Assignment 7

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EE20BTECH11042

Importing Libraries

[1]:
```
import emcee
import corner
import nestle
import numpy as np
import pandas as pd
from scipy import optimize
import matplotlib.pyplot as plt
from sklearn.neighbors import KernelDensity
```

Question 1

We have to fit the linear funciton on the given data as:

$$f_{gas} = f_0(1 + f_1 z)$$

with the bounds on f_0 and f_1 as

$$0 < f_0 < 0.5$$

$$-0.5 < f_1 < 0.5$$

[2]:
```
# Functions from JVDPs blog on performing Bayesian analysis
def log_prior(theta):
    f0, f1 = theta
    if (f0 > 0 and f0 < 0.5) and (f1 > -0.5 and f1 < 0.5):
        return 0
    else:
        return -np.inf

def log_likelihood(theta, x, y, sigma_y):
    f0, f1 = theta
    y_model = f0 * (1 + f1 * x)
    del_chi2 = np.sum(np.log(2 * np.pi * sigma_y ** 2) + ((y - y_model) /
 ↪sigma_y) ** 2)
    return -0.5 * del_chi2

def neg_log_likelihood(theta, x, y, sigma_y):
```

```
    return -log_likelihood(theta, x, y, sigma_y)

def log_posterior(theta, x, y, sigma_y):
    log_pr = log_prior(theta)
    if np.isfinite(log_pr):
        return log_pr + log_likelihood(theta, x, y, sigma_y)
    return -np.inf
```

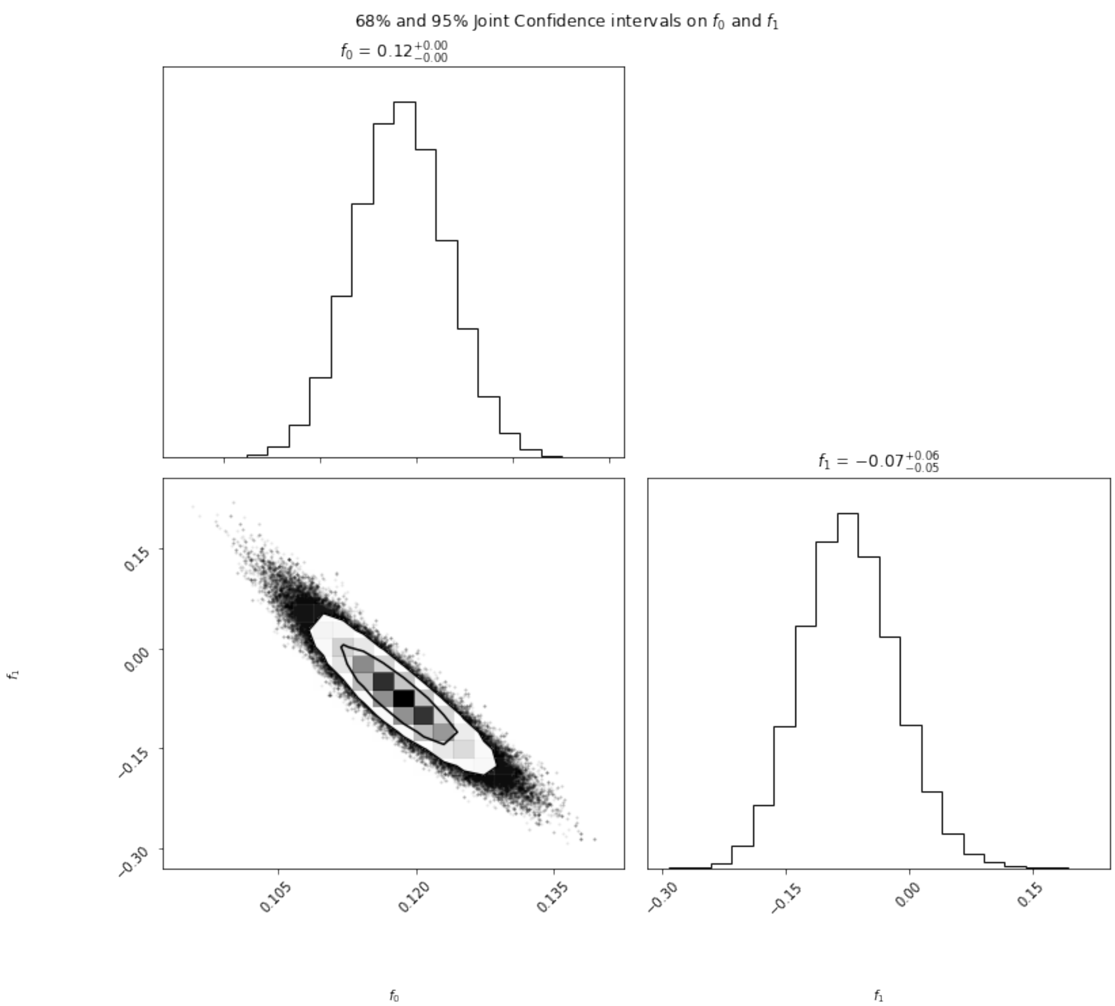
[3]:
```
q1_data = np.loadtxt("q1_data.csv", delimiter = " ", dtype = str)

z = []
f_gas = []
f_err = []

for data in q1_data:
    z.append(float(data[0]))
    f_gas.append(float(data[1]))
    f_err.append(float(data[2]))

z = np.array(z)
f_gas = np.array(f_gas)
f_err = np.array(f_err)

optim_solution = optimize.fmin(func = neg_log_likelihood, x0 = [0.0, 0.0], args␣
↪= (z, f_gas, f_err))

n_params = 2
n_walkers = 100
n_burn = 1000
n_steps = 5000

np.random.seed(11042)

initial_guesses = optim_solution + 1e-3 * np.random.random([n_walkers, n_params])
mcmc_sampler = emcee.EnsembleSampler(n_walkers, n_params, log_posterior, args =␣
↪[z, f_gas, f_err])

dump = mcmc_sampler.run_mcmc(initial_guesses, n_steps, progress = True)
mcmc_samples = mcmc_sampler.get_chain(discard = n_burn, flat = True)
```

```
Optimization terminated successfully.
         Current function value: -200.732529
         Iterations: 63
         Function evaluations: 120
```

