Wireless Communication

Homework - 3

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1) a) $f = 900\,MHz$

Transmission Rate $= 64$ kbps

Bit duration $= \dfrac{1}{64 \times 10^3}$

Bit duration $= 15.625\,\mu s = T_b$ (say)

Doppler frequency, $f_d = f_c \cdot \dfrac{v}{c} = 900 \times 10^6 \times \dfrac{v}{3 \times 10^8} = 3v$

$f_c \to$ carrier frequency

Coherence time, $T_c = \dfrac{0.4}{f_d} \to$ depends on speed.

Probability of error is used when $T_b \approx T_c$

Outage probability is used when $T_b << T_c$

i) When $v = 1$ mph $= 0.447$ m/s

$T_c = \dfrac{0.4}{3 \times 0.447} = 298.28\,ms \Rightarrow T_b << T_c$

So here we can use outage probability

ii) When $v = 10$ mph $= 4.47$ m/s

$T_c = \dfrac{0.4}{3 \times 4.47} = 29.828\,ms \Rightarrow T_b << T_c$

So here we can use outage probability

iii) When $v = 100$ mph $= 44.7$ m/s

$T_c = \dfrac{0.4}{3 \times 44.7} = 2.982\,ms \Rightarrow T_b << T_c \Rightarrow$ Here outage probability with average probability can be used.

1 b) MGF of SNR in Rayleigh fading is defined as -

$$M(s) = E[e^{sx}] = \int_0^\infty \exp[s x] \, p(x) \, dx$$

$p(x) \rightarrow$ p.d.f. of r.v. X

For Rayleigh fading, $p(x) = \frac{1}{\alpha} \exp\left(-\frac{x}{\alpha}\right)$

where $\alpha = E[x]$

$$\therefore M(s) = \int_0^\infty e^{sx} \frac{1}{\alpha} e^{-x/\alpha} \, dx = \frac{1}{\alpha} \int_0^\infty \exp\left(\left(s - \frac{1}{\alpha}\right)x\right) dx$$

$$M(s) = \frac{1}{\alpha s - 1} \exp\left(\left(s - \frac{1}{\alpha}\right)x\right)\Big|_0^\infty$$

$$M(s) = \frac{1}{1 - \alpha s}, \text{ given } \quad s < \frac{1}{\alpha}$$

$$\therefore MGF(SNR) = \frac{1}{1 - \alpha s}$$

2) Given cell radius of 100m, $K = 1$, $d_0 = 1m$, $\gamma = 3$

Log-normal shadowing with $\sigma_{\psi_{dB}} = 4$ dB

with Rayleigh Fading

$P_t = 100$ mW, $B = 30$ KHz, $N_0 = 10^{-14}$ W/Hz

$N_0 \rightarrow$ Noise PSD

We need average $P_b \leq 10^{-4}$

a) From path loss model, $\frac{P_r}{P_t} = \frac{1}{K} \left(\frac{d_0}{d}\right)^\gamma$, $d \rightarrow$ cell radius

$$\Rightarrow P_r = \frac{10^{-1}}{1} \times \left(\frac{1}{100}\right)^3 = 10^{-7} \text{ W}$$

$$P_r \text{ in dBm} = 10\log_{10}\left(\frac{10^{-7}}{10^{-3}}\right) = -40\,\text{dBm}$$

b) For Rayleigh fading using BPSK:

$$P_b = \frac{1}{2}\left[1 - \sqrt{\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}}\right] \approx \frac{1}{4\bar{\gamma}_b} < 10^{-4} \Rightarrow \bar{\gamma}_b > 2500$$

w.k.t. $\bar{\gamma}_b = \frac{E_b}{N_0}$ and $B \times E_b = P_o \Rightarrow P_o = B N_o \bar{\gamma}_b$

$$P_{power\,(min)} = \bar{\gamma}_b \times N_o \times B = 2500 \times 10^{-14} \times 30 \times 10^3 = 75 \times 10^{-8}\,W$$

