

# CHAPTER

# 2

## SKETCHING

### OBJECTIVES

After completing this chapter, you should be able to

- Explain the importance of sketching in the engineering design process
- Make simple sketches of basic shapes such as lines, circles, and ellipses
- Use 3-D coordinate systems, particularly right-handed systems
- Draw simple isometric sketches from coded plans
- Make simple oblique pictorial sketches
- Use advanced sketching skills for complex objects

**2.01****INTRODUCTION**

Sketching is one of the primary modes of communication in the initial stages of the design process. Sketching also is a means to creative thinking. It has been shown that your mind works more creatively when your hand is sketching as you are engaged in thinking about a problem.

This chapter focuses on one of the fundamental skills required of engineers and technologists—freehand sketching. The importance of sketching in the initial phases of the design process is presented, as are some techniques to help you create sketches that correctly convey your design ideas. The definition of 3-D coordinate systems and the way they are portrayed on a 2-D sheet of paper will be covered, along with the difference between right- and left-handed coordinate systems. The chapter will investigate how to create simple pictorial sketches. Finally, the advanced sketching techniques of shading and cartooning will be presented with a framework for creating sketches of complex objects. You will begin to explore these topics in this chapter and will further refine your sketching abilities as you progress through your graphics course.

**2.02 Sketching in the Engineering Design Process**

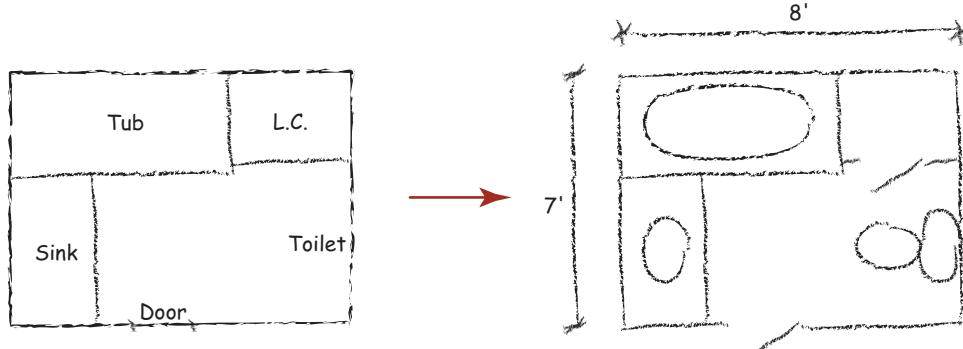
As you may remember from Chapter 1, engineers communicate with one another primarily through graphical means. Those graphical communications take several forms, ranging from precise, complex drawings to simple sketches on the back of an envelope. Most of this text is focused on complex drawings; however, this chapter focuses on simple sketches.

Technically speaking, a sketch is any drawing made without the use of drawing instruments such as triangles and T squares. Some computer graphics packages allow you to create sketches; however, you will probably be more creative (and thus more effective) if you stick to hand sketching, particularly in the initial stages of the design process. In fact, carefully constructed, exact drawings often serve as a hindrance to creativity when they are employed in the initial stages of the design process. Typically, all you need for sketching are a pencil, paper, an eraser, and your imagination.

Your initial sketches may be based on rough ideas. But as you refine your ideas, you will want to refine your sketches, including details that you left out of the originals. For example, suppose you were remodeling the bathroom in your house. Figure 2.01 shows two sketches that define the layout of the bathroom, with details added as ideas evolve. Once you have completed the layout to your satisfaction, you can create an official engineering drawing showing exact dimensions and features that you can give to the contractor who will perform the remodeling work for you.

When engineers sit down to brainstorm solutions to problems, before long, one of them usually takes out a sheet of paper and sketches an idea on it. The others in

**FIGURE 2.01.** Sketches for a bathroom remodel.



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the discussion may add to the original sketch, or they may create sketches of their own. The paper-and-pencil sketches become media for the effective exchange of ideas. Although few “rules” regulate the creation of sketches, you should follow some general guidelines to ensure clarity.

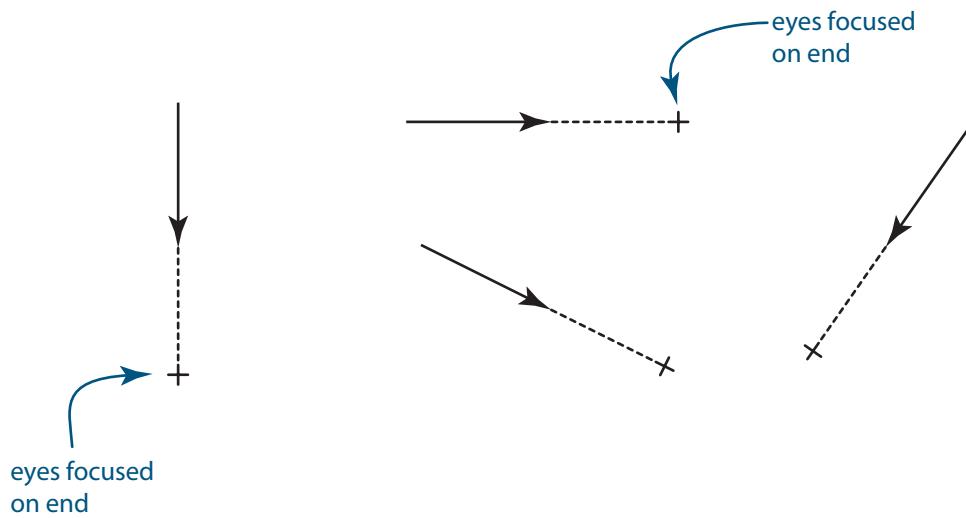
## 2.03 Sketching Lines

Most of your sketches will involve basic shapes made from lines and circles. Although you are not expected to make perfect sketches, a few simple techniques will enable you to create understandable sketches.

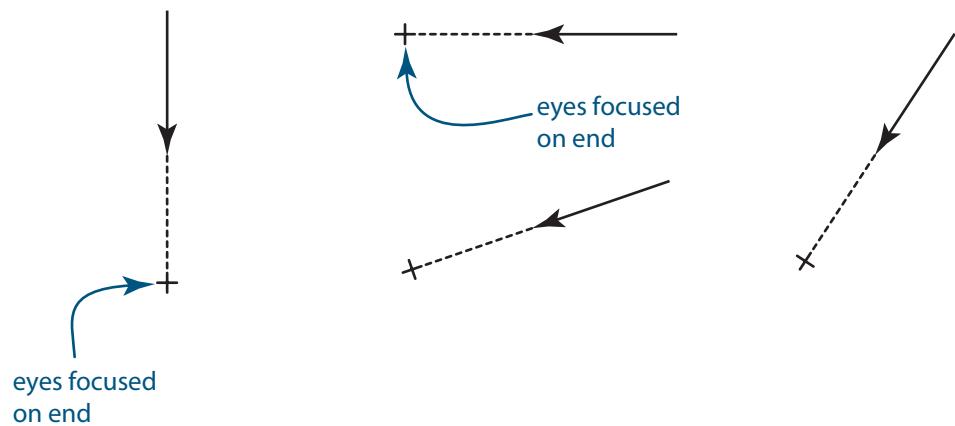
When sketching **lines**, the key is to make them as straight as possible. If you are right-handed, you should sketch your vertical lines from top to bottom and your horizontal lines from left to right. If you are sketching an angled line, choose a direction that matches the general inclination of the line—for angled lines that are mostly vertical, sketch them from top to bottom; for angled lines that are mostly horizontal, sketch them from left to right. If you are left-handed, you should sketch your vertical lines from top to bottom, but your horizontal lines from right to left. For angled lines, left-handed people should sketch from either right to left or top to bottom, again depending on the inclination of the line. To keep your lines straight, focus on the endpoint as you sketch. The best practices for sketching straight lines are illustrated in Figure 2.02.

**FIGURE 2.02.** Techniques for sketching straight lines.

For right-handed people:



For left-handed people:



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You also can try rotating the paper on the desk to suit your preferences. For example, if you find that drawing vertical lines is easiest for you and you are confronted with an angled line to sketch, rotate the paper on the desk so you can sketch a “vertical” line. Or you can rotate the paper 90 degrees to sketch a horizontal line. Figure 2.03 illustrates rotation of the paper to create an angled line.

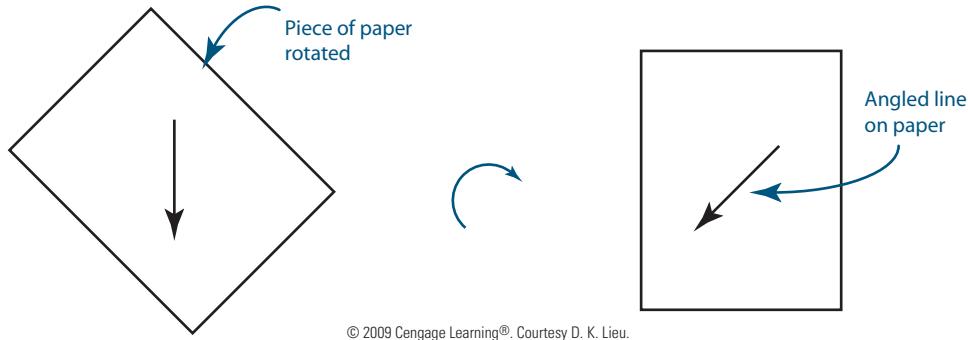
One last point to consider when sketching lines is that you initially may have to create “long” lines as a series of connected segments. Then you can sketch over the segments in a continuous motion to make sure the line appears to be one entity and not several joined end to end. Using segments to define long lines is illustrated in Figure 2.04.

## 2.04 Sketching Curved Entities

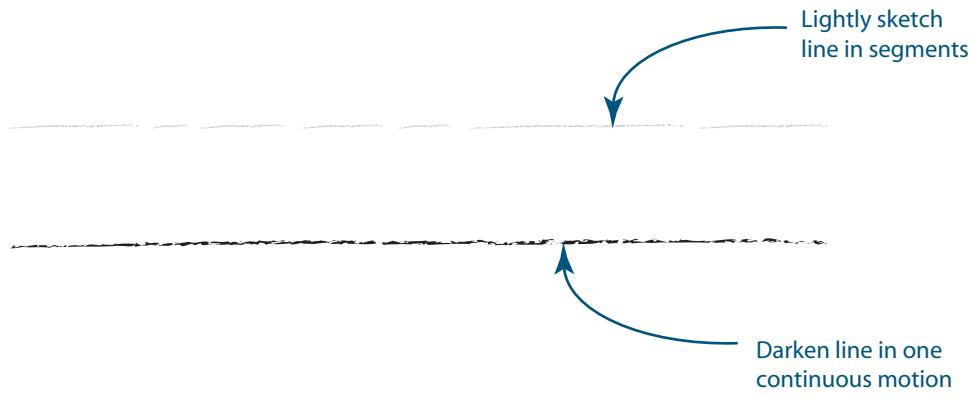
**Arcs** and **circles** are other types of geometric entities you often will be required to sketch. When sketching arcs and circles, use lightly sketched square **bounding boxes** to define the limits of the curved entities and then construct the curved entities as tangent to the edges of the bounding box. For example, to sketch a circle, you first lightly sketch a square (with straight lines). Note that the length of the sides of the bounding box is equal to the diameter of the circle you are attempting to sketch. At the centers of each edge of the box, you can make a short **tick mark** to establish the point of tangency for the circle, then draw the four arcs that make up the circle. Initially, you may find it easier to sketch one arc at a time to complete the circle; but as you gain experience, you may be able to sketch the entire circle all at once. Figure 2.05 shows the procedure used to sketch a circle by creating a bounding box first.

One problem you may have when using a bounding box to sketch a circle occurs when the radius of the circle is relatively large. In that case, the arcs you create may be too flat or too curved, as shown in Figure 2.06. To avoid this type of error, you might try marking the radius at points halfway between the tick marks included on the bounding box. Using simple geometry, when you draw a line between the center of the circle and the corner of the bounding box, the radius is about two-thirds of the distance

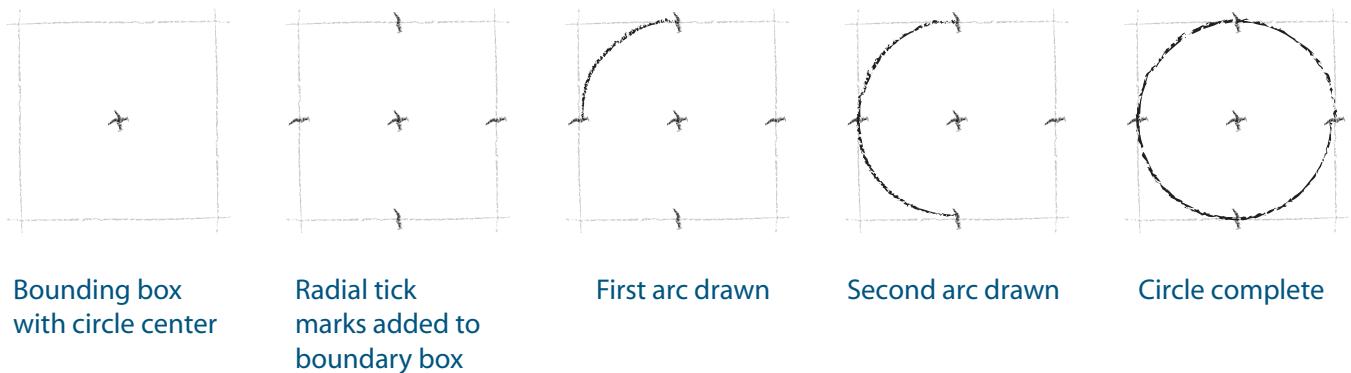
**FIGURE 2.03.** Rotating the paper to draw an angled line.



**FIGURE 2.04.** Sketching long lines in segments.



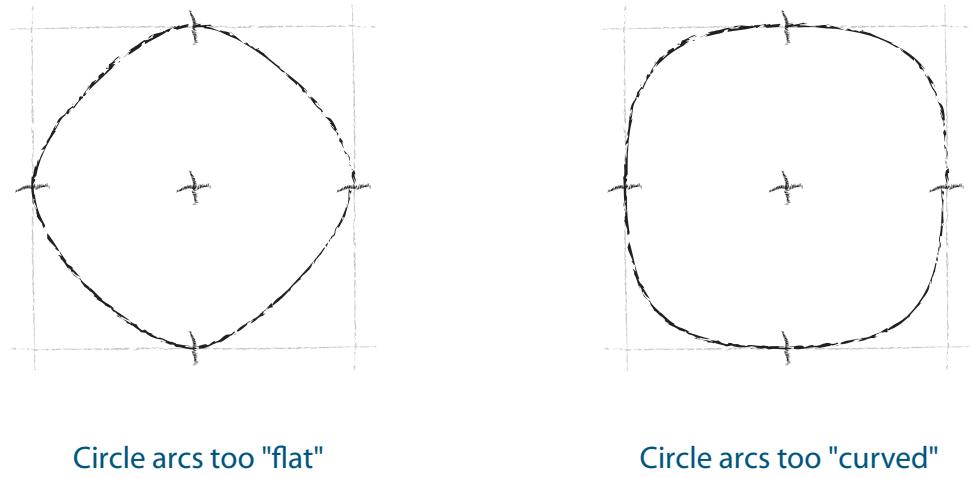
(technically, the radius is 0.707, but that number is close enough to two-thirds for your purposes). Then you can include some additional tick marks around the circle to guide your sketching and to improve the appearance of your circles. This technique is illustrated in Figure 2.07.



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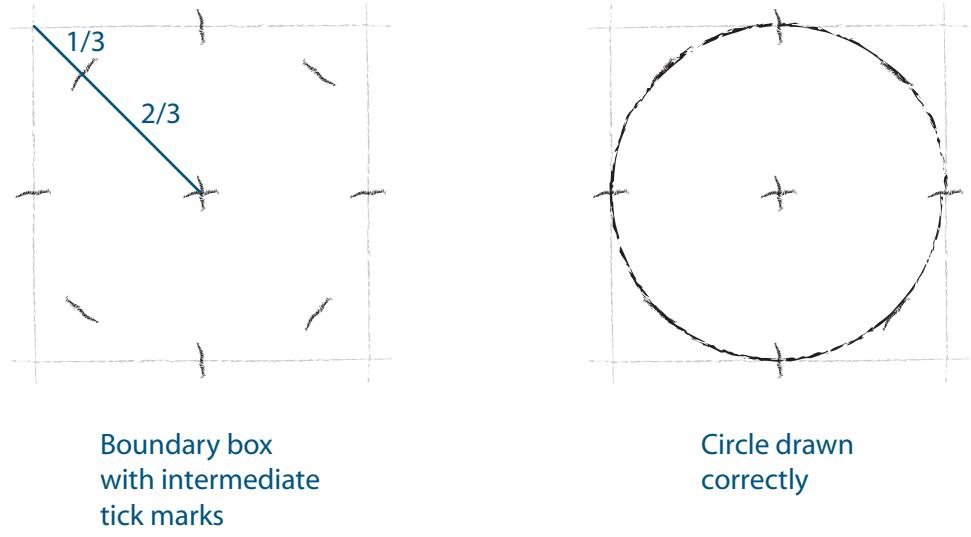
**FIGURE 2.05.** Sketching a circle using a bounding box.

**FIGURE 2.06.** Circles sketched either too flat or too curved.



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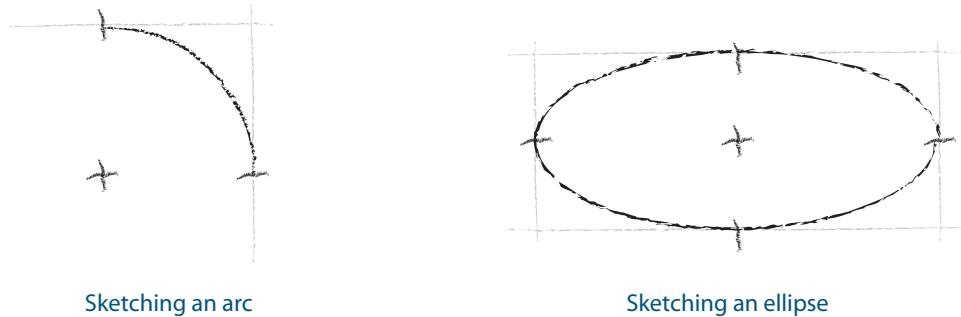
**FIGURE 2.07.** Using intermediate radial tick marks for large circles.



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Sketching an arc follows the same general procedure as sketching a circle, except that your curved entity is only a portion of a circle. Sketching an **ellipse** follows the same general rules as sketching a circle, except that your bounding box is a rectangle and not a square. Sketching arcs and ellipses is illustrated in Figure 2.08.

**FIGURE 2.08.** Using boundary boxes to sketch arcs and ellipses.



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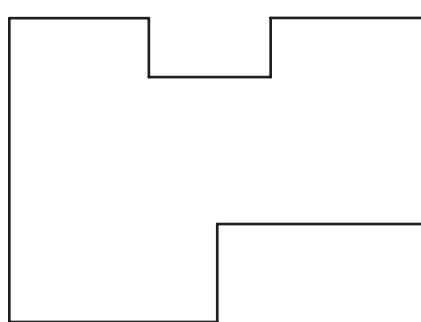
## 2.05 Construction Lines

Similar to the way you used bounding boxes to create circles and ellipses, other **construction lines** can help with your sketching. Using construction lines, you outline the shape of the object you are trying to sketch. Then you fill in the details of the sketch using the construction lines as a guide. Figure 2.09 shows the front view of an object you need to sketch. To create the sketch, you lightly draw the construction lines that outline the main body of the object and then create the construction lines that define the prominent features of it. One rule of thumb is that construction lines should be drawn so lightly on the page that when it is held at arm's length, the lines are nearly impossible to see. The creation of the relevant construction lines is illustrated in Figure 2.10.

Using construction lines as a guide, you can fill in the details of the front view of the object until it is complete. The final result is shown in Figure 2.11.

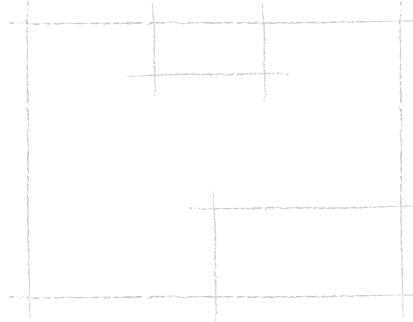
Another way you can use construction lines is to locate the center of a square or rectangle. Recall from your geometry class that the diagonals of a box (either a rectangle or a square) intersect at its center. After you create construction lines for the edges of the box, you sketch the two diagonals that intersect at the center. Once you find the center of the box, you can use it to create a new centered box of smaller dimensions—a kind of concentric box. Locating the center of a box and creating construction lines for a newly centered box within the original box are illustrated in Figure 2.12.

Once you have created your centered box within a box, you can sketch a circle using the smaller box as a bounding box, resulting in a circle that is centered within the larger box as shown in Figure 2.13. Or you can use these techniques to create a square with four holes located in the corners of the box as illustrated in Figure 2.14.



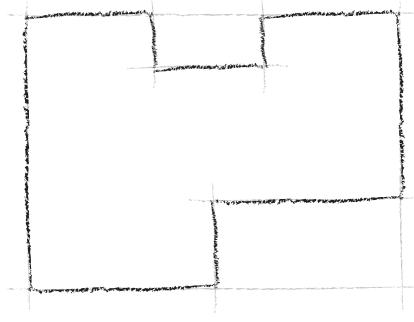
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**FIGURE 2.09.** The front view of an object to sketch.



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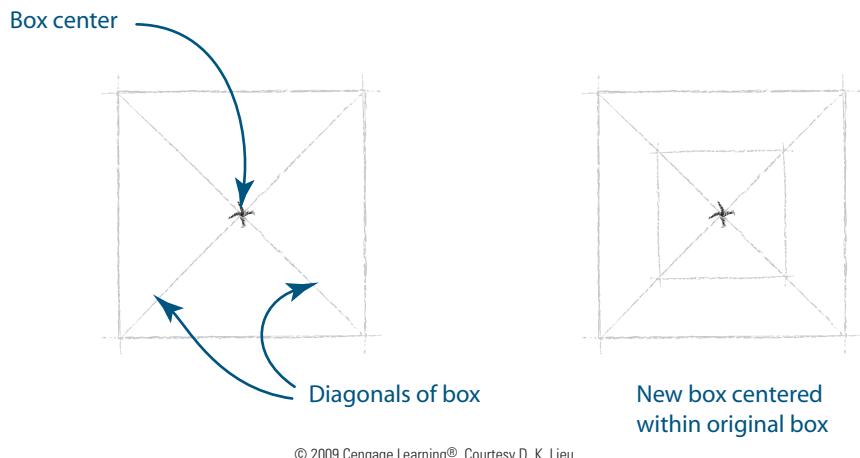
**FIGURE 2.10.** Construction lines used to create a sketch.



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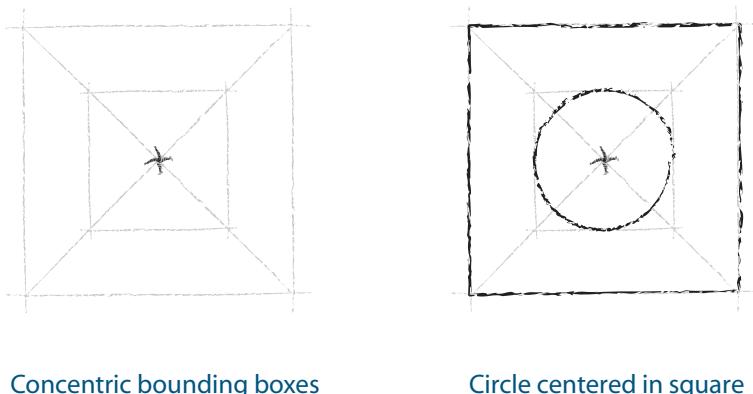
**FIGURE 2.11.** Completed sketch using construction lines as a guide.

**FIGURE 2.12.** Creating concentric bounding boxes.



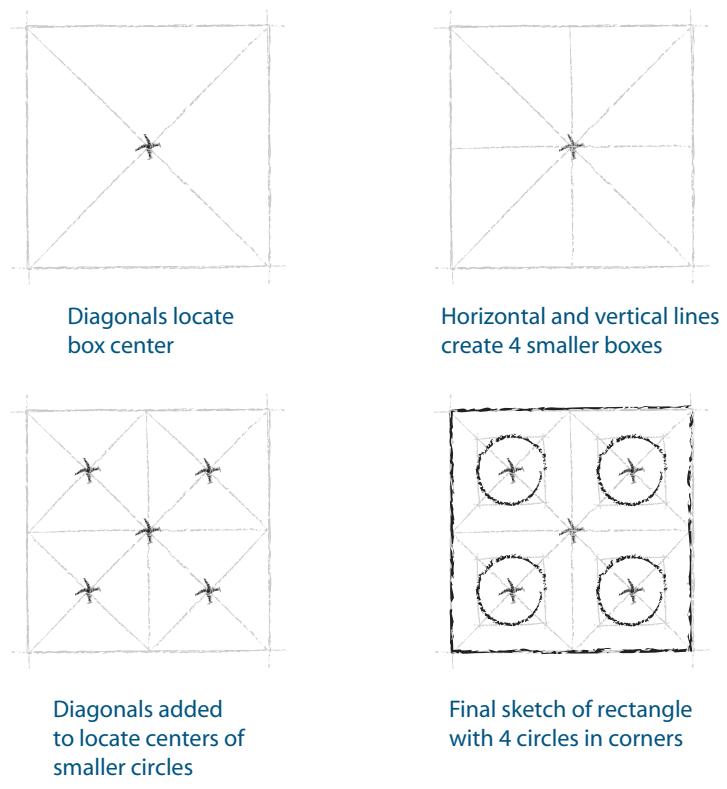
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**FIGURE 2.13.** Sketching a circle in a box.



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**FIGURE 2.14.** Using diagonal construction lines to locate centers.



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## 2.06 Coordinate Systems

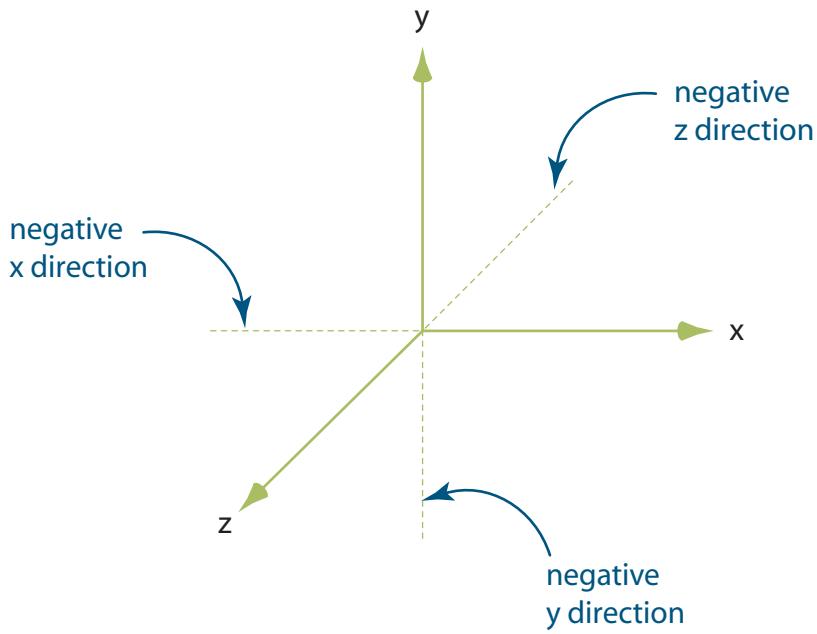
When sketching, you often have to portray 3-D objects on a flat 2-D sheet of paper. As is usually the case with graphical communication, a few conventions have evolved over time for representing 3-D space on a 2-D sheet of paper. One convention, called the **3-D coordinate system**, is that space can be represented by three mutually perpendicular coordinate axes, typically the x-, y-, and z-axes. To visualize those three axes, look at the bottom corner of the room. Notice the lines that are formed by the intersection of each of the two walls with the floor and the line that is formed where the two walls intersect. You can think of these lines of intersection as the x-, y-, and z-coordinate axes. You can define all locations in the room with respect to this corner, just as all points in 3-D space can be defined from an origin where the three axes intersect.

