n^{th} order ODEs

- UCM & variation of parameter
- $L(y)$, $L = D^2 + 4D + 1$

$A(f(x)) = 0$ $A = D^2$, $Ax = 0$

$(D+3)e^{-3x} = -3e^{-3x} + 3e^{-3x} = 0$

$(D-2)^2(xe^{2x}) = 0$, $m^2 + 2m + 5$

$m = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm 2i$

$L(y) = r(x)$, $A\,L(y) = 0$

$y^4 - 16y = 0$

Remove the part which is sdn of $L(y) = 0$

The remaining part is the candidate sdn

$24c_5 - 16c_1 = 1$, $c_5 = -\frac{1}{16}$

$$A\ L(y) = (D-1)^2 (D-2)(D-3)y = 0$$

$$y_p = (ax+b)e^x$$

$$L\left(axe^x + be^x + ce^{2x} + de^{3x}\right)$$

$$= L\left(axe^x + be^x\right) = xe^x$$

$$L\left((ax+b)e^x\right) = xe^x$$

$$e^{5x}\left(q_1(x)\cos 2x + q_2(x)\sin 2x\right)$$

$$x\cos x + \sin x = x\left(\frac{e^{ix}+e^{-ix}}{2}\right) - i\,\frac{e^{ix}-e^{-ix}}{2}$$

$$= \frac{1}{2}\left(\underbrace{xe^{ix} - ie^{ix}}_{\overset{\|}{f(x)}}\right) + \overline{f(x)}$$

$$L(y) = f(x), \qquad y'' + y = f(x), \quad Y$$

$$\text{''} \qquad = \overline{f(x)}, \quad \overline{Y}$$

$$y'' + y = \frac{1}{2}\left(x e^{ix} - i e^{ix}\right)$$

$$A = (D+i)^2$$

$$AL = (D+i)^2 (D+i)(D-i)$$

$$y_p = c x^2 e^{ix} + d x e^{ix}$$

$$L\left(c x^2 e^{ix} + d x e^{ix}\right) = (D-i)(D+i)\left(c x^2 e^{ix} + d x e^{ix}\right)$$

$$(D+i)(D-i)\left(Axe^{ix} + Bx^2 e^{ix}\right)$$

$$A(D+i)\,e^{ix} \quad + 2B\,(D+i)\,x e^{ix}$$

$$A\,(D-i+2i)\,e^{ix} + 2B\,(D-i+2i)\,x e^{ix}$$

$$2iA\,e^{ix} + 4iBxe^{ix} \quad + 2B\,e^{ix}$$

$$= \frac{1}{2}\left(xe^{ix} - i\,e^{ix}\right)$$

$$y(x) = -\frac{1}{8}\left(x\,e^{-ix} - ix^2 e^{-ix}\right)$$