EE5801: CSP Lab/EE5301: DSP Lab
EE3701: Communication Systems Lab
(Aug – Nov 2022)

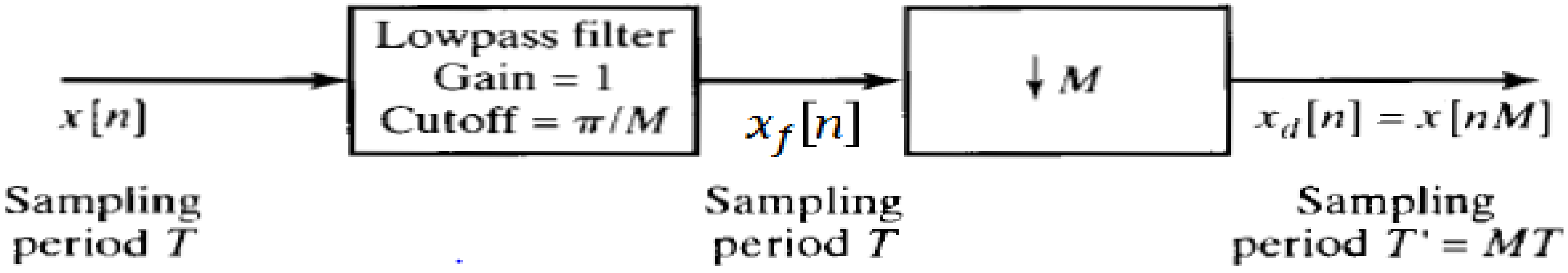
Lecture 4

Today's Topics

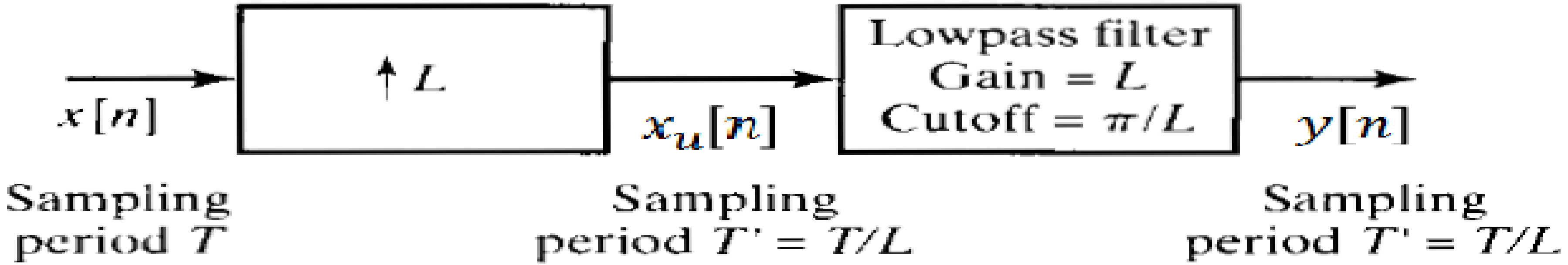
- Review of decimation and interpolation
- Cascading decimator and interpolator
- Time and Frequency domain plots from matlab simulation
- Generalizing decimation and interpolation to any factor M or L

Review of decimation and interpolation

- Decimation



- Interpolation



Review of decimation and interpolation (contd..)

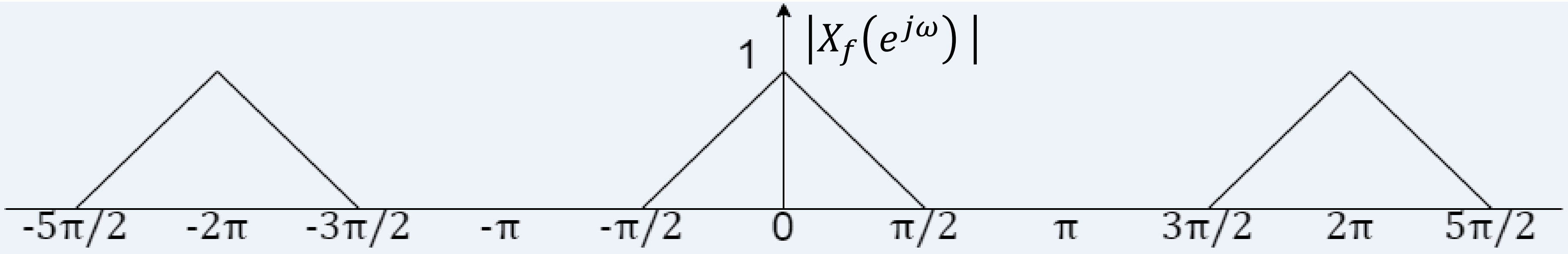


fig 1: Input to downsampler

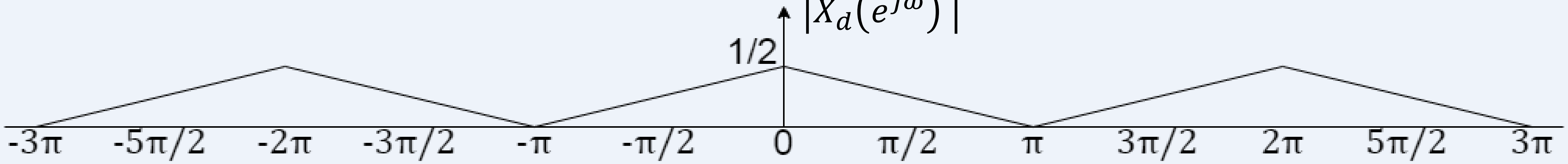
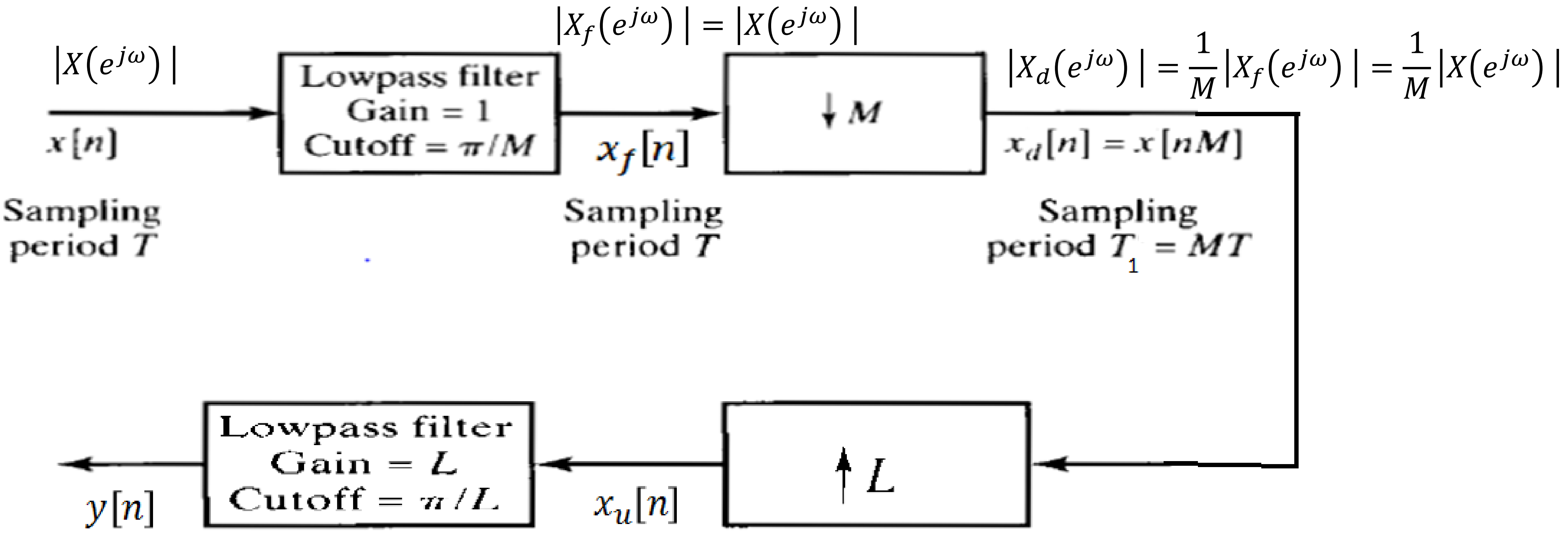


