Outage Performance for Non-Orthogonal Multiple Access With Fixed Power Allocation Over Nakagami-*m* Fading Channels

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Abstract—In this letter, we study the outage performance of non-orthogonal multiple access (NOMA) with fixed power allocation over Nakagami-m fading channels, where the fading parameters of NOMA users are different. In this scenario, new closed-form expressions are derived for the outage behavior of individual users and the system, respectively. Next, the diversity orders are obtained under high signal-to-noise ratio condition. The derived analytical expressions are exact and unprecedented in the earlier literature. Finally, simulations are conducted to confirm the validity of the analysis and show the outage performance of NOMA under different fading parameters of Nakagami-m fading channels.

Index Terms—Non-orthogonal multiple access, Nakagami-*m* fading, outage behavior, diversity order.

I. Introduction

S A promising multiple access technology for future wireless communications, non-orthogonal multiple access (NOMA) has received considerable attention because of its superior spectral efficiency. In NOMA, clustered users multiplex in the power domain, while at the receivers, the composite signal of different users is separated by successive interference cancellation (SIC) [1].

Recently, a few different forms of NOMA have been proposed in the literature. Ding et al. [2] investigated the outage performance of individual users and ergodic sum rate with randomly deployed users. The analytical results show that it is more preferable to group users whose channel gains are more distinctive under Rayleigh fading channels. Ding et al. [3], estimated the performance of NOMA with fixed power allocation (F-NOMA) and cognitive radio inspired NOMA over Rayleigh fading channels, which has demonstrated that only the user with higher channel gain influences the outage performance. However, Rayleigh fading is just a special case of fading channels. Considering the line of sight (LoS) communication, it is more advantageous to investigate the outage performance of NOMA with Nakagamim fading channels. To our best knowledge, the general case, namely, Nakagami-m fading channels, has not been well considered yet in previous literature.

Motivated by such observations, in this letter, we study the outage performance of F-NOMA in downlink scenario over Nakagami-*m* fading channels. In particular, the individual

Manuscript received January 6, 2018; revised January 24, 2018; accepted January 24, 2018. Date of publication January 30, 2018; date of current version April 7, 2018. This work was supported by the China Postdoctoral Science Foundation under Grant 2016M600911. The associate editor coordinating the review of this paper and approving it for publication was Y. Liu. (Corresponding author: Zhengyu Song.)

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Digital Object Identifier 10.1109/LCOMM.2018.2799609

user and system probabilities that NOMA achieves lower data rate than traditional orthogonal multiple access (OMA) are analyzed, and the impact of fading parameters on the outage performance is specially evaluated to better show the differences between Rayleigh fading channels and Nakagami-*m* fading channels. Interestingly, the analytical results show that the diversity order of the system depends on both users for Nakagami-*m* fading channels, which is different from [3], where the diversity order only depends on the user with higher channel gain. Besides, the outage probabilities of individual users and the system for F-NOMA over Nakagami-*m* fading channels are derived, which shows that F-NOMA increases the data rate of the user with higher channel gain while sacrificing the user with poorer channel gain.

II. NOMA WITH FIXED POWER ALLOCATION

Consider a downlink NOMA system with one base station (BS) and *M* users, where the channel state information (CSI) of users is perfectly known at the BS [4]. Without loss of generality, it is assumed that the channel gains of users in the system follow the order as

$$|h_1|^2 \le |h_2|^2 \le \dots \le |h_M|^2.$$
 (1)

Based on the NOMA protocol, the power allocated to users should satisfy $\alpha_1 \geq \cdots \geq \alpha_M$, and $\sum_{i=1}^M \alpha_i = 1$. In [3], it is indicated that grouping all the users in NOMA is not preferable in practice. Therefore, we consider a situation that two users, i.e., user w and user v with v < w, are paired to perform NOMA. In this case, the power allocated to the two users satisfies $\alpha_v > \alpha_w$ and $\alpha_v + \alpha_w = 1$. Accordingly, by applying SIC, the data rates of user w and user v are

$$R_w^N = \log\left(1 + \alpha_w \rho |h_w|^2\right),\tag{2}$$

$$R_{v}^{N} = \log \left(1 + \frac{\alpha_{v} |h_{v}|^{2}}{\alpha_{w} |h_{v}|^{2} + 1/\rho} \right), \tag{3}$$

where ρ is the transmit signal-to-noise ratio (SNR).

