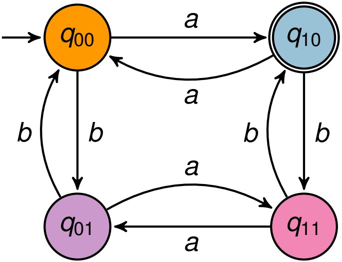
CS 228 : Logic in Computer Science

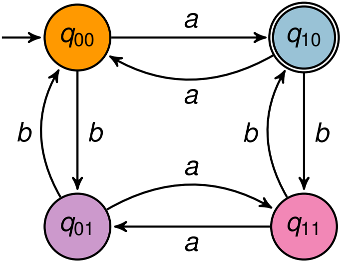
Krishna. S

Language Acceptance : Proof



- Prove by induction on $|w|$

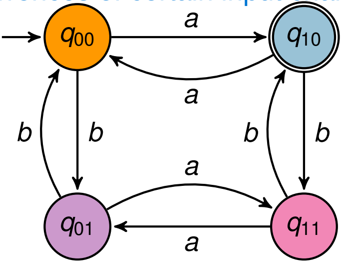
Language Acceptance : Proof



- Prove by induction on $|w|$
- Base case : For $|w| = \epsilon$, $\hat{\delta}(q_{00}, \epsilon) = q_{00}$

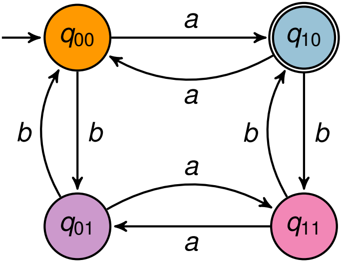
Language Acceptance : Proof

The goal of the proof is to show that, for any string w, the DFA transitions into a certain state based on the parity (even or odd) of the number of occurrences of certain input characters



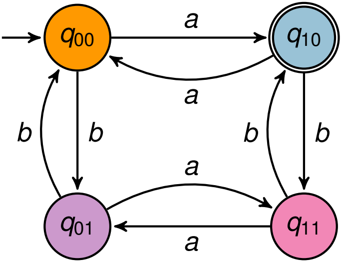
- Prove by induction on $|w|$
- Base case : For $|w| = \epsilon$, $\hat{\delta}(q_{00}, \epsilon) = q_{00}$
- Assume the claim for $x \in \Sigma^*$, and show it for $xc, c \in \{a, b\}$.

Language Acceptance : Proof



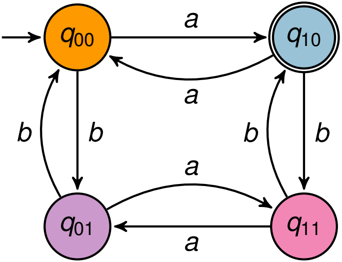
- $\hat{\delta}(q_{00}, xc) = \delta(\hat{\delta}(q_{00}, x), c)$

Language Acceptance : Proof



- $\hat{\delta}(q_{00}, xc) = \delta(\hat{\delta}(q_{00}, x), c)$
- By induction hypothesis, $\hat{\delta}(q_{00}, x) = q_{ij}$ iff
  - parity of i and $|x|_a$ are the same
  - parity of j and $|x|_b$ are the same

Language Acceptance : Proof



- Case Analysis : If $|x|_a$ odd and $|x|_b$ even, then $i = 1, j = 0$
  - $\delta(q_{10}, a) = q_{00}, \delta(q_{10}, b) = q_{11}$
  - $|xa|_a$ is even and $|xa|_b$ is even
  - $|xb|_a$ is odd and $|xb|_b$ is odd
- Other Cases : Similar
- $\hat{\delta}(q_{00}, x) = q_{10}$ iff $|x|_a$ odd and $|x|_b$ even

Yes, the proof was indeed intended to show that the DFA's transitions are based on the parity (even or odd) of the number of occurrences of the characters a and b in the input string

Closure Properties : DFA

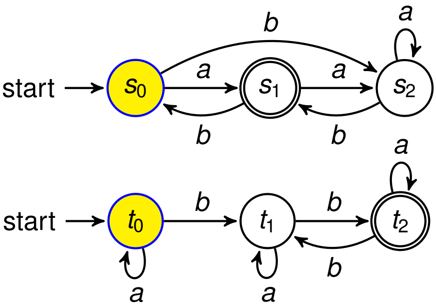
Closure under Complementation

Complementation Construction: To build the DFA for the complement language, we keep the same states, transitions, and initial state as the original DFA. The only change is in the accepting states, where we take the complement of the original accepting states (i.e., all states that were not accepting in the original DFA become accepting in the complement DFA).