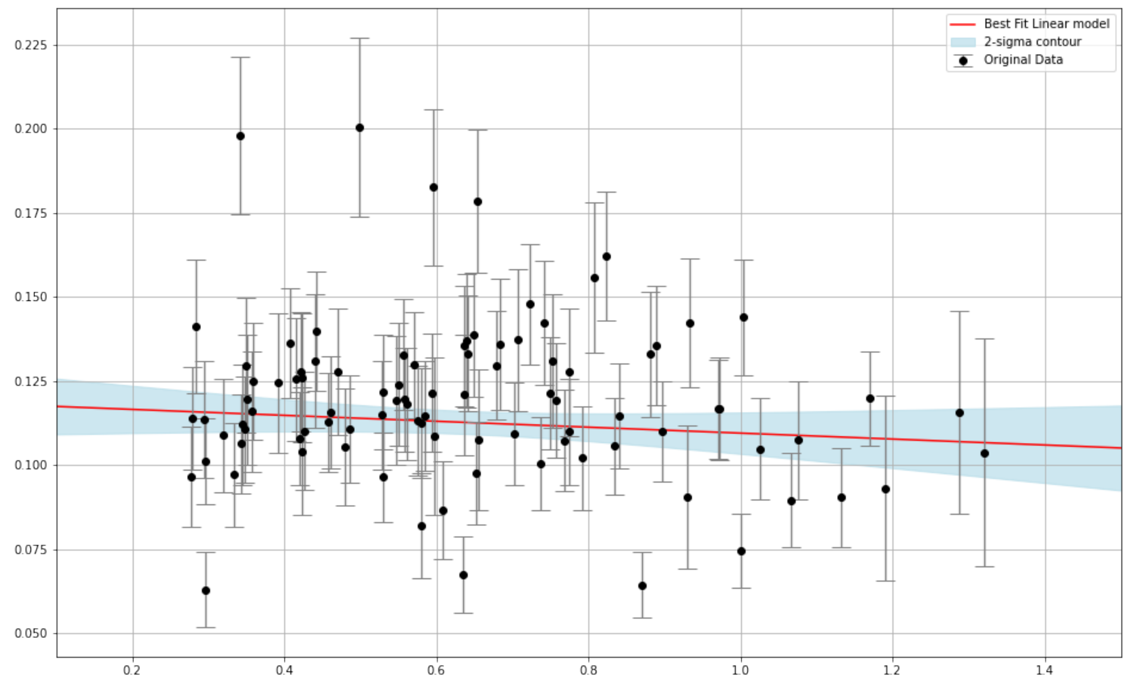
```
100%|| 5000/5000 [00:11<00:00,
419.32it/s]
```

