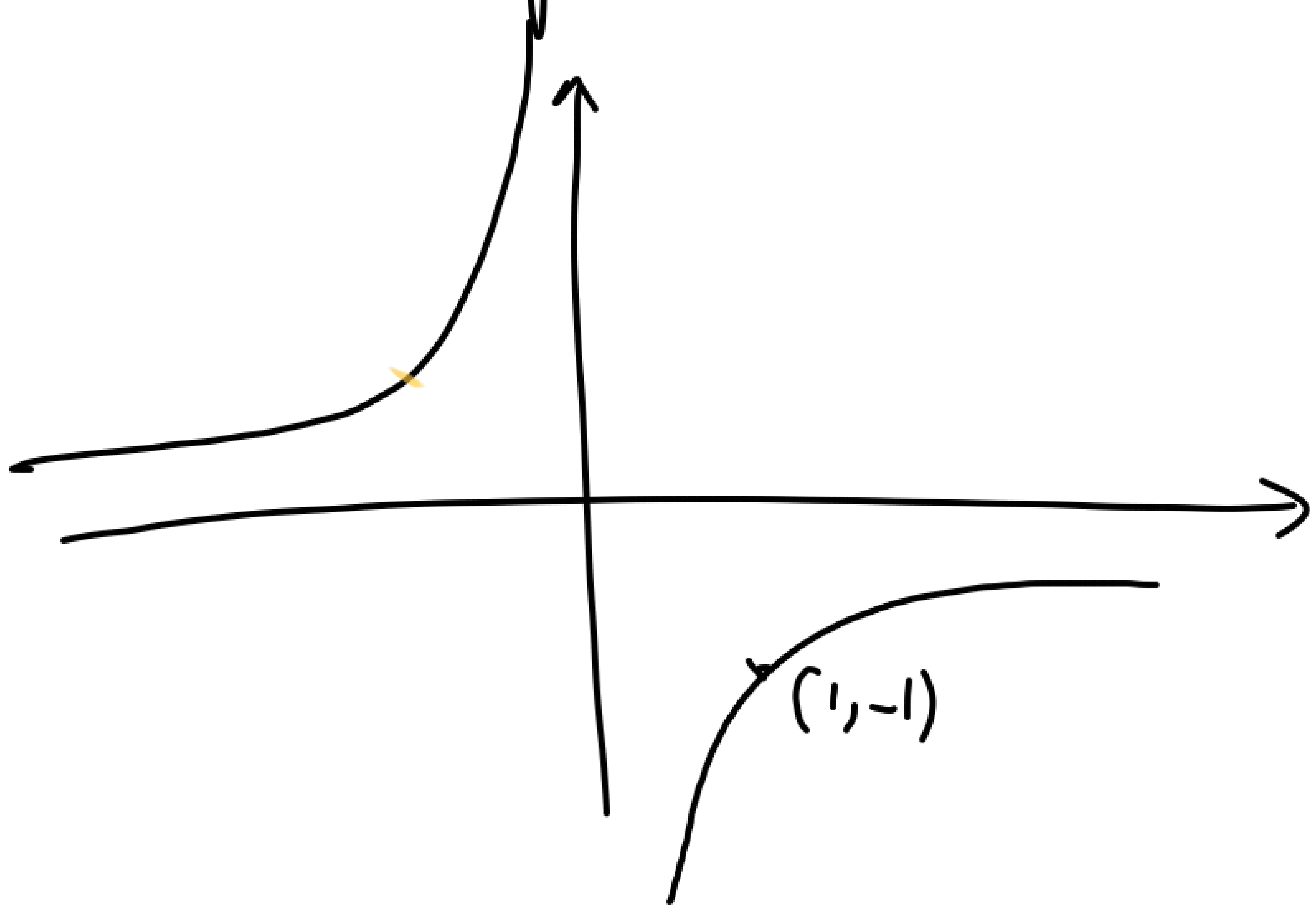
$\boxed{y' = f(x, y)}$, $y(x_0) = y_0$, $R \ni (x_0, y_0)$

existence: $\exists \alpha$ s.t. $|x - x_0| < \alpha$

uniqueness: Lipschitz condition



$y' = y^2, y(1) = -1$

$-\frac{1}{y} = x$

$$y' = -p(x)y + q(x) \quad , \quad y(x_0) = y_0$$

$p(x)$, $q(x)$ cont over I , $x_0 \in I$

$J \ni y_0$ $\quad$ $U(x) = \int_{x_0}^{x} |\phi_1(t) - \phi_2(t)|\, dt$

$\phi_1(x)$, $\phi_2(x)$ are sol^n of IVP

$$\phi_i(x) = y_0 + \int_{x_0}^{x} f(t, \phi_i(t))\, dt \quad \forall i = 1, 2$$

$$|\phi_1(x) - \phi_2(x)| \leq M \int_{x_0}^{x} |\phi_1(t) - \phi_2(t)|\, dt$$

$$u(x_0) = 0, \quad u(x) \geq 0 \quad \forall x \geq x_0$$

$$u'(x) - M u(x) \leq 0$$

$$\Rightarrow e^{-Mx}\left(u'(x) - M u(x)\right) \leq 0$$

$$\Rightarrow \left(e^{-Mx} u(x)\right)' \leq 0$$

$$\Rightarrow \int_{x_0}^{x} \left(e^{-Mx} u(x)\right)' dx \leq 0$$

$$\Rightarrow e^{-Mx} u(x) - \cancel{e^{-Mx_0} u(x_0)} \leq 0$$

$$\Rightarrow u(x) \leq 0 \quad \forall x \geq x_0 \Rightarrow u(x) \equiv 0$$

$$a_2(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_0(x)\,y = g(x)$$

$$\Rightarrow \boxed{\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + q(x)\,y = r(x)}$$

std form.

IVP $\rightarrow$ $y(x_0) = a,\ y'(x_0) = b$

---

y_1, y_2 , $W(y_1, y_2)(x) = \begin{vmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{vmatrix}$

$$y'' + p(x)y' + q(x)y = 0$$

y_1, y_2 are sol^n lin dep. iff $W(x_0) = 0$ for some x_0

$\exists k$ s.t. $y_1(x) = k\, y_2(x)$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

Conversely, $\exists x_0 \in I$ s.t. $W(x_0) = 0$

$$k_1 y_1(x_0) + k_2 y_2(x_0) = 0$$

$$k_1 y_1'(x_0) + k_2 y_2'(x_0) = 0$$

$\exists$ nontrivial $\boxed{k_1, k_2}$

Define $y = k_1 y_1 + k_2 y_2$, $y(x_0) = 0$, $y'(x_0) = 0$

By Ext-ang. $y \equiv 0$

So, y_1, y_2 lin. dep.

$$E = y^2 + (y')^2$$

$$E(t)' \le (1 + 3M)E(t) \quad \left(u'(x) \le k\, u(x) \right)$$