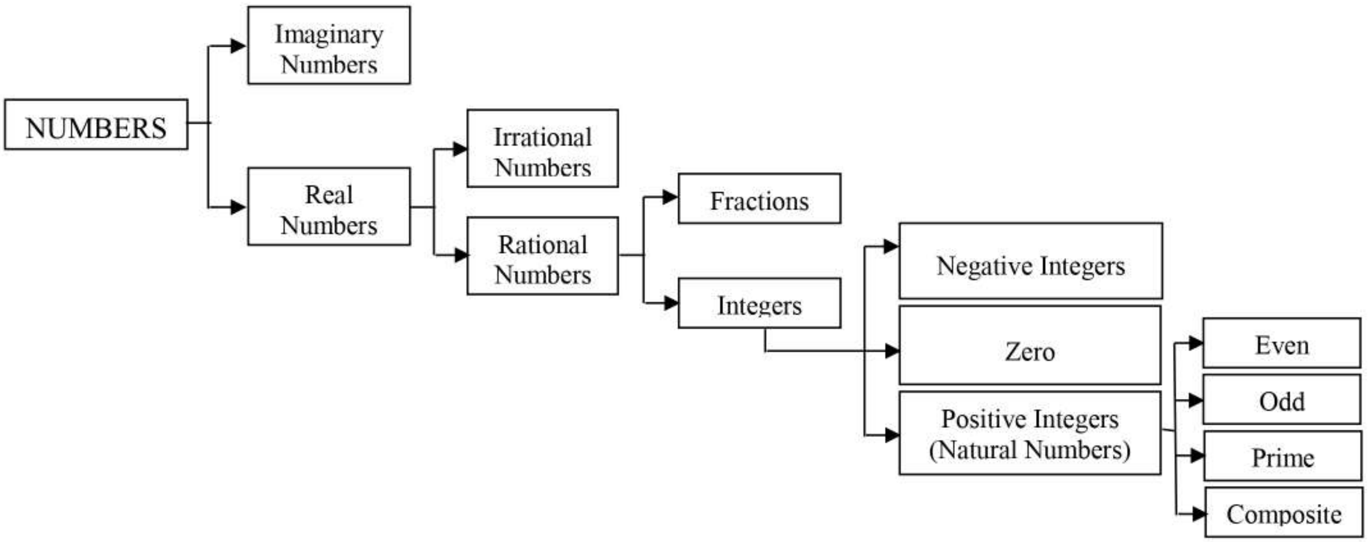
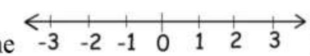
Unit – I: Number system: Base System, Exponents, Factorials, LCM & HCF, Properties of Numbers, Remainders, Successive Divisions – **Sequence & Series:** Arithmetic Progression, Harmonic Progression, Geometric Progression

NUMBER SYSTEM



I. Imaginary Numbers: An imaginary number is a real number multiplied by the imaginary unit ‘i'. Ex: 3i,

II. Real Numbers: Any number that can be plotted on a number line 

is called as real number. A real number is a number that can be used to measure a continuous one-dimensional quantity such as a distance, duration or temperature. Real numbers are denoted as R. Real numbers are numbers that include both rational and irrational numbers.

1. **Irrational Numbers:** An irrational number is a real number that cannot be expressed in the form of fraction or ratio. For example: $\sqrt{2}$, π, etc.

2. **Rational Numbers:** A rational number is any number that can be written as a fraction or ratio, where both the numerator (the top number) and the denominator (the bottom number) are integers, and the denominator is not zero. Some examples of rational numbers include:
   - → 3/4 (three quarters)
   - → 1/2 (one half)
   - → 5/8 (five eighths)

   2.1: Fractions: Fractions represent the parts of a whole or collection of objects. A fraction has two parts. The number on the top of the line is called the numerator. It tells how many equal parts of the whole or collection are taken. The number below the line is called the denominator. It shows the total number of equal parts the whole is divided into or the total number of the same objects in a collection.

   - ***Types of Fractions:***

Types of fractions	Definition	Example
Unit fractions	Fractions with numerator 1 .	$\frac{1}{7}$
Proper Fractions	Fractions in which the numerator is less than the denominator.	$\frac{2}{7}$
Improper Fractions	Fractions in which the numerator is more than or equal to the denominator.	$\frac{5}{3}$
Mixed Fractions	Mixed fractions consist of a whole number along with a proper fraction.	$8\frac{2}{3}$
Like Fractions	Fractions with the same denominators.	$\frac{1}{4}$ and $\frac{3}{4}$
Unlike Fractions	Fractions with different denominators.	$\frac{1}{3}$ and $\frac{3}{4}$
Equivalent Fractions	Fractions that have the same value after being simplified or reduced.	$\frac{6}{4}$ and $\frac{12}{8}$

2.2: Integers: An integer is a whole number (not a fractional number) that can be positive, negative, or zero. The set of integers, denoted Z, is formally defined as follows:

$$Z = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$$

→ Integers come in three types:

- Negative Integers (Additive inverse of Natural Numbers)
- Positive Integers (Natural numbers)
- Zero (0)

2.2.1: Negative Integers: Negative integers are integers with a value less than zero. They are represented by Z^-. The negative integers lie on the left side of zero on the number line. Example: $Z^- = \{-\infty, ..., -3, -2, -1\}$

Note: - Negative integers are also called the additive inverse of positive integers.

