EE 6340/3861 Wireless Communications
HW 2
Due: 6/2/2023 by 11.59 am.

1. On fading distributions: Prove, for X and Y independent zero-mean Gaussian random variables with variance σ^2, that $Z = |X + j\,Y|$ is Rayleigh distributed and that the distribution of Z^2 is exponential.
2. In problem 2, find the distribution when X has a non-zero mean.
3. Average signal power: Assume an application that requires a power outage probability of .05 for the threshold P_0=-70 dBm. For Rayleigh fading, what value of the average signal power is required?
4. Macrodiversity: In order to improve the performance of cellular systems, multiple base stations can receive the signal transmitted from a given mobile unit and combine these multiple signals either by selecting the strongest one or summing the signals together, perhaps with some optimized weights. This typically increases SNR and reduces the effects of shadowing. Combining of signals received from multiple base stations is called macrodiversity, and here we explore the benefits Diversity will be covered in more detail in Chapter 7. Consider a mobile at the midpoint between two base stations in a cellular network. The received signals (in dBW) from the base stations are given by
   $P_{r,1} = W + Z_1$, $P_{r,2} = W + Z_2$,
   where Z_1 , Z_2 are N $(0, \sigma^2)$ random variables. We define outage with macrodiversity to be the event that both $P_{r,1}$, $P_{r,2}$ fall below a threshold T .
   (a) Interpret the terms W, Z1 , Z2 in Pr,1 and Pr,2 . If Z_1 *and* Z_2 are independent, show that the outage probability is given by
   $P_{out} = [Q(\Delta/\sigma)]^2$,
   where $\Delta = W - T$ is the fade margin at the mobile's location.
5. Now suppose that Z_1 *and* Z_2 are correlated in Problem 4, with a correlation coefficient, b. Find the outage probability?
6. Channel fading simulator (Problem 3-11): The goal of this problem is to develop a Rayleigh fading simulator for a mobile communications channel using the method of filtering Gaussian processes that is based on the in-phase and quadrature PSDs described in Section 3.2.1. In this problem you must do the following.
   (a) Develop simulation code to generate a signal with Rayleigh fading amplitude over time. Your sample rate should be at least 1 mega samples per second, the average received envelope should be 1, and your simulation should be parameterized by the Doppler frequency f_D . Matlab is the easiest way to generate this simulation, but any code is fine. Write a description of your simulation that clearly explains how your code generates the fading envelope; use a block diagram and any necessary equations. Turn in any code that you use.
   (b) Provide plots of received amplitude (dB) versus time for fD = 1, 10, and 100 hertz over 2 seconds.
   (c) What is the power outage probability of for the threshold P_0=-70 dBm.

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