EE5801: CSP Lab/EE5301: DSP Lab
EE3701: Communication Systems Lab
(Aug – Nov 2022)

Lecture 2

Today’s Topics

- Analog vs Digital frequencies
- Digital filter design
    1. Low pass filter(LPF)
    2. High pass filter(HPF)
    3. Band pass filter(BPF)
- Half band filter and M band filter

Analog vs Digital frequencies

- Consider the continuous time signal $Sin(\Omega t)$,
  where $\Omega = 2\pi f, -\infty < f < \infty$, Ω is known as analog frequency
- To convert it into discrete time signal put $t = nT_s$, where $T_s =$ sampling interval and $f_s = \frac{1}{T_s} = sampling\ frequency$

$$Sin(2\pi f\ nT_s) = Sin\left(\frac{2\pi f}{f_s}n\right) = Sin(\omega n)$$

where ω is known as digital frequency

- Ω represents analog frequencies and ω represents digital frequencies.
- $\omega = \Omega T_s$, $\Omega \in (-\infty, \infty)$, $\omega \in [-\pi, \pi]$

Analog vs Digital frequencies(contd…)

- Why digital frequency ω has finite range?

Reason : As per sampling theorem $f_s \geq 2f$

We know, $\omega = \frac{2\pi f}{f_s}$

min value of f_s can be $2f$, so max value of $\omega = \frac{2\pi f}{2f} = \pi$

max value of f_s can be ∞ , so min value of $\omega \rightarrow 0$

Similarly for –ve frequencies, min value is $-\pi$ and max is 0

So $-\pi \leq \omega \leq \pi$

Digital Filters

- Two types of digital filter,
  1. FIR(Finite Impulse Response)
  2. IIR(Infinite Impulse Response)
- FIR filters are easy to design in discrete time than IIR filters.
- FIR filters may have linear or non linear phase response.
- The simplest method of FIR filter design is called the window method.
- In window method we always try to design **Linear Phase FIR filter** to avoid phase distortions.

Window method for FIR filter design

- Let the desired ideal frequency response is $H_d(e^{j\omega})$.
- Take IFFT of $H_d(e^{j\omega})$ to get $h_d[n]$.
- Since $h_d[n]$ has infinite length, truncate it using a finite length window function $w[n]$ to get $h[n]$.

$$h[n] = h_d[n] \times w[n]$$

- To see your practical filter frequency response you can take FFT of $h[n]$ which is $\mathrm{H}(e^{j\omega})$ and you can plot magnitude and phase response.

Some commonly used window functions

Rectangular

$$w[n] = \begin{cases} 1, & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases}$$

Bartlett (triangular)

$$w[n] = \begin{cases} 2n/M, & 0 \le n \le M/2, \\ 2 - 2n/M, & M/2 < n \le M, \\ 0, & \text{otherwise} \end{cases}$$

Hanning

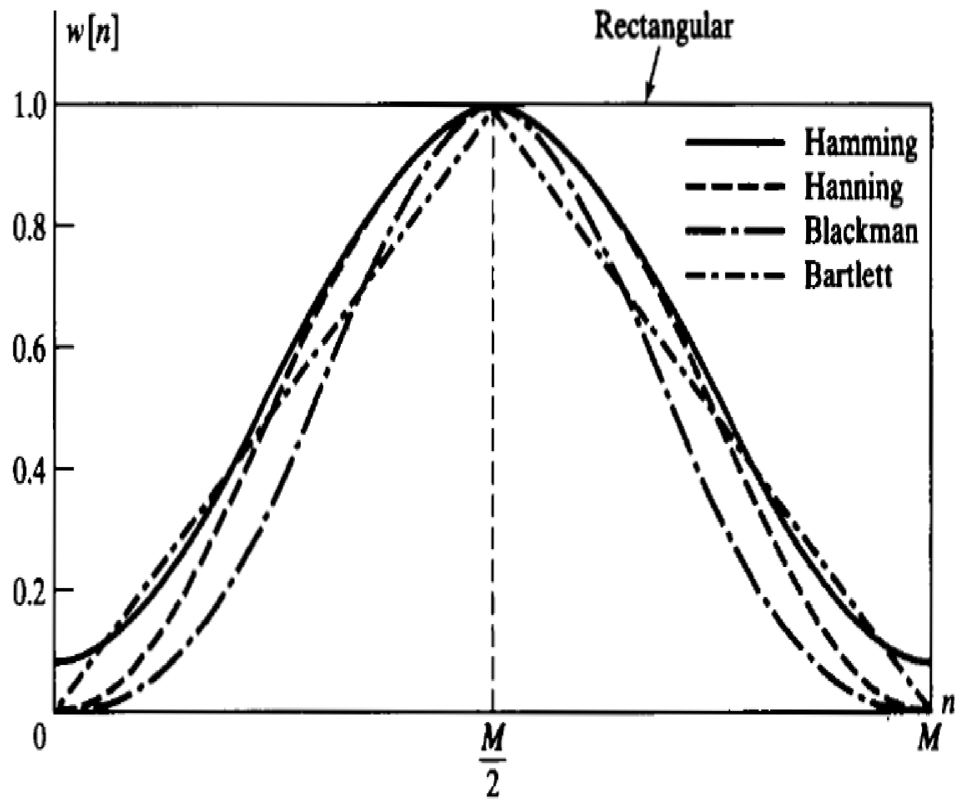
$$w[n] = \begin{cases} 0.5 - 0.5\cos(2\pi n/M), & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases}$$

Hamming

$$w[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/M), & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases}$$

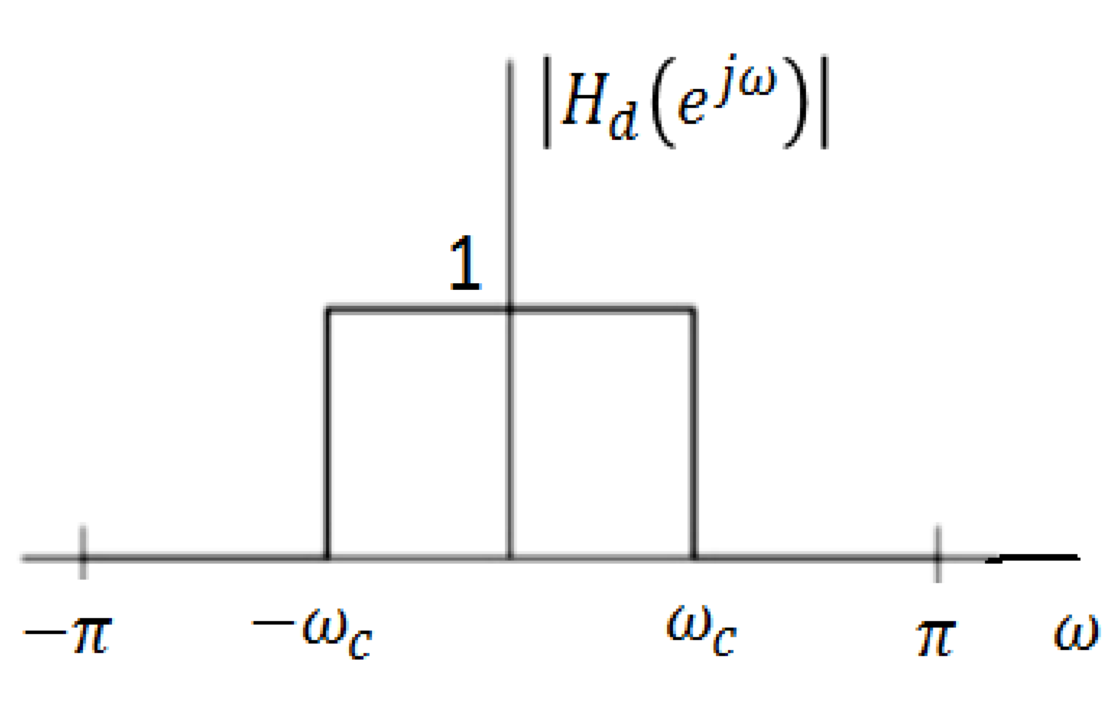
Blackman

$$w[n] = \begin{cases} 0.42 - 0.5\cos(2\pi n/M) + 0.08\cos(4\pi n/M), & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases}$$

