Indian Institute of Technology Hyderabad
EE6310: Image and Video Processing
Quiz 2, 17.03.2023, 10 points

1. The SSIM index between image patches $\mathbf{x}, \mathbf{y}$ can be defined as $\text{SSIM}(\mathbf{x}, \mathbf{y}) = \frac{\mathbf{2\mu_x\mu_y+C_1}}{\mathbf{\mu_x^2+\mu_y^2+C_1}} \cdot \frac{\mathbf{2\sigma_{xy}+C_2}}{\mathbf{\sigma_x^2+\sigma_y^2+C_2}}$. μ, σ correspond to local patch mean and standard deviation respectively, and C_1, C_2 are stabilizing constants. If the patches are zero-mean, can $\text{SSIM}(\mathbf{x}, \mathbf{y})$ be related to the MSE between $\mathbf{x}$ and $\mathbf{y}$? If yes, show your work. If not, justify. What happens when the patches have non-zero but equal means? (2)

2. Give an example of a pair of 3×3 patches $\mathbf{x}, \mathbf{y}$ such that $\text{SSIM}(\mathbf{x}, \mathbf{y}) =$ -1. (1)

3. For a continuous 2-D function $f(x, y)$, the Laplacian is defined as $\nabla^2 f(x, y) = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}$. Show that $\nabla^2 f(x, y)$ remains the same irrespective of the choice of the orthogonal basis. (3)

4. Show that the convolution operation is commutative, i.e., $(h * g) * f = (g * h) * f$ where h, g are linear and time-invariant systems, f is a signal. For simplicity, work with 1-D digital signals and systems. (2)

5. Use the above result to derive the efficient form of the LoG kernel. Assume the Gaussian low-pass filter to be $G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$. (2)