- If L is regular, so is $\overline{L}$
  - Let $A = (Q, q_0, \Sigma, \delta, F)$ be the DFA such that $L = L(A)$
  - For every $w \in L$, $\hat{\delta}(q_0, w) = f$ for some $f \in F$
  - For every $w \notin L$, $\hat{\delta}(q_0, w) = q$ for some $q \notin F$
  - Construct $\overline{A} = (Q, q_0, \Sigma, \delta, Q - F)$
    - $w \in L(\overline{A})$ iff $\hat{\delta}(q_0, w) \in Q - F$ iff $w \notin L(A)$
    - $L(\overline{A}) = \overline{L(A)}$

Closure under Intersection

- *aaab*



Closure under Intersection

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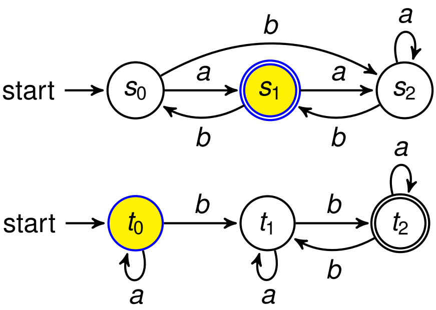

Closure under Intersection

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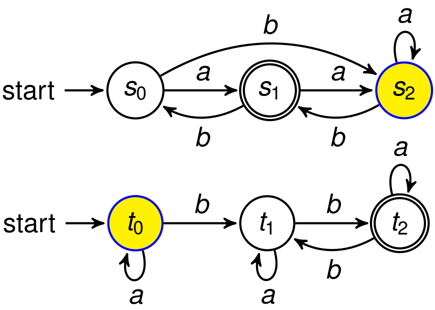
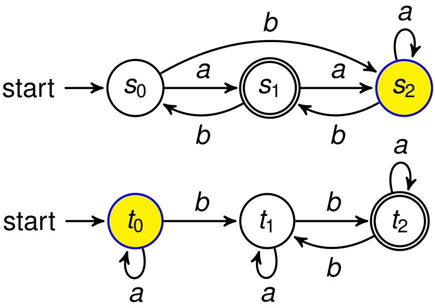


Closure under Intersection

- *aa**a**b*



Closure under Intersection

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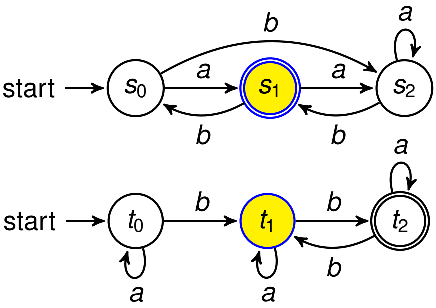

Closure under Intersection

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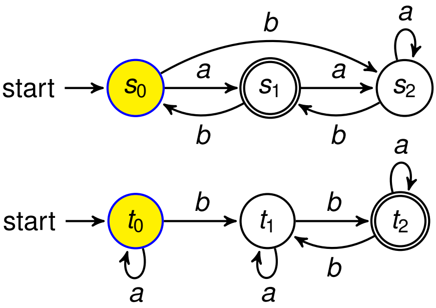

