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Connected and automated vehicles: A cooperative eco-driving strategy for heterogeneous vehicle platoon among multiple signalized intersections

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Abstract: With the development of vehicle-to-everything (V2X) communication and connected and automated vehicles (CAVs), multi-vehicle cooperative eco-driving shows great energy efficiency and transportation sustainability. As a multi-objective optimal control problem (OCP), how to coordinate the ecological velocity, motion planning and optimal energy management is a challenge. To address this issue, a cooperative eco-driving and energy management control strategy is developed for heterogeneous vehicle platoon among multiple signalized intersections. Firstly, a global eco-speed feasible area is derived by comprehensively considering signal phases and optimal terminal velocity that maximizes the passing number of following vehicles and collective fuel economy. Then, a soft actor-critic (SAC)-based cooperative eco-driving and energy management control strategy is proposed to keep a safe inter-vehicle distance and ecological velocity with minimum energy consumption and acceptable driving comfort.

Keywords: multi-objective cooperative optimization, heterogeneous vehicle platoon, reinforcement learning, energy management, multiple signalized intersections

1. INTRODUCTION

Thanks to vehicle-to-everything (V2X) communication and connected and automated vehicles (CAVs), multi-vehicle cooperative eco-driving has gradually been highlighted, which shows great energy-saving potential and transportation sustainability (Li et al., 2024). However, cooperative eco-driving faces three main challenges in practical applications.

The first challenge is the multi-agent multi-objective cooperative optimization. To a certain extent, the cooperative eco-driving in multi-vehicle scenarios is a multi-objective optimal control problem (OCP), with the goal of ecological velocity, optimal energy management, ensuring driving safety, acceptable driving comfort and maximizing traffic throughput. Currently, majority studies are proposed to fully coordinate the potential conflict between energy efficiency and driving safety or ecological velocity. Sun et al., proposed an adaptive co-optimization method of speed planning and energy management strategy, which can reduce fuel consumption by 15.8% while ensuring safety and comfort (Sun et al., 2022). For heterogeneous fleets, the coordination mechanism should be investigated to realize multi-objective optimization.

The second challenge is the trade-off between optimality and applicability. Due to the powertrain complexity of heterogeneous vehicles, the computational burden of energy consumption estimation increases notably in multi-vehicle scenario. Meanwhile, most existing studies mainly focuses on ecological speed planning and optimal energy management for leading vehicles, while the remaining vehicles are controlled through an adaptive cruise control (ACC) system, which overlooks collective energy and traffic efficiency. To achieve cooperative control for the powertrain and car-following behaviors, Wang et al., proposed an optimal energy-saving strategy based on multi-agent reinforcement learning (MARL) (Wang et al., 2023). For mixed platoon, Zhang et al., proposed a nested parallel optimization method to realize transportation efficiency-oriented velocity and energy management coordinated control (Zhang et al., 2024). Thence, how to balance the energy optimization efficiency between multi-vehicle and individual vehicle as well as platoon level real-time optimization is crucial for cooperative eco-driving.

The third challenge is the adaptability of cooperative control. The shared look-ahead traffic and road state determine the speed limits or power demand over the whole journey, such as traffic lights, road curvature and slope, which can be transferred to spatiotemporal constraints in multi-objective OCPs of eco-driving. Ma et al., proposed an ecological cooperative adaptive cruise control (Eco-CACC) method for the connected vehicles platoon driving through signalized intersections (Ma et al., 2021). In the dynamic traffic environment, the speed planning of fleets is unavoidably affected by the behaviors of surrounding vehicles and traffic flow. Considering random disturbance in traffic scenarios, Zhou et al., developed a chance constraint stochastic model predictive control (CC-MPC) method for simultaneously optimizing the speed planning and the powertrain energy management strategy (Zhou et al., 2022). How to make full use of prior information and traffic environment interaction to achieve adaptive eco-driving is a challenge.