fig 2: downsampling with factor 2

Note : For downsampling with factor 2, $|X_d(e^{j\omega})| = \frac{1}{2}|X_f(e^{j\omega})|$

For downsampling with factor M, $|X_d(e^{j\omega})| = \frac{1}{M}|X_f(e^{j\omega})|$

Cascading of decimator and interpolator



Sampling period

$$T_2 = \frac{T_1}{L} = \frac{MT}{L}$$

$$|Y(e^{j\omega})| = |X_u(e^{j\omega})| = \frac{L}{M}|X(e^{j\omega})|$$

Sampling period

$$T_2 = \frac{T_1}{L} = \frac{MT}{L}$$

$$|X_u(e^{j\omega})| = |X_d(e^{j\omega})| = \frac{1}{M}|X(e^{j\omega})|$$

Time and Frequency domain plots from matlab simulation

Example-1 :

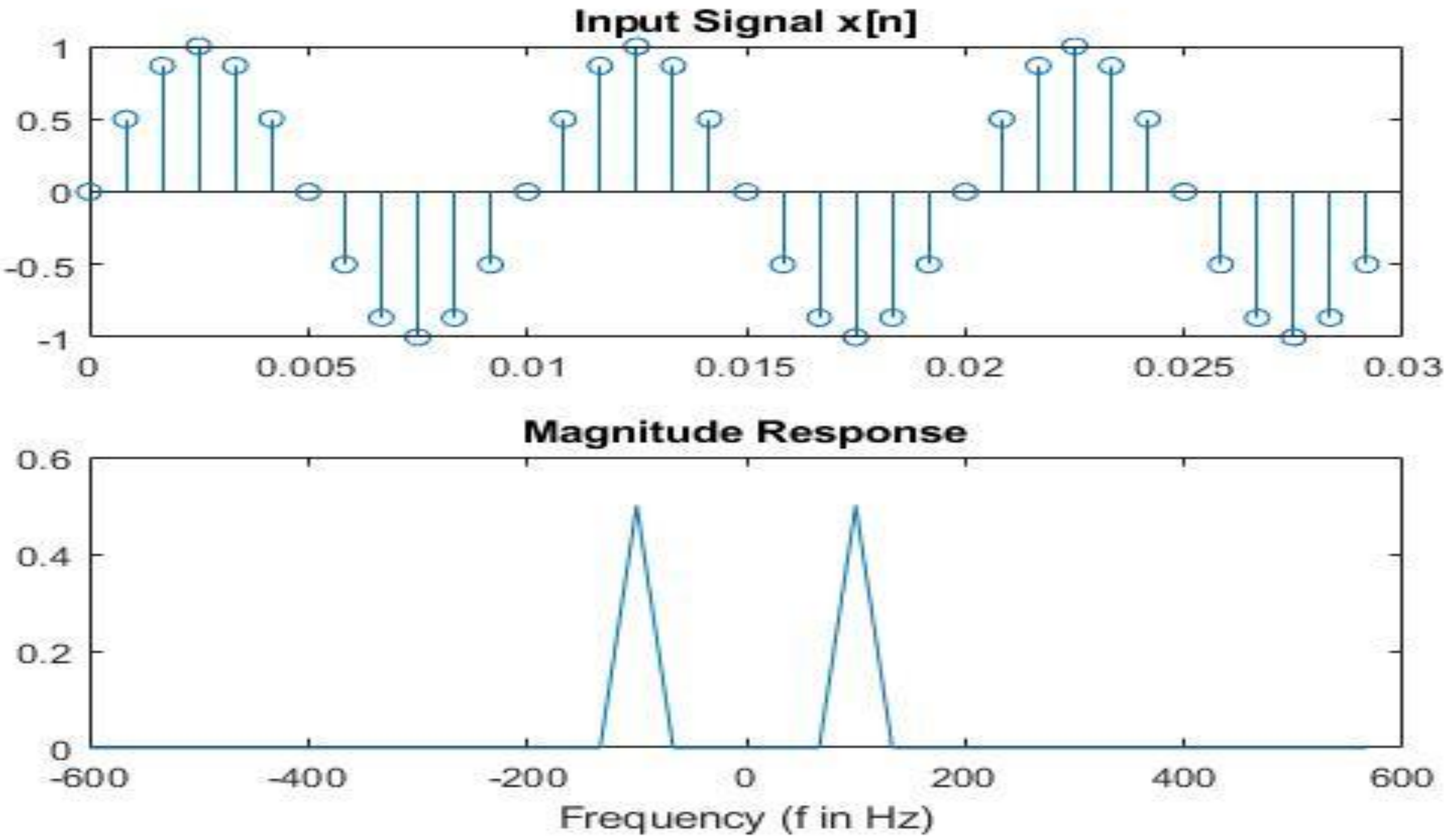
$x[n] = 1 * \sin(2\pi f n / f_s)$

Signal parameters :