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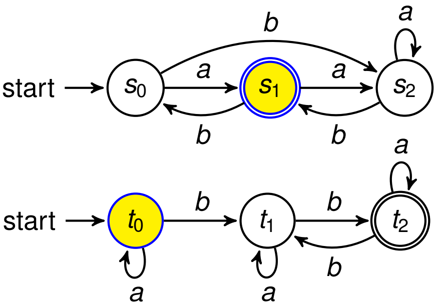
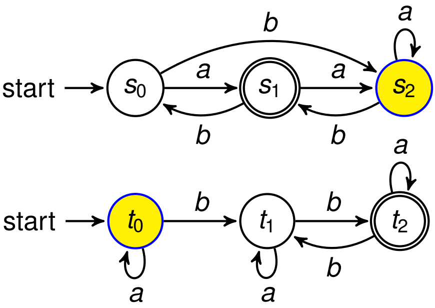


Closure under Intersection

- *a**a**bba*



Closure under Intersection

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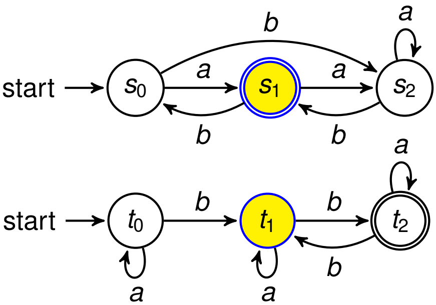

Closure under Intersection

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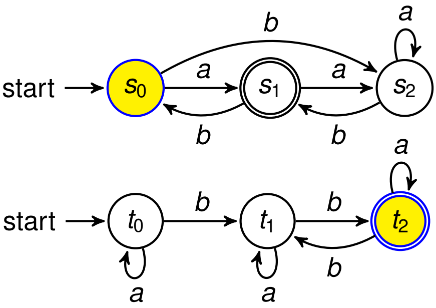

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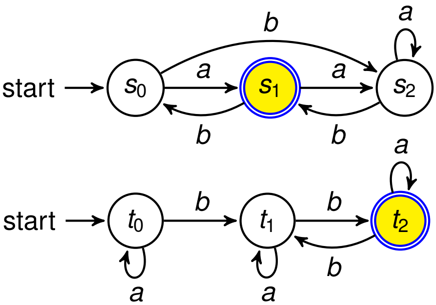

Closure under Intersection

- $A_1 = (Q_1, \Sigma, \delta_1, q_0, F_1)$
- $A_2 = (Q_2, \Sigma, \delta_2, s_0, F_2)$
- $A = (Q_1 \times Q_2, \Sigma, \delta, (q_0, s_0), F)$,
    - $\delta((q, s), a) = (\delta_1(q, a), \delta_2(s, a))$
    - $F = F_1 \times F_2$

Closure under Intersection

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  - $\delta((q, s), a) = (\delta_1(q, a), \delta_2(s, a))$
  - $F = F_1 \times F_2$
- Show that for all $x \in \Sigma^*$, $\hat{\delta}((p, q), x) = (\hat{\delta_1}(p, x), \hat{\delta_2}(q, x))$

$x \in L(A)$ iff $\hat{\delta}((q_0, s_0), x) \in F$

Closure under Intersection

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Closure under Intersection

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Closure under Intersection

- $A_1 = (Q_1, \Sigma, \delta_1, q_0, F_1)$
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$x \in L(A)$ iff $\hat{\delta}((q_0, s_0), x) \in F$ iff $(\hat{\delta}_1(q_0, x), \hat{\delta}_2(s_0, x)) \in F_1 \times F_2$ iff $\hat{\delta}_1(q_0, x) \in F_1$ and $\hat{\delta}_2(s_0, x) \in F_2$ iff $x \in L(A_1)$ and $x \in L(A_2)$

Closure under Union

- $A_1 = (Q_1, \Sigma, \delta_1, q_0, F_1)$
- $A_2 = (Q_2, \Sigma, \delta_2, s_0, F_2)$
- $A = (Q_1 \times Q_2, \Sigma, \delta, (q_0, s_0), F)$,
  - $\delta((q, s), a) = (\delta_1(q, a), \delta_2(s, a))$