To address the above issues, a cooperative control strategy is proposed for heterogeneous vehicle platoon among multiple signalized intersections. The major contributions are two-fold:

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(1) By maximizing the number of following vehicles passing through intersections and minimizing the collective energy consumption, the optimal terminal velocity and arrival time among multiple signalized intersections are derived, forming a global eco-speed feasible area for co-optimization; (2) A cooperative eco-driving and energy management control strategy is proposed for heterogeneous fleet based on soft actor-critic (SAC) algorithm, which generates stochastic policies based on global average constraints to accelerate learning and prevent a bad local optimum.

2. HETEROGENEOUS PLATOON MODELING

This paper considers a scenario where a platoon consisting of '$N+1$' CAVs passing through a road with multiple signal-controlled intersections. The heterogeneous vehicle platoon is taken as the research object, including plug-in hybrid electric vehicles (PHEVs), electric vehicles (EVs) and internal combustion engine vehicles (ICEVs). Driving on the curved road, the longitudinal control and lateral control of the CAVs are considered. Vehicle emergency situation is not taken into account, thus, the longitudinal and lateral motion are controlled independently.

2.1 Dynamical modeling

The car-following model is adopted to describe the vehicle dynamics of CAV platoon. The leading vehicle is indexed $i = 0$, and the following vehicles are indexed from 1 to N. At the moment t, the position and velocity of vehicle i are defined as $x_i(t)$ and $v_i(t)$, respectively. The headway distance of vehicle i from vehicle $i-1$ is defined as $d_i(t) = x_{i-1}(t) - x_i(t)$, and the relative velocity $\Delta v_i(t) = \dot{d}_i(t) = v_{i-1}(t) - v_i(t)$.

The FVDM model (Jiang et al, 2001) is regarded as the specific car-following model, and the CAV's car-following dynamics can be expressed as (Chen C et al, 2021):

$$\dot{v}_i = F(d_i, \dot{d}_i, v_i) = \kappa[V(d_i) - v_i] \tag{1}$$

Where κ is a sensitivity constant, $V(\cdot)$ is the optimal velocity function, given by $V(d_i) = V_1 + V_2 \tanh[C_1(d_i - L_c) - C_2]$, with L_c denoting the vehicle length and the rest of the symbols are constants. The parameter values are $\kappa = 0.41\, s^{-1}$, $V_1 = 6.75$ m/s, $V_2 = 7.91$ m/s, $C_1 = 0.13$, $C_2 = 1.57$, $L_c = 4.8$ m.

The heterogeneous platoon passes each intersection at a pre-specified equilibrium velocity v^* with a corresponding headway distance d^*. The deviation of the current state ($d_i(t)$, $v_i(t)$) of vehicle i from the equilibrium state (d^*, v^*) are taken as state variables, that is,

$$\begin{cases} \tilde{d}_i(t) = d_i(t) - d^* \\ \tilde{v}_i(t) = d_i(t) - d^* \end{cases} \tag{2}$$

Applying the first-order Taylor expansion to (1), the linearized dynamics of CAVs around the equilibrium state is given by:

$$\begin{cases} \dot{\tilde{d}}_i(t) = v_{i-1}(t) - v_i(t) \\ \dot{\tilde{v}}_i(t) = \alpha_1 \tilde{d}_i(t) - \alpha_2 \tilde{v}_i(t) + \alpha_3 \tilde{v}_{i-1}(t) \end{cases} \tag{3}$$

Where $\alpha_1 = \frac{\partial F}{\partial d}$, $\alpha_2 = \frac{\partial F}{\partial \dot{d}} - \frac{\partial F}{\partial v}$, $\alpha_3 = \frac{\partial F}{\partial \dot{d}}$.

2.2 Powertrain modelling

The combination of different powertrain components and placement positions forms different powertrain systems. The details of powertrain modeling are as follows:

The engine model is simplified as a static map to calculate the fuel consumption, which depends on engine speed n_e and engine torque T_e, that is, $\dot{m}_f = f(T_e, n_e)$.

Similarly, the motor efficiency η_m is modeled by a 3-D MAP, which is expressed as a nonlinear function of motor speed n_m and motor torque T_m, i.e., $\eta_m = g(T_m, n_m)$.

