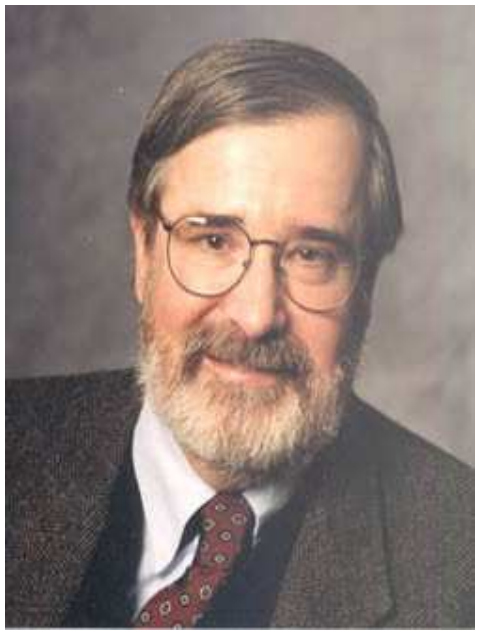
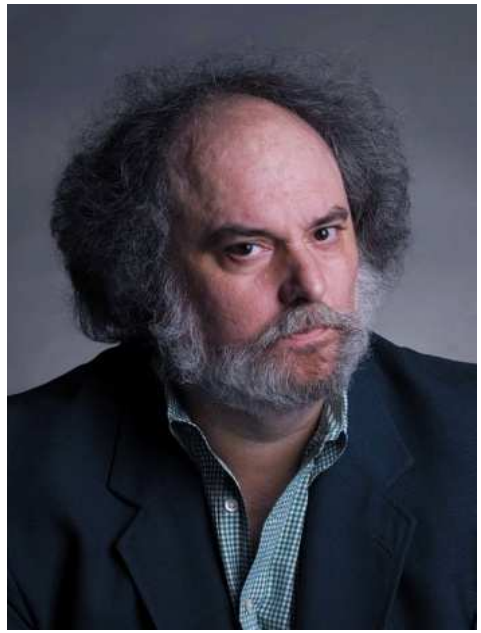
CS 228 : Logic in Computer Science

Krishna. S

Linear Temporal Logic

Model Checking

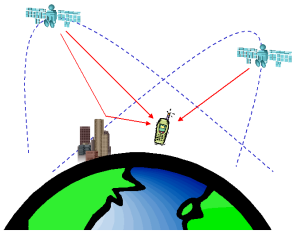



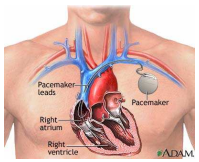

- Year 2007 : ACM confers the Turing Award to the pioneers of Model Checking: Ed Clarke, Allen Emerson, and Joseph Sifakis
- `https://amturing.acm.org/award_winners/clarke_1167964`

Model checking

- Model checking has evolved in last 25 years into a widely used verification and debugging technique for software and hardware.
- Model checking used (and further developed) by companies/institutes such as IBM, Intel, NASA, Cadence, Microsoft, and Siemens, and has culminated in many freely downloadable software tools that allow automated verification.

What is Model Checking?





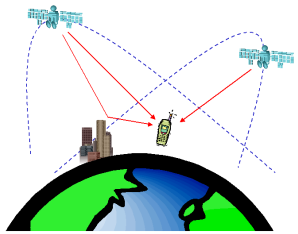
System

satisfy?

specification

good/bad properties

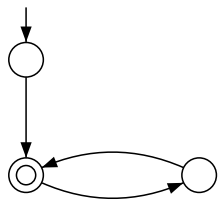
What is Model Checking?



System —— satisfy? ——> specification

System Model —— model-checking ——> Spec Model

$\models$?



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Model Checker as a Black Box

- Inputs to Model checker : A finite state system M, and a property P to be checked.
- Question : Does M satisfy P?
- Possible Outputs
  - Yes, M satisfies P
  - No, here is a counter example!.

What are Models?

Transition Systems

- States labeled with propositions
- Transition relation between states
- Action-labeled transitions to facilitate composition

What are Properties?

Example properties

- Can the system reach a deadlock?
- Can two processes ever be together in a critical section?
- On termination, does a program provide correct output?

Notations for Infinite Words

- Σ is a finite alphabet
- Σ^* set of finite words over Σ
- An infinite word is written as $\alpha = \alpha(0)\alpha(1)\alpha(2)\ldots$, where $\alpha(i) \in \Sigma$
- Such words are called ω-words
- a^ω, $a^7.b^\omega$

Transition Systems

A Transition System is a tuple $(S, Act, \rightarrow, I, AP, L)$ where

- S is a set of states
- Act is a set of actions
- $s \xrightarrow{\alpha} s'$ in $S \times Act \times S$ is the transition relation
- $I \subseteq S$ is the set of initial states
- AP is the set of atomic propositions
- $L : S \rightarrow 2^{AP}$ is the labeling function

Traces of Transition Systems

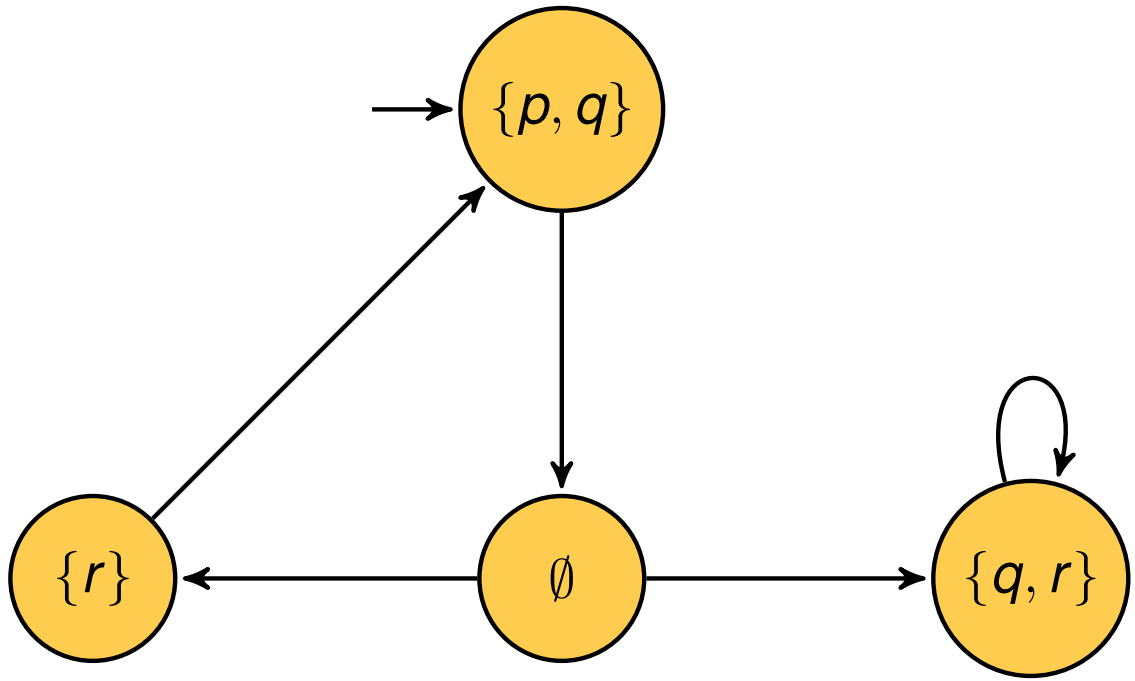
- Labels of the locations represent values of all observable propositions $\in AP$
- Captures system state
- Focus on sequences $L(s_0)L(s_1)\ldots$ of labels of locations
- Such sequences are called traces
- Assuming transition systems have no terminal states,
  - Traces are infinite words over 2^{AP}
  - Traces $\in (2^{AP})^{\omega}$
  - Go to the example slide and define traces

