CHAPTER 11

Feedback

IN THIS CHAPTER YOU WILL LEARN

1. The general structure of the negative-feedback amplifier and the basic principle that underlies its operation.
2. The advantages of negative feedback, how these come about, and at what cost.
3. The appropriate feedback topology to employ with amplifiers of each of the four types: voltage, current, transconductance, and transresistance.
4. An intuitive and insightful approach for the analysis of practical feedback-amplifier circuits.
5. Why and how negative-feedback amplifiers can become unstable (i.e., oscillate) and how to design the circuit to ensure stable performance.

Introduction

Most physical systems incorporate some form of feedback. It is interesting to note, though, that the theory of negative feedback has been developed by electronics engineers. In his search for methods for the design of amplifiers with stable gain for use in transatlantic telephone repeaters, Harold Black, an electronics engineer with the Western Electric Company, invented the feedback amplifier in 1928. Since then, the technique has been so widely used that it is almost impossible to think of electronic circuits without some form of feedback, either implicit or explicit. Furthermore, the concept of feedback and its associated theory are currently used in areas other than engineering, such as in the modeling of biological systems.

Feedback can be either **negative** or **positive**. In amplifier design, negative feedback is applied to effect one or more of the following goals:

1. *Desensitize the gain:* that is, make the value of the gain less sensitive to variations in the values of circuit components, such as might be caused by changes in temperature.
2. *Reduce nonlinear distortion:* that is, make the output proportional to the input (in other words, make the gain constant, independent of signal level).
3. *Reduce the effect of noise:* that is, minimize the contribution to the output of unwanted electric signals generated, either by the circuit components themselves or by extraneous interference.
4. *Control the input and output resistances:* that is, raise or lower the input and output resistances by the selection of an appropriate feedback topology.
5. *Extend the bandwidth* of the amplifier.

All of the desirable properties above are obtained at the expense of a reduction in gain. It will be shown that the gain-reduction factor, called the **amount of feedback**, is the factor by which the circuit is desensitized, by which the input resistance of a voltage amplifier is increased, by which the bandwidth is extended, and so on. In short, *the basic idea of negative feedback is to trade off gain for other desirable properties*. This chapter is devoted to the study of negative-feedback amplifiers: their analysis, design, and characteristics.

Under certain conditions, the negative feedback in an amplifier can become positive and of such a magnitude as to cause oscillation. In fact, in Chapter 18 we will study the use of positive feedback in the design of oscillators and bistable circuits. Here, in this chapter, however, we are interested in the design of stable amplifiers. We shall therefore study the stability problem of negative-feedback amplifiers and their potential for oscillation.

It should not be implied, however, that positive feedback always leads to instability. In fact, positive feedback is quite useful in a number of nonregenerative applications, such as the design of active filters, which are studied in Chapter 17.

Before we begin our study of negative feedback, we wish to remind the reader that we have already encountered negative feedback in a number of applications. Almost all op-amp circuits (Chapter 2) employ negative feedback. Another popular application of negative feedback is the use of the emitter resistance R_E to stabilize the bias point of bipolar transistors and to increase the input resistance, bandwidth, and linearity of a BJT amplifier. In addition, the source follower and the emitter follower both employ a large amount of negative feedback. The question then arises about the need for a formal study of negative feedback. As will be appreciated by the end of this chapter, the formal study of feedback provides an invaluable tool for the analysis and design of electronic circuits. Also, the insight gained by thinking in terms of feedback can be extremely profitable.

11.1 The General Feedback Structure

11.1.1 Signal-Flow Diagram

Figure 11.1 shows the basic structure of a feedback amplifier. Rather than showing voltages and currents, Fig. 11.1 is a signal-flow diagram, where each of the quantities x can represent either a voltage or a current signal. The basic amplifier is unilateral and has a gain A, known as the **open-loop gain**; thus its output x_o is related to the input x_i by

$$x_o = Ax_i \tag{11.1}$$

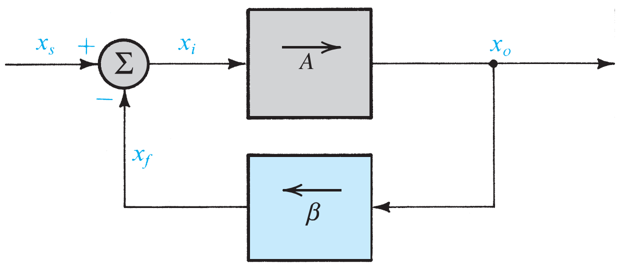

Figure 11.1 General structure of the feedback amplifier. This is a signal-flow diagram, and the quantities x represent either voltage or current signals.