In dBm, $P_{min} = 10\log_{10}\left(\frac{75\times10^{-8}}{10^{-3}}\right)$

$$P_{min} = -31.24\,\text{dBm}$$

c) $P_{min} = 7.5 \times 10^{-7}\,W$, $P_t = 100\,mW$

By Path loss model -

$$\frac{P_{min}}{P_t} = \frac{1}{K}\left(\frac{d_0}{d}\right)^{\gamma} \Rightarrow d^3 = \frac{10^{-1}}{7.5\times10^{-7}} = \frac{10^6}{7.5} \Rightarrow d = 51.0873\,m$$

$$\therefore \text{ \% of coverage area} = \frac{\pi d^2}{\pi r^2} = \frac{(51.0873)^2}{10^4} = 26.1\%$$

26.1% of area covered has $P_b < 10^{-4}$.

3) Given -

Delay Spread = 100 ns

Doppler Spread = 90 Hz

Required $P_{floor} = 10^{-4}$

We know that for BPSK : $P_{floor} \le \left(\frac{\sigma_{T_M}}{T_s}\right)^2$

$$\Rightarrow \left(\frac{\sigma_{T_M}}{T_s}\right)^2 \le 10^{-4} \Rightarrow T_s \ge 100\,\sigma_{T_M} = 10\mu s$$

$$\therefore \text{Data Rate} = \frac{1}{T_s} \le 100 \text{ Kbps}$$

For uniform scattering , $P_c = J_0(2\pi \overbrace{f_D}^{\text{doppler spread}} \underbrace{T_b}_{\text{bit time}})$

So we can write , $P_{floor} \approx \frac{1}{2}(\pi f_D T_b)^2 < 10^{-4}$ (given)

$$\Rightarrow T_b < \frac{\sqrt{2 \times 10^{-4}}}{\pi f_D} = 5.001 \times 10^{-5} s$$

$$\text{Data Rate} = \frac{1}{T_b} = 20 \text{ kbps}$$

$$\therefore \quad 20 \text{ kbps} < \text{Data Rate} \le 100 \text{ Kbps}$$

4) We use the following results -

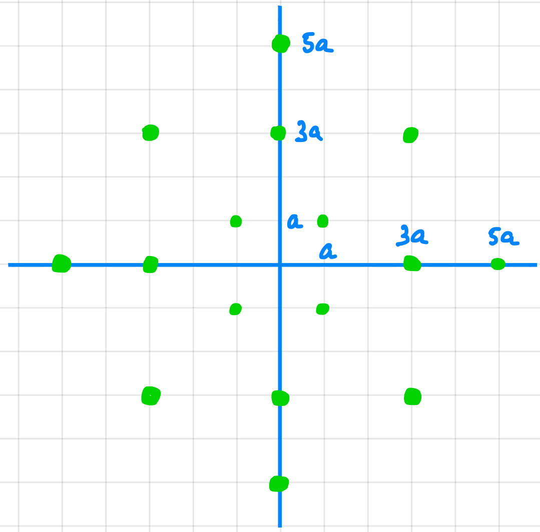
Union Bound : $\Pr\left[\bigcup_{i=1}^{n} E_i\right] \le \sum_{i=1}^{n} \Pr[E_i]$

$$\Pr(\text{Symbol Error}) \le \frac{1}{M} \sum_{i=0}^{M-1} \sum_{j \in N(i)} Q\left(\sqrt{\frac{d_{min}^2}{2N_0}}\right)$$

This can also be approximated as nearest neighbour -

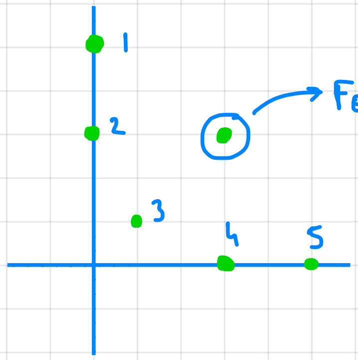
$$\Pr(\text{Symbol Error}) \approx \frac{N_{min}}{M} Q\left(\sqrt{\frac{d_{min}^2}{2N_0}}\right)$$

a) Given signal constellation :



Types of nearest neighbours -

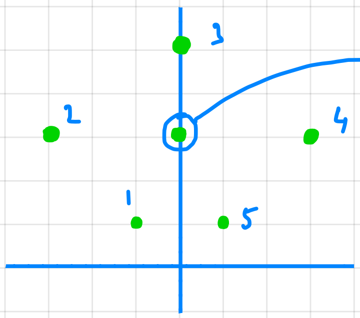
1)



→ For this point, there are 5 nearest neighbours.