Traces of Transition Systems

Given a transition system $TS = (S, Act, \rightarrow, I, AP, L)$ without terminal states,

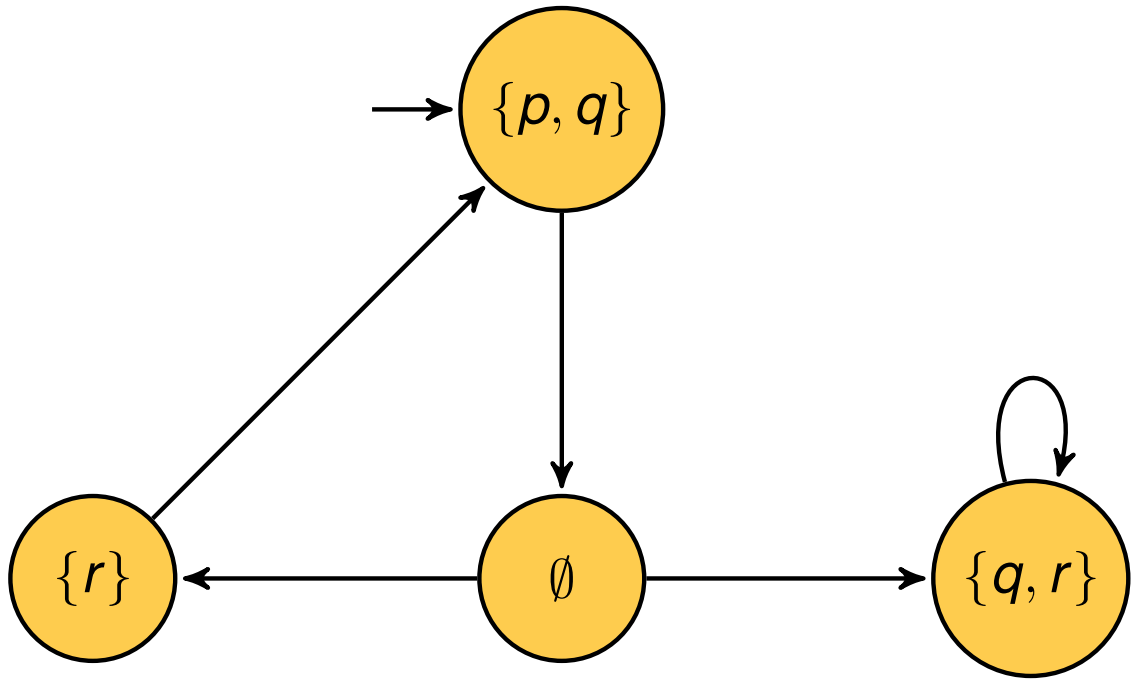
- All maximal executions/paths are infinite
- Path $\pi = s_0 s_1 s_2 \ldots$, $trace(\pi) = L(s_0)L(s_1)\ldots$
- For a set Π of paths, $Trace(\Pi) = \{trace(\pi) \mid \pi \in \Pi\}$
- For a location s, $Traces(s) = Trace(Paths(s))$
- $Traces(TS) = \bigcup_{s \in I} Traces(s)$