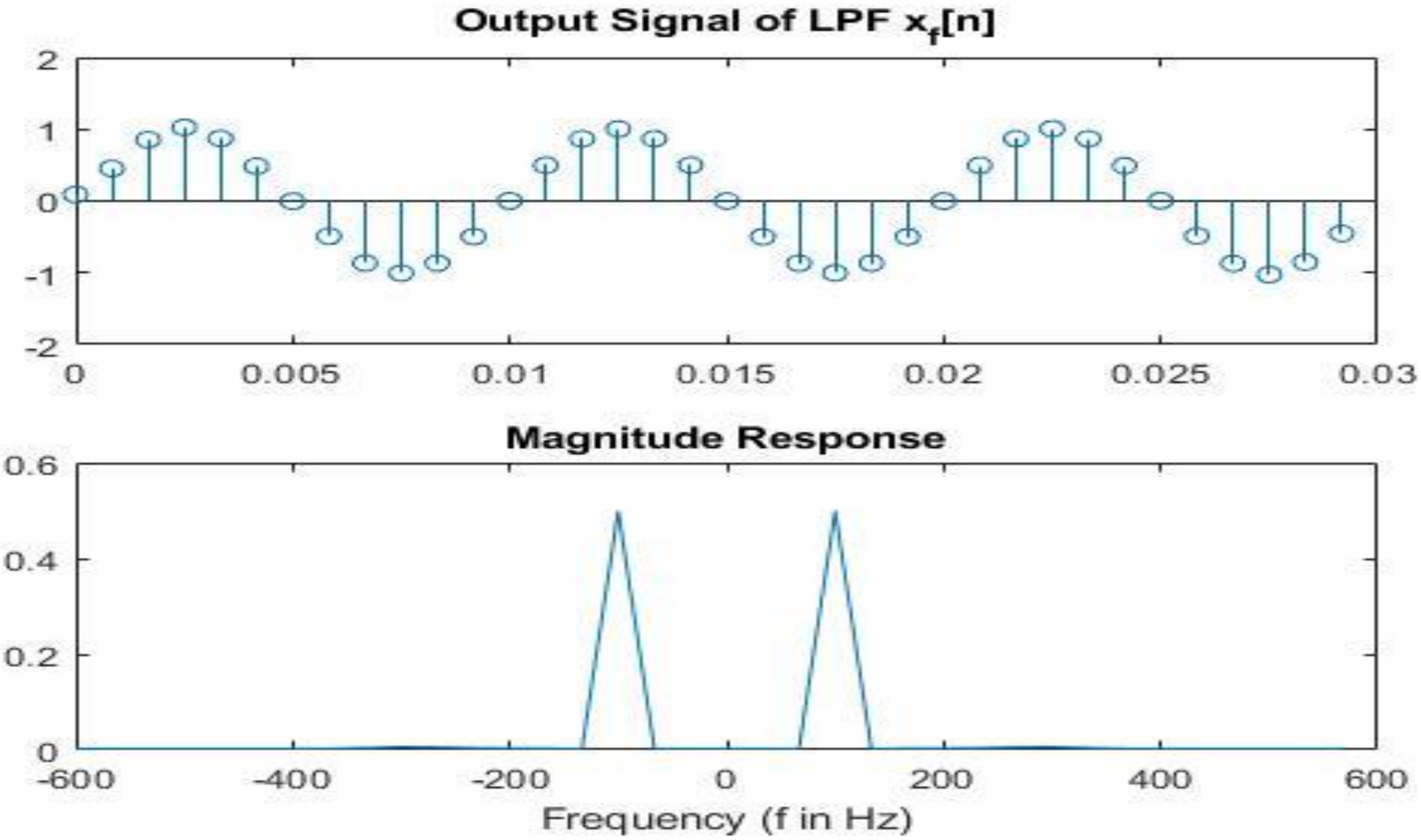
- $f = 100Hz,\ \omega = \frac{2\pi f}{f_s} = \frac{\pi}{6}$
- Sampling frequency $f_s = 1200\ Hz$

LPF has cutoff freq $f_c = 300\ Hz, \omega_{cutoff} = \frac{\pi}{2} = \frac{3\pi}{6}$

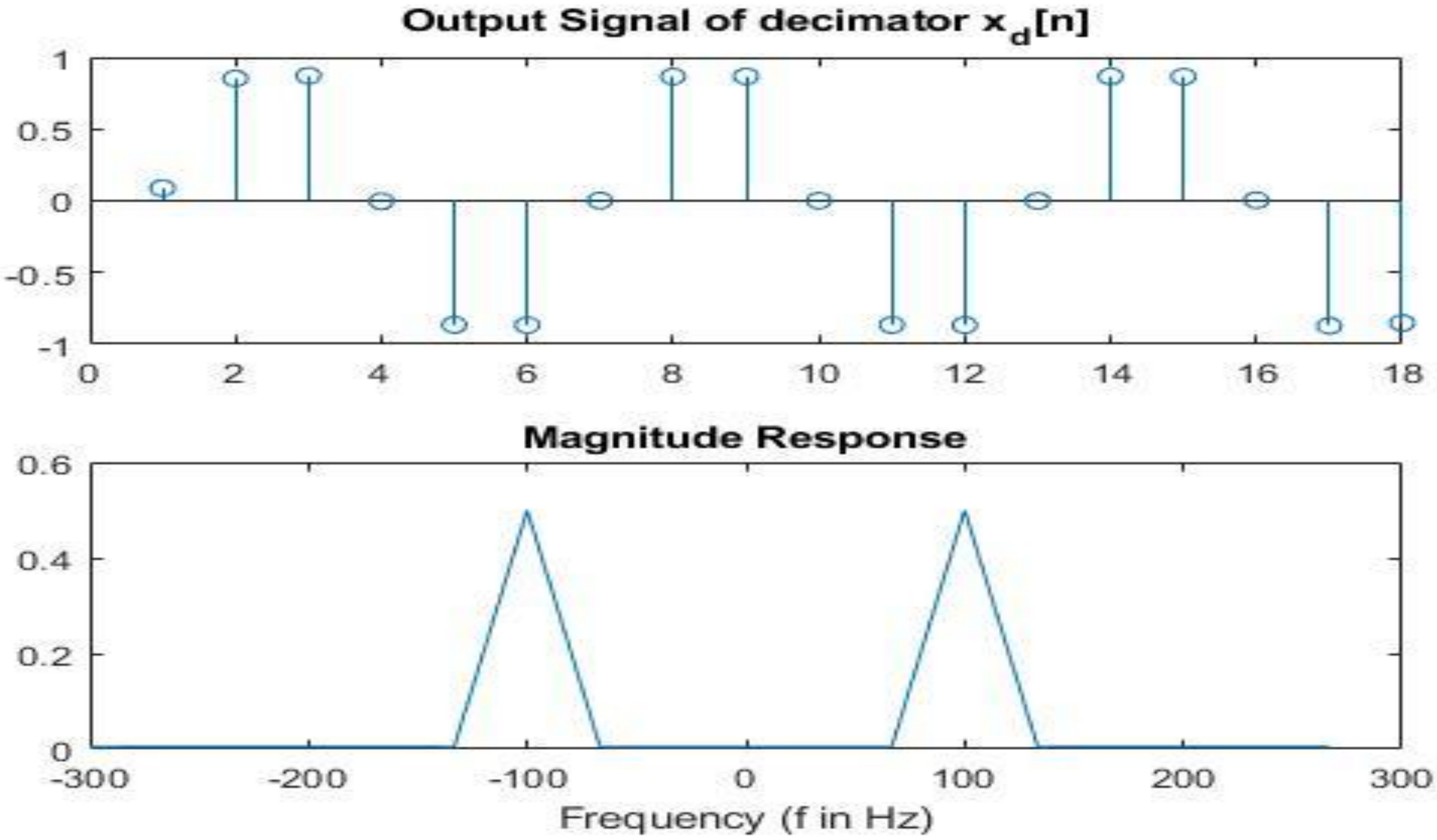
Time and Frequency domain plots from matlab simulation