You are probably familiar with the concept of the three coordinate axes from your math classes. In Figure 2.15, a set of coordinate axes, notice the positive and negative directions for each of the axes. Typically, arrows at the ends of the axes denote the positive direction along the axes.

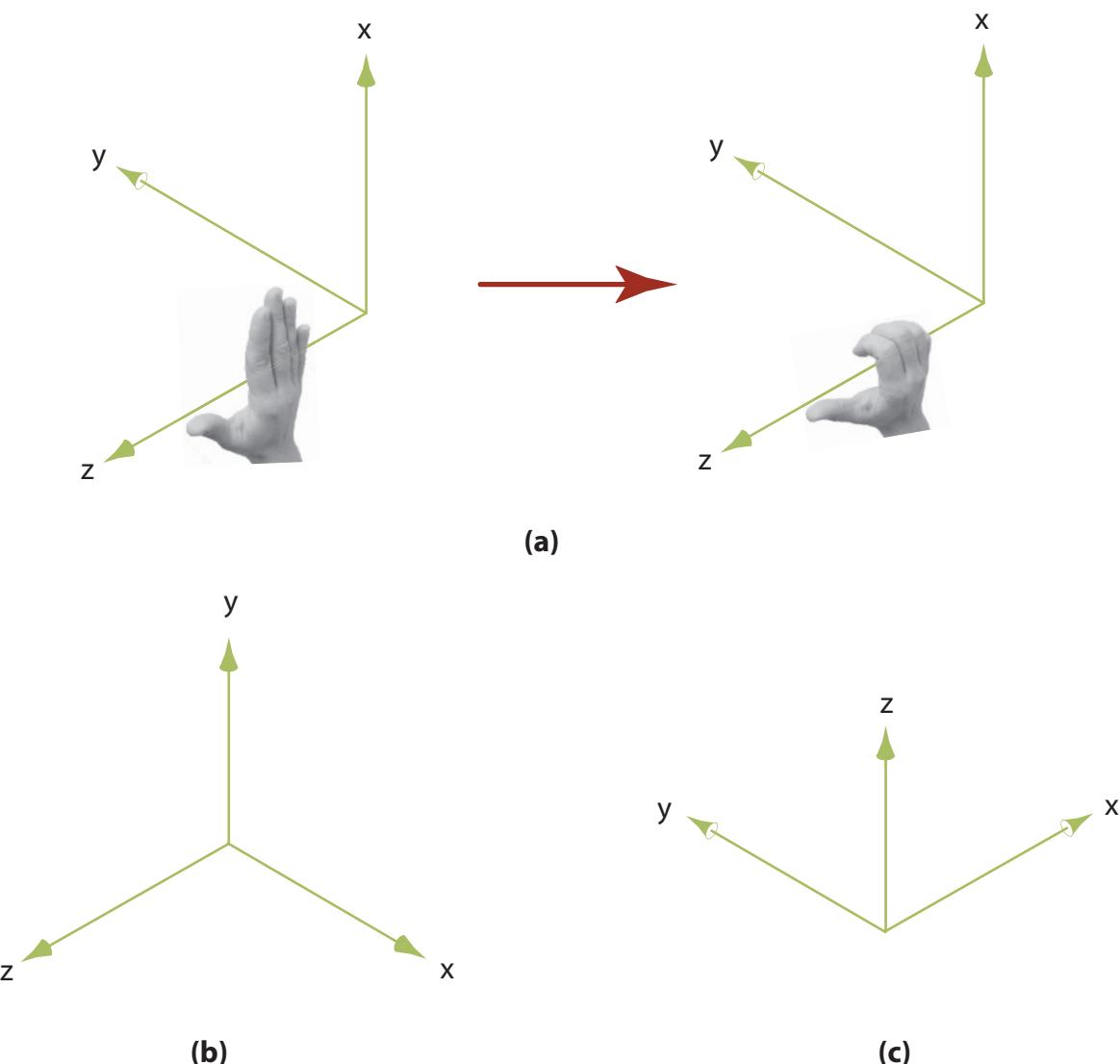
For engineering, the axes usually define a right-handed coordinate system. Since most engineering analysis techniques are defined by a right-handed system, you should learn what this means and how to recognize such a system when you see it. A **right-handed system** means that if you point the fingers of your right hand down the positive x-axis and curl them in the direction of the positive y-axis, your thumb will point in the direction of the positive z-axis, as illustrated in Figure 2.16. This procedure is sometimes referred to as the **right-hand rule**.

Another way to think about the right-hand rule is to point your thumb down the positive x-axis and your index finger down the positive y-axis; your middle finger will then automatically point down the positive z-axis. This technique is illustrated in Figure 2.17. Either method for illustrating the right-hand rule results in the same set of coordinate axes; choose the method that is easiest for you to use.

**FIGURE 2.15.** The x-, y-, and z-coordinate axes.



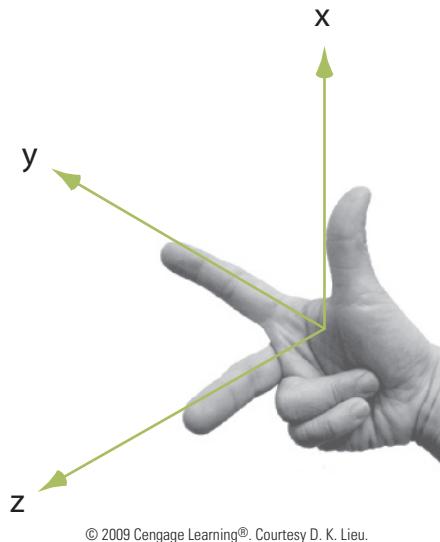
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**FIGURE 2.16.** Curling the fingers to check for a right-handed coordinate system in (a) and alternative presentations of right-handed coordinate systems in (b) and (c).

**FIGURE 2.17.** An alternative method to check for a right-handed coordinate system.

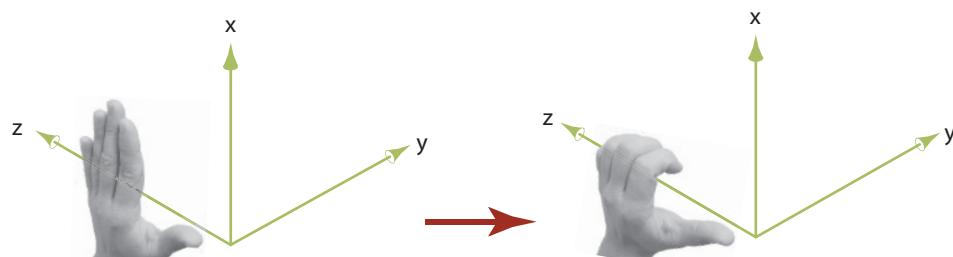


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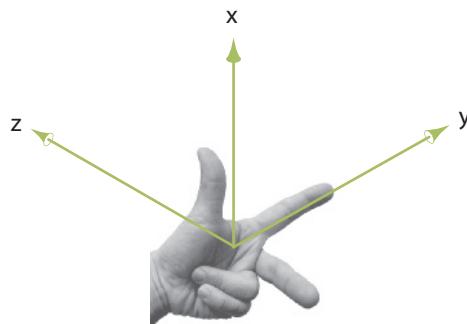
Notice that if you try either technique with your left hand, your thumb (or middle finger) will point down the negative z-axis, as illustrated in Figure 2.18.

A **left-handed system** is defined similarly to a right-handed system, except that you use your left hand to show the positive directions of the coordinate axes. Left-handed systems are typically used in engineering applications that are geologically based—positive z is defined as going down into the earth. Figure 2.19 illustrates left-handed coordinate systems. (Use the left-hand rule to verify that these are left-handed coordinate systems.)

**FIGURE 2.18.** The result of using the left hand to test for a right-handed coordinate system.



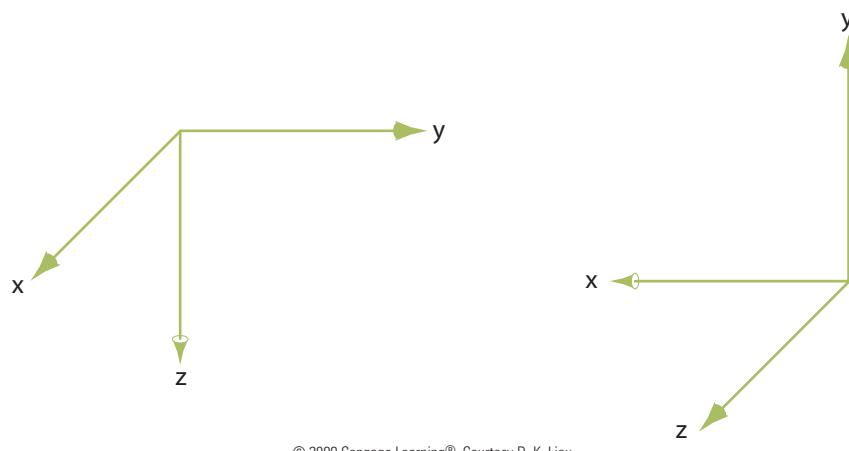
(a)



(b)

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**FIGURE 2.19.** Left-handed coordinate systems.



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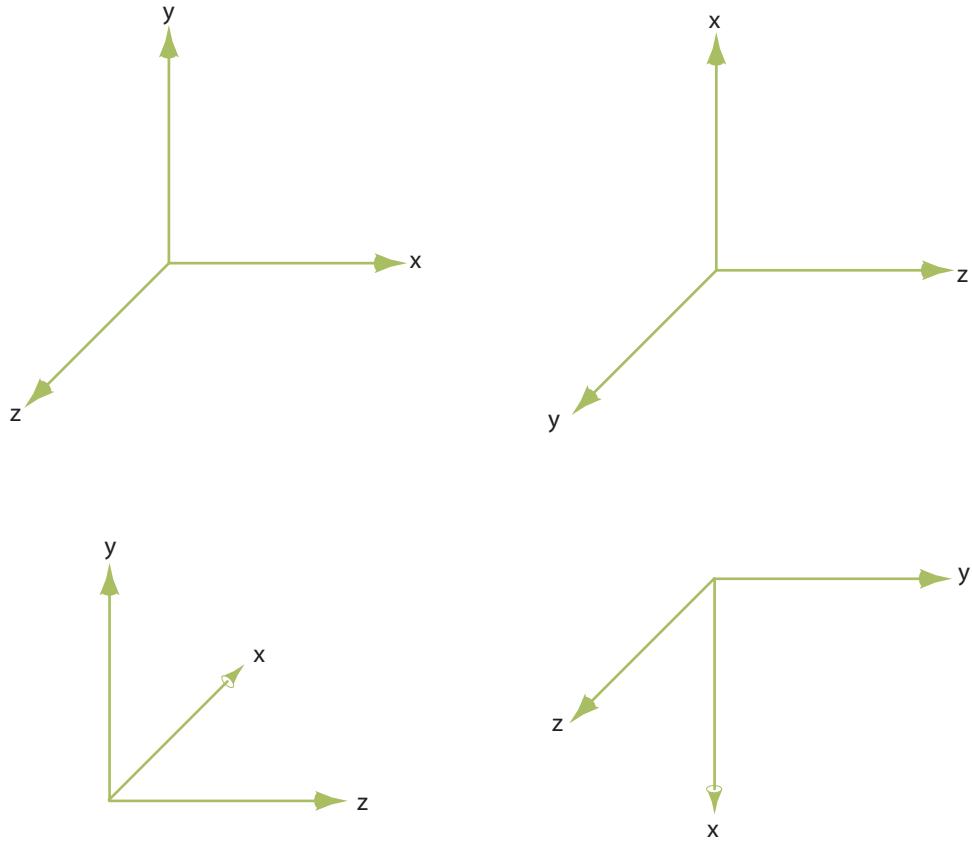
The question remains about how to represent 3-D space on a 2-D sheet of paper when sketching. The answer is that the three coordinate axes are typically represented as oblique or isometric, depending on the preferences of the person making the sketch. You are probably most familiar with oblique representation of the coordinate axes, which seems to be the preferred method of many individuals. With this method, two axes are sketched perpendicular to each other and the third is drawn at an angle, usually 45 degrees to both axes. The angle of the inclined line does not have to be 45 degrees, but it is usually sketched that way. Your math teachers probably sketched the three coordinate axes that way in their classes. Figure 2.20 shows multiple sets of coordinate axes drawn as oblique axes. Notice that all of the coordinate systems are right-handed systems. (Verify this for yourself by using the right-hand rule.)

Another way of portraying the 3-D coordinate axes on a 2-D sheet of paper is through isometric representation. With this method, the axes are projected onto the paper as if you were looking down the diagonal of a cube. When you do this, the axes appear to be 120 degrees apart, as shown in Figure 2.21. In fact, the term **isometric** comes from the Greek *iso* (meaning “the same”) and *metric* (meaning “measure”). Notice that for **isometric axes** representations, the right-hand rule still applies.

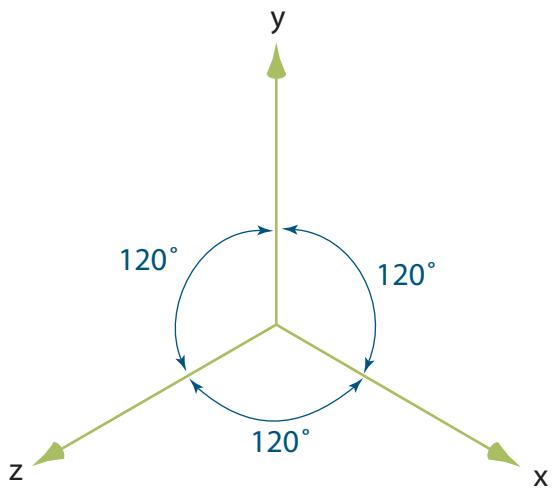
Isometric axes also can be sketched with one of the axes extending in the “opposite” direction. This results in angles other than 120 degrees, depending on the orientation of the axes with respect to the paper, as shown in Figure 2.22.

Grid or dot paper can help you make isometric sketches. With **isometric dot paper**, the dots are oriented such that when you sketch lines through the dots, you end up with standard 120-degree axes. With grid paper, the lines are already drawn at an angle of 120 degrees with respect to one another. Isometric grid paper and isometric dot paper are illustrated in Figure 2.23.

**FIGURE 2.20.** An oblique representation of right-handed coordinate systems.

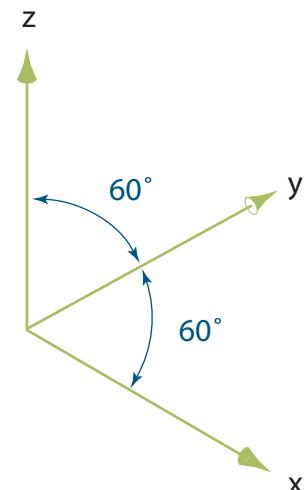


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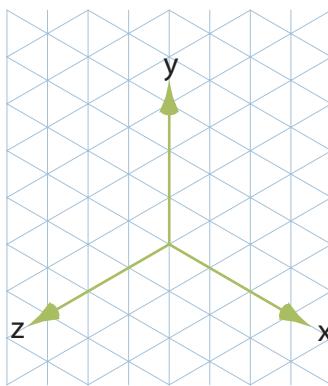
**FIGURE 2.21.** An isometric representation of a right-handed coordinate system.



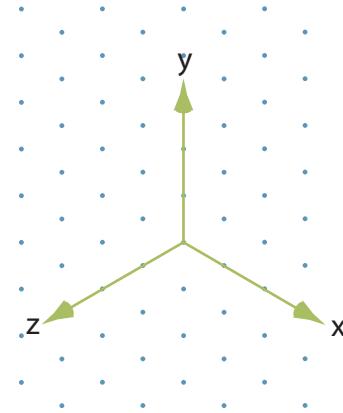
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**FIGURE 2.22.** An isometric representation of axes with angles less than 120 degrees.

**FIGURE 2.23.** Isometric grid and dot paper.



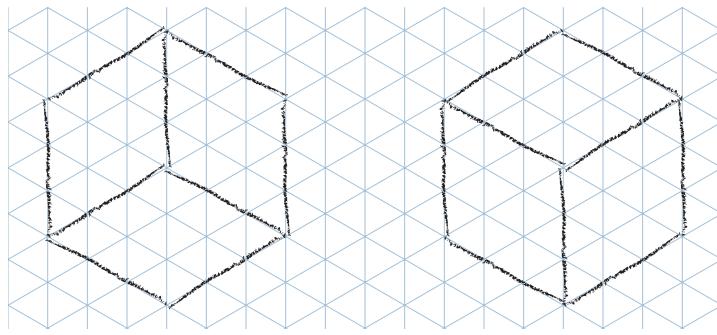
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## 2.07 Isometric Sketches of Simple Objects

Creating isometric drawings and sketches of complex objects will be covered in more detail in a later chapter; however, this section serves as an introduction to the topic for simple objects. Mastering the techniques used to create isometric sketches of simple objects may help as you branch out to tackle increasingly complex objects. Figure 2.24 shows how **isometric grid paper** is used to sketch a  $3 \times 3 \times 3$  block. Notice that there

**FIGURE 2.24.** Using isometric grid paper to sketch a block.



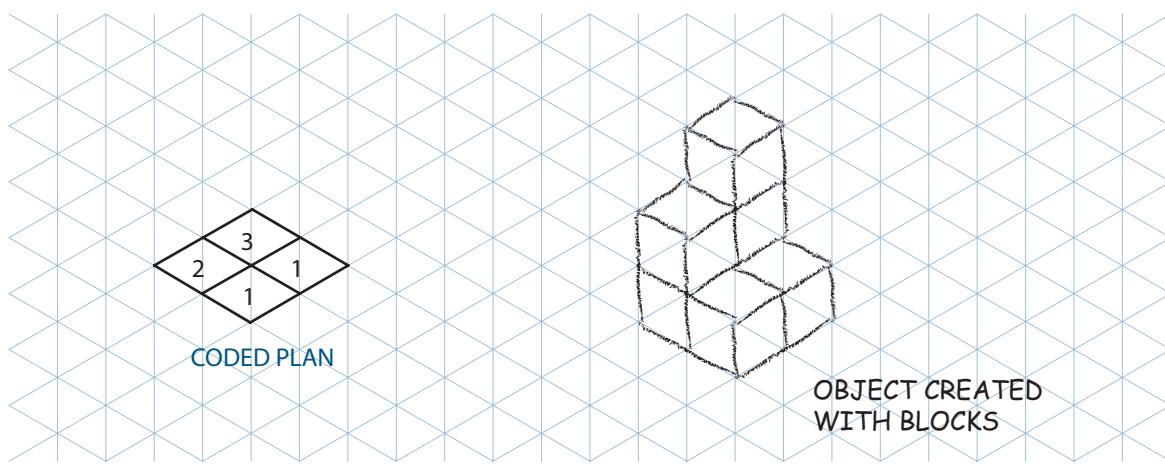
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is more than one orientation from which the block can be sketched on the same sheet of grid paper. Ultimately, the orientation you choose depends on your needs or preferences.

Coded plans can be used to define simple objects that are constructed entirely out of blocks. The numerical values in the coded plan represent the height of the stack of blocks at that location. The object then “grows” up from the plan according to the numbers specified. Figure 2.25 shows a coded plan on isometric grid paper and the object that results from it.

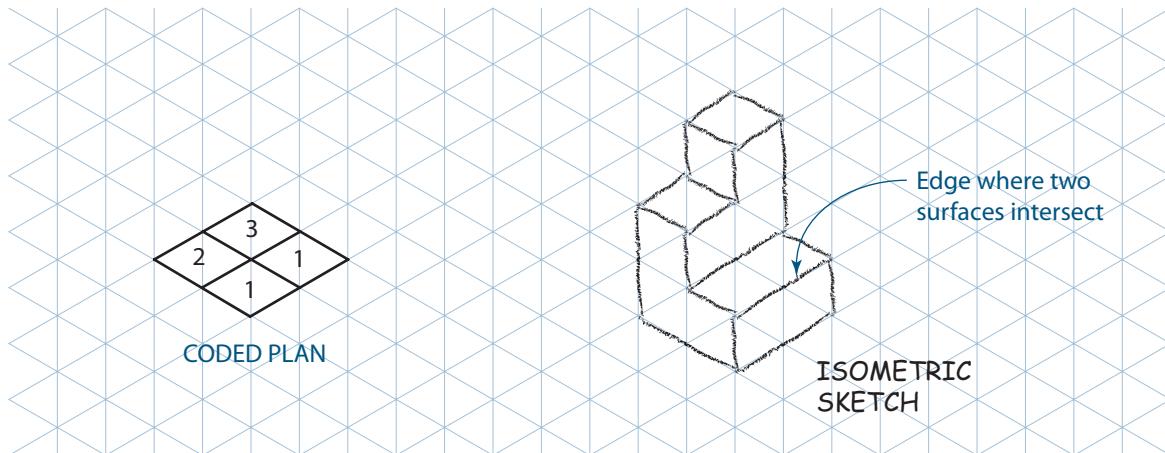
The object shown in Figure 2.25 clearly outlines all of the blocks used to create it. When isometric sketches of an object are made, however, standard practice dictates that lines appear only where two surfaces intersect—lines between blocks on the same surface are not shown. Figure 2.26 shows the object from Figure 2.25 after the unwanted lines have been removed. Notice that the only lines on the sketch are those formed from the intersection of two surfaces. Also notice that object edges hidden from view on the back side are not shown in the sketch. Not showing hidden edges on an **isometric pictorial** also is standard practice in technical sketching.

Sometimes when you are creating an isometric sketch of a simple object, part of one surface is obscured by one of the more prominent features of the object. When creating the sketch, make sure you show only the visible part of the surface in question, as illustrated in Figure 2.27.



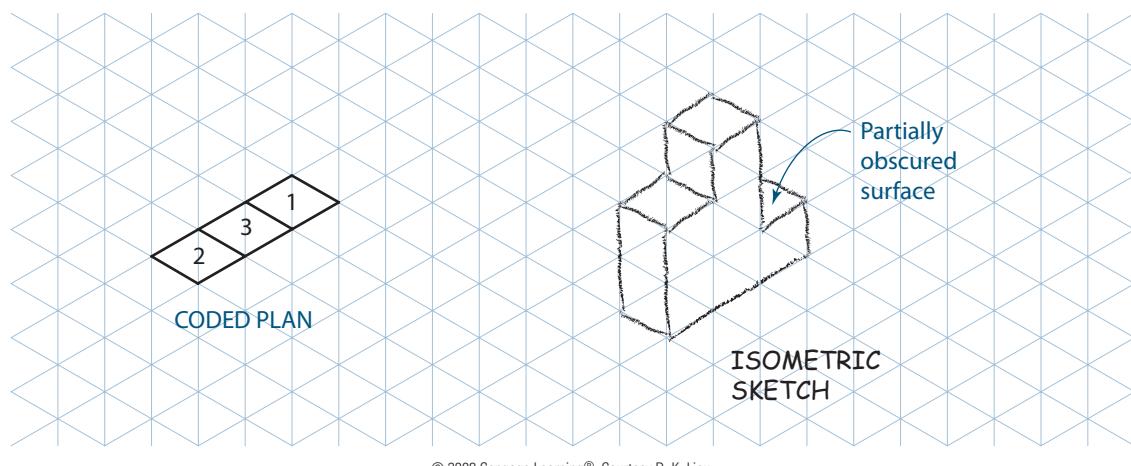
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**FIGURE 2.25.** A coded plan and the resulting object.



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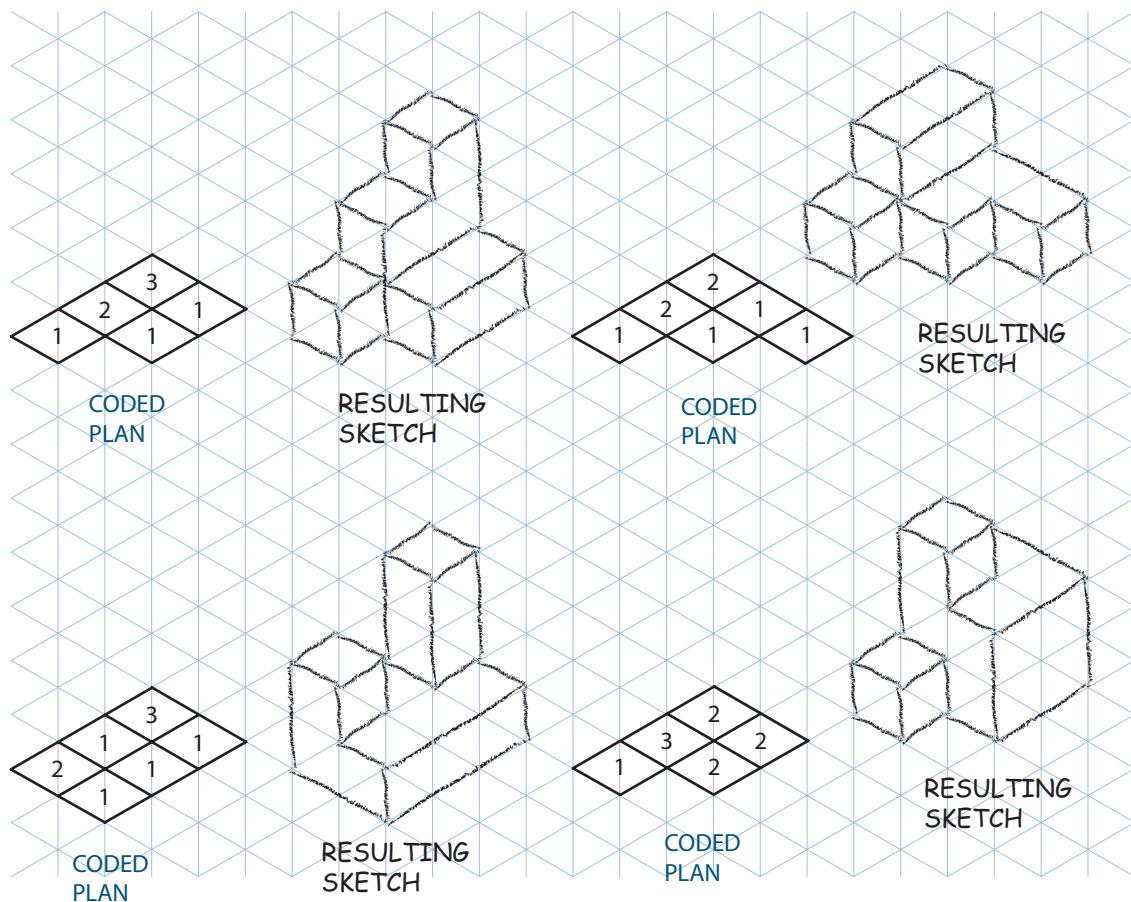
**FIGURE 2.26.** A properly drawn isometric sketch of the object from the coded plan.