Correspondingly, the data rates of user w and user v in OMA are

$$R_w^O = \frac{1}{2} \log \left(1 + \rho |h_w|^2 \right),\tag{4}$$

$$R_{v}^{O} = \frac{1}{2} \log \left(1 + \rho |h_{v}|^{2} \right). \tag{5}$$

III. OUTAGE BEHAVIOR

A. The Outage Probability of Individual Users

First, we focus on the probability that the v-th user achieves lower data rate than OMA, which can be

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expressed as

$$P(R_v^N < R_v^O) \qquad \text{sion on } \Lambda, \text{ we have}$$

$$= P\left(\log\left(1 + \frac{\alpha_v |h_v|^2}{\alpha_w |h_v|^2 + 1/\alpha}\right) < \frac{1}{2}\log(1 + \rho |h_v|^2)\right). \quad (6) \quad (\sum_{r=0}^{m_1 - 1} q_r y^r)^s = \sum_{s_1 = 0}^s \sum_{s_2 = 0}^{s - s_1} \cdots \sum_{s_{m_1 - 1} = 0}^s \sum_{s_2 = 0}^{s_2 - s_2} \cdots \sum_{s_{m_2 - 1} = 0}^s \sum_{s_3 = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_{m_3 - 1} = 0}^s \sum_{s_3 = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_3 = 0}^s \sum_{s_3 = 0}^s \sum_{s_3 = 0}^s \cdots \sum_{s_3 = 0}^s \sum_{s_$$

After some algebraic manipulations, we have

$$P(R_v^N < R_v^O) = P\left(\left(1 + \rho |h_v|^2\right) < \left(1 + \rho \alpha_w |h_v|^2\right)^2\right)$$

$$= P\left(|h_v|^2 > \frac{1 - 2\alpha_w}{\rho \alpha_w^2}\right). \tag{7}$$

Note that the marginal probability density function (PDF) of $|h_v|^2$ is given by [5]

$$f_{|h_{\nu}|^{2}}(y) = \varpi_{1} f(y) (F(y))^{\nu-1} (1 - F(y))^{M-\nu}, \tag{8}$$

and the joint PDF of $|h_v|^2$ and $|h_w|^2$ is given by

$$f_{|h_w|^2,|h_v|^2}(x,y) = \varpi_2 f(x) f(y) F(x)^{v-1} (1 - F(y))^{M-w} \times (F(y) - F(x))^{w-v-1}, \tag{9}$$

where $\varpi_1 = \frac{M!}{(v-1)!(M-v)!}$ and $\varpi_2 = \frac{M!}{(v-1)!(w-v-1)!(M-w)!}$. Since $|h_v|$ and $|h_w|$ follow Nakagami-m distribution with fading parameters m_1 and m_2 , respectively, the channel gain also follows a Gamma distribution, with PDF as [6]

$$f_1(y) = \frac{m_1^{m_1} y^{m_1 - 1}}{\Omega_1^{m_1} \Gamma(m_1)} e^{-\frac{m_1 y}{\Omega_1}},$$
 (10)

$$f_2(y) = \frac{m_2^{m_2} y^{m_2 - 1}}{\Omega_2^{m_2} \Gamma(m_2)} e^{-\frac{m_2 y}{\Omega_2}},$$
 (11)

where f_1 means the parameters of PDF are m_1 and w_1 . Note that $\Gamma(a) = (a-1)!$ when a is an integer [7]. Hence, the cumulative distribution function is

$$F_1(y) = 1 - e^{-\frac{m_1 y}{\Omega_1}} \sum_{r=0}^{m_1 - 1} \frac{(y m_1 / \Omega_1)^r}{r!}.$$
 (12)

Denote $I=\frac{1-2\alpha_w}{\alpha_w^2}$, which is a constant in F-NOMA. By applying the above density functions and binomial expansion, the outage probability in (7) can be expressed as

$$P(|h_{v}|^{2} > \frac{I}{\rho}) = \int_{I/\rho}^{\infty} \varpi_{1} f(y) (F(y))^{v-1} (1 - F(y))^{M-v} dy$$

$$= \varpi_{1} \int_{I/\rho}^{\infty} \sum_{i=1}^{M-v} \binom{M-v}{i} (-1)^{i} f(y) F(y)^{v+i-1} dy$$

$$= \frac{\varpi_{1}}{v+i} \sum_{i=0}^{M-v} \binom{M-v}{i} (-1)^{i} \left((1 - F(I/\rho)^{v+i}) \right). \quad (13)$$