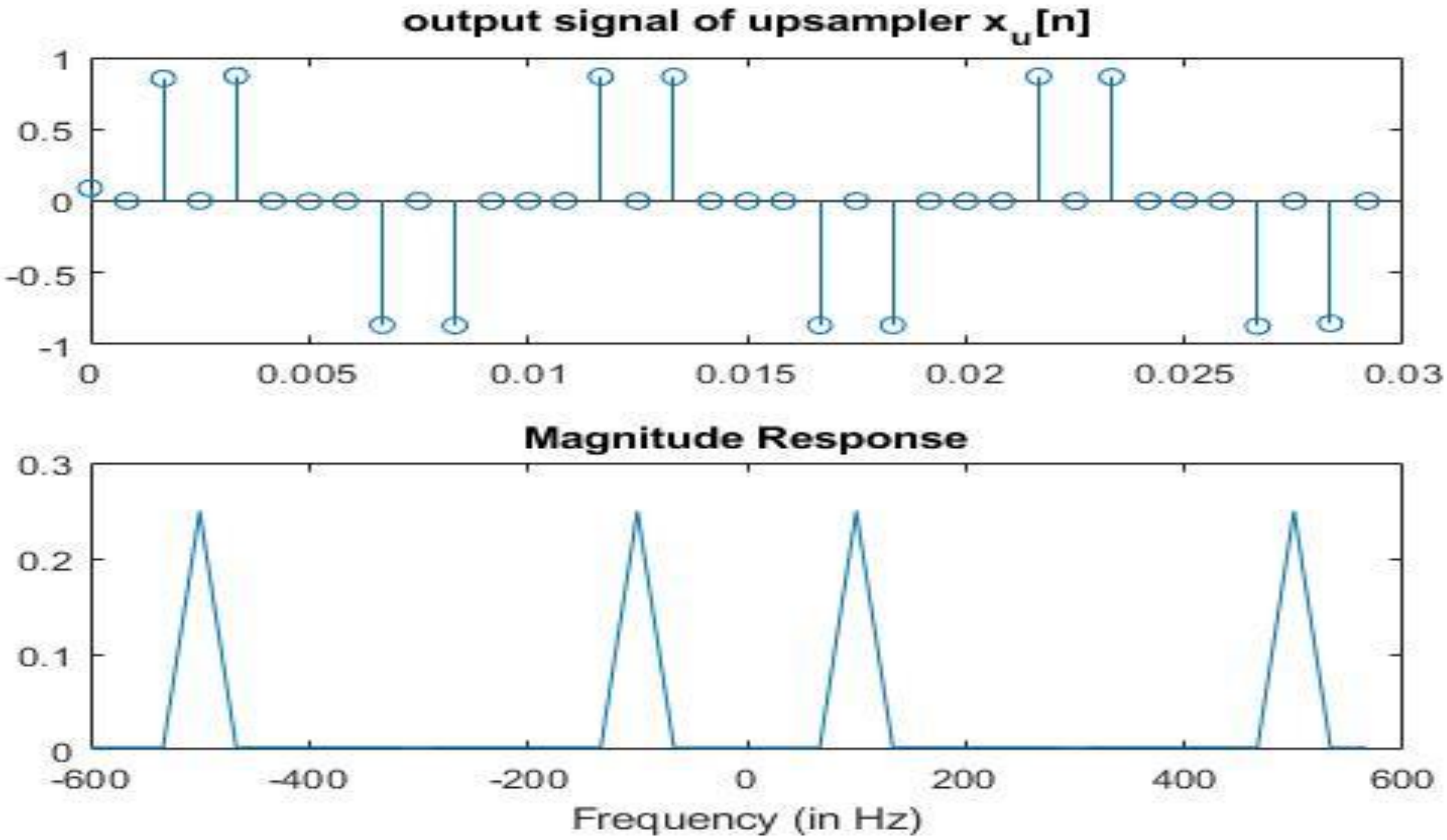
Time and Frequency domain plots from matlab simulation



