CS 228 : Logic in Computer Science

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GNBA

- Generalized NBA, a variant of NBA
- Only difference is in acceptance condition
- Acceptance condition in GNBA is a set $\mathcal{F} = \{F_1, \ldots, F_k\}$, each $F_i \subseteq Q$
- An infinite run ρ is accepting in a GNBA iff

$$\forall F_i \in \mathcal{F}, \mathit{Inf}(\rho) \cap F_i \neq \emptyset$$

- Note that when $\mathcal{F} = \emptyset$, all infinite runs are accepting
- GNBA and NBA are equivalent in expressive power.

The condition that requires visiting final states infinitely often becomes vacuous when there are no final states. As a result, any infinite sequence of states (i.e., any infinite run) satisfies the acceptance condition

Word View (On the board)

a must hold continuously for a period of time, but eventually, a point will be reached where a becomes false, and simultaneously, b becomes true. The formula is only true if this transition happens at some point in the future.

Think of it as a machine is running till task completes.

- $w = \{a\}\{a, b\}\{\}\ldots,$
- $\varphi = a\,\mathsf{U}(\neg a \wedge b)$

  Doubt, Why neg b and neg phi not included?
- Subformulae of $\varphi = \{a, \neg a, b, \neg a \wedge b, \varphi\}$
- Parse trees to compute all subformulae

Closure of φ, $cl(\varphi)$

- $cl(\varphi)$=all subformulae of φ and their negations, identifying $\neg\neg\psi$ to be ψ.
- Example for $\varphi = a \, \mathsf{U}(\neg a \wedge b)$
- $cl(\varphi) = \{a, \neg a, b, \neg b, \neg a \wedge b, \neg(\neg a \wedge b), \varphi, \neg\varphi\}$

Elementary Sets

Let φ be an LTL formula. Then $B \subseteq cl(\varphi)$ is elementary provided:

- B is maximally consistent : for all $\varphi_1 \wedge \varphi_2, \psi \in cl(\varphi)$,
  - $\varphi_1 \wedge \varphi_2 \in B \Leftrightarrow \varphi_1 \in B \wedge \varphi_2 \in B$ if conjunct belongs to set then individual cl. belong to set.
  - $\psi \in B \Leftrightarrow \neg\psi \notin B$ a cl. and its negation can't be in the set simultaneously.
  - $true \in cl(\varphi) \Rightarrow true \in B$
- B is locally consistent wrt U. That is, for all $\varphi_1 \, \mathsf{U} \varphi_2 \in cl(\varphi)$,
  - $\varphi_2 \in B \Rightarrow \varphi_1 \, \mathsf{U} \varphi_2 \in B$ if post until is in set, then whole formula is in set.
  - $\varphi_1 \, \mathsf{U} \varphi_2 \in B, \varphi_2 \notin B \Rightarrow \varphi_1 \in B$ if post until is not in set, then pre until has to be in set given until formula is in set.
- B is elementary : B is maximally and locally consistent
- Given a $B \subseteq cl(\varphi)$, how can you check if B is elementary?

Check Elementary

Let $\varphi = a \, \mathsf{U}(\neg a \wedge b)$

- $B_1 = \{a, b, \neg a \wedge b, \varphi\}$

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- $B_2 = \{\neg a, b, \varphi\}$

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- $B_2 = \{\neg a, b, \varphi\}$ No, not maximal as $\neg a \wedge b \notin B_2$, $\neg(\neg a \wedge b) \notin B_2$
- $B_3 = \{\neg a, b, \neg a \wedge b, \neg\varphi\}$

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- $B_3 = \{\neg a, b, \neg a \wedge b, \neg\varphi\}$ No, not locally consistent for U
- $B_4 = \{\neg a, \neg b, \neg(\neg a \wedge b), \neg\varphi\}$

Check Elementary

Let $\varphi = a \, \mathsf{U}(\neg a \wedge b)$

- $B_1 = \{a, b, \neg a \wedge b, \varphi\}$ No, propositionally inconsistent
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- $B_3 = \{\neg a, b, \neg a \wedge b, \neg\varphi\}$ No, not locally consistent for U
- $B_4 = \{\neg a, \neg b, \neg(\neg a \wedge b), \neg\varphi\}$ Yes, elementary

  But it does not contain neg a and neg b, i.e. conjunct.

LTL φ to GNBA G_φ

- States of G_φ are elementary sets B_i
- For a word $w = A_0 A_1 A_2 \ldots$ the sequence of states $\sigma = B_0 B_1 B_2 \ldots$ will be a run for w
- σ will be accepting iff $w \models \varphi$ iff $\varphi \in B_0$ Word makes the formula true iff the formula is from set.
- In general, a run $B_i B_{i+1} \ldots$ for $A_i A_{i+1} \ldots$ is accepting iff $A_i A_{i+1} \ldots \models \psi$ for all $\psi \in B_i$.