Example Traces



$AP = \{p, q, r, t\}$

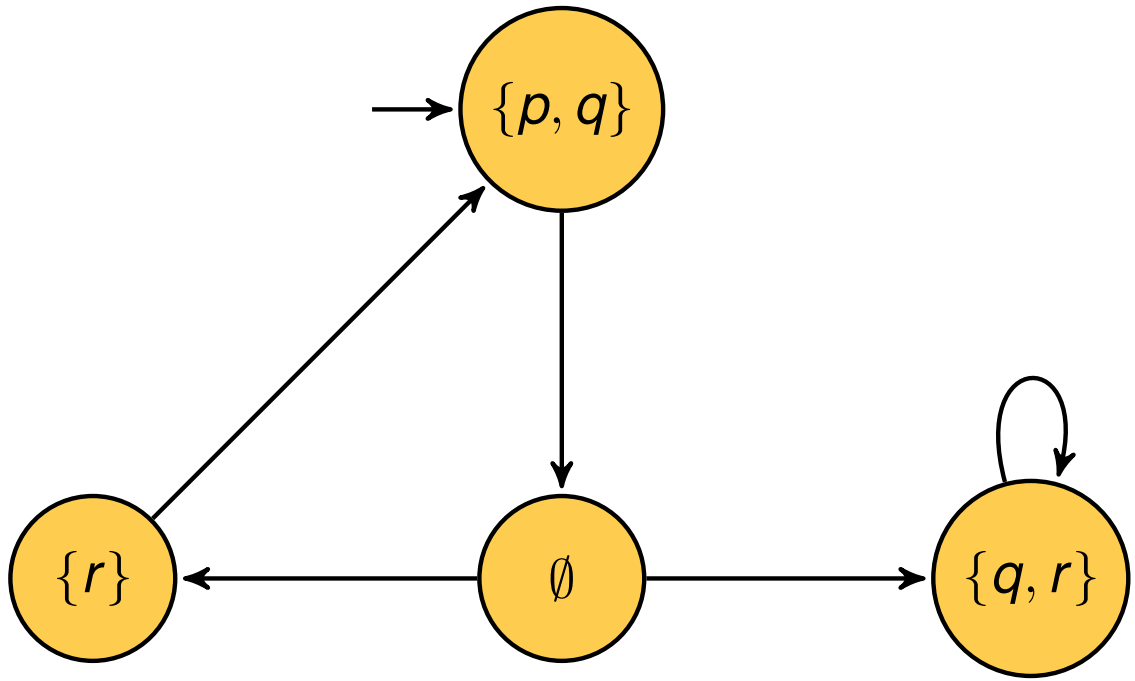
Example Traces



$AP = \{p, q, r, t\}$

- $\{p, q\}\emptyset\ \{q, r\}^{\omega}$

Example Traces



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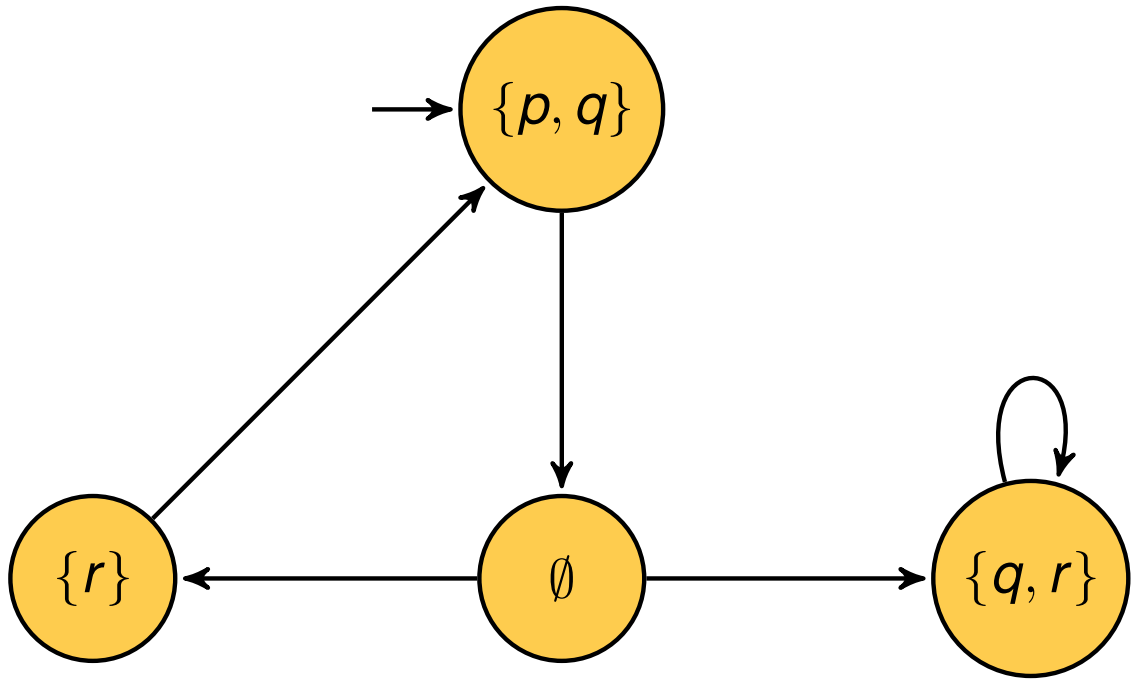
Example Traces



$AP = \{p, q, r, t\}$

- $\{p, q\}\emptyset\ \{q, r\}^\omega$
- $(\{p, q\}\emptyset\{r\})^\omega$
- $(\{p, q\}\emptyset\{r\})^*\ \{p, q\}\emptyset\ \{q, r\}^\omega$

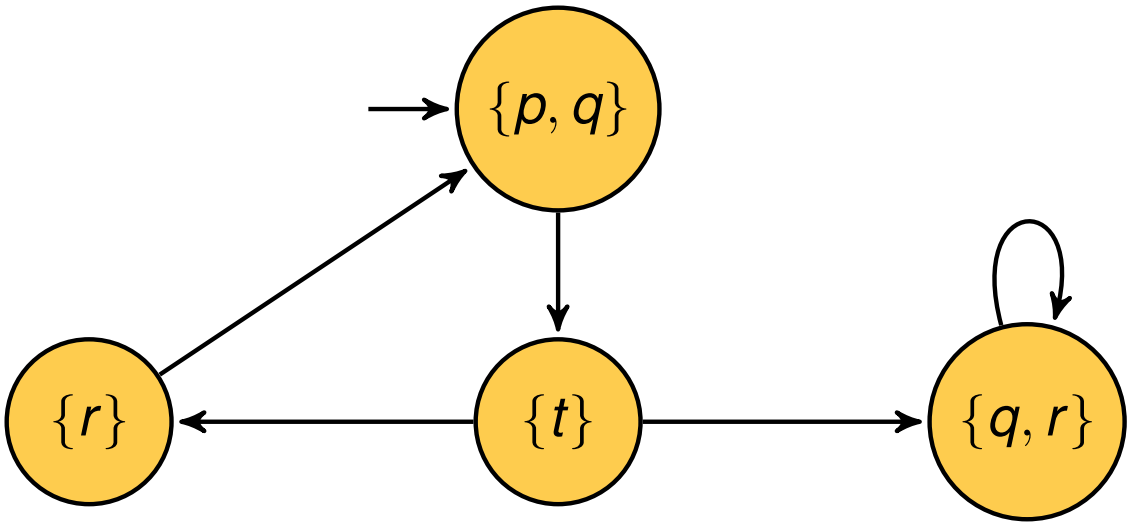
Linear Time Properties

- Linear-time properties specify traces that a TS must have
- A LT property P over AP is a subset of $(2^{AP})^{\omega}$
- TS over AP satisfies a LT property P over AP

$$TS \models P \text{ iff } \mathit{Traces}(TS) \subseteq P$$

- $s \in S$ satisfies LT property P (denoted $s \models P$) iff $\mathit{Traces}(s) \subseteq P$

Specifying Traces



- Whenever p is true, r will eventually become true
  - $\{A_0 A_1 A_2 \cdots \mid \forall i \geqslant 0, p \in A_i \rightarrow \exists j \geqslant i, r \in A_j\}$
- q is true infinitely often
  - $\{A_0 A_1 A_2 \cdots \mid \forall i \geqslant 0, \exists j \geqslant i, q \in A_j\}$
- Whenever r is true, so is q
  - $\{A_0 A_1 \cdots \mid \forall i \geqslant 0, r \in A_i \rightarrow q \in A_i\}$

Syntax of Linear Temporal Logic

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Syntax of Linear Temporal Logic

Given AP, a set of propositions,

- Propositional logic formulae over AP
  - $a \in AP$ (atomic propositions)
  - $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$
- Temporal Operators
  - $\bigcirc\varphi$ (Next φ)
  - $\varphi \, \mathrm{U} \psi$ (φ holds until a ψ-state is reached)
- LTL : Logic for describing LT properties

Semantics (On the board)

LTL formulae φ over AP interpreted over words $w \in \Sigma^\omega$, $\Sigma = 2^{AP}$, $w \models \varphi$

Derived Operators

- $true = \varphi \vee \neg\varphi$
- $false = \neg true$
- $\Diamond\varphi = true\,\mathrm{U}\varphi$ (Eventually φ)
- $\Box\varphi = \neg\Diamond\neg\varphi$ (Forever φ)

What does true until phi means? Does it mean some other propos. will hold until we see phi to be true.

Precedence

- Unary Operators bind stronger than Binary
- $\bigcirc$ and $\neg$ equally strong
- U takes precedence over $\wedge, \vee, \rightarrow$
  - $a \vee b\,\mathrm{U}c \equiv a \vee (b\,\mathrm{U}c)$
  - $\bigcirc a\,\mathrm{U}\neg b \equiv (\bigcirc a)\,\mathrm{U}(\neg b)$

starting at a state move to next state, at next state 'a' must be true and it must continue to hold in each subsequent state until we find '-b'. So when we are done at a position where 'a' and '-b' both will hold.

Examples

- Whenever the traffic light is red, it cannot become green immediately:

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  $\Box(\mathit{red} \rightarrow \neg \bigcirc \mathit{green})$
- Eventually the traffic light will become yellow
  $\Diamond \mathit{yellow}$
- Once the traffic light becomes yellow, it will eventually become green

Examples

Important note about the until operator, aUb means a holds until b becomes true but it does not asserts that a would be false thereafter.

