Mathematical Model of Starlings

Pushpam Anand (2016CS10347)
Nikhil Kulkarni (2016CS10340)

April'16 2018

1 Objectives

-To model and simulate the fascinating phenomenon of starling murmurations.

-Computationally simulate the phenomenon by modelling each bird as an independent agent communicating and cooperating with other neighbouring agents.

-To measure from a realistic simulation the average energy spend by each bird, the angular momentum and the force that each bird has to withstand in a typical flight ritual.

2 Overall Approach

2.1 Modelling of Birds

- Each individual bird is modelled as a tetrahedron and is identified by the coordinates of the center of the tetrahedron (x_0, y_0).
let the coordinates of the apex is (x_1, y_1).

$$(x_1, y_1) = (x_0 + r.cos\theta, y_0 + r.sin\theta \tag{1}$$

where,
r= circum-radius of the triangle.
θ = direction of the movement of the bird.

The other two vertices are determined by

$$(x_2, y_2) = (x_0 + r.cos(\theta + 5\pi/6), y_0 + r.sin(\theta + 5\pi/6)) \tag{2}$$

$$(x_3, y_3) = (x_0 + r.cos(\theta + 7\pi/6), y_0 + r.sin(\theta + 7\pi/6) \tag{3}$$

2.2 Movement of Birds

Every bird is moving in the direction of its apex. The initial velocity of individual birds is defined by a fixed speed and an angle of movement(called "offset (θ)" hereafter). velocity at any time :

$$\vec{v} = v_0 cos(\theta)\vec{i} + v_0 sin(\theta)\vec{j} \tag{4}$$

2.3 Modelling of Forces

Also , there are three forces acting on the bird which is affecting the motion of the bird .This forces depends upon the positions and velocities of the birds in local flock .There are three forces –
1.Separation
2.Alignment
3.Cohesion

a. Separation

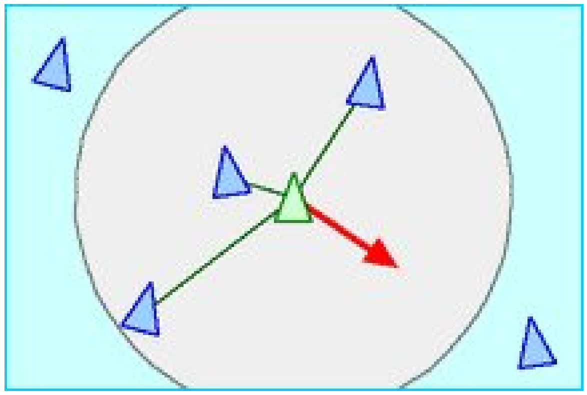
When two birds come too close then its direction is steered away from the average position of the birds in the neighbourhood (i.e. birds within certain radius) .
The neighbourhood is defined in a rectangle of small dimension 0.1×0.04

The Centre of mass of all the birds in the neighbourhood is calculated (say C)
$C_x = (\Sigma(P_x)_i)/n$ $\qquad C_y = (\Sigma(P_y)_i)/n$

New offset due to separation property:

$$\theta_1 = tan^{-1}\Big(\frac{C_y - P_{0y}}{C_x - P_{0x}}) \tag{5}$$

where,
n = total number of birds in local flock .
C$_x = x - coordinate of com of the neighbourhood$
$P_{0x} = x - coordinate of the position of the bird.$



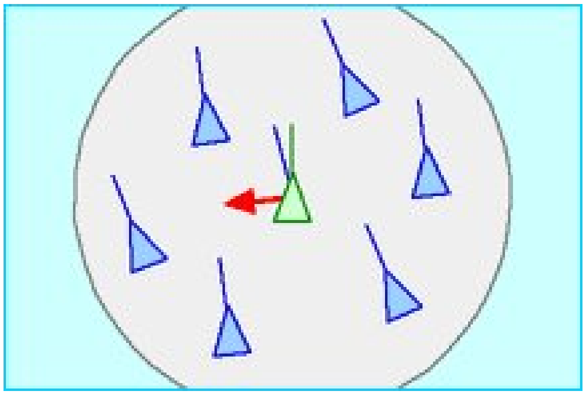
b. Alignment

Alignment: steer towards the average heading of local flockmates
Here we have calculated average direction vector of the flockmates (i.e. birds within certain radius) .
The neighbourhood is defined in a rectangle of dimension 0.3×0.1

New offset due to alignment property:

$$(\theta_2) = (\Sigma\theta_i)/n. \tag{6}$$

where,
n = total number of birds in local flock .
θ_i = direction vector of ith $bird$



c. Cohesion

Cohesion: steer to move toward the average position of local flockmates

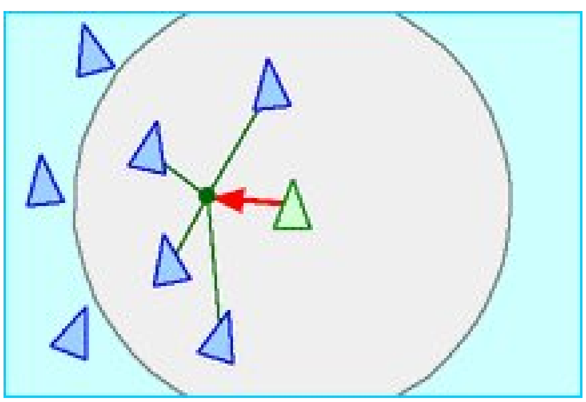
The neighbourhood is defined in a rectangle of small dimension 0.1×0.04

The Centre of mass of all the birds in the neighbourhood is calculated (say C)

$C_x = (\Sigma(P_x)_i)/n$ $\qquad$ $C_y = (\Sigma(P_y)_i)/n$

New offset due to cohesion property:

$$\theta_1 = tan^{-1}\Big(\frac{C_y - P_{0y}}{C_x - P_{0x}}\Big) \tag{7}$$



The absolute effect of the individual forces is decided, and corresponding weight is given to the properties as:

(* HERE GIVE THE WEIGHTS TO THE THREE OFFSETS *)

3 Calculating the Kinetic Energy and Angular Momentum

We think that starlings do it for many reasons. Grouping together offers safety in numbers , also there may be some interesting variations in the kinetic energy and the angular momentum of the flock. We have simulated this starling murmuration with the feature that at every momet we can see the variation in the K.E. of the flock .

3.1 Average Energy of individual birds

If the individual boid have velocity (v_x, v_y, v_z) then we have represented the energy as –

$$K.E. = (v_x^2 + v_y^2 + v_z^2)/2 \tag{8}$$

3.2 Angular momentum of the birds

As we have assumed mass of the every bird to b unit mass then ,if the individual boid have velocity (v_x, v_y, v_z) then we have represented the Angular Momentum as –

$$\vec{M} = (x\vec{i} + y\vec{j}) * (v_x cos(\theta)\vec{i} + v_y sin(\theta)\vec{j}) \tag{9}$$

3.3 Force on each Bird

$$\vec{F} = (\overrightarrow{Cohesion} + \overrightarrow{Separation} + \overrightarrow{Alignment}) \tag{10}$$