Without the consideration of temperature change and battery aging, a simple internal resistance battery model is used to calculate the battery power P_b. The battery state-of-charge (SOC) can be calculated by (Kim et al., 2010):

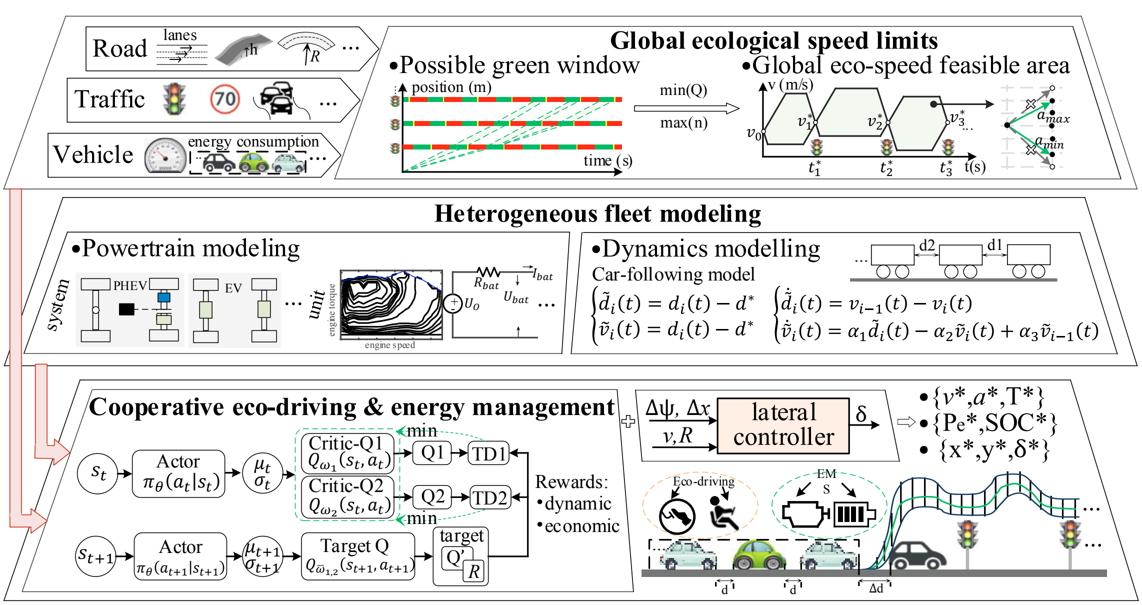


Figure 1. Overall framework of cooperative eco-driving and energy management control for heterogeneous fleet.

$$S\dot{O}C = \frac{U_b - \sqrt{U_b^2 - 4R_b \cdot P_b}}{2R_b \cdot C_b} \tag{4}$$

Where U_b, R_b, C_b are the open-circuit voltage, the internal resistance, and the capacity of battery, respectively.

A connected PHEV equipped with P2 configuration () is taken as the leading vehicle. A motor-generator (MG) is placed between the engine and a four-shift transmission (i_g = [3.45, 1.98, 1, 0.75]), which can work as electric motor or generator alternately. The main parameters are concluded in Table 1.

Table 1. Vehicle Parameters

Type	Description
Engine	Power: 63 KW @124 N m
Motor	Motor: 60 KW @ 458 N m
Battery	U_b = 328 V, capacity=10.57 kW h
Vehicle	m = 1500 kg, r_{wh} = 0.334 m, A = 2.25 m^2, C_D = 0.28, i_0 = 3.63

2.3 Collective energy consumption estimation

Considering the vehicle longitudinal dynamics, the vehicle power demand P_v can be expressed as:

$$P_v = \frac{v}{3600\eta_t}\left(mgf\cos\varphi + \frac{C_D A v^2}{21.15} + mg\sin\varphi + \zeta ma\right) \tag{5}$$

Where g is gravity acceleration, φ is the slope, v is velocity, ζ is the rotation mass conversion factor, a is acceleration.