→ Total 4 points in constellation

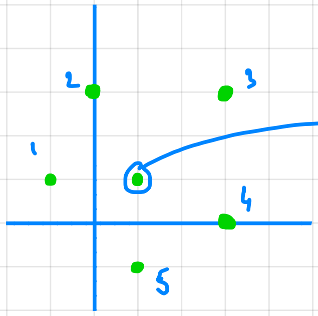
2)



→ For this point there are 5 nearest neighbours

→ Total 4 points like this in the constellation

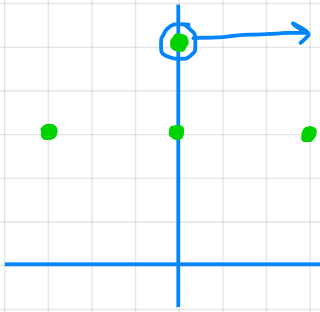
3)



→ For this point there are 5 nearest neighbours

→ Total 4 points like this in the constellation.

4)



For this point there are 3 nearest neighbours

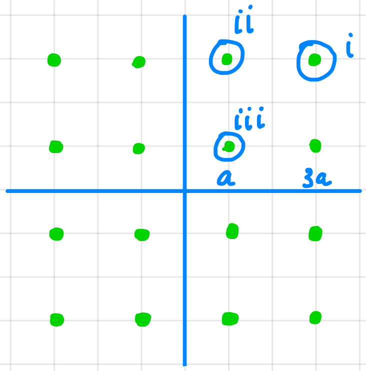
→ Total 4 points like this in the constellation

$$\therefore P_s = \frac{4\times5 + 4\times5 + 4\times5 + 4\times3}{16} Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

Here, $d_{min} = 2a$

$$P_s = 4.5\, Q\left(a\sqrt{\frac{2}{N_0}}\right)$$

4) b) Given signal constellation -



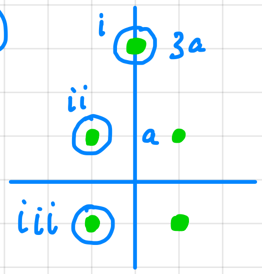
Points of (i) ⇒ 2 nearest neighbours × 4 in total

Points of (ii) ⇒ 3 nearest neighbours × 8 in total

Points of (iii) ⇒ 4 nearest neighbours × 4 in total

$$\therefore P_s = \frac{4\times2 + 8\times3 + 4\times4}{16} Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = 3\, Q\left(a\sqrt{\frac{2}{N_0}}\right)$$

4) c)



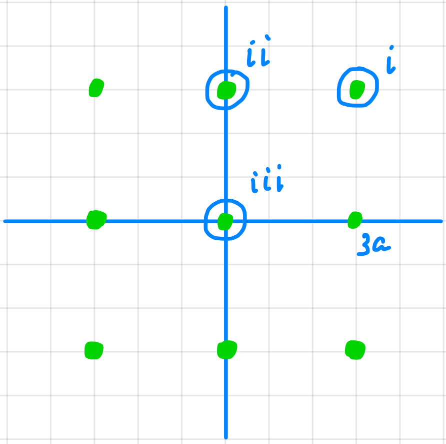
Points of type (i) ⇒ 2 nearest neighbours × 1 in total

Points of type (ii) ⇒ 3 nearest neighbours × 2 in total

Points of type (iii) ⇒ 2 nearest neighbours × 2 in total

$$\therefore P_s = \frac{2\times1 + 3\times2 + 2\times2}{5} Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = 2.4\, Q\left(a\sqrt{\frac{2}{N_0}}\right)$$