LTL to GNBA

- Let $\varphi = \bigcirc a$. assume next a.
- Subformulae of φ : $\{a, \bigcirc a\}$. Let $A = \{a, \bigcirc a, \neg a, \neg \bigcirc a\}$.
- Possibilities at each state
  - $\{a, \bigcirc a\}$
  - $\{\neg a, \bigcirc a\}$
  - $\{a, \neg \bigcirc a\}$
  - $\{\neg a, \neg \bigcirc a\}$
- Our initial state(s) must guarantee truth of $\bigcirc a$. Thus, initial states: $\{a, \bigcirc a\}$ and $\{\neg a, \bigcirc a\}$

LTL to GNBA

$\{a, \bigcirc a\}$

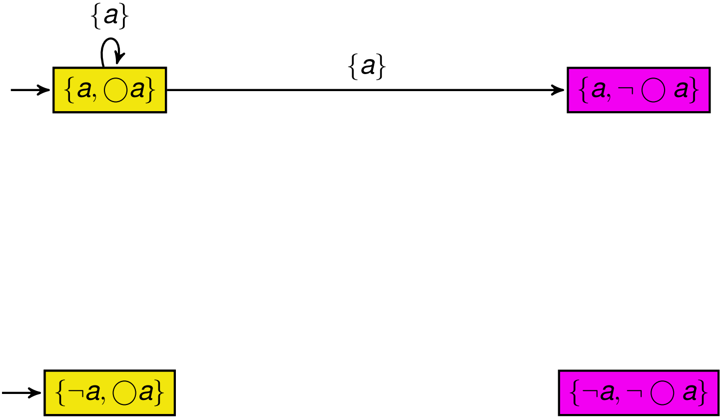
$\{a, \neg \bigcirc a\}$

$\{\neg a, \bigcirc a\}$

$\{\neg a, \neg \bigcirc a\}$

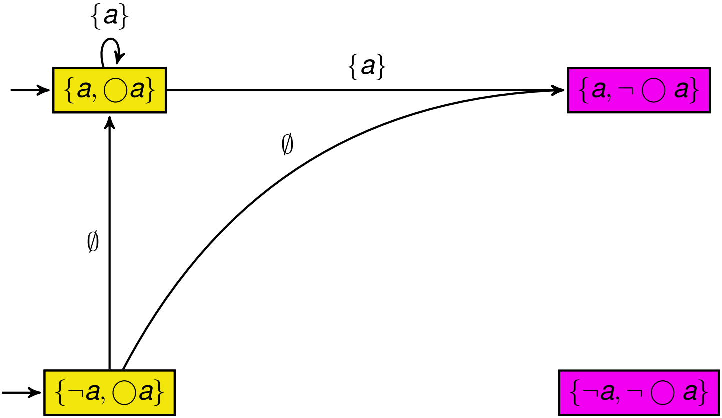
LTL to GNBA

LTL to GNBA



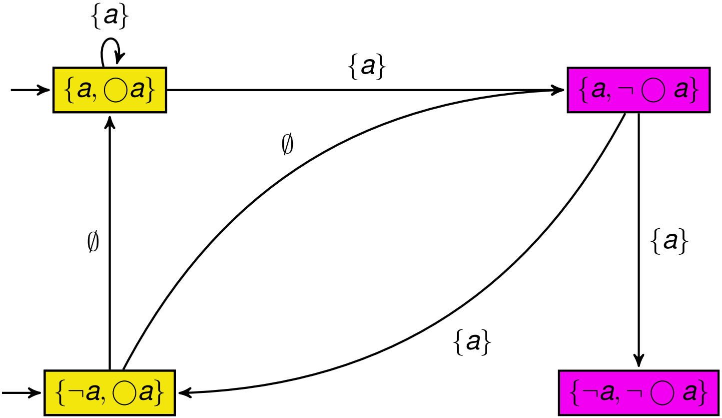
$\{a\}$
$\{a\}$
$\{a, \bigcirc a\}$
$\{a, \neg \bigcirc a\}$
$\{\neg a, \bigcirc a\}$
$\{\neg a, \neg \bigcirc a\}$

LTL to GNBA



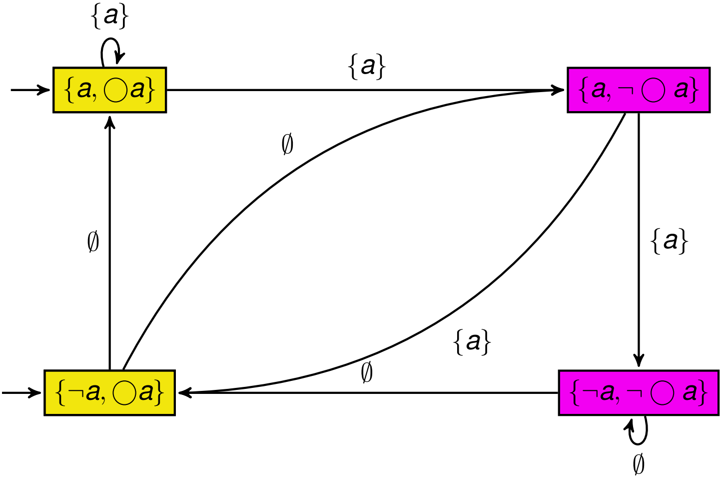
$\{a\}$
$\{a\}$
$\{a, \bigcirc a\}$
$\{a, \neg \bigcirc a\}$
$\emptyset$
$\emptyset$
$\{\neg a, \bigcirc a\}$
$\{\neg a, \neg \bigcirc a\}$

LTL to GNBA



$\{a\}$
$\{a\}$
$\{a, \bigcirc a\}$
$\{a, \neg \bigcirc a\}$
$\emptyset$
$\emptyset$
$\{a\}$
$\{a\}$
$\{\neg a, \bigcirc a\}$
$\{\neg a, \neg \bigcirc a\}$