- Whenever the traffic light is red, it cannot become green immediately:
  $\Box(red \rightarrow \neg \bigcirc green)$
- Eventually the traffic light will become yellow
  $\Diamond yellow$
- Once the traffic light becomes yellow, it will eventually become green
  $\Box(yellow \rightarrow \Diamond green)$

Semantics over Infinite Words

Given LTL formula φ over AP,

$$L(\varphi) = \{\sigma \in (2^{AP})^\omega \mid \sigma \models \varphi\}$$

Let $\sigma = A_0 A_1 A_2 \ldots$.

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- $\sigma \models a$ iff $a \in A_0$

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Semantics over Infinite Words

Doubt on first bullet.

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Semantics over Infinite Words

A_i, represents the set of propositions that are true in the ith position of infinite sequence.

Given LTL formula φ over AP,

$$L(\varphi) = \{\sigma \in (2^{AP})^{\omega} \mid \sigma \models \varphi\}$$

Let $\sigma = A_0 A_1 A_2 \ldots$. Each A_i is a subset of atomic proposition.

- $\sigma \models a$ iff $a \in A_0$ atomic proposition 'a' holds at the first position (state) of the word sigma
- $\sigma \models \varphi_1 \wedge \varphi_2$ iff $\sigma \models \varphi_1$ and $\sigma \models \varphi_2$
- $\sigma \models \neg\varphi$ iff $\sigma \not\models \varphi$ If it satisfies negation of a formula, then it won't satisfy the formula at all.
- $\sigma \models \bigcirc\varphi$ iff $A_1 A_2 \ldots \models \varphi$ It is compulsory for phi, to hold true from the next state.
- $\sigma \models \varphi \, U \psi$ iff after certain position psi is true before that state phi has to be true.
  $\exists j \geqslant 0$ such that $A_j A_{j+1} \ldots \models \psi \wedge \forall 0 \leqslant i < j, A_i A_{i+1} \ldots \models \varphi$

First point implies if word satisfies any atomic proposition then it must have to be in the very first state.

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Semantics over Infinite Words

Given LTL formula φ over AP,

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- $\sigma \models \Diamond\varphi$ iff $\exists j \geqslant 0$, $A_j A_{j+1} \ldots \models \varphi$ \exists a 'j' after which phi will hold == eventually
- $\sigma \models \Box\varphi$ iff $\forall j \geqslant 0$, $A_j A_{j+1} \ldots \models \varphi$ \all j phi holds == forever
- $\sigma \models \Box\Diamond\varphi$ iff $\forall j \geqslant 0$, $\exists i \geqslant j$, $A_i A_{i+1} \ldots \models \varphi$ Starting any state phi will hold true.
- $\sigma \models \Diamond\Box\varphi$ iff $\exists j \geqslant 0$, $\forall i \geqslant j$, $A_i A_{i+1} \ldots \models \varphi$ eventually for all phi, word will hold meaning there will be a 'i' after which phi will continuously hold.

If $\sigma = A_0 A_1 A_2 \ldots$, $\sigma \models \varphi$ is also written as $\sigma, 0 \models \varphi$. This simply means $A_0 A_1 A_2 \ldots \models \varphi$. One can also define $\sigma, i \models \varphi$ to mean $A_i A_{i+1} A_{i+2} \ldots \models \varphi$ to talk about a suffix of the word σ satisfying a property.

Transition System Semantics $TS \models \varphi$

Let $TS = (S, S_0, \rightarrow, AP, L)$ be a transition system, and φ an LTL formula over AP

- For an infinite path fragment π of TS,

$$\pi \models \varphi \text{ iff } \mathit{trace}(\pi) \models \varphi$$

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- For $s \in S$,

$$s \models \varphi \text{ iff } \forall \pi \in \mathit{Paths}(s), \pi \models \varphi$$

Transition System Semantics $TS \models \varphi$

L: The labeling function, which assigns to each state a set of atomic propositions that hold in that state.

Let $TS = (S, S_0, \rightarrow, AP, L)$ be a transition system, and φ an LTL formula over AP

- For an infinite path fragment π of TS,

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An infinite path fragment PI in a transition system is a sequence of states that follows the transition relation, starting from a particular state and
traces have the same condition except it requires all the atomic proposition in all the states of sequence have to hold true i.e. it removes all the AP of state not holding true for current word in its TS.

- For $s \in S$,

$$s \models \varphi \text{ iff } \forall \pi \in \mathit{Paths}(s), \pi \models \varphi$$

A state s satisfies an LTL formula phi if and only if all infinite paths starting from s satisfy phi.

- $TS \models \varphi$ iff $\mathit{Traces}(TS) \subseteq L(\varphi)$

In other words, all behaviors of the system (represented by traces of all infinite paths in the system) must satisfy the formula phi. If any trace generated by the system fails to satisfy phi, then the system does not satisfy phi.

Transition System Semantics $TS \models \varphi$

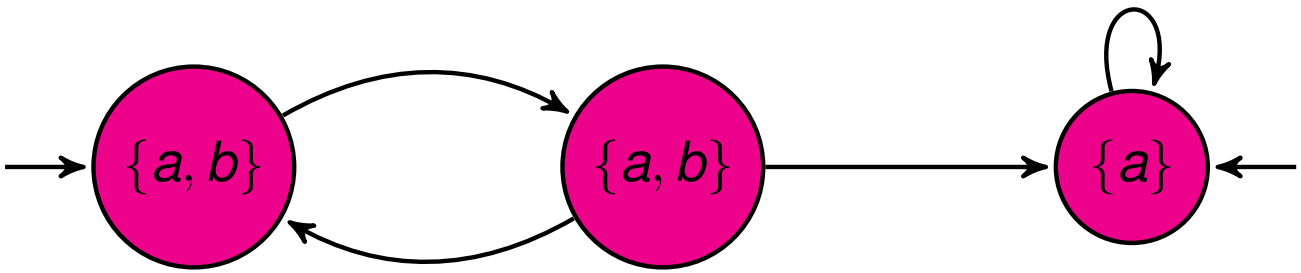
Assume all states in TS are reachable from S_0.

- $TS \models \varphi$ iff $TS \models L(\varphi)$ iff $Traces(TS) \subseteq L(\varphi)$
- $TS \models L(\varphi)$ iff $\pi \models \varphi \; \forall \pi \in Paths(TS)$
- $\pi \models \varphi \; \forall \pi \in Paths(TS)$ iff $s_0 \models \varphi \; \forall s_0 \in S_0$

2) Thus, to verify if the transition system satisfies the LTL formula, you can check all paths starting from any state within the system. If every path satisfies phi, then the system satisfies L(phi)

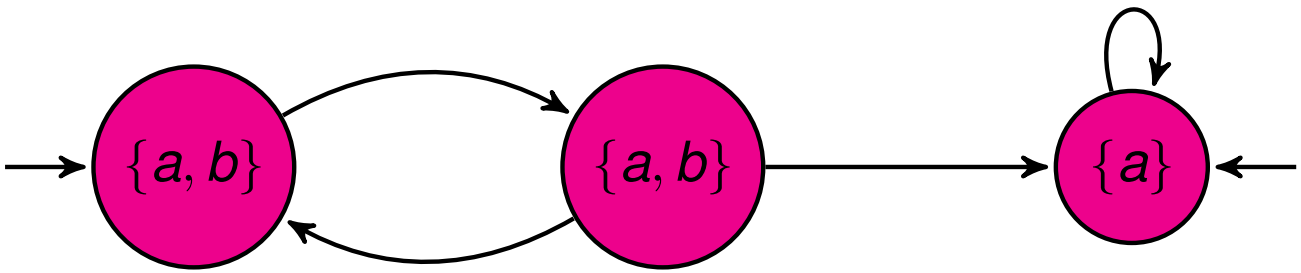
3) if every path from any initial state s0 in the system satisfies phi, then the initial state itself must also fulfill the requirements of the formula. This connection ensures that the properties expressed by phi are guaranteed from the starting point of the system.

