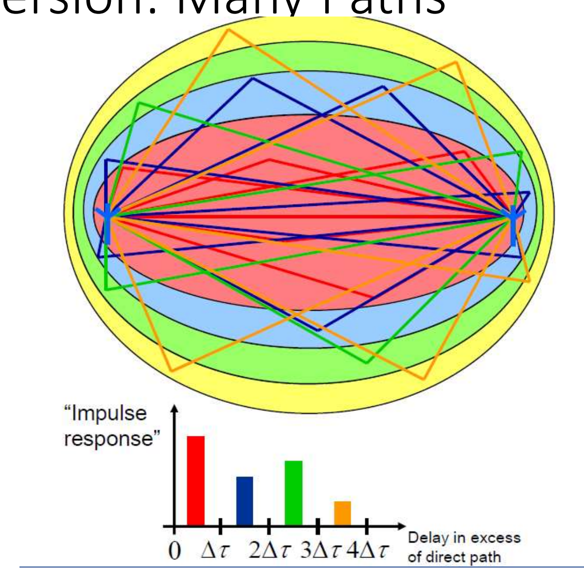
$$= a_1 e^{-j2\pi f\ \ 1} + a_2 e^{-j2\pi f\ \ 2}$$

$$|H(f)| = \sqrt{|a_1|^2 + |a_2|^2 + 2|a_1||a_2|\cos\{2\pi f\Delta\tau - \Delta\phi\}}$$

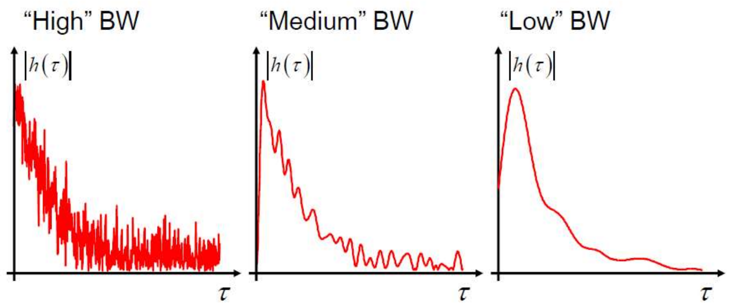


$|a_1| = 1.0$, $|a_2| = 0.95$, $\Delta\phi = 0$, $\tau_1 = 4\mu s$, $\tau_2 = 6\ \mu s$ at the 900-MHz carrier frequency

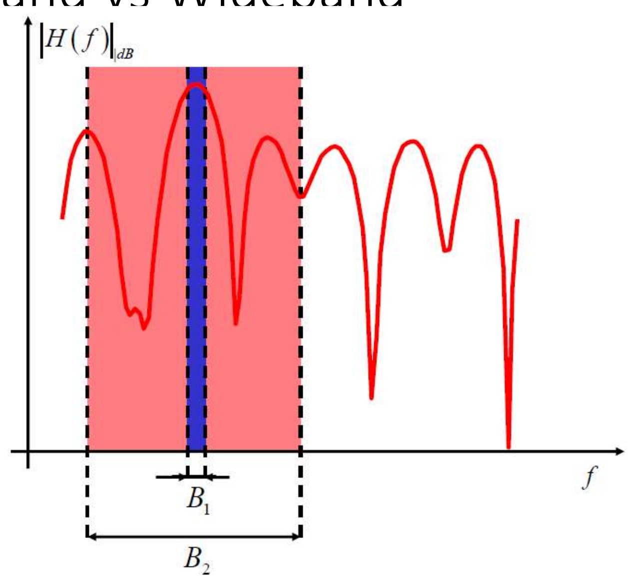
Delay Dispersion: Many Paths



Narrowband vs Wideband



Narrowband vs Wideband



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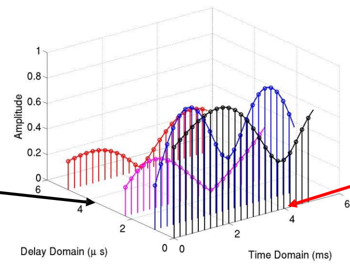
FADING: IMPULSE RESPONSE

- **The impulse response of fading is time-varying!**



Relative multipath delay τ

Time variation t

N(t) multipath components

$$c(\tau,t)=\sum_{n=1}^{N(t)}\alpha_n(t)\cos\left[2\pi f_c t+\varphi_n(t)\right]\times\delta(\tau-\tau_n)$$