LTL to GNBA



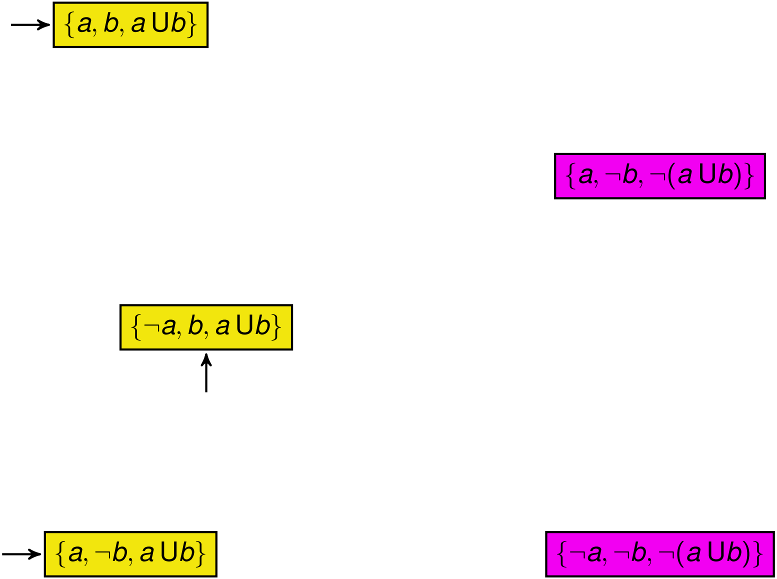
LTL to GNBA

- Claim : Runs from a state labelled set B indeed satisfy B
- No good states. All words having a run from a start state are accepted.
- Automaton for $\neg \bigcirc a$ same, except for the start states.

LTL to GNBA

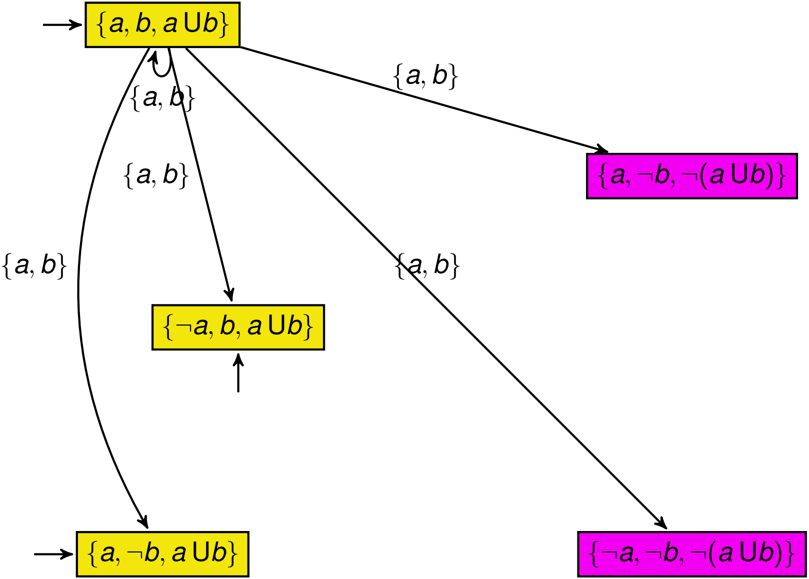
- Let $\varphi = a\,\mathsf{U}b$.
- Subformulae of φ : $\{a, b, a\,\mathsf{U}b\}$. Let $B = \{a, \neg a, b, \neg b, a\,\mathsf{U}b, \neg(a\,\mathsf{U}b)\}$.
- Possibilities at each state
  - $\{a, \neg b, a\,\mathsf{U}b\}$
  - $\{\neg a, b, a\,\mathsf{U}b\}$
  - $\{a, b, a\,\mathsf{U}b\}$
  - $\{a, \neg b, \neg(a\,\mathsf{U}b)\}$
  - $\{\neg a, \neg b, \neg(a\,\mathsf{U}b)\}$
- Our initial state(s) must guarantee truth of $a\,\mathsf{U}b$. Thus, initial states: $\{a, b, a\,\mathsf{U}b\}$ and $\{\neg a, b, a\,\mathsf{U}b\}$ and $\{a, \neg b, a\,\mathsf{U}b\}$.

LTL to GNBA



$\{a, b, a \,\mathsf{U}\, b\}$
$\{a, \neg b, \neg(a \,\mathsf{U}\, b)\}$
$\{\neg a, b, a \,\mathsf{U}\, b\}$
$\{a, \neg b, a \,\mathsf{U}\, b\}$
$\{\neg a, \neg b, \neg(a \,\mathsf{U}\, b)\}$

