2.13 For the circuit in Fig. E2.13 find the values of i_I, v_1, i_1, i_2, v_O, i_L, and i_O. Also find the voltage gain v_O/v_I, the current gain i_L/i_I, and the power gain P_L/P_I.
Ans. 0; 1 V; 1 mA; 1 mA; 10 V; 10 mA; 11 mA; 10 V/V (20 dB); ∞; ∞

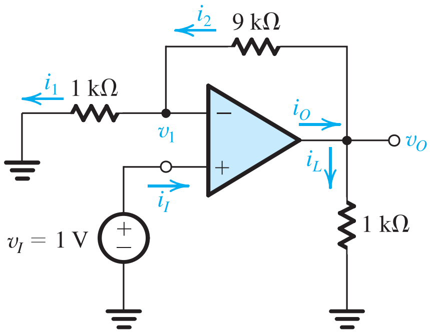

Figure E2.13

2.14 It is required to connect a transducer having an open-circuit voltage of 1 V and a source resistance of 1 MΩ to a load of 1-kΩ resistance. Find the load voltage if the connection is done (a) directly, and (b) through a unity-gain voltage follower.
Ans. (a) 1 mV; (b) 1 V

2.4 Difference Amplifiers

Having studied the two basic configurations of op-amp circuits together with some of their direct applications, we are now ready to consider a somewhat more involved but very important application. Specifically, we shall study the use of op amps to design difference or differential amplifiers.[2] A difference amplifier is one that responds to the difference between the two signals applied at its input and ideally rejects signals that are common to the two inputs. The representation of signals in terms of their differential and common-mode components was given in Fig. 2.4. It is repeated here in Fig. 2.15 with slightly different symbols to serve as the input signals for the difference amplifiers we are about to design. Although ideally the difference amplifier will amplify only the differential input signal v_{Id} and reject completely the common-mode input signal v_{Icm}, practical circuits will have an output voltage v_O given by

$$v_O = A_d v_{Id} + A_{cm} v_{Icm} \tag{2.13}$$

where A_d denotes the amplifier differential gain and A_{cm} denotes its common-mode gain (ideally zero). The efficacy of a differential amplifier is measured by the degree of its rejection of common-mode signals in preference to differential signals. This is usually quantified by a measure known as the **common-mode rejection ratio (CMRR)**, defined as

$$\text{CMRR} = 20 \log \frac{|A_d|}{|A_{cm}|} \tag{2.14}$$

[2]The terms *difference* and *differential* are usually used to describe somewhat different amplifier types. For our purposes at this point, the distinction is not sufficiently significant. We will be more precise near the end of this section.

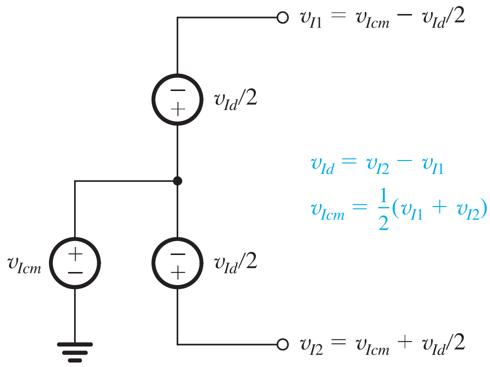

Figure 2.15 Representing the input signals to a differential amplifier in terms of their differential and common-mode components.

The need for difference amplifiers arises frequently in the design of electronic systems, especially those employed in instrumentation. As a common example, consider a transducer providing a small (e.g., 1 mV) signal between its two output terminals while each of the two wires leading from the transducer terminals to the measuring instrument may have a large interference signal (e.g., 1 V) relative to the circuit ground. The instrument front end obviously needs a difference amplifier.

Before we proceed any further we should address a question that the reader might have: The op amp is itself a difference amplifier; why not just use an op amp? The answer is that the very high (ideally infinite) gain of the op amp makes it impossible to use by itself. Rather, as we did before, we have to devise an appropriate feedback network to connect to the op amp to create a circuit whose closed-loop gain is finite, predictable, and stable.

