Department of Electrical Engineering
IIT Hyderabad

EE 6340/3801
Wireless Communications

Channel

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Lecture 4 Outline

- Announcements
  - TA: **ANNU SURAJPU,** ee21resch01010@iith.ac.in
- Review of Last Lecture
  - Path Loss Models
    - Free-space and 2-Ray Models
    - Simplified Path Loss Model
    - General Ray Tracing
    - Empirical Models
    - mmWave Models
  - Narrowband Channel Model
- Doppler

Narrowband Model

$$r(t) = Re\{v(t)e^{j(2fct+\emptyset 0)}\}$$
$$= Re\left\{\left[\frac{\lambda\sqrt{G}e^{-j2\pi d/\lambda}}{4\pi d}u(t) + n(t)\right]e^{j(2fct \quad \emptyset 0)}\right\}$$

- Assume delay spread $\max_{m,n}|\tau_n(t)-\tau_m(t)| << 1/B$
- Then $u(t-\tau) \approx u(t)$.
- Received signal given by

$$r(t) = \Re\left\{u(t)e^{j(2fct \quad \emptyset 0)}\boxed{\left[\sum_{n=0}^{N(t)} \alpha_n(t)e^{-j\varphi_n(t)}\right]} + n(t)e^{j(2fct \quad \emptyset 0)}\right\}$$

- No signal distortion (spreading in time)
- Multipath affects complex scale factor in brackets.
- Characterize scale factor by setting $u(t)=e^{j\phi_0}$: *or 1*

$$s(t) = \Re\{e^{j2\pi f_c t}\} = \cos 2\pi f_c t,$$

NarrowBand Channel Model

- The received signal

$$r(t) = \Re\left\{\left[\sum_{n=0}^{N(t)} \alpha_n(t)e^{-j\phi_n(t)}\right]e^{j2\pi f_c t}\right\}$$

$$= r_I(t)\cos 2\pi f_c t + r_Q(t)\sin 2\pi f_c t,$$

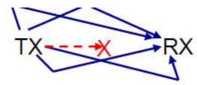
$$r_I(t) = \sum_{n=1}^{N(t)} \alpha_n(t)\cos\phi_n(t),$$

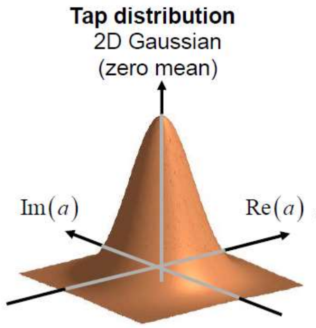
$$r_Q(t) = \sum_{n=1}^{N(t)} \alpha_n(t)\sin\phi_n(t).$$

For large $N(t)$, $r_I(t)$ and $r_Q(t)$ jointly Gaussian by CLT

Rayleigh Fading Channel

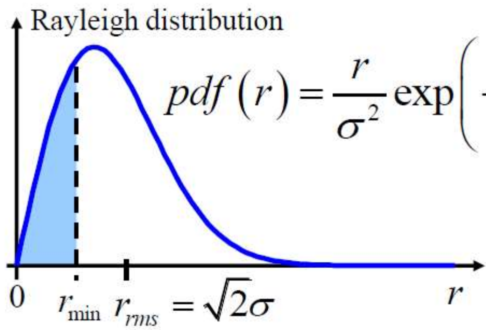
No dominant component
(no line-of-sight)