LTL to GNBA



$\{a, b, a\,\mathsf{U}\,b\}$
$\{a, b\}$
$\{a, b\}$
$\{a, b\}$
$\{a, b\}$
$\{a, b\}$
$\{a, \neg b, \neg(a\,\mathsf{U}\,b)\}$
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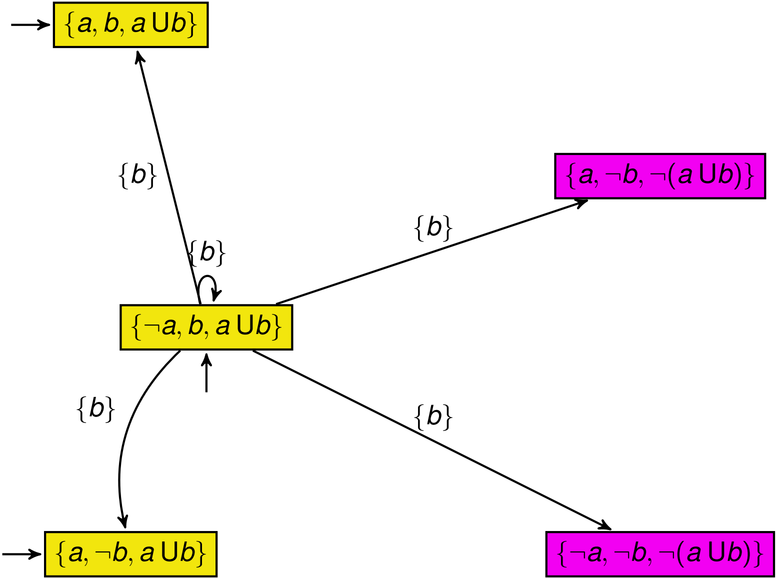

$\{a, b, a \,\mathsf{U}\, b\}$
$\{b\}$
$\{a, \neg b, \neg(a \,\mathsf{U}\, b)\}$
$\{b\}$
$\{b\}$
$\{\neg a, b, a \,\mathsf{U}\, b\}$
$\{b\}$
$\{b\}$
$\{a, \neg b, a \,\mathsf{U}\, b\}$
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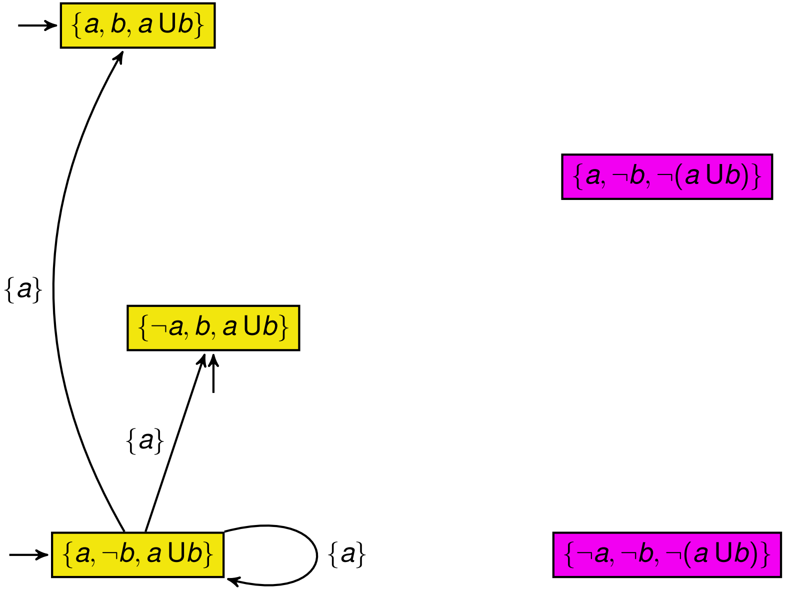
LTL to GNBA



$\{a, b, a \,\mathsf{U}\, b\}$
$\{a, \neg b, \neg(a \,\mathsf{U}\, b)\}$
$\{a\}$
$\{\neg a, b, a \,\mathsf{U}\, b\}$
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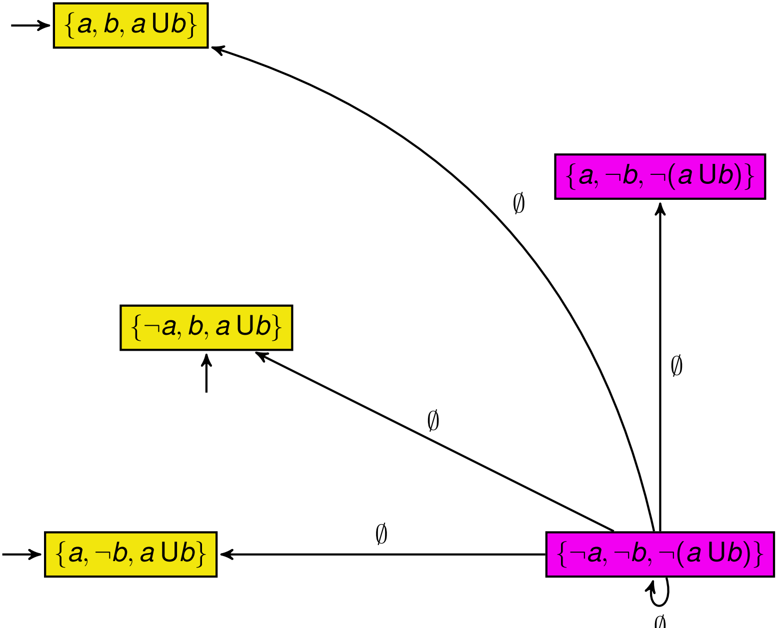
$\{a, b, a \mathsf{U} b\}$
$\{a, \neg b, \neg(a \mathsf{U} b)\}$
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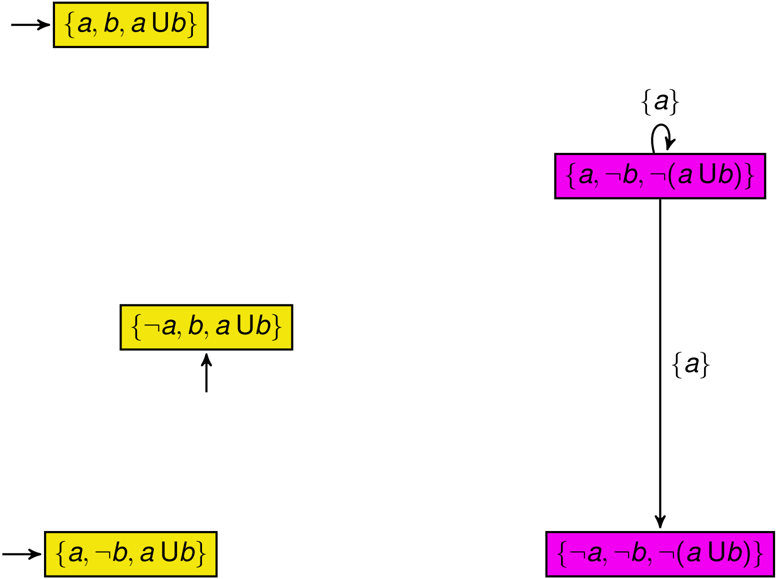
$\emptyset$
$\emptyset$
$\emptyset$

