Week 12: Assignment 1

The due date for submitting this assignment has passed.

Due on 2022-10-19, 23:59 IST.

Assignment submitted on 2022-10-19, 23:06 IST

1) You have a sequence of n colored blocks. ***2 points***

The color of the i-th block is $c[i]$, an integer between 1 and n.

You will place the blocks down in sequence on an infinite coordinate grid in the following way. Initially, you place block 1 at (0,0). For $2 \le i \le n$, if the $(i-1)$-th block is placed at position (x,y), then the i-th block can be placed at one of positions $(x+1,y)$, $(x-1,y)$, $(x,y+1)$ (but **not** at position $(x,y-1)$), as long no previous block was placed at that position.

A tower is formed by s blocks such that they are placed at positions $(x,y),(x,y+1),\ldots,(x,y+s-1)$ for some position (x,y) and integer s. The size of the tower is s, the number of blocks in it.

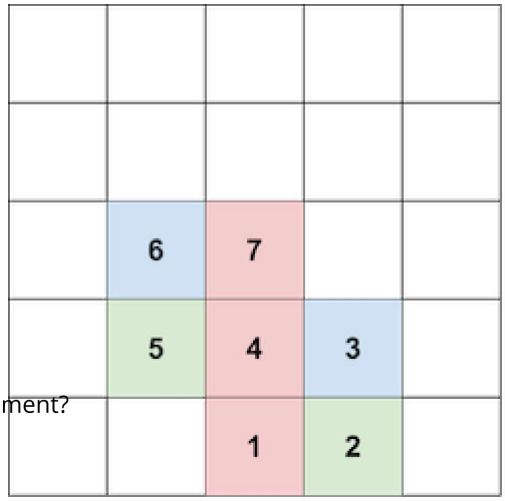
A tower of color r is a tower such that all blocks in it have the color r.

For each color r from 1 to n, solve the following problem *independently*: Find the maximum size of a tower of color r that you can form by placing down the blocks according to the rules.

For example, suppose we have the following sequence of blocks with n = 3.

1 2 3 1 2 3 1

Then notice that we can form a tower of height 3 and color 1 in the following way. The numbers in the image indicate the sequence (e.g, the tile colored 5 refers to the 5-th block being placed).



Suppose the maximum height tower we can form out of colors 2 and 3 are A and B respectively. What are the values of A and B? Remember you can use a different configuration!

- A = 1, B = 1
- A = 2, B = 2
- A = 1, B = 2
- A = 2, B = 1

Yes, the answer is correct.
Score: 2

Accepted Answers:
A = 2, B = 2

2) Suppose the sequence of blocks in the following:

3 3 3 1 3 3 2

What is the height of the largest tower that you can build of color 3?

4

Yes, the answer is correct.
Score: 2

Accepted Answers:
(Type: Numeric) 4

2 points

3) If we have two blocks of the same color at indices i and j such that $i<j$, how can we tell if it is possible to place them at $(x[i],y[i])$ and $(x[i],y[i+1])$ respectively? ***2 points***

i and j should have the same parity

i and j should have different parity

The answer depends on things other than i and j (for example, the value of n and the nature of the blocks between i and j)

Yes, the answer is correct.
Score: 2

Accepted Answers:
i and j should have different parity

4) Fix a color c. Let DP[i] denote the maximum size of a tower with color c, such that the last block of this tower was placed was at index i. What is the recurrence? ***2 points***

dp[i] = 1 + **sum** (dp[j]), where the sum is over all j < i such that j and i have different parities and the j-th block is the same color as i.

dp[i] = 1 + **sum** (dp[j]), where the sum is over all j < i such that j and i have the same parity and the j-th block is the same color as i.

dp[i] = 1 + **max** (dp[j]), where the sum is over all j < i such that j and i have different parities and the j-th block is the same color as i.

dp[i] = 1 + **max** (dp[j]), where the sum is over all j < i such that j and i have the same parity and the j-th block is the same color as i.

Yes, the answer is correct.
Score: 2

Accepted Answers:
*dp[i] = 1 + **max** (dp[j]), where the sum is over all j < i such that j and i have different parities and the j-th block is the same color as i.*

5) To optimize the recurrence proposed before, let us consider the following DP that computes the answers for all colors independently and simultaneously. ***2 points***

We will iterate through all blocks a[i], maintaining dp[p][c], where p is either 0 or 1, with the following semantics:

dp[0][c] = the maximum size of a tower with color c, where the last block included in the tower had an even index

dp[1][c] = the maximum size of a tower with color c, where the last block included in the tower had an odd index

If the current index is even, and the block at the current index has color c, then we set

dp[0][c] = max(dp[0][c], dp[1][c]+1)

If the current index is odd, and the block at the current index has color c, then we set

dp[1][c] = max(dp[1][c], dp[0][c]+1)

dp[1][c] = max(dp[1][c], dp[1][c]+1)

dp[1][c] = max(dp[0][c], dp[0][c]+1)

dp[1][c] = max(dp[0][c], dp[1][c]+1)

Yes, the answer is correct.
Score: 2

Accepted Answers:

dp[1][c] = max(dp[1][c], dp[0][c]+1)