An application of Abel's formula

Consider the second order linear homogeneous ODE

$$y'' + p(x)y' + q(x)y = 0.$$

As we remarked earlier, there is no general method to find a basis of solutions. Let one non-zero solution $y_1(x)$ be known to us.
If y_2 is any other solution of the ODE, the Abel's formula (also called as Abel-Liouville formula) tells us that

$$W(y_1, y_2)(x) \;=\; W(y_1, y_2)(x_0)e^{-\int_{x_0}^{x} p(t)dt}, \; x \in I.$$

For $W(y_1, y_2)(x_0) \neq 0$, y_2 satisfies the first order ODE

$$y_2' - \frac{y_1'(x)}{y_1(x)}y_2 = W(y_1, y_2)(x_0)\frac{1}{y_1(x)}e^{-\int_{x_0}^{x} p(t)dt}, \; x \in I.$$

Abel formula:Method of reduction of order

Integrating factor is

$$e^{\int -\frac{y_1'(x)}{y_1(x)}dx} = \frac{1}{y_1(x)}.$$

Hence

$$y_2(x) \;=\; y_1(x)\int \frac{e^{-\int p(x)dx}}{y_1(x)^2}.$$

This formula gives the method of reduction of order