Graphs

A graph is a data structure that consists of a finite set of vertices (or nodes) and a collection of edges connecting these vertices. The edges may or may not have a direction, and they may have weights or labels. Graphs are used to represent relationships and connections between different entities.

Components of a Graph:

1. **Vertices (Nodes):**
    - The fundamental entities in a graph.
    - Represented by points in a graph.
2. **Edges:**
    - Connections between vertices that represent relationships.
    - An edge can be directed (arrow indicating a one-way connection) or undirected (no direction).

Types of Graphs:

- **Directed Graph (DiGraph):**
    - Edges have a direction, indicating a one-way connection.
- **Undirected Graph:**

- Edges have no direction; connections are bidirectional.
- **Weighted Graph:**
  - Edges have weights or costs assigned to them.
- **Cyclic Graph:**
  - Contains at least one cycle (a path that starts and ends at the same vertex).
- **Acyclic Graph:**
  - Does not contain any cycles.
- **Connected Graph:**
  - There is a path between every pair of vertices.
- **Disconnected Graph:**
  - Contains at least two vertices with no path between them.

Graph Representation:

- **Adjacency Matrix:**
  - A 2D array where the entry `matrix[i][j]` represents whether there is an edge between vertices `i` and `j`.

Graph Operations:

- **Add Vertex and Edge:**
  - Adding new vertices and connecting them with edges.
- **Remove Vertex and Edge:**

    - Removing vertices and edges from the graph.
- **Traversal:**
    - Visiting all vertices and edges in the graph following a specific order.
    - Common traversal algorithms include Depth-First Search (DFS) and Breadth-First Search (BFS).

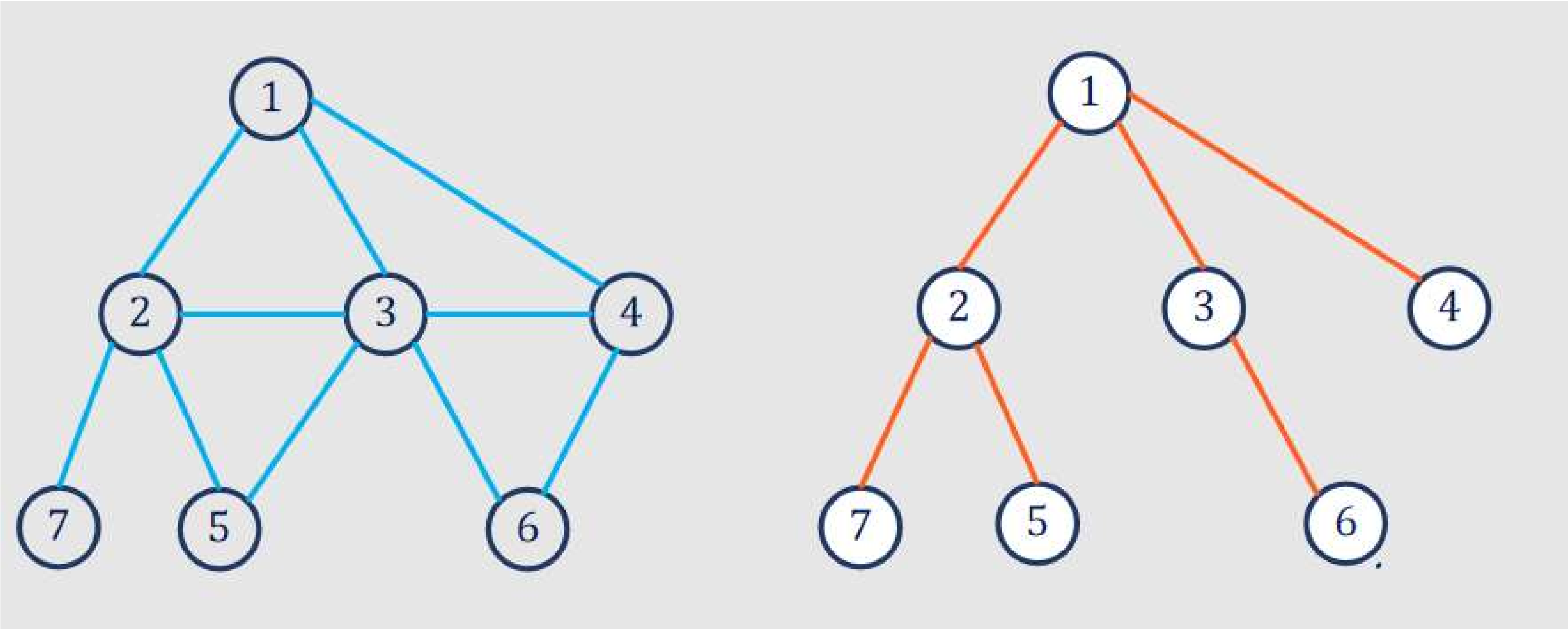
Applications of Graphs:

- **Networks:**
    - Modeling social networks, computer networks, transportation networks.
- **Routing Algorithms:**
    - Finding the shortest path between two vertices.
- **Dependency Analysis:**
    - Analyzing dependencies between different components.
- **Circuit Design:**
    - Representing connections in electronic circuits.
- **Recommendation Systems:**
    - Providing recommendations based on connections in user data.

Graph Traversal Techniques:

Breadth-First Search (BFS):

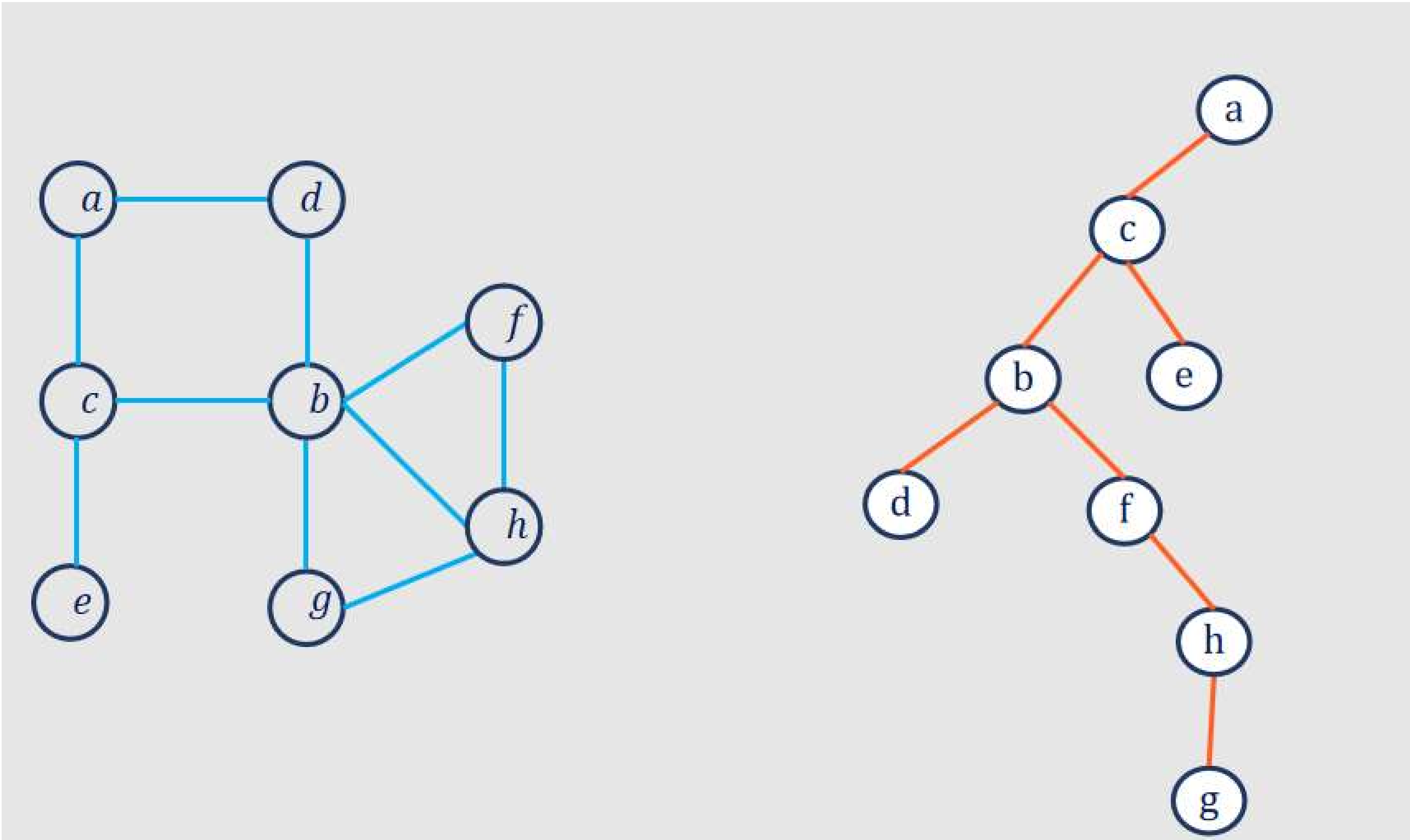
- **Description:**
  - BFS explores all the vertices at the current level before moving on to the next level.
  - Uses a queue to maintain the order of vertex exploration.
- **Algorithm:**
  1. Start at a source vertex and mark it as visited.
  2. Enqueue the source vertex.
  3. Dequeue a vertex and visit its unvisited neighbors.
  4. Enqueue unvisited neighbors.
  5. Repeat steps 3-4 until the queue is empty.