In addition, Λ can be rewritten as

$$\Lambda = \left(1 - e^{-\frac{m_1 y}{\Omega_1}} \sum_{r=0}^{m_1 - 1} \frac{(y m_1 / \Omega_1)^r}{r!}\right)^{s-r}$$

$$= \sum_{s=0}^{v+i} {v+i \choose s} (-1)^s e^{-\frac{m_1 ys}{\Omega_1}} \left(\sum_{r=0}^{m_1 - 1} \frac{(y m_1 / \Omega_1)^r}{r!}\right)^s. \quad (14)$$

Note that $q_r = \frac{(m_1/\Omega_1)^r}{r!}$. Applying successive binomial expansion on Λ , we have

$$(\sum_{r=0}^{m_{1}-1} q_{r} y^{r})^{s} = \sum_{s_{1}=0}^{s} \sum_{s_{2}=0}^{s-s_{1}} \cdots \sum_{s_{m_{1}-1}=0}^{s-s_{1}-\cdots s_{m_{1}-2}} \times \binom{s}{s_{1}} \binom{s-s_{1}}{s_{2}} \cdots \binom{s-s_{1}-\cdots s_{m_{1}-2}}{s_{m_{1}-1}} \times \prod_{r=0}^{m_{1}-2} q_{r}^{s_{r+1}} y^{r s_{r+1}} (q_{m_{1}-1} y^{m_{1}-1})^{s-s_{1}-\cdots s_{m_{1}-2}}.$$

$$(15)$$

Denote
$$U_{s,m_1} = \sum_{s_1=0}^{s} \sum_{s_2=0}^{s-s_1} \cdots \sum_{s_{m_1}=1=0}^{s-s_1-\cdots s_{m_1}-2}, A_{s,m_1} = \left(\frac{s}{s_1}\right) \left(\frac{s-s_1}{s_2}\right) \cdots \left(\frac{s-s_1-\cdots s_{m_1}-2}{s_{m_1-1}}\right), B_{s,m_1} = \prod_{r=0}^{m_1-2} q_r^{s_{r+1}} (q_{m_1-1})^{s-s_1-\cdots s_{m_1}-2}, \overline{s} = (m_1-1)(s-s_1) - (m_1-2)s_2 - \cdots - s_{m_1-1}, \text{ and we also apply the series expansion to the exponential function, i.e., } e^{-\frac{m_1 ys}{\Omega_1}} = \sum_{t=0}^{\infty} \frac{(-1)^t (m_1 ys)^t}{\Omega_1^t t!}, \text{ we can finally attain}$$

$$\Lambda = \sum_{s=0}^{v+i} \binom{v+i}{s} (-1)^s \sum_{t=0}^{\infty} \frac{(-1)^t (m_1 y s)^t}{\Omega_1^t t!} T_{m_1}^s y^{\overline{s}}, \quad (16)$$

where $T_{m_1}^s = \bigcup_{s,m_1} A_{s,m_1} B_{s,m_1}$. According to binomial coefficients, one can know that

$$\varpi_1 \sum_{i=0}^{M-v} \binom{M-v}{i} (-1)^i \frac{1}{v+i} = 1. \tag{17}$$

Substituting (16) and (17) into (13), the outage probability of the v-th user in (7) can be rewritten as

$$P_{v}^{out} = 1 - \frac{\varpi_{1}}{i+v} T_{m_{1}}^{s} \sum_{i=0}^{M-v} {M-v \choose i} (-1)^{i} \sum_{s=0}^{v+i} \times {v+i \choose s} (-1)^{s} \sum_{t=0}^{\infty} \frac{(-1)^{t} (m_{1} y s)^{t}}{\Omega_{1}^{t} t!} y^{\overline{s}}. \quad (18)$$

