INL/CON-23-75590-Revision-0

The Effect of Point Reactor Kinetics Solvers on the Uncertainties of Nuclear Reactor Simulations

November 2023

Thabit Abuqudaira, Pavel V. Tsvetkov, Piyush Sabharwall

Changing the World's Energy Future

Idaho National Laboratory

INL is a U.S. Department of Energy National Laboratory operated by Battelle Energy Alliance, LLC

INL/CON-23-75590-Revision-0

The Effect of Point Reactor Kinetics Solvers on the Uncertainties of Nuclear Reactor Simulations

Thabit Abuqudaira, Pavel V. Tsvetkov, Piyush Sabharwall

November 2023

Idaho National Laboratory
Idaho Falls, Idaho 83415

http://www.inl.gov

Prepared for the
U.S. Department of Energy
Under DOE Idaho Operations Office
Contract DE-AC07-05ID14517

The Effect of Point Reactor Kinetics Solvers on the Uncertainties of Nuclear Reactor Simulations

Thabit Abuqudaira,* Pavel V. Tsvetkov,* Piyush Sabharwall†

** Department of Nuclear Engineering, Texas A&M University, 423 Spence St, College Station, TX 77843, thabitne@tamu.edu, tsvetkov@tamu.edu*
† Nuclear Science & Technology, Idaho National Laboratory, 1955 N Fremont Ave, Idaho Falls, ID 83415, Piyush.Sabharwall@inl.gov

INTRODUCTION

In the next few decades, the nuclear industry is expected to expand and play a significant role in meeting global energy needs and addressing climate change concerns [1]. Currently, many advanced reactor technologies are being developed worldwide. Advanced nuclear reactors will incorporate enhanced safety features to minimize the possibility of accidents and mitigate their consequences. This trend of expansion requires extensive design and simulation tools development efforts.

Considerable interest is focused on the uncertainties of the simulation tools in nuclear reactor simulation. These uncertainties in the developed models significantly affect the reliability of the simulation results and, thus, the safety of nuclear reactors. Understanding and quantifying these uncertainties help ensure that the simulation results are reliable and can be used to assess the safety of reactor designs. Thus, analyzing the uncertainties of simulation tools will play a role in advanced nuclear reactors' licensing and demonstration efforts.

In reactor simulation, the neutronics behavior of the reactor can be modeled using deterministic or Monte Carlo codes. Deterministic codes are based on neutron balance equations. The point reactor kinetics equation is one of the simplest models describing a nuclear reactor's neutron balance. Simulation results obtained from the point reactor kinetics equation can provide insights into the dynamic behavior of the reactor and transients, such as the response to reactivity insertions.

A well-known challenge in reactor kinetics is the stiffness of the point reactor kinetics equations [2]. The stiffness of the model is mainly due to the difference in neutron lifetime magnitude between the prompt and the delayed neutrons. This behavior results in high uncertainties in solving the equations using numerical techniques and, thus, requires fine time steps in the numerical solvers. To overcome this issue, many algorithms and methods were developed [3]–[8].

Currently, various programming languages provide algorithms and solver packages to solve stiff equations. Such packages are essential when there is no analytical solution for the equation. Some of these packages were used to simulate the reactor dynamics using the point reactor kinetics model. Studying the uncertainties of using these packages in reactor simulation will help to design efficient and accurate simulation codes.

In the present study, the transient response of a reactor in the case of reactivity insertion with and without reactivity feedback was explored. The uncertainties resulting from using various numerical solvers to simulate the reactor response using the point reactor kinetics model were investigated.

METHODOLOGY

To investigate the capabilities of the available numerical solvers, the transient response of a benchmark was studied using the point reactor kinetics equations. The equations were solved numerically using several numerical solvers.

Point Reactor Kinetics

The space-independent point reactor kinetics equation with six groups of delayed neutron precursors can be written as follows:

$$\frac{dn(t)}{dt} = \frac{\rho-\beta}{\Lambda} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t) \quad (1)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t) \quad (2)$$

Where n(t) is the neutron density as a function of time, ρ is the net reactivity, β is the total delayed neutron fraction, Λ is the mean neutron generation time, λ_i is the decay constant for the i^{th} group of delayed neutrons, β_i is the delayed neutron fraction of the i^{th} group, and C_i is the delayed neutron precursors concentration for the i^{th} group as a function of time.