Depth-First Search (DFS):

- **Description:**
  - DFS explores as far as possible along each branch before backtracking.
  - Uses a stack (either explicitly or through recursion) to keep track of vertices.
- **Algorithm:**
  1. Start at a source vertex and mark it as visited.

2. Explore an unvisited neighbor of the current vertex.
3. If no unvisited neighbor, backtrack to the previous vertex.
4. Repeat steps 2-3 until all vertices are visited.



Minimum Spanning Tree (MST):

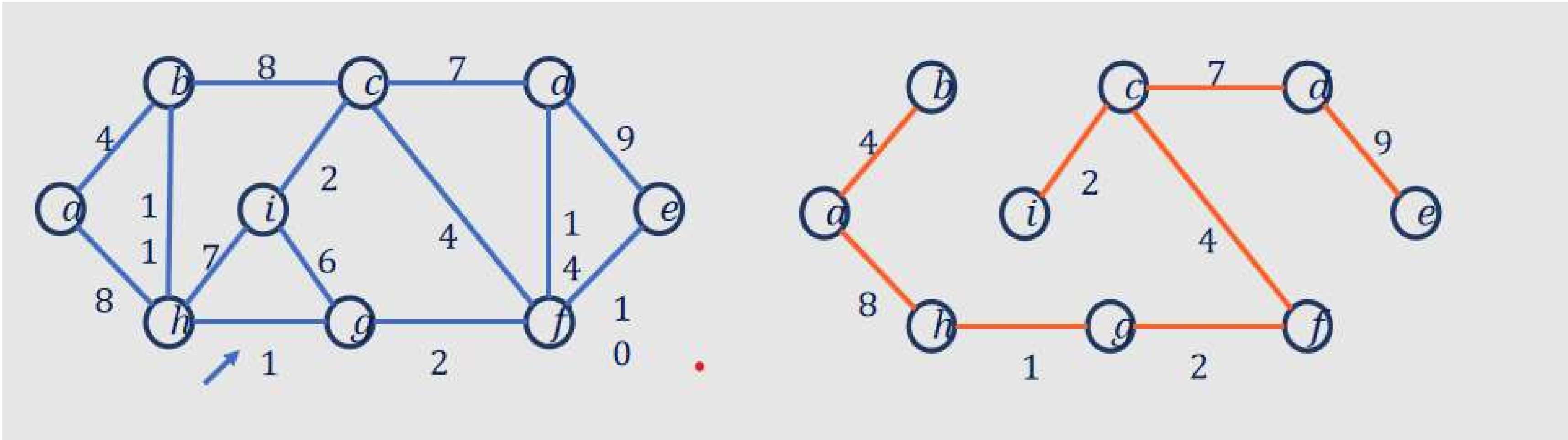
A Minimum Spanning Tree (MST) of a connected, undirected graph is a tree that spans all the vertices of the graph and has the minimum possible total edge weight. In other words, it is a subset of the edges of the graph that forms a tree and connects all the vertices with the minimum total edge weight.

Key properties of a minimum spanning tree:

1. **Connectivity:** It connects all the vertices in the original graph.
2. **Acyclic:** It forms a tree, meaning there are no cycles in the tree.
3. **Minimum Weight:** The sum of the edge weights in the tree is minimized.

1. Kruskal's Algorithm:

- **Algorithm Steps:**
    1. Initialize the MST as an empty set.
    2. Sort all the edges in non-decreasing order of their weights.
    3. Iterate through the sorted edges and add each edge to the MST if it does not form a cycle.
    4. Stop when the MST contains (V-1) edges, where V is the number of vertices.

