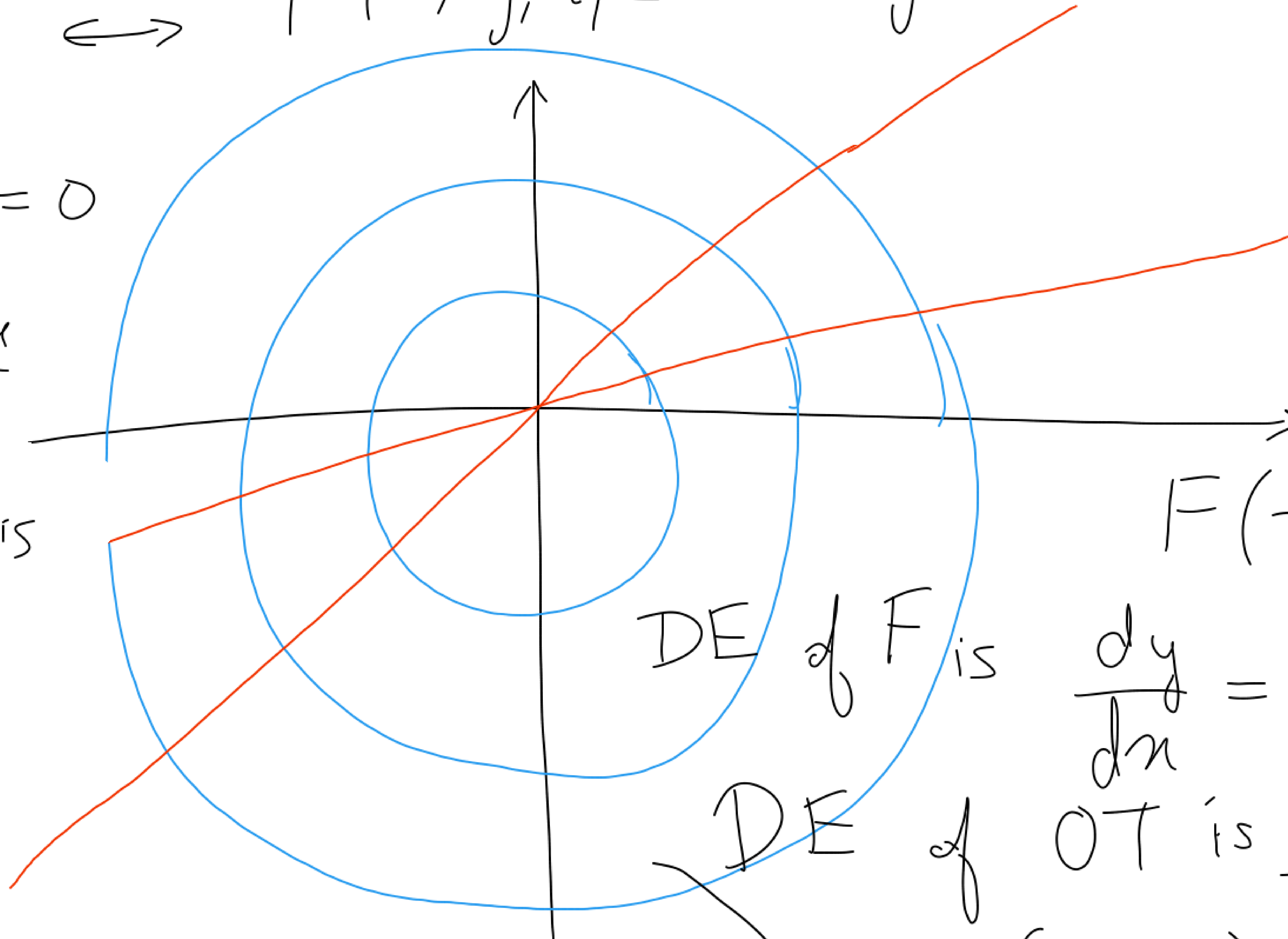
1) Separable ODE
2) Linear first order ODE
3) Exact ODE

Application — Orthogonal trajectory

$x^2+y^2=c^2 \longleftrightarrow F(x,y,c)=x^2+y^2-c^2$

$\Rightarrow 2x+2y\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=\frac{-\cancel{2}x}{\cancel{2}y}$