For heterogeneous fleets, the collective energy consumption Q includes fuel consumption Q_F and electricity consumption Q_E, which is expressed as:

$$Q = \sum_{i=1}^{N+1}\left(Q_F^i(T_e^i, n_e^i, \xi^i) + \lambda Q_E^i(T_m^i, n_m^i, \xi^i)\right) \tag{6}$$

Where λ is the equivalent fuel consumption coefficient, ξ is the power distribution ratio between the engine and the battery, which is defined as the ratio of engine power P_e to vehicle power demand P_v. The fuel consumption can be calculated by:

$$Q_F = \frac{P_e \dot{m}_f}{367.1\rho g} \tag{7}$$

Where ρ is fuel density (kg/L), ρg is selected to be 6.96~7.15 N/L for gasoline, $\dot{m}_f$ is fuel consumption rate (g/(Kw·h)).

Neglect the losses of the electric wire, inverter, and convertor, the battery power P_b can be calculated by:

$$P_b = P_m \eta_m^{-\mathrm{sign}(T_m)} \eta_b^{-\mathrm{sign}(T_m)} \tag{8}$$

Where $P_m = T_m n_m/9550$ is the motor power, η_b is the battery efficiency, sign(·) is the signum function. Thus, the electricity consumption is $Q_E = \int P_b dt$.

2.3 Problem formulation

The control objective of the '1+N' heterogeneous vehicle platoon is to maximize collective energy efficiency and traffic efficiency when crossing multiple signalized intersections, while ensuring driving safety and acceptable driving comfort.

This multi-objective OCP includes two primary levels, where the first level aims to plan global ecological velocity at signalized intersections as a spatiotemporal constraint for co-optimization, and the second level addresses the co-optimization of eco-driving and energy management.

Several constraints are considered to ensure the driving safety, driving comfort, and operation safety of power components (including the engine, the motor and the battery):

$$\begin{cases} v_{\min} \le v_i(t) \le v_{\max} \\ a_{\min} \le a_i(t) \le a_{\max} \\ d_{\mathrm{safe}} \le d_i(t) - L_c \le d_{\max} \\ SOC_{\min} \le SOC_i(t) \le SOC_{\max} \\ P_{\mathrm{b\_min}} \le P_b(t) \le P_{\mathrm{b\_max}} \\ T_{\mathrm{e\_min}} \le T_e(t) \le T_{\mathrm{e\_max}} \\ T_{\mathrm{m\_min}} \le T_m(t) \le T_{\mathrm{m\_max}} \\ u_g(t) \in [0,1,-1] \end{cases} \tag{9}$$

Where u_g is the gear shift command, d_{safe} is a safe distance, the subscripts max and min refer to the maximum and minimum values.

3. COOPERATIVE ECO-DRIVING CONTROL

A hierarchical structure is established to achieve cooperative eco-driving and energy management control among multiple signalized intersections, as depicted in Fig.1.

3.1 Global eco-speed limits at signalized intersections

We define a route with N_h signalized intersections. The road state, such as slope and curvature, and the traffic state, such as route distance, speed limits, the number of vehicles waiting at the intersection, can be accessed by communicating with Road Side Units or the Cloud. Thus, the set of the h-th traffic light collects its position P^h and SPaT information, defined as $\Omega_h = \{P^h, T_s^h, S^h, T_g^h, T_r^h, C^h, W^h\}$. Where T_s^h is the initial indication of the transition time, S^h is the initial indication of traffic signal; T_g^h and T_r^h are the intervals of the green and red signal, respectively; C^h is the accumulated cycle number of the traffic signal, and W^h is the waiting time of the vehicle queue. The yellow signal is simply treated as a red one for safety purposes. Let $S_h = 1$ denote the green signal and $S_h = 0$ denote the red signal.