No line-of-sight component

Rayleigh Fading



$$pdf(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

Fading Margin

$$M = \frac{{r_{rms}}^2}{{r_{\min}}^2}$$

$$M_{|dB} = 10\log_{10}\left(\frac{{r_{rms}}^2}{{r_{\min}}^2}\right)$$

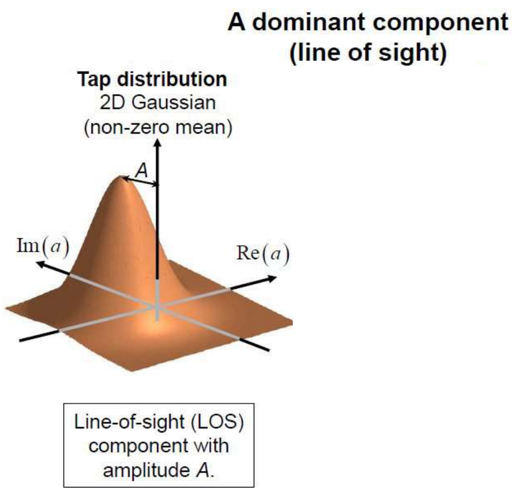
$$\Pr(r < r_{\min}) = \int_0^{r_{\min}} pdf(r)\,dr = 1 - \exp\left(-\frac{{r_{\min}}^2}{{r_{rms}}^2}\right)$$

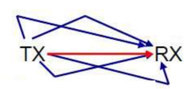
Ricean Fading: One Dominant Factor

- In case of Line-of-Sight (LOS) one component dominates.
- Assume it is $\mathrm{Re}(r) \in N(A, \sigma^2) \quad \mathrm{Im}(r) \in N(0, \sigma^2)$
- The received amplitude has now a Ricean distribution instead of a Rayleigh
- The ratio between the power of the LOS component and the diffuse components is called Ricean K-factor

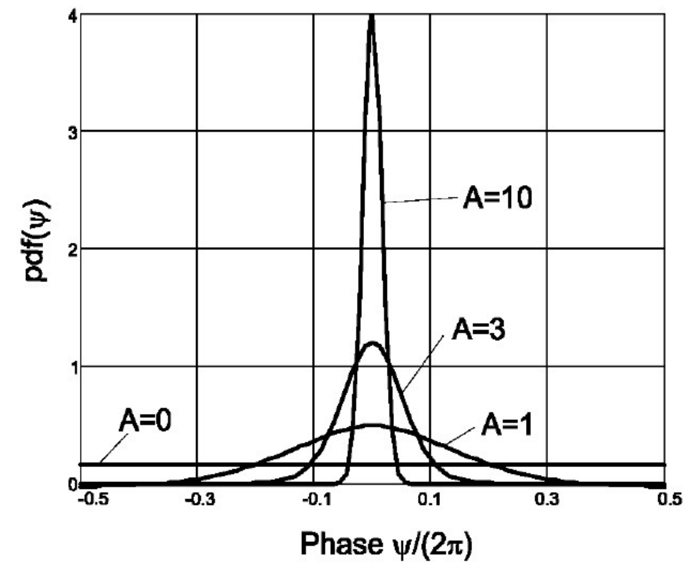
$$k = \frac{\text{Power in LOS component}}{\text{Power in random components}} = \frac{A^2}{2\sigma^2}$$