Equations 1 and 2 are Ordinary Differential Equations (ODEs). Solving the equations, the time-dependent neutron density and precursors concentration can be found. Thus, the reactor response can be assessed due to various transients.

Numerical Solvers

Developing numerical methods to solve stiff ODEs has been a broad research area. The advancement in computer software for solving ODEs allowed several codes to be used for solving the point reactor kinetics. Several packages of

numerical solvers are available in different programming languages. These include Visual Basic for Applications (VBA), MATLAB, and Python.

The most common type of numerical solvers is the ODE solver package in MATLAB. MATLAB ODE solvers are variable step-size solvers. Thus, the time step can not be optimized. However, error control can be performed using the relative error tolerance and absolute error tolerance commands, RelTol and AbsTol, respectively.

VBA is a programming language that is integrated into Microsoft Excel. VBA allows users to automate tasks, create custom functions, and extend the functionality of Excel beyond its built-in features. In addition, because of the user interface capabilities that VBA can provide along with Excel, it has been widely used for reactor simulators.

In Python, the odeint function is used to solve systems of ODEs numerically. It is part of the open-source SciPy library.

Benchmark

Benchmarks are used to test and compare new methods, tools, and nuclear data libraries. A thermal-spectrum nuclear reactor was chosen as the benchmark to assess the uncertainties using different numerical solvers for the point reactor kinetics model. The benchmark was widely used to evaluate and compare the results of new analytical and numerical methods to solve the point reactor kinetics equations [9]. The reactor has a mean neutron generation time of 5×10^{-4} (s). The remaining kinetics parameters of the reactor are listed in Table I.

TABLE I. Kinetics parameters of the thermal reactor.

Group i	β_i	λ_i
1	0.0002850	0.0127
2	0.0015975	0.0317
3	0.0014100	0.1150
4	0.0030525	0.3110
5	0.0009600	1.4000
6	0.0001950	3.8700

The reactor is assumed to be at a steady state with an initial value of neutron density equal to 1. The steady-state values of the delayed neutron precursors concentrations can be calculated from Equation 2.

RESULTS AND DISCUSSION

The neutron density was simulated in the reactor following step reactivity insertions using the previously mentioned solvers. Positive and negative values of step reactivity insertion were simulated.

First, a simple solution using Euler's method was developed in VBA. The simulation results following a step reactivity insertion of +β were compared with the exact solution of the point reactor kinetics equation with no reactivity feedback [10]. The effect of the step size adopted in Euler's method on the uncertainty of the simulated neutron density over time is shown in Fig. 1.

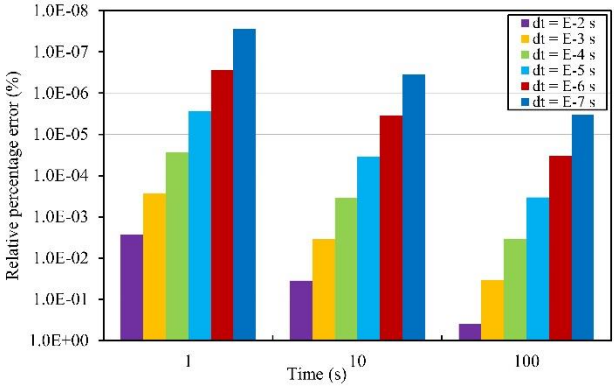

Fig. 1. The effect of step size in Euler's method on the uncertainty of the simulated neutron density.

It can be observed that using smaller time steps results in more accurate simulations. In addition, the truncation error seems to accumulate over time. Overall, results showed that even a basic first-order numerical solver could produce satisfactory results with the proper choice of the time step size, overcoming the stiffness issue of the point reactor kinetics equations.

The neutron densities in the reactor following a step reactivity insertion of +β using several MATLAB solvers are listed in Table II. Emboldened digits represent the digits that are unidentical to the exact solution.