2.2.2: Zero: Zero is an integer that is neither positive nor negative. It is called a neutral number. It is simply written as 0 without any positive or negative sign.

2.2.3: Positive Integers: Positive integers are natural counting numbers greater than zero. It denoted by Z^+. The positive integers lie on the right side of zero on the number line.

For example: $Z+ = \{1, 2, 3, ..., \infty\}$

- ***Types of Positive Integers:***
  - → **Even Number:** A counting number divisible by 2 is called as even number. Thus 0, 2, 4, …... are all even numbers.
  - → **Odd Number:** A counting number not divisible by 2 is called as odd number. Thus 1, 3, 5, …... are all odd numbers.
  - → **Prime Number:** Prime number is a number greater than 1 that only has two factors, 1 and the number itself. This means that a prime number is only divisible by 1 and itself. Example: 2, 3, 5, 7, 11, 13, etc.
  - → **Composite Number:** Composite numbers are numbers that have more than two factors. Example: 4, 6, 8, 9, etc.

- **Natural Numbers:** A set of all counting numbers which begins with 1 are known as natural numbers. It is denoted by the capital letter N and includes N = {1,2,3,4,5,6,7……infinite}
- **Whole Numbers:** A set of all countable numbers which begins with 0 are known as whole numbers. It is designated by the capital letter W and contains W = {0,1, 2…., 21…. 5551, …. infinite}

DIVISIBILITY RULES:

- **Divisibility by 2:** A number is divisible by 2 if the unit digit is zero or divisible by 2. Eg: 542 is divisible by 2, while 751 is not divisible by 2.
- **Divisibility by 3:** A number is divisible by 3 only when the sum of its digit is divisible by 3. Eg: In the number 342, the sum of digits is $3 + 4 + 2 = 9$, which is divisible by 3. Hence, the number 342 is divisible by 3.
- **Divisible by 4:** A number is divisible by 4 if its last two digits are divisible by 4. Eg: 6879376 is divisible by 4 since 76 is divisible by 4.
- **Divisible by 5:** A number is divisible by 5 only when its unit digit is 0 or 5. Eg: 789415 is divisible by 5 since it unit digit is 5.
- **Divisible by 6:** A number is divisible by 6 if the number is even and sum of digits is divisibly by 3. Eg: 4536 is divisible by 6 since it is an even number and also sum of digits $4+5+3+6=18$ is divisible by 3.
- **Divisible by 7:** Double the last digit and subtract it from a number made by the other digits. The result must be divisible by 7. Ex:
  - → 672 (Double 2 is 4, $67-4=63$, and $63\div7=9$) **Yes**
  - → 105 (Double 5 is 10, $10-10=0$, and 0 is divisible by 7) **Yes**
  - → 905 (Double 5 is 10, $90-10=80$, and $80\div7=11\ ^3/_7$) **No**

- **Divisible by 8:** A number is divisible by 8 if its last three digits are divisible by 8. In the number 16789352, the number formed by last 3 digits, namely 352 is divisible by 8. Hence, the entire number is divisible by 8.
- **Divisible by 9:** A number is divisible by 9 if the sum of its digit is divisible by 9. Eg: 108936 here 1+0+8+9+3+6 is 27 which is divisible by 9 and hence 108936 is divisible by 9.
- **Divisible by 10:** A number is divisible by 10 if its unit digit is ‘**0**’. Eg: 10, 90, 34920 etc.
- **Divisible by 11:** A number is divisible by 11 if the difference between the sum of its digits at odd places and the sum of its digits at even places is either ‘0’ or divisible by 11.

  Eg: Consider the number 29435417.
  = (Sum of its digits at odd places) – (Sum of its digits at even places)
  = (7+4+3+9) – (1+5+4+2) = 23 – 12 = 11, which is divisible by 11. Hence 29435417 is divisible by 11.

EXPONENTS

An exponent of a number, represents the number of times the number is multiplied by itself. For example, in a^n, 'a' is the base, 'n' is the exponent, and we say it as “**a** raised to the power **n**”. If 8 is multiplied by itself for **n** times, then, it is represented as:

$$8 \times 8 \times 8 \times 8 \times \ldots \text{ n times} = 8^n$$

The above expression, 8^n, is said as 8 raised to the power ‘n’. Therefore, exponents are also called power or sometimes indices.

Examples:

- $3 \times 3 \times 3 \times 3 = 3^4$
- $4 \times 4 \times 4 = 4^3$
- $7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^6$

Laws / Rules of Exponents: Here is the list of exponent rules.

Product Rule	$a^m \times a^n = a^{m+n}$
Quotient Rule	$a^m \div a^n = a^{m-n}$
Power of a Power Rule	$(a^m)^n = a^{mn}$
Power of a Product Rule	$(ab)^m = a^m b^m$
Power of a Quotient Rule	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
Zero Exponent Rule	$a^0 = 1$
Negative Exponent Rule	$a^{-m} = \frac{1}{a^m}$
Fractional Exponent Rule	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

- $a^1 = a$; According to this rule, if the exponent of any integer is 1, then the result is ‘**a**’.