Closure under Union

- $A_1 = (Q_1, \Sigma, \delta_1, q_0, F_1)$
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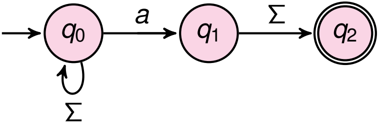
- $A = (Q_1 \times Q_2, \Sigma, \delta, (q_0, s_0), F)$,
  - $\delta((q,s), a) = (\delta_1(q,a), \delta_2(s,a))$
  - $F = (F_1 \times Q_2) \cup (Q_1 \times F_2)$
- Show that for all $x \in \Sigma^*$, $\hat{\delta}((p,q), x) = (\hat{\delta}_1(p,x), \hat{\delta}_2(q,x))$

$x \in L(A)$ iff $x \in L(A_1)$ or $x \in L(A_2)$

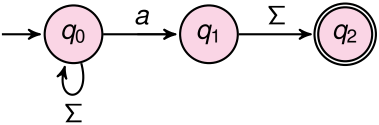
Moving on to Non-determinism

- We looked at DFA
- Showed closure under union, intersection and complementation
- Before we examine closure under concatenation, we look at a more relaxed model, which is as good as a DFA

The Comfort of Non-determinism



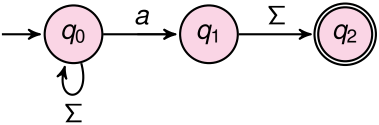
The Comfort of Non-determinism



- Assume we relax the condition on transitions, and allow
  - $\delta : Q \times \Sigma \to 2^Q$
  - $\delta(q_0, a) = \{q_0, q_1\}, \delta(q_2, a) = \emptyset$

Any alphabet we take will make a set of next state on transition

The Comfort of Non-determinism



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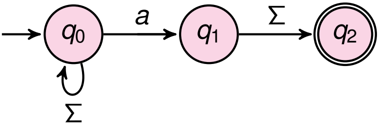
  - Is *aabb* accepted?

The Comfort of Non-determinism



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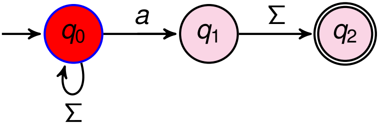
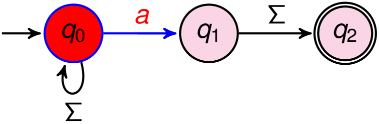


One run of *aabb*

Is *aabb* accepted?



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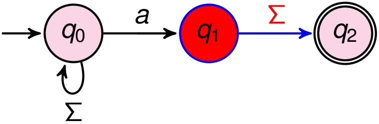

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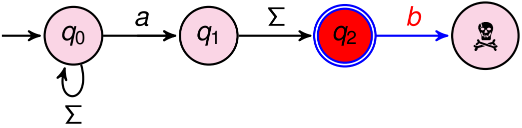
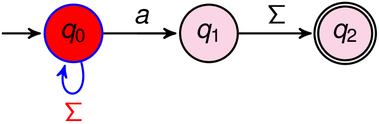


- A non-accepting run for *aabb*

Another run of *aabb*

Is *aabb* accepted?



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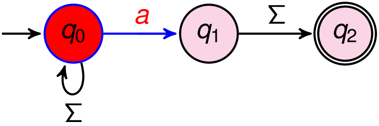

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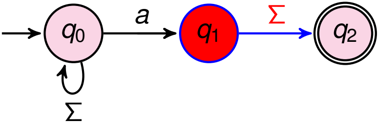

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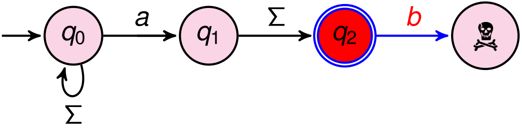

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A run of *aaab*

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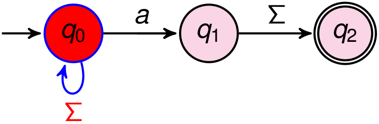
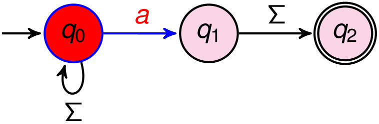


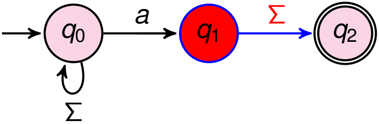
A run of *aaab*

Is *aaab* accepted?