For multiple signalized intersections, the traffic signal is usually operated with an independent signal-clock period (Dong et al., 2022). Assume that a full signal cycle starts with the beginning of the red light (including yellow) and ends with the green light. At any moment t, the accumulated cycle number of the traffic signal can be determined by:

$$C^h = \begin{cases} \text{floor}\left(\dfrac{t - T_0^h}{T_r^h + T_g^h}\right) + 1, & \dfrac{t - T_0^h}{T_r^h + T_g^h} = \text{integer} \\ \text{floor}\left(\dfrac{t - T_0^h}{T_r^h + T_g^h}\right) + 2, & \text{otherwise} \end{cases} \quad (10)$$

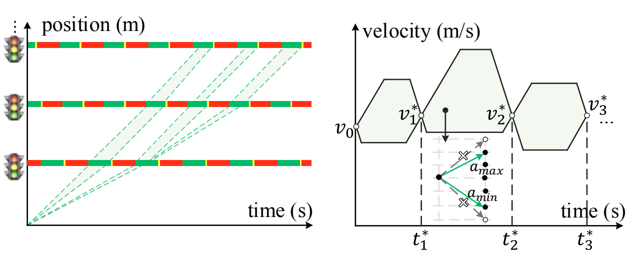
Where floor$(\cdot)$ is the least integer function, T_0^h is the length of initial signal cycle. When $S^h = 1$, $T_0^h = T_s^h$, otherwise, $T_0^h = T_s^h + T_g^h$.

Then, the starting times of the green and red signals for the C^h-th standard signal cycle ($C^h > 1$) are defined as t_{g,C^h}^h and t_{r,C^h}^h, respectively, formulated by:

$$t_{g,C^h}^h = \begin{cases} T_0^h + T_r^h(C^h - 1) + T_g^h(C^h - 1), S^h = 0 \\ T_0^h + T_r^h(C^h - 1) + T_g^h(C^h - 2), S^h = 1 \end{cases} \quad (11)$$

$$t_{r,C^h}^h = \begin{cases} T_0^h + T_r^h(C^h - 2) + T_g^h(C^h - 1), S^h = 0 \\ T_0^h + T_r^h(C^h - 2) + T_g^h(C^h - 2), S^h = 1 \end{cases} \quad (12)$$

At any time, the reachable state is subject to speed limits and acceleration/deceleration limits. Then, the possible green windows among multiple signalized intersections can be deduced, as shown in Fig.2 (a).



(a) (b)

Figure 2. Schematic diagram: (a) possible green windows; (b) global eco-speed feasible area.

Eco-driving at signalized intersections can avoid the stop-and-go operation to improve energy efficiency and traffic throughput. When a fleet drives through multiple signalized intersections, the optimization goals are to maximize the passing number n of the following vehicles during a constant green phase time and minimize the collective energy consumption. At the h-th signalized intersection, the target optimal velocity v_h^* can be obtained by solving (13).

$$\max_{h=1,\ldots N_h} n_h = \frac{v_h^* T_h^G}{d_h^*(v_h^*)} \quad (13)$$

Where the equilibrium state (d^*, v^*) satisfies $F(d^*, 0, v^*) = 0$, T_h^G is the green light phase time at the h-th light.

As a numerical problem, the green signal phase is meshed. Based on the optimal domain search algorithm (Xu et al, 2023), the terminal time arriving at each signalized intersection t_h^* and the average eco-speed between two consecutive intersections $\bar{v}_h^*$ are determined by minimizing the collective energy consumption. The above is taken as the terminal constraints at each signalized intersection, i.e., $x_0(t_h^*) = P^h$, $v_0(t_h^*) = v_h^*$, $h = 1, \ldots, N_h$.

As shown in Fig. 2 (b), a global eco-speed feasible area is obtained, which is imposed as the state space for subsequent co-optimization. Each reachable state is limited by the maximum acceleration and deceleration as well as the change rate of acceleration.

3.2 Co-optimization of eco-driving and energy management

Based on the derived global eco-speed limits, a cooperative optimization strategy for energy management and eco-driving is developed based on soft actor-critic (SAC) algorithm. Regarding lateral control, the desired yaw angle can be obtained by the reference trajectory, then, a PI (Proportional-Integral) controller can be utilized to reduce the lateral distance deviation and relative yaw angle to zero, which outputs the steering angle δ (front wheel).

