31/1/23 EE6310: Image & Video Processing

- Histogram equalization example (rounding)

I:

k	h(k)	p(k)
0	790	0.19
1	1023	0.25
2	850	0.21
3	656	0.16
4	329	0.08
5	245	0.06
6	122	0.03
7	81	0.01
	4096	

$K = 8$ → ① $J = 7 \times P_I(I)$ ✓

→ Round J to the closest integer ✓

→ ② $H(I) \approx 2$ ③ $\boxed{H(J) < 2}$ ✓

- Recall: $J = P_I(I)$

homework $\begin{cases} \text{• Data processing inequality} \\ \text{• } X \to Y \to Z \end{cases}$

I	→	J	$p_J(k)$
0	→	$7 \times 0.19 \approx 1$	
1	→	$7 \times 0.44 \approx 3$	
2	→	$7 \times 0.65 \approx 5$	
3	→	$7 \times 0.81 \approx 6$	
4	→	$7 \times 0.89 \approx 6$	
5	→	$7 \times 0.95 \approx 7$	
6	→	$7 \times 0.98 \approx 7$	
7	→	$7 \times 0.99 \approx 7$	

- Recall: Any discrete signal $x[n] = \sum_{k=-\infty}^{\infty} x[k]\, \delta[n-k]$ —①

$$I[m,n] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} I[k,l]\, \delta[m-k, n-l] \quad \text{—②}$$

$$\delta[m,n] = \begin{cases} 1, & m=0, n=0 \\ 0, & \text{otherwise} \end{cases}$$

- Derive the convolution sum for an LTI system for the 1D case
  - Given an LTI system $h[n]$ i.e. $h[n]$ satisfies the following: $h[n] = T[\delta[n]]$
    - Linearity: Let $T[\cdot]$ be the LTI system.

$\Rightarrow$ $T[a\,x_1[n] + b\cdot x_2[n]] = a\,y_1[n] + b\,y_2[n]$ where

$y_1[n] = T[x_1[n]]$; $y_2[n] = T[x_2[n]]$; a, b are arbit. scalars —③

- Time invariance: If $y[n] = T[x[n]]$, for any $n_0 \in \mathbb{Z}$,

$$T[x[n-n_0]] = y[n-n_0]. \quad -(4)$$

- We want to find the response of $T[\cdot]$ to an arbitrary input $x[n]$ as defined in (1)

$$y[n] = T[x[n]] = T\left[\sum_k x[k] . \delta[n-k]\right]$$

$$= \sum_k x[k] . T[\delta[n-k]] \qquad \text{due to (3)}$$

$$= \sum_k x[k] . h[n-k] \qquad \text{due to (4) and the defn of } h[n] = T[\delta[n]].$$

$$\boxed{y[n] = \sum_{k=-\infty}^{\infty} x[k] . h[n-k] = x[n] * \underbrace{h[n]}_{\text{Impulse response.}}} \rightarrow \text{Convolution sum.}$$

$$y = \sigma(Wx + b)\ ?$$

- $h[n] \xleftrightarrow{DFT} H[k]$

$\uparrow$ DFT of $h[n]$

$$\underset{N}{H[k]} = \sum_{n=0}^{L-1} h[n] . \underbrace{\left[e^{\frac{-j2\pi}{N} k}\right]^n}_{\text{basis}} \qquad (DFT)$$