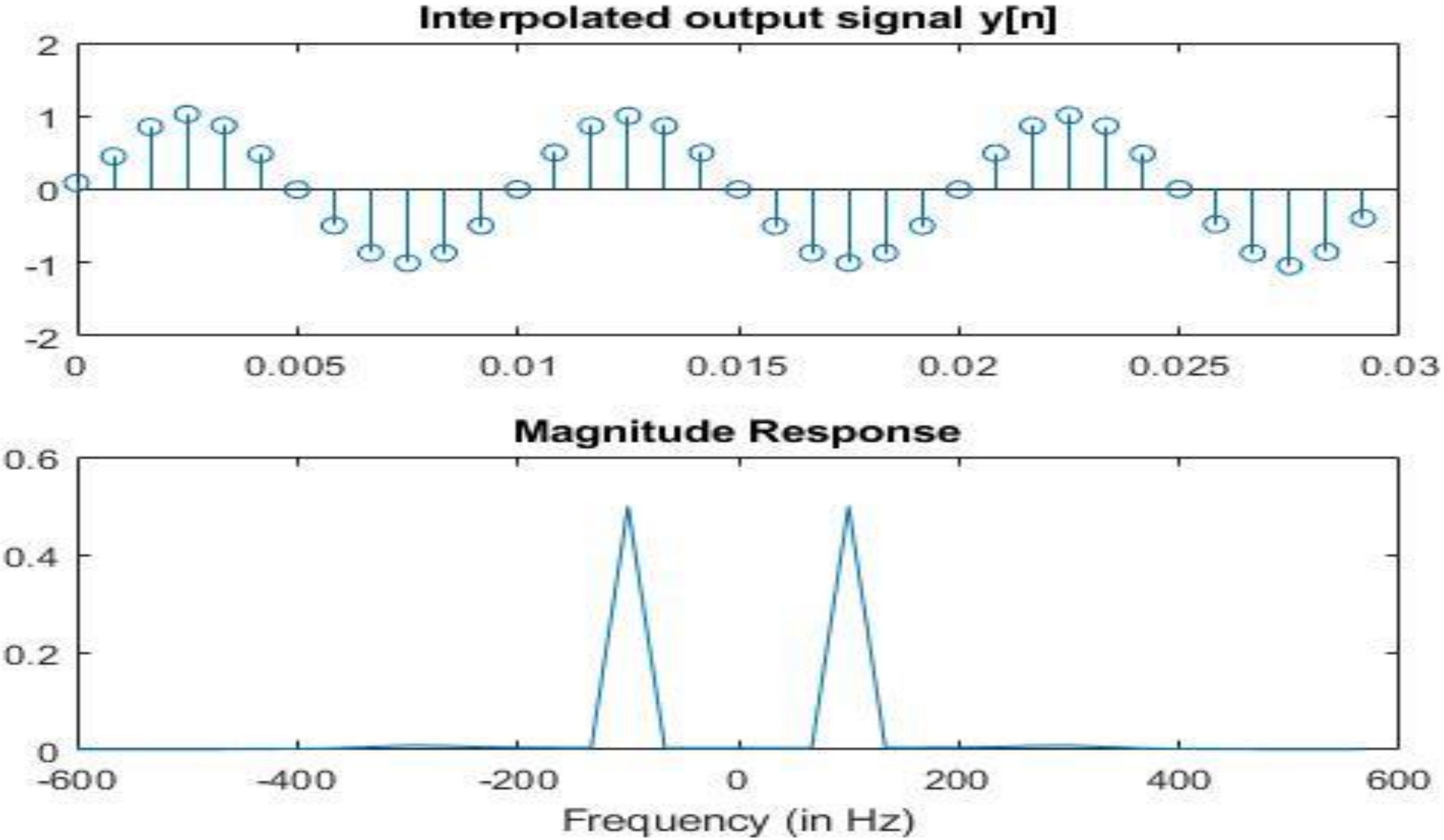
Time and Frequency domain plots from matlab simulation



Time and Frequency domain plots from matlab simulation



Time and Frequency domain plots from matlab simulation



Time and Frequency domain plots from matlab simulation

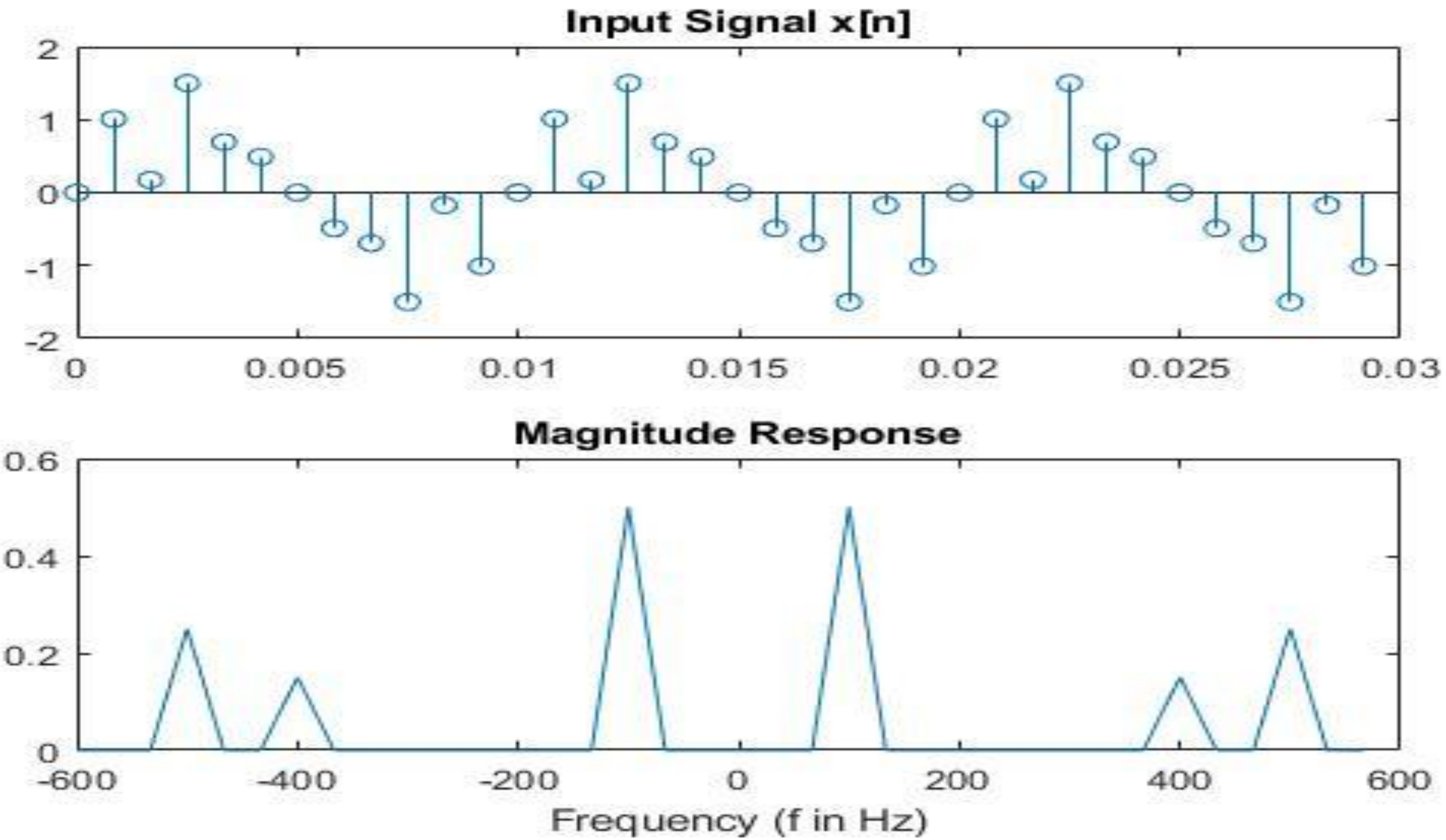
Example-2 :

$$x[n] = 1 * \sin(2\pi f_0 n/f_s) + 0.5 * \sin(2\pi f_1 n/f_s) + 0.3 * \sin(2\pi f_2 n/f_s)$$