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**FIGURE 2.27.** The partially obscured surface on an isometric sketch.

Figure 2.28 shows several coded plans and the corresponding isometric sketches. Look at each isometric sketch carefully to verify that it matches the defining coded plan; that lines are shown only at the edges between surfaces (not to define each block), that no hidden edges are shown, and that only the visible portions of partially obscured surfaces are shown.



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**FIGURE 2.28.** Four coded plans and the resulting isometric sketches.

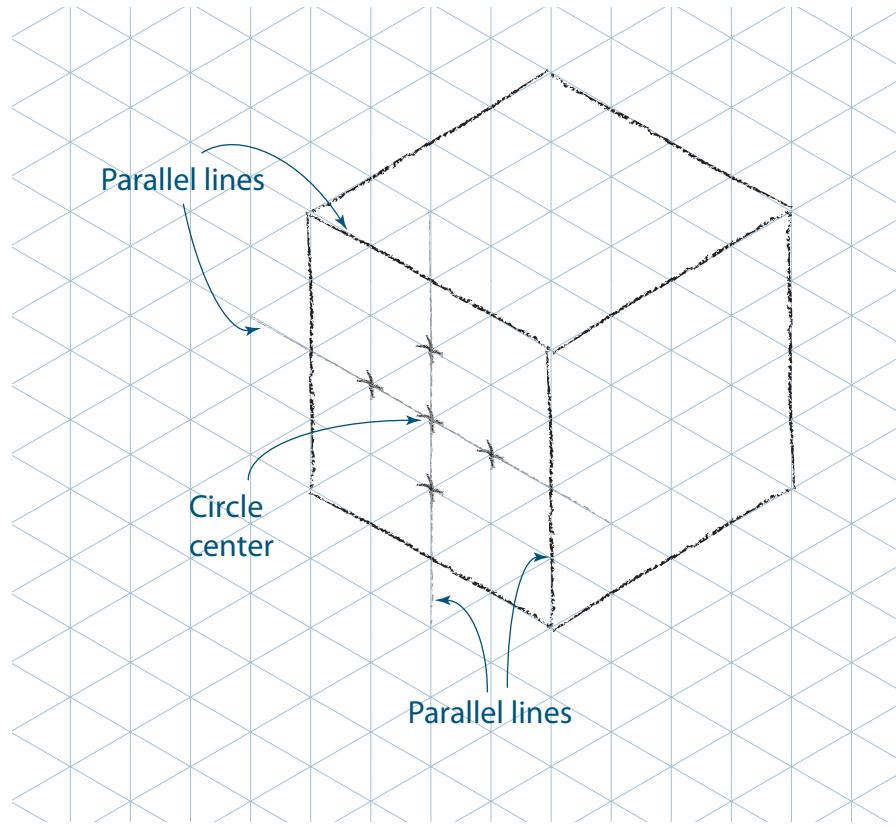
### 2.07.01 Circles in Isometric Sketches

Look back at the  $3 \times 3 \times 3$  block shown in Figure 2.24. In reality, you know that all of the surfaces of the block are  $3 \times 3$  squares; yet in the isometric sketch, each surface is shown as a parallelogram. The distortion of planar surfaces is one disadvantage of creating isometric sketches. The isometric portrayal of circles and arcs is particularly difficult. Circles appear as ellipses in isometric sketches; however, you will not be able to create a rectangular bounding box to sketch the ellipse in isometric as described earlier in this chapter. To create an ellipse that represents a circle in an isometric sketch, you first create a square bounding box as before; however, the bounding box will appear as a parallelogram in the isometric sketch. To create your bounding box, locate the center of the circle first. From the center, locate the four radial points. The direction you move on the grid corresponds to the lines that define the surface. If you are sketching the circle on a rectangular surface, look at the sides of the rectangle as they appear in isometric and move that same direction on the grid. Figure 2.29 shows a  $4 \times 4 \times 4$  cube with a circle center and four radial points located on one of the sides.

Once you have located the center of the circle and the four radial points, the next step is to create the bounding box through the radial points. The edges of the bounding box should correspond to the lines that define this particular surface. The edges will be parallel to the edges of the parallelogram that define the surface if that surface is square or rectangular. Figure 2.30 shows the cube with the circle center and the bounding box located on its side.

Four arcs that go through the radial points define the ellipse, just like an ellipse drawn in a regular rectangular bounding box. The difference is that for the isometric ellipse, the arcs are of varying curvatures—two long arcs and two short

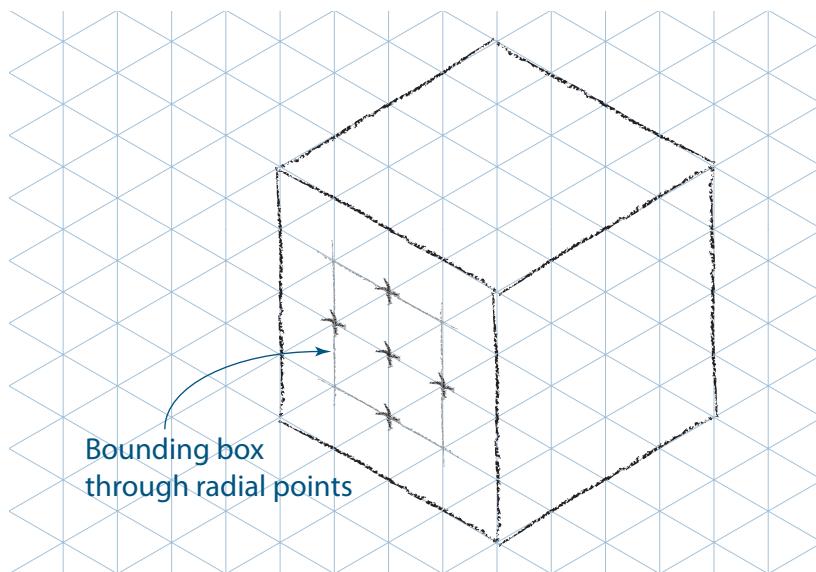
**FIGURE 2.29.** A cube with a circle center and radial points located.



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## 2-16 section one Laying the Foundation

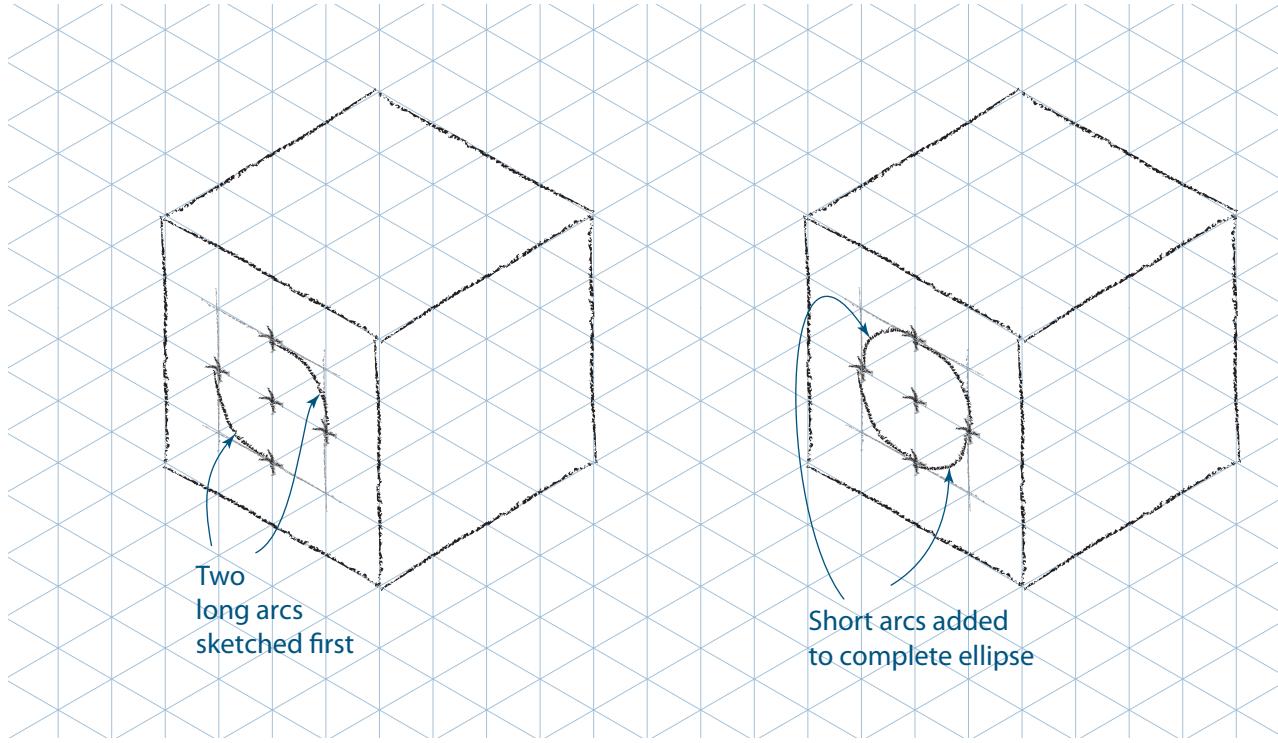
**FIGURE 2.30.** A cube with the circle center and bounding box on the side.



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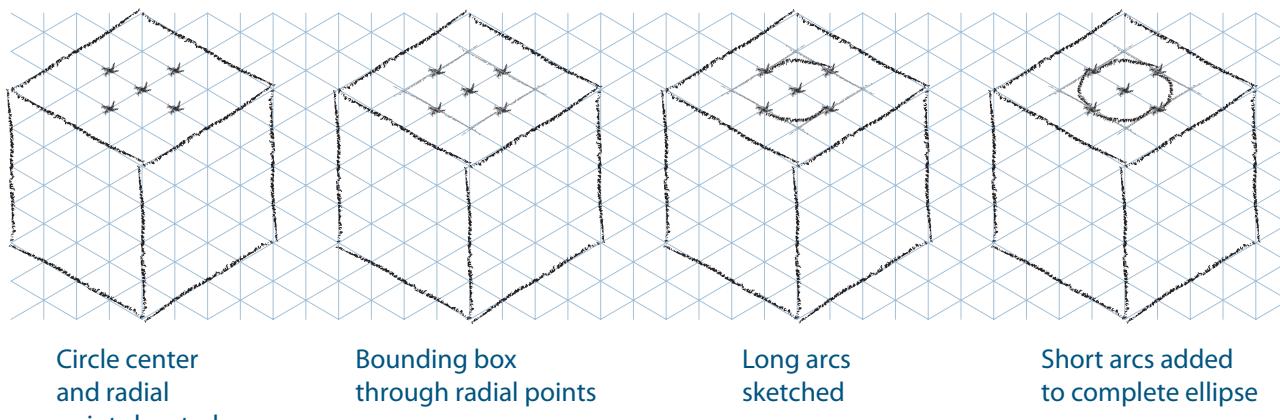
arcs in this case. The arcs are tangent to the bounding box at the radial points, as before. It is usually best if you start by sketching the long arcs, and then add the short arcs to complete the ellipse. Sketching the arcs that form the ellipse is illustrated in Figure 2.31.

Creating ellipses that represent circles on the other faces of the cube is accomplished in a similar manner, as illustrated in Figures 2.32 and 2.33.



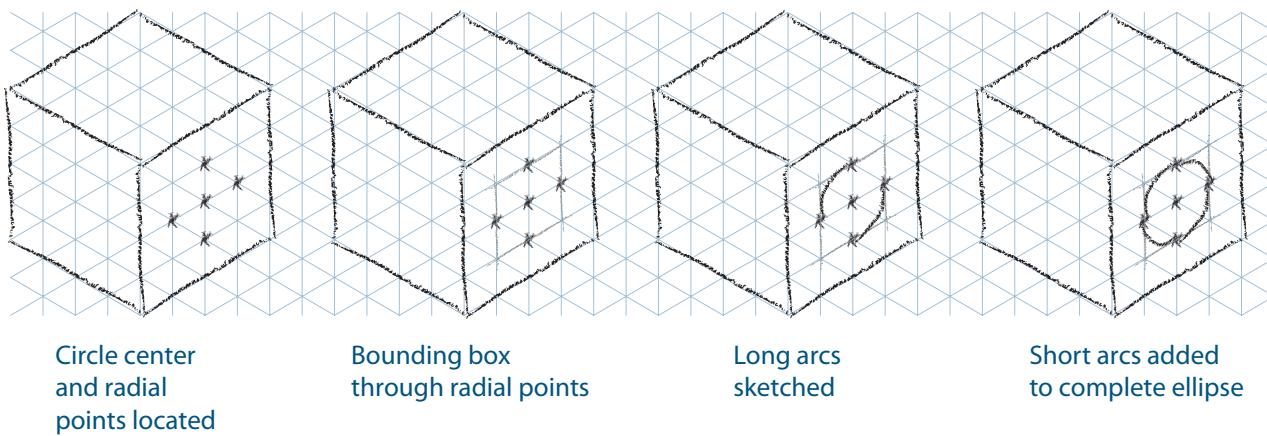
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**FIGURE 2.31.** Sketching arcs to form an ellipse.



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**FIGURE 2.32.** Sketching an ellipse on the top surface of a cube.



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**FIGURE 2.33.** Sketching an ellipse on the side face of a cube.

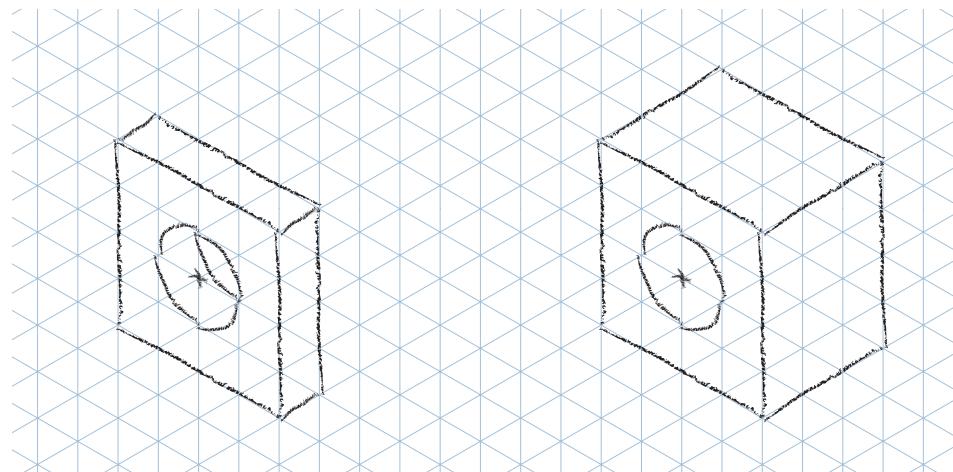
### 2.07.02 Circular Holes in Isometric Sketches

One of the most common occurrences that produces a circular feature in an isometric sketch is a hole in the object. You will learn more about circular holes and object features in a later chapter, but a short introduction follows here. A circular hole usually extends all the way through an object. In an isometric pictorial, a portion of the “back” edge of a circular hole is often visible through the hole and should be included in your sketch. As a rule of thumb, the back edge of a hole is partially visible when the object is relatively thin or the hole is relatively large; when the object is thick or the diameter of the hole is small, the back edge of the hole is not visible. Figure 2.34 shows two blocks with circular holes going through them. Notice in the “thin” block that you can see a portion of the back edge of the hole; in the thicker block, though, the back edge is not visible.

To determine whether a part of the back edge of a hole is visible in an isometric sketch, you first need to locate the center of the back hole. To locate the back center, start from the center of the hole on the front surface and move in a direction perpendicular to the front surface toward the back of the object a distance equal to the object’s dimension in that direction. Figure 2.35 shows the location of the center of the two back circles for the objects in Figure 2.34.

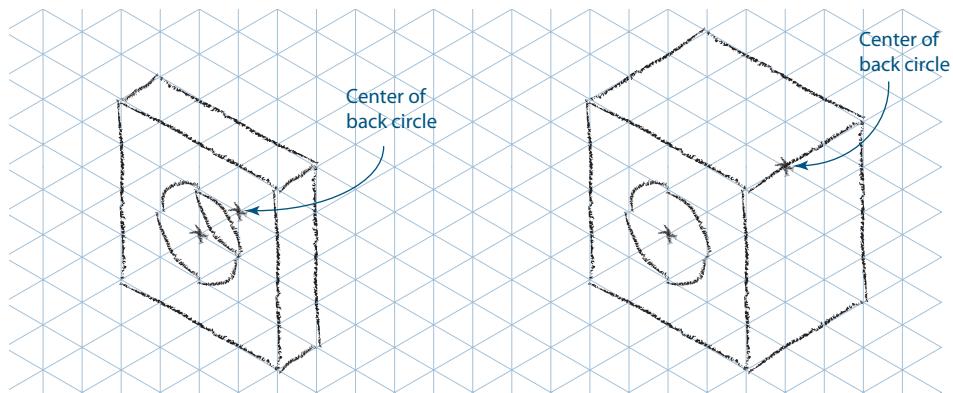
## 2-18 section one Laying the Foundation

**FIGURE 2.34.** Blocks with circular holes in them.



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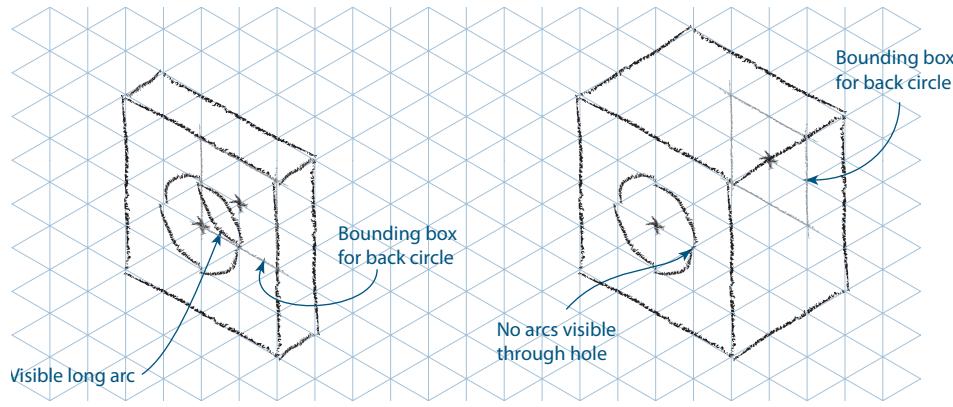
**FIGURE 2.35.** Centers of back circles located.



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Starting from the back center point, lightly sketch the radial points and the bounding box for the back circle similar to the way you did for the front circle. Then add the long arc that is visible through the hole. (Note that only *one* of the long arcs is typically visible through the hole.) Add segments of the short arcs as needed to complete the visible portion of the back edge of the hole. Conversely, if after you sketch the back bounding box you notice that no portion of the ellipse will be visible on the sketch, do not include any arcs within the hole on the sketch and erase any lines associated with the bounding box. Figure 2.36 illustrates the inclusion and noninclusion of segments of the back edges of holes for the objects in Figures 2.34 and 2.35.

**FIGURE 2.36.** Determining visibility of back circles.



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## 2.08 Oblique Pictorials

**Oblique pictorials** are another type of sketch you can create to show a 3-D object. Oblique pictorials are often preferred for freehand sketching because a specialized grid is not required. With oblique pictorials, as with **oblique axes**, the three dimensions of the object are shown with the height and width of the object in the plane of the paper and the third dimension (the depth) receding off at an angle from the others. Although the angle is usually 45 degrees, it can be any value.

The advantage that oblique pictorials have over isometric pictorials is that when one face of the object is placed in the plane of the paper, the object will appear in its true shape and size in that plane—it will be undistorted. This means that squares remain squares, rectangles remain rectangles, and circles remain circles. Figure 2.37 shows two pictorial representations of simple objects—one in isometric and one in oblique. Notice that the rules established for isometric pictorial sketches also hold true for oblique pictorial sketches—you do not show the hidden back edges, you show lines only where two surfaces intersect to form an edge, and you show only the visible parts of partially obstructed surfaces.

When making oblique sketches, the length of the **receding dimension** is not too important. In fact, oblique pictorials typically look better when the true length of the receding dimension is not shown. When the true length of an object's receding dimension is sketched, the object often appears distorted and unrealistic. Figure 2.38a shows the true length of a cube's receding dimension (use a ruler to make sure), and Figure 2.38b shows the same cube with the receding dimension drawn at about one-half to three-fourths its true length. Notice that the sketch in Figure 2.38a appears distorted—it does not look very much like a cube—whereas the sketch in Figure 2.38b looks like a cube even though the length of the receding dimension is less than the two sides of the square.

**FIGURE 2.37.** A comparison of isometric and oblique pictorials.

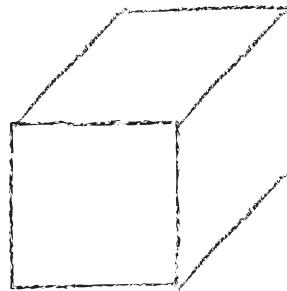


ISOMETRIC PICTORIAL

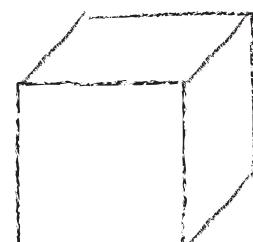
OBLIQUE PICTORIAL

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**FIGURE 2.38.** Oblique pictorials of a cube.



(a) Receding dimension drawn true length.



(b) Receding dimension drawn less than true length.

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Other conventions pertain to the way the receding dimension is portrayed in an oblique sketch; you will learn about them in a later chapter. For now, you will concentrate on trying to make a sketch that looks proportionally correct.

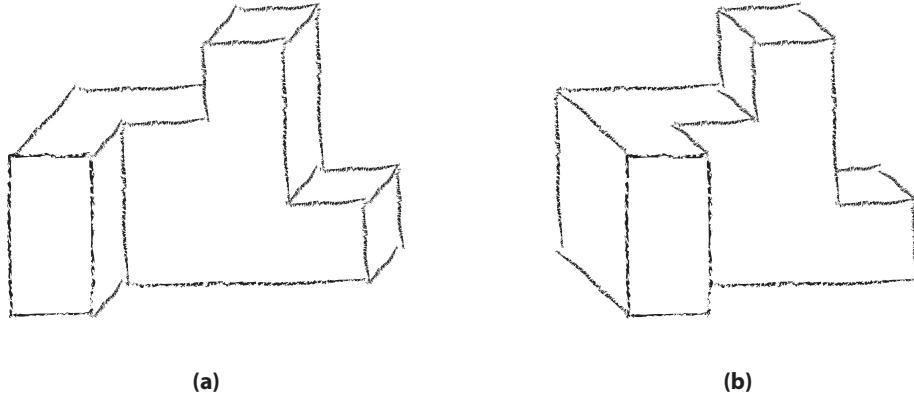
When creating oblique pictorials, you can choose to have the receding dimension going back and to the left or back and to the right. The direction you choose should be the one that produces the fewest obstructed surfaces in the resulting sketch. Figure 2.39 shows two possible sketches of the same object—one with the receding dimension to the left and one with the receding dimension to the right. Notice that the first sketch (Figure 2.39a) is preferable since none of the surfaces are obscured as they are with the second sketch (Figure 2.39b).

When creating an oblique pictorial, you should put the most irregular surface in the plane of the paper. This is particularly true about any surface that has a circular feature on it. Figure 2.40 shows two different oblique pictorials of the same object. In the first sketch (Figure 2.40a), the most irregular surface is placed in the plane of the paper as it should be; in the second sketch (Figure 2.40b), the irregular surface is shown in the receding dimension. Notice that the first sketch shows the features of the object more clearly than the second sketch does.

### **2.08.01 Circular Holes in Oblique Pictorial Sketches**

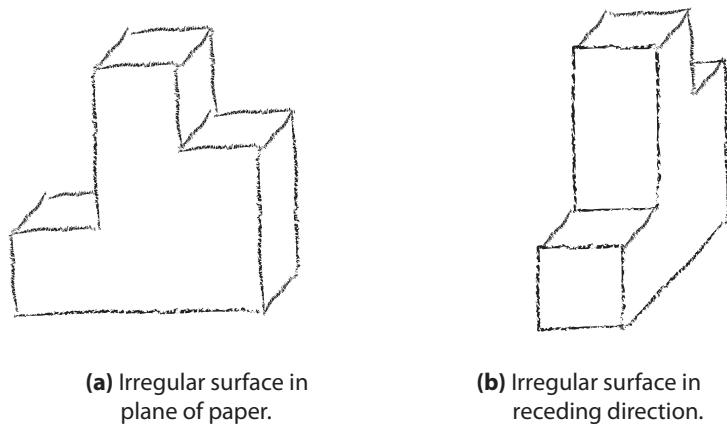
When circular holes appear in an oblique pictorial sketch, as with isometric sketches, you show the partial edges of the back circle where they are visible through the hole. Once again, partial circles are visible when the object is relatively thin or when the hole has a relatively large diameter; otherwise, partial edges are not shown. Figure 2.41 shows two oblique sketches—one in which a portion of the back edge of the hole is visible and the other in which it is not.

**FIGURE 2.39.** Two possible orientations for an oblique pictorial.



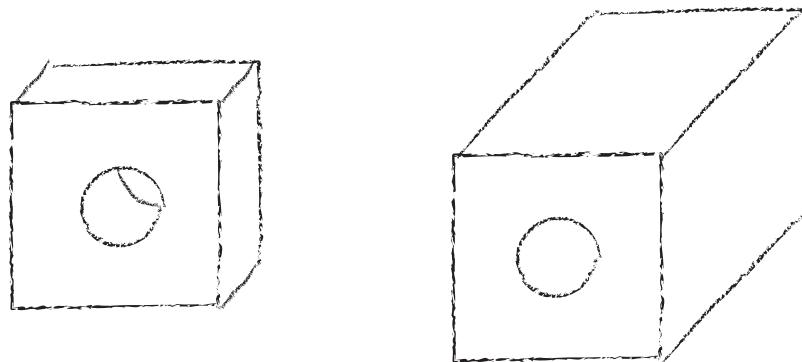
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**FIGURE 2.40.** Two possible orientations for an oblique pictorial.



