Wireless Communication

Homework - 1

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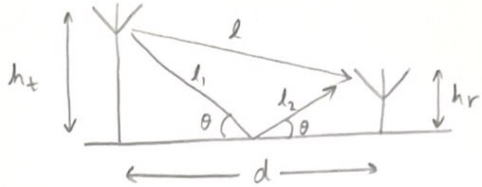
1) Since the two triangles are similar, we can write -



$$l_1 + l_2 = \sqrt{d^2 + (h_t + h_r)^2}$$

$$\Delta x = l_1 + l_2 - l$$

$$\Delta x = \sqrt{d^2 + (h_t + h_r)^2} - \sqrt{d^2 + (h_t - h_r)^2}$$

For large d compared to height of both transmitter and receiver -

$$\Delta x \approx d\left[1 + \frac{1}{2}\left(\frac{h_t + h_r}{d}\right)^2 - 1 - \frac{(h_t - h_r)^2}{2d^2}\right]$$

$$\Delta x = \frac{2h_t h_r}{d}$$

$\therefore$ The phase difference caused by this distance difference in LOS and reflected component is -

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{4\pi h_t h_r}{\lambda d}$$

By superposition of LOS & reflected components at the receiver and considering the surface to be purely reflective -

$$R_{Rx}(t) = Re\left\{\frac{\lambda}{4\pi}\left[\frac{\sqrt{G_l}\, e^{-j2\pi d/\lambda}}{d}u(t) - \frac{\sqrt{G_r}\, e^{-\frac{j2\pi(l_1+l_2)}{\lambda}}}{l_1 + l_2}u(t-\tau)\right]e^{j2\pi f_c t}\right\}$$

$(R = -1)$

We can assume that the transmitted signal is narrowband, i.e. $u(t) \approx u(t-\tau)$, and for large d, $l \approx l_1 + l_2 \approx d$, $G_l \approx G_R$.

Using the above approximations, the received power can be written as -

$$P_r \approx \left(\frac{\lambda}{4\pi d}\sqrt{G_l}\right)^2 \times \underbrace{\left|1 - e^{-j\Delta\phi}\right|^2}_{\left(\frac{4\pi h_t h_r}{\lambda d}\right)^2} \times P_t ,$$

$$\Rightarrow \quad \frac{P_r}{P_t} = \frac{G_l h_t^2 h_r^2}{d^4}$$

$$P_r\,dB = P_t\,dB + 10\log_{10} G_l + 20\log_{10}(h_t h_r) - 40\log_{10}(d)$$

2) From two ray model,

$$\Delta d = \sqrt{(h_t+h_r)^2+d^2} - \sqrt{(h_t-h_r)^2+d^2}$$

We need ~~a~~ distance values below d_c for which signal ~~s~~ null occurs, i.e. $\Delta\phi = k\pi = (2n+1)\pi$, $n \in \mathbb{N}$

$$\frac{2\pi}{\lambda}\Delta d = (2n+1)\pi \Rightarrow \quad \Delta d = \frac{(2n+1)}{2}\lambda = \frac{k\lambda}{\cancel{4}}$$

$$\sqrt{(h_t-h_r)^2+d^2} + \frac{k\lambda}{2} = \sqrt{(h_t+h_r)^2+d^2}$$

$$(h_t-h_r)^2 + d^2 + \frac{k^2\lambda^2}{4} + k\lambda\sqrt{d^2+(h_t-h_r)^2} = d^2 + (h_t+h_r)^2$$

$$k\lambda\sqrt{d^2+(h_t-h_r)^2} = 4h_t h_r - \frac{k^2\lambda^2}{4}$$

$$\sqrt{d^2+(h_t-h_r)^2} = \frac{4h_t h_r}{k\lambda} - \frac{k\lambda}{4}$$

3) Given $u(t) = e^{j2\pi f_s t}$

$G_1 = 2,\ G_2 = 1,\ d = 10\,km,\ h_t = 2h_r = 50\,m$

Clearly $d \gg (h_t + h_r)$

a) Delay spread $= \dfrac{x + x' - l}{c} \approx \dfrac{2 h_t h_r}{d \times c}$ (as $d \gg h_t + h_r$)

$$= \frac{2 \times 50 \times 50}{10 \times 10^3 \times 3 \times 10^8 \times 2} = 8.33 \times 10^{-9}\,s$$

