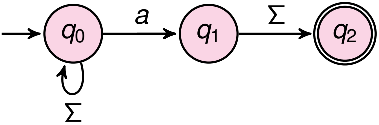
CS 228 : Logic in Computer Science

Krishna. S

Recap

- FOL over words : Satisfiability
- Translation from formulae φ to equivalent DFA A, $L(\varphi) = L(A)$
- Proof by structural induction, with $\neg, \wedge, \vee$ mapping to complementation, intersection and union
  - Union in DFA–> disjunction in logic
  - Intersection in DFA–> conjunction in logic
  - Complementation in DFA –> Negation in logic
- How to handle quantifiers?

Non-determinism



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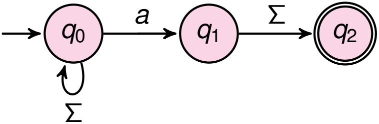

- Assume we relax the condition on transitions, and allow
  - $\delta : Q \times \Sigma \to 2^Q$
  - $\delta(q_0, a) = \{q_0, q_1\}, \delta(q_2, a) = \delta(q_2, b) = \emptyset$
  - Is *aabb* accepted?

One run of *aabb*

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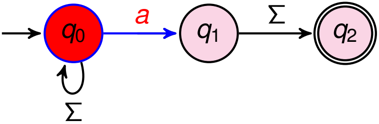

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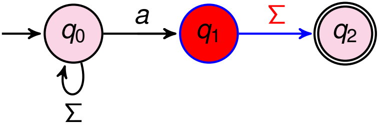

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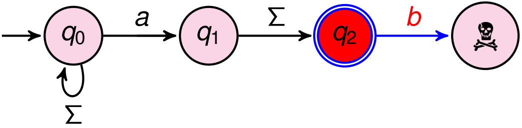

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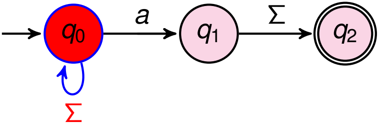

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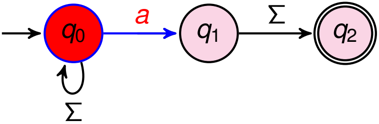

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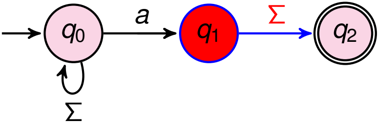

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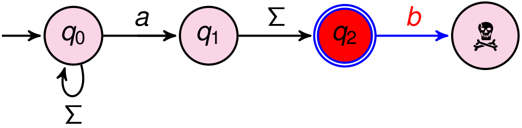

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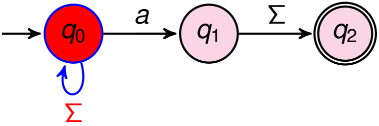

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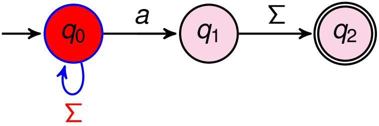

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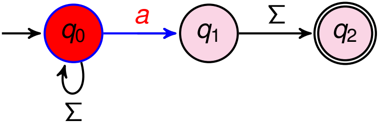

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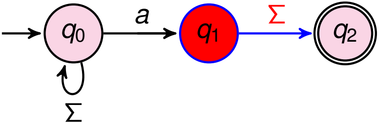
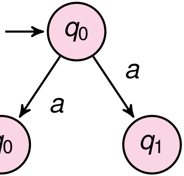


- An accepting run for *aaab*

Nondeterministic Finite Automata(NFA)

- $N = (Q, \Sigma, \delta, Q_0, F)$
  - Q is a finite set of states
  - $Q_0 \subseteq Q$ is the set of initial states
  - $\delta : Q \times \Sigma \rightarrow 2^Q$ is the transition function
  - $F \subseteq Q$ is the set of final states
- Acceptance condition : A word w is accepted iff it has atleast one accepting path

Run Tree of *aaab*



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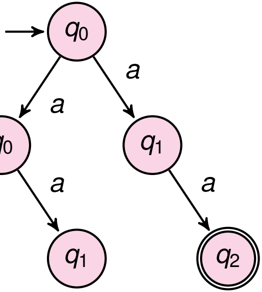

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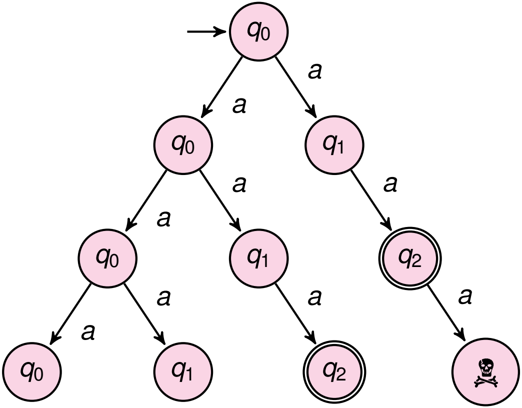

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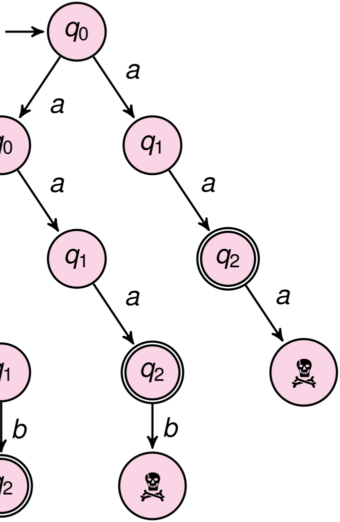

