CS 228 : Logic in Computer Science

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Finite State Machines

A deterministic finite state automaton (DFA) $A = (Q, \Sigma, \delta, q_0, F)$

- Q is a finite set of states
- Σ is a finite alphabet
- $\delta : Q \times \Sigma \to Q$ is the transition function
- $q_0 \in Q$ is the initial state
- $F \subseteq Q$ is the set of final states
- $L(A)$=all words leading from q_0 to some $f \in F$

Languages, Machines and Logic

A language $L \subseteq \Sigma^*$ is called regular iff there exists some DFA A such that $L = L(A)$.
A language $L \subseteq \Sigma^*$ is called FO-definable iff there exists an FO formula φ such that $L = L(\varphi)$.

Is it Regular? Is it FO-definable?

$\Sigma = \{a, b\}$. Consider the following languages $L \subseteq \Sigma^*$:

- Begins with *a*, ends with *b*, and has a pair of consecutive *a*'s
- Contains a *b* and ends with *aa*
- Contains *abb*
- There are two occurrences of *b* between which only *a*'s occur
- Right before the last position is an *a*
- Even length words

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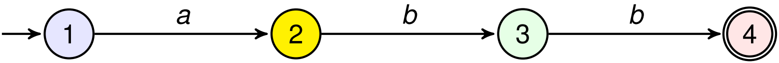

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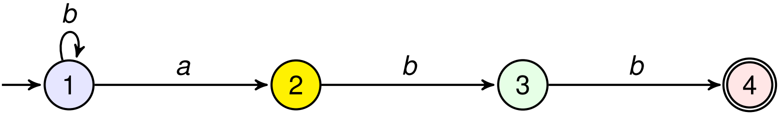

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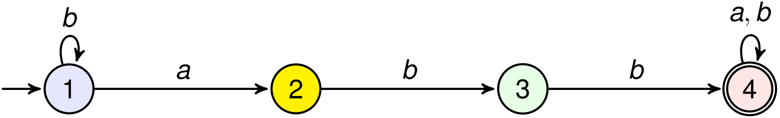

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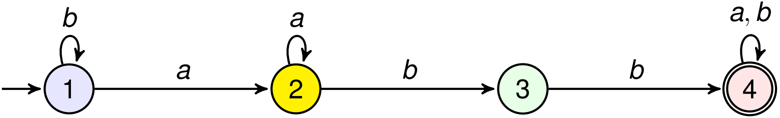

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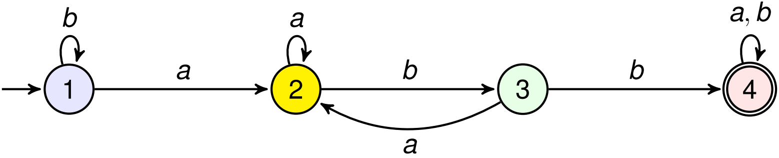

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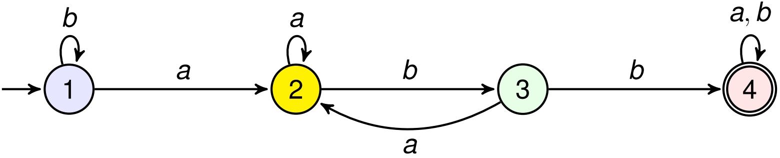

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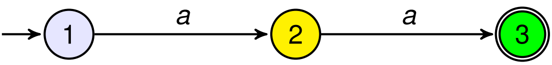

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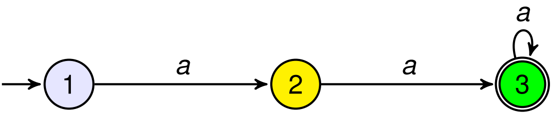

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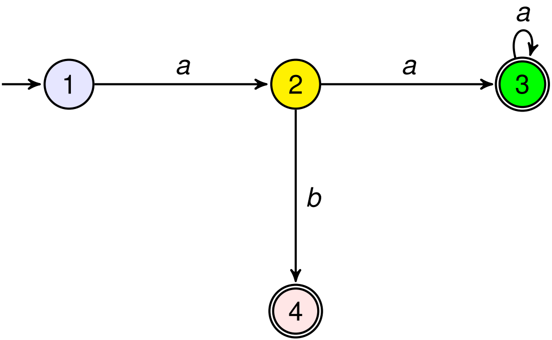

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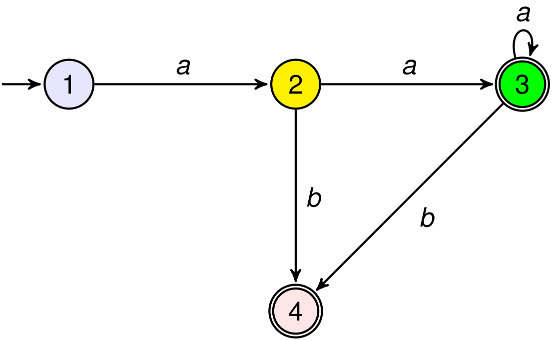