4) d) Given signal constellation -



Points of type (i) $\Rightarrow$ 2 nearest neighbours $\times$ 4 in total

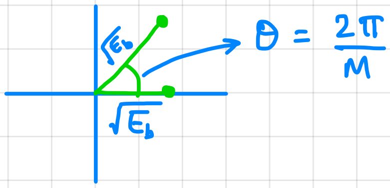
Points of type (ii) $\Rightarrow$ 3 nearest neighbours $\times$ 4 in total

Points of type (iii) $\Rightarrow$ 4 nearest neighbours $\times$ 1 in total

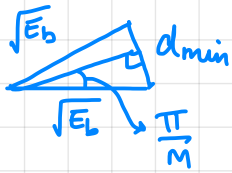
$$\therefore P_s = \frac{2\times4 + 3\times4 + 4\times1}{9} Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right), \quad d_{min} = 3a$$

$$P_s = 2.67\, Q\left(\frac{3a}{\sqrt{2N_0}}\right)$$

5) a) In M-ary PSK -



Redrawing the triangle :



From the inner right angled triangle : $\sin\left(\frac{\pi}{M}\right) = \frac{d_{min}}{2\sqrt{E_b}}$

$$\Rightarrow d_{min} = 2\sqrt{E_b}\,\sin\left(\frac{\pi}{M}\right)$$

For 16-PSK modulation, $M = 16 \Rightarrow d_{min} = 2\sqrt{E_b}\,\sin\left(\frac{\pi}{16}\right)$

$$d_{min} \approx 0.39\sqrt{E_b}$$

5) b) From union bound expression -

$$P_s = \frac{N_{min}}{M} Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

16-PSK is essentially 16 points on a circle with $r = \sqrt{E_b}$.

Now, every point has 2 nearest neighbours and there are 16 points as such $\Rightarrow N_{min} = 16\times2 = 32$

$$\therefore P_s = \frac{32}{16} Q\left(\frac{0.39\sqrt{E_b}}{\sqrt{2N_0}}\right) = 2Q\left(\sqrt{0.076 \times \frac{E_b}{N_0}}\right)$$

Also, $SNR = \gamma_s = \frac{E_b}{N_0} \Rightarrow P_s = 2Q\left(\sqrt{0.076\,\gamma_s}\right)$

$$\Rightarrow \alpha_M = 2 \text{ and } \beta_M = 0.076$$

5) c) Average Symbol Error : $P_e = \int_0^\infty P_s(\gamma_s)\, f(\gamma_s)\, d\gamma_s$

From previous part, $P_s(\gamma_s) = \alpha_M\, Q\left(\sqrt{\beta_M \gamma_s}\right)$

$$Q(z) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{z^2}{2\sin^2\phi}\right) d\phi \quad , \quad z > 0$$

$$Q\left(\sqrt{\beta_M \gamma_s}\right) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{\beta_M \gamma_s}{2\sin^2\phi}\right) d\phi \quad , \quad \beta_M \gamma_s > 0$$

$$\therefore P_s = \int_0^\infty \frac{\alpha_M}{\pi}\int_0^{\pi/2} \exp\left(-\frac{\beta_M \gamma_s}{2\sin^2\phi}\right) d\phi\; f(\gamma_s)\, d\gamma_s$$

$$P_s = \frac{\alpha_M}{\pi}\int_0^{\pi/2} \underbrace{\int_0^\infty \exp\left(-\frac{\beta_M}{2\sin^2\phi}\gamma_s\right) f(\gamma_s)\, d\gamma_s}_{\text{MGF of Rayleigh distribution with } s = -\frac{\beta_M}{2\sin^2\phi}}\; d\phi$$