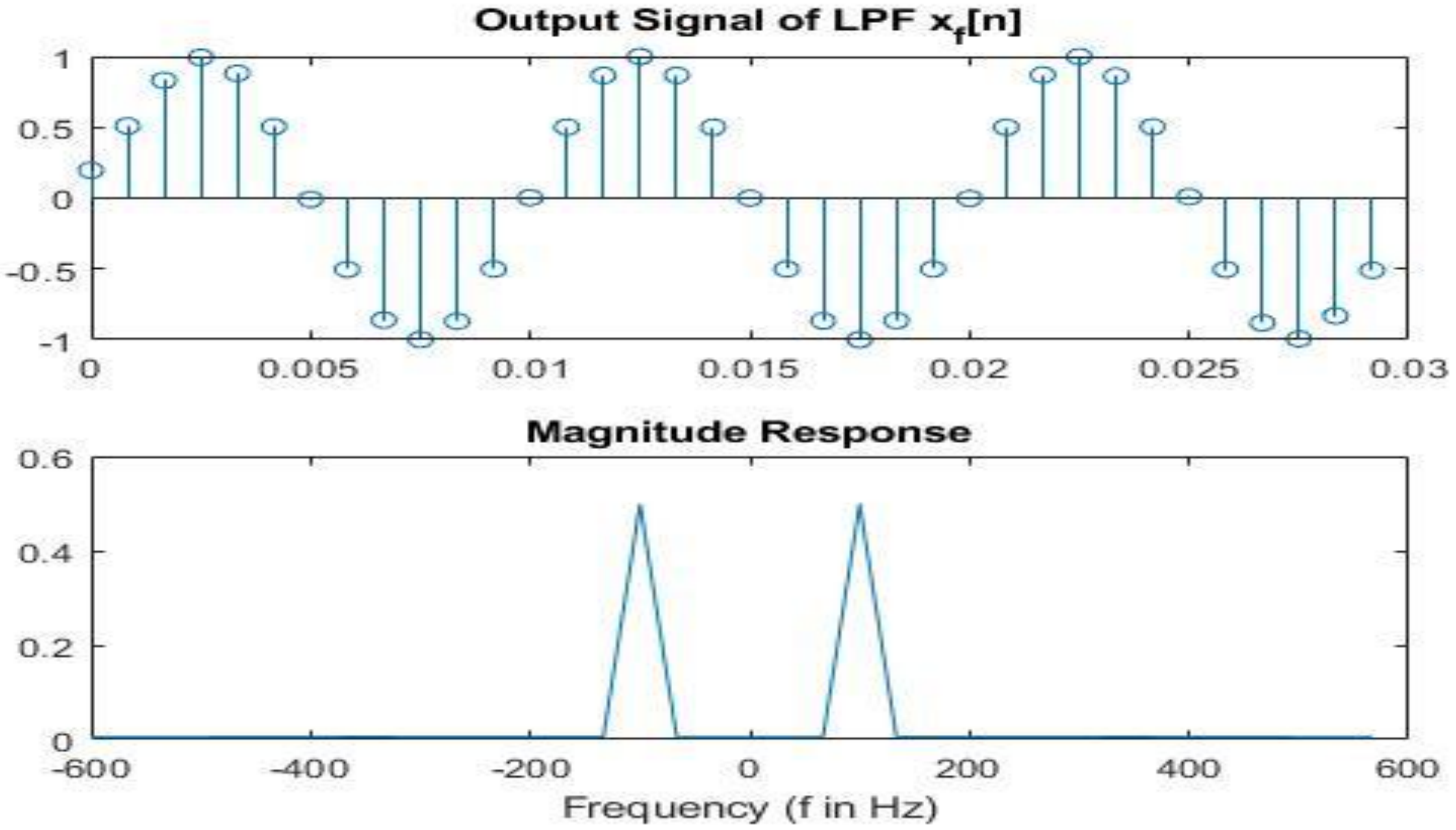
Signal parameters :

- $f_0 = 100Hz$, $f_1 = 500Hz$, $f_2 = 400Hz$
- $\omega_0 = \frac{\pi}{6}, \omega_1 = \frac{5\pi}{6}, \omega_2 = \frac{4\pi}{6}, \omega_{cutoff} = \frac{3\pi}{6}$
- Sampling frequency $f_s = 1200\ Hz$

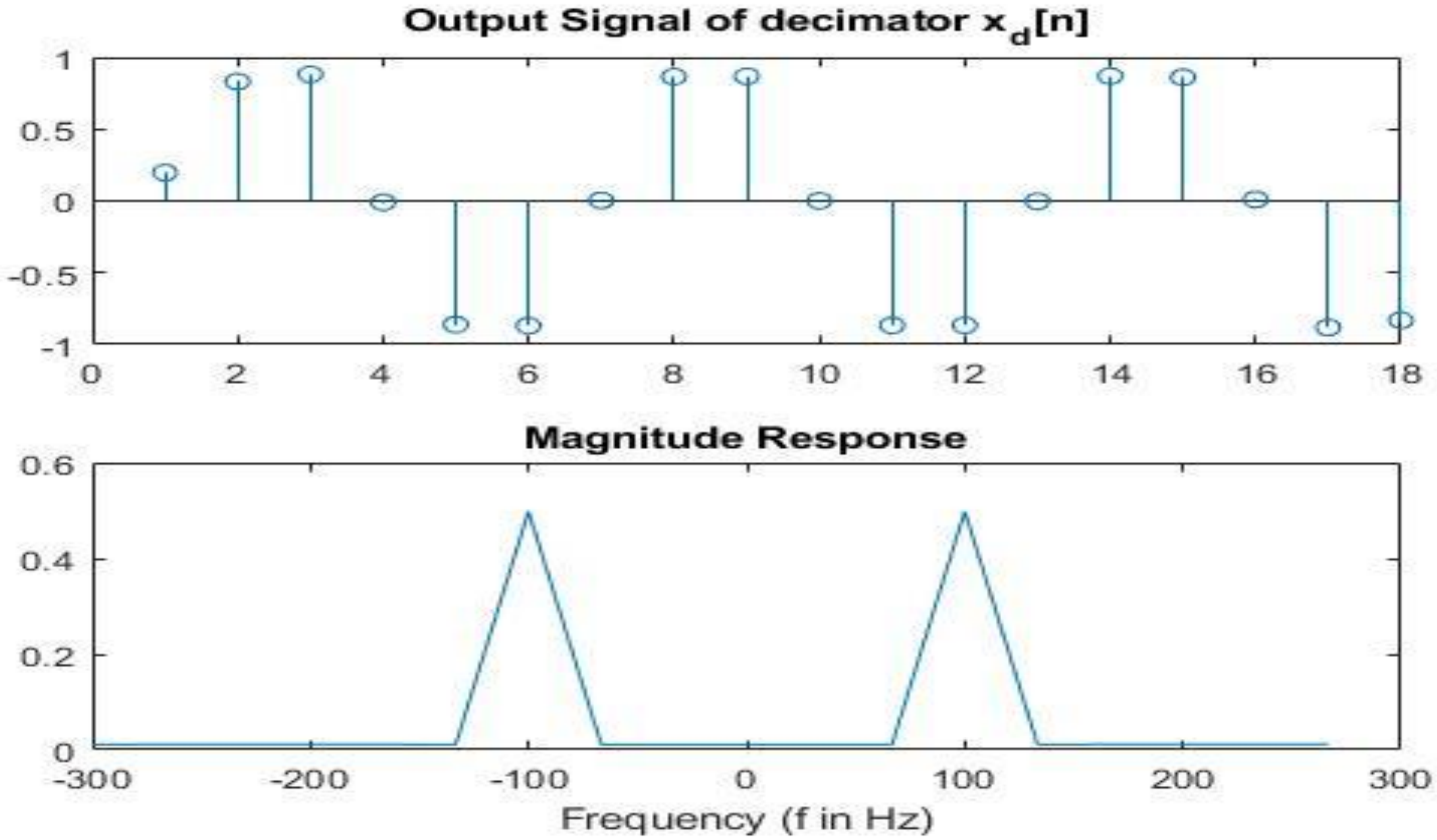
Time and Frequency domain plots from matlab simulation