[4]:
```
f0_median, f1_median = np.median(mcmc_samples, axis = 0)
print("Estimated parameter values for best fit are:")
print(f"f0 = {f0_median}")
print(f"f1 = {f1_median}")

fig = plt.figure(figsize = (13, 11))
figure = corner.corner(mcmc_samples, levels = [0.68, 0.90], labels = [r"$f_0$",␣
↪r"$f_1$"], fig = fig, show_titles = True)
plt.suptitle("68% and 95% Joint Confidence intervals on" + r" $f_0$ " + "and" +␣
↪" $f_1$")
plt.show()
```

```
Estimated parameter values for best fit are:
f0 = 0.1182968820994239
f1 = -0.07434616099756106
```



[5]:
```
f0, f1 = mcmc_samples.T[ : 2]
x_plot = np.linspace(0.1, 1.5, 300)
y_plot = f0[: , None] * (1 + f1[: , None] * x_plot)
mu = y_plot.mean(0)
sigma = 2 * y_plot.std(0)

plt.figure(figsize = (16, 10))
plt.plot(x_plot, mu, color = "red", label = "Best Fit Linear model")
plt.fill_between(x_plot, mu - sigma, mu + sigma, color = 'lightblue', alpha = 0.
 ↪6, label = "2-sigma contour")
plt.errorbar(z, f_gas, f_err, fmt = "ok", lw = 1.35, capsize = 8, ecolor =␣
 ↪"gray", label = "Original Data")
plt.xlim(0.1, 1.5)
plt.legend()
plt.grid()
plt.show()
```



Question 2

[6]:
```
q2_data = np.array([[ 0.42, 0.72, 0.0 ,  0.3 ,  0.15,
                      0.09, 0.19, 0.35,  0.4 ,  0.54,
                      0.42, 0.69, 0.2 ,  0.88,  0.03,
                      0.67, 0.42, 0.56,  0.14,  0.2 ],
                    [ 0.33, 0.41, -0.22, 0.01, -0.05,
```

```
                  -0.05, -0.12, 0.26, 0.29,  0.39,
                   0.31, 0.42, -0.01, 0.58, -0.2,
                   0.52, 0.15, 0.32, -0.13, -0.09 ],
                 [ 0.1,  0.1,  0.1,   0.1,   0.1,
                   0.1,  0.1,  0.1,   0.1,   0.1,
                   0.1,  0.1,  0.1,   0.1,   0.1,
                   0.1,  0.1,  0.1,   0.1,   0.1 ]])

def polynomial_fit(theta, x):
    return sum(t * x ** n for (n, t) in enumerate(theta))

def log_prior(theta):
    return 200 * theta - 100

def log_likelihood(theta, data = q2_data):
    x, y, sigma_y = data
    y_pred = polynomial_fit(theta, x)
    return -0.5 * np.sum(np.log(2 * np.pi * sigma_y ** 2) + (y - y_pred) ** 2 /
 ↪sigma_y ** 2)
```

[7]:
```
np.random.seed(11042)

linear_fit = nestle.sample(log_likelihood, log_prior, 2)
print("Summary of Linear Fit:")
print(linear_fit.summary())
print(f"\nThe Log-Evidence for Linear Model: {linear_fit.logz}\n")

quadratic_fit = nestle.sample(log_likelihood, log_prior, 3)
print("Summary of Quadratic Fit:")
print(quadratic_fit.summary())
print(f"\nThe Log-Evidence for Quadratic Model: {quadratic_fit.logz}")

print(f"\nFrom the fifth blog article of the Pythonic Perambulations Series, the
 ↪Log-Evidence values were:")
print("For Linear Model: 46942613.34886921")
print("For Quadratic Model: 111116773.89368105")
```

```
Summary of Linear Fit:
niter: 1618
ncall: 2772
nsamples: 1718
logz:  6.683 +/-  0.378
h: 14.285

The Log-Evidence for Linear Model: 6.683109087857267

Summary of Quadratic Fit:
niter: 2167
```