Example



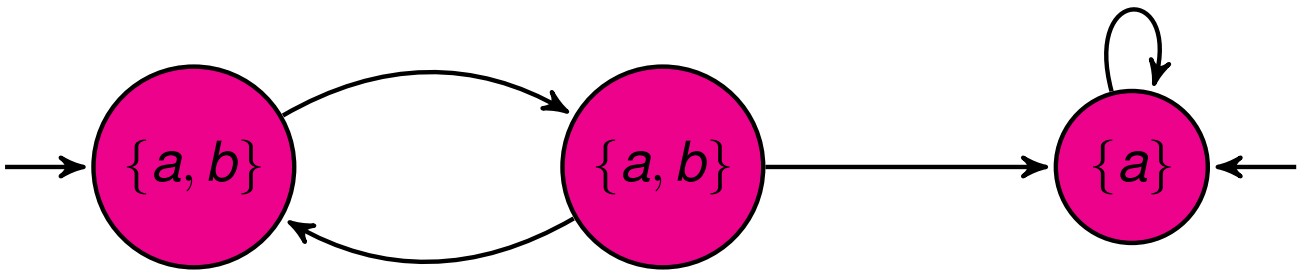
- $TS \models \Box a$,

Example



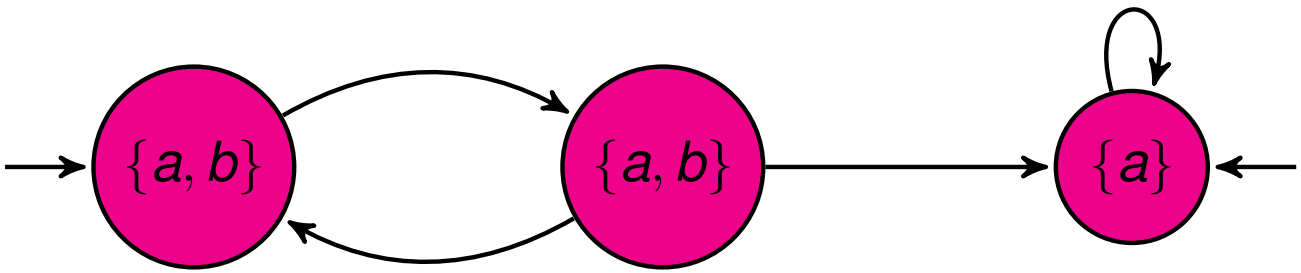
- $TS \models \Box a$,
- $TS \not\models \bigcirc(a \wedge b)$

Example



- $TS \models \Box a$,
- $TS \not\models \bigcirc(a \wedge b)$ Because all initial states does not guarantee it.
- $TS \not\models (b \,\mathrm{U}(a \wedge \neg b))$ All initial state doesn't hold as well as there is no negation b.

Example



- $TS \models \Box a$,
- $TS \not\models \bigcirc(a \wedge b)$
- $TS \not\models (b \,\mathsf{U}(a \wedge \neg b))$
- $TS \models \Box(\neg b \rightarrow \Box(a \wedge \neg b))$ forever when 'b' is not true it implies that forever only 'a' is true which is indeed the case.

More Semantics

- For paths π, $\pi \models \varphi$ iff $\pi \not\models \neg\varphi$

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- For paths π, $\pi \models \varphi$ iff $\pi \not\models \neg\varphi$
  $\mathit{trace}(\pi) \in L(\varphi)$ iff $\mathit{trace}(\pi) \notin L(\neg\varphi) = \overline{L(\varphi)}$
- $TS \not\models \varphi$ iff $TS \models \neg\varphi$?
  - $TS \models \neg\varphi \rightarrow \forall$ paths π of TS, $\pi \models \neg\varphi$
  - Thus, $\forall\pi$, $\pi \not\models \varphi$. Hence, $TS \not\models \varphi$

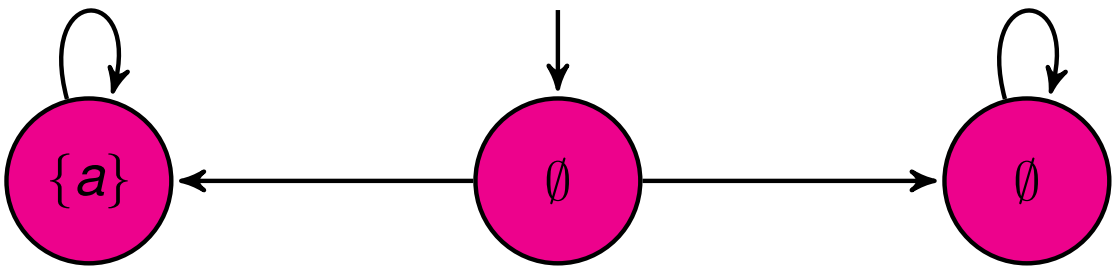
More Semantics

- For paths π, $\pi \models \varphi$ iff $\pi \not\models \neg\varphi$
  $trace(\pi) \in L(\varphi)$ iff $trace(\pi) \notin L(\neg\varphi) = \overline{L(\varphi)}$
- $TS \not\models \varphi$ iff $TS \models \neg\varphi$? So the argument that TS system does not satisfies phi, if and only if it satisfies negation phi, is incorrect because it doesn't matter.
  - $TS \models \neg\varphi \rightarrow \forall$ paths π of TS, $\pi \models \neg\varphi$
  - Thus, $\forall\pi$, $\pi \not\models \varphi$. Hence, $TS \not\models \varphi$
  - Now assume $TS \not\models \varphi$
  - Then $\exists$ some path π in TS such that $\pi \models \neg\varphi$
  - However, there could be another path π' such that $\pi' \models \varphi$
  - Then $TS \not\models \neg\varphi$ as well
- Thus, $TS \not\models \varphi \not\equiv TS \models \neg\varphi$. This whole argument is incorrect as there can be paths satisfying phi and neg(phi) for the same TS.

An Example

Doubt in this example.

left part satisfies the left of the
formula and right satisfies the right of the formula. But do we need the formula to be satisfied across all path to say TS satisfies the formula?



$TS \not\models \Diamond a$ and $TS \not\models \Box \neg a$

TS not satisfying both phi and neg-phi.

Equivalence of LTL Formulae

Even though they are syntactically different they must satisfy the same "Temporal Logic".

Equivalence

φ and ψ are equivalent ($\varphi \equiv \psi$) iff $L(\varphi) = L(\psi)$.