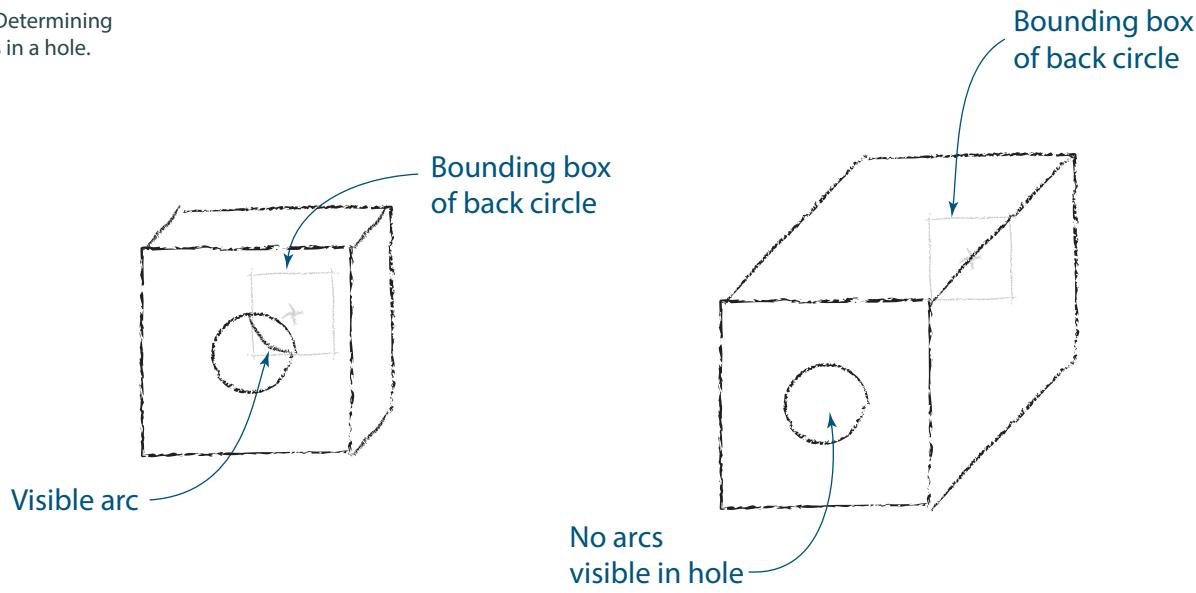
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**FIGURE 2.41.** Oblique pictorials with circular holes in objects.



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**FIGURE 2.42.** Determining visible back arcs in a hole.



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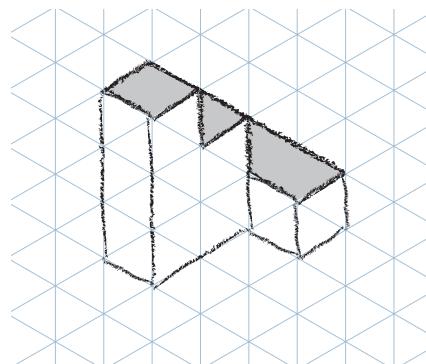
The procedure you use to determine whether a portion of the back circle edge is visible and, if so, which portion is visible follows the procedure outlined for isometric sketches. You start by locating the center of the back edge of the hole and marking off the four radial points. You then lightly sketch the bounding box that defines the circle. Finally, as needed, you sketch the visible portions of arcs within the circular hole. Figure 2.42 shows the procedure used to sketch the visible back edges of a circular hole in an oblique pictorial.

## 2.09 Shading and Other Special Effects

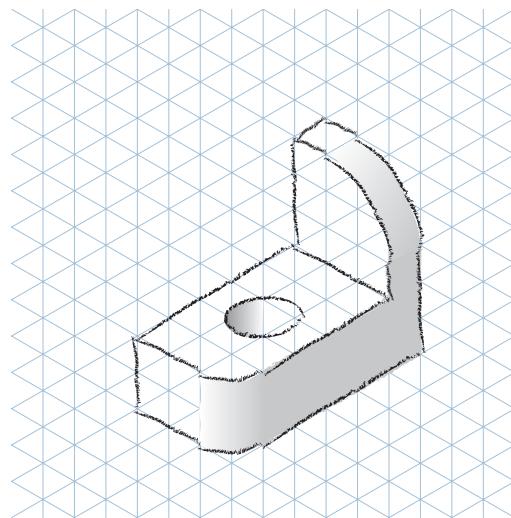
One thing you can do to improve the quality of your pictorial sketches is to include **shading** on selected surfaces to make them stand out from other surfaces or to provide clarity for the viewer. Figure 2.43 shows an isometric sketch with all of the top surfaces shaded. Notice that the shading better defines the object for the viewer. When including shading on a pictorial sketch, try not to overdo the shading. Too much shading can be confusing or irritating to the viewer—two things you should avoid in effective graphical communication.

Another common use of shading is to show curvature of a surface. For example, the visible portion of a hole's curved surface might be shaded in a pictorial sketch. A curved surface on an exterior corner also might be shaded to highlight its curvature.

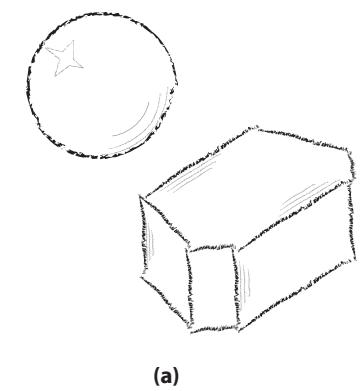
**FIGURE 2.43.** An object with the top surface shaded.



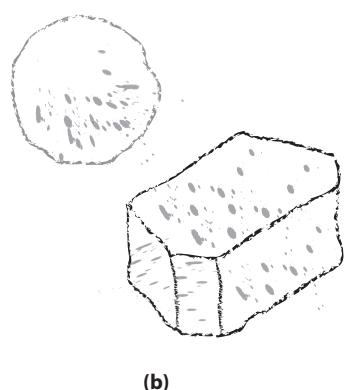
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**FIGURE 2.45.** The addition of surface treatments to convey smooth surfaces (a) and rough surfaces (b).

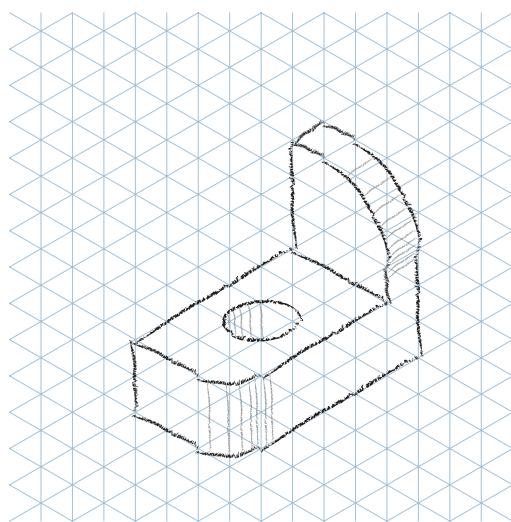


(a)



(b)

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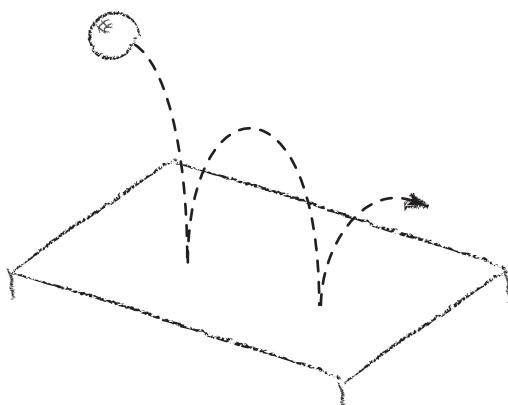
**FIGURE 2.44.** A simple object with two possible types of progressive shading used to emphasize the curvature of surfaces.

Figure 2.44 shows a pictorial sketch of a simple object with curved surfaces that are shaded.

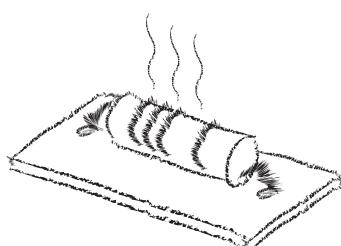
Other sketching techniques can be used to convey features such as smooth or rough surfaces. Figure 2.45 shows different types of surface treatments that are possible for sketched objects.

You are probably familiar with techniques used in cartoons to convey ideas such as motion, temperature, and sound. Figure 2.46 shows typical cartooning lines that convey concepts not easily incorporated in a static sketch. Many of these same markings can be used in technical sketches. For example, Figure 2.47 uses action lines to convey motion for the sketch of linkages.

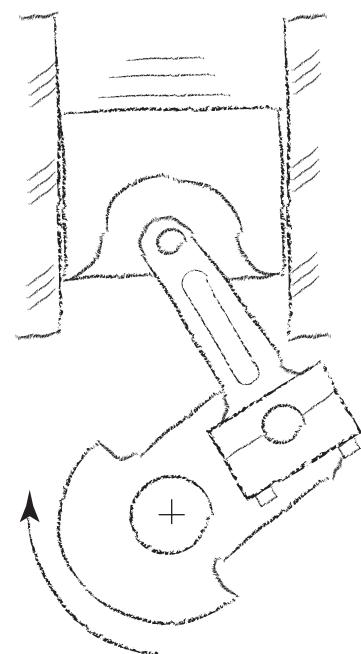
**FIGURE 2.46.** Some sketching techniques that can be used to convey motion (a), temperature (b), and sound (c).



(a)

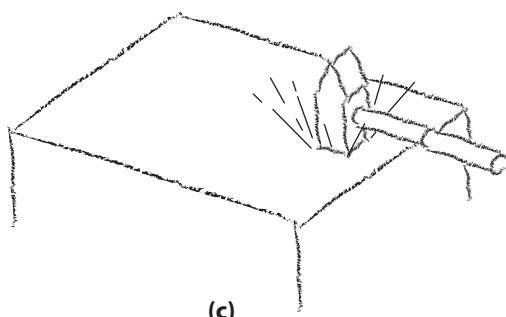


(b)



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**FIGURE 2.47.** Action lines used to convey the motion of linkages.

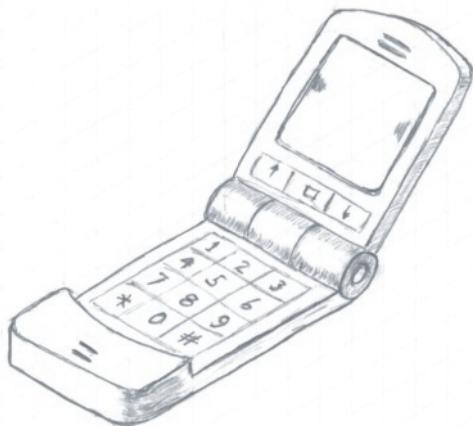


(c)

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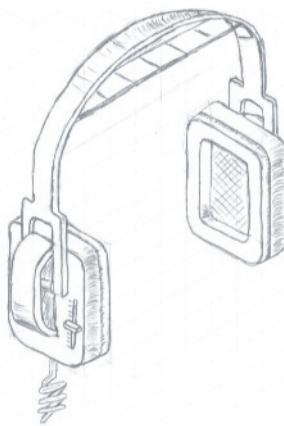
## 2.10 Sketching Complex Objects

As you refine your sketching skills, you will be able to tackle increasingly complex objects. Figures 2.48, 2.49, and 2.50 show pictorial sketches of small electronic devices. These sketches were not made to any particular scale, but were constructed so the object features appear proportionally correct with respect to one another. Notice the use of shading to enhance object appearance and to make the objects look more realistic. Being able to sketch relatively complicated objects such as these will improve your ability to communicate with colleagues throughout your career. To develop this



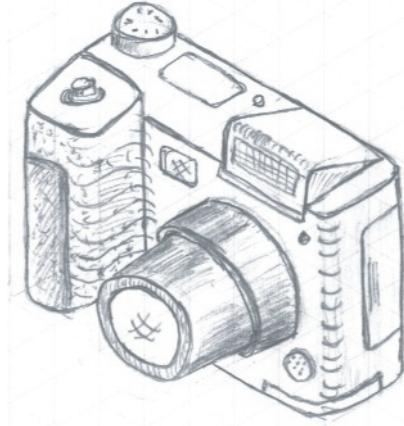
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**FIGURE 2.48.** A sketch of a cell phone.



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**FIGURE 2.49.** A sketch of a set of headphones.

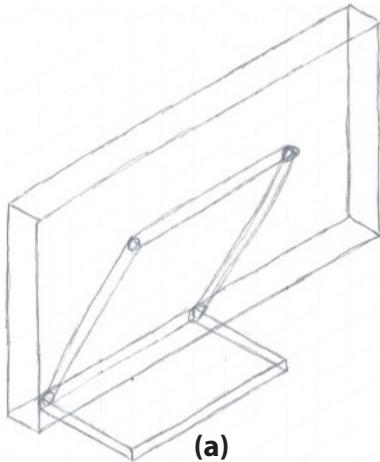


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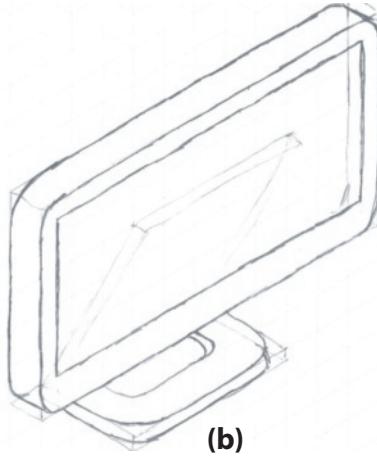
**FIGURE 2.50.** A sketch of a camera.

important skill, you should practice often. Do not be afraid to make mistakes—just keep trying until you get the results you want.

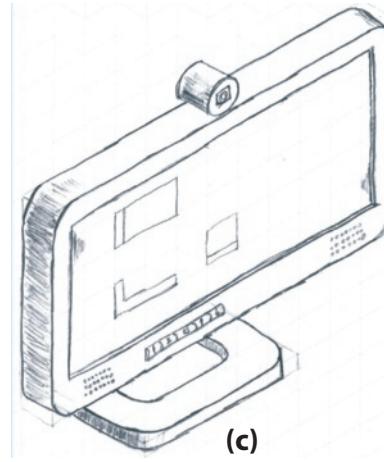
One way to tackle sketching a complex object is to think about it in the same way that a house is constructed—namely, “foundation, frame, finish.” Using this method, you start with the “foundation” of the sketch, which usually consists of multiple guidelines and construction lines. When creating the sketch foundation, think about outlining the volume taken up by the entire object. You next “frame” the object by darkening some of the construction lines to define the basic shape of the object and its features. Once the basic frame is complete, you “finish” the sketch by adding necessary details and special features such as shading, especially on curved surfaces. Figure 2.51 shows a sketch of a flat panel computer monitor by the “foundation, frame, finish” method. Several of the exercises at the end of this chapter ask you to use this technique to develop your skills in sketching complex objects.



**(a)**



**(b)**



**(c)**

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**FIGURE 2.51.** A sketch of a computer monitor using the method of “foundation (a), frame (b), finish (c).”

## 2.11 Chapter Summary

In this chapter, you learned about technical sketching and about some techniques to help you master this important form of communication. Specifically, you:

- Learned about the importance of sketching for engineering professionals and the link between creativity and freehand sketching.
- Developed techniques for successfully sketching basic shapes such as lines, arcs, circles, and ellipses.
- Learned about the right-hand rule and the way it is used to define 3-D coordinate systems in space. The axes can be portrayed on paper in either isometric or oblique format.
- Discovered how to make basic isometric sketches of objects from coded plans and about some of the rules that govern the creation of these sketches. You also learned about creating ellipses in isometric to represent circular holes in objects.
- Developed techniques for creating oblique pictorials. You also learned that for this type of pictorial, you should not show the receding dimension of the object true to size in order to avoid a distorted image.

### 2.12

### GLOSSARY OF KEY TERMS

**arc:** A curved entity that represents a portion of a circle.

**bounding box:** A square box used to sketch circles or ellipses.

**circle:** A closed curved figure where all points on it are equidistant from its center point.

**construction line:** A faint line used in sketching to align items and define shapes.

**ellipse:** A closed curved figure where the sum of the distance between any point on the figure and its two foci is constant.

**isometric axes:** A set of three coordinate axes that are portrayed on the paper at 120 degrees relative to one another.

**isometric dot paper:** Paper used for sketching purposes that includes dots located along lines that meet at 120 degrees.

**isometric grid paper:** Paper used for sketching purposes that includes grid lines at 120 degrees relative to one another.

**isometric pictorial:** A sketch of an object that shows its three dimensions where isometric axes were used as the basis for defining the edges of the object.

**left-handed system:** Any 3-D coordinate system that is defined by the left-hand rule.

**line:** A spatial feature that marks the shortest distance between two points. A line has location, orientation, and length, but no area.

**oblique axes:** A set of three coordinate axes that are portrayed on the paper as two perpendicular lines, with the third axis meeting them at an angle, typically 45 degrees.

**oblique pictorial:** A sketch of an object that shows one face in the plane of the paper and the third dimension receding off at an angle relative to the face.

**receding dimension:** The portion of the object that appears to go back from the plane of the paper in an oblique pictorial.

**right-hand rule:** Used to define a 3-D coordinate system whereby by pointing the fingers of the right hand down the x-axis and curling them in the direction of the y-axis, the thumb will point down the z-axis.

**right-handed system:** Any 3-D coordinate system that is defined by the right-hand rule.

**shading:** Marks added to surfaces and features of a sketch to highlight 3-D effects.

**3-D coordinate system:** A set of three mutually perpendicular axes used to define 3-D space.

**tick mark:** A short dash used in sketching to locate points on the paper.

**2.13****QUESTIONS FOR REVIEW**

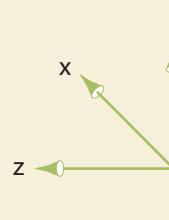
1. What is the role of sketching in engineering design? In creativity?
2. Describe which procedure you should use to sketch straight lines. (Are you right- or left-handed?)
3. How do circles appear on an isometric pictorial? On an oblique pictorial?
4. What is a bounding box?
5. How are construction lines used in sketching?
6. Why is it important to know the right-hand rule?

**2.14****PROBLEMS**

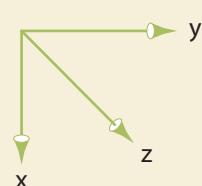
1. For each of the coordinate axes shown below, indicate whether they are isometric or oblique and whether they represent right- or left-handed systems.



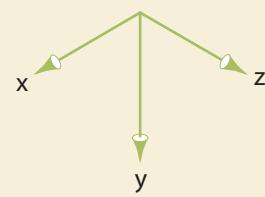
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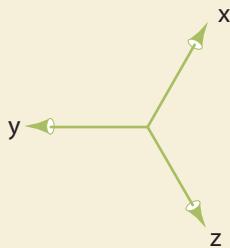
(b)



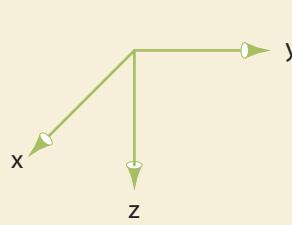
(c)



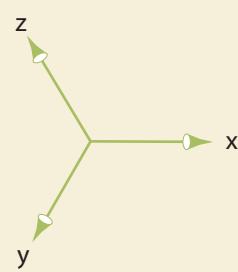
(d)



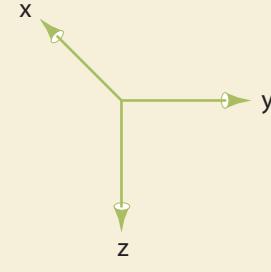
(e)



(f)



(g)



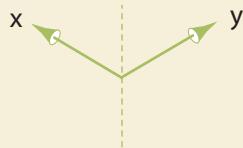
(h)

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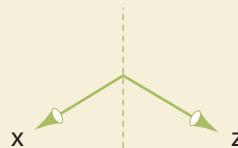
**FIGURE P2.1.**

**2.14 PROBLEMS (CONTINUED)**

2. Label the third axis in each of the following figures to define a right-handed system.



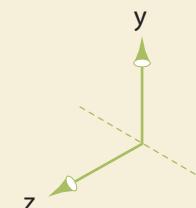
(a)



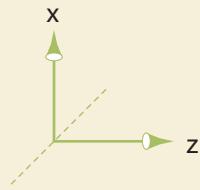
(b)



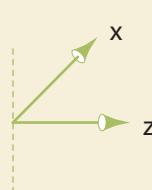
(c)



(d)



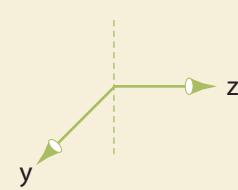
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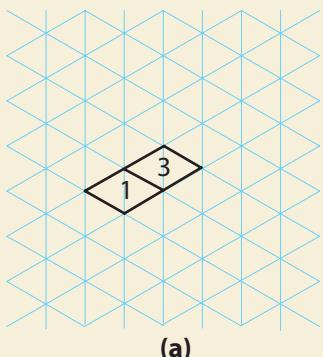
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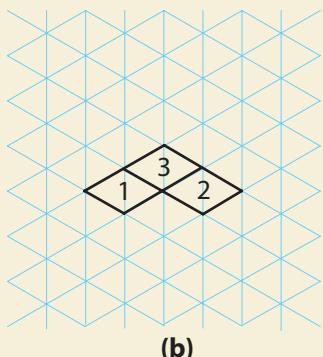
**FIGURE P2.2.**

**2.14 PROBLEMS (CONTINUED)**

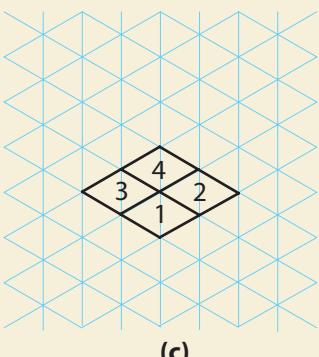
3. Create isometric sketches from the coded plans shown below.



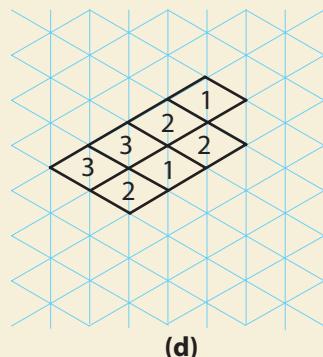
(a)



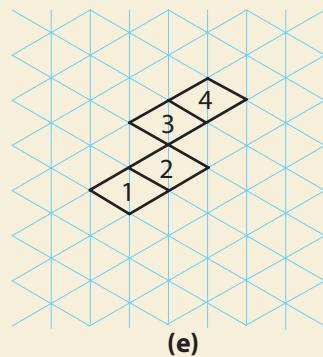
(b)



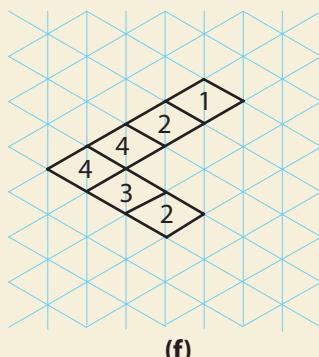
(c)



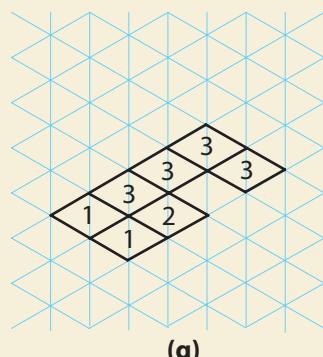
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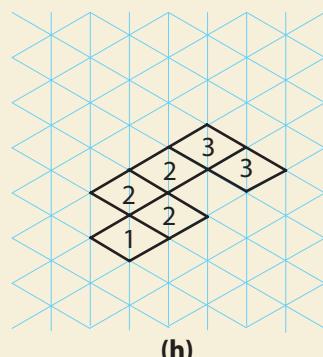
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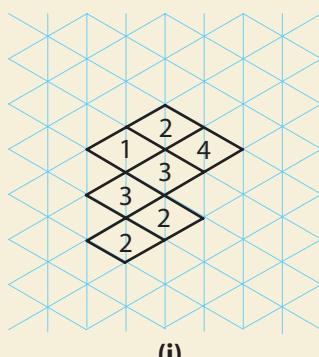
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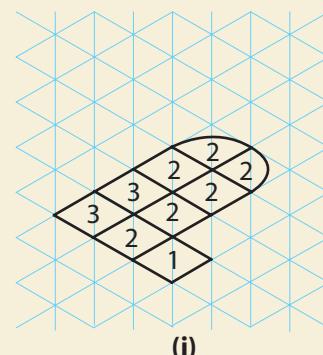
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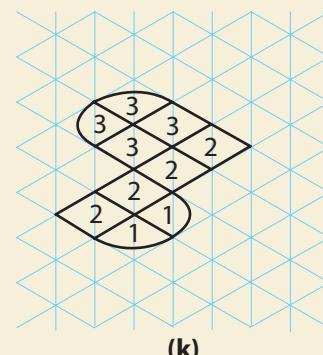
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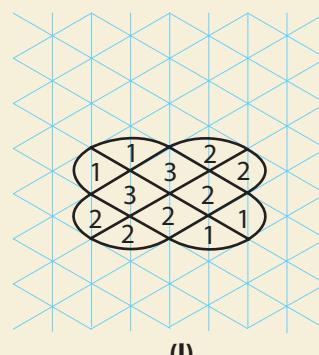
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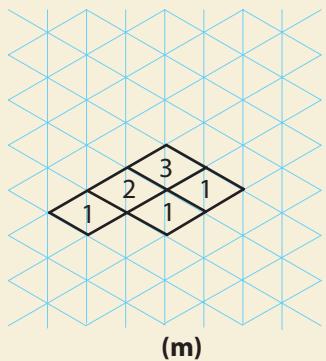


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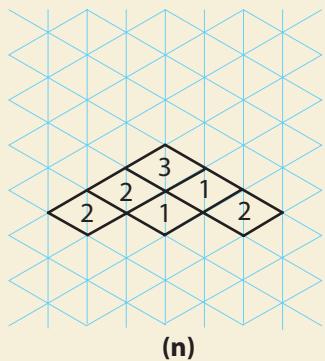
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**FIGURE P2.3.**

