Department of Electrical Engineering
IIT Hyderabad

EE 6340/3801
Wireless Communications

Modulation

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Lecture 6 Outline

- Announcements
- Last few Lecture
  - Wireless Channel
- **Todays lecture**
  - **Modulation**
  - **Linear Modulation I**
  - **Complex Signal Representation**
  - Pulse Shaping
  - Linear Modulation II
  - Non-linear Modulation
  - FDMA

MODULATION: DEFINITION

- **What is modulation?**
  - The process of modifying a carrier signal (usually a sinusoid) in accordance with an information-bearing signal (modulating signal), such that the modified carrier signal (modulated signal) can be used to convey information.
  - Carrier signal:

$$c(t) = A_c \sin(2\pi f_c t + \theta)$$

    - Three parameters: amplitude, frequency, phase
    - One or all of them can be modified to carry information.
      - E.g. the amplitude is modified to carry information $m(t)$

$$s(t) = m(t) A_c \sin(2\pi f_c t + \theta)$$

$$\boldsymbol{m(t)} \longrightarrow \boxed{\textbf{Modulator}} \longrightarrow \boldsymbol{s(t)}$$

MODULATION: DEFINITION

- **What is modulation? (cont'd)**
  - Modulating signal $m(t)$
    - Original electrical information-bearing signal
    - E.g. electrical waveform representing voice, ASCII, digital video, etc.
    - Usually at low frequency
    - Also called baseband signal
  - Carrier signal $c(t)$
    - High frequency sinusoid to carry the information.
  - Modulated signal $s(t)$
    - Carrier signal modified by modulating signal
    - At the output of the modulator.
    - Also called bandpass signal, or RF signal

$$\boldsymbol{m(t)} \longrightarrow \boxed{\textbf{Modulator}} \longrightarrow \boldsymbol{s(t)}$$

$$\uparrow$$

$$\boldsymbol{c(t)}$$

MODULATION: WHY?

- **Why modulation?**
  - Shift the frequency of the message signal to the pre-allocated channel.
    - For example:
      - speech signal: 300 ~ 3100 Hz $m(t)$
      - Shift the signal to the allocated frequency range: 900MHz $c(t)$
  - Transfer the message into a form more suitable for wireless transmission.
    - Limited bandwidth
    - Make better use of the limited spectrum
    - High frequency signals are more suitable for wireless transmission.
  - Enables multiple access
    - Signals from different users can be shifted to different frequencies.
    - Modulation allows the simultaneous transmission of multiple users.

MODULATION: TERMS

- **Demodulation**
  - Recover the original message signal $m(t)$ from the modulated signal $s(t)$.
- **Modulator**
  - Device used to perform modulation
- **Demodulator**
  - Device used to perform demodulation.

***r*(*t*): modulated signal impaired by fading and noise.**

MODULATION: CLASSIFICATIONS

- **Linear modulation v.s. non-linear modulation**
- **Analog modulation v.s. Digital modulation**
- **Amplitude modulation v.s. Angle modulation**

MODULATION: LINEAR V.S. NON-LINEAR

- **Principle of superposition**

  - If input is $m_1(t)+m_2(t)$, then output is $s_1(t)+s_2(t)$
  - If input is $a \cdot m_1(t)$, then output is $a \cdot s_1(t)$
- **If the input-output of the modulator satisfies principle of superposition, then the modulator is called linear modulator.**
  - E.g.

$$s(t) = m(t)A_c \sin(2\pi f_c t + \theta)$$

- **If the principle of superposition is not satisfied, the modulator is non-linear modulator.**

MODULATION: ANALOG V.S. DIGITAL

- **Analog modulation**
  - $m(t)$ is an analog signal
    - $m(t)$ is a continuous function of time t.
    - $m(t)$ can take infinite number of values
  - Analog modulation is also called continuous-wave (CW) modulation.
  - AM radio, FM radio, first generation cell phone system
- **Digital modulation**
  - $m(t)$ is a digital signal
    - Take finite number of values
    - E.g. {-1, 1}, {-2, -1, 1, 2}, …
  - 2^{nd} generation cell phone system
  - Digital modulation systems have become more and more popular.

