

Second order non-hom. ODE

Variation of parameters

$$y'' + p y' + q y = r$$

Asso. homogeneous ODE has sol^r y_1, y_2 (l. ind.)

$$y_p = v_1 y_1 + v_2 y_2$$

$$y_p' = v_1 y_1' + v_2 y_2'$$

$$y_p''$$

$$\boxed{v_1' y_1 + v_2' y_2 = 0}$$

$$v_1' y_1' + v_2' y_2' = r$$

$$Y(\zeta) = v_1(\zeta) y_1(\zeta)$$

$$+ v_2(\zeta) y_2(\zeta)$$

$$\zeta = \zeta$$

$$y_p(\zeta) = Y(\zeta)$$

$$y_p'(\zeta) = \frac{\partial Y}{\partial \zeta}(u, \zeta) \Big|_{u=\zeta}$$

$$u = \zeta_1$$

$$y_p(u) = v_1(u) y_1(u)$$

$$+ v_2(u) y_2(u)$$

$$Y(u, \zeta) = v_1(\zeta) y_1(u)$$

$$+ v_2(\zeta) y_2(u)$$

$$v_1'(\zeta) y_1(\zeta) + v_2'(\zeta) y_2(\zeta) = 0$$

$$y_p' = v_1' y_1 + v_2' y_2$$

$$+ v_1 y_1' + v_2 y_2'$$

$$y'_p(u) = v_1(u) y_1'(u)$$

$$+ v_2(u) y_2'(u)$$

envelope