2.4.1 A Single-Op-Amp Difference Amplifier

Our first attempt at designing a difference amplifier is motivated by the observation that the gain of the noninverting amplifier configuration is positive, $(1 + R_2/R_1)$, while that of the inverting configuration is negative, $(-R_2/R_1)$. Combining the two configurations together is then a step in the right direction—namely, getting the difference between two input signals. Of course, we have to make the two gain magnitudes equal in order to reject common-mode signals. This, however, can be easily achieved by attenuating the positive input signal to reduce the gain of the positive path from $(1+R_2/R_1)$ to (R_2/R_1). The resulting circuit would then look like that shown in Fig. 2.16, where the attenuation in the positive input path is achieved by the voltage divider (R_3, R_4). The proper ratio of this voltage divider can be determined from

$$\frac{R_4}{R_4+R_3}\left(1+\frac{R_2}{R_1}\right)=\frac{R_2}{R_1}$$

which can be put in the form

$$\frac{R_4}{R_4+R_3}=\frac{R_2}{R_2+R_1}$$

This condition is satisfied by selecting

$$\frac{R_4}{R_3}=\frac{R_2}{R_1} \qquad (2.15)$$

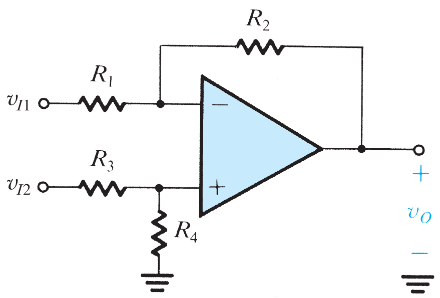

Figure 2.16 A difference amplifier.

This completes our work. However, we have perhaps proceeded a little too fast! Let's step back and verify that the circuit in Fig. 2.16 with R_3 and R_4 selected according to Eq. (2.15) does in fact function as a difference amplifier. Specifically, we wish to determine the output voltage v_O in terms of v_{I1} and v_{I2}. Toward that end, we observe that the circuit is linear, and thus we can use superposition.

To apply superposition, we first reduce v_{I2} to zero—that is, ground the terminal to which v_{I2} is applied—and then find the corresponding output voltage, which will be due entirely to v_{I1}. We denote this output voltage v_{O1}. Its value may be found from the circuit in Fig. 2.17(a), which we recognize as that of the inverting configuration. The existence of R_3 and R_4 does not affect the gain expression, since no current flows through either of them. Thus,

$$v_{O1} = -\frac{R_2}{R_1} v_{I1}$$

Next, we reduce v_{I1} to zero and evaluate the corresponding output voltage v_{O2}. The circuit will now take the form shown in Fig. 2.17(b), which we recognize as the noninverting configuration with an additional voltage divider, made up of R_3 and R_4, connected to the input v_{I2}. The output voltage v_{O2} is therefore given by

$$v_{O2} = v_{I2} \frac{R_4}{R_3 + R_4}\left(1 + \frac{R_2}{R_1}\right) = \frac{R_2}{R_1} v_{I2}$$

where we have utilized Eq. (2.15).

The superposition principle tells us that the output voltage v_O is equal to the sum of v_{O1} and v_{O2}. Thus we have

$$v_O = \frac{R_2}{R_1}(v_{I2} - v_{I1}) = \frac{R_2}{R_1} v_{Id} \tag{2.16}$$

Thus, as expected, the circuit acts as a difference amplifier with a differential gain A_d of

$$A_d = \frac{R_2}{R_1} \tag{2.17}$$

Of course this is predicated on the op amp being ideal and furthermore on the selection of R_3 and R_4 so that their ratio matches that of R_1 and R_2 (Eq. 2.15). To make this matching requirement a little easier to satisfy, we usually select

$$R_3 = R_1 \quad \text{and} \quad R_4 = R_2$$

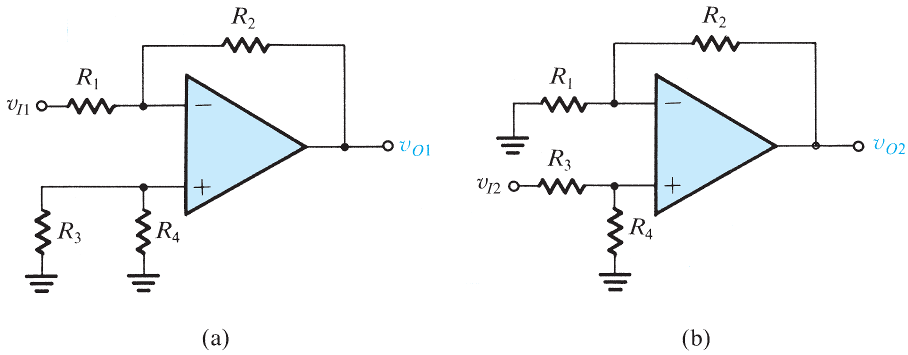

Figure 2.17 Application of superposition to the analysis of the circuit of Fig. 2.16.