From Q1, MGF for SNR in Rayleigh fading is given as -

$$M_{\gamma_s}\left(-\frac{\beta_M}{2\sin^2\phi}\right) = \frac{1}{1 + \frac{\beta_M}{2\sin^2\phi}\bar{\gamma}_s} = \left(1 + \frac{\beta_M \bar{\gamma}_s}{2\sin^2\phi}\right)^{-1}$$

$$\therefore P_s = \frac{\alpha_M}{\pi} \int_0^{\pi/2} \left(1 + \frac{\beta_M}{2\sin^2\phi} \bar{\gamma}_s\right)^{-1} d\phi$$

$$P_s = \frac{\alpha_M}{2}\left(1 - \sqrt{\frac{\beta_M \bar{\gamma}_s}{2+\beta_M \bar{\gamma}_s}}\right) = 1 - \sqrt{\frac{0.38 \cdot \bar{\gamma}_s}{1+0.38\,\bar{\gamma}_s}}$$

$$P_s \approx \frac{1}{0.076\,\bar{\gamma}_s}$$

5) d) Assuming for Gray coding:

$$P_{gray} = \frac{P_{avg}}{\log_2 M} = \frac{P_{avg}}{\log_2 16}$$

$$P_{gray} = \frac{P_{avg}}{4}$$

5) e) For BPSK: $BER = SER = \frac{E_s}{d_{min}^2\,\bar{\gamma}_b}$

$$d_{min} = 2\sqrt{E_s}\,\sin\left(\frac{\pi}{2}\right) = 2\sqrt{E_s}$$

$$\therefore 10^{-3} = \frac{E_s}{4E_s\,\bar{\gamma}_b} \Rightarrow \bar{\gamma}_b = 250$$

For 16-PSK, $P_s = \frac{1}{0.076\,\bar{\gamma}_s} \longrightarrow P_b = \frac{1}{\underbrace{\log_2 M}_{4} \cdot 0.076 \cdot \bar{\gamma}_s} = 10^{-3}$

$$\therefore \bar{\gamma}_s = \frac{1000}{4 \times 0.076}$$

$$\bar{\gamma}_s = 3289.47$$

Power penalty $= \frac{3289.47}{250} = 13.15$ times $\approx 11.27\,dB$

6) Symbol Error : $P_s = \frac{\alpha}{\pi} \int_0^{\pi/2} \underbrace{M\left(\frac{-\beta_M}{2\sin^2\phi}\right)}_{= \left(1 + \frac{\beta_M \bar{\gamma}_s}{m \sin^2\phi}\right)^{-m}} d\phi$

$$\Rightarrow P_s = \frac{\alpha}{\pi} \int_0^{\pi/2} \left(1 + \frac{\beta_M \bar{\gamma}_s}{m \sin^2\phi}\right)^{-m} d\phi$$

For BPSK , $P_b = Q\left(\sqrt{2\bar{\gamma}_b}\right) \Rightarrow \alpha_M = 1 , \beta_M = 2$

$$\therefore P_s = \frac{1}{\pi} \int_0^{\pi/2} \left(1 + \frac{\bar{\gamma}_b}{m \sin^2\phi}\right) d\phi$$

Code , Output and Plot is given below.