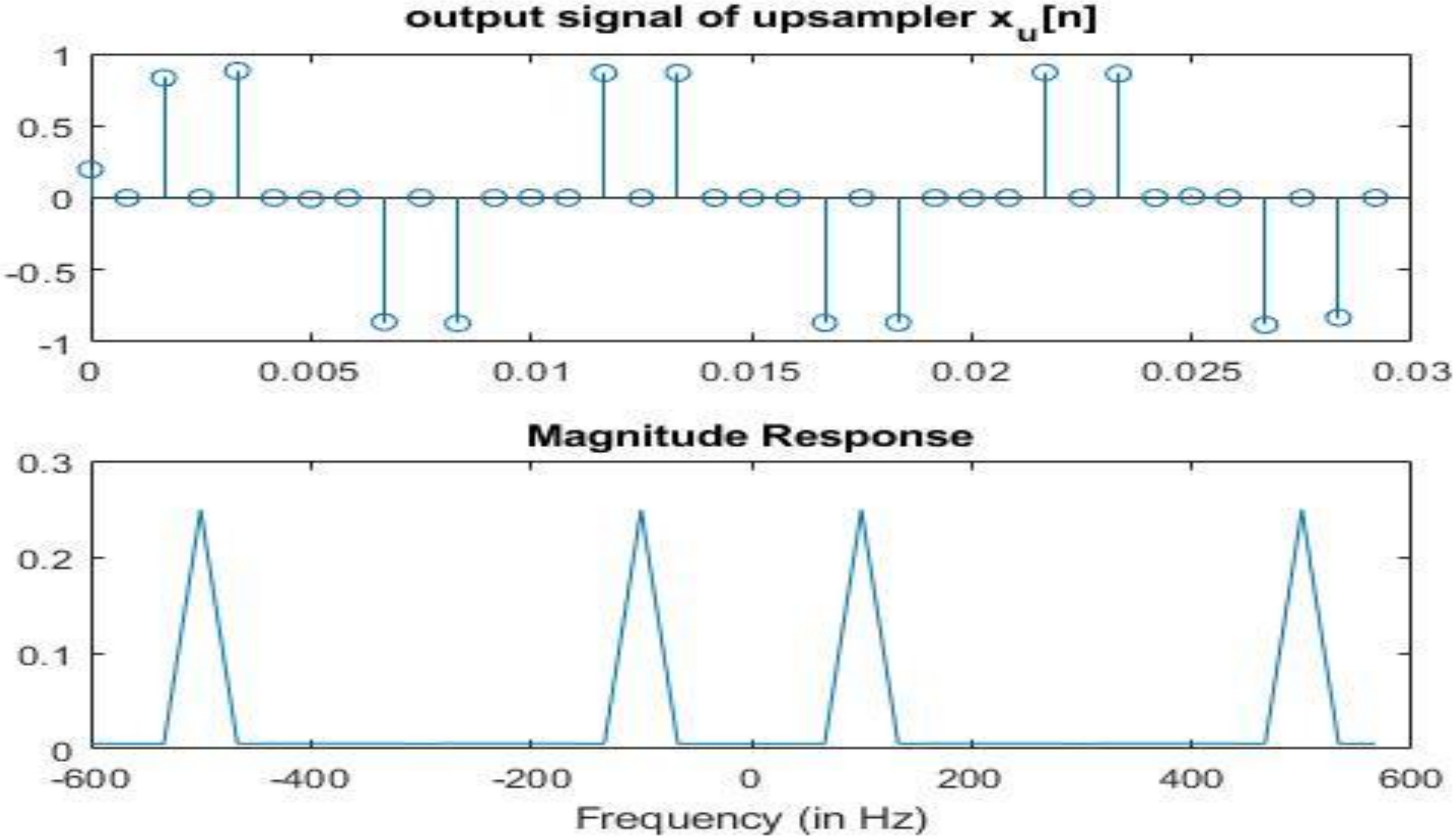
Time and Frequency domain plots from matlab simulation