- f_c : system operating frequency (e.g. 900MHz, 1.8GHz)
- t: the time variation (both amplitude and phase changes with respect to time)
- τ : relative delay between multipath components
- $\varphi_n(t)$: depends on path distance and Doppler shift $(2\pi f_D t)$

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FADING: IMPULSE RESPONSE

- **Complex baseband representation**

$$c(t,\tau)=\sum_{n=1}\alpha_n(t)\,\mathrm{Re}\left[e^{j2\pi f_c t+\varphi_n(t)}\right]\times\delta(\tau-\tau_n)$$

$$=\mathrm{Re}\left\{e^{j2\pi f_c t}\left[\sum_{n=1}^{N}\alpha_n(t)e^{j\varphi_n(t)}\times\delta(\tau-\tau_n)\right]\right\}$$

$$h(\tau,t)=\sum_{n=1}^{N}\alpha_n(t)e^{j\varphi_n(t)}\times\delta(\tau-\tau_n)$$

- **Maximum delay spread**
  - The time interval between the first multipath and the last multipath

$$\tau_{\max}=\tau_N-\tau_1$$

Time Varying Impulse Response

- Response of channel at t to impulse at t-τ:

$$h(\tau,t)=\left(\sum_{n=0}^{N(t)}\alpha_n(t)e^{-j\phi_n(t)}\,\delta(\tau-\tau_n(t))\right)$$

$$\boxed{\phi_n(t)=2\pi f_c\tau_n(t)-\phi_{D_n}-\phi_0}$$

- t is time when impulse response is observed
- t-τ is time when impulse put into the channel
- τ is how long ago impulse was put into the channel for the current observation
  - path delay for MP component currently observed

In-Phase and Quadrature under CLT Approximation

- In phase and quadrature signal components:

$$r_I(t)=\sum_{n=0}^{N(t)}\alpha_n(t)e^{-j\phi_n(t)}\cos(2\pi f_c t),$$

$$r_Q(t)=\sum_{n=0}^{N(t)}\alpha_n(t)e^{-j\phi_n(t)}\sin(2\pi f_c t)$$

- For *N(t)* large, $r_I(t)$ and $r_Q(t)$ jointly Gaussian by CLT (sum of large # of random vars).
- Received signal characterized by its mean, autocorrelation, and cross correlation.
- If $\varphi_n(t)$ uniform, the in-phase/quad components are mean zero, independent, and stationary.

Auto and Cross Correlation

- Assume ϕ_n~U[0,2π]
- Recall that θ_n is the multipath arrival angle
- Autocorrelation of inphase/quad signal is

$$A_{r_I}(\tau) = A_{r_Q}(\tau) = PE_{\theta_n}[\cos 2\pi f_{D_n}\tau], \qquad f_{D_n} = v\cos\theta_n/\lambda$$

- Cross Correlation of inphase/quad signal is

$$A_{r_I,r_Q}(\tau) = PE_{\theta_n}[\sin 2\pi f_{D_n}\tau] = -A_{r_I,r_Q}(\tau)$$

- Autocorrelation of received signal is

$$A_r(\tau) = A_{r_I}(\tau)\cos(2\pi f_c\tau) - A_{r_I,r_Q}(\tau)\sin(2\pi f_c\tau)$$

Cross Correlation

- Cross Correlation of inphase/quad signal is

$$r_I(t) = \sum_{n=0}^{N(t)} \alpha_n(t)e^{-j\phi_n(t)}\cos(2\pi f_c t),\ r_Q(t) = \sum_{n=0}^{N(t)} \alpha_n(t)e^{-j\phi_n(t)}\sin(2\pi f_c t),\quad \phi_n\text{~U}[0,2\pi]$$

  - Thus, $A_{r_I,r_Q}(0) = 0$ so $r_I(t)$ and $r_Q(t)$ independent
- Autocorrelation of received signal is

$$A_{r_I,r_Q}(\tau) = E[r_I(t)r_Q(t+\tau)] = P_r E_{\theta_n}[\sin 2\pi f_{D_n}\tau] = -A_{r_I,r_Q}(\tau)$$