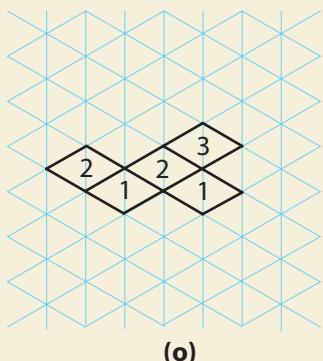
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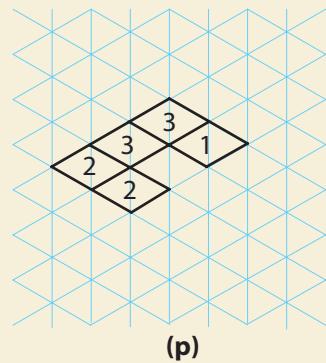
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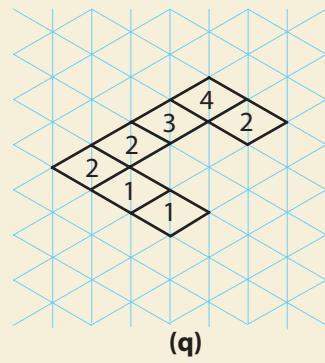
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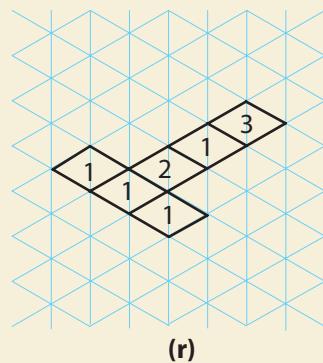
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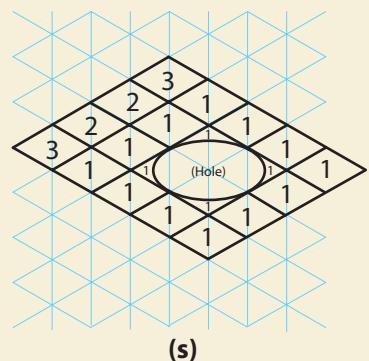
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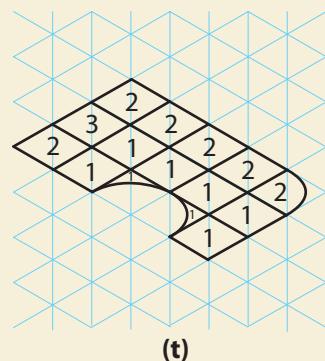
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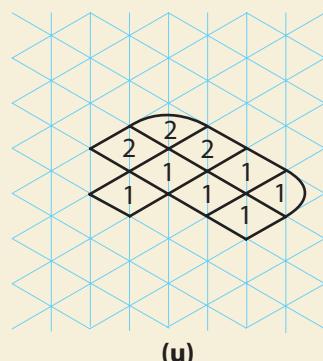
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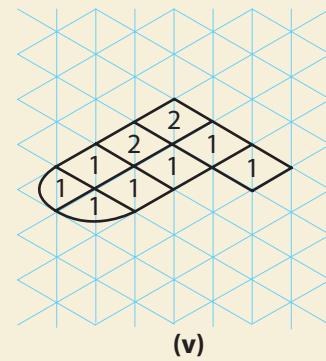
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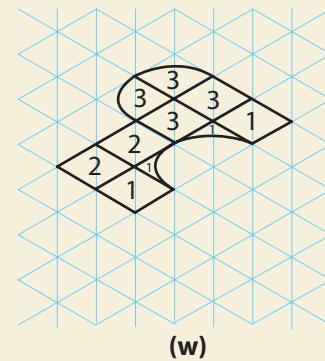
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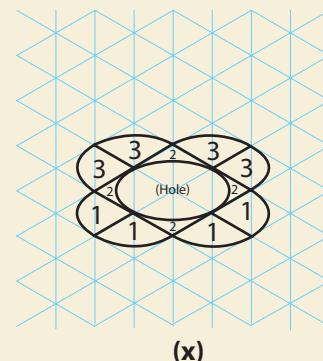
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(v)



(w)



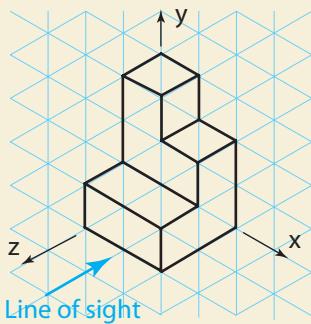
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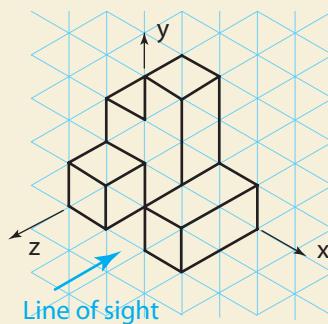
**FIGURE P2.3.** (Concluded)

## 2.14 PROBLEMS (CONTINUED)

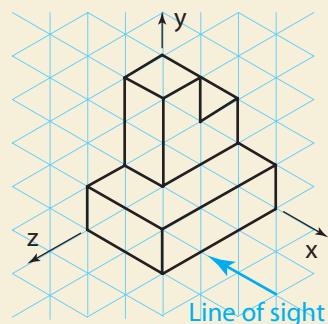
4. Sketch a  $6 \times 6 \times 2$  block in isometric. On the  $6 \times 6$  side, sketch a hole of diameter 4, making sure you include back edges of the hole as appropriate. Also create an oblique pictorial of the block.
5. Sketch a  $6 \times 6 \times 2$  block in isometric. On the  $6 \times 6$  side, sketch a hole of diameter 2, making sure you include back edges of the hole as appropriate. Also create an oblique pictorial of the block.
6. Sketch a  $6 \times 6 \times 4$  block in isometric. On the  $6 \times 6$  side, sketch a hole of diameter 2, making sure you include back edges of the hole as appropriate. Also create an oblique pictorial of the block.
7. From the isometric pictorials and viewing directions defined in the following sketches, create oblique pictorial sketches that look proportionally correct.



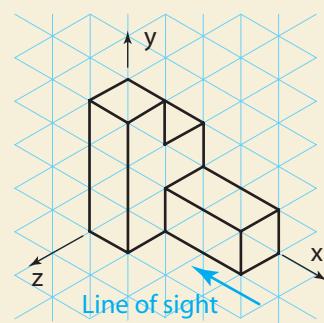
(a)



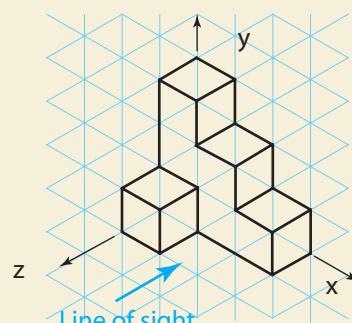
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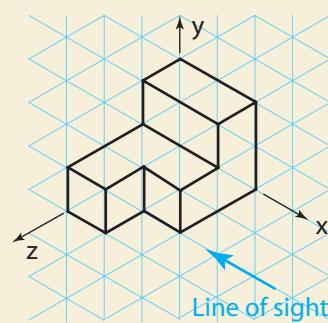
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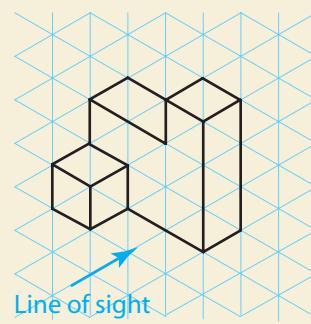
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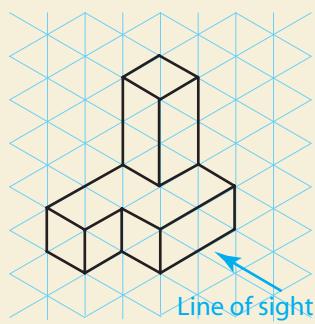
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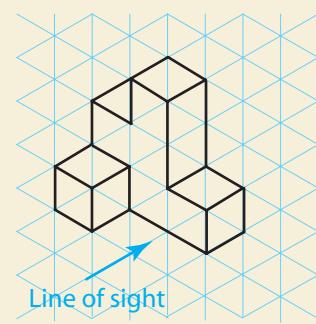
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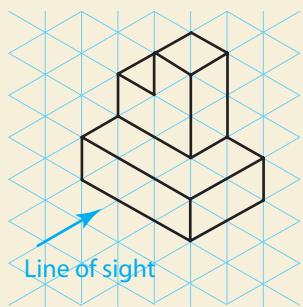


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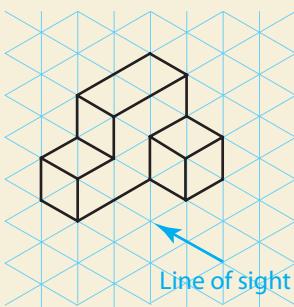
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**FIGURE P2.4.**

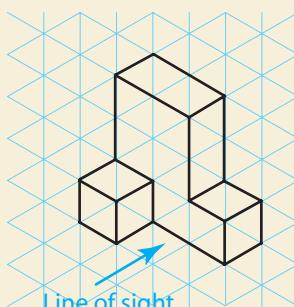
## 2.14 PROBLEMS (CONTINUED)



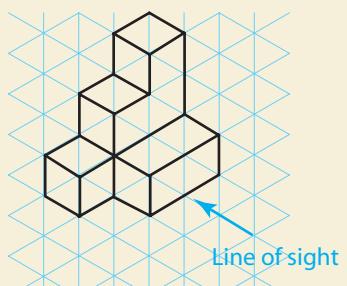
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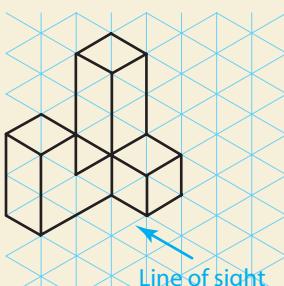
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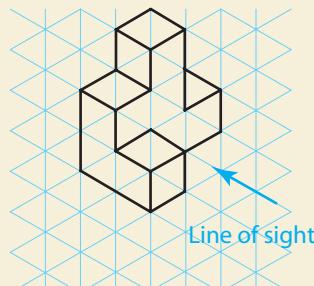
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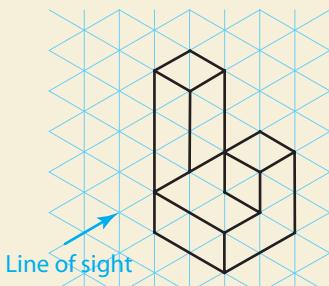
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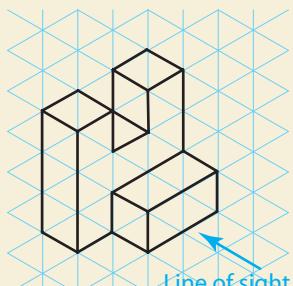
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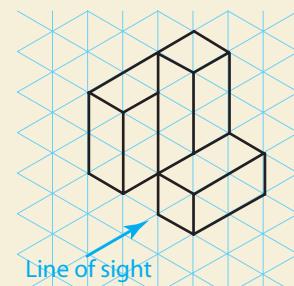
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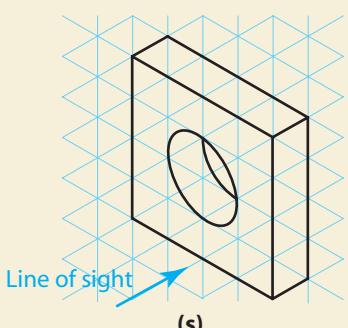
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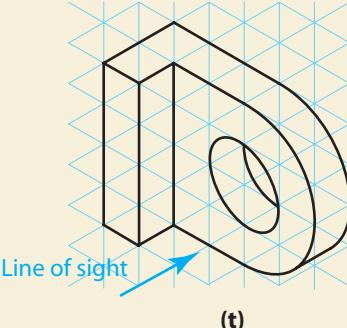
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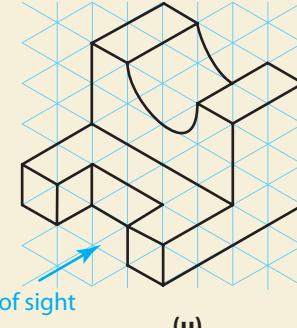
(r)



(s)



(t)



(u)

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**FIGURE P2.4.** (Concluded)

8. Use the “foundation, frame, finish” method to create sketches of the following:

- |               |                    |                 |
|---------------|--------------------|-----------------|
| a. stapler    | d. bicycle         | g. simple table |
| b. speedboat  | e. calculator      | h. mobile phone |
| c. coffee mug | f. laptop computer | i. bookshelf    |



# CHAPTER 3

## VISUALIZATION

### OBJECTIVES

After completing this chapter, you should be able to

- Recognize that 3-D spatial skills are necessary for success in engineering or technology
- Describe how a person's spatial skills develop as they age
- Examine the types of questions used to assess a person's spatial skill level
- Show how you can improve your 3-D spatial skills through techniques that include
  - Drawing different corner views of an object.
  - Rotating objects about one or more axes.
  - Sketching object reflections and making use of symmetries.
  - Considering cross sections of objects.
  - Combining two objects to form a third object through Boolean operations.

**3.01****INTRODUCTION**

When you start your first job in the real world, an engineer or a technologist is likely to hand you a drawing and expect you to understand what is on the page. Imagine your embarrassment if you have no clue what all of the lines and symbols on the drawing mean. One of the fundamental skills you need to understand is that drawing is the ability to visualize in three dimensions. The ability to visualize in three dimensions is also linked to creativity in design. People who think creatively are able to “see” things in their minds that others cannot. Their imaginations are not confined by traditional boundaries.

In this chapter, you will learn about the different types of three-dimensional (3-D) spatial skills and ways they can be developed through practice. The chapter will begin with an introduction to the background research conducted in education and to 3-D spatial skills. Then the chapter will take you through several types of visualization activities to further develop your 3-D skills through practice.

**3.02 Background**

Beginning in the early part of the twentieth century, IQ testing was developed to categorize a person based on his or her intelligence quotient. Anyone who took the IQ test was defined by a number that identified a level of intelligence. IQ scores over 140 identified geniuses; scores below 100 identified slow thinkers. Beginning in the 1970s, scholars began to perceive problems with this one-number categorization of a person’s ability to think. One scholar in particular, Howard Gardner, theorized that there were multiple human intelligences and the one-number-fits-all theory did not accurately reflect the scope of human thought processes. Although some of his theories might be subject to scrutiny, they have gained acceptance within the scientific and educational communities. Gardner theorized that there are eight distinct human intelligences; he identified them as:

- Linguistic—the ability to use words effectively in speaking or in writing.
- Logical-Mathematical—the ability to use numbers effectively and to reason well.
- Spatial—the ability to perceive the visual-spatial world accurately and to perform transformations on those perceptions.
- Bodily-Kinesthetic—the capacity of a person to use the whole body to express ideas or feelings and the facility to use the hands to produce or transform things.
- Musical—the capacity to perceive, discriminate, transform, and express musical forms.
- Interpersonal—the ability to perceive and make distinctions in the moods, intentions, motivations, and feelings of other people.
- Intrapersonal—self-knowledge and the ability to act adaptively on the basis of that knowledge.
- Naturalist—the ability to recognize plant or animal species within the environment.

You may be acquainted with someone who has a high level of linguistic intelligence but a low level of musical intelligence. Or you might know someone who has a high level of logical-mathematical intelligence but who lacks interpersonal intelligence relationships. You may even have a friend who is generally smart but who lacks intrapersonal intelligence and attempts stunts that are beyond his or her limitations.

Most people are born with one or more of the intelligences listed. As a child, Tiger Woods was gifted with a natural ability in bodily-kinesthetic intelligence.

Mozart was born with a high level of musical intelligence. However, just because a person naturally has a high level of intelligence in one area does not mean that he or she cannot learn and improve his or her abilities in weaker areas. A person might naturally have strength in linguistics and musical intelligences, but he or she can still learn and improve in logical-mathematical endeavors. The goal of this chapter is to help those of you who were not born with a high level of spatial intelligence as defined by Gardner.

Learning in general and spatial skills in particular have been the subjects of education research studies over the past several decades. The following are a few important questions that the research raised in the area of spatial intelligence:

- How does a person develop spatial skills?
- Why does a person need well-developed spatial skills?
- How are spatial skills measured?

The next few sections will examine researchers' answers to these questions.

### **3.03 Development of Spatial Skills**

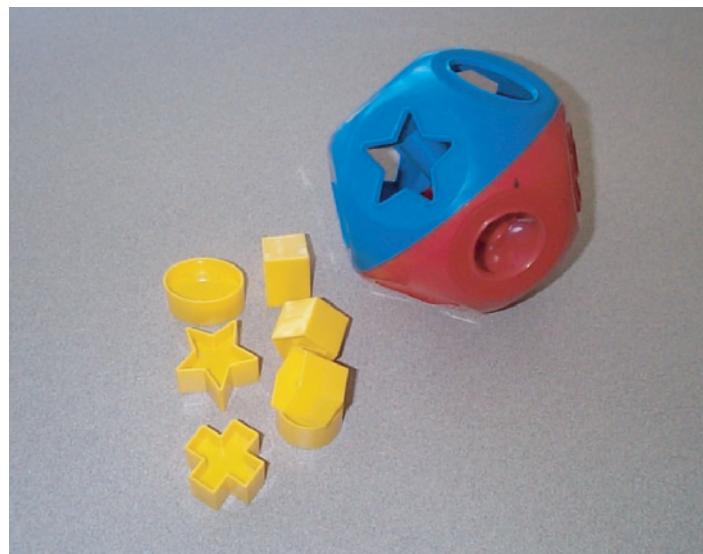
As a child grows, the brain develops in ways that enable the child to learn. If you think of each of the eight intelligences described by Gardner, you can understand how these skills and abilities develop as a child grows to maturity. Consider kinesthetic intelligence. Newborn infants cannot move on their own during the first few weeks of life. Within a few months, they can hold up their heads without support. By the age of four months, they can roll over; by six months, they can crawl; by one year, they can walk. Children learn to run, skip, and jump within the next year or so. Eventually, they usually develop all sorts of kinesthetic abilities that enable them to enjoy physical activities such as basketball, swimming, ballet, and bike riding. Nearly every child goes through this natural progression. However, some children develop more quickly than others; some even skip a step and go directly from rolling over to walking without ever crawling. As with most types of intelligence, some individuals—such as professional athletes—have exceptional kinesthetic skills, while others have poorly developed skills and struggle to perform the simplest tasks. However, even people who naturally have little kinesthetic ability can improve through practice and perseverance.

The remaining intelligences (mathematics, verbal, etc.) also have a natural progression; for example, to develop your mathematical intelligence, you have to learn addition before you can learn algebra. Children also acquire spatial skills through a natural progression; however, you may not be as aware of that progression of development as you are of the progressions for the other intelligences. Educational psychologists theorize that there are three distinct stages of development for spatial skills.

The first stage of development involves 2-D spatial skills. As children develop these skills, they are able to recognize 2-D shapes and eventually are able to recognize that a 2-D shape has a certain orientation in space. If you watched *Sesame Street* as a child, you may remember the game where four pictures of 2-D objects are shown on the TV screen—three objects are identical; the fourth is different in some way. A song urges you to pick out the object that does not belong with the other three. A child who can accomplish this task has developed some of the spatial skills at the first stage. You also may remember playing with a toy similar to a Tupperware™ ShapeSorter, shown in Figure 3.01. The toy is a ball that is half red and half blue with ten holes in it, each hole a different shape. A child playing this game not only has to recognize that the star-shaped piece corresponds to the star-shaped hole but also has to turn the piece to the correct orientation to fit the piece through the hole. This game challenges different 2-D skills found at the first stage of development of spatial intelligence—a child must recognize the 2-D shape of the object and then must be able to recognize its orientation in 2-D space to complete the task.

### 3-4 section one Laying the Foundation

**FIGURE 3.01.** A Tupperware™ ShapeSorter toy.



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Three-dimensional spatial skills are acquired during the second stage of development. Children at this stage can imagine what a 3-D object looks like when it is rotated in space. They can imagine what an object looks like from a different point of view, or they can imagine what an object would look like when folded up from a 2-D pattern. People who are adept at solving the Rubik's cube puzzle have well-developed 3-D spatial skills. Computer games such as 3D Tetris require well-developed 3-D spatial skills to perform the manipulations required to remain "alive." Soccer players who can imagine the trajectory that puts the ball in the goal from any angle on the playing field typically have well-developed 3-D spatial skills. Children have usually acquired 3-D spatial skills by the time they are in middle school. For some children, it may take a few more years, depending on their natural predisposition toward spatial intelligence and their childhood experiences.

People at an advanced stage of the development of spatial intelligence can combine their 3-D skills with concepts of measurement. Assume you are buying sand for a turtle-shaped sandbox. You go to the local gravel pit where an employee loads the sand in the back of your pickup using a big "scoop." How many scoopfuls will you need? If you can successfully visualize the volume of sand as it is transformed from the 3-D volume of one full scoop to the 3-D volume of the turtle-shaped sandbox, you have acquired this advanced 3-D visualization skill.

Many people never develop the advanced level in spatial intelligence, just like the many people who never achieve advanced skill levels in mathematics or kinesthetic intelligence. Not achieving advanced levels in some of the intelligence areas is not likely to hamper your ability to become a productive, well-adjusted member of society. However, just as a lack of basic development in verbal intelligence is likely to hurt your chances professionally, a lack of basic skills in spatial intelligence may limit your ability to be successful, especially in engineering or a technical field.

Schools help students develop most of the intelligence types; however, schools do not usually provide formal training to develop spatial intelligence. You began learning mathematics in kindergarten and are likely continuing your education in math at the present time. If you get a graduate degree in a technical area, you will probably be developing your mathematical intelligence for many years thereafter. The focus on developing spatial skills from an early age, continuing through high school and beyond, is typically absent in the U.S. educational system. Developing spatial intelligence is largely ignored in schools for a variety of reasons; however, those reasons are not the subject of this text.

The lack of prior spatial training may not be a problem for you—you developed your spatial skills informally through everyday experiences or you naturally have a high level of ability in spatial intelligence. However, poorly developed 3-D spatial skills may hinder your success in fields such as engineering and technology. This is especially true as you embark on a journey through an engineering graphics course. Poorly developed spatial skills will leave you frustrated and possibly discouraged about engineering graphics. The good news is that if you do not have a naturally high level of ability in 3-D spatial skills you can develop them through practice and exercise.

## 3.04 Types of Spatial Skills

According to McGee (1979), spatial ability is “the ability to mentally manipulate, rotate, twist, or invert pictorially presented visual stimuli.” McGee identifies five components of spatial skills:

- **Spatial perception**—the ability to identify horizontal and vertical directions.
- **Spatial visualization**—the ability to mentally transform (rotate, translate, or mirror) or to mentally alter (twist, fold, or invert) 2-D figures and/or 3-D objects.
- **Mental rotations**—the ability to mentally turn a 3-D object in space and then be able to mentally rotate a different 3-D object in the same way.
- **Spatial relations**—the ability to visualize the relationship between two objects in space, i.e., overlapping or nonoverlapping.
- **Spatial orientation**—the ability of a person to mentally determine his or her own location and orientation within a given environment.

A different researcher proposed a classification scheme for spatial skills based on the mental processes that are expected to be used in performing a given task. She believes that there are two distinct categories of 3-D spatial skills—spatial visualization and spatial orientation. Spatial visualization is mentally moving an object. Spatial orientation is mentally shifting the point from which you view the object while it remains fixed in space.

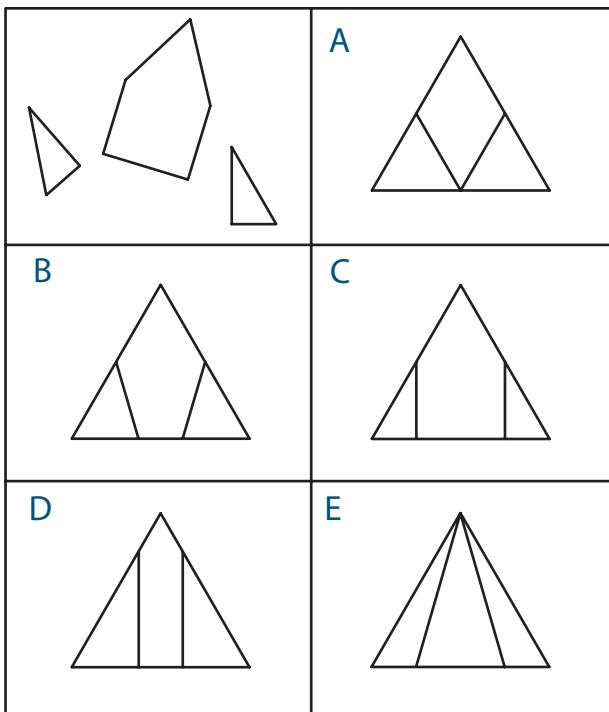
Regardless of the classification scheme you choose to believe, it is clear that more than one component skill makes up the broad category of human intelligence known as spatial visualization. Thus, you cannot do just one type of activity and expect to develop all of the components of spatial skills equally. You need to do a variety of tasks to develop your spatial intelligence, just as developing linguistic intelligence requires you to speak, read, write, and listen.

## 3.05 Assessing Spatial Skills

As with the seven other intelligence types, standardized tests have been developed to determine your level of achievements in spatial intelligence. There are many different tests—some are for 2-D shapes, and some are for 3-D objects. Some evaluate mental rotation skills, and others measure spatial relations skills. The standardized tests usually measure only one specific component of visualization skill. If you were to take a number of different visualization tests, you might find that you have a high level of ability in one component (perhaps paper folding) relative to a low ability in a different component, such as 3-D object rotations. That is normal. Many educators and psychologists believe there is no one-size-fits-all measure of spatial intelligence, just as a single IQ number does not give a clear indication of a person’s overall intelligence.

One of the tests designed to measure your level of 2-D spatial skills is the Minnesota Paper Form Board (MPFB) test. Figure 3.02 shows a visualization problem similar to those found on the MPFB test. This problem tests a person’s ability to determine which set of five 2-D shapes, A through E, is the composite of the 2-D fragments given in the upper left corner of the figure. The way to solve this test is to

**FIGURE 3.02.** A problem similar to that found on the Minnesota Paper Form Board test.



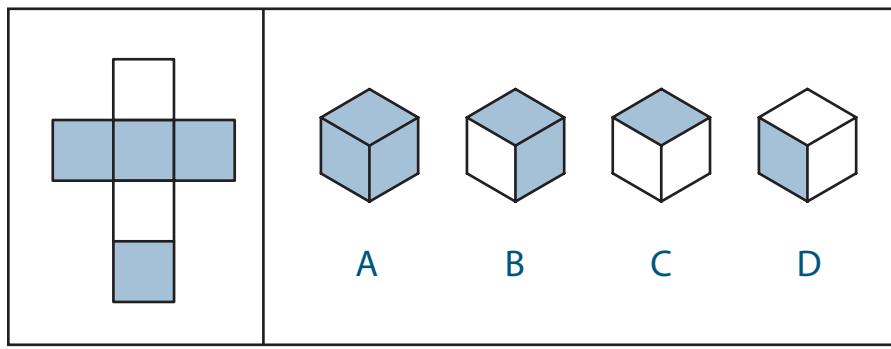
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mentally rotate or move the three pieces to visualize how to put them together to coincide with the combined shape that contains the pieces. The test may seem easy, but you should have fully developed the 2-D spatial skills needed to solve this test when you were four or five years old. During the years since then, you should have developed more advanced 2-D visualization skills. For example, you may now be able to follow a map and determine whether to make a right or a left turn without turning the map.

Figure 3.03 shows a visualization problem similar to what is found on the Differential Aptitude Test: Space Relations. This test is designed to measure your ability to move from the 2-D to the 3-D world. The objective is to mentally fold the 2-D pattern along the solid lines, which designate the fold lines, so the object will result in the 3-D shape. You then choose the correct 3-D object from the four possibilities shown in the figure. In your previous math classes, these 2-D patterns may have been referred to as *nets*. In engineering, the 2-D figures are called *flat patterns* or *developments*.

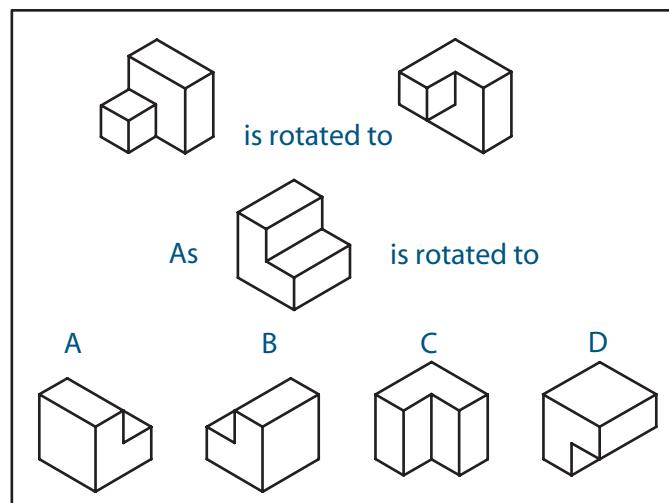
Mental rotation—the ability to visualize the rotation of 3-D objects—is a necessary component skill in engineering graphics and in the use of 3-D modeling software. Figures 3.04 and 3.05 show problems similar to those found on two widely used 3-D spatial tests for rotations.