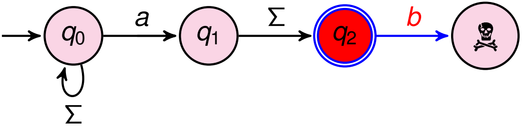
A run of *aaab*

Is *aaab* accepted?



A run of *aaab*

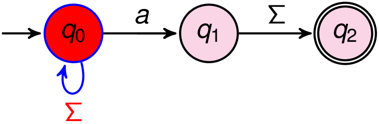
Is *aaa**b* accepted?



- A non-accepting run for *aaab*

Another run of *aaab*

Is *aaab* accepted?



Another run of *aaab*

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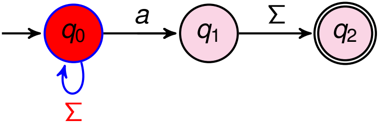

Another run of *aaab*

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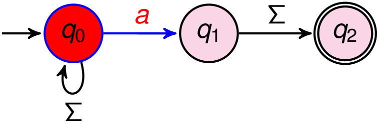

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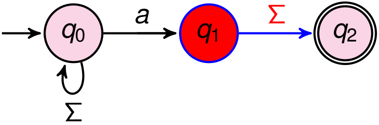


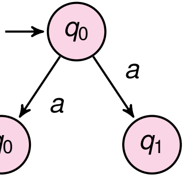
- An accepting run for *aaab*

Nondeterministic Finite Automata(NFA)

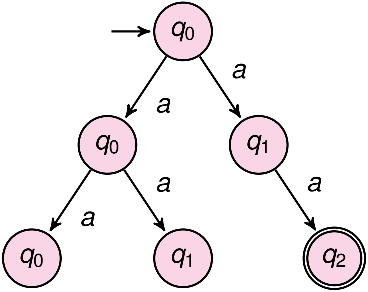
Here 2^Q represents the subset of Q.

- $N = (Q, \Sigma, \delta, Q_0, F)$
  - Q is a finite set of states
  - $Q_0 \subseteq Q$ is the set of initial states
  - $\delta : Q \times \Sigma \rightarrow 2^Q$ is the transition function
  - $F \subseteq Q$ is the set of final states
- Acceptance condition : A word w is accepted iff it has atleast one accepting path

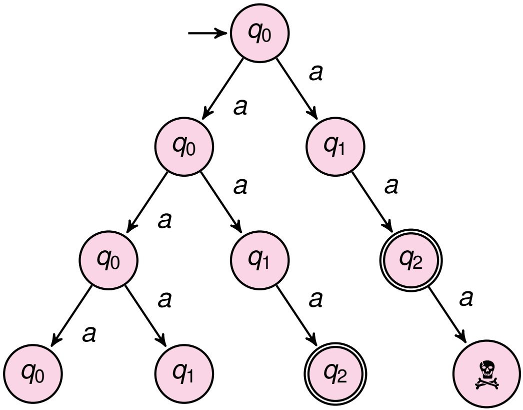
Run Tree of *aaab*



Run Tree of *aaab*



Run Tree of *aaab*



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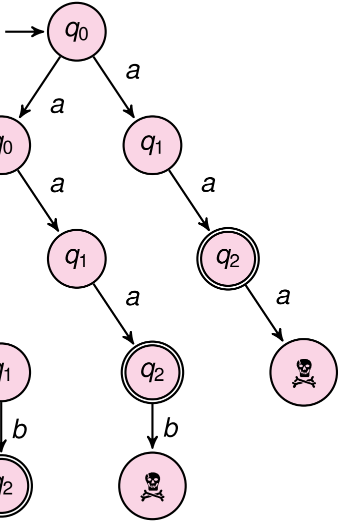

