$$M\,dx + N\,dy = 0$$

$$\boxed{M_y = N_x}$$

$$M = xy^2 + bx^2y$$

$$N = (x+y)x^2$$

$$M_y = 2xy + bx^2$$

$$N_x = 3x^2 + 2xy$$

$$\boxed{b = 3}$$

$$y'' + y' - 6y = 0$$

$$\Rightarrow (r^2 + r - 6)e^{rt} = 0$$

$$\Rightarrow r^2 + r - 6 = 0$$

$$\Rightarrow (r+3)(r-2) = 0$$

$$\Rightarrow r = -3, 2$$

$$(D^2 + D - 6)\,y = 0$$

$$\Leftrightarrow L(y) = 0 \quad (L(D)y = 0)$$

$$L(x^3) = x^2(3 \times 2x) + 2x(3x^2) + 2x^3$$
$$= 6x^3 + 6x^3 + 2x^3 = 14x^3$$

$$D \equiv \frac{d}{dx}$$

$$M\,dx + N\,dy = 0$$

$$u(x,y) = \int M\,dx + k(y)$$

$$\frac{\partial u}{\partial y} = N \text{ . Determine } k(y)$$

$$\boxed{u(x,y) = c}$$

$$M\,dx + N\,dy = 0 \qquad M_y \neq N_x$$

$$\mu M\,dx + \mu N\,dy = 0 \qquad \mu(x, y)$$

$$(\mu M)_y = (\mu N)_x$$

$$\Rightarrow \boxed{\mu_y M + \mu M_y - \mu_x N - \mu N_x = 0}$$

$$\Rightarrow \mu_y M - \underline{\mu_x N} + \mu(M_y - N_x) = 0$$

$$\frac{d\mu}{\mu} = \left(\frac{M_y - N_x}{N}\right) dx, \quad \mu = e^{\int \frac{M_y - N_x}{N} dx}$$

$$M_y - N_x = 8x - 18y - (4x - 6y)$$

$$= 4x - 12y$$

$$\frac{M_y - N_x}{N} = \frac{4x - 12y}{2x^2 - 6xy} = \frac{2(2x - 6y)}{x(2x - 6y)} = \frac{2}{x}$$

$$IF = e^{\int \frac{2}{dx} dx} = x^2$$

$$\underbrace{x^2(8xy - 9y^2)}_{\tilde{M}} + \underbrace{x^2(2x^2 - 6xy)}_{\tilde{N}} \frac{dy}{dx} = 0$$

$$M_y - N_x = -2, \qquad \frac{N_x - M_y}{M} = -\frac{2}{y}$$

$$IF = e^{-\int \frac{2dy}{y}} = \frac{1}{y^2}$$

$$\boxed{x = cy}$$

$$\frac{dy}{dx} + P(x)\,y = Q(x)\,y^n$$

$$v = y^{1-n}$$

$$\frac{dv}{dx} = \frac{dv}{dy}\frac{dy}{dx} = (1-n)\,y^{-n}\frac{dy}{dx}$$

$$\frac{1}{(1-n)}y^n\frac{dv}{dx} + P(x)\,y = Q(x)\,y^n$$

$$\frac{dy}{dx} + P(x)\, y = Q(x)\, y^n$$

$$\Rightarrow y^{-n}\frac{dy}{dx} + P(x)\, \underline{\underline{y^{1-n}}} = Q(x)$$