Expansion Laws

- $\varphi \, \mathsf{U} \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \psi))$ phi holds true and continue to hold true from the next state as well until it finds psi to be true.
- $\Diamond \varphi \equiv \varphi \vee \bigcirc \Diamond \varphi$
- $\Box \varphi \equiv \varphi \wedge \bigcirc \Box \varphi$

Equivalence of LTL Formulae

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Equivalence of LTL Formulae

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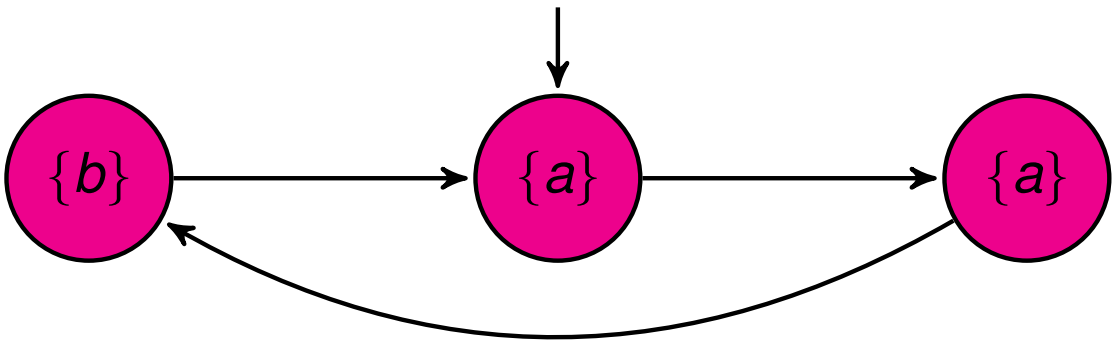
Doubt in this.

Distribution

$\bigcirc(\varphi \vee \psi) \equiv \bigcirc\varphi \vee \bigcirc\psi$,
$\bigcirc(\varphi \wedge \psi) \equiv \bigcirc\varphi \wedge \bigcirc\psi$,
$\bigcirc(\varphi \,\mathsf{U}\, \psi) \equiv (\bigcirc\varphi)\,\mathsf{U}\,(\bigcirc\psi)$,
$\Diamond(\varphi \vee \psi) \equiv \Diamond\varphi \vee \Diamond\psi$,
$\Box(\varphi \wedge \psi) \equiv \Box\varphi \wedge \Box\psi$

Equivalence of LTL Formulae



$$TS \models \Diamond a \wedge \Diamond b, TS \not\models \Diamond(a \wedge b)$$

$$TS \models \Box(a \vee b), TS \not\models \Box a \vee \Box b$$

You can get any one out of 'a' or 'b' but you won't be able to get both forever a and forever b at the same state.

Satisfiability, Model Checking of LTL

Two Questions

Given transition system TS, and an LTL formula φ. Does $TS \models \varphi$?
Given an LTL formula φ, is $L(\varphi) = \emptyset$?

How we go about this:

- Translate φ into an automaton A_φ that accepts infinite words such that $L(A_\varphi) = L(\varphi)$.
- Check for emptiness of A_φ to check satisfiability of φ.
- Check if $TS \cap \overline{A_\varphi}$ is empty, to answer the model-checking problem.

ω-automata

An ω-automaton is a tuple $\mathcal{A} = (Q, \Sigma, \delta, q_0, Acc)$ where

- Q is a finite set of states
- Σ is a finite alphabet
- $\delta : Q \times \Sigma \to 2^Q$ is a state transition function (if non-deterministic, otherwise, $\delta : Q \times \Sigma \to Q$) Any transition will give set of states.
- $q_0 \in Q$ is an initial state and Acc is an acceptance condition

There is always an accepting condition and initial states.

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Run

A run ρ of $\mathcal{A}$ on an ω-word $\alpha = a_1 a_2 \cdots \in \Sigma^\omega$ is an infinite state sequence $\rho(0)\rho(1)\rho(2)\ldots$ such that

- $\rho(0) = q_0$, Initial condition put.
- $\rho(i) = \delta(\rho(i-1), a_i)$ if $\mathcal{A}$ is deterministic,
- $\rho(i) \in \delta(\rho(i-1), a_i)$ if $\mathcal{A}$ is non-deterministic,

Transition happening.

ω-automata

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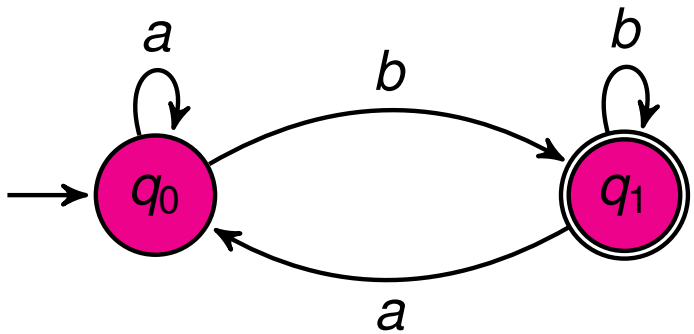
Accepting states should come infinite times in Buchi acceptance run.

Büchi Acceptance

For Büchi Acceptance, Acc is specified as a set of states, $G \subseteq Q$. The ω-word α is accepted if there is a run ρ of α such that $Inf(\rho) \cap G \neq \emptyset$.

ω-Automata with Büchi Acceptance

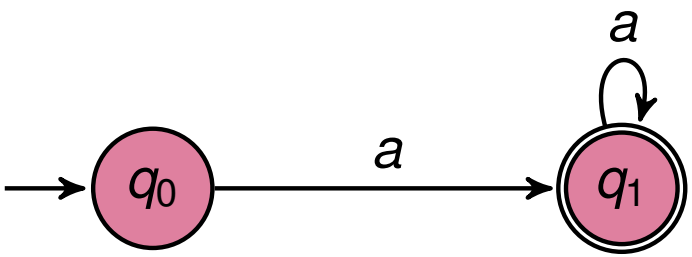


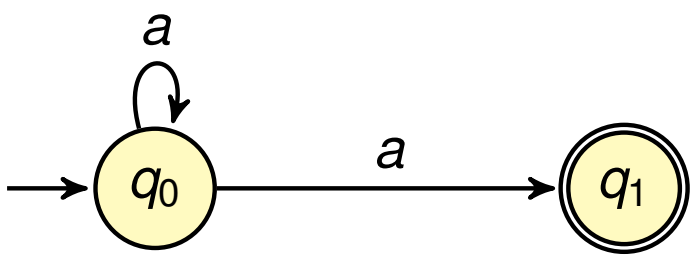
aUb buchi automaton.

$L(\mathcal{A}) = \{\alpha \in \Sigma^\omega \mid \alpha \text{ has a run } \rho \text{ such that } \mathit{Inf}(\rho) \cap G \neq \emptyset$

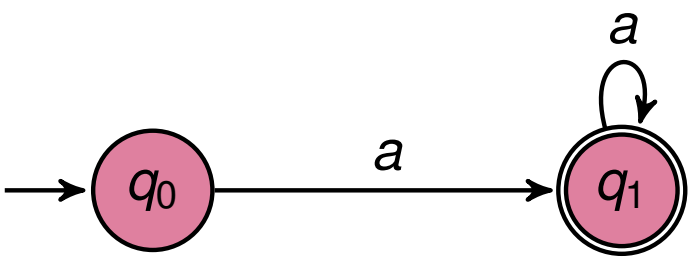
Language accepted=Infinitely many *b*'s.