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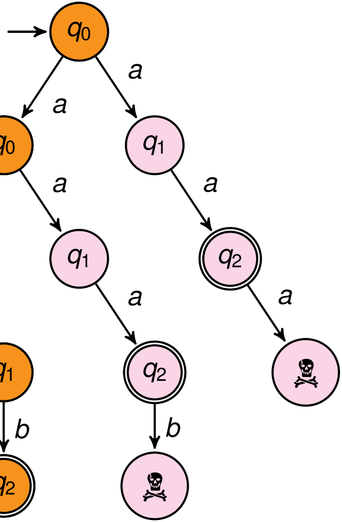

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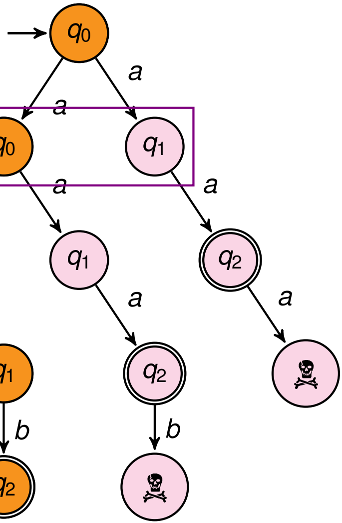

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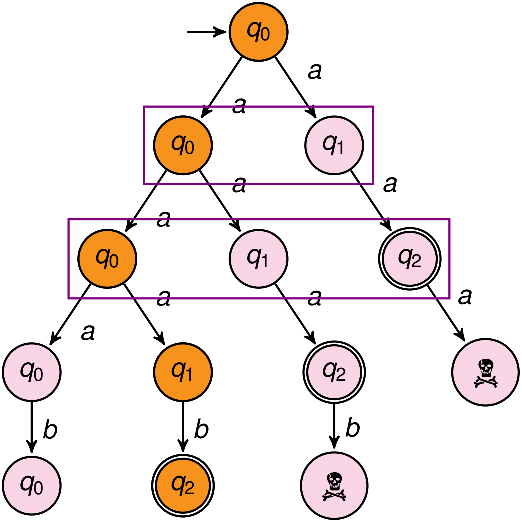
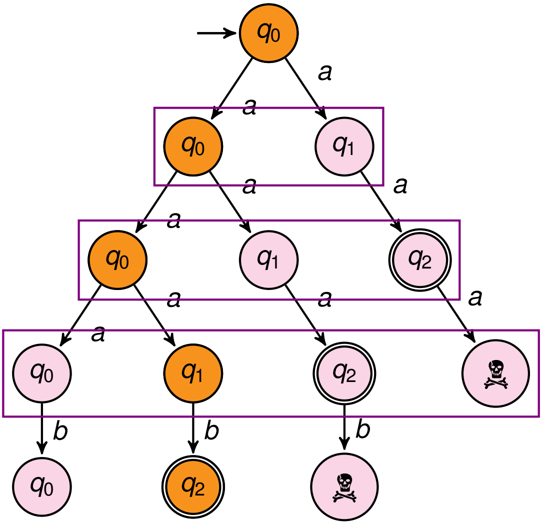
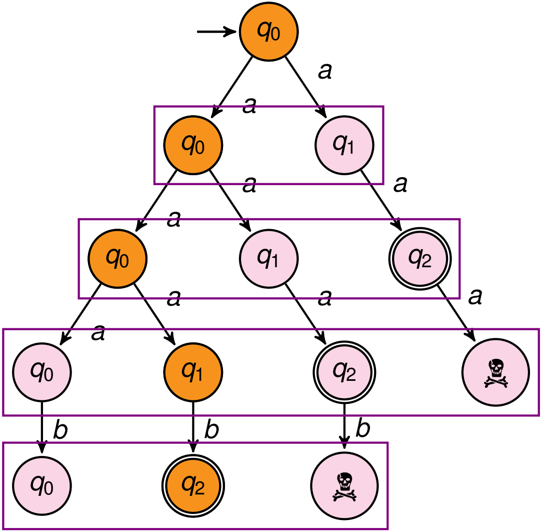


Run Tree of *aaab*



Run Tree of *aaab*



The Single Run