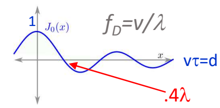
- Thus, r(t) is stationary (WSS)

$$A_r(\tau) = A_{r_I}(\tau)\cos(2\pi f_c\tau) - A_{r_I,r_Q}(\tau)\sin(2\pi f_c\tau)$$

Uniform AOAs

- Under uniform scattering, in phase and quad comps have no cross correlation and autocorrelation is

$$A_{r_I}(\tau) = A_{r_Q}(\tau) = P_r J_0(2\pi f_D \tau)$$

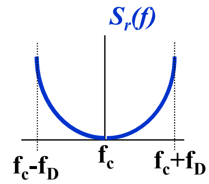


Decorrelates over roughly half a wavelength

- The PSD of received signal is

$$S_r(f) = .25[S_{r_I}(f - f_c) + S_{r_I}(f + f_c)]$$

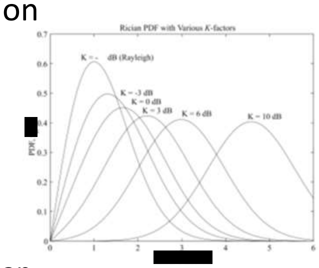
$$S_{r_I}(f) = \mathcal{F}[P_r J_0(2\pi f_D \tau)]$$

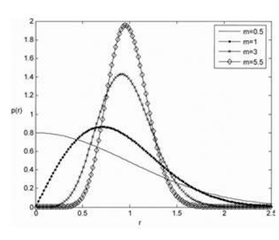


Used to generate simulation values

Signal Envelope Distribution

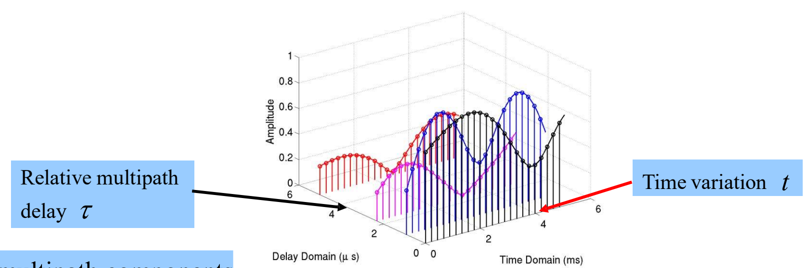
- CLT approx. leads to Rayleigh distribution (power is exponential)
- When LOS component present, Ricean distribution is used
- Measurements support Nakagami distribution in some environments
  - Similar to Ricean, but models “worse than Rayleigh”
  - Lends itself better to closed form BER expressions





FADING: IMPULSE RESPONSE

- **The impulse response of fading is time-varying!**



N(t) multipath components

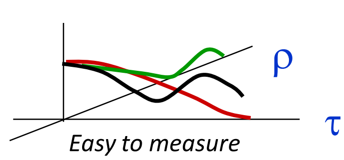
$$c(\tau,t)=\sum_{n=1}^{N(t)}\alpha_n(t)\cos\left[2\pi f_c t+\varphi_n(t)\right]\times\delta(\tau-\tau_n)$$

– f_c : system operating frequency (e.g. 900MHz, 1.8GHz)
– t: the time variation (both amplitude and phase changes with respect to time)
– τ : relative delay between multipath components
– $\varphi_n(t)$: depends on path distance and Doppler shift $(2\pi f_D t)$

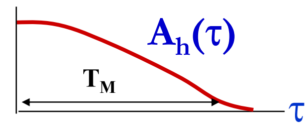
Scattering Function

- Typically characterize h(τ,t) by its statistics, since it is a random process
- Underlying process WSS and Gaussian, so only characterize mean (0) and correlation
- Autocorrelation is $A_c(\tau_1,\tau_2,\Delta t)=A_c(\tau,\Delta t)$
  - Correlation for single mp delay/time difference
- Statistical scattering function:
  - Average power for given mp delay and doppler

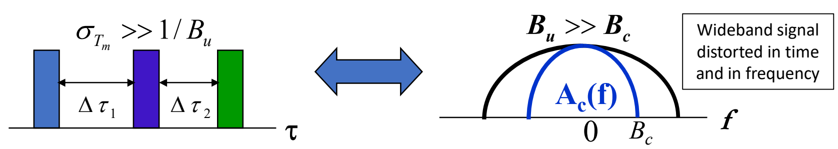
$$s(\tau,\rho)=\mathcal{F}_{\Delta t}[A_c(\tau,\Delta t)]$$