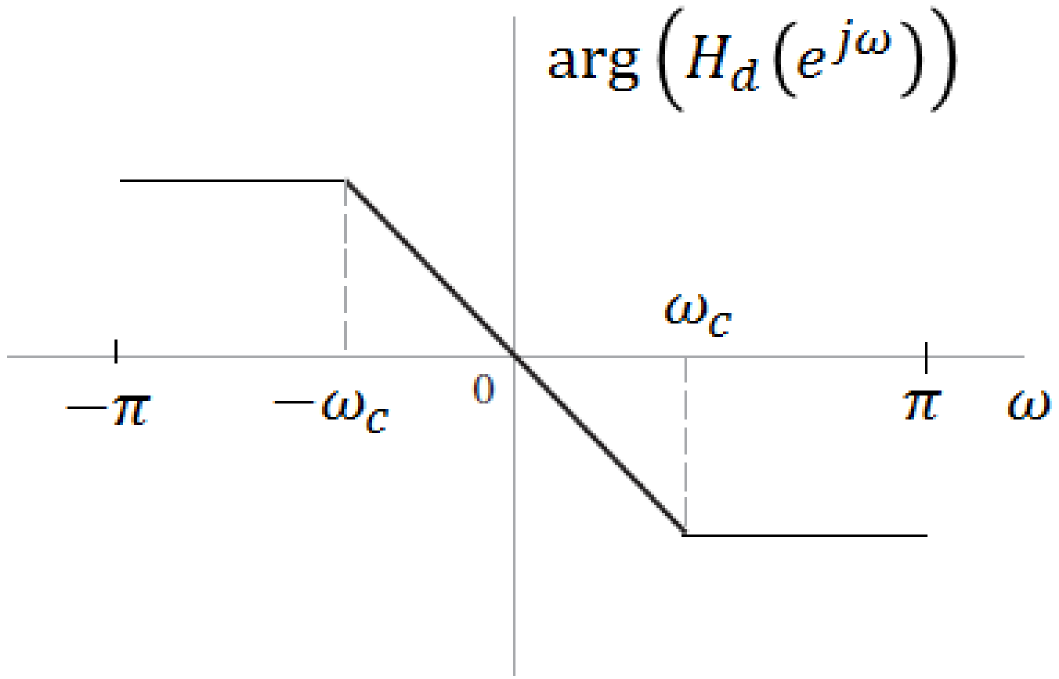


Design of LPF

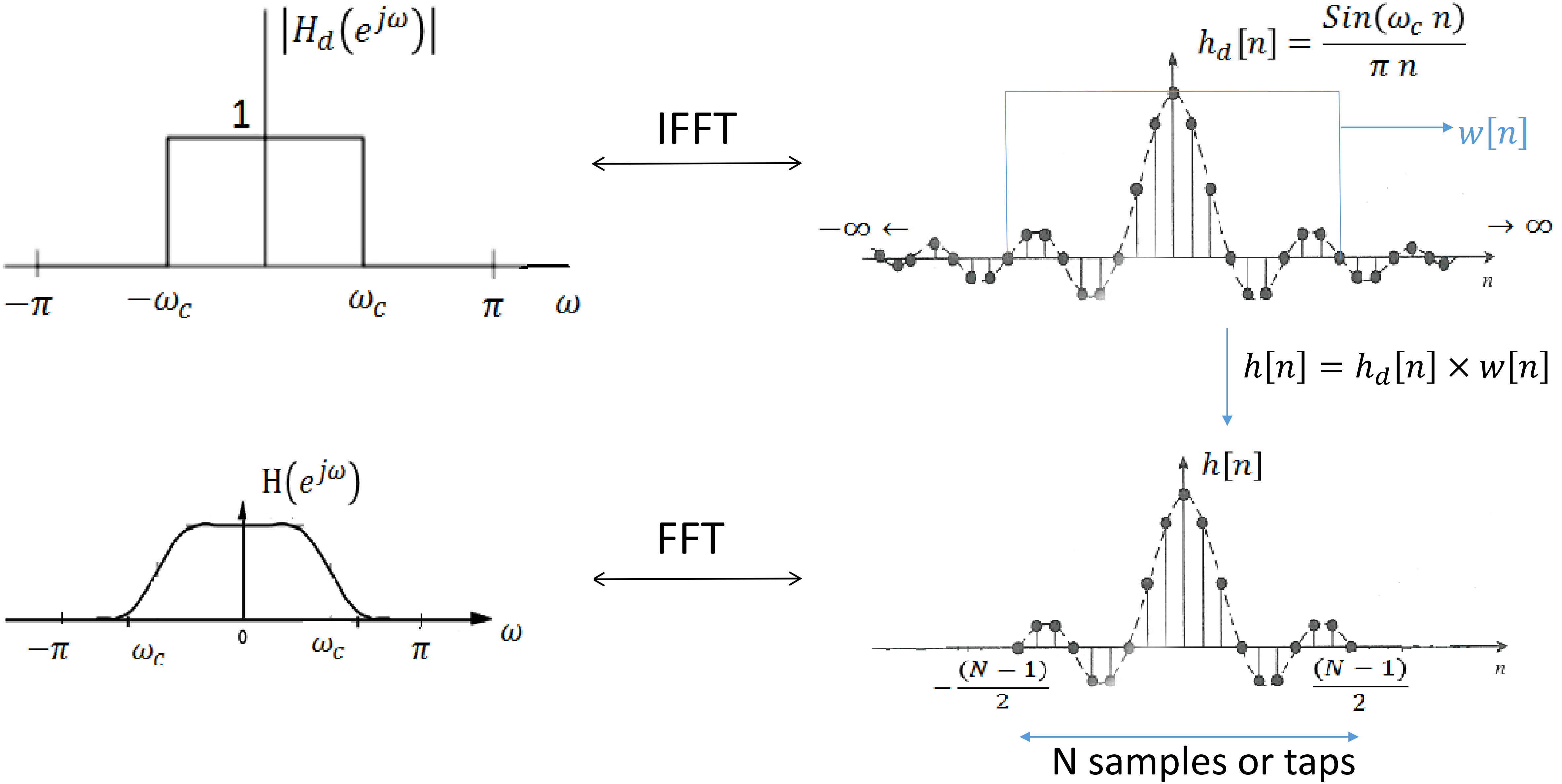
Ideal LPF magnitude response



Ideal LPF phase response



Design of LPF(contd…)