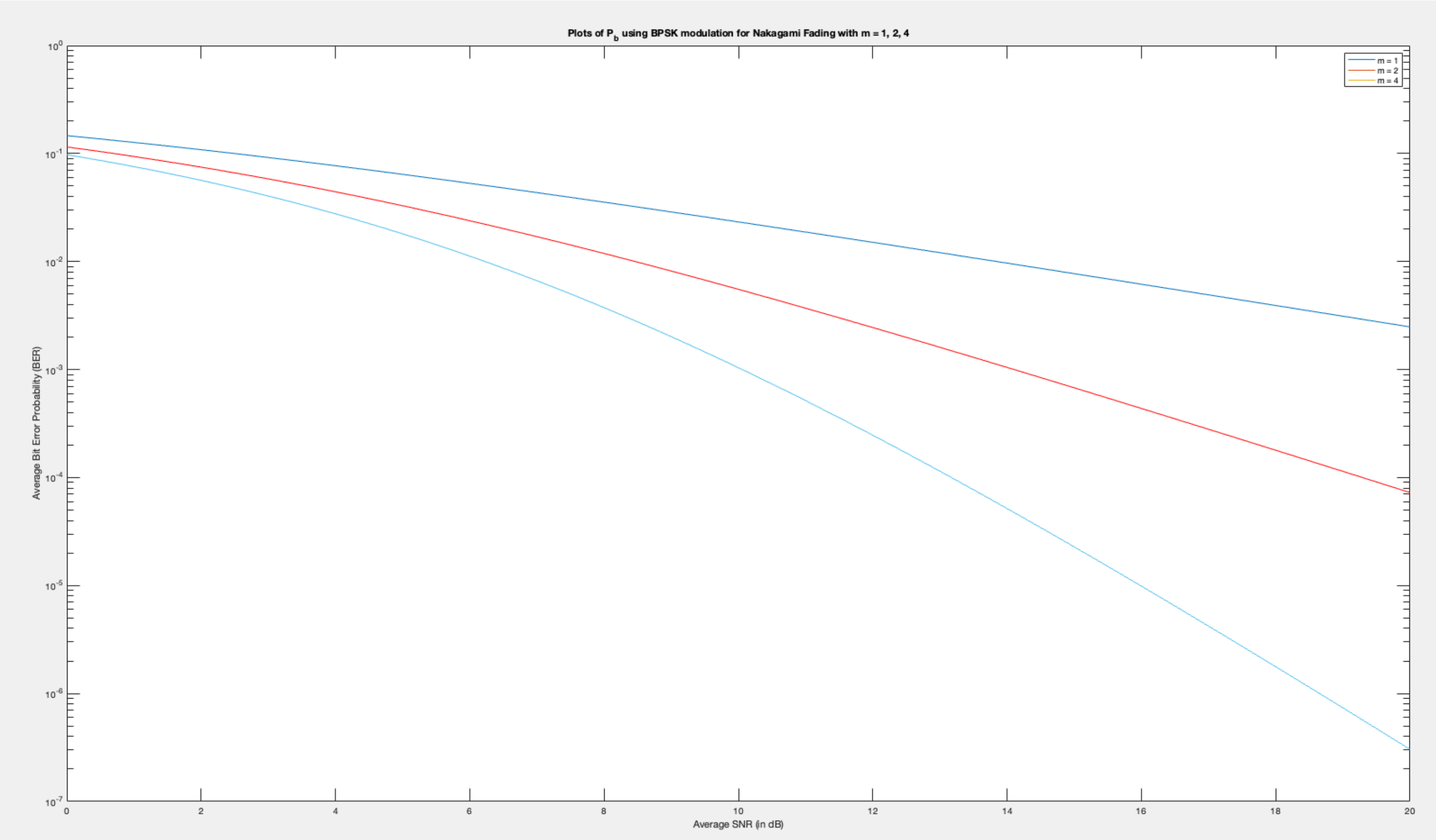
Q6-

```
clc;
clear all;
close all;

m = [1, 2, 4];
SNR = 0:0.01:20;
P_b_bar = zeros(length(SNR));
gamma_b = 10 .^ (SNR / 10);

f1 = figure(1);
line_style = ['-r', '-b', '-k'];

fprintf('At SNR = 10dB \n');
fprintf('m     BER\n');

for i = 1 : length(m)

   for j = 1 : length(gamma_b)

       syms phi
       integrand = (1 + (gamma_b(j) / (m(i) * (sin(phi) .^ 2)))) .^ (-m(i));
       integral = (1 / pi) * vpaintegral(integrand, phi, [0 (pi / 2)]);
       P_b_bar(j) = vpa(integral);

       if SNR(j) == 10

           fprintf('%d   %f\n', m(i), integral);

       end

   end

   semilogy(SNR, P_b_bar, line_style(i));
   hold on;

end

legend('m = 1', 'm = 2', 'm = 4');
title('Plots of P_b using BPSK modulation for Nakagami Fading with m = 1, 2, 4');
ylabel('Average Bit Error Probability (BER)');
xlabel('Average SNR (in dB)');
```

OUTPUT-

```
Command Window
At SNR = 10dB
m      BER
1   0.023269
2   0.005528
4   0.001039
fx >>
```

PLOT-



Plots of P_b using BPSK modulation for Nakagami Fading with m = 1, 2, 4
m = 1
m = 2
m = 4
Average Bit Error Probability (BER)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
2
4
6
8
10
12
14
16
18
20
Average SNR (in dB)