**FIGURE 3.03.** A problem similar to that found on the Differential Aptitude Test: Space Relations.



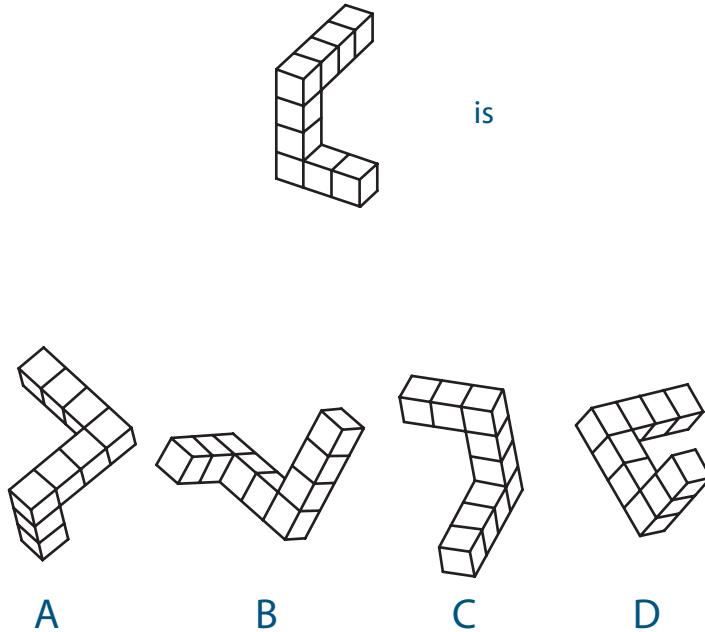
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**Figure 3.04.** A problem similar to that found on the Purdue Spatial Visualization Test: Rotations.



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**FIGURE 3.05.** A problem similar to that found on the Mental Rotation Test.



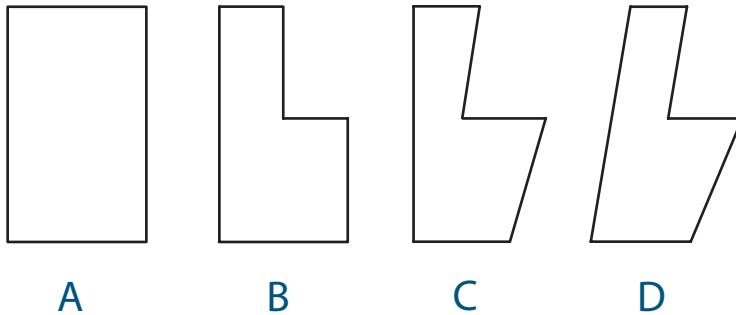
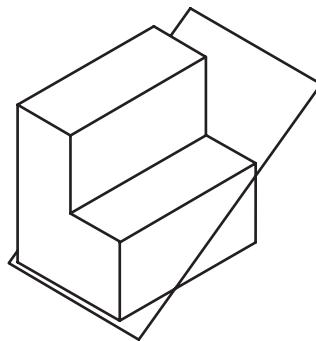
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In the Purdue Spatial Visualization Test: Rotations, an object such as is shown in Figure 3.04 is given on the top line before and after it has been rotated in 3-D space. You then have to mentally rotate a different object on the second line by the same amount and select the correct result from the choices given on the third line.

In the Mental Rotation Test, you are given an object such as shown in Figure 3.05 at the top. Of the four choices given, you pick the two that show correct possible rotations in space of the original object. (Note that two choices are the same object and two choices are different objects.)

Another type of spatial skill that is often tested is the ability to visualize the **cross section** that results from “slicing” a 3-D object with a **cutting plane**. One popular test of this type is the Mental Cutting Test. Figure 3.06 shows the type of problem found on this test, which challenges you to imagine the 2-D shape that is the intersection between the cutting plane and the 3-D object.

**FIGURE 3.06.** A problem similar to that found on the Mental Cutting Test.



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## 3.06 The Importance of Spatial Skills

Engineers and technologists communicate with each other largely through graphical means. They use drawings, sketches, charts, graphs, and CAD models to convey ideas. Design solutions commonly have a graphical component that is backed up by pages of calculations and analysis. Your designs will not be complete without graphics. Even chemical and electrical engineers use drawings for the processes and circuits they design.

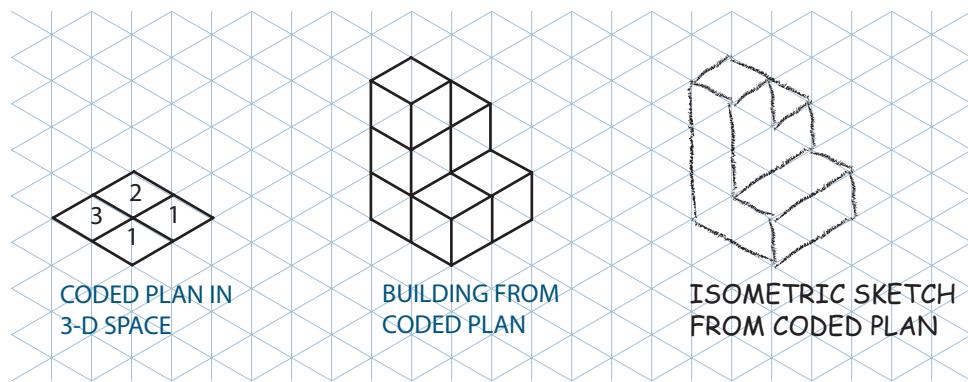
So to communicate as an engineer, you must be able to visualize and interpret the images represented in the drawings. Besides satisfying the need for effective communication, a side benefit to having well-developed 3-D spatial skills is that your brain works better when *all* parts of it are focused on solving a problem. Sketching and visualization have been shown to improve the creative process. Well-developed spatial skills contribute to your ability to work innovatively, as well as to learn to use 3-D modeling software.

The remaining sections of this chapter will provide exercises for your brain—exercises that develop your 3-D spatial skills; exercises that help you think differently from the way you are thinking in your math and science courses; exercises that will help you improve your sketching skills.

## 3.07 Isometric Corner Views of Simple Objects

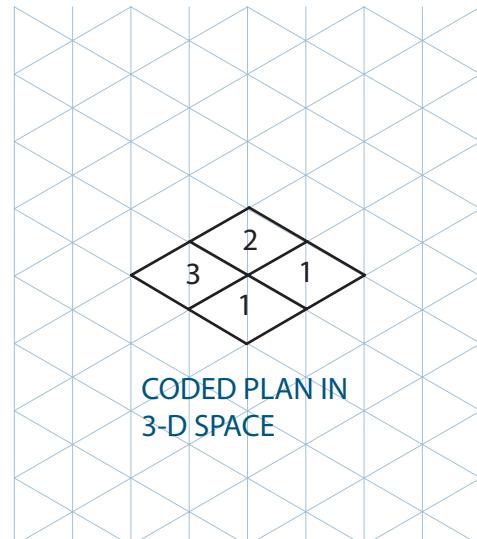
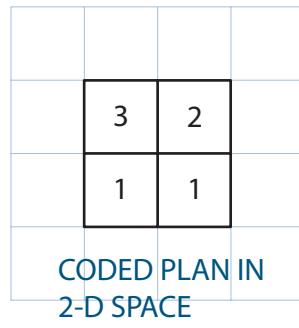
In Chapter 2, you learned how to create a simple isometric sketch of an object made out of blocks as specified by a coded plan. The coded plan is a 2-D portrayal of the object, using numbers to specify the height of the stack of blocks at a given location. Figure 3.07 illustrates the relationship between the coded plan, the object constructed

**FIGURE 3.07.** A coded plan and its resulting isometric sketch.



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**FIGURE 3.08.** The relationship between a coded plan in 2-D space and 3-D space.



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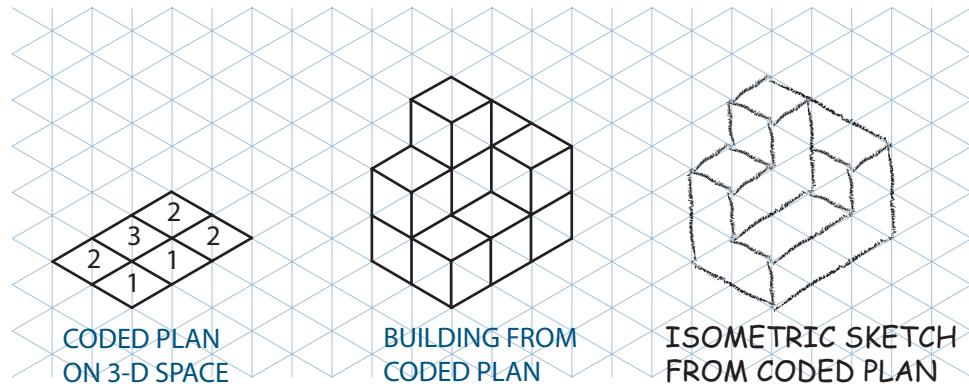
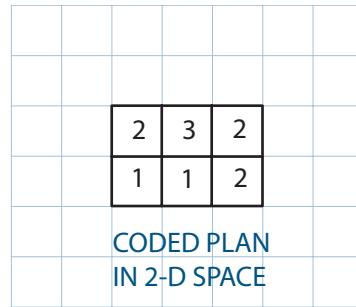
out of blocks, and the resulting isometric sketch of the object—remember, you show edges only between surfaces on the isometric sketch.

The coded plans you viewed in Chapter 2 were constructed on isometric grid paper. The building “grew up” from the plan into the isometric grid. In the previous exercises, the coded plan was oriented in 3-D space on the isometric grid, which represents 3-D space. Now think about laying the coded plan flat on a 2-D sheet of paper. Figure 3.08 shows the coded plan for the object shown in Figure 3.07 laid flat in a 2-D orientation. Figure 3.09 shows the relationship between a coded plan in 2-D space, the coded plan in 3-D space, the object made of blocks, and the resulting isometric sketch.

When you orient the coded plan in 2-D space, everything you learned about these plans still applies: you “build up” from the plan. The numbers represent the height of the stack of blocks at a given location, and you show lines only where two surfaces intersect. However, now one more consideration has been introduced into the isometric sketching equation—the orientation of your “eye” with respect to the object itself. (Note that *the orientation of your eye* is often referred to as *your viewpoint*.)

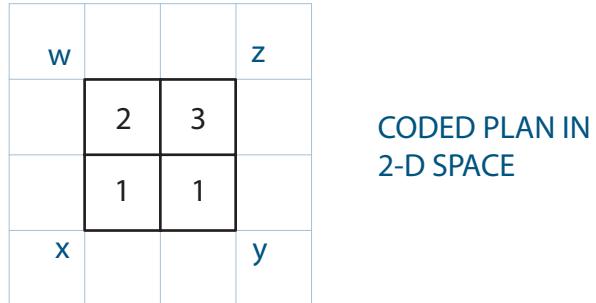
Examine again the coded plans in 2-D space. Figure 3.10 shows a simple coded plan with its four corners labeled as w, x, y, and z. A **corner view** of the object represented by the coded plan in Figure 3.10 is the view from a given corner when

**FIGURE 3.09.** The relationship between coded plans, a building, and an isometric sketch.



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**FIGURE 3.10.** A simple coded plan with corners labeled.



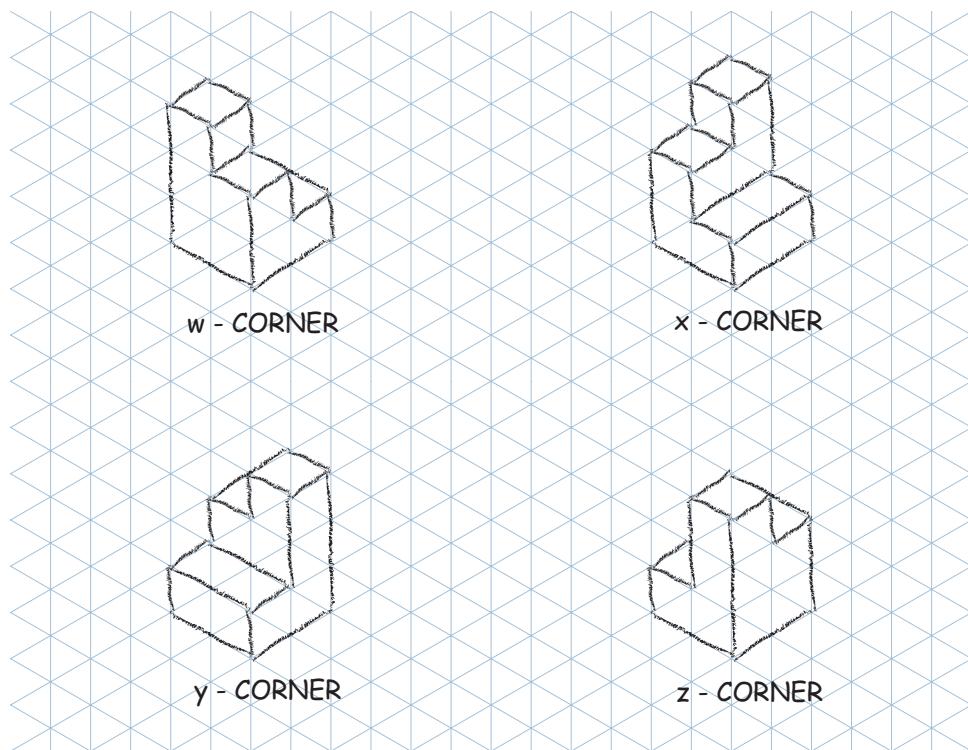
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the viewpoint is *above* the object in question. This view is sometimes referred to as *the bird's-eye view*, because the viewpoint is above the object. A worm's-eye view is the viewpoint from *below* the object. Figure 3.11 shows the four corner views for the coded plan from Figure 3.10.

When the four corner views of the object are created, the object does not change—just your viewpoint of the object. The importance of viewpoint in visualization is readily apparent when you think about a complex system such as an automobile. When you are looking at a car from the front, you may have an entirely different mental image of the car than if you look at it from the side or rear. What you "see" depends largely on where your eye is located relative to the object.

With more practice, you will find it easier to make corner views from coded plans. At first, you may need to turn the paper to visualize what an object will look like from a given corner. With continued practice, however, you should be able to mentally turn the paper to sketch the object from the vantage point of any corner.

**FIGURE 3.11.** Sketched isometric views from the corners of the coded plan in Figure 3.10.



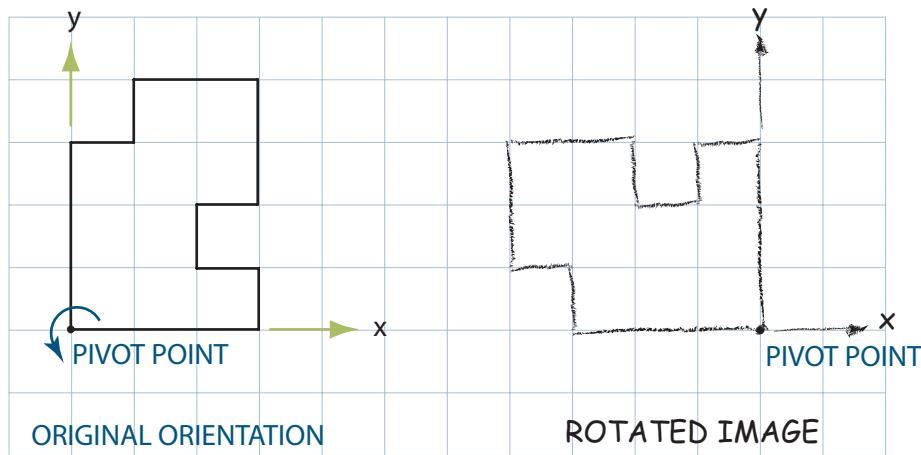
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### 3.08 Object Rotations about a Single Axis

Being able to mentally visualize an object as it rotates in space is an important skill for you to acquire as an engineer or a technologist. You already have had limited exposure to the concept of rotating objects through your work with mentally rotating coded plans to obtain different corner views. In the preceding section, you started with the y-corner view to draw the isometric. Having done that, you should be able to imagine what the object will look like from the x-corner view. If you can see in your mind what the object looks like from the x-corner view, you are mentally rotating the object in space. In this section, you will continue to work with object rotations, tackling increasingly complex objects and using increasingly complex manipulations.

You probably learned in your math classes how 2-D shapes are rotated in 2-D space about a pivot point, as illustrated in Figure 3.12. In this figure, the shape has

**FIGURE 3.12.** A shape rotated about a pivot point in 2-D space.



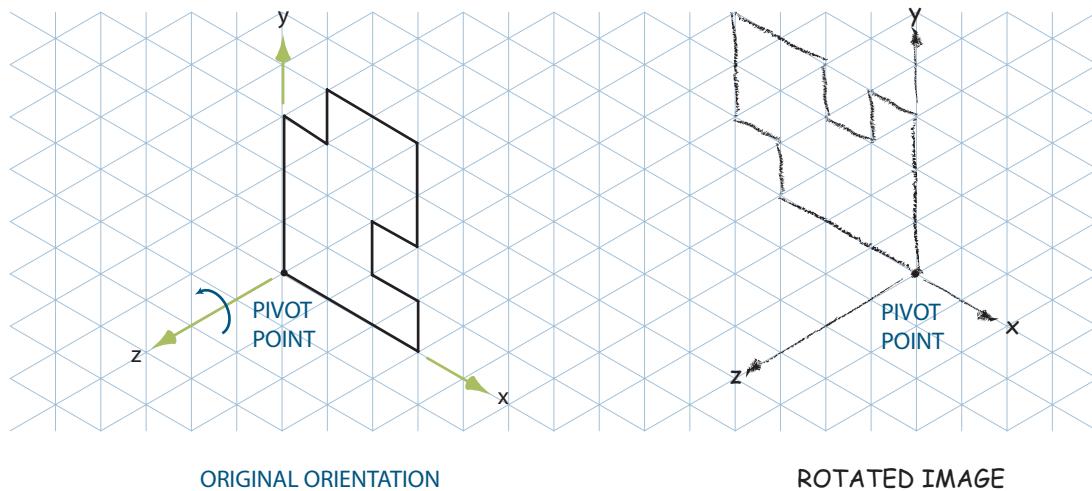
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been rotated 90 degrees counterclockwise (CCW) about the pivot point, which is the origin of the 2-D xy-coordinate system. After rotation, the newly oriented shape is referred to as the “image” of the original shape. Notice that when the 2-D shape is rotated about the pivot point, each line on the shape is rotated by the same amount—in this case, 90 degrees CCW about the pivot point. Also notice that the point on the shape that was originally located at the pivot point, the origin, remains at that same location after rotation.

In Chapter 2, you learned about 3-D coordinate systems and how three axes (the  $x$ -,  $y$ -, and  $z$ -axes) can be used to describe 3-D space. When you rotate an object in 3-D space, the same principles apply as for 2-D rotations. In fact, you can reexamine the rotation of the shape in Figure 3.12 from a 3-D perspective. Figure 3.13 shows the 2-D shape drawn in 3-D space before and after it was rotated 90 degrees CCW about the pivot point, which is the origin of the xyz-coordinate system.

Observe and understand how each line on the shape is rotated the same amount—90 degrees CCW about the origin—and that the point on the shape originally in contact with the origin remains at the origin after rotation. One other thing you may notice is that the pivot point is the point view of the  $z$ -axis. The point view of a line is what you see as you look down the length of the axis. To illustrate this principle, take a pen or pencil and rotate it so you are looking directly at its point; notice that the length of the pen “vanishes” and only the “point” remains visible, as shown in Figure 3.14. As such, the original rotation of the 2-D shape, as shown in Figure 3.12, could be considered a 90-degree CCW rotation about the  $z$ -axis in 3-D space.

Think back to what you learned in Chapter 2 about the right-hand rule. If you point the thumb of your right hand in the positive direction of the  $z$ -axis and curl your fingers, you will see that the 90-degree CCW rotation mimics the direction that your fingers curl, as illustrated in Figure 3.15. This CCW rotation of the 2-D shape represents a *positive* 90-degree rotation about the  $z$ -axis. The CCW rotation is positive because the thumb of your right hand was pointing in the positive direction of the  $z$ -axis as the shape was rotated. If you point the thumb of your right hand in the negative direction of the  $z$ -axis and the shape is rotated in the direction the fingers of your right hand curl, your fingers indicate a clockwise (CW) rotation of the shape about the  $z$ -axis, as shown in Figure 3.16. A CW



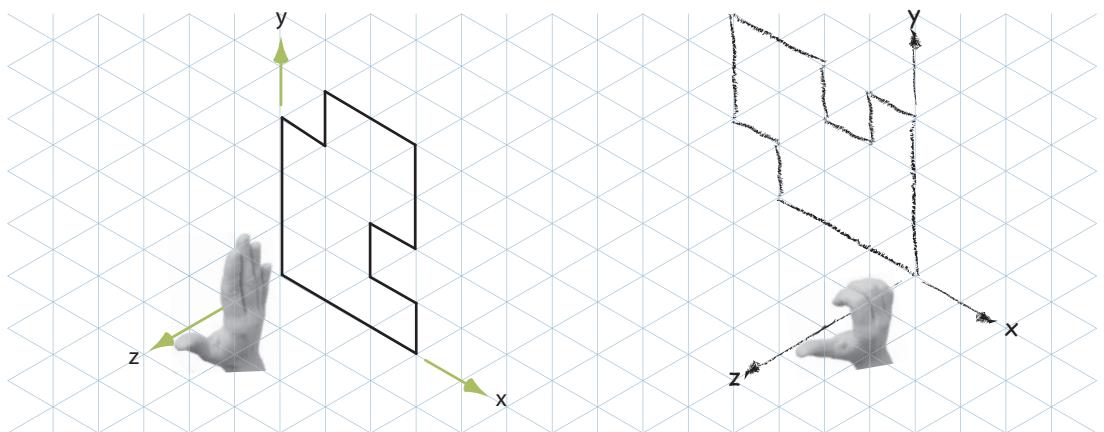
**FIGURE 3.13.** A 2-D shape rotated in 3-D space.

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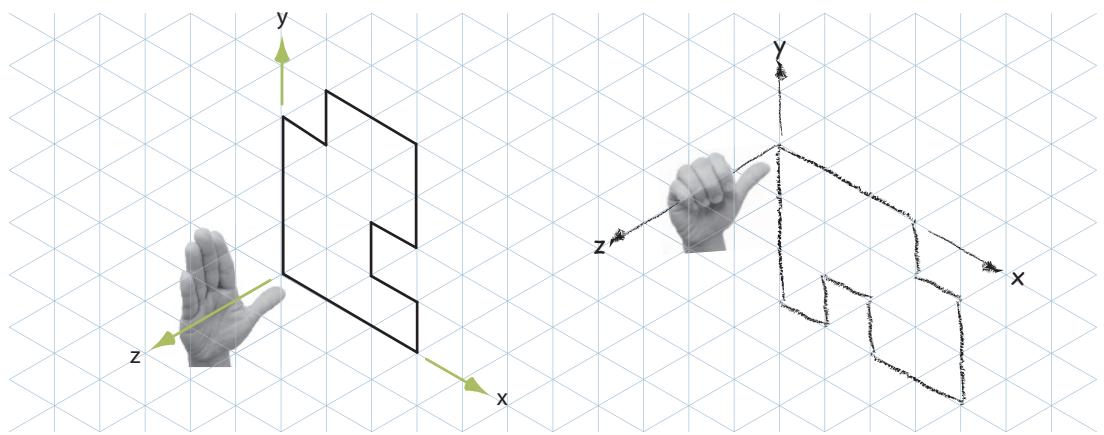
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**FIGURE 3.14.** Looking down the end of a pencil.



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**FIGURE 3.15.** Positive rotation of a 2-D shape about the z-axis.



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**FIGURE 3.16.** Negative rotation of a 2-D shape about the z-axis.

rotation about an axis is defined as a negative rotation. Remember that the thumb of your right hand is pointing in the negative z-direction. Also remember that the pivot point of the shape remains at a fixed location in space as it is rotated in the negative z-direction.

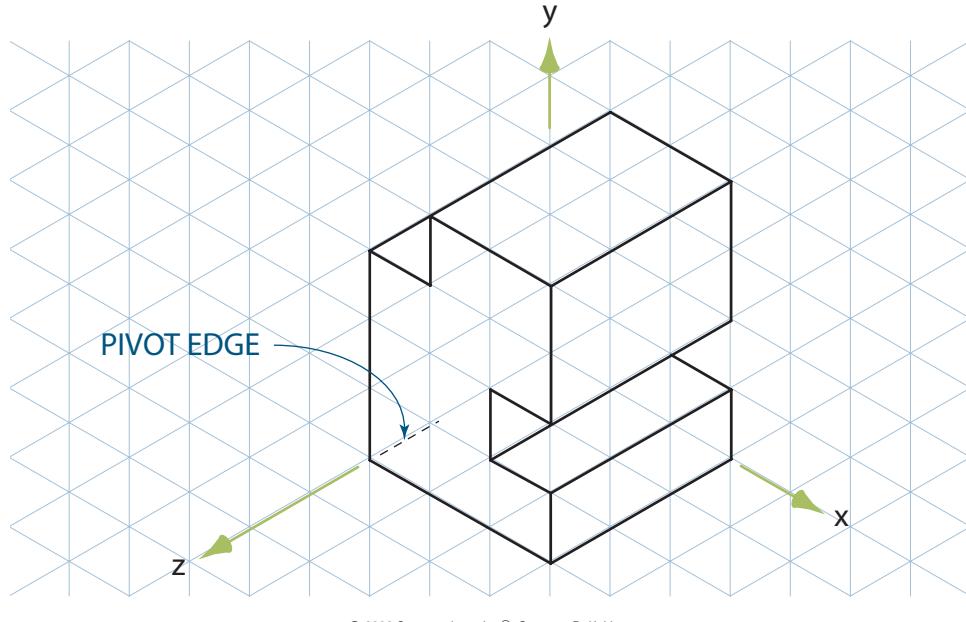
You should now be ready to tackle rotations of 3-D objects in 3-D space. Imagine the 2-D shape from the past several figures is a surface view of a 3-D object. Assume you can extend the surface you have been seeing in the xy-plane into the z-dimension. The result of extending that surface in the third dimension is a solid object. The terminology of 3-D CAD software says that the shape was extruded. You will learn more about extrusion later in this text. If this shape is “extruded” three units into the z-direction, the object will appear as shown in Figure 3.17. In this figure, notice that instead of a single point located on the axis of rotation (the z-axis in the figure), an entire edge of the object is located on that axis. The edge is hidden from sight in this view, but you can imagine it nonetheless. Now think about rotating the entire object about the z-axis in a positive direction (or CCW) 90 degrees from its original position. When this happens, the image shown in Figure 3.18 appears. Instead of a single pivot point, the 3-D rotation has a pivot edge. Throughout the rotation, the edge remained in contact with the axis of rotation. All parts of the object also rotated by the same amount (90 degrees CCW about z) just as all parts of the surface were rotated when you were considering 2-D shapes.

