n^{th} order linear ODEs

$$y_g = c_1 y_1 + \cdots + c_2 y_n + y_p$$

$$e^{0x} = 1, \quad m^2, \quad c_1 + c_2 x$$

$$(m+1)^3, \quad c_4 e^{-x} + c_5 x e^{-x} + c_6 x^2 e^{-x}$$

$$e^{2x}\left(c_2 \cos x + c_3 \sin x\right) + x\, e^{2x}\left(c_4 \cos x + c_5 \sin x\right)$$

$$y_p = y = v_1 y_1 + \cdots + v_n y_n$$

$$y' = \cdots$$

$$y'' = \cdots$$

$$y^{(n-1)} = \cdots$$

$$y^{(n)}$$

$$v_1' y_1 + \cdots + v_n' y_n = 0$$

$$v_1' y_1' + \cdots + v_n' y_n' = 0$$

$$v_1' y_1^{(n-2)} + \cdots + v_n' y_n^{(n-2)} = 0$$

$$v_1' y_1^{(n-1)} + \cdots + v_n' y_n^{(n-1)} = \gamma$$

$$L(y)=0 \iff y \in \ker L$$

$$x \sin x \longrightarrow y_p = (\alpha x+\beta)\sin x + (\gamma x+\delta)\cos x$$

$$x^2 e^x \longrightarrow x^2(ax^2+bx+c)e^x$$

$$e^{2x} \longrightarrow d e^{2x}$$

$$y'' + \left(1+\frac{1}{y}\right)(y')^2 = 0$$

Let $v = y'$, $\quad y'' = \dfrac{dv}{dx} = \dfrac{dv}{dy}\dfrac{dy}{dx} = v\dfrac{dv}{dy}$

So, $v\dfrac{dv}{dy} + \left(1+\frac{1}{y}\right)v^2 = 0$

$\Rightarrow \dfrac{dv}{v} + \left(1+\frac{1}{y}\right)dy = 0 \Rightarrow \ln|v| + y + \ln|y| = \ln c$

$\Rightarrow v e^y y = c \Rightarrow v = \dfrac{c e^{-y}}{y}$, $\quad v = \frac{1}{e}$ at $y = 1$

$\Rightarrow yv = e^{-y} \Rightarrow y e^y dy = dx \Rightarrow ye^y - e^y = x + c$

$\Rightarrow (y-1)e^y = x$

$$W = y_1 y_2' - y_1' y_2$$

$$\Rightarrow y_2' - \frac{y_1'}{y_1} y_2 = \frac{W}{y_1}$$

$$\Rightarrow \quad y_2 = y_1 \int \frac{W}{y_1^2} \, dx$$

$$= x \int \frac{\cancel{x} e^x}{\cancel{x^2}} \, dx = x e^x$$

$$(x^2+1)(y''-2y+1) = e^x$$

$$m^2 - 2 = 0 \Rightarrow m = \pm\sqrt{2}$$

$$c_1 e^{\sqrt{2}x} + c_2 e^{-\sqrt{2}x}, \qquad r = \frac{e^x}{x^2+1} - 1$$

$$v_1 = -\int \frac{y_2 r}{W}\,dx, \quad v_2 = \int \frac{y_1 r}{W}\,dx$$