Annihilator method (for y_p)

$L(y) = r(x)$, Find A s.t. $A\,r(x) = 0$

Solve $AL(y) = 0$. Remove sol^n of $L(y) = 0$ from this to get the candidate sol^n y_c

Proceed with $L(y_c) = r$ to find y_p

$$x^2 y'' + 2x y' - 6y = 0$$

$$m(m-1) + 2m - 6 = 0$$

$$(xD - 2)x^2 = 2x x - 2x^2 = 0$$

$$(xD - n)x^n = 0$$

$$xDxD = xD + x^2D^2$$

$$xD(xD-1) = x^2D^2$$

$$xD(xD-1)(xD-2) = x^2D^2(xD-2) = x^3D^3$$

$$(xD-2)(xD+3)(xD-2)y_p = 0$$

$$x^2D^2 + 2xD - 6 = xD(xD-1) + 2xD - 6$$

$$= xDxD + xD - 6$$

$$= xDxD + 3xD - 2xD - 6$$

$$= (xD+3)(xD-2)$$

constant coeff	Cauchy-Euler
• 2, 2, 2 root of cha. eq	2, 2, 2, root of A. eq.
$c_1e^{2x} + c_2xe^{2x} + c_3x^2e^{2x}$	$c_1x^2 + c_2x^2\ln x + c_3x^2(\ln x)^2$
• $2+3i$, $e^{2x}(c_1\cos 3x + c_2\sin 3x)$	• $2+3i$, $x^2(c_1\cos(3\ln x) + c_2\sin(3\ln x))$

$y_2 = Ax^2 \ln x$

$L(y_p) = r \Rightarrow (xD-2)(xD+3)\, y_p = 10x^2$

$\Rightarrow (xD+3)(Ax^2) = 10x^2$

$\Rightarrow (xD-2+5)(Ax^2) = 10x^2$

$\Rightarrow 5Ax^2 = 10x^3$

$$L(c)(s) = \int_0^\infty c\, e^{-st}\, dt = \left. \frac{c\, e^{-st}}{-s} \right|_0^\infty$$

$$= 0 + \frac{c}{s}$$

$$\left. \frac{e^{-st} \cos at}{a} \right|_{t=0} = \frac{1}{a}$$

$$L(t^2)(s) = \int_0^\infty e^{-st} t^2 \, dt$$

$$= \frac{t^2 e^{-st}}{-s}\Bigg|_0^\infty - \int_0^\infty \frac{2t\, e^{-st}}{(-s)}\, dt$$

$$= 2\int_0^\infty \frac{t\, e^{-st}}{s}\, dt = \dots = \frac{2}{s^3}$$

$$L(\sin at) = \frac{a}{s^2 + a^2}, \quad L(\cos at) = \frac{s}{s^2+a^2}$$

$$L(t^n) = \frac{n!}{s^{n+1}}$$

$$\int_0^\infty e^{-st} f(t)\,dt$$

$$L\left(\frac{1}{\sqrt{t}}\right)$$