The *feedback network* measures or samples the output signal x_o and provides a *feedback signal* x_f that is related to x_o by the **feedback factor** β,

$$x_f = \beta x_o \tag{11.2}$$

It is assumed that connecting the feedback network to the amplifier output does not change the gain A or the value of x_o; that is, *the feedback network does not load the amplifier output*. Also, the feedback network is unilateral.

The feedback signal x_f is *subtracted* from the source signal x_s, which is the input to the complete feedback amplifier,[1] to produce the signal x_i, which is the input to the basic amplifier,

$$x_i = x_s - x_f \tag{11.3}$$

Here we note that it is this subtraction that makes the feedback negative. In essence, *negative feedback reduces the signal that appears at the input of the basic amplifier*. Here, too, we assume that connecting the output of the feedback network to the amplifier input, through the subtractor or differencing circuit, does not change the gain A; that is, *the feedback network does not load the amplifier input.*

11.1.2 The Closed-Loop Gain

The gain of the feedback amplifier, known as the closed-loop gain or the **gain-with-feedback** and denoted A_f, is defined as

$$A_f \equiv \frac{x_o}{x_s}$$

Combining Eqs. (11.1) through (11.3) provides the following expression for A_f:

$$A_f = \frac{A}{1 + A\beta} \tag{11.4}$$

The quantity $A\beta$ is called the **loop gain**, a name that follows from Fig. 11.1. For the feedback to be negative, the loop gain $A\beta$ must be positive; that is, the feedback signal x_f should have the same sign as x_s, thus resulting in a smaller difference signal x_i. Equation (11.4) indicates that for positive $A\beta$ the gain with feedback A_f will be smaller than the open-loop gain A by a factor equal to $1 + A\beta$, which is called the **amount of feedback**.

If, as is the case in many circuits, the loop gain $A\beta$ is large, $A\beta \gg 1$, then from Eq. (11.4) it follows that

$$A_f \simeq \frac{1}{\beta} \tag{11.5}$$

which is a very interesting result: *When the loop gain is large, the gain of the feedback amplifier is almost entirely determined by the feedback network*. Since the feedback network usually consists of passive components, which usually can be chosen to be as accurate as one wishes, the advantage of negative feedback in obtaining accurate, predictable, and stable gain

[1] In earlier chapters, we used the subscript "sig" for quantities associated with the signal source (e.g., v_{sig} and R_{sig}). We did that to avoid confusion with the subscript "*s*," which is usually used with FETs to denote quantities associated with the source terminal of the transistor. At this point, however, it is expected that readers have become sufficiently familiar with the subject that the possibility of confusion is minimal. Therefore, we will revert to using the simpler subscript *s* for signal-source quantities.

should be apparent. In other words, the overall gain will have very little dependence on the gain of the basic amplifier, A, a desirable property because the gain A is usually a function of many manufacturing and application parameters, some of which might have wide tolerances. We have seen a dramatic illustration of all of these effects in op-amp circuits in Chapter 2, where the closed-loop gain is almost entirely determined by the feedback elements. Generally, we will consider $(1/\beta)$ to be the ideal value of A_f.

Equations (11.1) through (11.3) can be combined to obtain the following expression for the feedback signal x_f:

$$x_f = \frac{A\beta}{1+A\beta} x_s \tag{11.6}$$

Thus for $A\beta \gg 1$ we see that $x_f \simeq x_s$, which implies that the signal x_i at the input of the basic amplifier is reduced to almost zero. Thus if a large amount of negative feedback is employed, the feedback signal x_f becomes an almost identical replica of the input signal x_s. The difference between x_s and x_f, which is x_i, is sometimes referred to as the **error signal**.[2] Accordingly, the **input differencing circuit** is often also called a **comparison circuit**. (It is also known as a **mixer**.) An expression for x_i can be easily determined as

$$x_i = \frac{1}{1+A\beta} x_s \tag{11.7}$$

from which we can verify that for $A\beta \gg 1$, x_i becomes very small. An outcome of this property is the tracking of the two input terminals of an op amp. Observe that negative feedback reduces the signal that appears at the input terminals of the basic amplifier by the amount of feedback $(1 + A\beta)$. As will be seen later, it is this reduction of input signal that results in the increased linearity of the feedback amplifier.[3]

11.1.3 The Loop Gain

From the discussion above we see that the loop gain $A\beta$ is a very important—in fact, the most important—characteristic parameter of a feedback amplifier:

1. The sign of $A\beta$ determines the polarity of the feedback; the loop gain $A\beta$ must be positive for the feedback to be negative.
2. The magnitude of $A\beta$ determines how close the closed-loop gain A_f is to the ideal value of $1/\beta$.
3. The magnitude of $A\beta$ determines the amount of feedback $(1+A\beta)$ and hence, as we shall see in the next section, the magnitude of the various improvements in amplifier performance resulting from the negative feedback.
4. As we shall see in later sections, the inevitable variation of $A\beta$ with frequency can cause $A\beta$ to become negative, which in turn can cause the feedback amplifier to become unstable. It follows that the design of a stable feedback amplifier may involve modifying the frequency behaviors of its loop gain $A\beta$ appropriately (Section 11.10).

[2]This terminology is more common in feedback control systems than in feedback amplifiers.

[3]We have in fact already seen examples of this: adding a resistance R_e in the emitter of a CE amplifier (or a resistance R_s in the source of a CS amplifier) increases the linearity of these amplifiers because for the same input signal as before, v_{be} and v_{gs} are now smaller (by the amount of feedback).

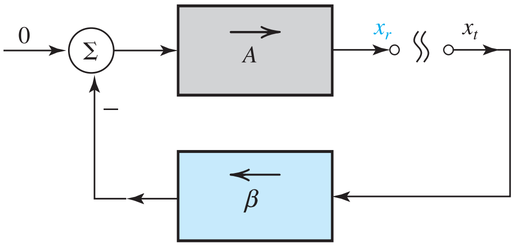

Figure 11.2 Determining the loop gain by breaking the feedback loop at the output of the basic amplifier, applying a test signal x_t, and measuring the returned signal x_r : $A\beta \equiv -x_r/x_t$.

The significance of the loop gain requires us to consider its determination. Reference to Fig. 11.1 indicates that the value of the loop gain $A\beta$ can be determined as follows:

1. Set $x_s = 0$.
2. Break the feedback loop at a convenient location, ensuring that the values of A and β *do not change.* Since we assumed that the feedback network does not load the amplifier output, we can break the loop at the amplifier output (see Fig. 11.2) without causing A to change.
3. Apply a test signal x_t to the input of the loop (where the break has been made) and determine the *returned signal* x_r at the loop output (i.e., at the other side of the break). From Fig. 11.2 we see that

$$x_r = -A\beta x_t$$

and the loop gain $A\beta$ is obtained as

$$A\beta = -\frac{x_r}{x_t} \tag{11.8}$$

We observe that since $A\beta$ is positive, the returned signal x_r will be out of phase with the test signal x_t, verifying that the feedback is indeed negative. In fact, this approach is used qualitatively to ascertain the polarity of the feedback. We will have a lot more to say about the loop gain in subsequent sections.

Example 11.1

The noninverting op-amp configuration shown in Fig. 11.3(a) provides a direct implementation of the feedback loop of Fig. 11.1.

(a) Assume that the op amp has infinite input resistance and zero output resistance. Find an expression for the feedback factor β.

(b) Find the condition the open-loop gain A must satisfy so that the closed-loop gain A_f is almost entirely determined by the feedback network. Also, give the value of A_f in this case.