LTL to GNBA



$\{a, b, a\,\mathsf{U}\,b\}$
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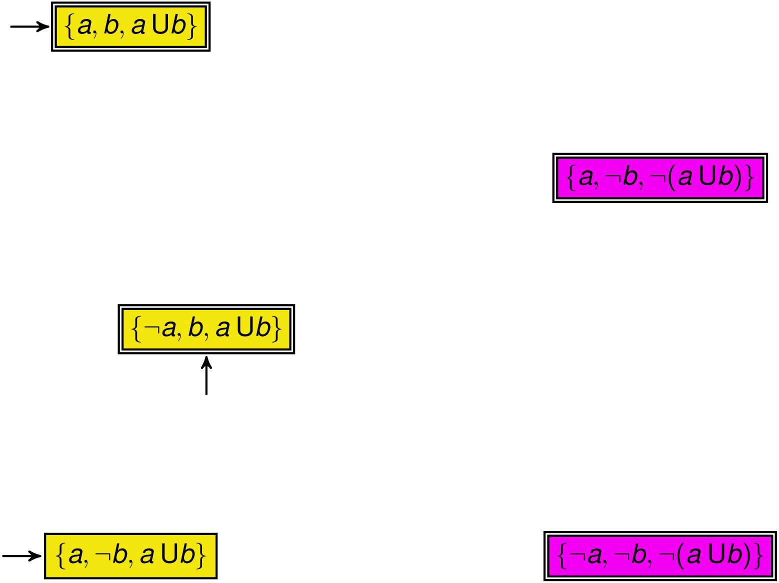
LTL to GNBA : Accepting States



LTL to GNBA

Construct GNBA for $\neg(a \mathbin{\mathsf{U}} b)$.

LTL to GNBA

- Let $\varphi = a \, \mathsf{U} (\neg a \, \mathsf{U} c)$. Let $\psi = \neg a \, \mathsf{U} c$
- Subformulae of φ : $\{a, \neg a, c, \psi, \varphi\}$. Let $B = \{a, \neg a, c, \neg c, \psi, \neg\psi, \varphi, \neg\varphi\}$.
- Possibilities at each state
  - $\{a, c, \psi, \varphi\}$
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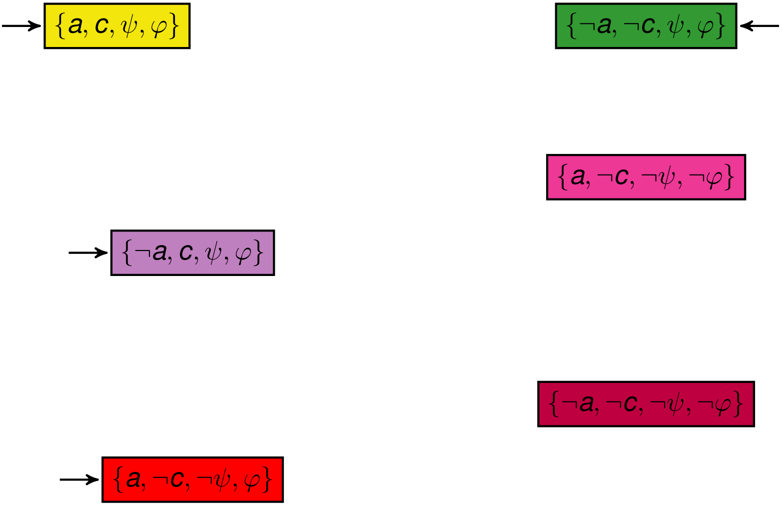
LTL to GNBA



$\{a, c, \psi, \varphi\}$
$\{\neg a, \neg c, \psi, \varphi\}$
$\{a, \neg c, \neg\psi, \neg\varphi\}$
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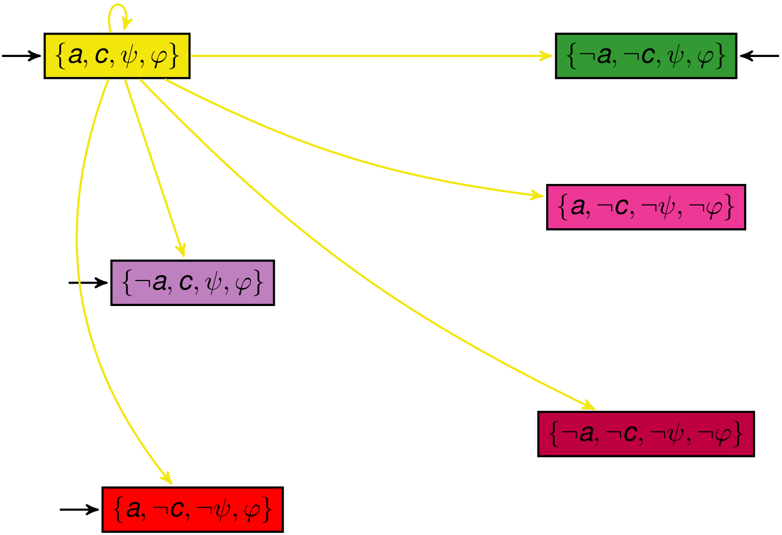
LTL to GNBA



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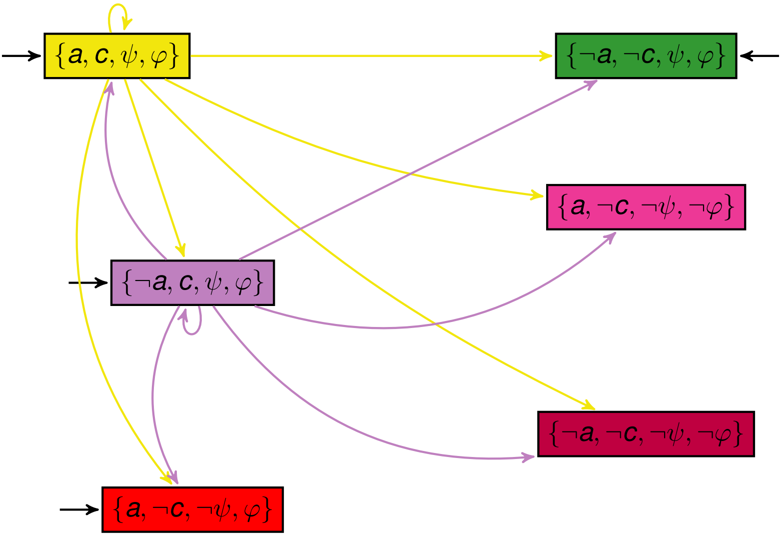
LTL to GNBA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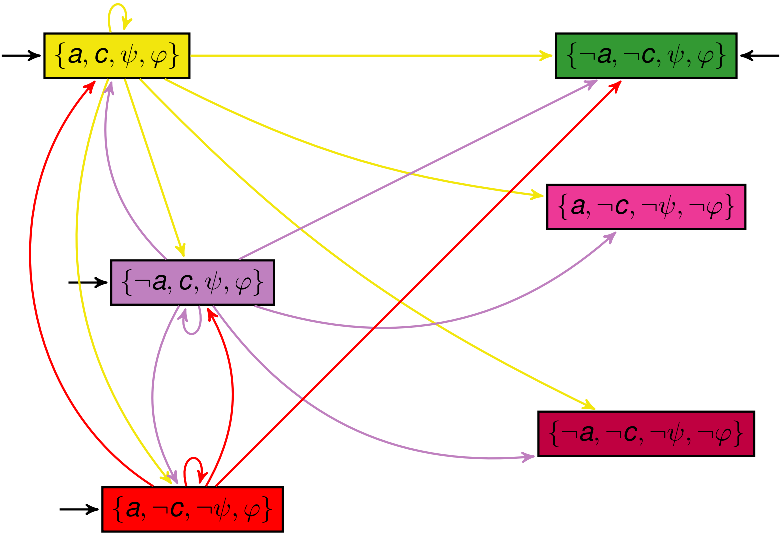