Delay spread $= 8.33 \times 10^{-9}\,s = 8.33\,ns$

$$\frac{\Delta \text{ delay spread}}{\text{delay spread}} = \frac{\Delta d}{d} = \frac{1}{100} \text{ (given)}$$

Error in delay spread is $83.3\,ps$

b) $\Delta\phi = \dfrac{2\pi}{\lambda}(x + x' - l) = \dfrac{2\pi c}{\lambda}(\text{delay spread})$

$\Delta\phi = 2\pi \times \text{delay spread} \times \text{frequency}$

i) $f_s = \dfrac{1}{8.33 \times 10^{-9}} \Rightarrow \Delta\phi = 2\pi = 360°$

ii) $f_s = \dfrac{1}{2m} \Rightarrow \Delta\phi = 2\pi \times \dfrac{1}{2} = \pi = 180°$

iii) $f_s = \dfrac{1}{100\,m} \Rightarrow \Delta\phi = \dfrac{2\pi}{1} \times \dfrac{1}{100} = \dfrac{\pi}{50} = 7.2°$

The approximation $u(t) \approx u(t-\tau)$ holds when $\tau \ll \dfrac{1}{B_u}$,

i.e. in case 3 where $(f_s)^{-1}$ is 100 times the delay spread (τ)

$$d = \sqrt{\left(\frac{4h_t h_r}{k\lambda} - \frac{k\lambda}{4}\right)^2 - (h_t - h_r)^2}$$

for $k = 1, 3, 5, 7, \ldots\ldots$

Clearly, all distances are less than $d_c \left(\frac{4h_t h_r}{\lambda}\right)$.

$$\therefore d = \sqrt{\left(\frac{4h_t h_r}{\lambda(2n+1)} - \frac{(2n+1)\lambda}{4}\right)^2 - (h_t - h_r)^2} \quad \forall n = 0, 1, 2, 3, 4, \ldots\ldots$$

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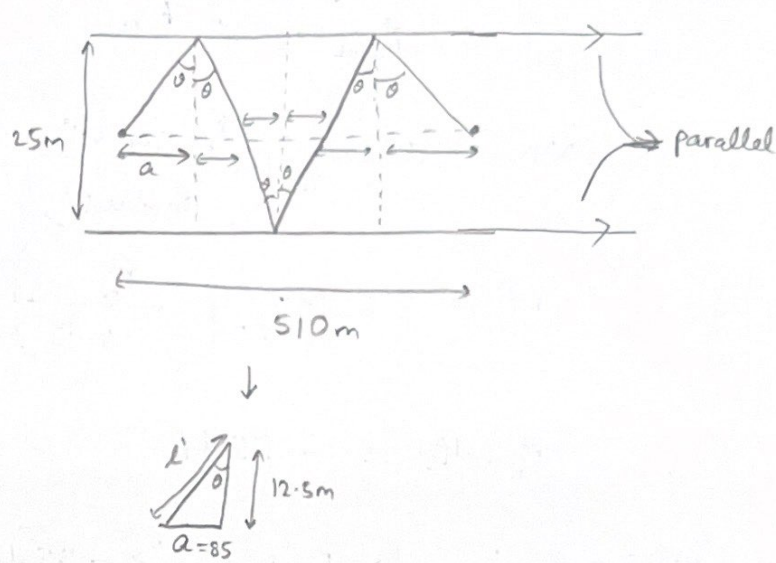
4) a) Power falloff is proportional to d^{-2} and is insensitive to transmitter height.

b) Since all 6 triangles are similar, we can consider the component along LOS as a.



$$l(LOS) = 6a = 510 \text{ m}$$

$$a = 85\text{m}$$

$$l(3 \text{ reflections}) = 6 \times l'$$

$$= 6\sqrt{85^2 + 12.5^2} = 515.4852083 \text{ m}$$

Here we consider the model with 3 reflections because it gives the maximum delay spread due to maximum distance covered.