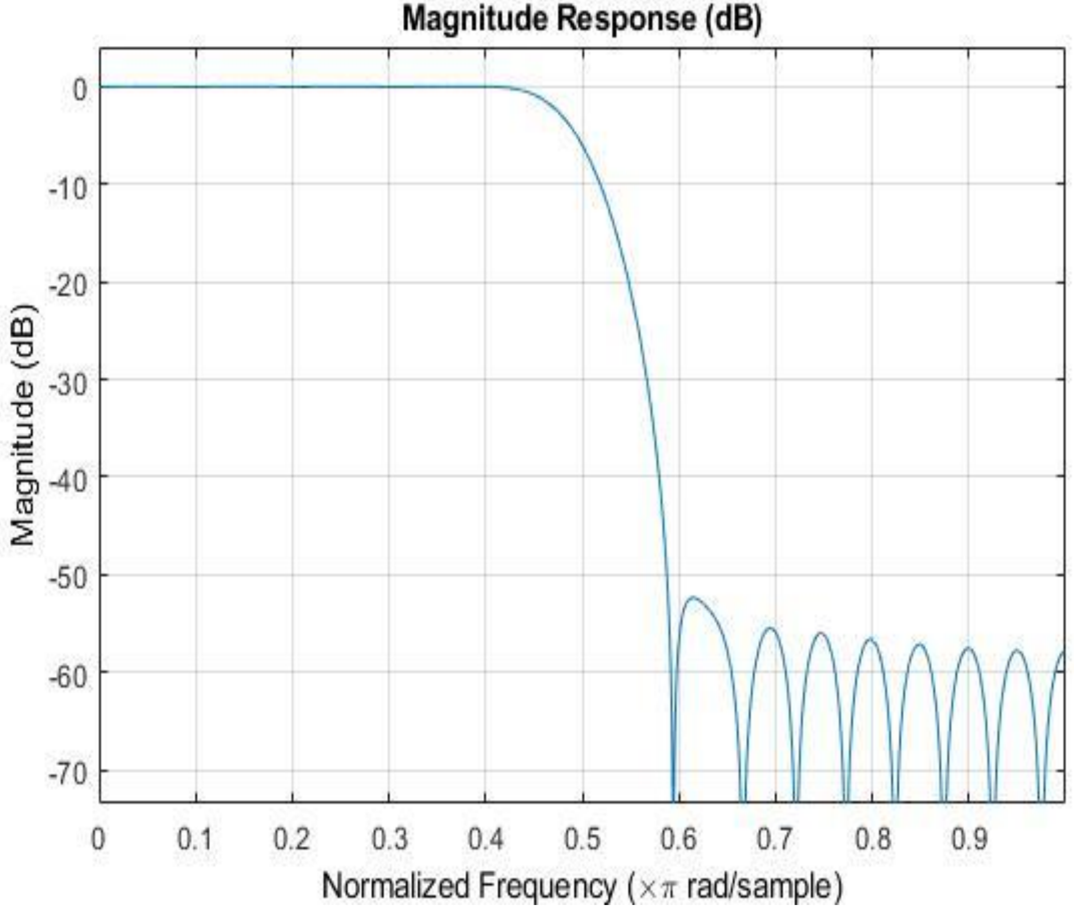
Design of LPF(contd…)

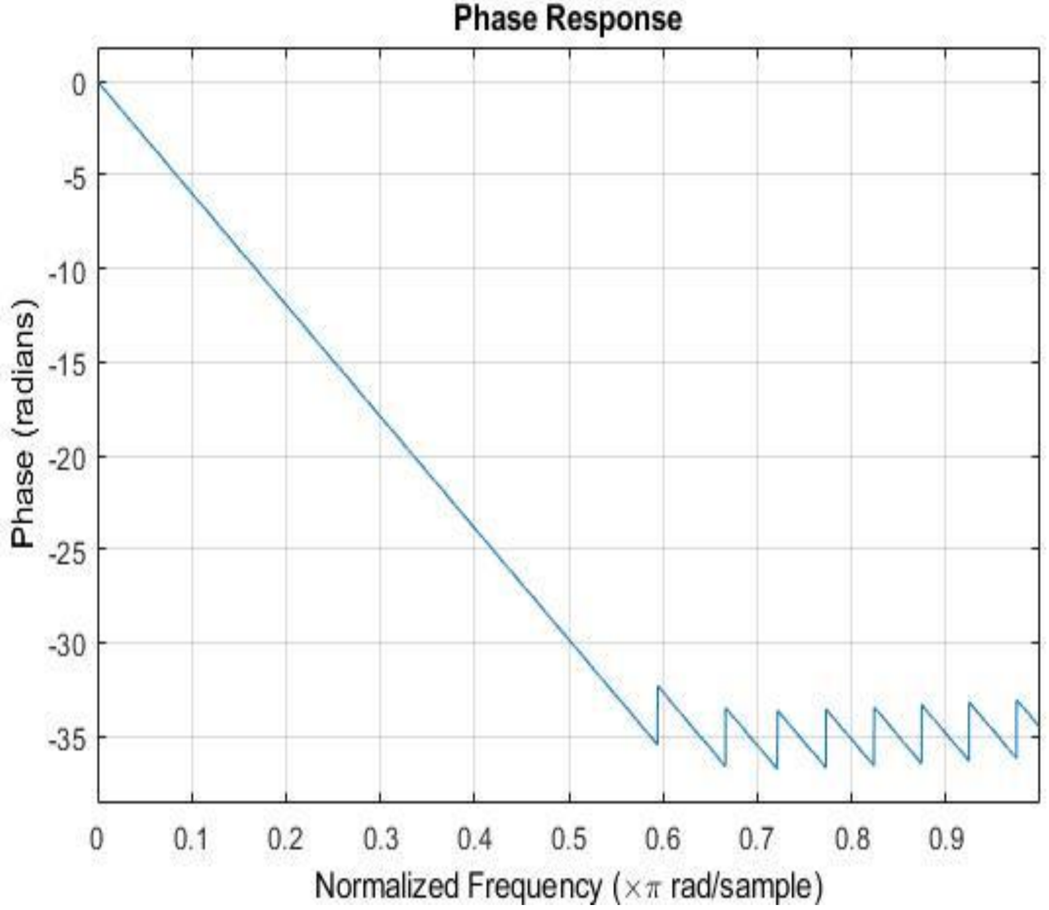
- Now we have $\mathrm{h_d}[n] = \frac{Sin(\omega_c\, n)}{\pi\, n}, where\ -(N-1)/2 \le n \le (N-1)/2$

  and $\omega_\mathrm{c} = \frac{2\pi f_c}{f_s}$
- What happens to $h_d[n]$ at n=0 ?

  Ans: $\lim_{n\to 0} h_d[n] = \lim_{n\to 0} \frac{\omega_c cos(\omega_c\, n)}{\pi} = \frac{\omega_c}{\pi}$
- So for LPF $h_d[n] = \begin{cases} \frac{Sin(\omega_c\, n)}{\pi\, n}, & -(N-1)/2 \le n \le (N-1)/2 \\ \frac{\omega_c}{\pi}, & n = 0 \end{cases}$
- The impulse response of practical LPF is $h[n] = h_d[n] \times w[n]$