Example 11.1 *continued*

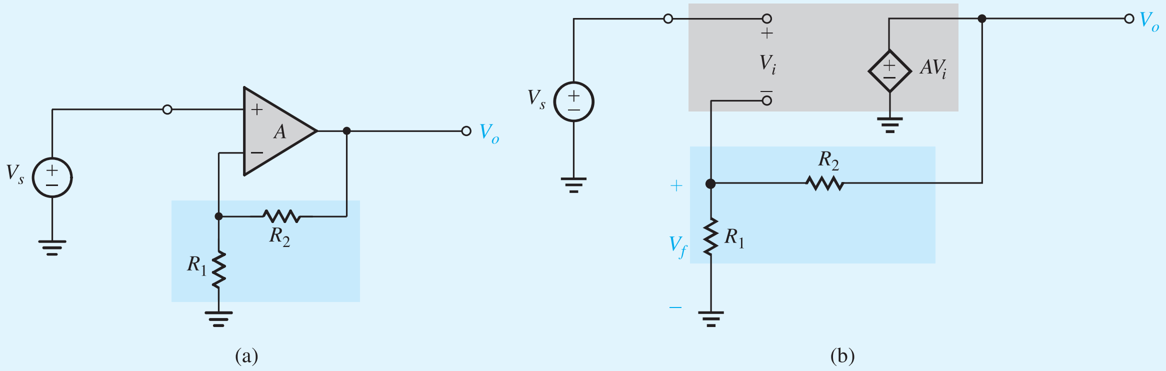

Figure 11.3 **(a)** A noninverting op-amp circuit for Example 11.1. **(b)** The circuit in **(a)** with the op amp replaced with its equivalent circuit.

(c) If the open-loop gain $A = 10^4$ V/V, find R_2/R_1 to obtain a closed-loop gain A_f of 10 V/V.
(d) What is the amount of feedback in decibels?
(e) If $V_s = 1$ V, find V_o, V_f, and V_i.
(f) If A decreases by 20%, what is the corresponding decrease in A_f?

Solution

(a) To be able to see more clearly the direct correspondence between the circuit in Fig. 11.3(a) and the block diagram in Fig. 11.1, we replace the op amp with its equivalent-circuit model, as shown in Fig. 11.3(b). Since the op amp is assumed to have infinite input resistance and zero output resistance, its model is simply an ideal voltage-controlled voltage source of gain A. From Fig. 11.3(b) we observe that the feedback network, consisting of the voltage divider (R_1, R_2), is connected directly to the output and feeds a signal V_f to the inverting input terminal of the op amp. It is important at this point to note that the zero output resistance of the op amp causes the output voltage to be AV_i irrespective of the values of R_1 and R_2. That is what we meant by the statement that in the block diagram of Fig. 11.1, the feedback network is assumed to not load the basic amplifier. Now we can easily determine the feedback factor β from

$$\beta \equiv \frac{V_f}{V_o} = \frac{R_1}{R_1 + R_2}$$

Let's next examine how V_f is subtracted from V_s at the input side. The subtraction is effectively performed by the differential action of the op amp; by its very nature, a differential-input amplifier takes the difference between the signals at its two input terminals. Observe also that because the input resistance of the op amp is assumed to be infinite, no current flows into the negative input terminal of the op amp and that the feedback network does not load the amplifier at the input side.

(b) The closed-loop gain A_f is given by

$$A_f = \frac{A}{1 + A\beta}$$

To make A_f nearly independent of A, we must ensure that the loop gain $A\beta$ is much larger than unity

$$A\beta \gg 1$$

in which case

$$A_f \simeq 1/\beta$$

Thus,

$$A/A_f \gg 1$$

or equivalently,

$$A \gg A_f$$

and

$$A_f \simeq \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

(c) For $A = 10^4$ V/V and $A_f = 10$ V/V, we see that $A \gg A_f$, thus we can select R_1 and R_2 to obtain

$$\beta \simeq \frac{1}{A_f} = 0.1$$

Thus,

$$\frac{1}{\beta} = 1 + \frac{R_2}{R_1} = A_f = 10$$

which yields

$$R_2/R_1 = 9$$

A more exact value for the required ratio R_2/R_1 can be obtained from

$$A_f = \frac{A}{1 + A\beta}$$

$$10 = \frac{10^4}{1 + 10^4\beta}$$

which results in

$$\beta = 0.0999$$

and,

$$\frac{R_2}{R_1} = 9.01$$

Example 11.1 *continued*

(d) The amount of feedback is

$$1 + A\beta = \frac{A}{A_f} = \frac{10^4}{10} = 1000$$

which is 60 dB.
(e) For $V_s = 1$ V,

$$V_o = A_f V_s = 10 \times 1 = 10 \text{ V}$$
$$V_f = \beta V_o = 0.0999 \times 10 = 0.999 \text{ V}$$
$$V_i = \frac{V_o}{A} = \frac{10}{10^4} = 0.001 \text{ V}$$

Note that if we had used the approximate value of $\beta = 0.1$, we would have obtained $V_f = 1$ V and $V_i = 0$ V.
(f) If A decreases by 20%, thus becoming

$$A = 0.8 \times 10^4 \text{ V/V}$$

the value of A_f becomes

$$A_f = \frac{0.8 \times 10^4}{1 + 0.8 \times 10^4 \times 0.0999} = 9.9975 \text{ V/V}$$

that is, it decreases by 0.025%, which is less than the percentage change in A by approximately a factor $(1 + A\beta)$.