Ricean fading



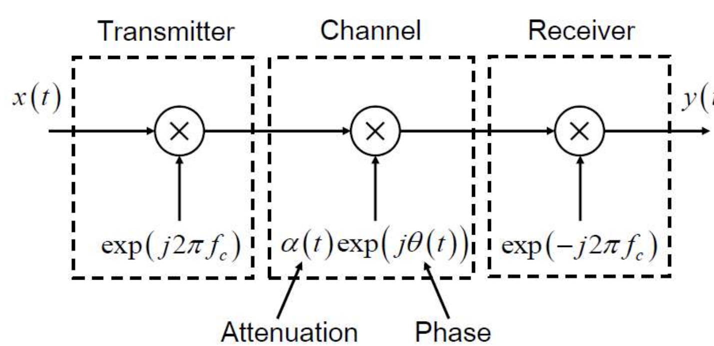


Phase Distribution



The Narowband Multipath Channel without Noise

$$\text{In}: \quad x(t) = \boxed{A(t)}\exp(j\boxed{\phi(t)})$$

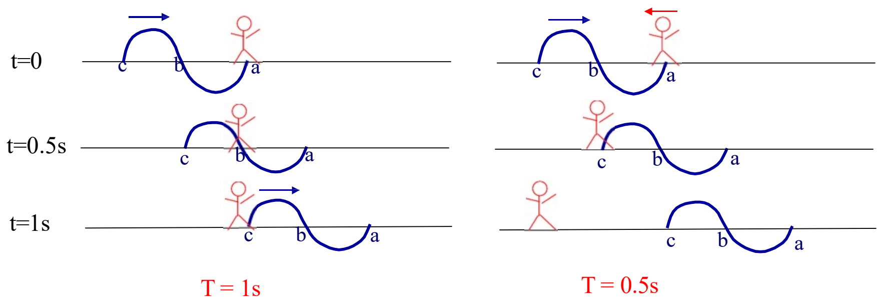


$$\text{Out}: \; y(t) = A(t)\exp(j\phi(t))\exp\cancel{(j2\pi f_c t)}\,\alpha(t)\exp(j\theta(t))\exp\cancel{(-j2\pi f_c t)}$$
$$= \boxed{A(t)\alpha(t)}\exp(j(\boxed{\phi(t)+\theta(t)}))$$

6

FADING: DOPPLER

- **What is Doppler?**
  - ignal frequency change due to the relative movement between Tx and Rx is called Doppler effect



7

FADING: DOPPLER

- **Consider Tx sends out a sinusoid with frequency 1Hz**
  - If Rx moves toward Tx, the signal observed by Rx will have a shorter period ➔ frequency increased
  - If Rx moves away from Tx, the signal observed by Rx will have a longer period ➔ frequency decreased
- **The amount of frequency change is called Doppler shift**
  - Doppler shift depends on
    - Relative speed between Tx and Rx
    - The frequency of the original signal

Doppler Shift

- How large is the maximum Doppler frequency at
  - pedestrian speeds for 5.2 GHz WLAN with v= 5Km/Hr and at
  - highway speeds using GSM 900 with v=110 Km/Hr?

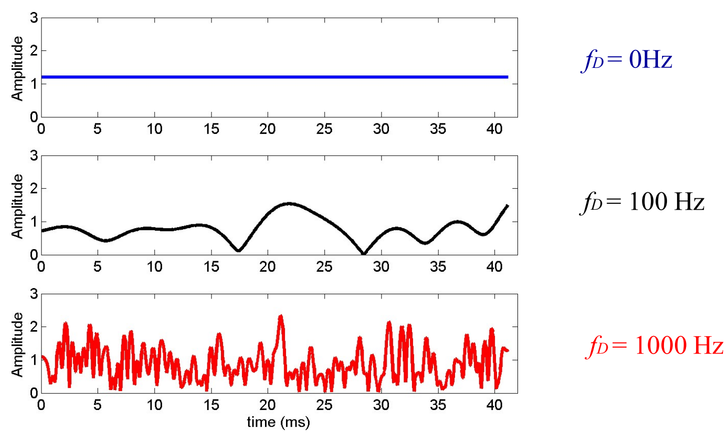
5.2 10^9 Hz, v=5 km/h, (1.4 m/s) ➡ 24 Hz

900 10^6 Hz, v=110 km/h, (30.6 m/s) ➡ 92 Hz

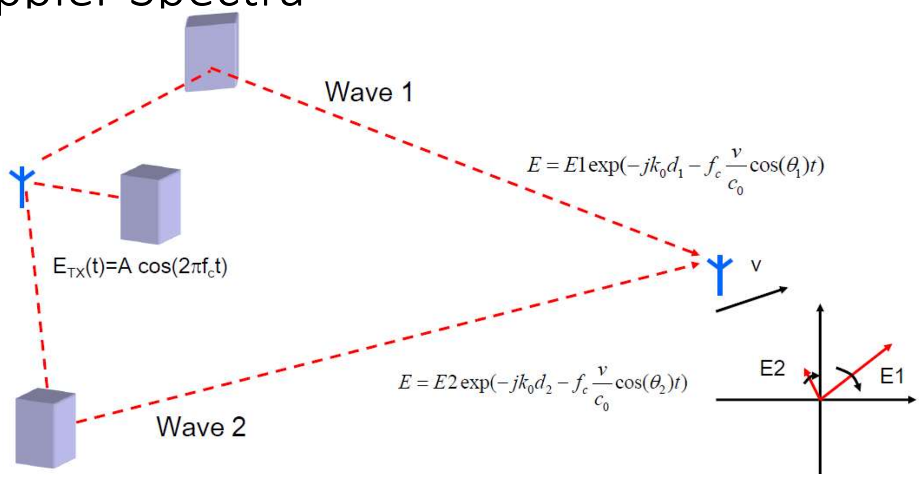
9

FADING: DOPPLER

- **At given frequency**
  – $v \uparrow \rightarrow f_D \uparrow \rightarrow$ channel changes more rapidly