  Example: $4^1 = 4$
- $a^0 = 1$; According to this rule, if the exponent of any integer is zero, then the result is ‘**1**’.

  Example: $5^0 = 1$
- $a^m \times a^n = a^{m+n}$; This is the law for multiplication of exponents with the same base and different powers.

  Example: $2^2 \times 2^3 = (2)^5 = 64$
- $a^m \times b^m = (ab)^m$; The product of 'a' raised to the power 'm' and 'b' raised to the power 'm' is equal to the product of 'a' and 'b' both raised to the power 'm'.

  Example: $2^2 \times 3^2 = (2 \times 3)^2$

$= 6^2 = 36$

- $a^m \div a^n = a^{m-n}$; This law is used for the division of exponents with the same base and different powers.

  Example: $2^5 \div 2^2 = (2)^{5-2} = 2^3 = 8$

- $a^m \div b^m = (a/b)^m$; The division of 'a' raised to the power 'm' and 'b' raised to the power 'm' is equal to 'a' divided by 'b', all raised to the power 'm'.

  Example – 1: $2^2 / 3^2 = (2/3)^2$
  Example – 2: Simplify $25^3/5^3$
  Solution:

  Using Law: $a^m/b^m = (a/b)^m$
  $25^3/5^3$ can be written as $(25/5)^3 = 5^3 = 125$.

- $(a^m)^n = a^{m \times n}$; Here, 'a' raised to the power 'm' and then to the power 'n' is equal to 'a' raised to the product of 'm' and 'n'.

  Example: $(2^2)^3 = 2^{2x3}$

FACTORIALS

The factorial of a number is the multiplication of all the numbers between 1 and the number itself. For example, the factorial of 3 represents the multiplication of numbers 3, 2, 1, i.e. 3! = 3 × 2 × 1 and is equal to 6. Exclamation mark (!) indicates the factorial. Factorials are used in topics like Permutations and combinations, series, sequences and many more.

Factorials of Numbers 1 to 10 Table:

n	Factorial of a Number (n!)	Expansion	Value
0	0!		1
1	1!	1	1
2	2!	2 × 1	2
3	3!	3 × 2 × 1	6
4	4!	4 × 3 × 2 × 1	24
5	5!	5 × 4 × 3 × 2 × 1	120
6	6!	6 × 5 × 4 × 3 × 2 × 1	720
7	7!	7 × 6 × 5 × 4 × 3 × 2 × 1	5,040
8	8!	8 × 7 × 6 × 5 × 4 × 3 × 2 × 1	40,320
9	9!	9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1	362,880
10	10!	10 × 9 ×8 × 7 × 6 × 5 ×4 × 3 × 2 × 1	3,628,800

Useful Factorial Properties:

- n! = n x (n – 1)!
- (n x m)! ≠ n! x m!
- (n + m)! ≠ n! + m!
- (n – m)! ≠ n! – m!

Exercise:

i. What is the value of 7!

Ans: 7! = 7 x 6 x 5 x 4 x 3 x 2 x 1
= 42 x 120
= 5040

ii. Simplify (6! × 3!) ÷ (3!)

Ans:

$= (6! \times \cancel{3!}) \div (\cancel{3!})$
= 6! = 720

iii. $\frac{8!}{5! \times 2!}$

$= \frac{8 \times 7 \times 6 \times \cancel{5!}}{\cancel{5!} \times 2!}$

$= \frac{8 \times 7 \times 6}{2}$

= 168

iv. How many ways can you arrange the letters in the word **"movies"** without repeating them?

Ans:

For this problem, count the number of letters in the word **"movies"** to find there are **six** letters. Then, find the factorial of the number six. It should look like this:

$$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$$
$$= 720$$

v. How many ways can you arrange the letters A, B, & C without repeating them?

Ans:

$$3! = 3 \times 2 \times 1$$
$$= 6$$

LCM AND HCF

HCF:

Highest Common Factor of two or more numbers is the greatest number that divided each one of them exactly. HCF is also called as Greatest Common Divisor (GCD) or Greatest Common Measure (GCM).

Example: Find the HCF of 18 and 24.

- Factors: 18 – 1, 2, 3, 6, 9, 18.

  24 – 1, 2, 3, 4, 6, 8, 12, 24.

$$\therefore \text{HCF} = 6$$

- **Methods of Finding HCF:**

1. HCF by Factorization
2. HCF by Division

1. HCF by Factorization:

- ***Step – 1:*** Breakdown each one of the given numbers as the product of prime factors. Eg: $8 = 2 \times 4$
  $= 2 \times 2 \times 2$
- ***Step – 2:*** Choose Common Factors
- ***Step – 3:*** Find the product of lowest power of those factors on this is the required HCF of the given numbers.