MODULATION: AMPLITUDE V.S. ANGLE

- **Amplitude modulation**
  - The amplitude of the carrier, A_c , is modified by the message signal $m(t)$.
- **Angle modulation**
  - The angle of the carrier is modified by the message signal $m(t)$.

$$\psi(t) = 2\pi f_c t + \theta$$

  - Frequency modulation: frequency is modified by $m(t)$.
  - Phase modulation: phase is modified by $m(t)$

LINEAR MODULATION: AM

- **Amplitude modulation (AM)**
  - The term, AM, is usually used when $m(t)$ is analog

$$s(t) = A_c \underbrace{m(t)}_{\text{message signal}} \cos(2\pi \underbrace{f_c}_{\text{carrier frequency}} t)$$

- **Spectrum of AM**
  - $m(t) \rightarrow M(f)$
  - $A_c \cos(2\pi f_c t) \rightarrow \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]$
  - $s(t) \rightarrow S(f) = ?$

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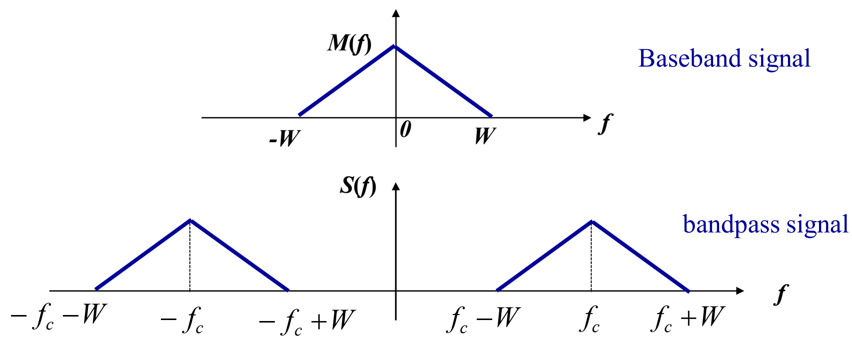
LINEAR MODULATION: AM

- **Spectrum of AM**



- **Center frequency shifted from 0 to f_c**
- **The bandwidth is doubled from *W* to 2*W***

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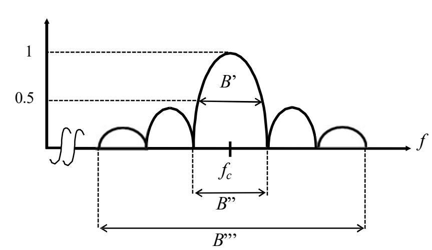
LINEAR MODULATION: AM

- **Bandwidth**
  - Baseband bandwidth (bandwidth of baseband signal): from 0 to W
  - Passband bandwidth (bandwidth of modulated signal): from *Fc-W* to *Fc+W.*
    - Passband bandwidth is twice of baseband bandwidth.
- **How do we define bandwidth?**
  - There are many different definitions of bandwidth
  - B': half power bandwidth (3dB bandwidth).
  - B": null-to-null bandwidth
  - B"': absolute bandwidth
  - FCC definition: bandwidth contains 99% of signal power

power spectrum



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LINEAR MODULATION: BPSK

- **Binary phase shift keying (BPSK) ➔ Digital modulation**

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t) & \text{for binary 1} \\ A_c \cos(2\pi f_c t + \pi) & \text{for binary 0} \end{cases}$$

  - Or alternatively

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t) & \text{for binary 1} \\ -A_c \cos(2\pi f_c t) & \text{for binary 0} \end{cases}$$

  - Also called binary amplitude shift keying (BASK)