Just as 2-D shapes can be rotated positively (CCW) or negatively (CW) about the z-axis, 3-D objects can be rotated in either direction. Figure 3.19 shows the same object after it has been rotated negative 90 degrees (CW) about the z-axis. This figure also makes clear that the pivot edge of the object, which is now visible in the rotated view, remains in contact with the axis of rotation as the object is rotated.

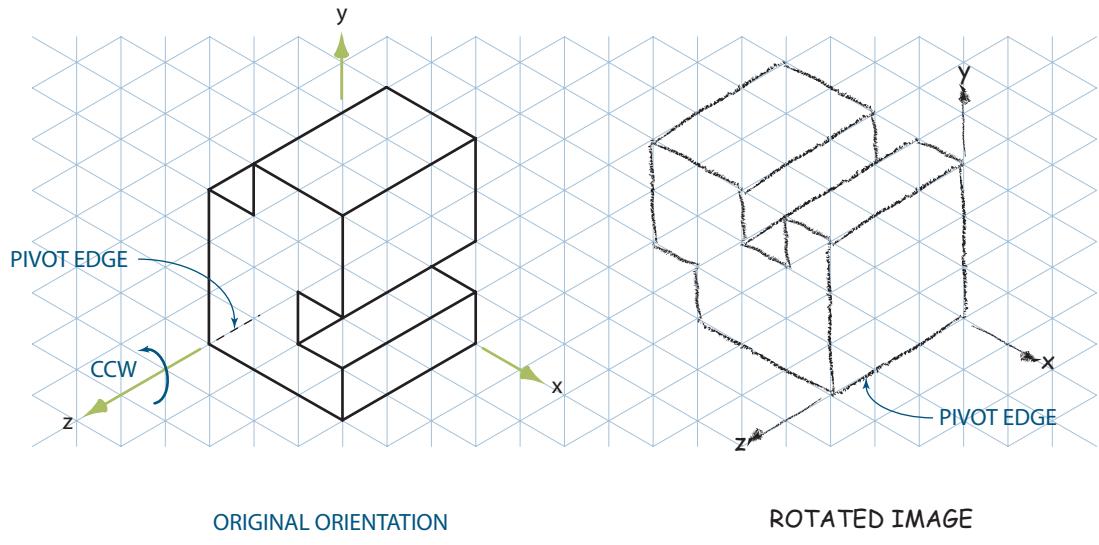
Any object can be rotated about the x- or y-axis by following the same simple rules established for rotation about the z-axis:

1. The edge of the object originally in contact with the axis of rotation remains in contact after the rotation. This edge is called the pivot edge.
2. Each point, edge, and surface on the object is rotated by exactly the same amount.

**FIGURE 3.17.** A 2-D shape from Figure 3.12 extruded three units in the z-direction.

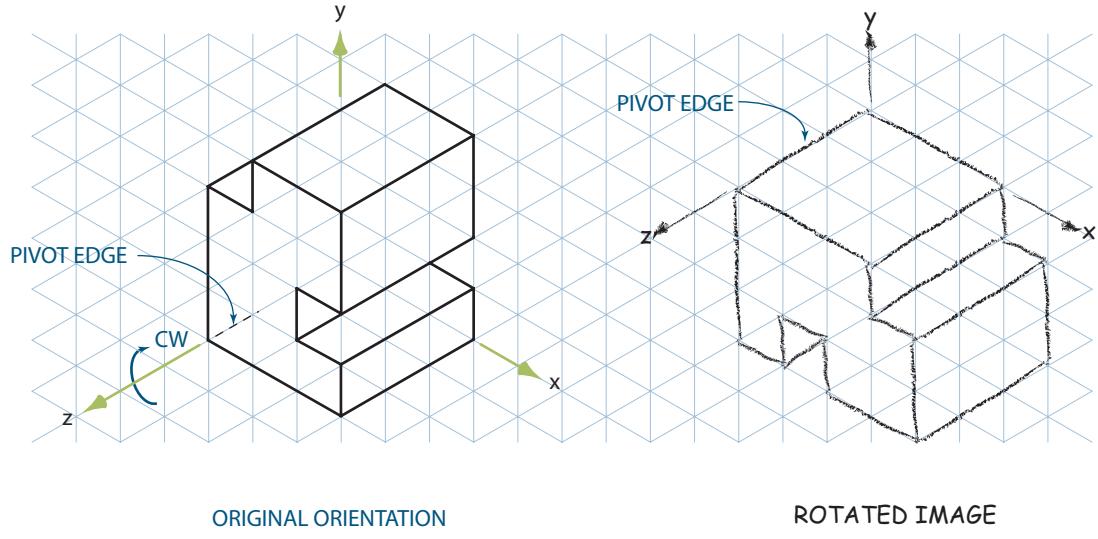


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**FIGURE 3.18.** A 3-D object rotated 90 degrees counterclockwise about the z-axis.

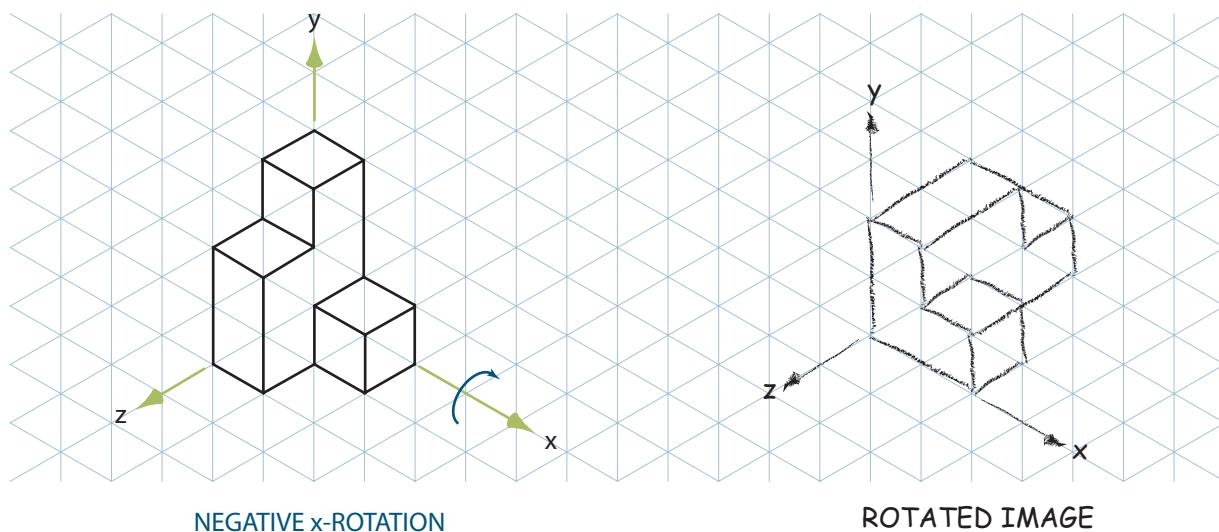
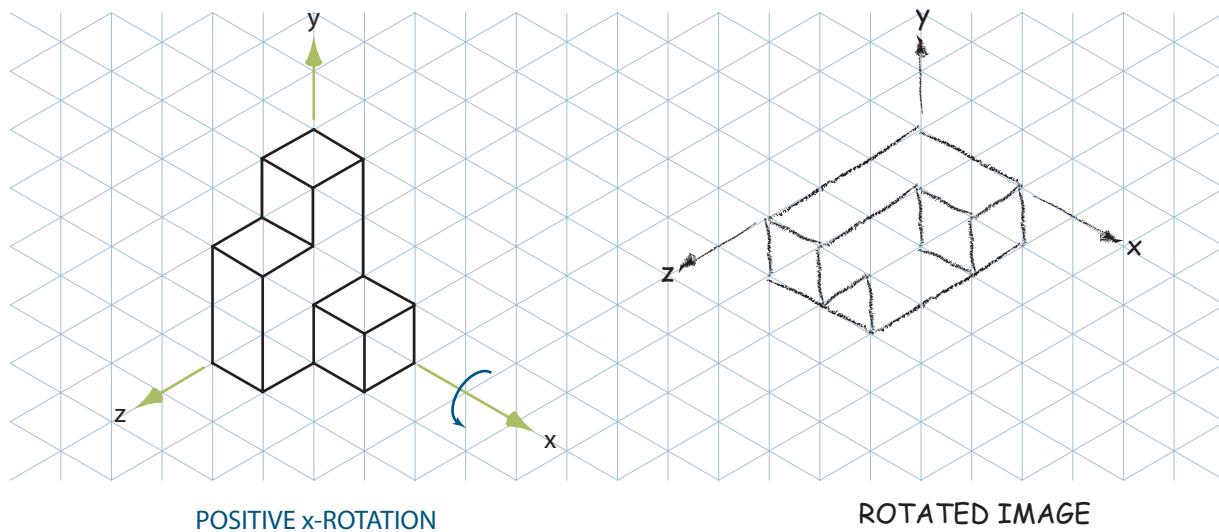


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**FIGURE 3.19.** A 3-D object rotated 90 degrees clockwise about the z-axis.

3. The rotation is positive when it is CCW about an axis and negative when it is CW about an axis. The direction is determined by looking directly down the positive end of the axis of rotation.
4. An alternative method for determining the direction of the rotation is the right-hand rule. Point the thumb of your right hand into the axis of rotation—into either the positive or negative end of the axis of rotation—and curl your fingers in the direction the object is rotated. The direction you obtain from the right-hand rule is the same as the direction defined in number 3 above, positive is CCW and negative is CW.

Figures 3.20 and 3.21 illustrate the positive and negative 90-degree rotations obtained about the x- and y-axis, respectively.



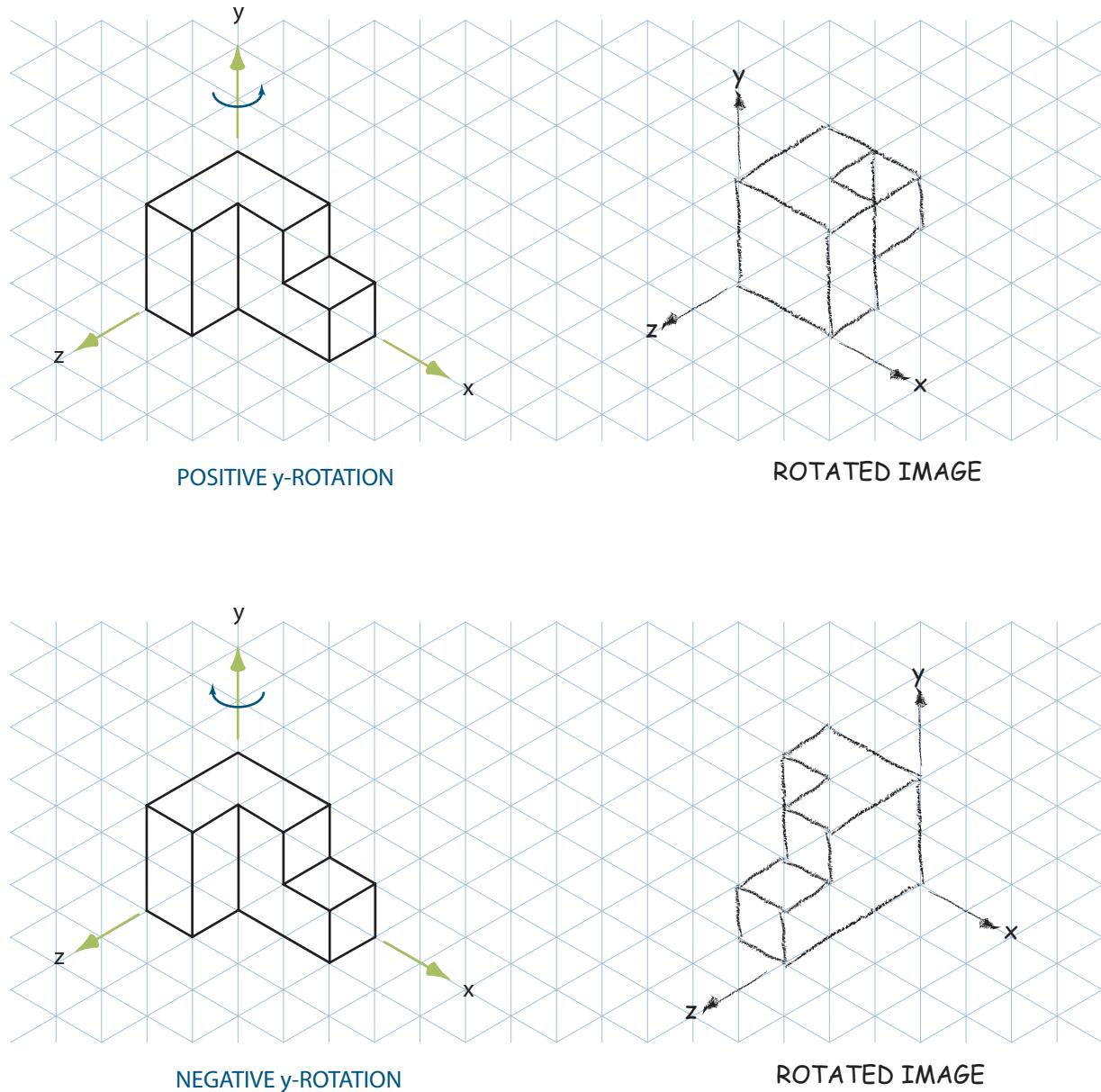
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**FIGURE 3.20.** Positive and negative rotations about the x-axis.

### 3.08.01 Notation

Specifying in writing a positive, or CCW, rotation about any axis is cumbersome and time-consuming. For this reason, the following notations will be used to describe object rotations in this text:

- To denote positive rotations of an object about the indicated axis.
- To denote negative rotations of an object about the indicated axis.
- Also, for simplicity in sketching, this text will always rotate an object in increments of 90 degrees about the indicated axis. Figure 3.22 illustrates the result when you rotate the object according to the notation given.

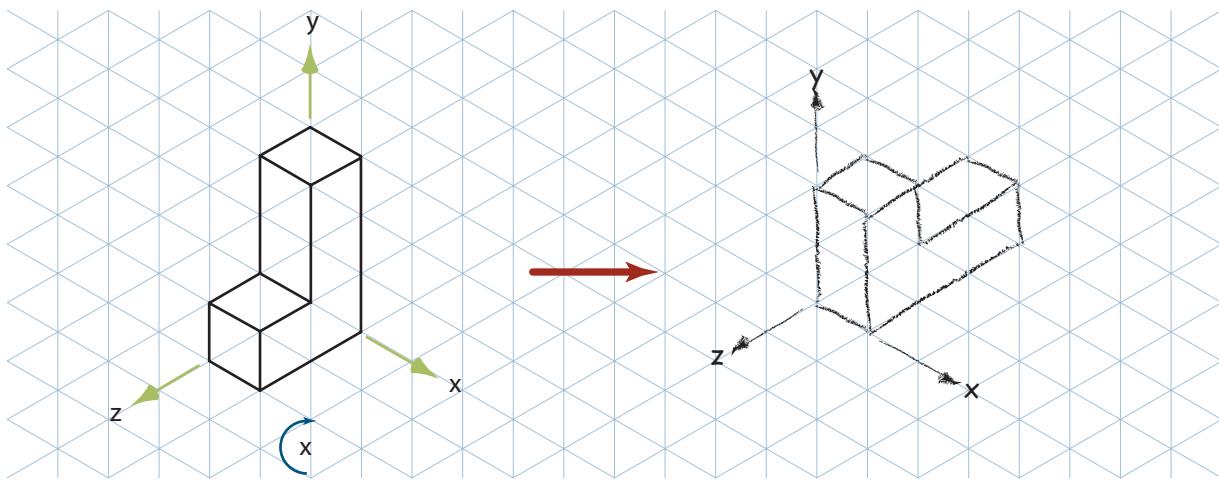
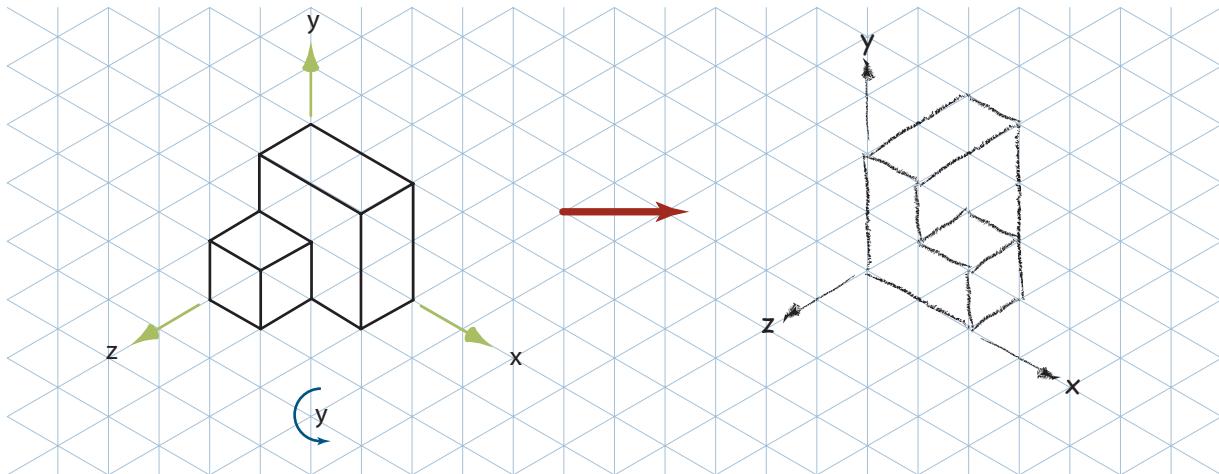


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**FIGURE 3.21.** Positive and negative rotations about the y-axis.

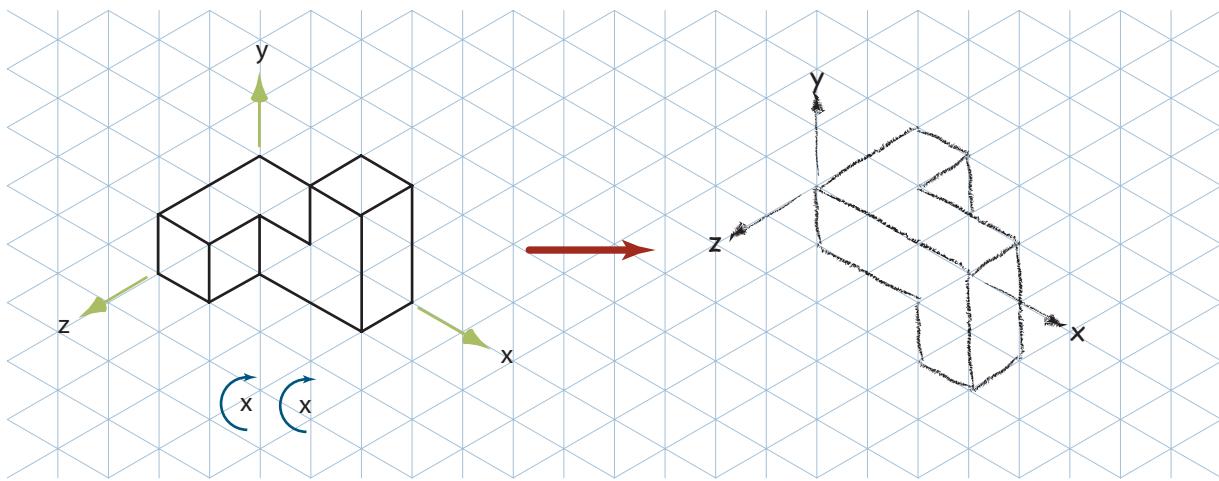
### 3.08.02 Rotation of Objects by More Than 90 Degrees about a Single Axis

In all examples and figures in the preceding sections, objects were rotated exactly 90 degrees about a single axis. In reality, you can rotate objects by any number of degrees. If you rotate an object in two increments of 90 degrees about the same axis, the total rotation will be 180 degrees. Similarly, if you rotate an object in three increments, the total rotation will be 270 degrees. Figure 3.23 shows an object that has been rotated 180 degrees about a single axis, along with the symbol denoting the amount and direction of rotation. Notice that the two 90-degree positive x-axis rotations indicate the total 180-degree rotation achieved.



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**FIGURE 3.22.** Object rotations specified by notation.



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**FIGURE 3.23.** An object rotated 180 degrees about an axis.

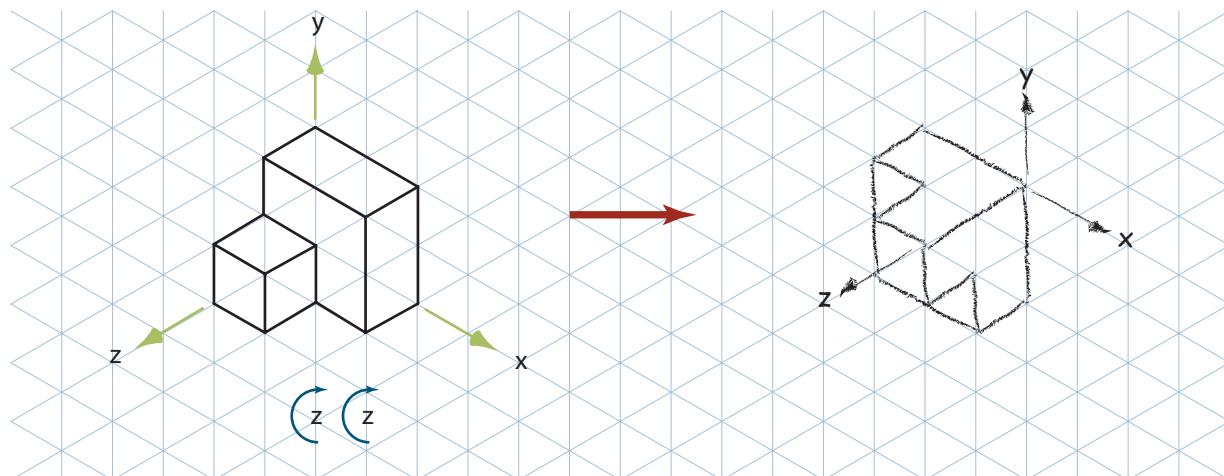
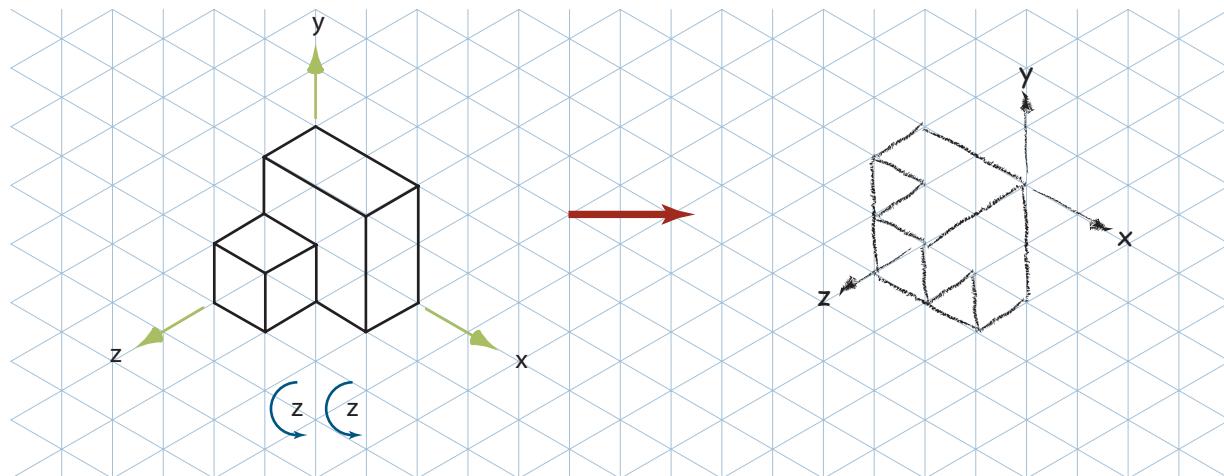
Once you are free to rotate objects in multiple increments of 90 degrees, you can achieve several equivalent rotations. The term *equivalent rotations* means that two different sets of rotations produce the same result.

### 3.08.03 Equivalencies for Rotations about a Single Axis

When an object is rotated in multiple increments about an axis, the following equivalencies can be observed:

- A positive 180-degree rotation is equivalent to a negative 180-degree rotation.
- A negative 90-degree rotation is equivalent to a positive 270-degree rotation.
- A positive 90-degree rotation is equivalent to a negative 270-degree rotation.

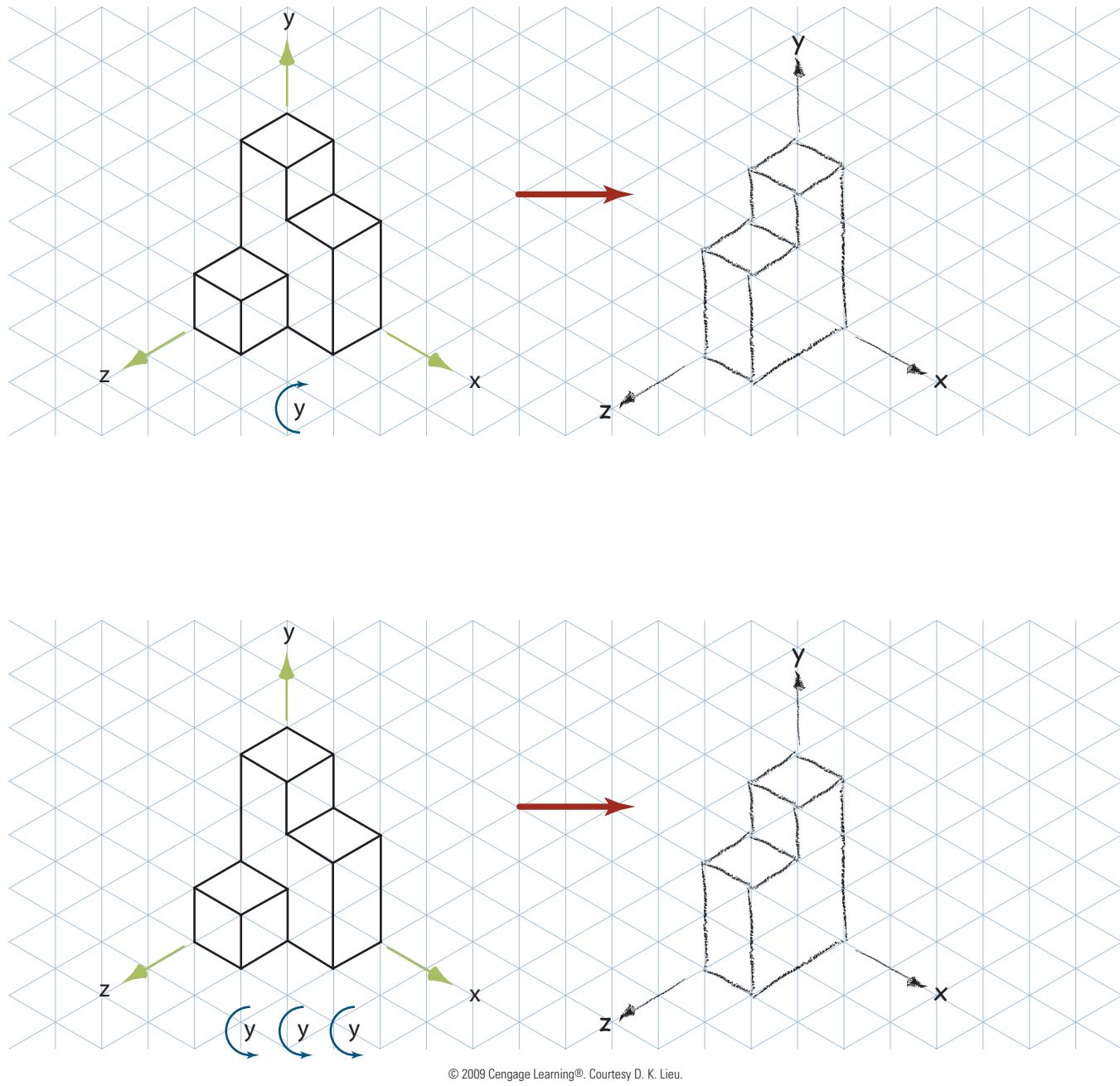
These equivalencies are illustrated in Figures 3.24, 3.25, and 3.26, respectively.