Let's next consider the circuit with only a common-mode signal applied at the input, as shown in Fig. 2.18. The figure also shows some of the analysis steps. Thus,

$$i_1 = \frac{1}{R_1}\left[v_{Icm} - \frac{R_4}{R_4+R_3}v_{Icm}\right]$$

$$= v_{Icm}\frac{R_3}{R_4+R_3}\frac{1}{R_1} \tag{2.18}$$

The output voltage can now be found from

$$v_O = \frac{R_4}{R_4+R_3}v_{Icm} - i_2 R_2$$

Substituting $i_2 = i_1$ and for i_1 from Eq. (2.18),

$$v_O = \frac{R_4}{R_4+R_3}v_{Icm} - \frac{R_2}{R_1}\frac{R_3}{R_4+R_3}v_{Icm}$$

$$= \frac{R_4}{R_4+R_3}\left(1 - \frac{R_2}{R_1}\frac{R_3}{R_4}\right)v_{Icm}$$

Thus,

$$A_{cm} \equiv \frac{v_O}{v_{Icm}} = \left(\frac{R_4}{R_4+R_3}\right)\left(1 - \frac{R_2}{R_1}\frac{R_3}{R_4}\right) \tag{2.19}$$

For the design with the resistor ratios selected according to Eq. (2.15), we obtain

$$A_{cm} = 0$$

as expected. Note, however, that any mismatch in the resistance ratios can make A_{cm} nonzero, and hence CMRR finite.

In addition to rejecting common-mode signals, a difference amplifier is usually required to have a high input resistance. To find the input resistance between the two input terminals

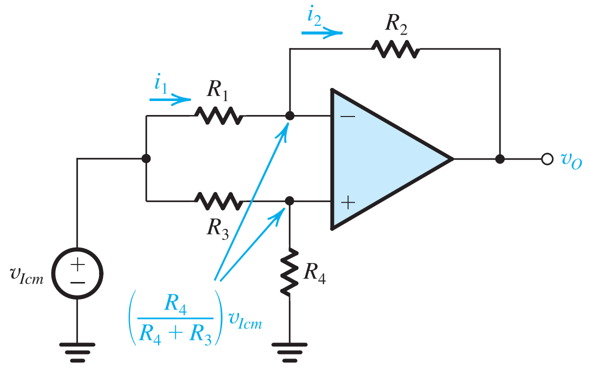

Figure 2.18 Analysis of the difference amplifier to determine its common-mode gain $A_{cm} \equiv v_O/v_{Icm}$.

(i.e., the resistance seen by v_{Id}), called the **differential input resistance** R_{id}, consider Fig. 2.19. Here we have assumed that the resistors are selected so that

$$R_3 = R_1 \quad \text{and} \quad R_4 = R_2$$

Now

$$R_{id} \equiv \frac{v_{Id}}{i_I}$$

Since the two input terminals of the op amp track each other in potential, we may write a loop equation and obtain

$$v_{Id} = R_1 i_I + 0 + R_1 i_I$$

Thus,

$$R_{id} = 2R_1 \tag{2.20}$$

Note that if the amplifier is required to have a large differential gain (R_2/R_1), then R_1 of necessity will be relatively small and the input resistance will be correspondingly low, a drawback of this circuit. Another drawback of the circuit is that it is not easy to vary the differential gain of the amplifier. Both of these drawbacks are overcome in the instrumentation amplifier discussed next.

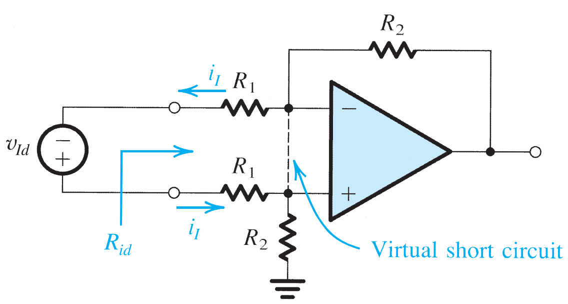

Figure 2.19 Finding the input resistance of the difference amplifier for the case $R_3 = R_1$ and $R_4 = R_2$.