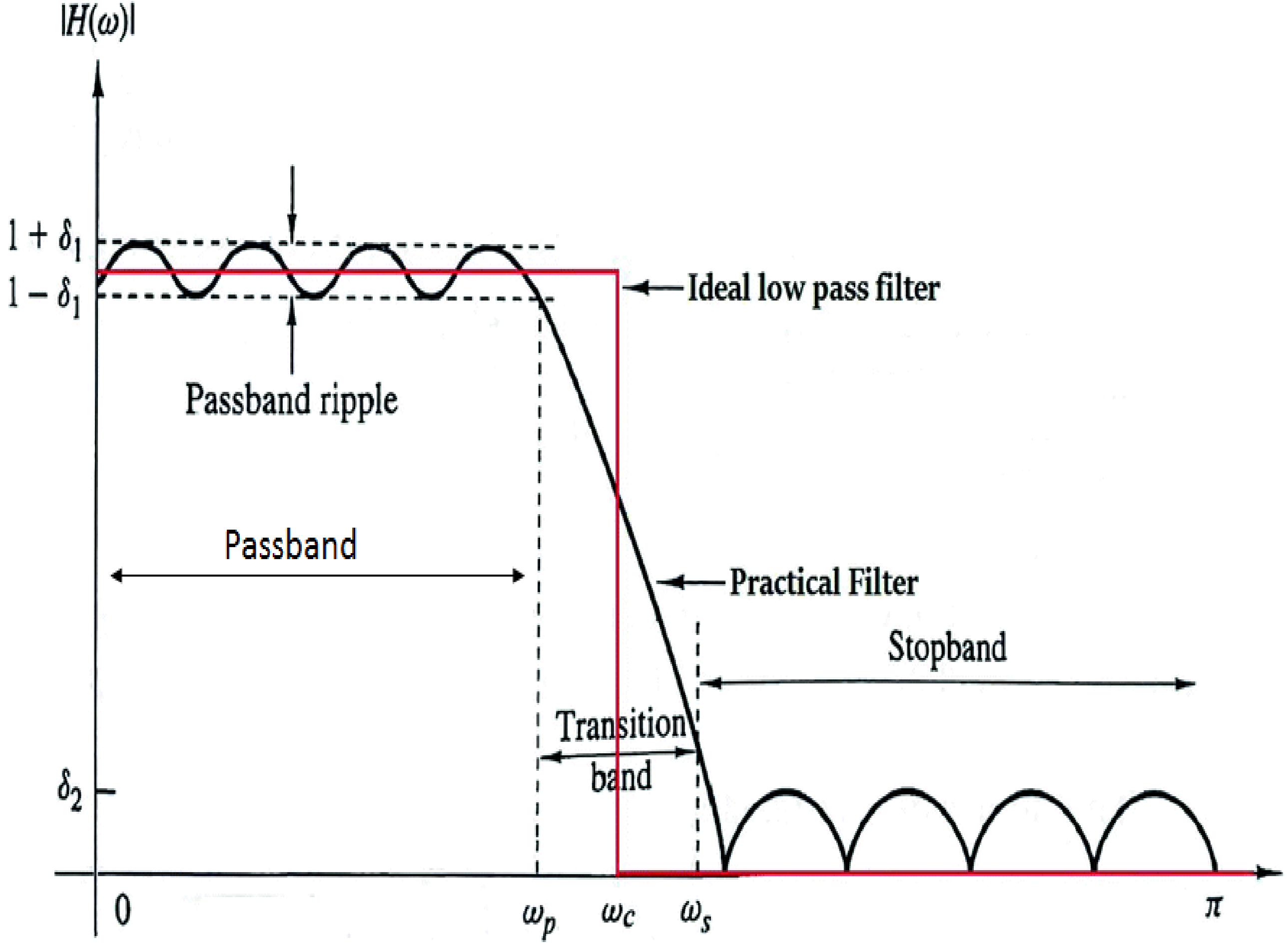
Magnitude and phase response of practical LPF from Matlab simulation





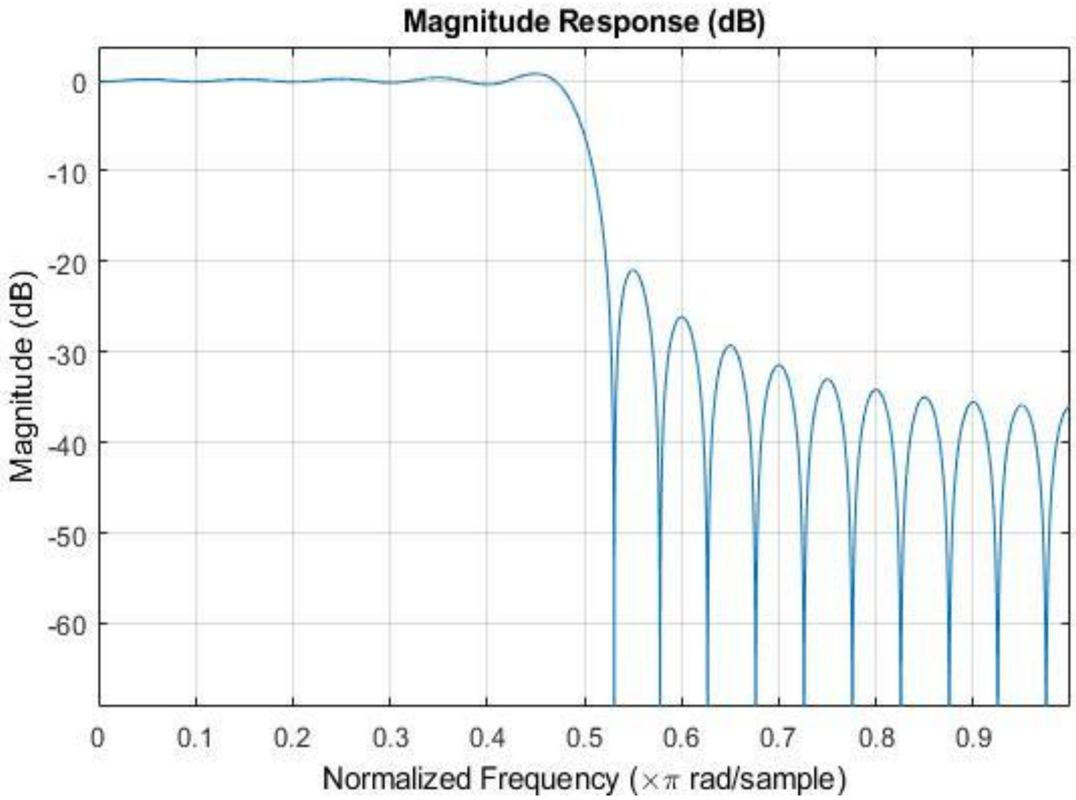
Filter specifications

- δ_1 is passband ripple.
- δ_2 is stopband ripple.
- ω_p is passband edge ripple.
- ω_s is stopband edge ripple.
- Cutoff frequency ω_c lies in between ω_p and ω_s.
- Passband and stopband ripples should be as low as possible.
- Width of transition band should be as small as possible.