Exercise – 1: Find the HCF of 84 and 540

$$84 = 7 \times 12$$
$$= 7 \times 3 \times 4$$
$$= 7 \times 3 \times 2 \times 2$$
$$= 7^1 \times 3^1 \times 2^2$$

$$540 = 10 \times 54$$
$$= 2 \times 5 \times 6 \times 9$$
$$= 2 \times 5 \times 2 \times 3 \times 3 \times 3$$
$$= 2^2 \times 3^3\ 5^1$$

$$\text{HCF} = 2^2 \times 3^1$$
$$= 2 \times 2 \times 3 = 12$$

→ **Prime Numbers:** A number is greater than one and it is divisible by one and itself.
Ex: 2, 3, 5, 7, 11, 13, 17, 19, 23, …..

→ **Twin Primes:** The difference between two prime numbers is "2" then those numbers are called as twin primes.
(3, 5), (5, 7), (11, 13), (17, 19), …..etc.

→ **Canonical Form:** Product of primes are called as canonical form
Ex: $2 \times 3 \times 7 = 42$

Exercise – 2: Find the HCF of $2^2 \times 3^1 \times 7^1$ and $2^2 \times 3^3 \times 5^1$

Ans:

$$\text{HCF} = 2^2 \times 3^1$$
$$= 2 \times 2 \times 3$$
$$= 12$$

Exercise – 3: Find the HCF of $2^5 \times 3^7 \times 5^3$ and $2^2 \times 3^{10} \times 7^2$

Ans: HCF = $2^2 \times 3^7$

$= 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 8748$

2. ***HCF by Division:*** This method is also known as **Successive Division Method**.

- ***Step – 1:*** Consider two different numbers.
- ***Step – 2:*** Divide the larger number by the smaller one.
- ***Step – 3:*** Now divide the divisor by the remainder.
- ***Step – 4:*** Repeat the process of dividing the preceding divisor by the last reminder obtained, till a remainder zero. The last division is HCF of two numbers.

Ex: Find the HCF of 42, 70, 98 by successive division method.

```
42) 70 (1            14) 98 (7
    42                   98
28) 42 (1                 0
    28
  14) 28 (2          ∴ HCF = 14
      28
       0
```

Other Method:

```
2 | 42, 70, 98
7 | 21, 35, 49
  |  3,  5,  9
```

$\therefore$ HCF = $2 \times 7 = 14$

LCM:

LCM stands for Least Common Multiple. The least number which is exactly divisible by each one of the given numbers is called their LCM.

Methods of Finding LCM:

1. LCM by Factorization
2. LCM by Division

1. ***LCM by Factorization:***

- ***Step – 1:*** Breakdown each one of the given numbers as the product of prime factors. Eg: $8 = 2 \times 4$
$= 2 \times 2 \times 2$
- ***Step – 2:*** Choose Common Factors
- ***Step – 3:*** Find the product of highest power of those factors on this is the required LCM of the given numbers.

2. ***LCM by Division:*** Arrange the given numbers in a row in any order. Divide by a number which divided exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of the divisors and the undivided numbers is the required LCM of the given numbers.

HCF and LCM of Fractions:

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$$\boxed{\text{HCF} = \frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}} \qquad \boxed{\text{LCM} = \frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}}$$

Ex:

Find the HCF of $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$

Ans:

HCF = $\frac{1}{12}$

Note – 1: When any two or more numbers are given, to find the HCF, the least common value of all the given numbers can be taken as HCF. In this problem we need the HCF for 1, 2 and 3. Least value of which "one" is common in all three can be taken as HCF. "1" can be taken as HCF if there is no common value.

Note – 2: To find the LCM of any two or more numbers, the highest common value of all the given numbers can be taken as the LCM. If there is no highest value common to all of them, if there is any common to some of them, cancel that number and take the product of the remaining numbers as LCM. In this calculation we need LCM for 2, 3, 4. But there is no common highest value in all these three. But 2 is divisible of 4 so cancel "2" and take the product of remaining two numbers 3 and 4 i.e. 3 x 4 = 12 as LCM.

Ex:

Find the HCF of $\frac{2}{3}, \frac{6}{15}, \frac{4}{5}$,

Ans:

$$\frac{2}{3}, \frac{6}{15}, \frac{4}{5}$$

$$\text{HCF} = \frac{2}{15}$$

<u>HCF of Decimals</u>:

Ex: Find the HCF of 1.7, 0.51, 0.0153.

Ans:

$$\frac{17}{10}, \frac{51}{100}, \frac{153}{10000}$$

$$\text{HCF} = \frac{17}{10000} = 0.0017$$

Ex: Find the HCF of 16.5, 0.45, 0.075?

Ans:

$$\frac{165}{10}, \frac{45}{100}, \frac{75}{1000}$$

$$\frac{\overset{33}{\cancel{165}}}{\underset{2}{\cancel{10}}}, \frac{\overset{9}{\cancel{45}}}{\underset{20}{\cancel{100}}}, \frac{\overset{3}{\cancel{75}}}{\underset{40}{\cancel{1000}}}$$

$$\frac{33}{2}, \frac{9}{20}, \frac{3}{40}$$

$$\text{HCF} = \frac{3}{40}$$

$$= \frac{3}{4} \times \frac{1}{10}$$

$$= 0.75 \times 0.1 = 0.075$$

1. The least number which is exactly divisible by 5, 8, 10, 12, and 15 is ________.

Ans:

$\cancel{5}$, 8, 10, 12, 15

$$\begin{array}{r|l} 2 & 8,\ 10,\ 12,\ 15 \\ \hline 2 & 4,\ 5,\ 6,\ 15 \\ \hline 5 & 2,\ 5,\ 3,\ 15 \\ \hline 3 & 2,\ 1,\ 3,\ 3 \\ \hline & 2,\ 1,\ 1,\ 1 \end{array} = 120$$

2. The LCM and HCF of two numbers are 1056 and 16 respectively and one of these number is 128. Find the second number.

Ans:

$$\boxed{\text{Product of two numbers} = \text{LCM x HCF}}$$

$$\overset{8}{\cancel{128}} \times X = \overset{132}{\cancel{1056}} \times \cancel{16}$$

$$X = 132$$

$$\boxed{\text{Required No.} = \frac{\text{LCM x HCF}}{\text{Given Number}}}$$

$$\text{Required No.} = \frac{1056 \times 16}{128}$$

$$= 132$$

3. Four bells ring at 10, 12, 15 and 20 minutes respectively beginning together. After what interval of time do they ring again together?

Ans: $\cancel{10}$, 12, 15, 20

$$\begin{array}{c|l}
2 & 12,\ 15,\ 20 \\ \hline
2 & 6,\ 15,\ 10 \\ \hline
3 & 3,\ 15,\ 5 \\ \hline
5 & 1,\ 5,\ 5 \\ \hline
 & 1,\ 1,\ 1 \ = 60 \text{ Minutes}
\end{array}$$

4. Find the smallest number when divided by 6, 9, 12, and 15 leaves 5 as remainder in each case.

Ans:

$\cancel{6}, 9, 12, 15$

$$\begin{array}{c|l}
3 & 9,\ 12,\ 15 \\ \hline
 & 3,\ 4,\ 5 \ = 180
\end{array}$$

$$\begin{array}{r}
180 \\
+\ 5 \\ \hline
185
\end{array}$$

5. Find the smallest number which when increased by 7, divisible by 8, 15, 16, and 25.

Ans:

$\cancel{8}, 15, 16, 25$

$$\begin{array}{c|l}
5 & 15,\ 16,\ 25 \\ \hline
 & 3,\ 16,\ 5 \ = 1200
\end{array}$$

$$\begin{array}{r}
1200 \\
-\ 7 \\ \hline
1193
\end{array}$$

6. Find the least number of 5 digits which is exactly divisible by 5, 10, 15, and 25.

Ans:

$\cancel{5}, 10, 15, 25$

$$\begin{array}{c|l}
5 & 10,\ 15,\ 25 \\ \hline
 & 2,\ 3,\ 5 \ = 150
\end{array}$$

$$\begin{array}{r}
150)\ 10000\ (66 \\
900 \\ \hline
1000 \\
900 \\ \hline
100
\end{array}$$

50

$\therefore 10050$

7. Find the greatest no. of 4 digits which is exactly divisible by 7, 14, and 21.

Ans:

$\cancel{7}, 14, 21$

$$\begin{array}{c|l}
7 & 14,\ 21 \\ \hline
 & 2,\ 3 \ = 42
\end{array}$$

$$\begin{array}{r}
42)\ 9999\ (238 \\
84 \\ \hline
159 \\
126 \\ \hline
339 \\
336 \\ \hline
3
\end{array}$$

$\therefore$ Required Number = Dividend – Remainder

= 9999 – 3 = 9996

REMAINDER

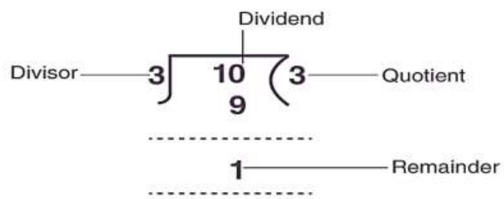
The word 'remainder' is derived from the French word 'Remaindre', which means 'to remain'. Remainder is a part of a division.

Remainder is the value that is left over after the division. If a number (dividend) is not completely divisible by another number (divisor) then we are left with a value once the division is done. This value is called the remainder.

In division, the four main parts are:

- Dividend – the number or value that is divided
- Divisor – the value that divides the other number
- Quotient – an answer we get when one value is divided by another value
- Remainder – the number that is left when a dividend is not completely divisible by the divisor

For example, 10 is not exactly divided by 3. Since the closest value, we can get 3 x 3 = 9. Hence, $10 \div 3 \rightarrow 3$ R 1, where 3 is the quotient and 1 is the remainder.



Formulas:

→ Dividend = Divisor x Quotient + Remainder

→ Remainder = Dividend – (Divisor x Quotient)

Examples are:

- $12 \div 5 = 2$ Remainder 2 since $5 \times 2 = 10$ and $10 + 2 = 12$
- $33 \div 10 = 3$ Remainder 3, since $10 \times 3 = 30$ and $30 + 3 = 33$
- $46 \div 5 = 9$ Remainder 1, since $5 \times 9 = 45$ and $45 + 1 = 46$

Properties of Remainder: The properties of the remainder are as follows:

- The remainder is always less than the divisor.
- If one number (divisor) divides the other number (dividend) exactly, then the remainder is 0.
- The remainder can be either greater, lesser, or equal to the quotient.