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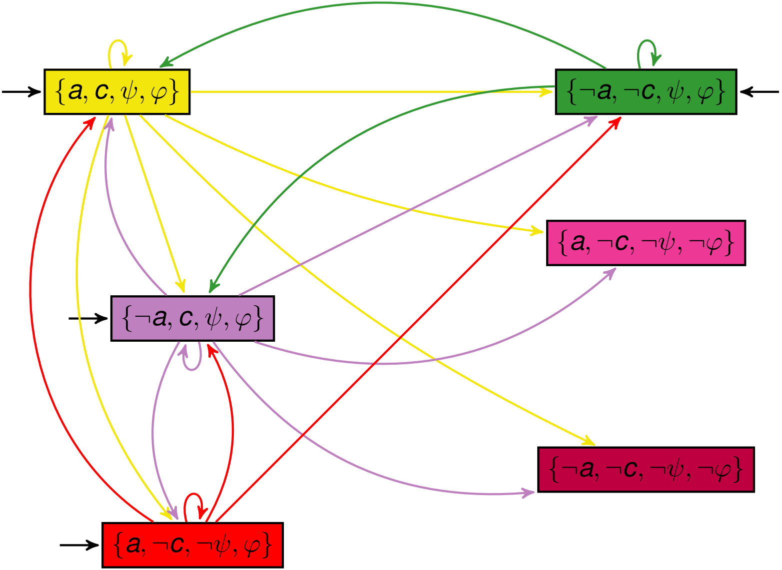
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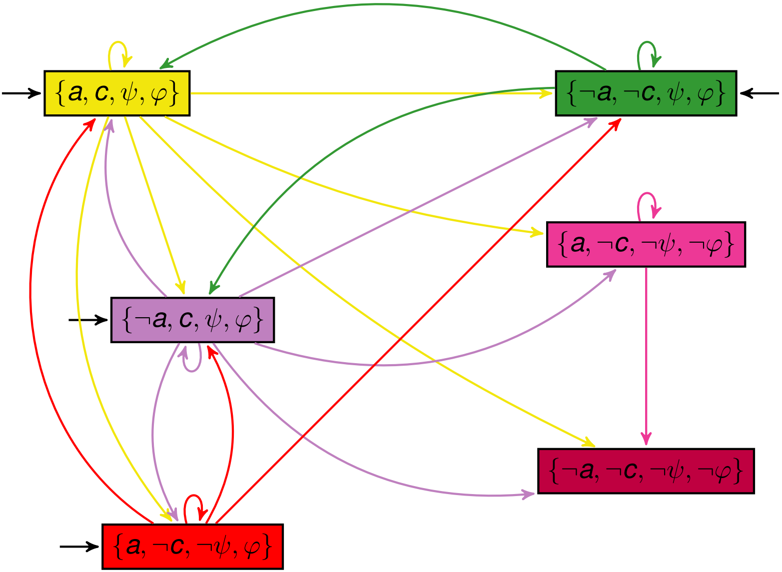

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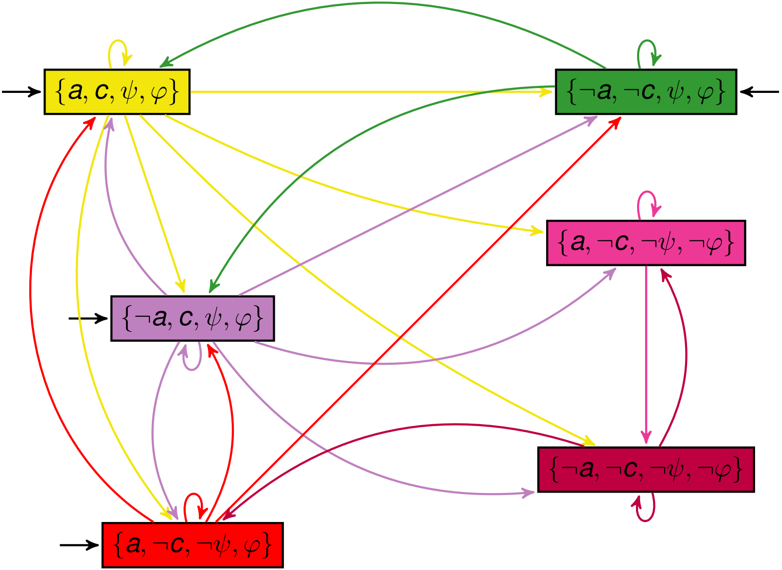

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GNBA Acceptance Condition

- $\psi = \neg a \, \mathsf{U} c$
- $\varphi = a \, \mathsf{U} \psi$
- $F_1 = \{B \mid \psi \in B \to c \in B\}$
- $F_2 = \{B \mid \varphi \in B \to \psi \in B\}$
- $\mathcal{F} = \{F_1, F_2\}$

Final States

$\rightarrow$ $\{a, c, \psi, \varphi\} \in F_1, F_2$

$\{\neg a, \neg c, \psi, \varphi\} \in F_2$ $\leftarrow$

$\{a, \neg c, \neg\psi, \neg\varphi\} \in F_1, F_2$

$\rightarrow$ $\{\neg a, c, \psi, \varphi\} \in F_1, F_2$

$\{\neg a, \neg c, \neg\psi, \neg\varphi\} \in F_1, F_2$

$\rightarrow$ $\{a, \neg c, \neg\psi, \varphi\} \in F_1$

Putting Together

- Given φ, build $Cl(\varphi)$, the set of all subformulae of φ and their negations

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- Consider those $B \subseteq Cl(\varphi)$ which are consistent
  - $\varphi_1 \wedge \varphi_2 \in B \leftrightarrow \varphi_1 \in B$ and $\varphi_2 \in B$

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Putting Together

- Given φ, build $Cl(\varphi)$, the set of all subformulae of φ and their negations
- Consider those $B \subseteq Cl(\varphi)$ which are consistent
  - $\varphi_1 \wedge \varphi_2 \in B \leftrightarrow \varphi_1 \in B$ and $\varphi_2 \in B$
  - $\psi \in B \rightarrow \neg\psi \notin B$ and $\psi \notin B \rightarrow \neg\psi \in B$
  - Whenever $\psi_1 \, \mathrm{U} \psi_2 \in Cl(\varphi)$,
    - $\psi_2 \in B \rightarrow \psi_1 \, \mathrm{U} \psi_2 \in B$
    - $\psi_1 \, \mathrm{U} \psi_2 \in B$ and $\psi_2 \notin B \rightarrow \psi_1 \in B$