Doppler Spectra

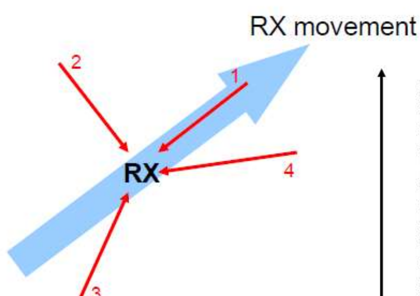


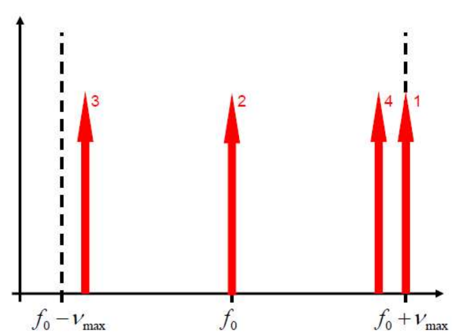
The two components have different Doppler shifts!
The Doppler shifts will cause a random frequency modulation

Doppler Spectrum

Incoming waves from several directions
(relative to movement or RX)

Spectrum of received signal
when a f_0 Hz signal is transmitted.

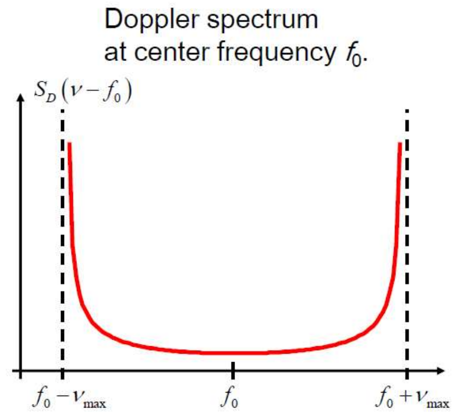




All waves of equal strength in
this example, for simplicity.

Doppler Spectrum

Classical or *Jakes* spectrum

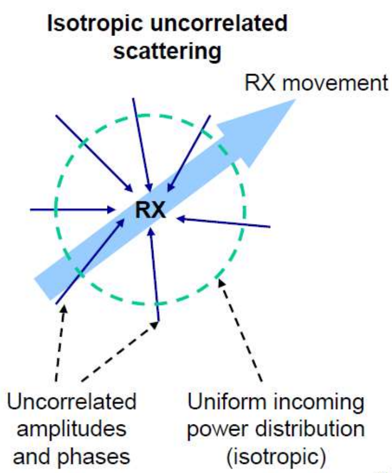


$$S_D(\nu) = \int \rho(\Delta\tau) e^{-j2\pi\nu\Delta\tau} d\Delta\tau$$
$$\propto \frac{1}{\pi\sqrt{\nu_{max}^{2} - \nu^2}}$$

for $-\nu_{max} < \nu < \nu_{max}$

It describes *frequency dispersion*

Doppler Spectrum

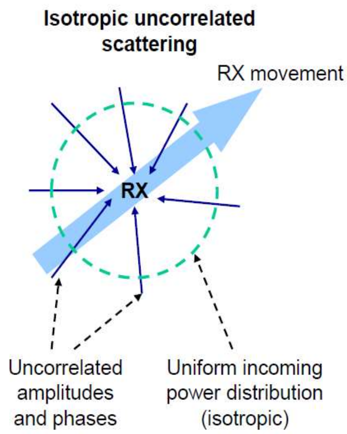


Time correlation - how static is the channel?

$$\rho(\Delta t) = E\left\{a(t)a^*(t+\Delta t)\right\} \propto J_0(2\pi\nu_{max}\Delta t)$$