(1) Soft actor-critic algorithm

Regarding the soft actor-critic algorithm, to accelerate learning and prevent a bad local optimum, the objective of agent is to maximize the expected discounted reward and the regularized entropy, that is,

$$J(\pi) = \sum_{t=0}^{T} \gamma^t \left(r(s_t, a_t, s_{t+1}) + \alpha H\big(\pi(\cdot\,|s_t)\big) \right) \quad (14)$$

Where $\alpha > 0$ is the trade-off coefficient, $H\big(\pi(\cdot\,|s_t)\big)$ is the entropy of the policy, which is computed from its distribution according to $H\big(\pi(\cdot\,|s_t)\big) = E[-\log \pi(\cdot\,|s_t)]$.

Similar to conventional reinforcement learning, the recursive Bellman equation for Q-value function $Q^\pi(s_t, a_t)$ is:

$$Q^\pi(s_t, a_t) = E_{s_{t+1}\sim P}[r(s_t, a_t, s_{t+1}) + \gamma V^\pi(s_{t+1})] \quad (15)$$

Where value function $V^\pi(s_{t+1}) = E_{a_{t+1}\sim\pi}\big[Q^\pi(s_{t+1}, a_{t+1}) + \alpha H\big(\pi(\cdot\,|s_{t+1})\big)\big]$.

SAC concurrently learns a policy and two Q-functions. The soft Q network and target Q network constitute of multilayer perceptron (MLP), which utilizes TD algorithm (the squared loss of the soft Bellman residual) to update the Q network parameters, that is,

$$J_Q(\theta) = E_{(s_t, a_t, s_{t+1})\sim\mathcal{D}} \Big[\frac{1}{2} \Big(Q_\theta(s_t, a_t) - \big(r(s_t, a_t) + \gamma V_{\bar{\theta}}(s_{t+1}) \big) \Big)^2 \Big] \quad (16)$$

Where the parameters of the target Q network $\bar{\theta}$ is obtained by exponentially moving average of the soft Q-function weights.

The policy improvement is realized by minimizing Kullback-Leibler (KL) divergence, that is,

$$\pi' = \text{argmin}_{\pi_k \in \prod} D_{KL} \left(\pi_k(\cdot\,|s_t) \| \frac{\exp(Q^\pi(s_t, \cdot)/\alpha)}{Z^\pi(s_t)} \right) \quad (17)$$

(2) Reward function

The goal of multi-vehicle cooperative eco-driving is to keep a safe inter-vehicle distance and ecological velocity with minimum energy consumption and acceptable driving comfort.

The tracking performance can be evaluated by keeping the desired relative distance and relative velocity for driving safety. Thence, the state (x_0, v_0) of the leading connected PHEV and the relative distance d and relative velocity Δv of the following vehicles are regarded as state variables, and the power split ratio ξ_0 and acceleration a_0 of the leading vehicle are taken as the control variables.

Considering dynamic indexes and economic indexes, the total cost function are expressed as:

$$J = \int_0^{t_f} \left\{ \sum_{i=1}^{N+1} \left[w_1 \Delta v_{i\_t} + w_2 d_{i\_t} + w_3 |\dot{a}_t| \right] + \sum_{i=0}^{N+1} \left[Q\left(T_t^i, n_t^i\right) + \beta \left(SOC_t^i - SOC_r^i\right)^2 \right] \right\} \tag{18}$$

Where $w_i (i = 1,2,3)$ is the weighting factors, $|\dot{a}|$ (jerk) is the change rate of acceleration; SOC_r is SOC terminal constraint, β is a weight coefficient, which is utilized to maintain charge-sustaining.

(3) Solving procedure

Making use of the reparameterization trick, the action is generated as follows:

$$a_t = \tanh\left(\mu_\phi(s) + \sigma_\phi \odot \varepsilon\right), \varepsilon \sim \mathcal{N}(0,1) \tag{19}$$

Where ε is an input noise vector, the activation function is $\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$; μ_ϕ and σ_ϕ are the outputs of the policy network, which is a squashed Gaussian policy.