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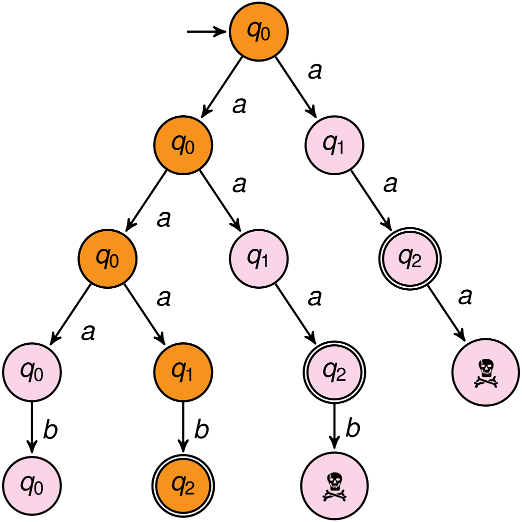

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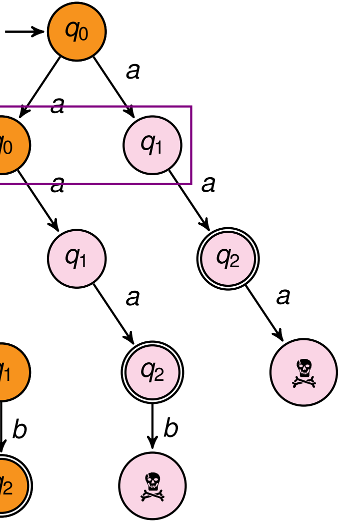

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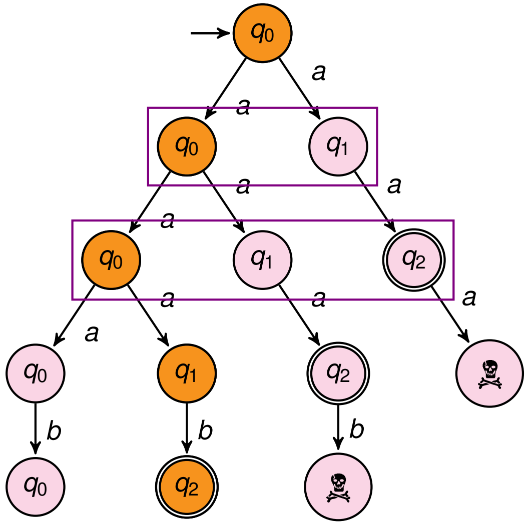

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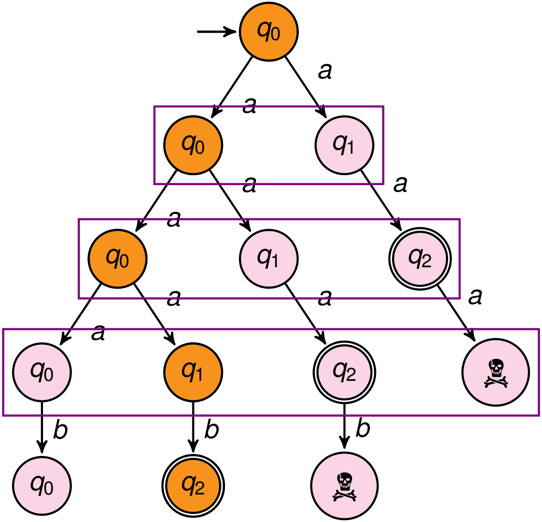

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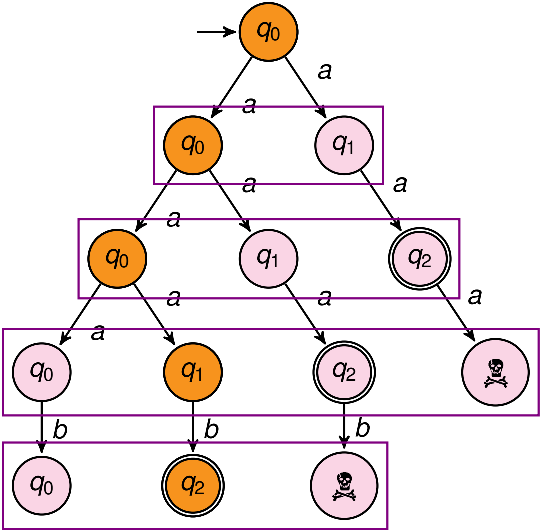
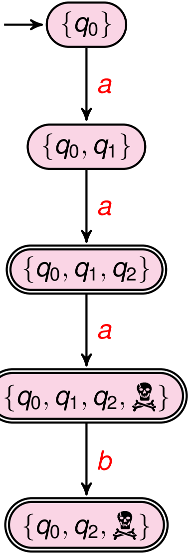
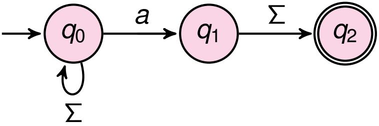


The Single Run



As soon as we get the path which is accepted we are done.

NFA to DFA : On the board



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  - Accept if the obtained set of states contains a final state

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  Union of all the new states after the transition.

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So see after the transition on individual states of DFS state-set we take the union of reached set. And accepting state is that which has atleast one accepting state from NFA.

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NFA = DFA

NFA and DFA are equivalent if they accept the same language.

$$x \in L(D) \leftrightarrow \hat{\Delta}(Q_0, x) \in F'$$

$$\leftrightarrow$$

$$\hat{\delta}(Q_0, x) \in F'$$

$$\leftrightarrow$$

DFA reaches an accepting state if and only if the NFA also reaches an accepting state on some computation path.

$$\hat{\delta}(Q_0, x) \cap F \neq \emptyset$$

$$\leftrightarrow$$

$$x \in L(N)$$

Regularity

A language L is regular iff there exists an NFA A such that $L = L(A)$