Logic in CS Autumn 2024

Problem Sheet 4

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1. If DPLL always chooses to assign 0 to a decision variable before assigning 1 (if needed) to a variable, then show that DPLL will never need to backtrack when given a Horn formula encoded in CNF as input.

2. Let's try to solve the HornSAT problem using DPLL. We can convert each Horn clause into a CNF clause by simply rewriting $(a \to b)$ as $(\neg a \vee b)$, which preserves the semantics of the formula. The resultant formula is in CNF.

   Suppose our version of DPLL always assigns 1 to a decision variable before assigning 0 (if needed). How many backtrackings are needed if we run this version of DPLL on the (CNF-ised version of) the following Horn formulae, assuming DPLL always chooses the unassigned variable with the smallest subscript when choosing a decision variable?

   (a)
$$\left(\left(\bigwedge_{i=0}^{n-1} x_i\right) \to x_n\right) \wedge \bigwedge_{i=0}^{n-1} (x_n \to x_i) \wedge \left(\left(\bigwedge_{i=0}^{n} x_i\right) \to \bot\right)$$

   (b)
$$\bigwedge_{i=0}^{n-1} ((x_i \to x_{n+i}) \wedge (x_{n+i} \to x_i) \wedge (x_i \wedge x_{n+i} \to \bot))$$

3. Let P, Q, R be propositional variables. Convert the formula

$$\neg(P \vee (\neg Q \wedge R)) \to (\neg P \wedge (Q \vee \neg R))$$

   to an equisatisfiable Conjunctive Normal Form (CNF) formula using Tseitin encoding.

4. Consider the following CNF formula:

$$\begin{aligned}
(\neg p_1 \vee p_2 \vee p_3) & \wedge (\neg p_1 \vee p_3 \vee p_9) \\
& \wedge (\neg p_2 \vee \neg p_3 \vee p_4) \\
& \wedge (\neg p_4 \vee p_5) \\
& \wedge (\neg p_4 \vee p_6 \vee \neg p_8) \\
& \wedge (\neg p_5 \vee \neg p_6) \\
& \wedge (p_7 \vee p_1 \vee \neg p_{10}) \\
& \wedge (p_1 \vee p_8) \\
& \wedge (\neg p_7 \vee \neg p_8)
\end{aligned}$$

   (a) Check if this CNF formula is satisfiable using the DPLL method.

(b) Assume $p_9 = \bot$ and $p_{10} = \top$. Now, check if we can find a solution for the above formula using the DPLL method.

5. Discuss the best-case and worst-case time complexities of the DPLL algorithm. Provide a representative example for each scenario.