

Second order lin diff eqn.

• Homogeneous eq. (const. coeff., C-E eq)

• $y'' + p(n)y' + q(n)y = r(n) \rightarrow \textcircled{B}$

(1) Assu. hom. eqn $y'' + p(n)y' + q(n)y = 0$

Its solⁿ $c_1 y_1 + c_2 y_2$

(2) A particular solⁿ y_p of $\textcircled{*}$

Gen. Solⁿ of \propto is $c_1 y_1 + c_2 y_2 + y_p$

Variation of parameters

$$y'' + p(u) y' + q(u) y = \gamma(u)$$

Asso. h. eq. $y'' + p(u) y' + q(u) y = 0$

I-t. soln is $c_1 y_1 + c_2 y_2$

It $y_p(u) = v_1(u) y_1(u) + v_2(u) y_2(u)$

$$y_p' = v_1' y_1 + v_1 y_1' + v_2' y_2 + v_2 y_2'$$

Let $v_1' y_1 + v_2' y_2 = 0$. Then $\boxed{y_p' = v_1 y_1' + v_2 y_2'}$

$$y_p'' + p y_p' + q y_p = \gamma$$

$$y_p' = v_1 y_1' + v_2 y_2' \Rightarrow \underline{v_1}' y_1 + \underline{v_2}' y_2 = 0 \quad \textcircled{1}$$

$$\underline{v_1}' y_1 + \underline{v_2}' y_2 = \gamma(n) \rightarrow \textcircled{2}$$

$$\begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} v_1' \\ v_2' \end{bmatrix} = \begin{bmatrix} 0 \\ \gamma \end{bmatrix}$$

$$y_p = v_1 y_1 + v_2 y_2 + c_1 y_1 + c_2 y_2$$

$$e^{an}(\cos bn + \sin bn)$$

$$a + ib = 0 + i$$

$$y_p = \left(\ln |\sin n| \right) \sin n - n \cos n$$

$$y_g = c_1 \sin n + c_2 \cos n + y_p$$

$$\begin{aligned} m^2 - m - 2 &= 0 \\ m^2 - 3m + m - 2 &= 0 \end{aligned} \quad \left| \begin{array}{l} y_p = v_1 y_1 + v_2 y_2 \\ = (\) e^{3n} + (\) e^{-n} \end{array} \right.$$

$$m^2 + \zeta = 0$$

$$m = \pm 2i$$

$$y'' - 4y' + 5y = x^2 e^x$$

$$(an^2 + bn + c)e^{2x}$$

$$y'' - 4y' + 5y = e^{2x}$$

$$y_p = k e^{2x} \rightarrow \text{candidate sd}^n$$

$$\underbrace{9k - 15k + 6k}_{=0} = 1$$

$$y_p = kx e^{-3x}$$