Exercise: Find the remainder in the following cases.

i. $\frac{23}{5}$ ii. $\frac{17}{6}$ iii. $\frac{86 \times 23}{11}$ iv. $\frac{38 + 71 + 86}{16}$ v. $\frac{123 \times 124 \times 125}{9}$ vi. $\frac{7^5}{4}$

Ans: i. 3

ii. 2

iii. $\frac{\overset{®\,9}{\cancel{86}} \times \overset{1}{\cancel{23}}}{\cancel{11}} = 9 \times 1 = 9$

iv. $\frac{\overset{®\,6}{\cancel{38}} + \overset{7}{\cancel{71}} + \overset{6}{\cancel{86}}}{16} = \frac{6 + 7 + 6}{16} = \frac{19}{16} = 3$

v. $\frac{\overset{®\,6}{\cancel{123}} \times \overset{7}{\cancel{124}} \times \overset{8}{\cancel{125}}}{9} = \frac{6 \times 7 \times 8}{9} = \frac{\overset{®\,6}{42} \times 8}{9} = \frac{6 \times 8}{9} = \frac{48}{9} = 3$

vi. $\frac{7^5}{4} = \frac{\overset{®\,3}{7} \times \overset{3}{7} \times \overset{3}{7} \times \overset{3}{7} \times \overset{3}{7}}{4}$

$= \frac{\overset{1}{9} \times \overset{1}{9} \times 3}{4}$

$= \frac{1 \times 1 \times 3}{4} = 3$

vii. Find the remainder of $\frac{2^{75}}{5}$.

Ans:

$= \frac{2^{\overset{18}{\cancel{75/4}}}}{5} = \frac{(2^4)^{18} \times 2^3}{5}$

$= \frac{\cancel{16}^{\,18} \times 8}{\cancel{5}}$

$= \frac{1^{18} \times 8}{\cancel{5}}$

$= \frac{8}{\cancel{5}} = 3$

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16 \rightarrow$
$2^5 = 80$

More or less equal to 5 multiples i.e., $5 \times 3 = 15$

SEQUENCE AND SERIES

A sequence is also known as progression and a series is developed by sequence. Sequence and series are one of the basic concepts in Arithmetic. A sequence is an arrangement of any objects or a set of numbers in a particular order(manner) followed by some rule, whereas a series is the sum of the elements in the sequence.

For example:

- 2, 4, 6, 8 → Sequence
- 2 + 4 + 6+ 8 = 20 → Series

Types of Sequence / Progression:

i. Arithmetic Progression
ii. Geometric Progression
iii. Harmonic Progression

i. ***Arithmetic Progression:*** An arithmetic sequence is a sequence where **each term of the sequence** is formed either by adding or subtracting **a common term** from the preceding number, and the common term is called the common difference.

For example, 1, 4, 7, 10, ...is an arithmetic sequence. A series formed by using an arithmetic sequence is known as the arithmetic series. For example, 1 + 4 + 7 + 10... is an arithmetic series.

→ The arithmetic sequence is represented by a, a + d, a + 2d, a + 3d, …

→ So, the arithmetic series will be a + (a + d) + (a + 2d) + (a + 3d) + …

Here, a = first term, d = common difference (i.e., successive term – preceding term).

Formula:

→ Sequence formula for **nth** term in an arithmetic progression:

$$\boxed{T_n = a_1 + (n-1)\, d}$$

T_n = the nth term in the sequence
a_1 = the first term in the sequence
d = the common difference between terms.

→ The series for sum of **n** terms:

$$S_n = \frac{n}{2}\,(2a + (n-1)\,d)$$

Exercise: In the series 357, 363, 369, …., what will be the 10th item?

a) 405 b) 411 c) 413 d) 417

Ans:

→ $T_n = a + (n-1)\, d$

→ The given series is an A.P. in which a = 357 and d = 6.

→ 10th term as: $T_{10} = 357 + (10-1)6 = 357 + (9)6$
$= 357 + 54 = 411.$

Exercise: How many terms are there in the series 201, 208, 215, …., 369?

a) 23 b) 24 c) 25 d) 26

Ans:

→ $T_n = a + (n-1)\, d$
→ $369 = 201 + (n-1)\, 7$
→ $n = 25$

ii. Geometric Progression: A geometric progression is a sequence where each term of the sequence is formed either by multiplying or dividing a common term with the preceding number, and the common term is called the common ratio.

For example, 1, 4, 16, 64, ...is an geometric sequence. A series formed by using geometric sequence is known as the geometric series for example 1 + 4 + 16 + 64... is a geometric series.

→ Geometric sequence is given by $a, ar, ar^2, \ldots, ar^{(n-1)}$

→ So, Geometric series will be $a + ar + ar^2 + \ldots + ar^{(n-1)}$

Here, a = first term, r = common ration (i.e., successive term / preceding term), and n = number of terms

Formula:

→ Sequence formula for **n^th^** term in a geometric progression:

n^{th} term = $a\,r^{n-1}$

→ The series for sum of **n** terms:

$$S_n = \frac{a\,(1 - r^{n})}{(1 - r)}$$

Exercise: In the series 1, 4, 16, 64, 256, 1024, …. Find the common ratio and 9th term.