Recall the following binomial coefficient in [7]:

$$\sum_{s=0}^{v+i} \binom{v+i}{s} (-1)^s (s)^t = 0, \tag{19}$$

for $1 \le t < v + i$ or $t \ge v + i + 1$, and

$$\sum_{s=0}^{v+i} \binom{v+i}{s} (-1)^s (s)^t = (-1)^{v+i} (v+i)!, \tag{20}$$

for t = v + i. With these steps and substituting $y = I/\rho$ to the outage probability, (18) can be rewritten as

$$P_{v}^{out} = 1 - \sum_{i=0}^{M-v} {M-v \choose i} (-1)^{i} \frac{\varpi_{1} T_{m_{1}}^{v+i}}{v+i} \times \left(\frac{m_{1}}{\Omega_{1}}\right)^{v+i} \left(\frac{I}{\rho}\right)^{m_{1}(v+i)}.$$
(21)

The diversity order of the v-th user with high SNR approximation is given by

$$\lim_{\rho \to \infty} -\frac{\log P_v^{out}}{\log \rho} = m_1 v. \tag{22}$$

From (22), one can know that the outage probability of the v-th user depends on the fading parameter m_1 and v itself.

On the other hand, the outage probability of user w is also worth estimating. In NOMA, the w-th user needs to decode the signal intended for user v before decoding its own signal. Therefore, it is assumed that R_v is the target rate for the v-th user, and the probability that the w-th user performs poorer in NOMA than OMA is given by

$$P_{w}^{out} = P\left(\log\left(1 + \frac{\alpha_{v}|h_{w}|^{2}}{\alpha_{w}|h_{w}|^{2} + 1/\rho}\right) < R_{v}\right) + P\left(\log\left(1 + \frac{\alpha_{v}|h_{w}|^{2}}{\alpha_{w}|h_{w}|^{2} + 1/\rho}\right) > R_{v}, \log(1 + \alpha_{w}\rho|h_{w}|^{2}) < \frac{1}{2}\log(1 + \rho|h_{w}|^{2})\right). \quad (23)$$

After some algebraic manipulations and denoting $C = \frac{\varepsilon_v - 1}{|1 - \varepsilon_v a_w|}$, we finally have

$$P_w^{out} = P\left(|h_w|^2 < \frac{C}{\rho}\right) + P\left(\frac{C}{\rho} < |h_w|^2 < \frac{I}{\rho}\right)$$
$$= P\left(|h_w|^2 < \frac{I}{\rho}\right). \tag{24}$$

Similar to the argument from (13) to (22), the outage probability of the w-th user can be obtained as

$$P_{w}^{out} = \sum_{i=0}^{M-w} \binom{M-w}{i} (-1)^{i} \frac{\varpi_{3} T_{m_{2}}^{w+i}}{w+i} \left(\frac{m_{2}}{\Omega_{2}}\right)^{w+i} \left(\frac{I}{\rho}\right)^{m_{2}(w+i)}, \tag{25}$$

where $\varpi_3 = \frac{M!}{(w-1)!(M-w)!}$, and the diversity order is m_2w . One can observe that the diversity order of user w only depends on the fading parameter m_2 and w itself, although the w-th user decodes the signal of the v-th user first.

B. The Outage Probability of Sum Rate

Next, we consider the probability that the sum rate of NOMA is lower than OMA, which is given by

$$P_{SR}^{\text{out}} = 1 - P(R_v^N + R_w^N > R_v^O + R_w^O), \tag{26}$$

where $|h_w|^2 \ge |h_v|^2$, which means the w-th user can always decode the v-th user's signal successfully. By some algebraic handling, we have

$$P_{SR}^{\text{out}} = 1 - P(\rho |h_v|^2 + \rho |h_w|^2 + \rho^2 |h_v|^2 |h_w|^2 > I,$$
$$|h_v|^2 < |h_w|^2). \quad (27)$$