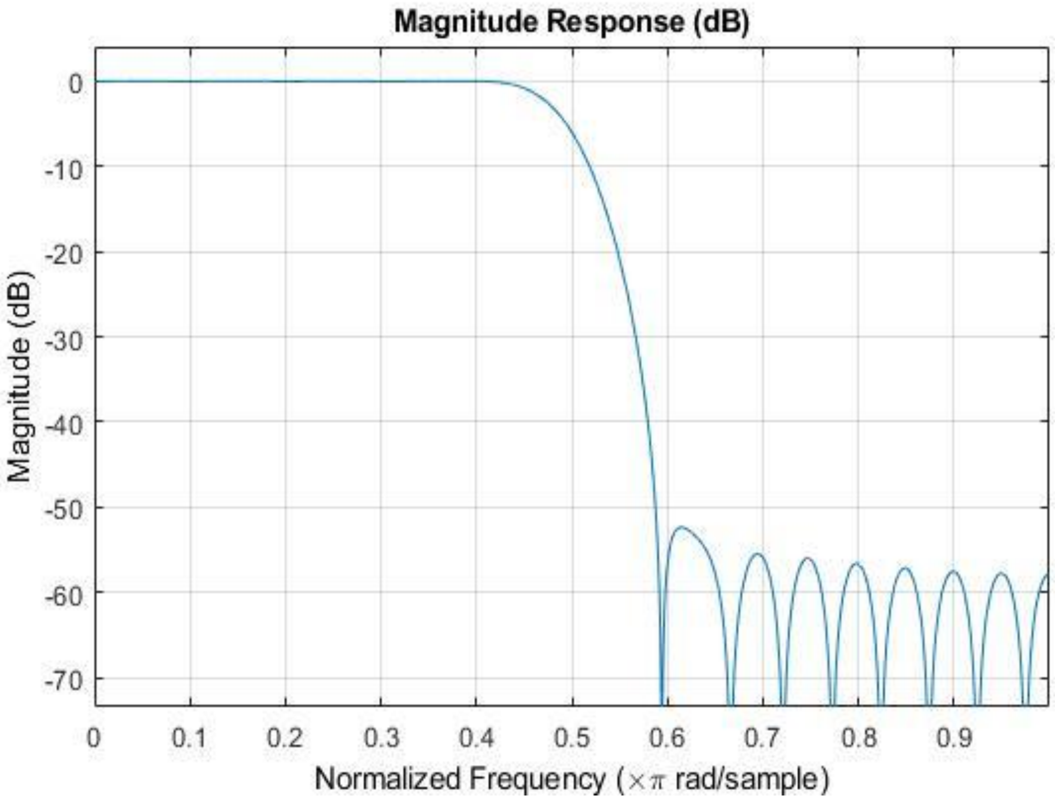


Comparison of two different window method

Rectangular window

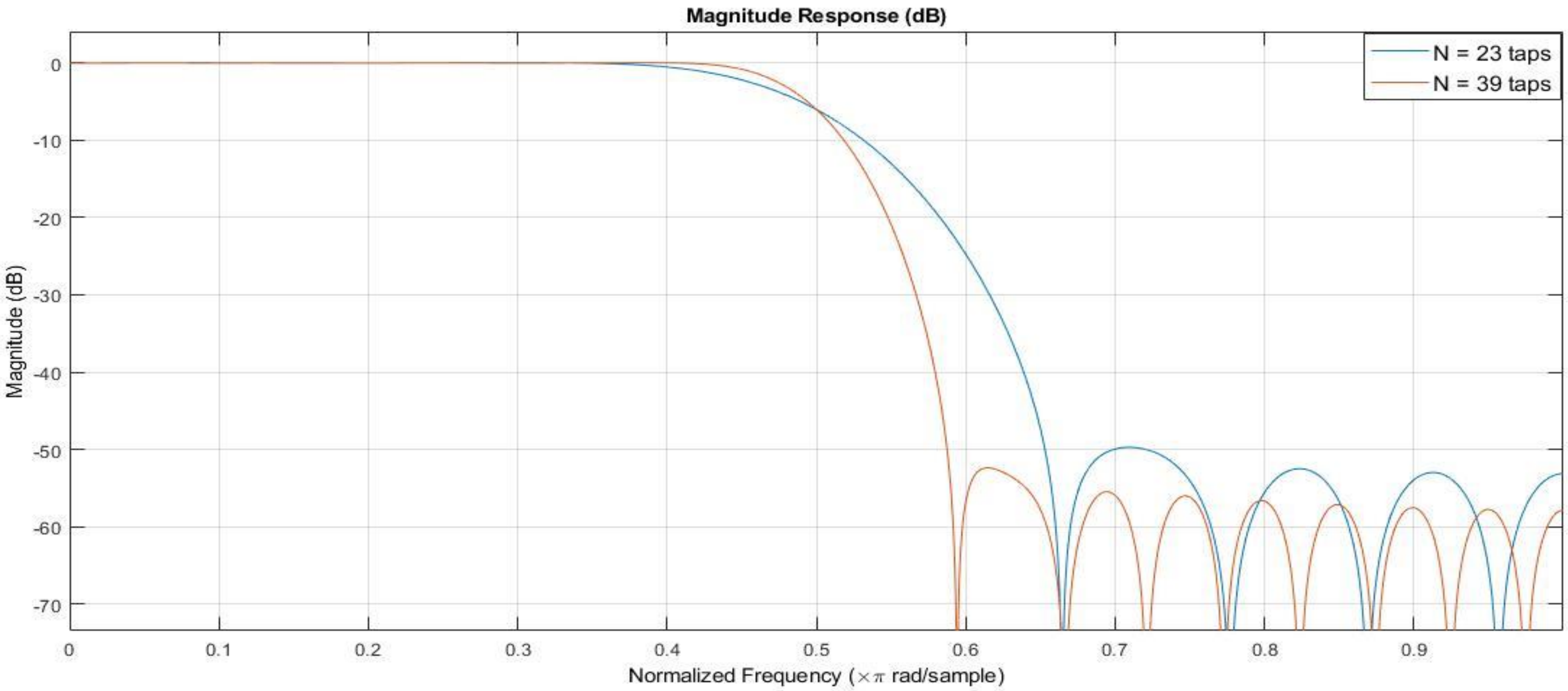


Hamming window



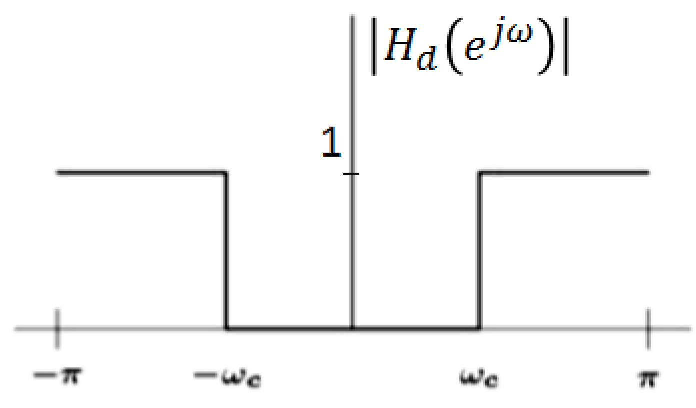
Observation: Hamming windowing results in less passband and stopband ripples than rectangular windowing.

Comparison of two different tap filter with Hamming windowing method



<u>Observation</u>: More taps in impulse response results in lesser transition band width.

Design of HPF



IFFT $\longleftrightarrow$

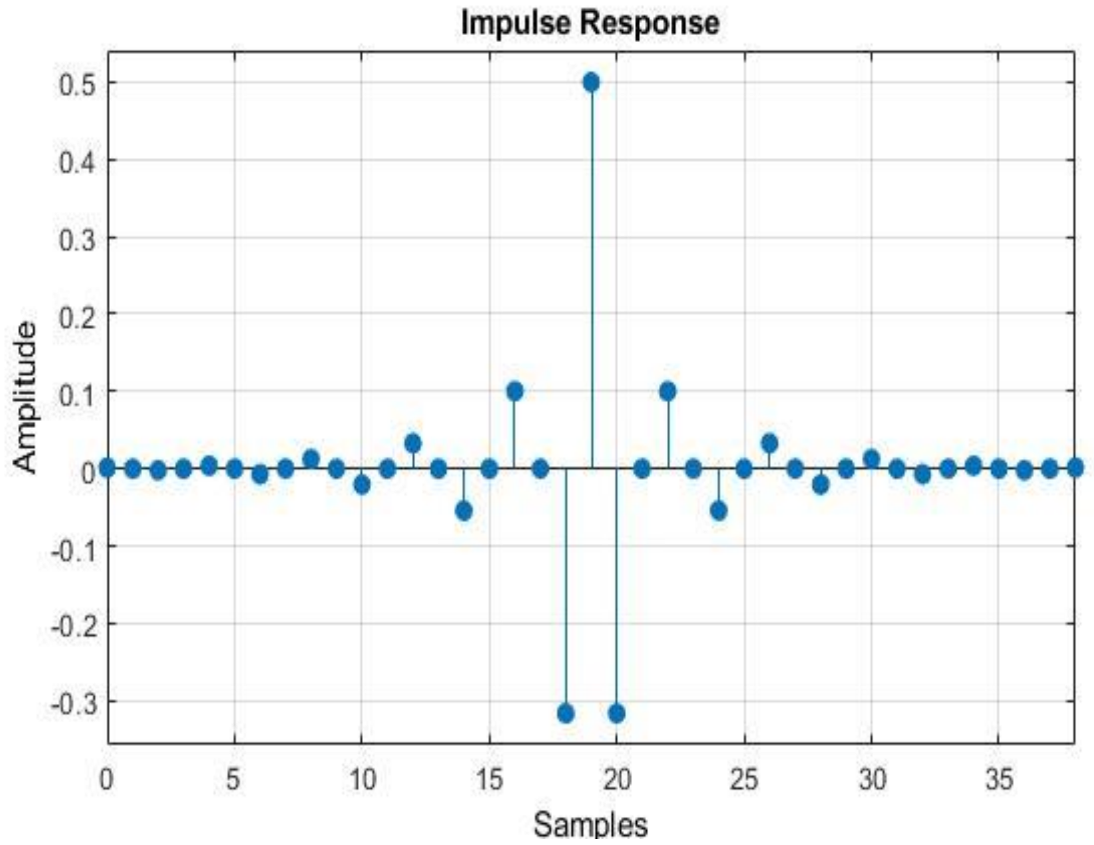
$$\mathrm{h_d}[n] = \frac{Sin(\pi n)}{\pi\, n} - \frac{Sin(\omega_c\, n)}{\pi\, n}$$