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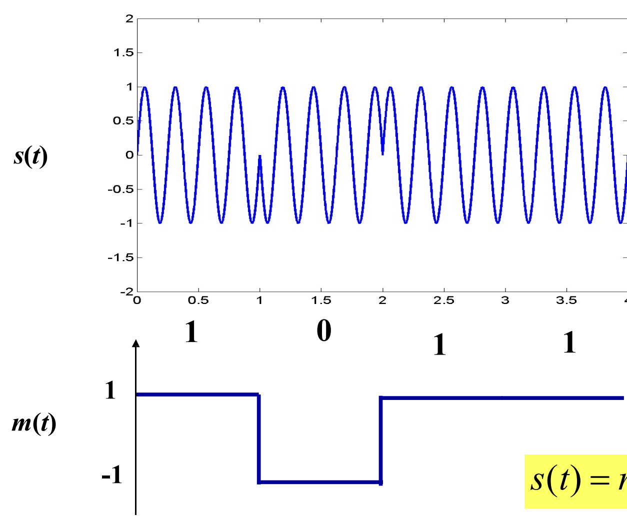
LINEAR MODULATION: BPSK

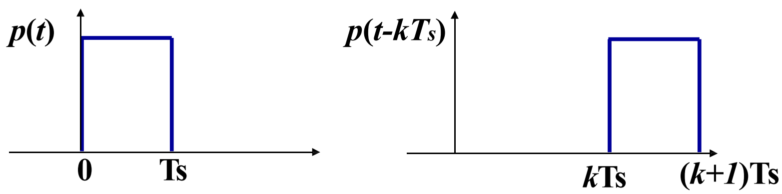


$$s(t) = m(t) \cdot \sin(2\pi f_c t)$$

LINEAR MODULATION: BPSK

$$m(t) = \sum_{k=1}^{\infty} b_k \, p(t - kT_s)$$

$$b_k = \begin{cases} 1 \\ -1 \end{cases} \qquad p(t) = \begin{cases} 1 & 0 \le t \le T \\ 0 & o.w. \end{cases}$$



Ts**:** symbol period. (The time duration to carry one symbol)

- usually inverse proportional to signal BW

LINEAR MODULATION: MASK

- **M-ary amplitude shift keying (MASK)**
  - The information can take $M = 2^n$ values

$$m(t) = \sum_{k=1}^{\infty} m_k \, p(t - kT_s)$$

$$m_k = \{s_1, s_2, \cdots, s_M\} \qquad p(t) = \begin{cases} 1 & 0 \le t \le T \\ 0 & o.w. \end{cases}$$

- **E.g. 4-ary amplitude shift keying (4ASK)**

$$m_k = \{-3, -1, 1, 3\}$$

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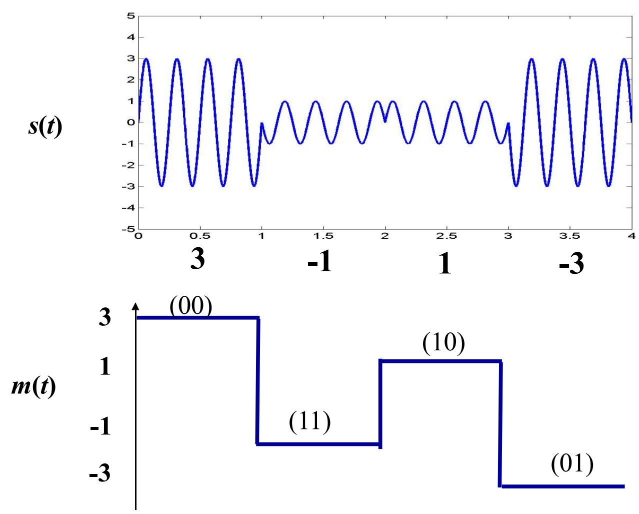
LINEAR MODULATION: MASK



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LINEAR MODULATION: MASK

- **Each symbol can take *M* values**

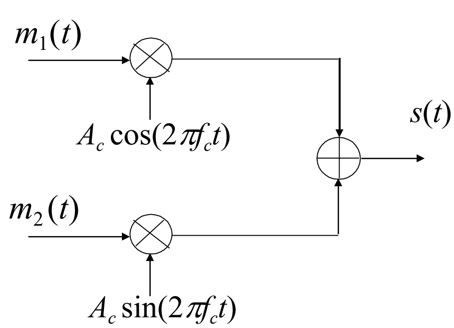
  ➔Each symbol can represent $n = \log_2 M$ bits of information.
- **E.g.**
  - 4ASK ➔ M = 4 ➔ 2 bits/sym
  - 8ASK ➔ M = 8 ➔ 3 bits/sym
  - 16ASK ➔ M = 16 ➔ 4 bits/sym
- **Symbol rate (baud) R_s :**
  - # of modulation symbols/second
  - Generally, signal bandwidth is proportional to symbol rate!
- **Bit rate R_b:**
  - Bits/second
- **$R_b = R_s$ x (# of bits/sym)**