The pseudo code of the proposed co-optimization for eco-driving and energy management can be found in Table 2.

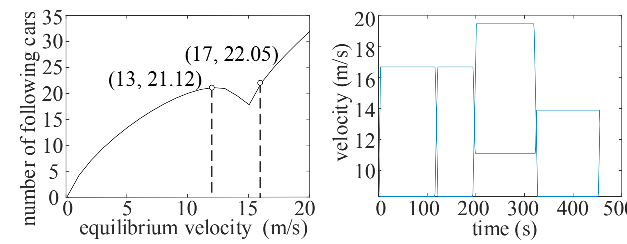
Table 2. Pseudo code of SAC-based co-optimization

Input: parameter initialization $\omega_1, \omega_2, \theta, \bar{\omega}_1, \bar{\omega}_2, \mathcal{D}$
for $e = 1 \to E$ **do**
  Observe initial states: $s_1 \leftarrow x_i, v_i, \tilde{d}_i, \tilde{v}_i (i = 1, \ldots, N)$
  for $t = 1 \to T$ **do**
    Select action $a_t \sim \pi_\theta(s_t) \to$ fleet dynamics (3) $\dot{x}_i, \dot{v}_i, \dot{d}_i, \dot{\tilde{v}}_i$
    Calculate (18) $r(s_t, a_t) \to$ Sample transition s_{t+1}
    Store in the replay pool $\mathcal{D} \leftarrow \mathcal{D} \cup \{(s_t, a_t, r(s_t, a_t), s_{t+1})\}$
  end for
  for $k = 1 \to K$ **do**
    Sample: $B = \{(s_i, a_i, r_i, s_{i+1})\}_{i \in (1,\ldots,N)}$ from $\mathcal{D}$
    Target: $y_i = r_i + \gamma \min_{j=1,2} Q_{\bar{\omega}_j}(s_{i+1}, a_{i+1}) - \alpha \log \pi_\theta (a_{i+1} | s_{i+1})$
    Update Q: $\nabla_{\omega_j} \frac{1}{|B|} \sum_{(s,a,r,s') \in B} \left(Q_{\omega_j}(s_i, a_i) - y_i \right)^2, j = 1,2$
    Update policy: $\nabla_\theta \frac{1}{|B|} \sum_{s \in B} \left(\min_{j=1,2} Q_{\omega_j}(s_i, \tilde{a}_i) - \alpha \log \pi_\theta (\tilde{a}_i | s_i) \right)$
    Adjust α
    Update target networks: $\bar{\omega}_j \leftarrow \tau \omega_j + (1 - \tau), j = 1,2$
  end for
end for

3.3 Simulation and analysis

Taking three vehicles (PHEV, EV, ICEV) passing through four signalized intersections as an example, the parameter values are set as follows: P =[1500, 2500, 4000, 5200], T_r =[55, 60, 50, 45], T_g =[35, 40, 30, 35], T_0 =[15, 15, 10, 20], S_0 =[0, 1, 1, 0], v_{max} =[60, 60, 70, 50], v_{min} =[30, 30, 40, 30].

As shown in Fig. 3 (a), the optimal terminal vehicle is set to 13m/s. Correspondingly, the terminal time arriving at each signalized intersection is shown in Fig. 3 (b).



(a) velocity- passing number (b) global eco-speed limits

Figure 3. Global terminal constraints: (a) velocity- passing number; (b) global eco-speed limits.

4. CONCLUSIONS

To coordinate traffic efficiency-oriented ecological velocity and energy management, a cooperative optimization control strategy is developed for heterogenous vehicle platoon based on soft actor-critic (SAC) algorithm. Among multiple signalized intersections, the global eco-speed limits are derived by maximizing the passing number of following vehicles and average energy efficiency, which is imposed as constraints for co-optimization. Then, a cooperative eco-driving and energy management control strategy is proposed based on SAC algorithm, the core of which is to introduce regularized entropy to conduct more exploration and avoid converging to a bad local optimum. Finally, a case is given to analyze the effectiveness of the proposed strategy in improving fuel economy and traffic efficiency.

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