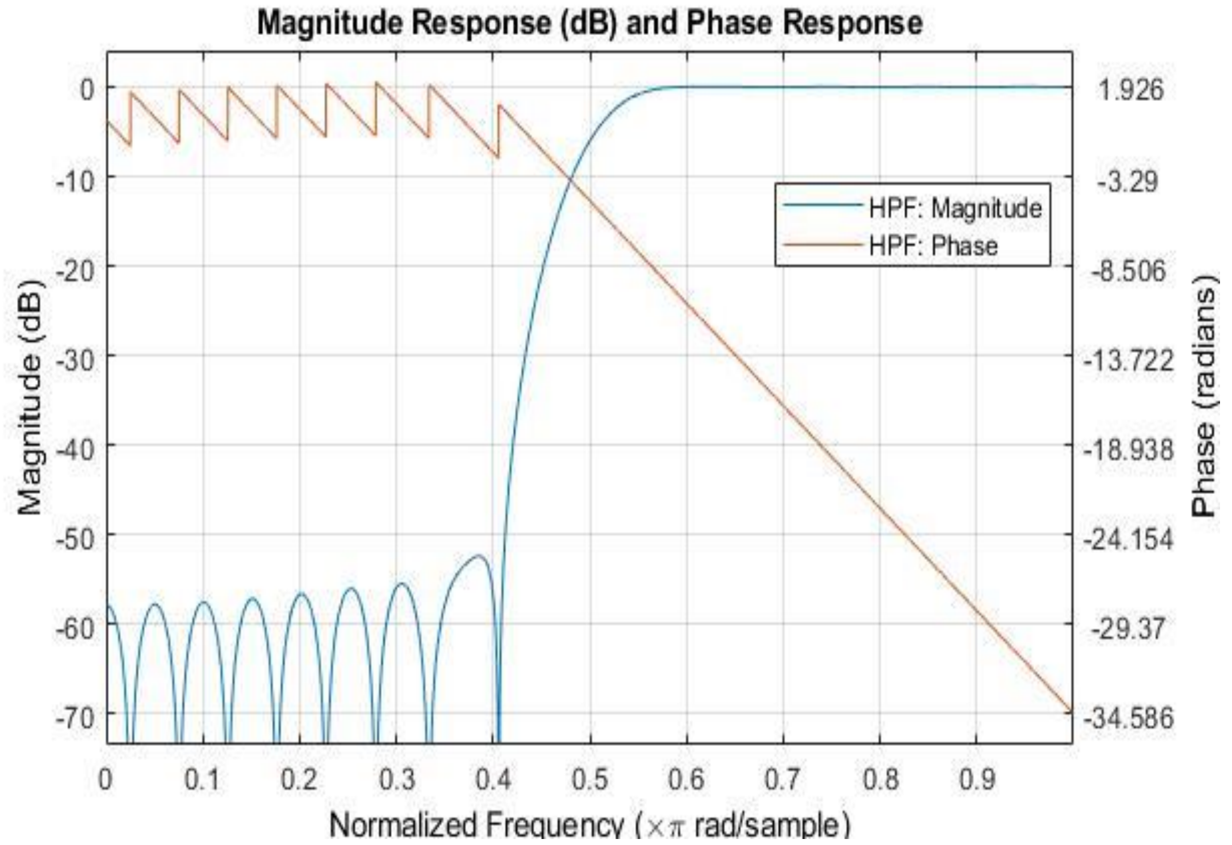
$$for\; n = 0,\quad h_d\,[0] = 1 - \frac{\omega_c}{\pi}$$

Idea: Subtract the LPF magnitude response with cutoff frequency ω_c from a LPF magnitude response with cutoff frequency π.

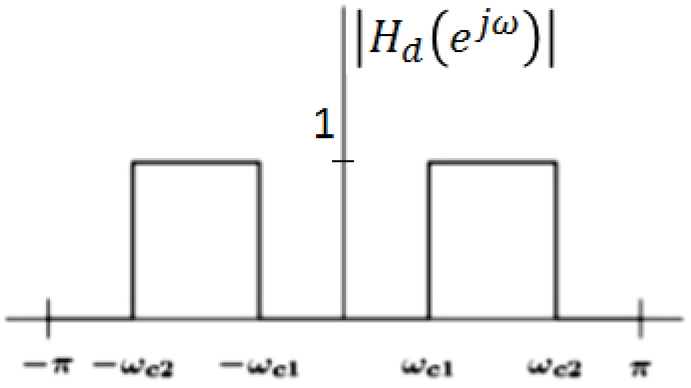
- So for HPF $h_d[n] = \begin{cases} \dfrac{Sin(\pi n)}{\pi\, n} - \dfrac{Sin(\omega_c\, n)}{\pi\, n}\,, & -(N-1)/2 \le n \le (N-1)/2 \\ 1 - \dfrac{\omega_c}{\pi}\,, & n = 0 \end{cases}$
- The impulse response of practical HPF is $h[n] = h_d[n] \times w[n]$

Magnitude, phase and impulse response of practical HPF from Matlab simulation





Design of BPF



IFFT

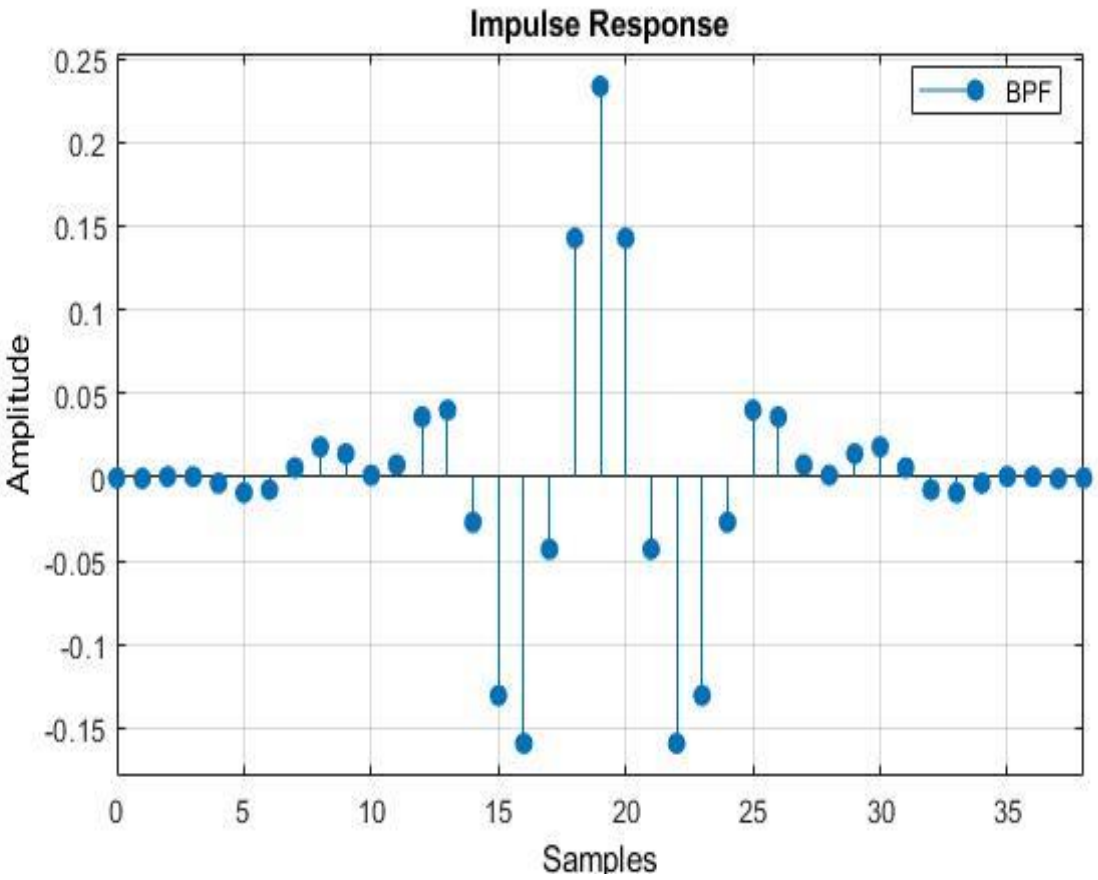
$$\mathrm{h_d}[n] = \frac{Sin(\omega_{c2} n)}{\pi\, n} - \frac{Sin(\omega_{c1}\, n)}{\pi\, n}$$