LINEAR MODULATION: MASK

- **M-ary modulation**
  - At the same symbol rate, (or fixed bandwidth)
    - Larger M ➔ more bits/symbol ➔ larger bit rate
  - M-ary modulation is good for band-limited system
    - In wireless systems, spectrum is precious
    - M-ary modulation is widely used in wireless systems!
    - Typical values used in wireless system: M = 2, 4, 8, 16
- **Why don't we use a very large M?**
  - If M ➔ infinity, then analog signal
  - At the same SNR
    - Larger M ➔ signals are more closed to each other ➔ it's harder to distinguish between all signals at receiver due to noise ➔ probability of error becomes larger!

LINEAR MODULATION: QPSK

- **Quadrature phase shift keying (QPSK)**
  - Combination of two BPSK streams
  - Quadrature: orthogonal (two carriers that are 90 degree apart)



$$s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)$$

Inphase signal

quadrature signal

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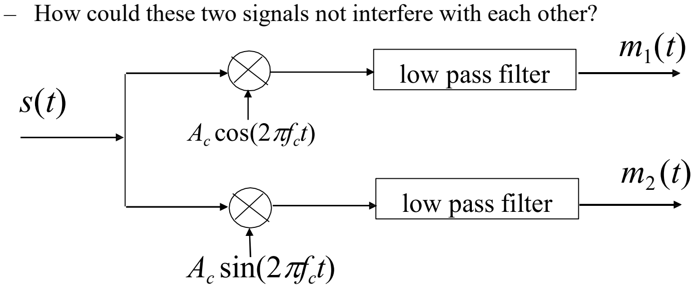
LINEAR MODULATION: QPSK

- **Demodulator**
  - How could these two signals not interfere with each other?



$$s(t)\cos(2\pi f_c t) = A_c m_1(t)\cos^2(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)\cos(2\pi f_c t)$$
$$= \frac{A_c}{2} m_1(t) + \frac{A_c}{2} m_1(t)\cos(4\pi f_c t) + \frac{A_c}{2} m_2(t)\sin(4\pi f_c t)$$

After low pass filter: $\frac{A_c}{2} m_1(t)$

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LINEAR MODULATION: QPSK

- **At each symbol period, two bits of information are transmitted**
  - 2 bits/symbol
  - Inphase signal $s_I(t) = A_c m_1(t)\cos(2\pi f_c t)$ has the same bandwidth as BPSK
  - Quadrature signal $s_Q(t) = A_c m_1(t)\sin(2\pi f_c t)$ has the same bandwidth as BPSK
  - The sum $s(t) = s_I(t) + s_Q(t)$ has the same bandwidth as BPSK
    - Sum in time domain ➔ sum in frequency domain ➔ bandwidth unchanged.
  - The same bandwidth as BPSK, but twice the bit rate of BPSK!

  - No interference between quadrature and inphase
    - Inphase has the same error performance as BPSK
    - Quadrature has the same error performance as BPSK
    - QPSK has the same error performance of BPSK!

One of the rare occasions that increase bit rate without sacrificing error performance!