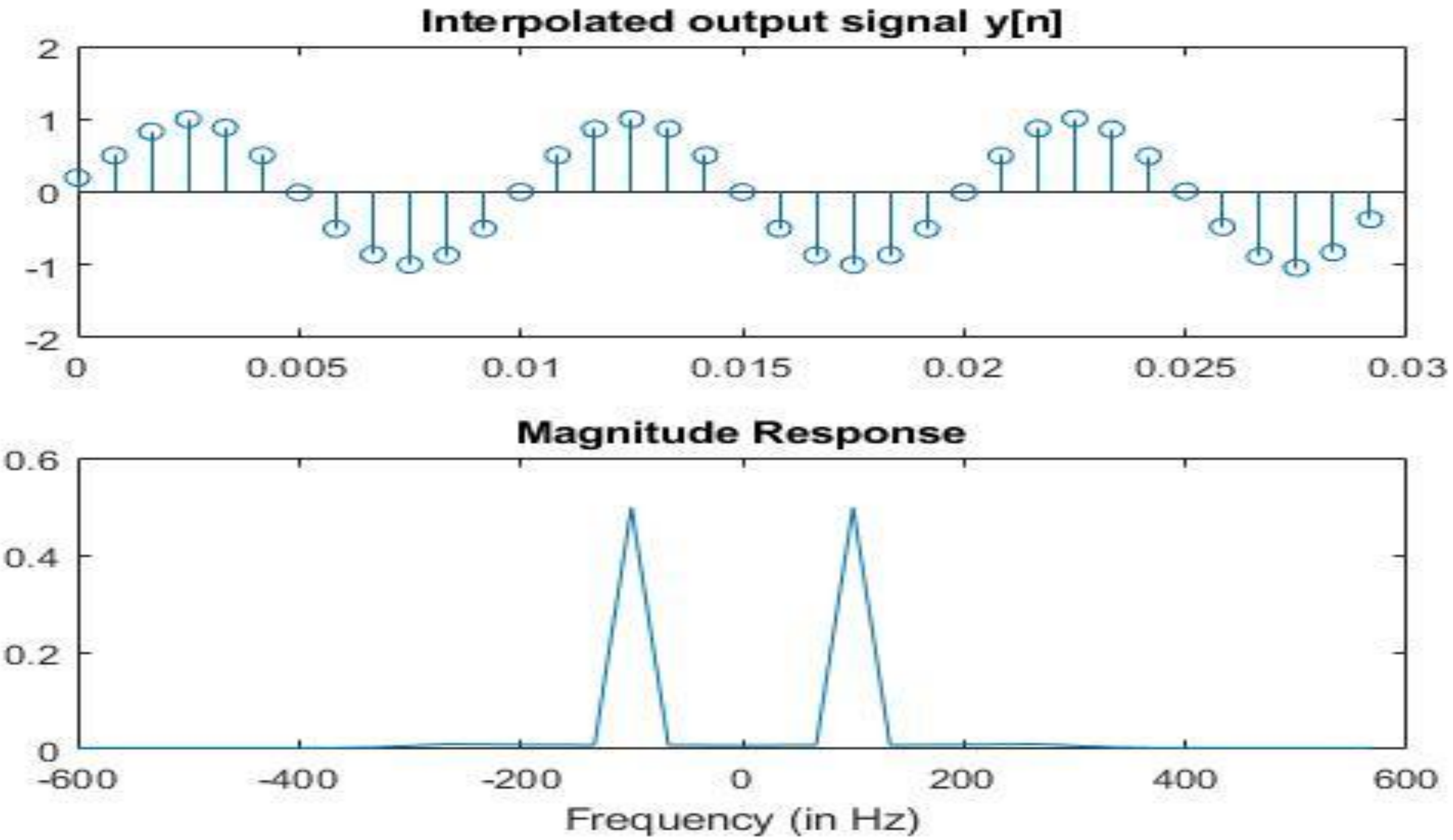
Time and Frequency domain plots from matlab simulation



Time and Frequency domain plots from matlab simulation



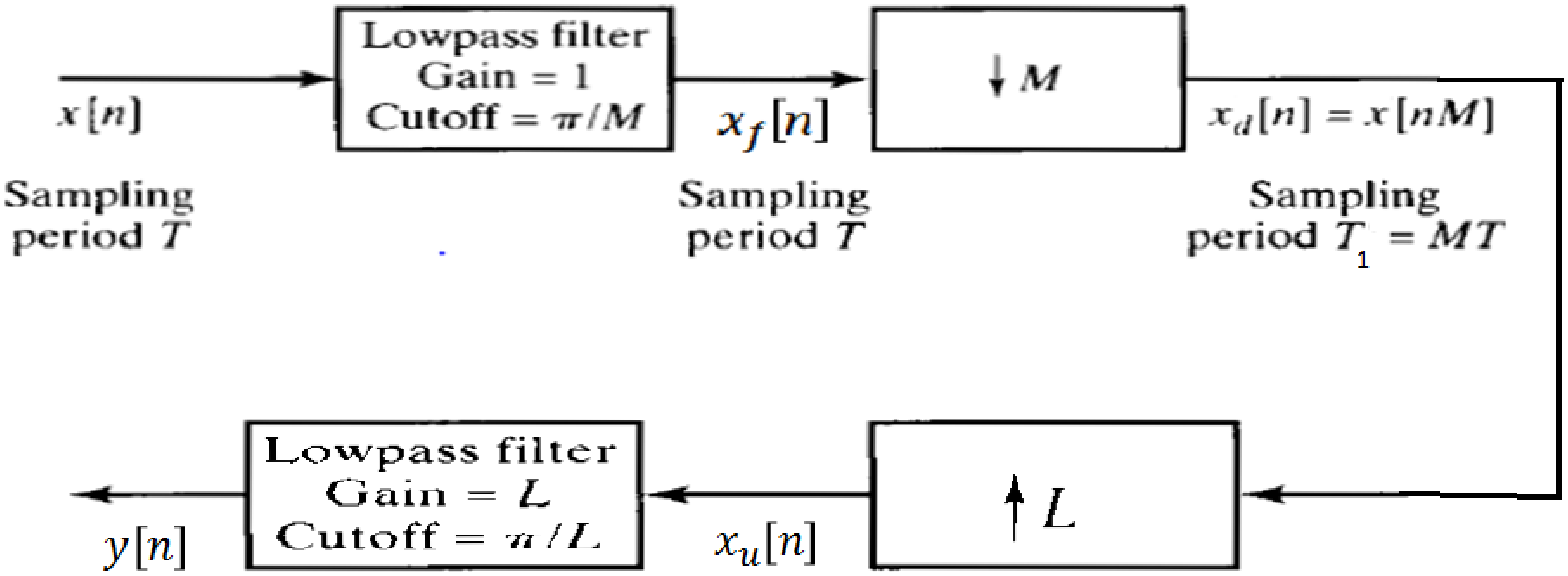
Time and Frequency domain plots from matlab simulation



Observation

- If input signal to decimator has **BW less than** $\frac{2\pi}{M}$ then, interpolator can reconstruct the input signal completely.
- If input signal to decimator has **BW more than** $\frac{2\pi}{M}$ then, only the information with in the BW limit can be reconstructed with the interpolator. i.e. the interpolator can reconstruct the signal which is input to downsampler.

Generalizing decimation and interpolation to any factor M or L



Sampling period

$$T_2 = \frac{T_1}{L} = \frac{MT}{L}$$

Sampling period

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Note: 1. We will generalize here for M = L

2. Sampling frequency of $x[n]$ and $h[n]$ should be same.

Generalizing decimation and interpolation to any factor M or L

- **Ex-1:** Let decimation and interpolation factors are M = L = 3, what are the filter gain and cutoff frequency of anti-aliasing filter and anti-imaging filter?

  Ans: For anti-aliasing filter, gain = 1, cutoff frequency $\omega_c = \pi/3$.

  For anti-imaging filter, gain = 3, cutoff frequency $\omega_c = \pi/3$.

- **Ex-2:** Let decimation and interpolation factors are M = L = 4, what are the filter gain and cutoff frequency of anti-aliasing filter and anti-imaging filter?

  Ans: For anti-aliasing filter, gain = 1, cutoff frequency $\omega_c = \pi/4$.

  For anti-imaging filter, gain = 4, cutoff frequency $\omega_c = \pi/4$.

Reference

- Discrete Time Signal Processing by Alan V. Oppenheim and Ronald W. Schafer - link