Tutorial:1 (Electrodynamics, PYL-205)

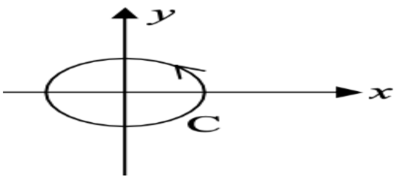
Problem 1: Find the value of integral $\int_{-\pi/2}^{\pi/2} dx \int_{-1}^{1} dy\ \delta(\sin 2x)\delta(x-y)$.

Problem 2: Find the value of integral $\int_{-\infty}^{\infty} dx\ e^{-\frac{|x|}{\pi}} \delta(\sin x)$,where δ(x) is the Dirac delta function.

Problem 3: Calculate the value of a and b for which the force $\vec{F}$=$(axy+z^3)\hat{\imath}+x^2\hat{\jmath}+bxz^2\hat{k}$ is conservative.

Problem 4: Find the value of integral $\oint \vec{A}.\overrightarrow{dl}$, where $\vec{A}$= $\hat{\imath}\,yz+\hat{\jmath}xz+\hat{k}xy$ along the perimeter of a rectangular are bounded by x=0, x=a and y=0, y=b.

Problem 5: Given $\vec{F}=\vec{r}\times\vec{B}$ where $\vec{B}=B_0(\hat{\imath}+\hat{\jmath}+\hat{k})$ is constant vector and $\vec{r}$ is the position vector. Find the value of $\oint \vec{F}.\overrightarrow{dr}$, along the circle of unit radius at origin.



Problem 6: Evaluate the following integrals

(a) $\int_1^5 (4x^3-5x-3)\ \delta(x-3)dx$
(b) $\int_{-3}^{3} (8x+3)\ \delta(4x)dx$
(c) $\int_0^3 t^2\ \delta(3t-6)dt$

Problem 7: Evaluate the following integrals

(a) $\int_0^5 tanx\ \delta(x-\pi)dx$
(b) $\int_{-\infty}^{\infty} \ln(x+3)\ \delta(x+2)dx$
(c) $\int_{-\infty}^{\infty} (\mathrm{x}^2+1)\ \delta(x^2-3x+2\ )dx$
(d) $\int_0^{\infty} dx\ e^{-x}\delta(sinx)$

Problem 8: Find the value of the integral

(a) $\int_{-\infty}^{+\infty} dx\ \delta(x^2-\pi^2)\ sinx$
(b) $\int_{-\infty}^{+\infty} dx\ \delta(x^2-\pi^2)\ cosx$
(c) $\pi\int_{-\infty}^{+\infty} \exp(-|x|)\ \delta(\sin(\pi x))\mathrm{dx}$

Problem 9: If $f(x)=\alpha\delta(x)+\beta\delta'(x)+\gamma\delta''(x)+\mu\delta'''(x)$, where $\delta(x)$ is Dirac-delta function and prime represent derivative. Find the value of integral $\int_{-\infty}^{+\infty} f\,(x)e^{ikx}dx$.

Problem 10: Calculate the divergence of the following vector functions

(a) $\boldsymbol{V_a}=x^4\hat{x}+3yz^3\hat{y}-2x^2z\hat{z}$
(b) $\boldsymbol{V_b}=xy\hat{x}+2yz\hat{y}+5zx\hat{z}$

Problem 11: Evaluate the following integral

(a) $\int_v [r^4 + r^2(\mathbf{r}.\mathbf{c}) + c^4]\, \delta^3(\boldsymbol{r} - \boldsymbol{c}) d\tau$, where v is a sphere of radius 6 about the origin, $\mathbf{c} = 5\hat{x} + 3\hat{y} + 2\hat{z}$, and c is its magnitude.

Problem 12: A fluid motion is given by

$$\mathbf{v} = (ysinz - sinx)\hat{\mathrm{x}} + (xsinz + 2yz)\hat{\mathrm{y}} + (xycosz + y^2)\hat{\mathrm{z}}$$

Check this motion is rotational or irrotational. If irrotational, find the velocity potential.