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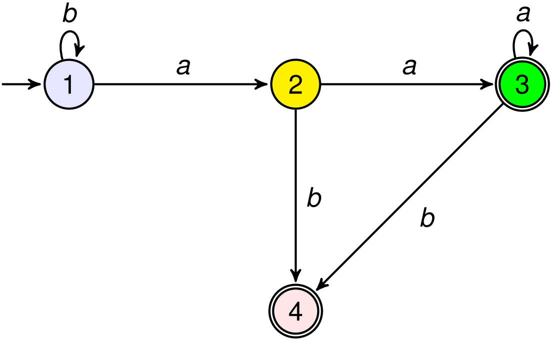

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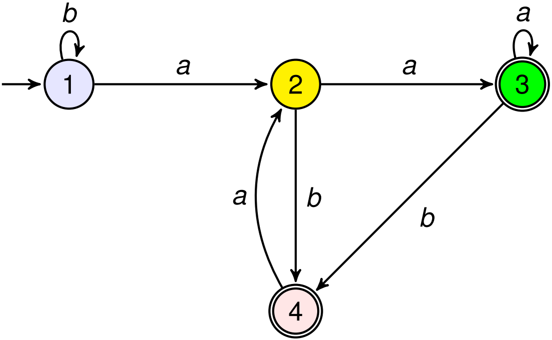

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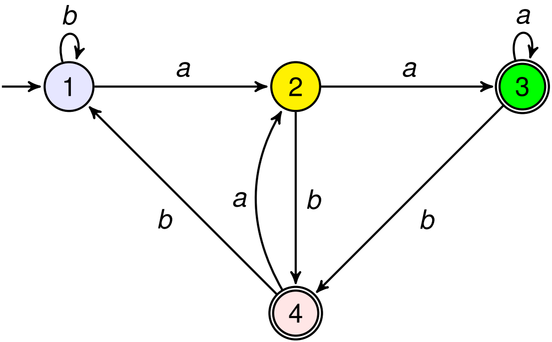

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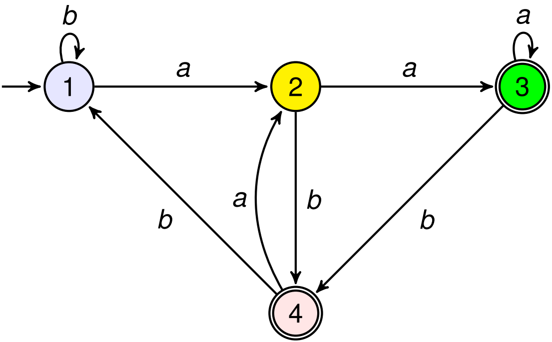

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Deterministic Finite Automata

- Every state on every symbol goes to a unique state
  - $\delta : Q \times \Sigma \to Q$ is a transition function

Any letter you put on the state changes the state.

- Given a string $w \in \Sigma^*$ and a state $q \in Q$, iteratively apply δ
  - $w = aab$
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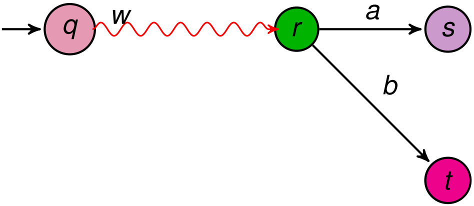
  - $\hat{\delta} : Q \times \Sigma^* \to Q$ extension of δ to strings
    - $\hat{\delta}(q, \epsilon) = q$
    - $\hat{\delta}(q, wa) = \delta(\hat{\delta}(q, w), a)$

DFA : Transition Function on Words



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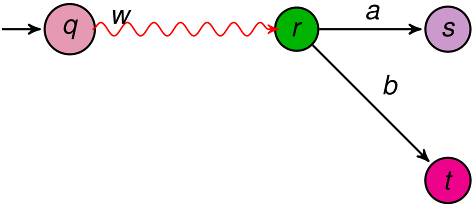

- $\hat{\delta}(q, wa) = s = \delta(\hat{\delta}(q, w), a) = \delta(r, a)$
- $\hat{\delta}(q, wb) = t = \delta(\hat{\delta}(q, w), b) = \delta(r, b)$

DFA Acceptance

- $w \in \Sigma^*$ is accepted iff $\hat{\delta}(q_0, w) \in F$
- $w \in \Sigma^*$ is rejected iff $\hat{\delta}(q_0, w) \notin F$
- Any string $w \in \Sigma^*$ is either accepted or rejected by a DFA A
- $L(A) = \{w \in \Sigma^* \mid \hat{\delta}(q_0, w) \in F\}$
- $\Sigma^* = L(A) \cup \overline{L(A)}$

DFA States

- Each state is a bucket holding infinitely many words
- Thus we have good and bad buckets
- The buckets partition Σ^*
- Good buckets determine the language accepted by the DFA
- Words that land in bad buckets are not accepted by the DFA