Solution:

$$\mathbf{T_n = ar^{(n-1)}}$$

Here a = 1, r = 4 and n = 9

So, 9^{th} term is can be calculated as $T_9 = 1 \times (4)^{(9-1)} = 4^8 = 65536$.

Exercise: In the series 7, 14, 28, ……, what will be the 10^{th} term.

Solution:

$$\mathbf{T_n = a\,r^{(n-1)}}$$

Here a = 7, r = 2 and n = 10

So, 10^{th} term is can be calculated as $T_{10} = 7 \times (2)^{(10-1)} = 7 \times 2^9 = 7 \times 512 = 3584$.

Exercise: In the series $1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \ldots\ldots,$ find the next term.

$$a = 1 \quad r = \frac{1}{4} \quad n = 5$$

$$5^{th}\text{ term} = 1 \times \left(\tfrac{1}{4}\right)^{(5-1)}$$

$$= \left(\tfrac{1}{4}\right)^{4}$$

$$= \tfrac{1}{256}$$

iii. Harmonic Progression: A harmonic sequence is a sequence where the sequence is formed by taking the reciprocal of each term of an arithmetic sequence.

For example, 1, 1/4, 1/7, 1/10, ... is a harmonic sequence. A series formed by using harmonic sequence is known as the harmonic series. For example, 1 + 1/4 + 1/7 + 1/10.... is a harmonic series.

Example of Harmonic Sequence: Here reciprocal of all the terms are in the arithmetic sequence: 3, 6, 9, 12, 15

$$\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \frac{1}{15}.$$

Also, if the sequence a, b, c, d, … is assumed to be an arithmetic sequence then the harmonic sequence can be written as:

$$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}$$

Formula:

→ Sequence formula for $\mathbf{n^{th}}$ term in harmonic progression:

$$\frac{1}{a^n} = \frac{1}{[a+(n-1)\ d]}$$

→ The series for sum of **n** terms:

$$S_n = \frac{1}{d} \ln \frac{[2a + (2n-1)\ d]}{(2a-d)}$$

Here, a = first term in A.P., d = common difference, and n = number of terms in A.P.

Exercise: Find the 4th and 8^{th} terms of the harmonic progression 6, 4, 3, …

a) 12/5 b) 7/10 c) 1/7 d) 20/11

Ans:

$$\frac{1}{a^n} = \frac{1}{[a+(n-1)\ d]}$$

Consider 1/6, 1/4, 1/3, ...
Here T2-T1 = T3-T2 = 1/12, so 1/12 is the common difference.
Here a = 1/6, d=1/12,
Therefore 1/6, 1/4, 1/3 is in A.P.
4th term of this Arithmetic Progression = 1/6 + (3 × 1/12) = 1/6 + 1/4 = 5/12,
Eighth term = 1/6 + (7 × 1/12) = 9/12.
Hence the 8th term of the H.P. = 12/9 = 4/3 and the 4th term = 12/5.

4th term = 1/6 + (3 × 1/12)
= 1/6 + 1/4 = 5/12

PROPERTIES OF NUMBERS

Number properties lay down some rules that we can follow while performing mathematical operations. There are four basic properties in math:

i. Commutative Property
ii. Associative Property
iii. Distributive Property
iv. Identity Property

We apply these properties while doing addition and multiplication operations.

i. ***Commutative / Order Property:*** The word commute means "to travel back and forth". If a number is commutative, that means it is movable. The commutative property states that changing the order of addends or factors does not change the sum or the product.

Formula:

- Addition: a + b = b + a
- Multiplication: a × b = b × a

→ **Commutative Property of Addition:** This property says that when we add two numbers, the order in which we add the numbers makes no difference to the answer.

Suppose we have to add 3 and 5. We can write 3+5=8. Similarly, we can also write it as 5+3=8. Thus, 3 + 5 = 5 + 3 = 8. Hence, the commutative property of addition for any two real numbers a and b is given as: **a + b = b + a**

→ **Commutative Property of Multiplication:** This property says that when we multiply two numbers, the order in which we multiply the numbers makes no difference to the answer.

Let's multiply 4 and 5. We get $4 \times 5 = 20$. Now, if we reverse the order of the numbers and multiply, we get $5 \times 4 = 20$. Thus, $4 \times 5 = 5 \times 4$

Hence, the commutative property of multiplication for any two real numbers a and b is given as:

$$\mathbf{a \times b = b \times a}$$

ii. ***Associative Property:*** Some math expressions with more than two terms can be solved easily by grouping the terms in the expression. To "associate" numbers means to group numbers. The associative property states that changing the grouping of addends or factors does not change the sum or the product.

Formula:

- Addition: $(a + b) + c = a + (b + c)$
- Multiplication: $(a \times b) \times c = a \times (b \times c)$

→ **Associative Property of Addition:** This property says that when we add three or more numbers, the order in which the numbers are grouped has no impact on the sum.