TABLE II. Neutron density after +β step reactivity insertion using several MATLAB solvers.

Solver	Neutron density (cm^{-3})	
	t = 1 (s)	t = 100 (s)
Exact	3.21835409455342E+01	2.59648464655087E+89
ode45	3.21835409455343E+01	2.59648464655216E+89
ode23	3.21835409455332E+01	2.59648464654076E+89
ode113	3.21835409455341E+01	2.59648464657050E+89
ode78	3.21835409455344E+01	2.59648464655307E+89
ode89	3.21835409455342E+01	2.59648464655104E+89
ode15s	3.21835409455573E+01	2.59648464755296E+89
ode23s	3.21835409540523E+01	2.59648473838132E+89
ode23t	3.21835409529999E+01	2.59648472676726E+89
ode23tb	3.21835409508072E+01	2.59648466787295E+89

All MATLAB solvers could correctly predict the neutron density following a step reactivity insertion. However, few solvers were able to predict the density with 11 or more correct decimal places. ode89 solver was shown to be the highest accuracy solver.

CONCLUSION

The integration of advanced numerical solvers within programming languages has dramatically simplified the solution of the point reactor kinetics equation. As a consequence, obtaining precise and efficient results has become straightforward. Identifying the numerical solvers that produce high-accuracy results in solving the point reactor kinetics equations will help advance reactor dynamics simulations' capabilities.

Extensive comparisons conducted on diverse numerical solvers in different programming languages have consistently demonstrated the high accuracy achievable with some solvers. Thus, such numerical solvers can be used for reactor simulations without building new algorithms or complicated codes. Results showed that MATLAB and Python have many installed ODE solvers packages that can be used to solve the point reactor kinetics equations.

ACKNOWLEDGMENTS

This paper is based upon work supported by the project funded by the Idaho National Laboratory.

REFERENCES

[1] S. Sadekin, S. Zaman, M. Mahfuz, and R. Sarkar, "Nuclear Power as Foundation of a Clean Energy Future: A Review," in *Energy Procedia*, **160**, pp. 513–518 (2019).

[2] Y. A. Chao and A. Attard, "Resolution of the Stiffness Problem of Reactor Kinetics," *Nuclear Science and Engineering*, **90**, pp. 40–46 (1985).

[3] A. E. Aboanber, "Analytical Solution of the Point Kinetics Equations by Exponential Mode Analysis," *Progress in Nuclear Energy*, **42** No. 2, pp. 179-197 (2003).

[4] M. Kinard and E. J. Allen, "Efficient Numerical Solution of the Point Kinetics Equations in Nuclear Reactor Dynamics," *Ann Nucl Energy*, **31** No. 9, pp. 1039–1051 (2004).

[5] A. A. Nahla, "Analytical Solution to Solve the Point Reactor Kinetics Equations," *Nuclear Engineering and Design*, **240** No. 6, pp. 1622–1629 (2010).

[6] A. A. Nahla, "An Efficient Technique for the Point Reactor Kinetics Equations with Newtonian Temperature Feedback Effects," *Ann Nucl Energy*, **38** no. 12, pp. 2810–2817 (2011).

[7] C. Z. Petersen, S. Dulla, M. T. M. B. Vilhena, and P. Ravetto, "An Analytical Solution of the Point Kinetics Equations with Time-Variable Reactivity by the Decomposition Method," *Progress in Nuclear Energy*, **53**, pp. 1091–1094 (2011).

[8] D. Suescún-Díaz and G. Espinosa-Paredes, "On the Numerical Solution of the Point Reactor Kinetics Equations," *Nuclear Engineering and Technology*, **52** no. 6, pp. 1340–1346 (2020).

[9] B. Ganapol, P. Picca, A. Previti, and D. Mostacci, "The Solution of the Point Kinetics Equations via Converged Accelerated Taylor Series (CATS)," *PHYSOR 2012 – Advances in Reactor Physics*, Knoxville, Tennessee (2012).

[10] B. D. Ganapol, "A Highly Accurate Algorithm for the Solution of the Point Kinetics Equations," *Ann Nucl Energy*, **62**, pp. 564–571 (2013).