Problem 13: Derive Coulomb's law of electrostatics with the help of Maxwell's first equation.

Problem 14: Find the value of a, if the vector $\boldsymbol{A} = x\hat{\imath} + ay\hat{\jmath} + 3z\hat{k}$ is solenoidal.

Problem 15: Calculate the flux of a vector $x^3\hat{\imath} + y^3\hat{\jmath} + z^3\hat{k}$ over the surface of a sphere of radius *R* with its center at the origin also verify the result by divergence theorem.

Problem 16: Evaluate $x^2 \sin 5z\, \hat{\imath} + xe^y\hat{\jmath} + 4y\hat{k}$ using Stoke's theorem over a circle of radius 2 in yz-plane whose center is located at $(0,2,2)$.

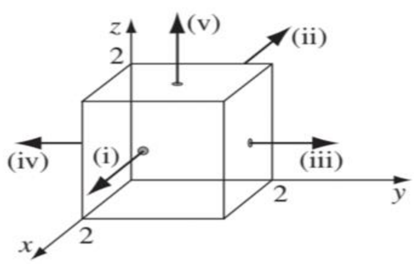
Problem 17: Find the charge density that gives rise to the Electric field in some region to be $\boldsymbol{E} = kr^3\hat{r}$, in spherical coordinates (where k is a constant).

Problem 18: Check the divergence theorem using the function $\boldsymbol{A} = y^2\hat{x} + (2xy + Z^2)\hat{y} + (2yz)\hat{z}$ and a unit cube at the origin.

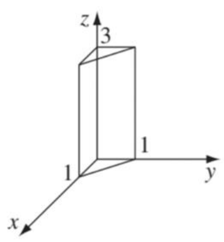
Problem 19: Prove that the divergence of a curl is always zero. Check it for the given functions

(a) $v_a = x^2\hat{x} + 3xz^2\hat{y} - 2xz\hat{z}$
(b) $v_b = xy\hat{x} + 2yz\hat{y} + 3zx\hat{z}$

Problem 20: Calculate the surface integral of $\boldsymbol{v} = 2xz\hat{x} + (x + 2)\hat{y} + y(z^2 - 3)\hat{z}$ over five sides (excluding the bottom) of the cubical box (side 2). (Let "upward and outward" be the positive direction, as indicated by the arrows).



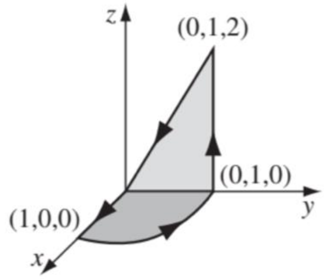
Problem 21: Calculate the volume integral of $T = xyz^2$ over the prism given below.



Problem22: Compute the line integral of

$$\boldsymbol{v} = (rcos^2\theta)\hat{r} - (rcos\theta sin\theta)\hat{\theta} + 3r\hat{\varphi}$$

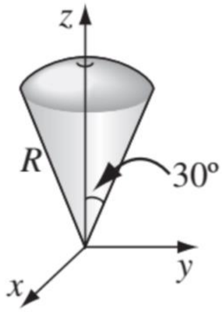
around the path shown in Fig. below (the points are labeled by their Cartesian coordinates). Do it either in cylindrical or in spherical coordinates. Check your answer, using Stokes' theorem.



Problem 23: Check the divergence theorem for the function

$$\boldsymbol{v} = r^2sin\theta\hat{r} + 4r^2cos\theta\hat{\theta} + r^2tan\theta\hat{\varphi}$$

using the volume of the "ice-cream cone" shown in Fig. below (the top surface is spherical, with radius R and centered at the origin).



Problem 24: Check Stokes' theorem for the function $\boldsymbol{v} = y\hat{z}$, using the triangular surface shown in Figure below.