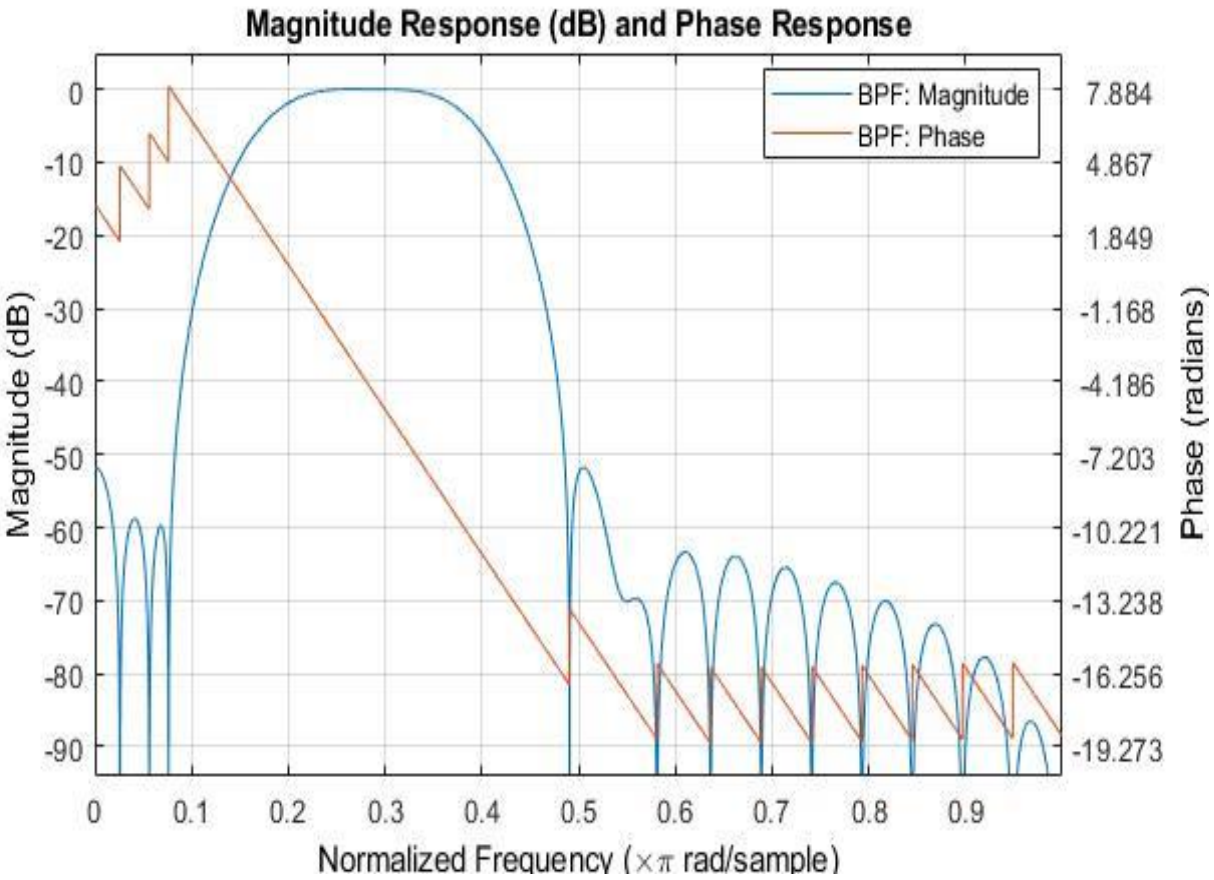
$$for\ n = 0, \quad h_d[0] = \frac{\omega_{c2} - \omega_{c1}}{\pi}$$

Idea: Subtract the LPF magnitude response with cutoff frequency ω_{c1} from a LPF magnitude response with cutoff frequency ω_{c2}.

- So for BPF $h_d[n] = \begin{cases} \frac{Sin(\omega_{c2} n)}{\pi\, n} - \frac{Sin(\omega_{c1}\, n)}{\pi\, n}, & -(N-1)/2 \le n \le (N-1)/2 \\ \frac{\omega_{c2} - \omega_{c1}}{\pi}, & n = 0 \end{cases}$
- The impulse response of practical BPF is $h[n] = h_d[n] \times w[n]$

Magnitude, phase and impulse response of practical BPF from Matlab simulation





Steps for C code implementation of filters

1. Decide the filter parameters such as cutoff frequency(f_c), sampling frequency(f_s), number of taps or samples(N).
2. Generate the N samples of $h_d[n]$ in time domain for the filter you want to design.
3. Multiply the window function $w[n]$ with $h_d[n]$ to get practical impulse response $h[n]$.

Exercise

- Design a digital LPF with gain = 1 and $\omega_c = \pi/3$

Steps: 1. decide f_c, f_s and N, but $f_c = f_s/6$.

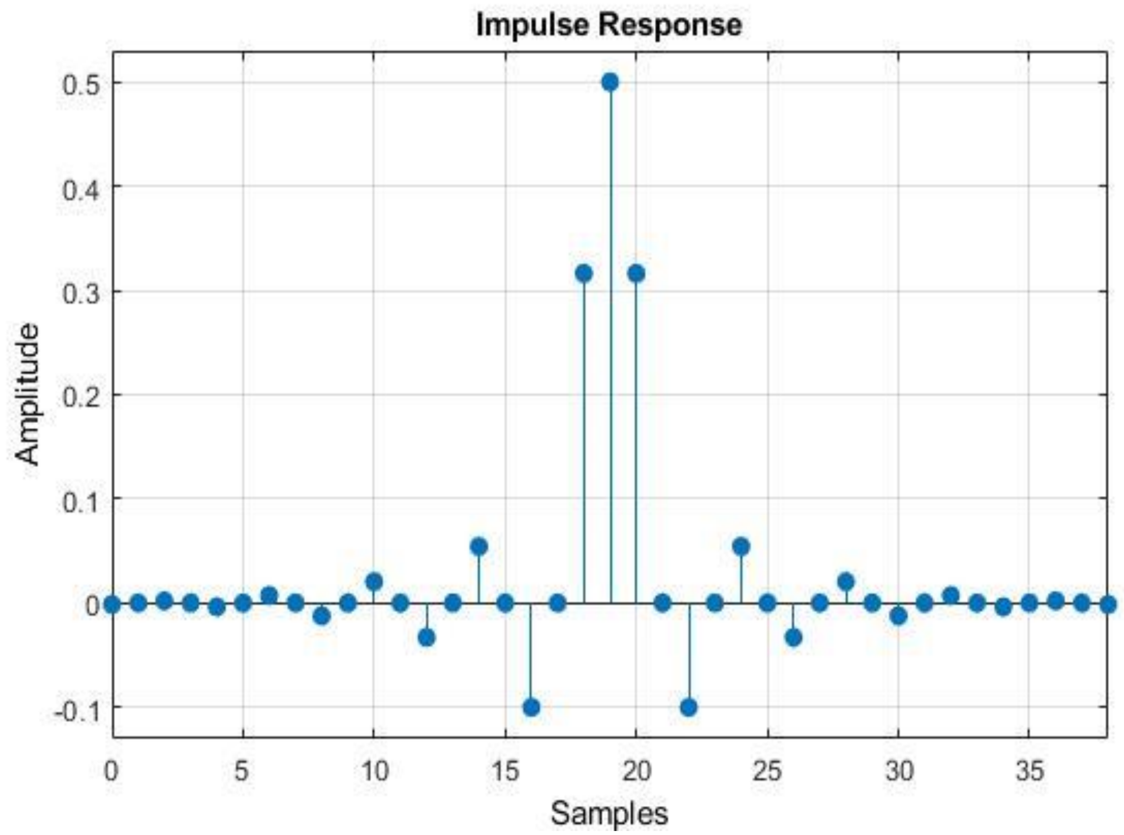
2. Generate the N samples of $h_d[n]$ using the parameters in step-1.