Doppler Spectrum

Jakes Model



$$v = -f_c \frac{v_r}{c} \cos(\theta)$$

A MPC arriving in the direction θ has to be multiplied by the pattern $G(\theta)$

$$S(\bar{\theta}) = \bar{\Omega}\left[pdf_{\theta}(\theta)G(\theta) + pdf_{\theta}(-\theta)G(-\theta)\right]$$

Change of variable from θ *to* v

$$\left|\frac{d\theta}{dv}\right| = \left|\frac{1}{\frac{dv}{d\theta}}\right| = \frac{1}{|v_{max}\sin\theta|} = \frac{1}{\sqrt{v_{max}^2 - v^2}}$$

Using $pdf(\theta) = 1/2\pi$ *and* $G(\theta)$=1.5

$$S_D(\nu) \propto \frac{1}{\pi\sqrt{\nu_{\max}{}^2 - \nu^2}}$$

Coherence Time

- Coherence time is the time domain dual of Doppler spread
- Used to characterize the time varying nature of the frequency dispersiveness of the channel in the time domain.
- The Maximum Doppler spread and coherence time are inversely proportional to one another

$$T_c \approx \frac{0.4}{v_{max}}$$

- **Slow fading** arises when the T_c>> the delay requirement of the application. The channel can be considered roughly constant over the period of use.
- **Fast fading** occurs when the T_c<< relative to the delay requirement of the application.

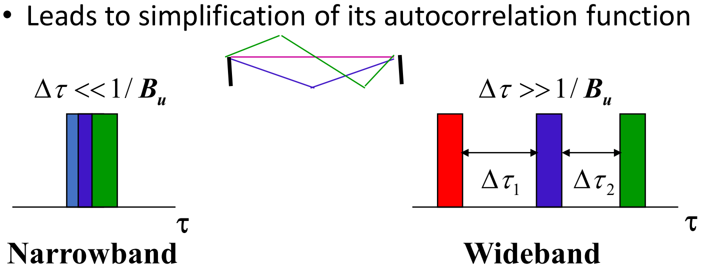
Received Signal Characteristics

- Received signal consists of many multipath components
- Amplitudes change slowly
- Phases change rapidly
  - Constructive and destructive addition of signal components
  - Amplitude fading of **received signal** (both wideband and narrowband signals)

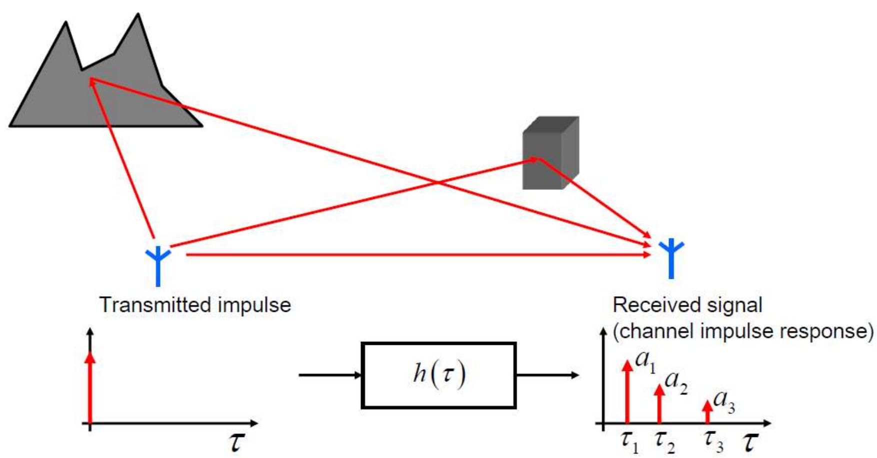
Wideband Channels

Wideband Channels

- Individual multipath components resolvable
- True when time difference between components exceeds signal bandwidth
- Requires statistical characterization of h(τ,t)
  - Assume CLT, stationarity and uncorrelated scattering
  - Leads to simplification of its autocorrelation function



Delay dispersion: A simple case



$$h(\tau) = a_1\delta(\tau-\tau_1) + a_2\delta(\tau-\tau_2) + a_3\delta(\tau-\tau_3)$$