Comparing NFA and NBA

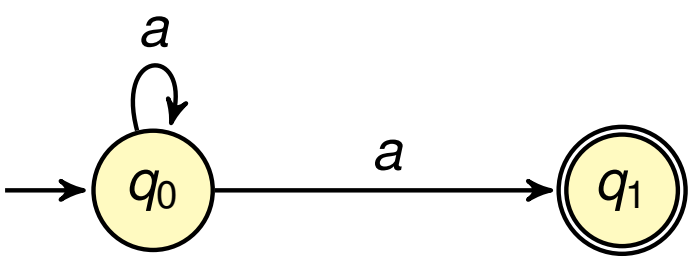




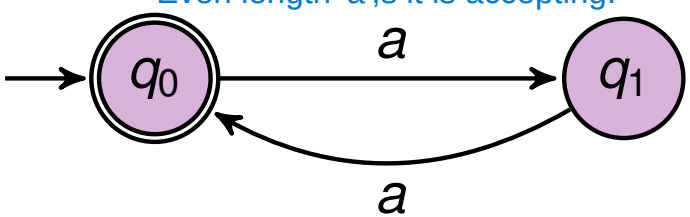
Comparing NFA and NBA



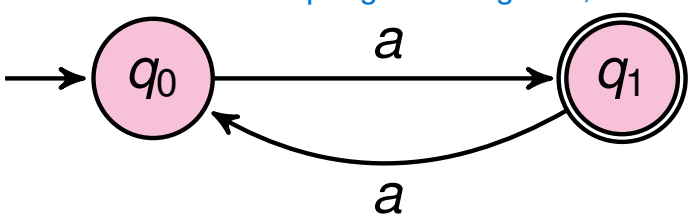
both these are equivalent.



Even length 'a',s it is accepting.

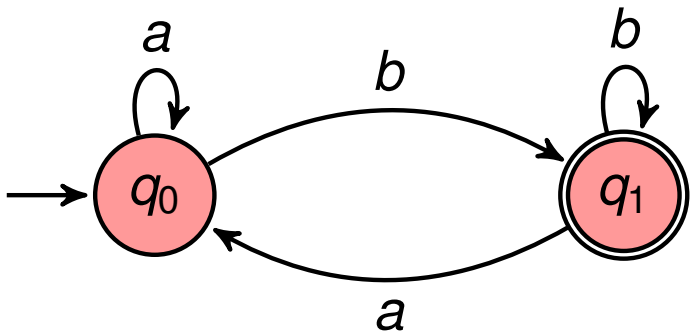


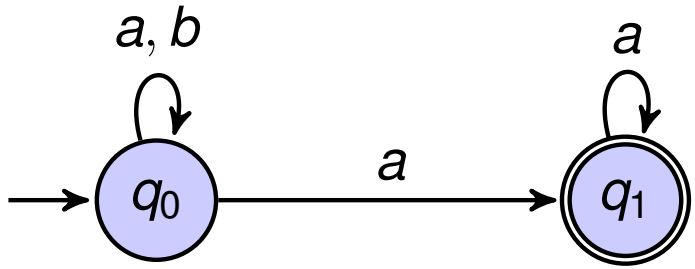
It is accepting odd length 'a',s

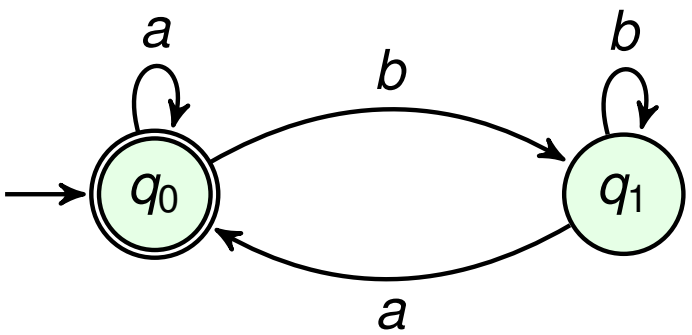


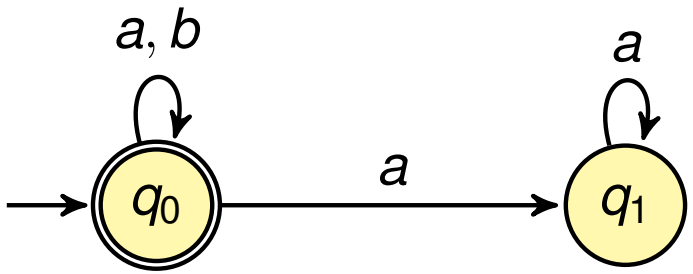
ω-Automata with Büchi Acceptance

One thing to observe in the buchi automaton is that accepting states have self loop, i.e. there can be infinite run before accepting the state.







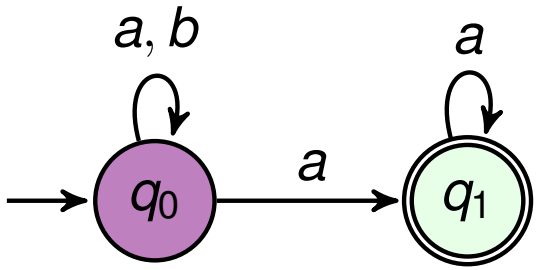


- Left (T-B): Inf many b's, Inf many a's
- Right (T-B): Finitely many b's, $(a+b)^\omega$

NBA and DBA

- Is every DBA as expressible as a NBA, like in the case of DFA and NFA? Don't know
- Can we do subset construction on NBA and obtain DBA?

No.



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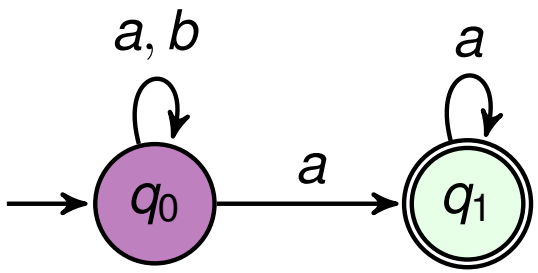

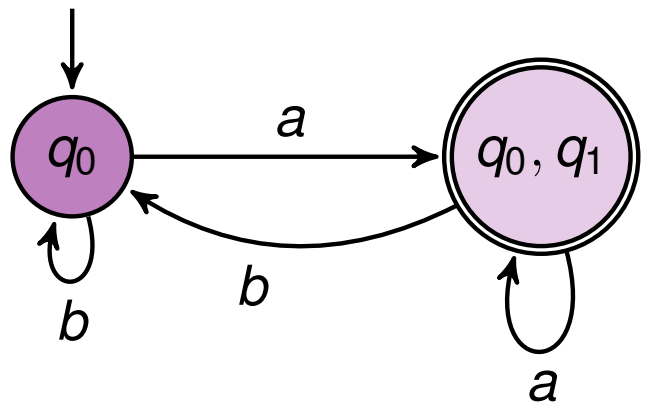

NBA and DBA

There does not exist a deterministic Büchi automata capturing the language finitely many *a*'s.

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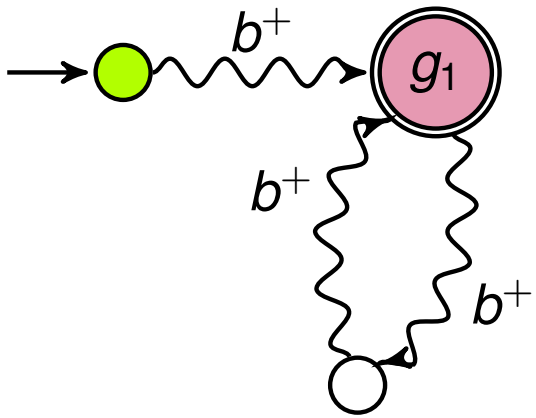

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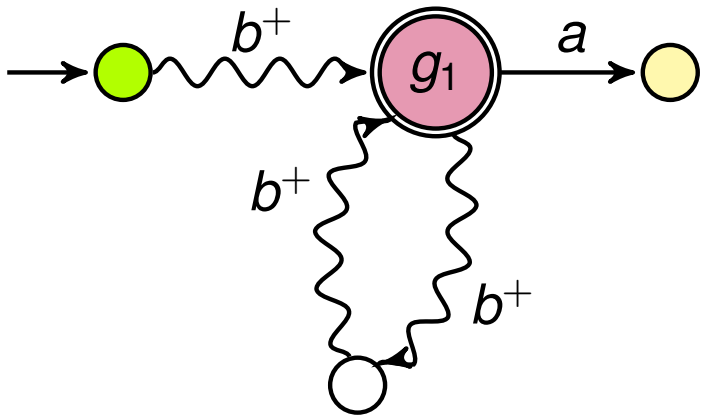

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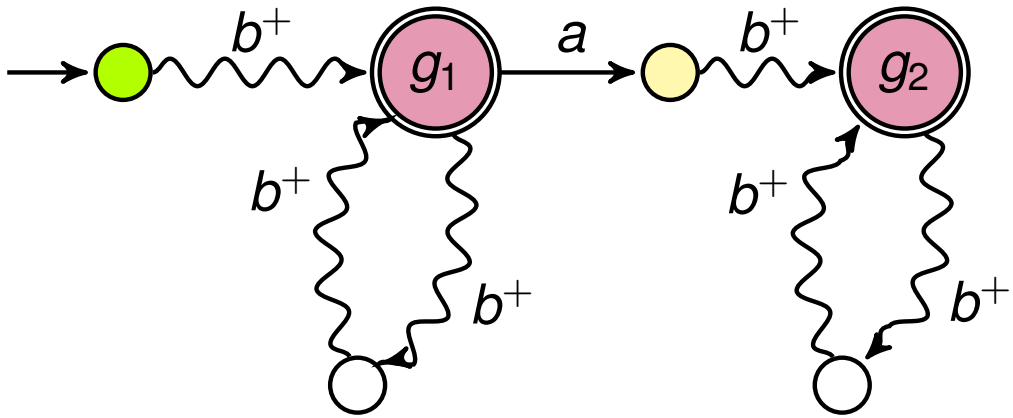

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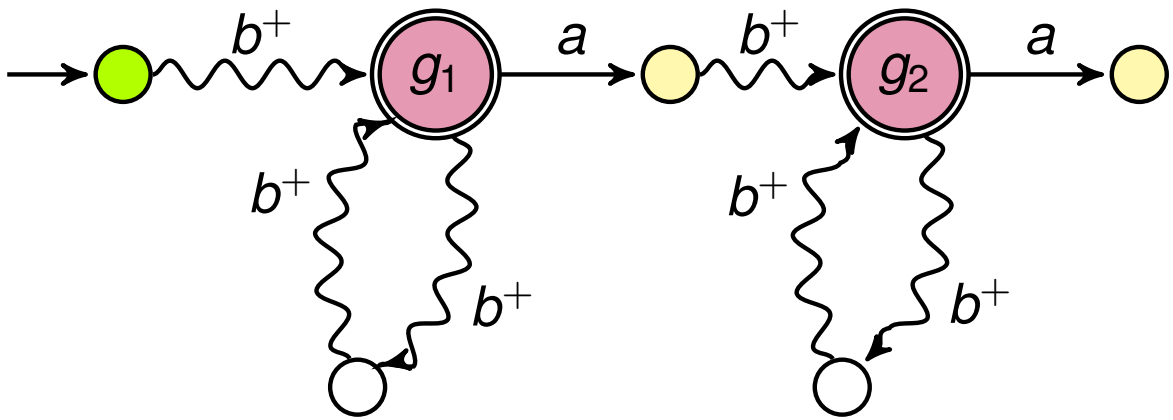
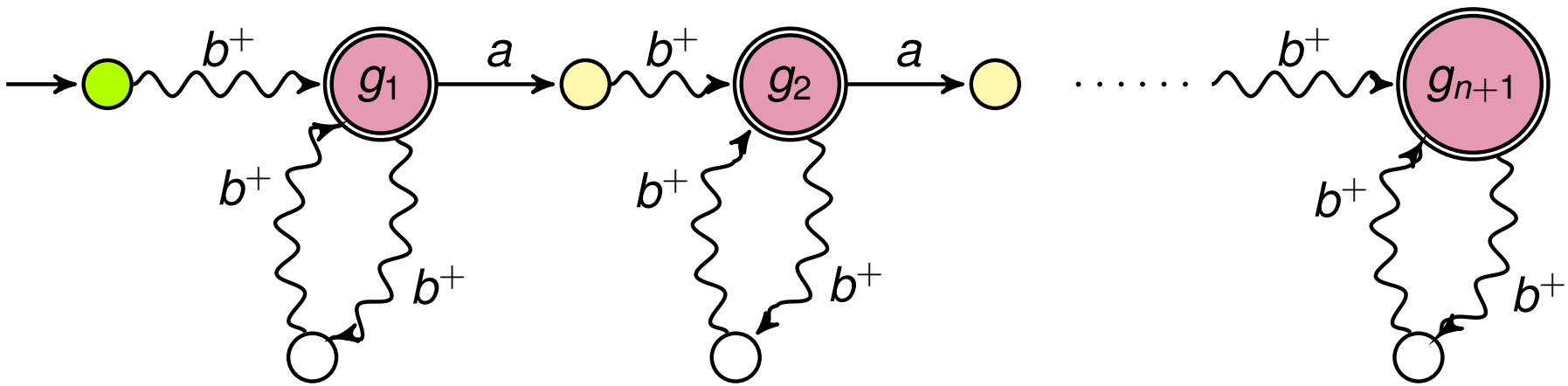


NBA and DBA

Showing limitation of DBA
this example illustrates that while deterministic Büchi automata are powerful for some types of infinite languages, they fail to capture languages where a certain action or condition happens only finitely many times. This difference in expressiveness between DBAs and NBAs is a key concept in automata theory.

There does not exist a deterministic Büchi automata capturing the language finitely many *a*'s.



In NBA due to non-determinism, they are allowed to make a guess when does the last occurrence of 'a' occurs.

In DBA it is unable to determine when does that last occurrence of 'a' happens.