3. Multiply $h_d[n]$ with window function $w[n]$ to get $h[n]$.

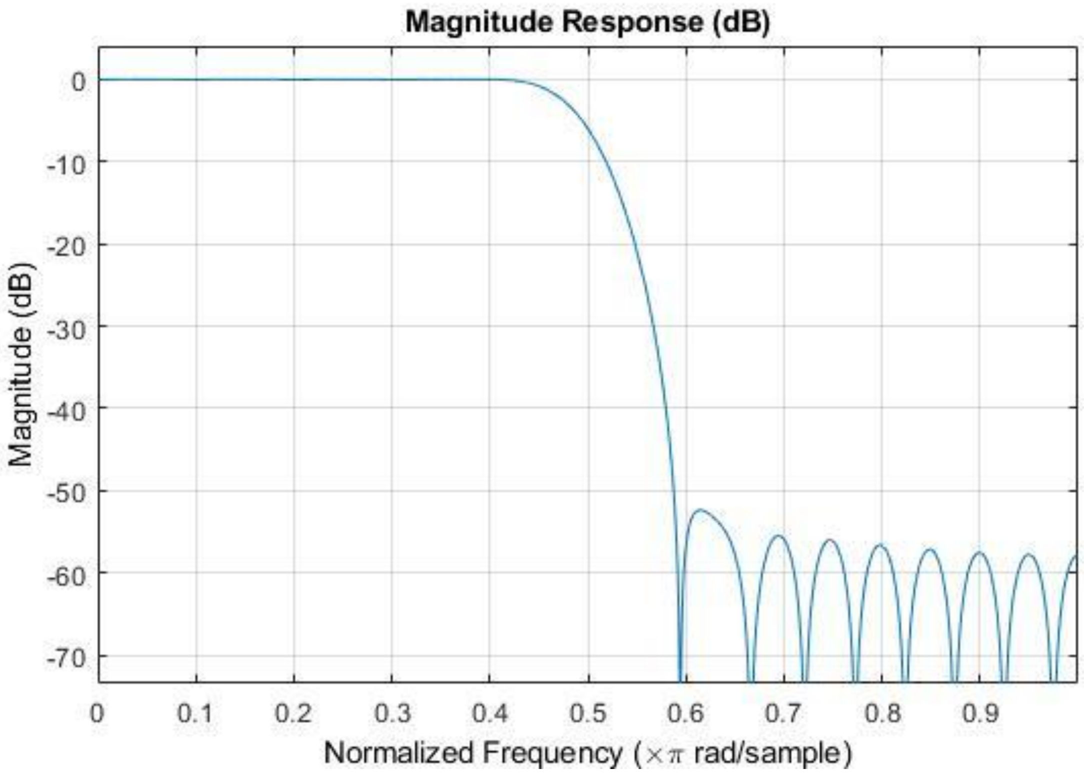
Half band filter (HBF)

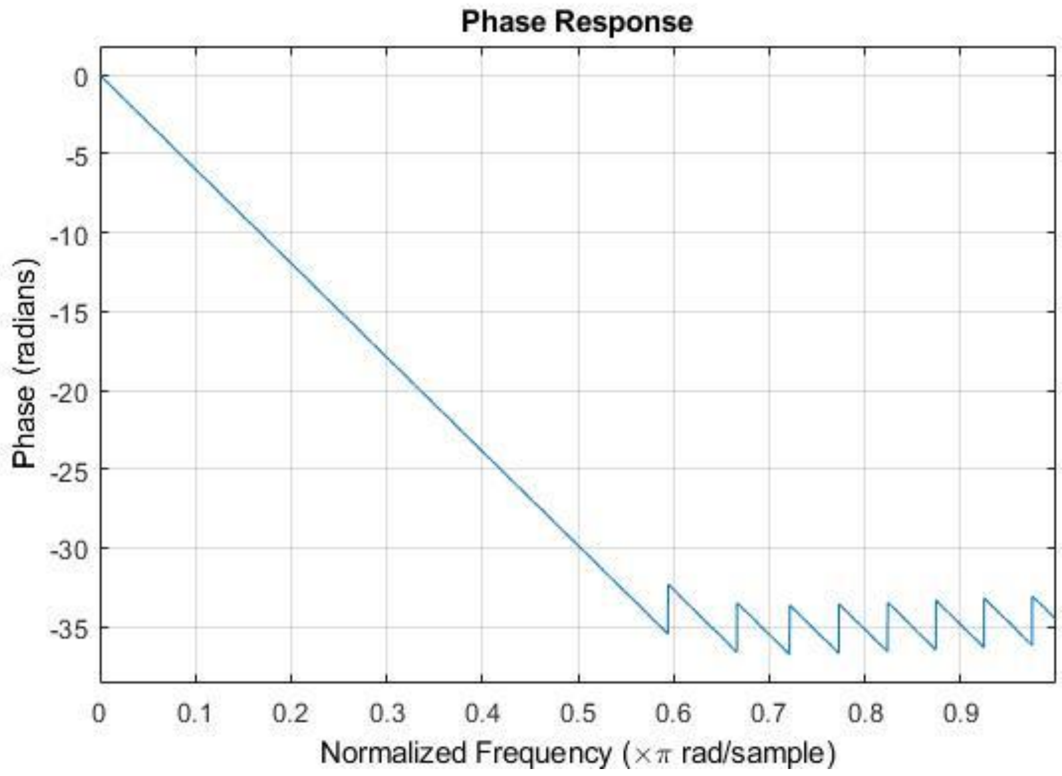
- HBF is a special case of LPF, whose $f_c = f_s/4$.
- Impulse response of half band filter is

$$h(2n) = \begin{cases} 1/2 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



Half band filter (HBF)





M band filter

- Generalization of half band filter is M band filter.
- Impulse response of M band filter is

$$h(Mn) = \begin{cases} c & n = 0 \\ 0 & n \neq 0 \end{cases}$$

- For half band filter, $c = \frac{1}{2}$, Cut-off frequency: $\omega_c = \frac{\pi}{2}$
- For M band filter, $c = \frac{1}{M}$, Cut-off frequency: $\omega_c = \frac{\pi}{M}$

Applications

- Pulse shaping: Raised cosine pulse filters are used to minimize ISI.

- Decimation and Interpolation

- Noise and interference suppression

Reference

- Discrete Time Signal Processing by Alan V. Oppenheim and Ronald W. Schafer [Chapter 7, section 7.2]
- Multirate Digital Filters, Filter Banks, Polyphase Networks, and Applications: A Tutorial, P.P. Vaidyanathan, senior member, ieee [section V. A]