Note that in this formula, $|h_v|^2$ has an integral range that $\frac{I-\rho|h_w|^2}{(1+\rho|h_w|^2)\rho} < |h_v|^2 < |h_w|^2$. To guarantee the upper bound is higher than the lower bound for $|h_v|^2$, an additional constraint is imposed on the $|h_w|^2$, i.e., $|h_w|^2 > \frac{\sqrt{1+I}-1}{\rho}$. Therefore, the outage probability in (27) can be rewritten as

$$P_{SR}^{\text{out}} = 1 - P\left(\frac{R_1}{\rho} < |h_v|^2 < |h_w|^2, \frac{R_2}{\rho} < |h_w|^2\right), (28)$$

where $R_1 = \frac{I - \rho |h_w|^2}{1 + \rho |h_w|^2}$, $R_2 = \sqrt{1 + I} - 1$. Again applying the joint PDF in (9), the outage probability is further expanded as

$$P_{SR}^{\text{out}} = 1 - \int_{\frac{R_2}{\rho}}^{\infty} \int_{\frac{R_1}{\rho}}^{y} \varpi_2 f_1(x) f_2(y) (F_1(x))^{v-1} \times (1 - F_2(y))^{M-w} (F_2(y) - F_1(x))^{w-v-1} dx dy.$$
 (29)

Now applying binomial expansion, the outage probability is shown in (38) at the bottom of the next page, where

$$Q_1 = \sum_{i=0}^{M-w} \binom{M-w}{i} (-1)^i \sum_{j=0}^{w-v-1} \binom{w-v-1}{j} (-1)^j \frac{\varpi_2}{v+j}.$$

Substituting (11) and (12) into P_1 , and deploying binomial expansion, P_1 can be converted as

$$P_{1} = \left(\frac{m_{1}}{\Omega_{1}}\right)^{v+j} \left(\frac{m_{2}}{\Omega_{2}}\right)^{a+m_{2}-1} \frac{T_{m_{2}}^{a-1} T_{m_{1}}^{v+j}}{\Gamma(m_{2})b} \left(1 - \left(\frac{R_{2}}{\rho}\right)^{b}\right), \quad (30)$$

where a = w - v + i - j, $b = m_2 a + m_1 (v + j)$. The diversity order of P_1 with high SNR approximation is

$$\lim_{\rho \to \infty} -\frac{\log P_1}{\log \rho} = m_2 w + (m_1 - m_2)v. \tag{31}$$

Similar to P_1 , P_2 can be rewritten as

$$P_2 = Q_2 \int_{\frac{R_2}{\rho}}^{\infty} y^{m_2 a - 1} \left(\frac{R_1}{\rho}\right)^{m_1(\nu + j)} dy, \tag{32}$$

where $Q_2 = (\frac{m_1}{\Omega_1})^{v+j} (\frac{m_2}{\Omega_2})^{a+m_2-1} \frac{1}{\Gamma(m_2)} T_{m_2}^{a-1} T_{m_1}^{v+j}$. In the high SNR situation, R_1 can be considered as a constant. Therefore, the diversity order of P_2 is shown as

$$\lim_{\rho \to \infty} -\frac{\log P_2}{\log \rho} = m_2 w + (m_1 - m_2)v. \tag{33}$$

From formula (31) and (33), we can know that the diversity order is $m_2w + (m_1 - m_2)v$.

C. The Outage Probability for the System

Finally, the outage probability for the system is given by

$$P_{system}^{\text{out}} = 1 - P(R_v^N > R_v^O, R_w^N > R_w^O, R_v^N + R_w^N > R_v^O + R_w^O). \quad (34)$$

By some algebraic handling, it is transformed into

$$P_{system}^{\text{out}} = 1 - P\left(\frac{R_1}{\rho} < |h_v|^2 < \frac{I}{\rho}, \max\left\{\frac{R_1}{\rho}, \frac{I}{\rho}, \frac{R_2}{\rho}\right\} < |h_w|^2\right). \tag{35}$$

In addition, $I > R_1$ and $I > R_2$ are always satisfied because of I > 0 and $|h_v|^2 > 0$. Therefore, the outage probability for the system can be rewritten as

$$P_{system}^{\text{out}} = 1 - P\left(\frac{R_1}{\rho} < |h_v|^2 < \frac{I}{\rho}, \frac{I}{\rho} < |h_w|^2\right).$$
 (36)

Similar to the argument from (29) to (33), the outage probability for the system is obtained as

$$P_{system}^{out} = 1 - Q_1 Q_2 \left(\left(\frac{I}{\rho} \right)^{m_1(v+j)} \frac{1}{m_2 a} \left(1 - \left(\frac{I}{\rho} \right)^{m_2 a} \right) - \int_{\frac{I}{\rho}}^{\infty} y^{m_2 a - 1} \left(\frac{R_1}{\rho} \right)^{m_1(v+j)} dy \right), \quad (37)$$