EXERCISES

11.1 Repeat Example 11.1 (c) to (f) for $A = 100$ V/V.
Ans. (c) 10.11; (d) 20 dB; (e) 10 V, 0.9 V, 0.1 V; (f) 2.44%

11.2 Repeat Example 11.1 (c) to (f) for $A_f = 10^3$ V/V. For (e) use $V_s = 0.01$ V.
Ans. (c) 1110.1; (d) 20 dB; (e) 10 V, 0.009 V, 0.001 V; (f) 2.44%

11.1.4 Summary

We conclude this section by presenting in Table 11.1 a summary of the important parameters and formulas that characterize the ideal negative-feedback amplifier structure of Fig. 11.1.

Table 11.1 Summary of the Parameters and Formulas for the Ideal Feedback-Amplifier Structure of Fig. 11.1

- Open-loop gain $\equiv A$
- Feedback factor $\equiv \beta$
- Loop gain $\equiv A\beta$ (positive number)
- Amount of feedback $\equiv 1 + A\beta$
- Closed-loop gain $\equiv A_f = \dfrac{x_o}{x_s} = \dfrac{A}{1+A\beta}$
- Feedback signal $\equiv x_f = \dfrac{A\beta}{1+A\beta} x_s$
- Input signal to basic amplifier $\equiv x_i = \dfrac{1}{1+A\beta} x_s$
- Closed-loop gain as a function of the ideal value $\dfrac{1}{\beta}$: $A_f = \left(\dfrac{1}{\beta}\right)\dfrac{1}{1+1/A\beta}$
- For large loop gain, $A\beta \gg 1$,

$A_f \simeq \dfrac{1}{\beta}$ $\qquad x_f \simeq x_s$ $\qquad x_i \simeq 0$

11.2 Some Properties of Negative Feedback

The properties of negative feedback were mentioned in the introduction. In the following, we shall consider some of these properties in more detail.

11.2.1 Gain Desensitivity

The effect of negative feedback on desensitizing the closed-loop gain was demonstrated in Example 11.1, where we saw that a 20% reduction in the gain of the basic amplifier gave rise to only a 0.025% reduction in the gain of the closed-loop amplifier. This sensitivity-reduction property can be analytically established as follows.

Assume that β is constant. Taking differentials of both sides of Eq. (11.4) results in

$$dA_f = \frac{dA}{(1+A\beta)^2} \tag{11.9}$$

Dividing Eq. (11.9) by Eq. (11.4) yields

$$\frac{dA_f}{A_f} = \frac{1}{(1+A\beta)}\frac{dA}{A} \tag{11.10}$$

which says that the percentage change in A_f (due to variations in some circuit parameter) is smaller than the percentage change in A by a factor equal to the amount of feedback. For this reason, the amount of feedback, $1 + A\beta$, is also known as the **desensitivity factor**.

EXERCISE

11.3 An amplifier with a nominal gain $A = 1000$ V/V exhibits a gain change of 10% as the operating temperature changes from 25°C to 75°C. If it is required to constrain the change to 0.1% by applying negative feedback, what is the largest closed-loop gain possible? If three of these feedback amplifiers are placed in cascade, what overall gain and gain stability are achieved?
Ans. 10 V/V; 1000 V/V, with a maximum variability of 0.3% over the specified temperature range.

11.2.2 Bandwidth Extension

Consider an amplifier whose high-frequency response is characterized by a single pole. Its gain at mid and high frequencies can be expressed as

$$A(s) = \frac{A_M}{1 + s/\omega_H} \tag{11.11}$$

where A_M denotes the midband gain and ω_H is the upper 3-dB frequency. Application of negative feedback, with a frequency-independent factor β, around this amplifier results in a closed-loop gain $A_f(s)$ given by

$$A_f(s) = \frac{A(s)}{1 + \beta A(s)}$$

Substituting for $A(s)$ from Eq. (11.11) results, after a little manipulation, in

$$A_f(s) = \frac{A_M/(1 + A_M\beta)}{1 + s/\omega_H(1 + A_M\beta)} \tag{11.12}$$

Thus the feedback amplifier will have a midband gain of $A_M/(1 + A_M\beta)$ and an upper 3-dB frequency ω_{Hf} given by

$$\omega_{Hf} = \omega_H(1 + A_M\beta) \tag{11.13}$$

It follows that the upper 3-dB frequency is increased by a factor equal to the amount of feedback.