$$\text{Delay spread, } \tau = \frac{l(3 \text{ reflections}) - l(LOS)}{c}$$

$$\tau = \frac{5.485208323}{3 \times 10^8}$$

$$\tau = 18.284 \text{ ns}$$

5) Given - Operating frequency at 910 MHz

$\sigma_{\psi_{dB}} = 6\,dB$, $SNR = 15\,dB$

$P_t = 1W$, Antenna Gain $= 3dB = G_T$, $G_R = 0dB$

$P_{noise} = -40dBm$

$$-40 = 10 \log_{10} \frac{P_A(mW)}{\cancel{10}\,1\,mW}$$

$$P_{noise} = 10^{-7}\ W$$

$$SNR = 10 \log_{10}\left(\frac{P_{received}(W)}{P_{noise}(W)}\right)$$

$$15 = 10 \log_{10}\left(\frac{P_R}{10^{-7}}\right) \Rightarrow P_R(W) = 10^{-5.5}\ W$$

$$P_R(W) = 3.162 \times 10^{-6}\ W$$

$$P_R\ (dB) = -55\,dB$$

We require $Pr(P_{received} > -55dB) = 0.9$

The distribution of this probability is gaussian.

$$P_{Rx}(d) = P_{Tx}\, G_{Tx}\, G_{Rx} \left(\frac{\lambda}{4\pi d}\right)^2$$

$$P_{Rx}(d) = 1 \times 10^{0.3} \times 1 \times \left(\frac{3\times 10^8}{910\times 10^6}\right)^2 \times \frac{1}{16\times 9.8696} \times \frac{1}{d^2}$$

$$P_{Rx}(d) = \frac{1.3726\times 10^{-3}}{d^2}\ W$$

We need the highest probability (occurs at μ) at the cell distance.

$$\therefore\ \mu = \frac{1.3726\times 10^{-3}}{d^2}$$

$Pr(X > x) = Q(x)$ for a gaussian.

$\Rightarrow x = Q^{-1}(0.9) = -1.2816$

$\hookrightarrow$ minimum power

$$\frac{-55 - \mu}{6} = -1.2816 \Rightarrow \mu = -47.3104 \, dB$$

$$\frac{1.3726 \times 10^{-3}}{d^2} = 10^{-4.73104} = 1.8576 \times 10^{-5}$$

$$d = 8.5959 \, m$$

6) For a two ray model -

$$r(t) = \frac{\lambda}{4\pi} Re\left[\left(\frac{\sqrt{G_l}\, e^{-\frac{2\pi}{\lambda}d}}{d} s(t-\tau_1) + \frac{R\sqrt{G_R}\, e^{-\frac{2\pi}{\lambda}l'}}{l'} s(t-\tau_2)\right) e^{j2\pi f_c t}\right]$$

Given $G_l = G_R = 1$, $R = -1$

$$d = Vt, \quad l' = d + \frac{h^2}{2d} = Vt + \frac{h^2}{2Vt}$$

The channel response, $h(t) = \underbrace{\alpha_1 e^{j\phi_1}\delta(t-\tau_1)}_{LOS} + \underbrace{\alpha_2 e^{j\phi_2}\delta(t-\tau_2)}_{reflected}$

$$\alpha_1 = \frac{\lambda}{4\pi d} = \frac{\lambda}{4\pi Vt}$$

$$\alpha_2 = \frac{\lambda}{4\pi\left(Vt + \frac{h^2}{2Vt}\right)}$$

$$\phi_1(t) = 2\pi f_c \tau_1(t) - \phi_{D_1}, \quad \tau_1 = \frac{d}{c}$$

$$\phi_{D_1}(t) = \int_t 2\pi f_{D_p}(t)\, dt$$

$$\Rightarrow \phi_q(t) = 2\pi f_c \frac{d}{c} - \int_t 2\pi f_{D_q}(t)\, dt$$

From dopler effect, $f_D = \frac{f_c}{c} v \cos\theta$

$$\phi_1(t) = \int_t 2\pi \frac{f_c}{c} v \cos\theta \, dt = 0$$

$$\phi_2(t) = 2\pi f_c \tau_2 - \phi_{D_2} = \frac{2\pi f_c}{c}\left(d + \frac{2h^2}{d}\right) - \phi_{D_2}$$

$$\phi_{D_2}(t) = \int_t 2\pi f_{D_2}(t)\, dt = \int_t 2\pi \frac{f_c v}{c} \cos\phi_2 \, dt$$

$$\phi_{D_2}(t) = \pi - \tan^{-1}\left(\frac{2h}{d}\right)$$

$\therefore$ The impulse response comes out -

$$h(t) = \frac{\lambda}{4\pi v t}\, \delta\left(t - \frac{d}{c}\right) + \frac{\lambda}{4\pi\left(vt + \frac{2h^2}{vt}\right)} \exp\left(j \frac{2\pi f_c}{c}\left[\left(d + \frac{2h^2}{d}\right) - \pi + \tan^{-1}\left(\frac{2h}{d}\right)\right]\right) \cdot \delta\left(t - \frac{d}{c} + \frac{2h^2}{cd}\right)$$