Putting Together

Given φ over AP, construct $A_\varphi = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$,

- $Q = \{B \mid B \subseteq Cl(\varphi)$ is consistent $\}$
- $Q_0 = \{B \mid \varphi \in B\}$
- $\delta : Q \times 2^{AP} \to 2^Q$ is such that

Putting Together

Given φ over AP, construct $A_\varphi = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$,

- $Q = \{B \mid B \subseteq Cl(\varphi) \text{ is consistent }\}$
- $Q_0 = \{B \mid \varphi \in B\}$
- $\delta : Q \times 2^{AP} \rightarrow 2^Q$ is such that
  - For $C = B \cap AP$, $\delta(B, C)$ is enabled and is defined as :
  - If $\bigcirc\psi \in Cl(\varphi)$, $\bigcirc\psi \in B$ iff $\psi \in \delta(B, C)$
  - If $\varphi_1 \,\mathsf{U} \varphi_2 \in Cl(\varphi)$,
    $\varphi_1 \,\mathsf{U} \varphi_2 \in B$ iff $(\varphi_2 \in B \vee (\varphi_1 \in B \wedge \varphi_1 \,\mathsf{U} \varphi_2 \in \delta(B, C)))$

Putting Together

Given φ over AP, construct $A_\varphi = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$,

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  - If $\bigcirc\psi \in Cl(\varphi)$, $\bigcirc\psi \in B$ iff $\psi \in \delta(B, C)$
  - If $\varphi_1 \mathsf{U} \varphi_2 \in Cl(\varphi)$,
    $\varphi_1 \mathsf{U} \varphi_2 \in B$ iff $(\varphi_2 \in B \vee (\varphi_1 \in B \wedge \varphi_1 \mathsf{U} \varphi_2 \in \delta(B, C)))$
- $\mathcal{F} = \{F_{\varphi_1 \mathsf{U} \varphi_2} \mid \varphi_1 \mathsf{U} \varphi_2 \in Cl(\varphi)\}$, with
  $F_{\varphi_1 \mathsf{U} \varphi_2} = \{B \in Q \mid \varphi_1 \mathsf{U} \varphi_2 \in B \to \varphi_2 \in B\}$

Putting Together

Given φ over AP, construct $A_\varphi = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$,

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    $\varphi_1 \mathsf{U} \varphi_2 \in B$ iff $(\varphi_2 \in B \vee (\varphi_1 \in B \wedge \varphi_1 \mathsf{U} \varphi_2 \in \delta(B, C)))$
- $\mathcal{F} = \{F_{\varphi_1 \mathsf{U} \varphi_2} \mid \varphi_1 \mathsf{U} \varphi_2 \in Cl(\varphi)\}$, with
  $F_{\varphi_1 \mathsf{U} \varphi_2} = \{B \in Q \mid \varphi_1 \mathsf{U} \varphi_2 \in B \to \varphi_2 \in B\}$
- Prove that $L(\varphi) = L(A_\varphi)$

GNBA Size

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GNBA Size

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- Maximum number of states $\leqslant 2^{|\varphi|}$

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- Maximum number of states $\leqslant 2^{|\varphi|}$
- Number of sets in $\mathcal{F} = |\varphi|$

GNBA Size

- States of A_φ are subsets of $Cl(\varphi)$
- Maximum number of states $\leqslant 2^{|\varphi|}$
- Number of sets in $\mathcal{F} = |\varphi|$
- LTL $\varphi \rightsquigarrow$ NBA A_φ : Number of states in $A_\varphi \leqslant |\varphi|.2^{|\varphi|}$
- There is no LTL formula φ for the language

$$L = \{A_0A_1A_2\cdots \mid a \in A_{2i}, i \geqslant 0\}$$

Complexity of LTL Modelchecking

- Given φ, $A_{\neg\varphi}$ has $\leqslant 2^{|\varphi|}$ states (to be proved)
- $TS \otimes A_{\neg\varphi}$ has $\leqslant |TS|.2^{|\varphi|}$ states
- Persistence checking : Checking $\square\lozenge\eta$ on $TS \otimes A_{\neg\varphi}$ takes time linear in $\eta.|TS \otimes A_{\neg\varphi}|$

A Weak Lower Bound

The hamiltonian path problem is polynomially reducible to the complement of the LTL modelchecking problem.

- Given graph $G = (V, E)$ synthesize in polynomial time a TS and an LTL formula φ
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- Show that G has a HP iff $TS \nvDash \varphi$
- TS is the graph itself, with one new node added, say b such all vertices of G have an edge to b, and b has a self loop. Let the label of a node in the TS be the name of the vertex.
- Write an LTL formula to capture absence of a HP in G. Assume $V = \{v_1, \dots, v_n\}$.
- The formula $\varphi = \neg\psi$ where ψ is

$$(\Diamond v_1 \wedge \Box(v_1 \rightarrow \bigcirc\Box\neg v_1)) \wedge \dots (\Diamond v_n \wedge \Box(v_n \rightarrow \bigcirc\Box\neg v_n))$$

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- So LTL model checking is co-NP hard as HP is NP-complete.
- Actual complexity of LTL model checking : PSPACE-complete. For this, show that given a LBTM M and a word w, construct in poly time a TS and an LTL formula φ such that M accepts w iff $TS \models \varphi$.