Similarly, it can be shown that if the open-loop gain is characterized by a dominant low-frequency pole giving rise to a lower 3-dB frequency ω_L, then the feedback amplifier will have a lower 3-dB frequency ω_{Lf},

$$\omega_{Lf} = \frac{\omega_L}{1 + A_M\beta} \tag{11.14}$$

Note that the amplifier bandwidth is increased by the same factor by which its midband gain is decreased, *maintaining the gain–bandwidth product at a constant value*. This point is further illustrated by the Bode plot in Fig. 11.4.

Finally, note that the action of negative feedback in extending the amplifier bandwidth should not be surprising: Negative feedback works to minimize the change in gain magnitude, including its change with frequency.

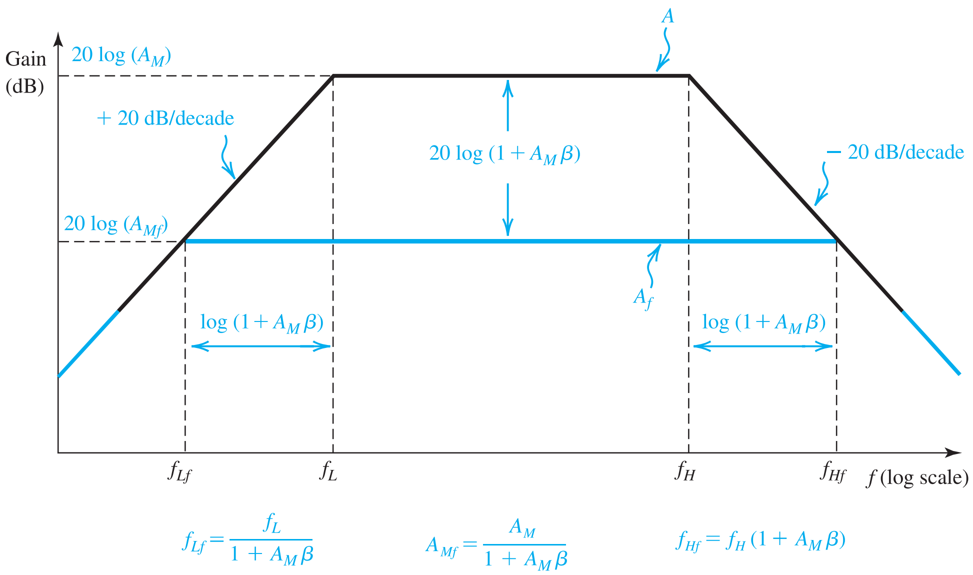

Figure 11.4 Application of negative feedback reduces the midband gain, increases f_H, and reduces f_L, all by the same factor, $(1 + A_M\beta)$, which is equal to the amount of feedback.

EXERCISE

11.4 Consider the noninverting op-amp circuit of Example 11.1. Let the open-loop gain A have a low-frequency value of 10^4 and a uniform –6-dB/octave rolloff at high frequencies with a 3-dB frequency of 100 Hz. Find the low-frequency gain and the upper 3-dB frequency of a closed-loop amplifier with $R_1 = 1\text{ k}\Omega$ and $R_2 = 9\text{ k}\Omega$.
Ans. 9.99 V/V; 100.1 kHz

11.2.3 Interference Reduction

Negative feedback can be employed to reduce the interference in an amplifier or, more precisely, to increase the ratio of signal to interference. However, as we shall now explain, this interference-reduction process is possible only under certain conditions. Consider the situation illustrated in Fig. 11.5. Figure 11.5(a) shows an amplifier with gain A_1, an input signal V_s, and interference, V_n. It is assumed that for some reason this amplifier suffers from interference and that the interference can be assumed to be introduced at the input of the amplifier. The **signal-to-interference ratio** for this amplifier is

$$S/I = V_s/V_n \tag{11.15}$$

Consider next the circuit in Fig. 11.5(b). Here we assume that it is possible to build another amplifier stage with gain A_2 that does not suffer from the interference problem. If this is the