Information Theory, Quiz 2 (Feb 27, 2023)

Serial Number: _____ **Roll Number:** _____

Important

- Any malpractice will lead to instant fail grade.
- This is a question paper and answer sheet. Please provide only final answers in the spaces provided. Do not include derivations or proofs. You will be given a 8-page booklet for rough work.
- **Write your roll number and "serial number" in this sheet.**
- **No breaks during the exam.** No books, notes or electronic devices allowed.

Have a good test, and try listening to 

Instructions: Provide only the final answers for the below questions.

1. (1 mark) Suppose X is a real-valued continuous random variable with density $f(x)$ such that $f(x) = 0$ for all $x < 0$ and all $x > 3$. Then the maximum possible value of $h(X)$ is

   Answer:

2. (1 mark) Suppose X is a real-valued continuous random variable with density $f(x)$ such that

   $$\int_{-\infty}^{\infty} x^2 f(x)\, dx = a$$

   for some real number $a > 0$. Then the maximum possible value of $h(X)$ is

   Answer:

3. (1 mark) Let X be a continuous random variable, and let Y be obtained by quantizing X using a step size of Δ, i.e., $Y = \left\lfloor \frac{X}{\Delta} \right\rfloor$, where $\lfloor x \rfloor$ denotes the largest integer smaller than or equal to x. As $\Delta \to 0$, what is the approximate relation between $h(X)$ and $H(Y)$.

   Answer:

4. (1 mark) The entropy power of a continuous random variable X is equal to

   Answer:

5. (1 mark) Consider any set $B \subset \mathbb{R}^n$, and assume that $X_1, \ldots, X_n$ are iid with each X_i having the same density as the continuous random variable X. If n is sufficiently large and $P\left[(X_1, \ldots, X_n) \in B\right] \geq 1 - \epsilon$, then the volume of B is lower bounded by

   Answer:

6. (1 marks) If (X_1, X_2) is a two-dimensional Gaussian random vector with covariance matrix

   $$\boldsymbol{K} = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix},$$

   then what is the value of $I(X_1; X_2)$?

   Answer:

7. (2 marks) Suppose X is a discrete random variable with $\mathcal{X} = \{0, 1, 2, 3, \ldots\}$ and U is a continuous random variable with uniform distribution in $[0, 0.5]$. Let $Y = X + U$, where X and U are independent. What is the relation between $h(Y)$ and $H(X)$?

   Answer:

8. (2 marks) Suppose X has density function $f(x) = 3x^2$ for $0 \leq x \leq 1$ and $f(x) = 0$ otherwise. Approximately draw/sketch the typical set $A_\epsilon^{(n)}$ for the two-dimensional case $n = 2$. Ensure that you clearly highlight the typical set in your sketch. You don't have to use the correct value of $h(X)$ to generate the sketch.

   Answer: