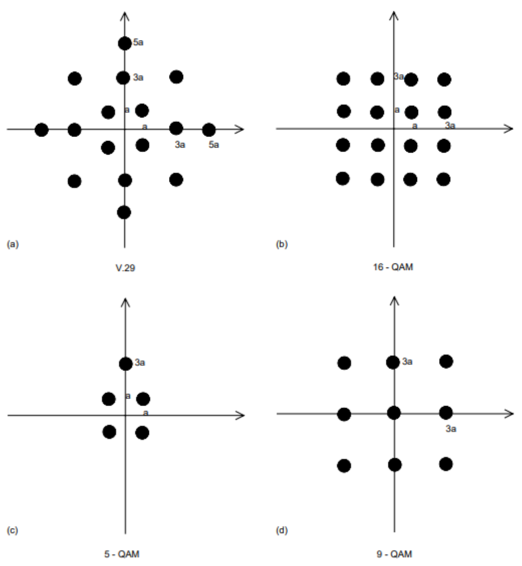
EE 6340/3861 Wireless Communications
HW 3
Due: 16/2/2023 by 11.59 am.

1. Outage probability and average probability of error
   (a) Consider a cellular system at 900 MHz with a transmission rate of 64 kbps and multipath fading. Explain which performance metric average probability of error or outage probability is more appropriate (and why) for user speeds of 1 mph, 10 mph, and 100 mph.
   (b) Derive the expression for the moment generating function for SNR in Rayleigh fading.
2. QoS in a cell: Consider a cellular system with circular cells of radius 100 meters. Assume that propagation follows the simplified path-loss model with $K = 1$, $d_0 = 1$m, and $\gamma = 3$. Assume the signal experiences (in addition to path loss) log-normal shadowing with $\sigma_{\psi dB} = 4$ as well as Rayleigh fading. The transmit power at the base station is $P_t = 100$ mW, the system bandwidth is $B = 30$ kHz, and the noise PSD $N_0/2$ has $N0 = 10^{-14}$ W/ Hz. Assuming BPSK modulation, we want to find the cell coverage area (percentage of locations in the cell) where users have average Pb of less than 10^{-4}.
   (a) Find the received power due to path loss at the cell boundary.
   (b) Find the minimum average received power (due to path loss and shadowing) such that, with Rayleigh fading about this average, a BPSK modulated signal with this average received power at a given cell location has $P_b < 10^{-4}$.
   (c) Given the propagation model for this system (simplified path loss, shadowing, and Rayleigh fading), find the percentage of locations in the cell where $P_b < 10^{-4}$ under BPSK modulation.

3. Error floors (similar to Problem 6-21*): Consider a wireless channel with average delay spread of 100ns and a Doppler spread of 90 Hz. Given the error floors due to Doppler and ISI and assuming BPSK modulation in Rayleigh fading and uniform scattering- approximately what range of data rates can be transmitted over this channel with a BER of less than 10^{-4}? Section 6.4 of the textbook has relevant content on error floors, equation (6.90) is especially useful

4. Find an approximation to P_s for the following signal constellations:



Mohammed Zafar Ali Khan
EE 6340/3801
zafar@ee.iith.ac.in
Phone: +91-40 2301 6454

5. In this problem we explore the power penalty involved in going to higher level signal modulations, i.e. from BPSK to 16PSK?
   (a) Find the minimum distance between constellation points in 16PSK modulation as a function of signal energy Es.
   (b) Find α_M and β_M such that the symbol error probability of 16PSK in AWGN is approximately

$$P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right).$$

   (c) Using your expression in part (b), find an approximation for the average symbol error probability of 16PSK in Rayleigh fading in terms of γ_s.
   (d) Convert the expressions for average symbol error probability of 16PSK in Rayleigh fading to expressions for average bit error probability assuming Gray coding.
   (e) Find the approximate value of γ_b required to obtain a BER of 10^{-3} in Rayleigh fading for BPSK and 16PSK. What is the power penalty in going to the higher level signal constellation at this BER?
6. The Nakagami distribution is parameterized by m, which ranges from m = .5 to m = ∞. The m parameter measures the ratio of LOS signal power to multipath power, so m = 1 corresponds to Rayleigh fading, m = ∞ corresponds to an AWGN channel with no fading, and m = .5 corresponds to fading that results in performance that is worse than with a Rayleigh distribution. In this problem we explore the impact of the parameter m on the performance of BPSK modulation in Nakagami fading.
   Plot the average bit error Pb of BPSK modulation in Nakagami fading with average SNR ranging from 0 to 20dB for m parameters m = 1 (Rayleigh), m = 2, and m = 4 (The Moment Generating Function technique of Section 6.3.3 should be used to obtain the average error probability). At an average SNR of 10 dB, what is the difference in average BER?

zafar@ee.iith.ac.in
Phone: +91-40 2301 6454