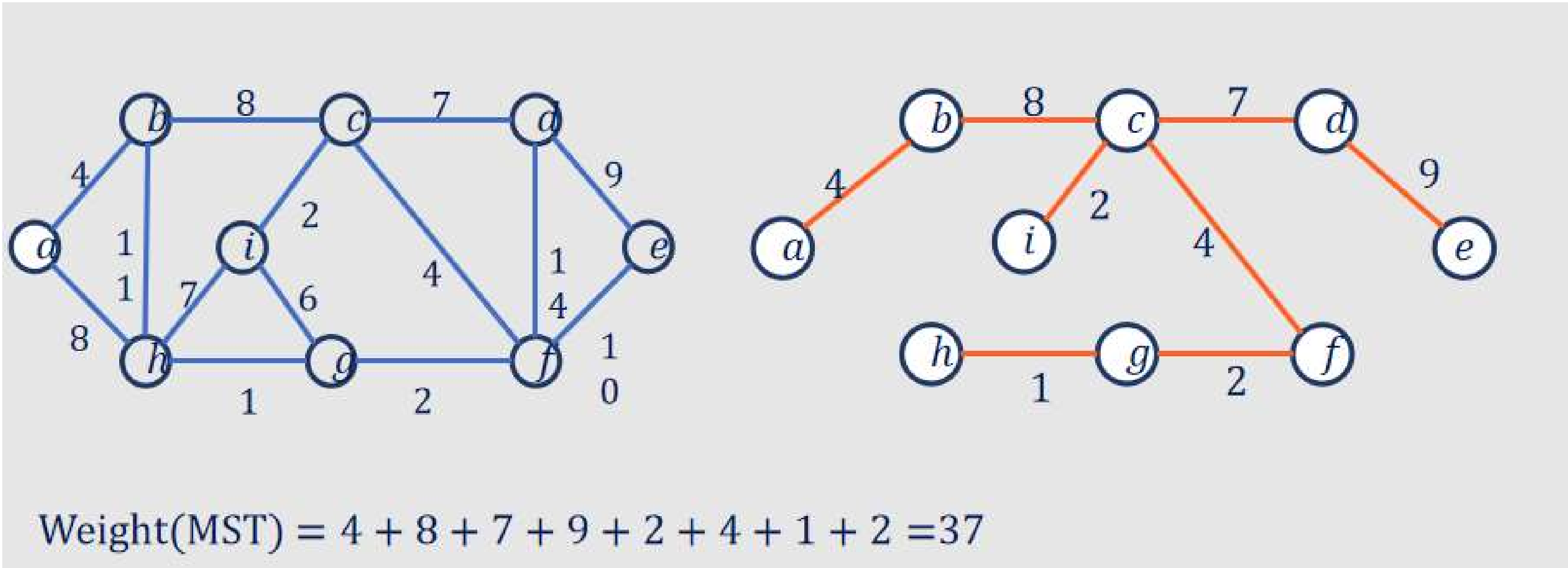


$$\text{Weight(MST)} = 1 + 2 + 2 + 4 + 4 + 7 + 8 + 9 = 37$$

2. Prim's Algorithm:

- **Algorithm Steps:**

  1. Start with an arbitrary vertex as the initial MST.
  2. At each step, add the minimum weight edge that connects a vertex in the MST to a vertex outside the MST.
  3. Repeat until all vertices are included in the MST.



Difference Between BFS and DFS:

Feature	Breadth-First Search (BFS)	Depth-First Search (DFS)
Exploration Order	Visits all neighbors at the current level before moving on to the next level.	Explores as far as possible along each branch before backtracking.
Data Structure Used	Uses a queue to maintain the order of vertex exploration.	Uses a stack (either explicitly or through recursion) to keep track of vertices.
Application	Often used for finding the shortest path in unweighted graphs.	Commonly used for detecting cycles, topological sorting, and solving maze problems.
Completeness	Guarantees the shortest path in unweighted graphs.	Does not guarantee the shortest path.

Order of Exploration	Explores vertices in the order they are enqueued.	Explores vertices in the order they are popped from the stack.
Example Use Case	Finding the shortest path between two nodes in an unweighted graph.	Detecting cycles in a graph or performing topological sorting.

Difference Between Kruskal's and Prism's:

Feature	Kruskal's Algorithm	Prim's Algorithm
Algorithm Type	Greedy algorithm that selects edges based on weight without forming cycles.	Greedy algorithm that grows the tree from an initial vertex.
Operation	Works by repeatedly adding the smallest edge that doesn't form a cycle.	Works by growing the tree from an initial vertex, adding the smallest edge to connect the tree with the rest of the graph.
Edge Selection	Chooses edges based on weight without concern for the source or destination vertices.	Chooses edges based on weight to connect the current tree with the closest non-tree vertex.
Deterministic Output	May have multiple valid solutions depending on the order edges are considered.	Always produces the same minimum spanning tree for a given starting vertex.
Use Cases	Suitable for sparse graphs and scenarios where edge weights are relatively uniform.	Suitable for dense graphs and scenarios where there is a clear central location or starting vertex.