Suppose we have to add 3, 4, and 5. We first group 3 and 4 as (3+4) and add the sum to 5. Next, we take 3 and add it to the group of 4 and 5 as (4+5). We get $(3 + 4) + 5 = 3 + (4 + 5) = 12$.
Hence, the associative property of addition for three real numbers a, b, and c is:

$$\mathbf{(a + b) + c = a + (b + c)}$$

→ **Associative Property of Multiplication:** This property says that when we multiply three or more numbers, the order in which the numbers are grouped has no impact on the product.

We take the numbers 2, 3, and 5. We first group 2 and 3 as (2×3) and multiply the product by 5. We get $(2 \times 3) \times 5 = 6 \times 5 = 30$. Next, we group 3 and 5 as (3×5) and multiply by 2. So, $2 (3 \times 5) = 2 \times 15 = 30$. Thus, $(2 \times 3) \times 5 = 2 \times (3 \times 5) = 30$. Hence, the associative property of multiplication for three real numbers a, b, and c is:

$$\mathbf{(a \times b) \times c = a \times (b \times c)}$$

iii. ***Distributive Property:*** The distributive property states that multiplying the sum of two or more addends by a number is the same as multiplying each addend individually by the number and then adding the products together.

Formula:

- $\mathbf{a \times (b + c) = (a \times b) + (a \times c)}$

Let's multiply 3 by the sum of 4 and 5. We get $3 \times (4 + 5) = 27$. Now we multiply 3 by 4 and 5 individually and then add the products. We get $(3 \times 4) + (3 \times 5) = 12 + 15 = 27$.
Thus, $3 \times (4 + 5) = (3 \times 4) + (3 \times 5) = 27$. Hence, the distributive property of multiplication over addition for three real numbers a, b, and c is:

$$\mathbf{a \times (b + c) = a \times b + a \times c}$$

iv. ***Identity Property:*** Identity property states that when a number is added, subtracted, multiplied or divided by a specific number, the result will be the same as the original number.

When you add zero to a number, or multiply a number by one, the result is the original number itself, unchanged. In other words, the identity property states that:

- Zero is the "identity" element for addition, as it doesn't change the value of the number.
- One is the "identity" element for multiplication, as it doesn't change the value of the number.

Formula:

- Addition: $a + 0 = a$
- Multiplication: $a \times 1 = a$

→ **Identity Property of Addition:** This property says that when we add 0 to any number, the sum is equal to the number itself. We call 0 the additive identity.

Let's take the 5 and add 0 to it. We get $5 + 0 = 5$ or $0 + 5 = 5$. Hence, the identity property of addition for any real number a is:

$$\mathbf{a + 0 = 0 + a = a}$$

→ **Identity Property of Multiplication:** This property says that when we multiply any number by 1, the product is equal to the number itself. We call 1 the multiplicative identity.

Let's take the 4 and multiply it by 1. We get $4 \times 1 = 4$ or $1 \times 4 = 4$. Hence, the identity property of multiplication for any real number a is:

$$\mathbf{a \times 1 = 1 \times a = a}$$

1. **Identify the number property used in the given equation:**

   $(12 \times 9) \times 4 = 12 \times (9 \times 4)$

   Solution:

   The property used is the associative property of multiplication, $(a \times b) \times c = a \times (b \times c)$

   The product is not affected by the way we group the numbers.

2. **Is 3yz=3zy**?

   Solution:

   Yes, $3yz = 3zy$.

   By the commutative property of multiplication, we have $a \times b = b \times a$

   We can say that $3yz = 3zy$.

3. **By the commutative property of addition, n + 3 =?**

   Solution:

   By the commutative property of addition, we have

   $a + b = b + a$; for any real numbers a and b

   Thus, $n + 3 = 3 + n$

   The order of numbers does not affects the sum.

4. **If 4m×1=20, find the value of m using the identity property of multiplication.**

   Solution:

   The given expression is $4m \times 1 = 20$.

   As per the identity property of multiplication, $a \times 1 = a$

   So, $4m = 20$

   $m = 204$

   $m = 5$.

   Hence, the value of m is 5.

5. **Solve the given expression using the distributive property.**

   $\mathbf{6 \times (5 + 8)}$

   Solution:

   The distributive property for three real numbers a, b, and c is:

   $a \times (b + c) = a \times b + a \times c$.

   Let's consider $a = 6$, $b = 5$, and $c = 8$, and substitute the values in the equation

   $a \times (b + c) = a \times b + a \times c$.

   We get $6 \times (5 + 8) = 6 \times 5 + 6 \times 8 = 30 + 48 = 78$.

   So, $6 \times (5 + 8) = 78$.

6. Which of the following is an example of identity property of addition?

   a) $7 + 1 = 8$

   b) $22 + 2 = 24$

   c) $20 + 0 = 20$

d) $1-1=0$

7. Select the number property that the given expression illustrates. $yz \times 1 = yz$
   a) Identity property of multiplication
   b) Associative property of multiplication
   c) Distributive Property
   d) None of the above
8. Which expression is equal to $(7 \times 3) \times 9$?
   a) $7 \times (3 + 9)$
   b) $7 \times (3 \times 9)$
   c) $7 + (3 \times 9)$
   d) None of the above
9. If you expand $5 \times (9+1)$ using the distributive property, which expression will you get?
   a) 5×10
   b) $5 + (9 \times 1)$
   c) $5 \times 9 + 5 \times 1$
   d) None of the above