DE of OT is

$\frac{dy}{dx}=\frac{y}{x}$

$\Rightarrow \frac{dy}{y}=\frac{dx}{x}$

$\Rightarrow \ln|y|=\ln|x|+\ln|k| \Rightarrow y=kx$

$F(x,y,c)=0$

DE of F is $\frac{dy}{dx}=f(x,y)$

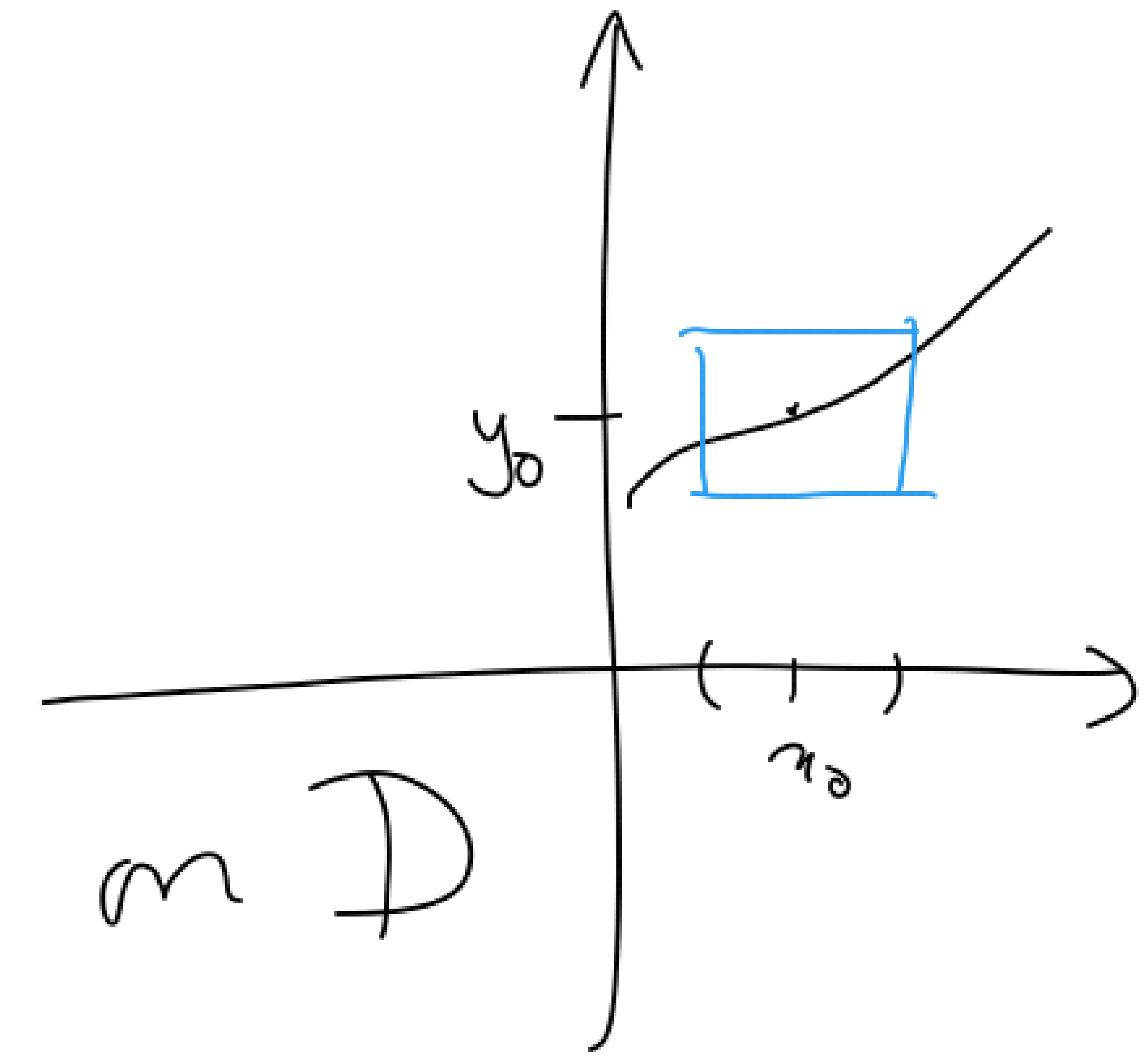
DE of OT is $\frac{dy}{dx}=\frac{-1}{f(x,y)}$