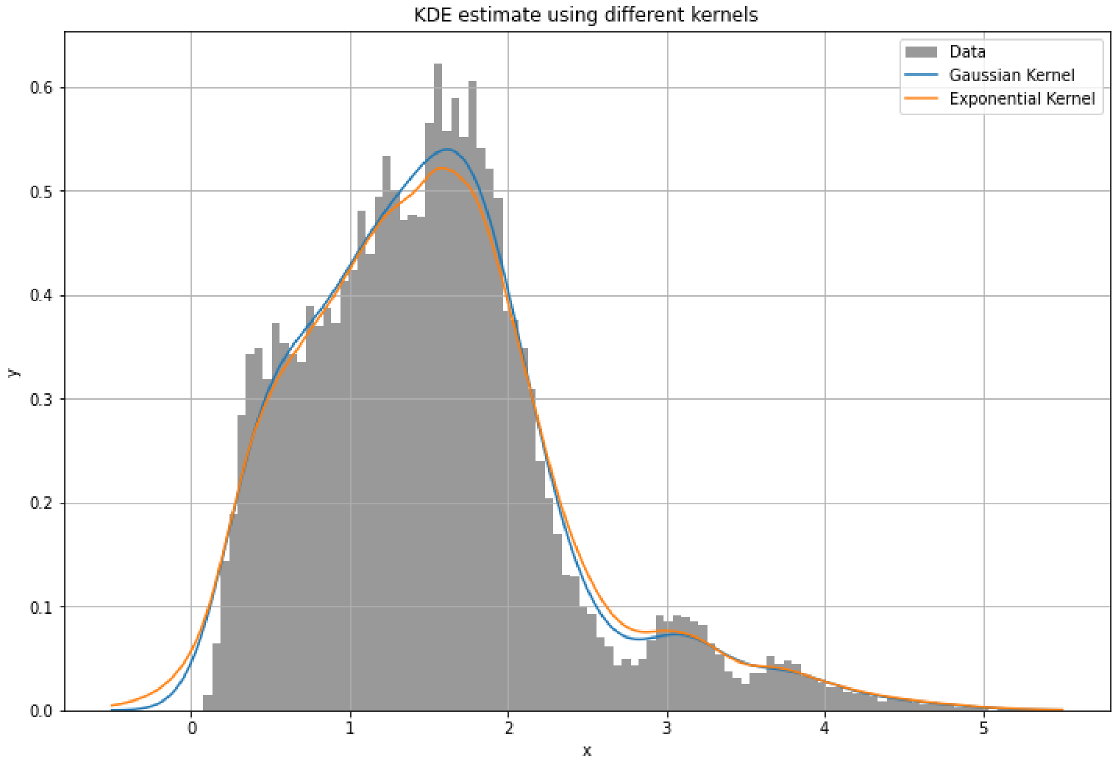
```
ncall: 3989
nsamples: 2267
logz:  2.080 +/-  0.440
h: 19.388

The Log-Evidence for Quadratic Model: 2.0795724779962583

From the fifth blog article of the Pythonic Perambulations Series, the Log-
Evidence values were:
For Linear Model: 46942613.34886921
For Quadratic Model: 111116773.89368105
```

Clearly these values do not match with the ones obtained from the Nested sampling by Nestle.

Question 3

[8]:
```
q3_data = pd.read_csv('q3_data.txt', sep = '\s+')

z = q3_data['z'].to_numpy().reshape([-1, 1])
x = np.linspace(-0.5, 5.5, 6000).reshape([-1, 1])

kde_gauss = KernelDensity(kernel = 'gaussian', bandwidth = 0.2).fit(z)
log_density_gauss = np.exp(kde_gauss.score_samples(x))

kde_exp = KernelDensity(kernel = 'exponential', bandwidth = 0.2).fit(z)
log_density_exp = np.exp(kde_exp.score_samples(x))
```

[9]:
```
fig = plt.figure(figsize = (12, 8))
plt.hist(z, bins = "auto", density = True, color = "black", alpha = 0.4, label = "Data")
plt.plot(x, log_density_gauss, label = "Gaussian Kernel")
plt.plot(x, log_density_exp, label = "Exponential Kernel")
plt.title("KDE estimate using different kernels")
plt.xlabel("x")
plt.ylabel("y")
plt.legend()
plt.grid()
plt.show()
```


KDE estimate using different kernels
Data
Gaussian Kernel
Exponential Kernel
0.0
0.1
0.2
0.3
0.4
0.5
0.6
y
0
1
2
3
4
5
x