Multipath Intensity Profile



- Defined as $A_h(\tau,\Delta t=0)= A_h(\tau)$
  - Determines average (μ_{T_m}) and rms (σ_{T_m}) delay spread
  - Approximates maximum delay of significant multipath
- Coherence bandwidth $B_c=1/\sigma_{T_m}$
  - Maximum frequency over which $A_c(\Delta f)=F[A_c(\tau)]>0$
  - $A_c(\Delta f)=0$ implies signals separated in freq. by Δf will be uncorrelated after going through channel: freq. distortion

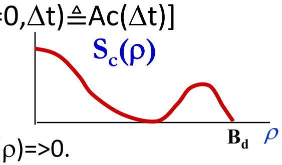


Doppler Power Spectrum

Scattering Function: $s(\tau,\rho)=\mathcal{F}_{\Delta t}[A_c(\tau,\Delta t)]$

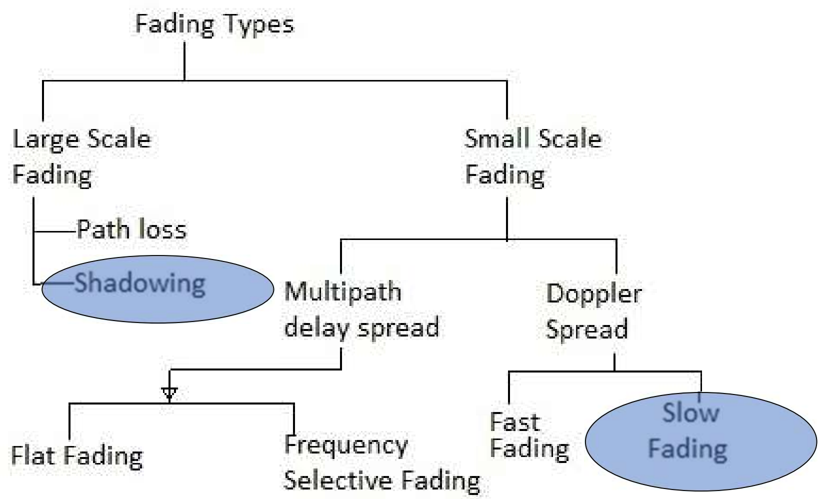
- Doppler Power Spectrum: $S_c(\rho)=\mathcal{F}_{\Delta t}\,[A_c(\Delta f=0,\Delta t)\triangleq Ac(\Delta t)]$

$$A_c(\Delta f,\Delta t)=\mathcal{F}_\tau[A_c(\tau,\Delta t)]$$



  - Power of multipath at given Doppler
  - Doppler spread B_d: Max. doppler for which $S_c(\rho)$=>0.
  - Coherence time $T_c=1/B_d$: Max time over which $A_c(\Delta t)>0$
    - $A_c(\Delta t)=0 \Rightarrow$ signals separated in time by Δt uncorrelated after passing through channel
- Why do we look at Doppler w.r.t. $A_c(\Delta f=0,\Delta t)$?
  - Captures Doppler associated with a narrowband signal
  - Autocorrelation over a narrow range of frequencies
  - Fully captures time-variations, multipath angles of arrival

Fading Summary



Main Points

- Narrowband model has in-phase and quad. comps that are zero-mean stationary Gaussian processes
  - Auto and cross correlation depends on AOAs of multipath
- Uniform scattering makes autocorrelation of inphase and quad comps of RX signal follow Bessel function
  - Signal components decorrelate over half wavelength
  - The PSD has a bowel shape centered at carrier frequency
- Fading distribution depends on environment
  - Rayleigh, Ricean, and Nakagami all common
- Wideband channels have resolvable multipath
  - Will statistically characterize $c(\tau,t)$ for WSSUS model

Main Points

- Wideband channels have resolvable multipath
  - Statistically characterize $h(\tau,t)$ for WSSUS model
  - Scattering function characterizes rms delay and Doppler spread. Key parameters for system design.
- Delay spread defines maximum delay of significant multipath components. Inverse is coherence BW
  - Signal distortion in time/freq. when delay spread exceeds inverse signal BW (signal BW exceeds coherence BW)
- Doppler spread defines maximum nonzero doppler, its inverse is coherence time
  - Channel decorrelates over channel coherence time