$\Rightarrow G(x,y,k)=0$

$$y' = f(x, y) \quad , \quad y(x_0) = y_0$$

Rectangle $R \ni (x_0, y_0)$

$$f : D \longrightarrow \mathbb{R} \quad , \quad D \subseteq \mathbb{R}^2$$



- f is continuous & bounded on D for existence of solution
- For uniqueness of solution : Lipschitz condition wrt y if $\exists M > 0$

$$|f(x, y_1) - f(x, y_2)| \leq M \, |y_1 - y_2|$$

$g : D \to \mathbb{R}, \quad D \subseteq \mathbb{R}$

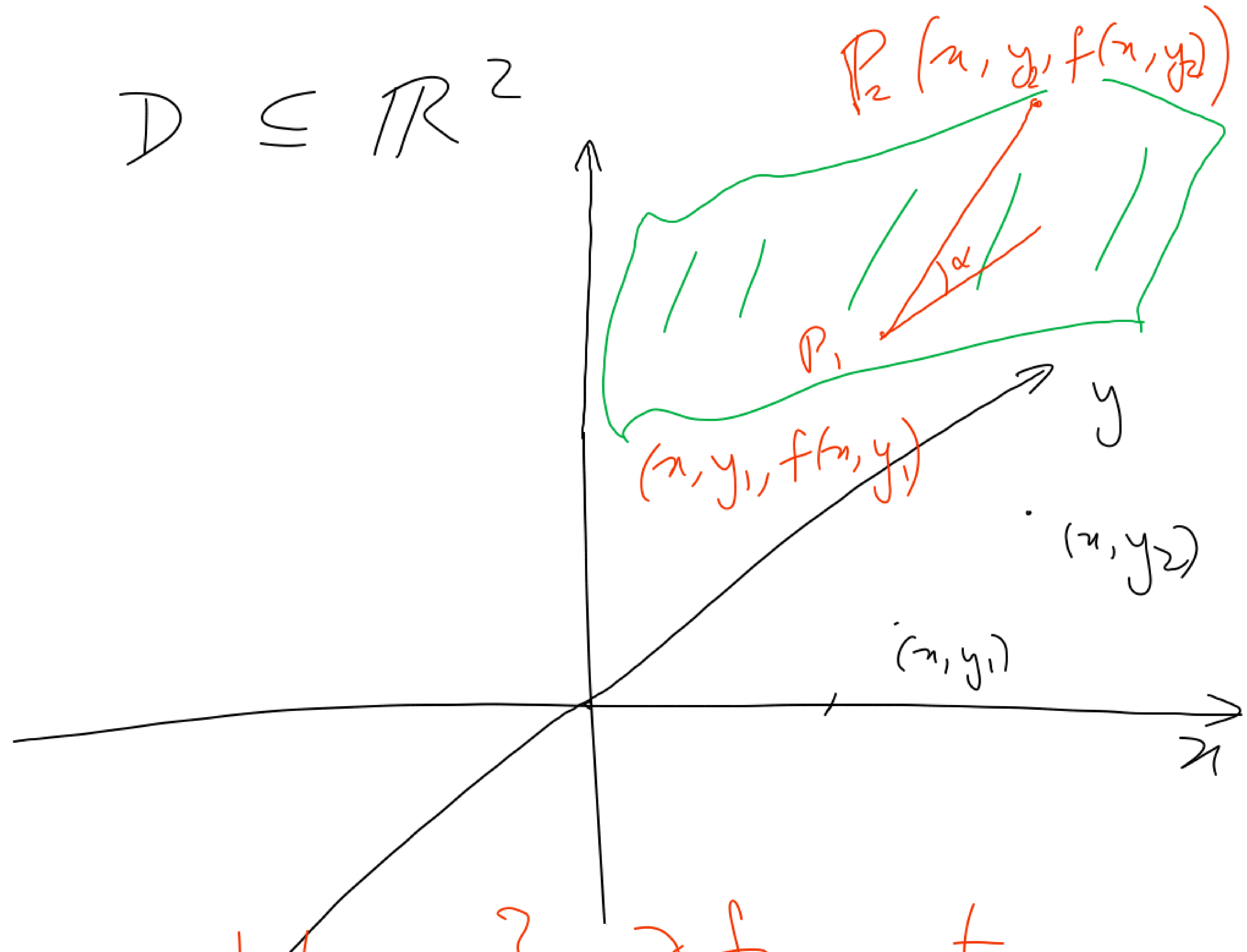
$$-M \leq \frac{g(x_2) - g(x_1)}{x_2 - x_1} \leq M$$

. $g(x) = x^2$ in $[1, 2]$

$$|g(x_1) - g(x_2)| = |x_2^2 - x_1^2| = |x_2 + x_1|\,|x_2 - x_1|$$
$$\leq 4\,|x_2 - x_1|$$

$f : D \to \mathbb{R}$, $\quad D \subseteq \mathbb{R}^2$

$z = f(x, y)$



Lipschitz w.r.t. y $\overset{?}{\Rightarrow}$ continuity w.r.t. x ✗

continuity y ✓

Continuity wrt y $\overset{?}{\Rightarrow}$ Lipschitz wrt y ✗

$\frac{\partial f}{\partial y}$ exist over D & bdd $\overset{?}{\Rightarrow}$ Lipschitz wrt y ✓