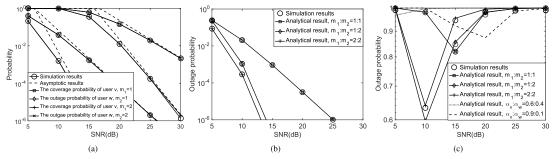


Fig. 1. (a) The probability of individual users. (b) The outage probability of sum rate. (c) The outage probability for the system.

where the diversity order is $m_2w + (m_1 - m_2)v$. Unlike [3], where the diversity order is only dependent on the user with better channel gain, (31), (33) and (38) show that the diversity order depends on both users under Nakagami-m fading channels. One can also observe that m_2 and w are the dominant part of the diversity order, and v can be omitted in the case of $m_1 = m_2$.

IV. NUMERICAL STUDIES

In this section, we demonstrate the outage performance of F-NOMA over Nakagami-m fading channels. In the simulations, the power allocation factors are set as $\alpha_v = 0.8$ and $\alpha_w = 0.2$. The distances from the BS to user v and user w are fixed to 1 and 0.5, respectively. Then, we can obtain $\Omega_1 = 1$ and $\Omega_2 = \left(\frac{1}{2}\right)^{-\gamma}$, where γ is the path loss exponent. Here, we set $\gamma = 3$. Without loss of generality, we set the number of users M = 5, w = 3, and v = 2.

Fig. 1(a) demonstrates two different but related probabilities for individual users versus transmit SNR in different fading parameters. One is the outage probability of the w-th user, i.e., NOMA user w performs worse than OMA. The other is the probability that the NOMA user with poorer channel gain achieves higher data rate than OMA. In Fig. 1(a), it can be seen that the probabilities decrease with the increase of transmit SNR. In addition, the increase of fading parameters m_1 and m_2 decreases the above probabilities dramatically. The asymptotic simulations are also provided to confirm the close agreement between analytical results and the simulations. Thus, the correctness of analytical results is verified.

In Fig. 1(b), the probability that F-NOMA achieves lower sum rate than OMA is shown, which decreases monotonically. It is demonstrated that when m_2 varies from 1 to 2, the decreasing rate of outage probability is much faster than the case when m_1 increases from 1 to 2, which shows that the existence of line of sight (LoS) for the user with higher channel gain is able to dramatically decrease the outage probability. Comparatively, the existence of LoS for the user with poorer channel gain only decreases the outage probability slightly.

The probability that the data rate of individual user and the sum rate in F-NOMA are both lower than OMA is illustrated in Fig. 1(c). As can be seen in this figure, the outage probability is not a monotonically decreasing function, and it is always larger than 0.6 no matter what the fading parameters are. In other words, F-NOMA cannot perform better than OMA on both individual user's data rate and sum rate simultaneously, which indicates that F-NOMA improves the performance of the user with higher channel gain while sacrificing the user with poorer channel gain. In addition, the dot curve and dash curve are the outage probabilities of $\alpha_v : \alpha_w = 0.6 : 0.4$ and $\alpha_v : \alpha_w = 0.9 : 0.1$, respectively, when the fading parameters $m_1 : m_2 = 2 : 2$. From this result, it is found that the power allocation factors α_v and α_w affect the system outage probability slightly.

V. CONCLUSIONS

In this letter, both analytical and numerical results have been provided to demonstrate that F-NOMA can offer larger individual rates for the users with higher channel gain, while the data rate of the user with poorer channel gain in F-NOMA is even lower than OMA in high transmit SNR situation. It is also shown that the outage probabilities of the sum rate and the system with F-NOMA over Nakagami-*m* fading channels depend on both users from the analytical results.

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$$P_{SR}^{\text{out}} = 1 - Q_1 \left(\underbrace{\int_{\frac{R_2}{\rho}}^{\infty} f_2(y) F_2(y)^{w-v-1-j+i} F_1(y)^{v+j} dy}_{P_1} - \underbrace{\int_{\frac{R_2}{\rho}}^{\infty} f_2(y) F_2(y)^{w-v-1-j+i} F_1\left(\frac{R_1}{\rho}\right)^{v+j} dy}_{P_2} \right). \tag{38}$$