7) a) $f_c = 900$ MHz, $d_0 = 1$m

Simple model -

$$10\log_{10} \frac{P_r}{P_t} = 10 \log_{10} K - 10 \nu \log_{10}\left(\frac{d}{d_0}\right) \quad (d_0 = 1\,m)$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{9 \times 10^8} = \frac{1}{3}$$

At $d = d_0$, $\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi d}\right)^2 = \frac{1}{(12\pi d)^2}$

$\therefore$ $K dB = 20 \log_{10} \frac{1}{12\pi d_0} = -31.5266$ dB

$K = 7.0362 \times 10^{-4}$

At $d = 5m$, $\frac{P_r}{P_t} = -60 dB$

$-60 \text{ dB} = -31.5266 \text{ dB} - 10 \nu \log_{10} 5$

For free space model, $\nu = 2$ or $4 \Rightarrow$ take $\nu = 4$

Best fit model-

$$F(\nu) = \sum_{i=1}^{6} \left(M_{measured}(d_i) - \underset{\downarrow}{M_{model}}(d_i) \right)^2$$

$$M_{model} = K - 10 \nu \log_{10}\left(\frac{d}{1}\right)$$

$\sigma^2_{\psi_{dB}} = F(\nu) = 1.4809 \text{ dB}$ (calculated in (c))

b) $P_{Loss}(2 \text{ Km}) = 10 \log_{10} K - 10 \nu \log_{10}\left(\frac{d}{d_0}\right)$

$P_{Loss}(2 \text{ Km}) = -31.5266 - 10 \times 4 \times \log_{10}(2000)$

$P_{Loss}(2 \text{ Km}) = -163.5667 \text{ dB}$

c) Received power is assumed to be gaussian with variance σ^2_{dB} (mean = 0).

$$P_r(x < -10) = P_r\left(\frac{x - \overset{0}{\mu}}{\sigma} < -\frac{10}{\sigma}\right)$$

$$= P_r\left(\frac{x}{\sigma} < -\frac{10}{\sigma}\right) = 1 - Q\left(\frac{P_{min}}{\sigma_{\psi_{dB}}}\right)$$

Model prediction -

At 5m $\Rightarrow$ $-31.5266 - 40\log_{10} 5 = -59.4854\,dB$

Predicted $= -60\,dB$

At 25 m $\Rightarrow$ $-31.5266 - 40\log_{10} 25 = -87.4442\,dB$

Predicted $= -85\,dB$

At 65m $\Rightarrow$ $-31.5266 - 40\log_{10} 65 = -104.0431\,dB$

Predicted $= -105\,dB$

At 110m $\Rightarrow$ $-31.5266 - 40\log_{10} 110 = -113.1823\,dB$

Predicted $= -115\,dB$

At 400m $\Rightarrow$ $-31.5266 - 40\log_{10} 400 = -135.6089\,dB$

Predicted $= -135\,dB$

At 1000m $\Rightarrow$ $-31.5266 - 40\log_{10} 1000 = -151.5266\,dB$

Predicted $= -150\,dB$

$$\sigma^2_{\psi_{dB}} = \frac{1}{6}\left((0.5146)^2 + (2.444)^2 + (0.9569)^2 + (1.8177)^2 + (0.6089)^2 + (1.5266)^2\right)$$

$$\sigma^2_{\psi_{dB}} = \frac{1}{6}\left(0.26481 + 5.9731 + 0.91565 + 3.3040 + 0.3707 + 2.3305\right)$$

$$\sigma^2_{\psi_{dB}} = 2.1931\,(dB)^2 \Rightarrow \sigma_{\psi_{dB}} = 1.4809\,dB$$

$$P_r(X < -10) = P_r\left(\frac{X}{\sigma_{dB}} < \frac{-10}{\sigma_{dB}}\right) = 1 - Q\left(\frac{-10}{1.4809}\right)$$

$$P_r(X < -10) = 1 - Q(-6.75265)$$

$$P_r(X < -10) = 7.258 \times 10^{-12}$$

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