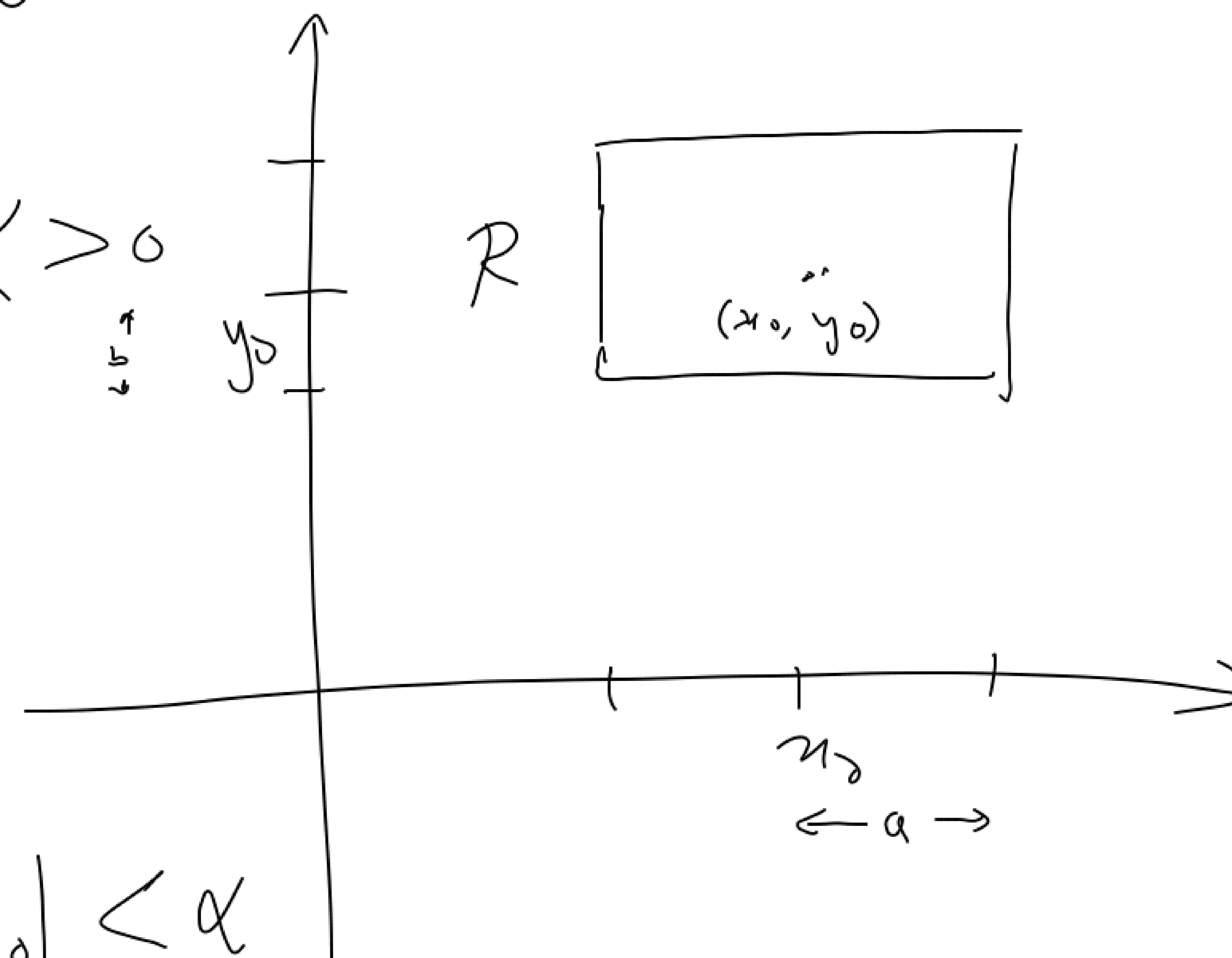
Lipschitz wrt y $\overset{?}{\Rightarrow}$ $\frac{\partial f}{\partial y}$ exist ✗

$$y' = f(x, y), \qquad y(x_0) = y_0$$

f conti on R

f bdd on R, i.e., $\exists K > 0$

$$|f(x,y)| \leq K$$



then $\alpha = \min\left\{a, \dfrac{b}{K}\right\}$

$\exists$ a solution on $|x - x_0| < \alpha$

Lipschitz condition $\Rightarrow$ uniqueness of sol^n IVP