- Modulation
- Linear Modulation Techniques I
- **Complex Signal Representation**
- Pulse Shaping
- Linear Modulation Techniques II
- Non-linear modulation
- FDMA

COMPLEX REPRESENTATION

- **Band-pass signal**

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

- **Complex base-band signal (also called the complex envelope of the signal)**

$$\tilde{s}(t) = s_I(t) + js_Q(t)$$

- **Relationship between complex baseband and band-pass**

$$s(t) = \mathrm{Re}\left\{\tilde{s}(t)\exp(j2\pi f_c t)\right\}$$

COMPLEX REPRESENTATION

- **Modulation can be decomposed into two steps:**
  - 1. complex baseband modulation, 2. frequency upconversion
    - The complex representation completely preserves the information of the modulating signal except the carrier frequency
  - It's suffice for us to examine baseband modulation only!

$$s(t) = \mathrm{Re}\{\tilde{s}(t)\exp(j2\pi f_c t)\}$$

$$\boxed{\text{Baseband modulation}} \longrightarrow \boxed{\text{Frequency upconversion}}$$

Modulation

COMPLEX REPRESENTATION: BASEBAND MODULATION

- **BPSK**

$$s_I(t) \in \{-1,1\} \qquad s_Q(t) = 0$$

$$\tilde{s}(t) \in \{-1,1\} \qquad (M = 2)$$

- **QPSK**

$$s_I(t) \in \{-1,1\} \qquad s_Q(t) \in \{-1,1\}$$

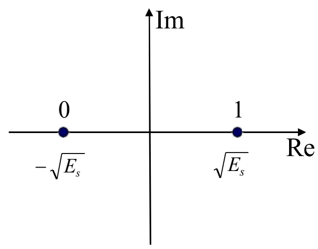
$$\tilde{s}(t) \in \{-1-j, -1+j, 1-j, 1+j\} \qquad (M = 4)$$

$$\boxed{\text{Baseband modulation}} \longrightarrow \boxed{\text{Frequency upconversion}}$$

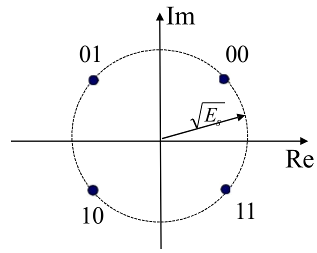
Modulation

COMPLEX REPRESENTATION: CONSTELLATION

- **Modulation Constellation**
  - The collection of all modulation symbols in their complex representation.
  - E_s : the energy of one symbol



BPSK



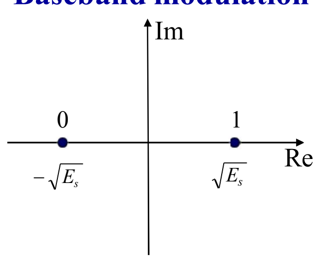
QPSK

$$\tilde{s}(t) \in \left\{ \sqrt{E_s}\exp(j0), \sqrt{E_s}\exp(j\pi) \right\}$$

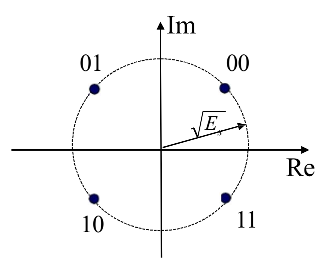
$$\tilde{s}(t) \in \left\{ \begin{array}{l} \sqrt{E_s}\exp(j\frac{\pi}{4}), \sqrt{E_s}\exp\left(j\frac{3\pi}{4}\right), \\ \sqrt{E_s}\exp(j\frac{5\pi}{4}), \sqrt{E_s}\exp\left(j\frac{7\pi}{4}\right) \end{array} \right\}$$

COMPLEX REPRESENTATION

- **Baseband modulation examples**



BPSK



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BPSK:

QPSK:

COMPLEX REPRESENTATION: SYMBOL ENERGY

- **Symbol energy *Es***
  - If modulation symbol is

$$\tilde{s}(t) = \sqrt{E_s}\exp\left[j\theta(t)\right]$$

  - The energy of one symbol is E_s .
    - Proof:

- **Bit energy *Eb***
  - The energy of one bit
  - # of bits per symbol: $\log_2 M$

$$E_b = E_s / \log_2 M$$

COMPLEX REPRESENTATION

- **Signals pass through channel with flat fading and AWGN**

h

$\tilde{h}(t) = h_I(t) + jh_Q(t)$ **: time-varying flat fading**

$\tilde{n}(t) = n_I(t) + jn_Q(t)$ **: AWGN**

Both the inphase component and quadrature component are going to be distorted.