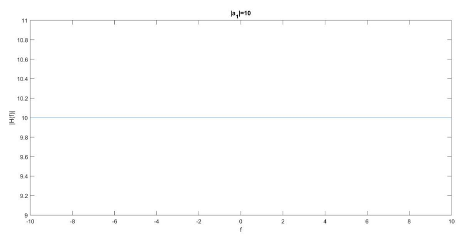
Effect of No Delay Dispersion: Frequency Flat Fading

- Single Path:

$$h(\tau) = a_1\delta(\tau - \tau_1)$$

Taking Fourier Transform, we have

$$H(f) = \int_{-\propto}^{\propto} h(\tau)e^{-j2\pi f} \;\; d\tau$$



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Effect of Delay Dispersion: Frequency Selective Fading

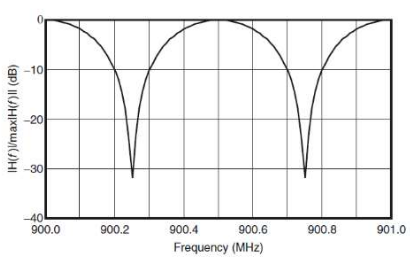
- Two Path:

$$h(\tau) = a_1\delta(\tau - \tau_1) + a_2\delta(\tau - \tau_2)$$

Taking Fourier Transform, we have $H(f) = \int_{-\propto}^{\propto} h(\tau)e^{-j2\pi f\tau}\, d\tau$

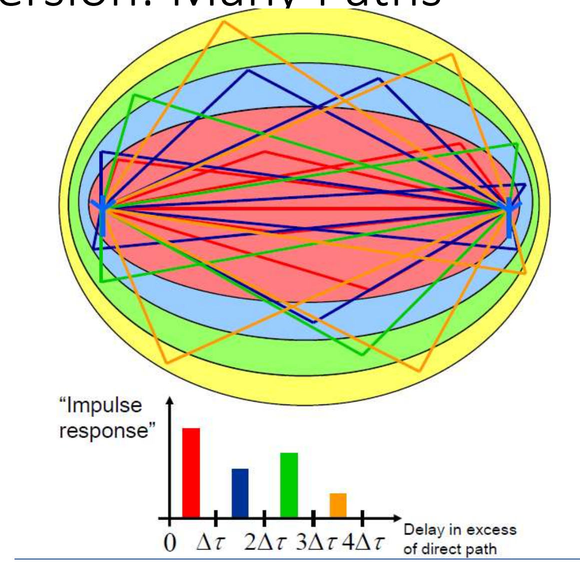
$$= a_1e^{-j2\pi}\;\;{}_1 + a_2e^{-j2\pi f}\;\;{}_2$$

$$|H(f)| = \sqrt{|a_1|^2 + |a_2|^2 + 2|a_1||a_2|\cos\{2\pi f\Delta\tau - \Delta\phi\}}$$

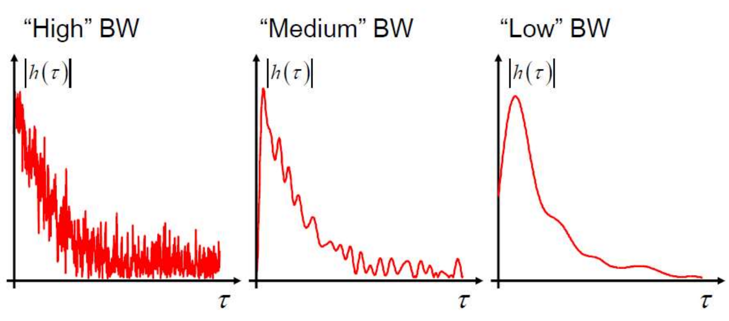


$|a_1| = 1.0$, $|a_2| = 0.95$, $\Delta\phi = 0$, $\tau_1 = 4\mu s$, $\tau_2 = 6\ \mu s$ at the 900-MHz carrier frequency

Delay Dispersion: Many Paths



Narrowband vs Wideband



Narrowband vs Wideband