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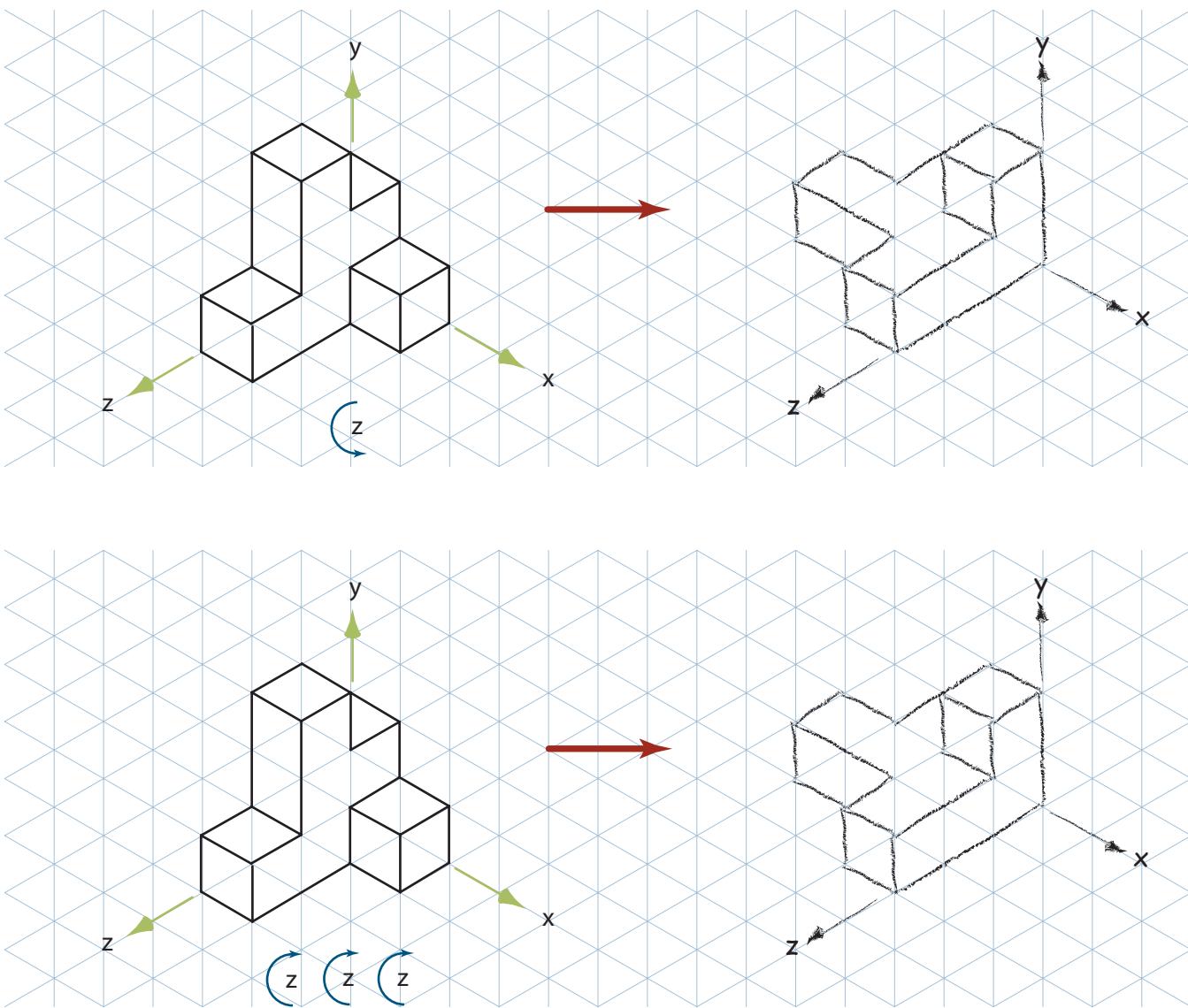
**FIGURE 3.24.** A positive 180-degree rotation is equivalent to a negative 180-degree rotation.

**3-20** section one Laying the Foundation



**FIGURE 3.25.** A negative 90-degree rotation is equivalent to a positive 270-degree rotation.

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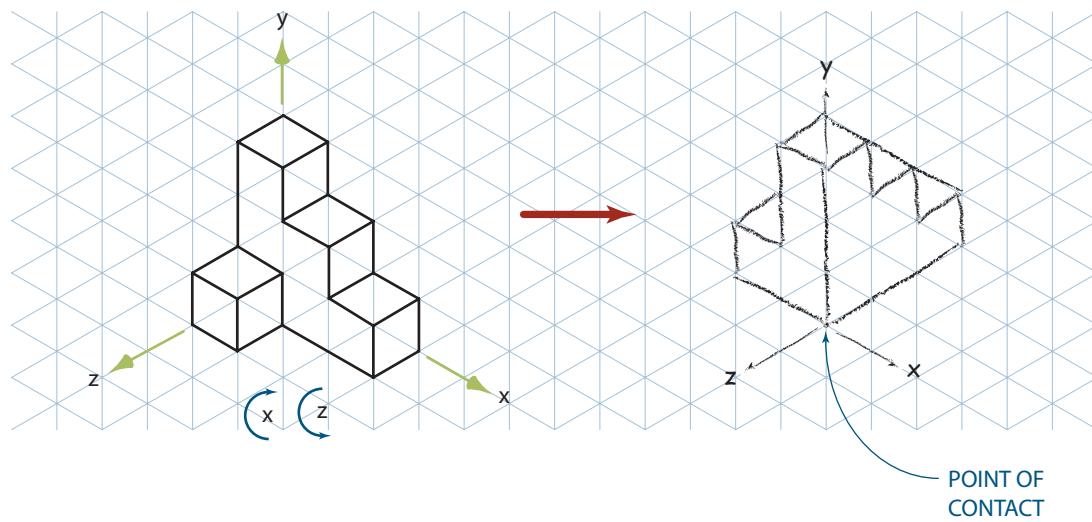


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**FIGURE 3.26.** A positive 90-degree rotation is equivalent to a negative 270-degree rotation.

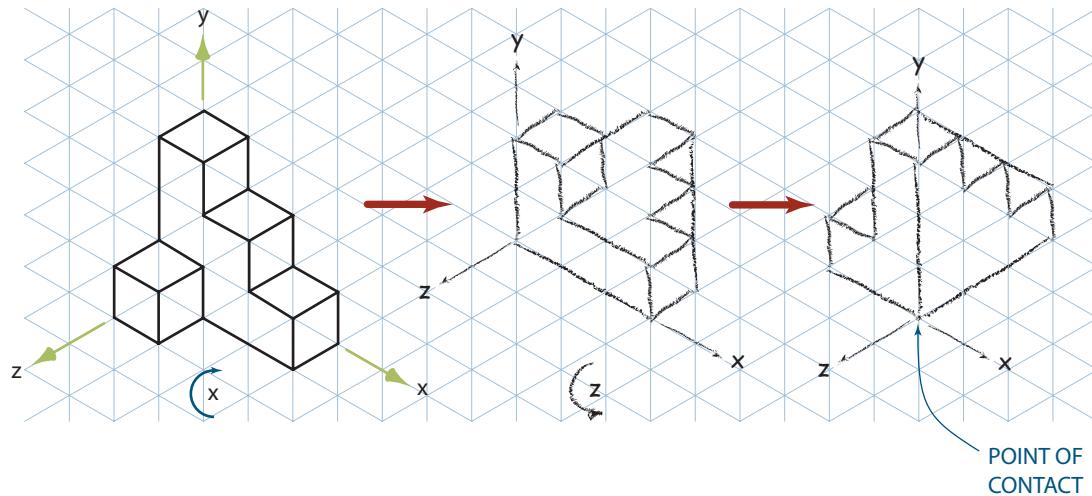
### 3.09 Rotation about Two or More Axes

In the same way you rotated an object about a single axis, you can also rotate the object about more than one axis in a series of steps. Figure 3.27 shows an object that has been rotated in the negative direction about the x-axis and then rotated in the positive direction about the z-axis. The rotation notation used in the figure indicates the specified two-step rotation. Figure 3.28 shows the same set of rotations, only this time they are shown in two single steps to achieve the final result. Notice that when an object is rotated about two different axes, a single edge no longer remains in contact with the axis of rotation (since there are now two of them). For rotations about two axes, only a single point remains in its original location, as shown in Figures 3.27 and 3.28.



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**FIGURE 3.27.** An object rotated about two axes.

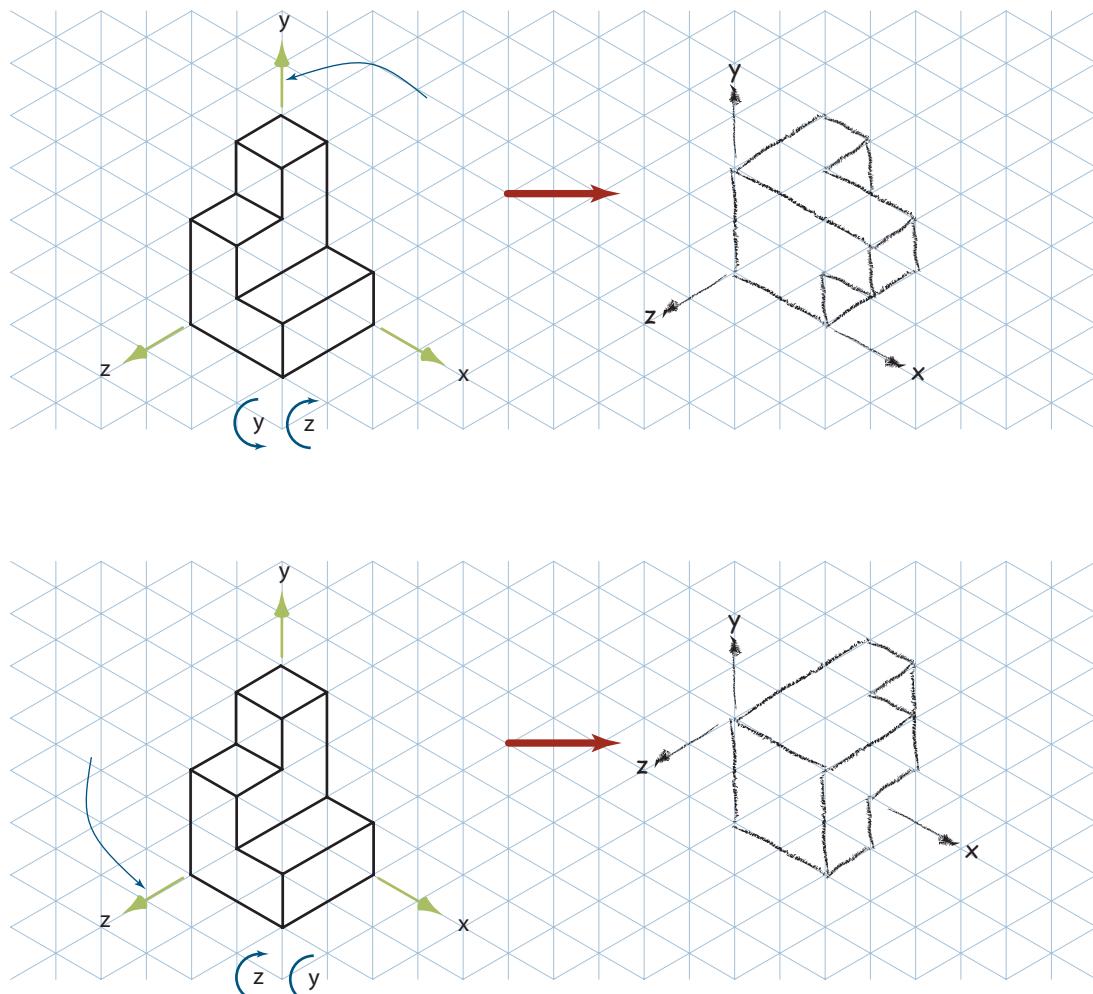


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**FIGURE 3.28.** An object rotated in two steps.

When rotating an object about two or more axes, you must be careful to perform the rotations in the exact order specified. If the rotations are listed such that you rotate the object CW in the negative direction about the x-axis and then rotate it CCW in the positive direction about the z-axis, you must perform the rotations in that order. Object rotations are not commutative. (Remember that the commutative property in math states that  $2 + 3 = 3 + 2$ .) For object rotations, rotating about the x-axis and then rotating about the y-axis is *not* the same as rotating about the y-axis and then rotating about the x-axis.

In the top portion of Figure 3.29, the object has been rotated about positive y and then rotated about negative z to obtain its image. In the bottom portion of the figure, the object has been rotated about negative z and then rotated about positive y to obtain a new image of the rotated object. The second image is obtained by reversing



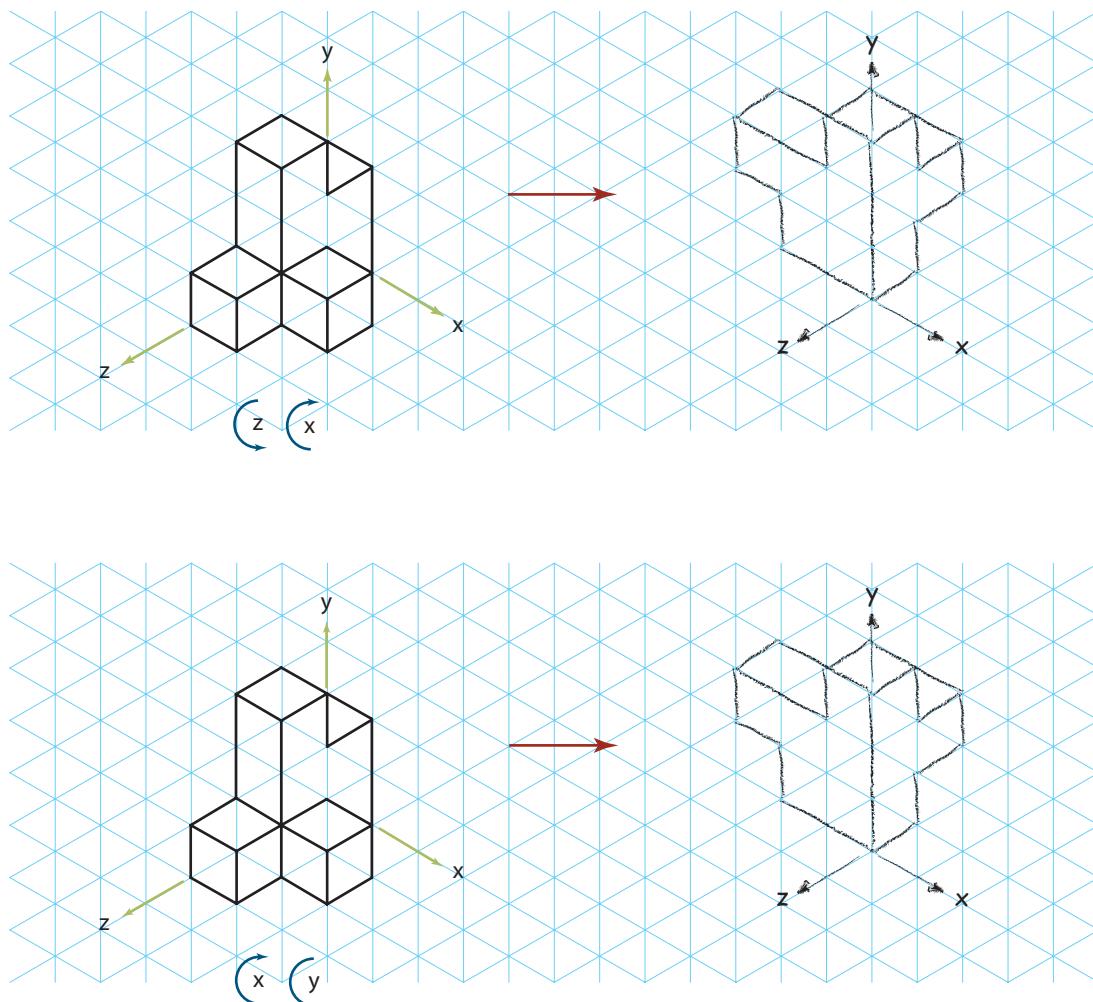
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**FIGURE 3.29.** Object rotations about two axes—order not commutative.

the order of the rotations. The resulting images are not the same when the order of rotation is changed. Why? Because with the first set of rotations, the edge of the object on the  $y$ -axis serves as the pivot line for the first rotation, which is about positive  $y$ . For the second set of rotations, the edge of the object on the  $z$ -axis serves as the pivot edge for the first of the two rotations. When you rotate first about negative  $z$ , you are using an entirely different object edge than the initial pivot line; hence, the difference in rotated images.

### 3.09.01 Equivalencies for Object Rotations about Two or More Axes

Just as there are equivalencies for rotations of an object about a single axis, there are equivalencies for object rotations about two axes. Figure 3.30 shows one pair of rotational equivalencies. Can you find another set? How about positive  $x$  and then negative  $z$ ? No! Or positive  $y$  and then positive  $z$ ? Yes! There are several possibilities for each pair of rotations. But it is impossible to come up with simple rules for equivalency, as in the previous discussion of equivalent rotations about a single axis. Equivalent rotations for objects about two or more axes are likely to be determined through trial and error and a great deal of practice.



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**FIGURE 3.30.** Equivalent rotations about two axes.

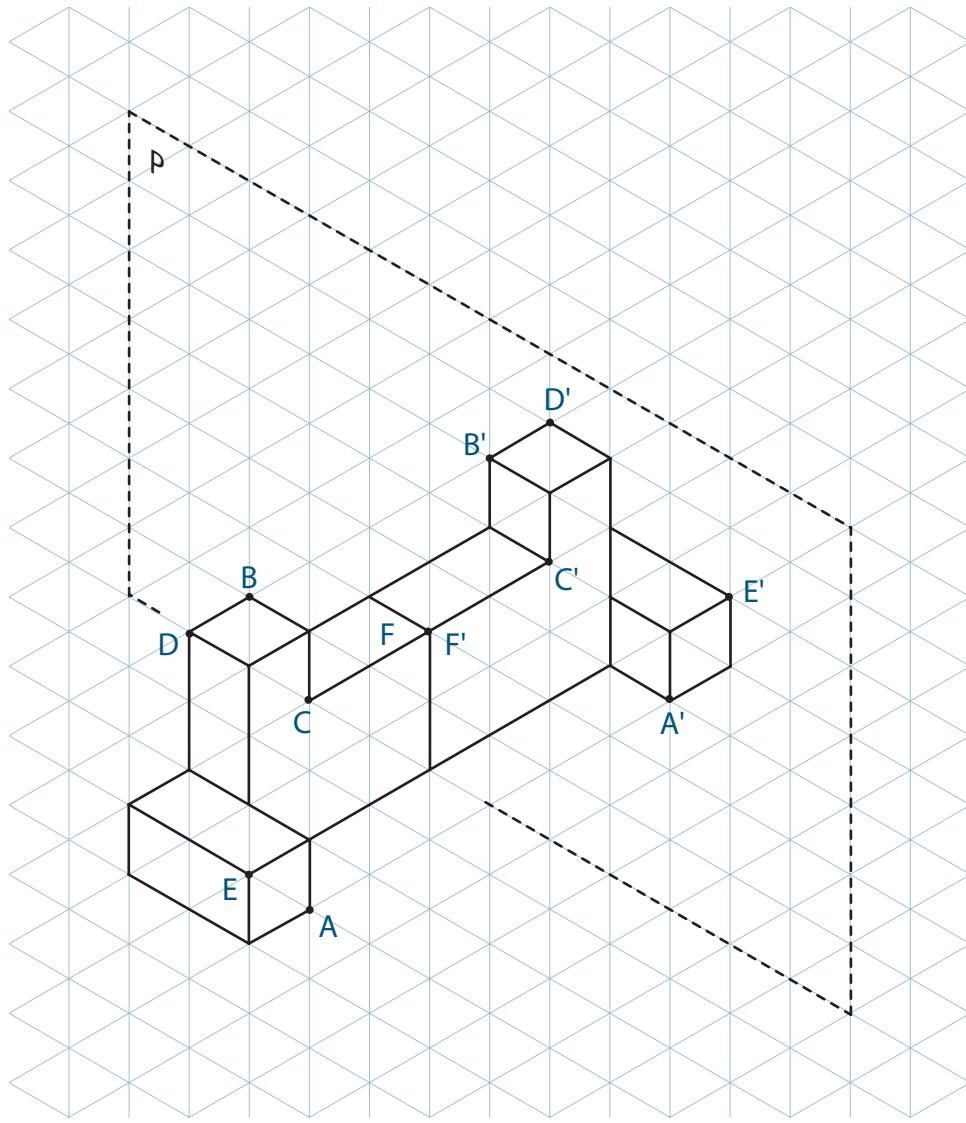
## 3.10 Reflections and Symmetry

Now that you know the basics of how to visualize an object rotated about an axis, you are ready to move on to visualizing reflections and symmetry. Visualizing planes of symmetry, for example, could save you a great deal of computation time when you are using tools such as Finite Element Analysis or FEA. You will learn more about FEA in later chapters of this text.

You are probably familiar with the concept of **reflections** because you are used to looking at your image reflected back to you from a mirror. With a mirror, you see a reflected 2-D image of your face. If you have a mole on your right cheek, you will see the mole on the right cheek of the reflection. Even though your face is three-dimensional, your face in the mirror is a 2-D reflection—as if your face were projected onto a 2-D plane with your line of sight perpendicular to the plane. You may be able to see somewhat in the third dimension from this mirror plane; however, your depth perception will be a bit off because the image is only two-dimensional. Three-dimensional reflection of objects is different from 2-D reflections with mirrors. For one thing, you reflect a 3-D object *across* the plane so that a 3-D image ends up on the other side of the reflection plane.

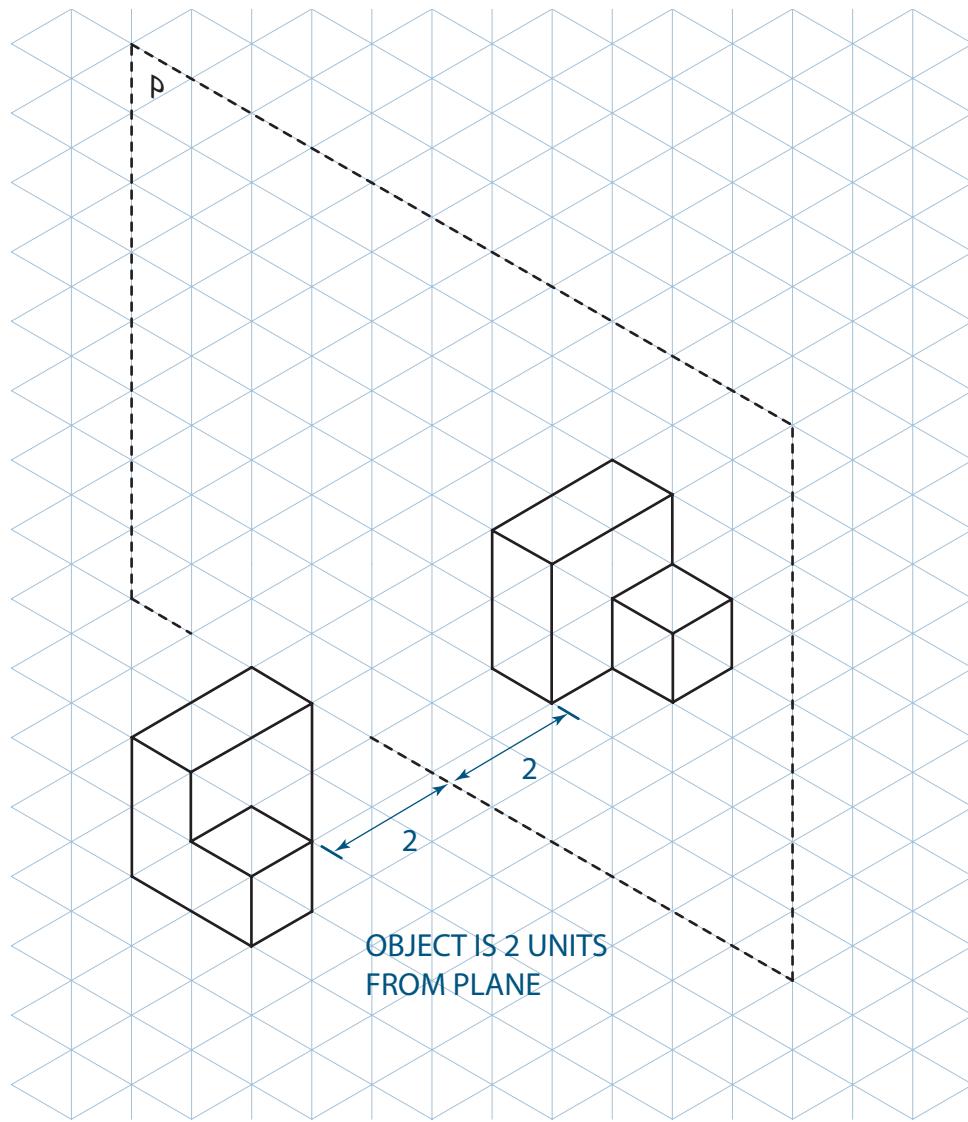
Figure 3.31 shows a simple object and its reflection across a reflection plane. Formally stated, in the case of 3-D object reflections, such as shown in Figure 3.31, each point A of the object is associated with an image point A' in the reflection such that the plane of reflection is a perpendicular bisector of the line segment AA'. What this means is the distance between a point on an object and the reflection plane is equal to the distance between the corresponding point on the image and the reflection plane. The distances are measured along a line perpendicular to the plane of reflection. In this figure, several points on the original object are labeled, as well as their corresponding points on the reflected image. In this case, the plane of reflection coincides with one planar end of the original object; therefore, the corresponding planar end of the reflected image also coincides with the reflection plane. If you measure the distance between point A on the object and the reflection plane, you will find that it is three units. Then if you measure the distance between A' and the reflection plane, you will find the distance to be three units again. It is also possible to reflect an object across a plane when the object is located some distance from the reflection plane, as illustrated in Figure 3.32.

**FIGURE 3.31.** An object and its 3-D reflection.



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**FIGURE 3.32.** An object located at a distance from the plane and its reflection.



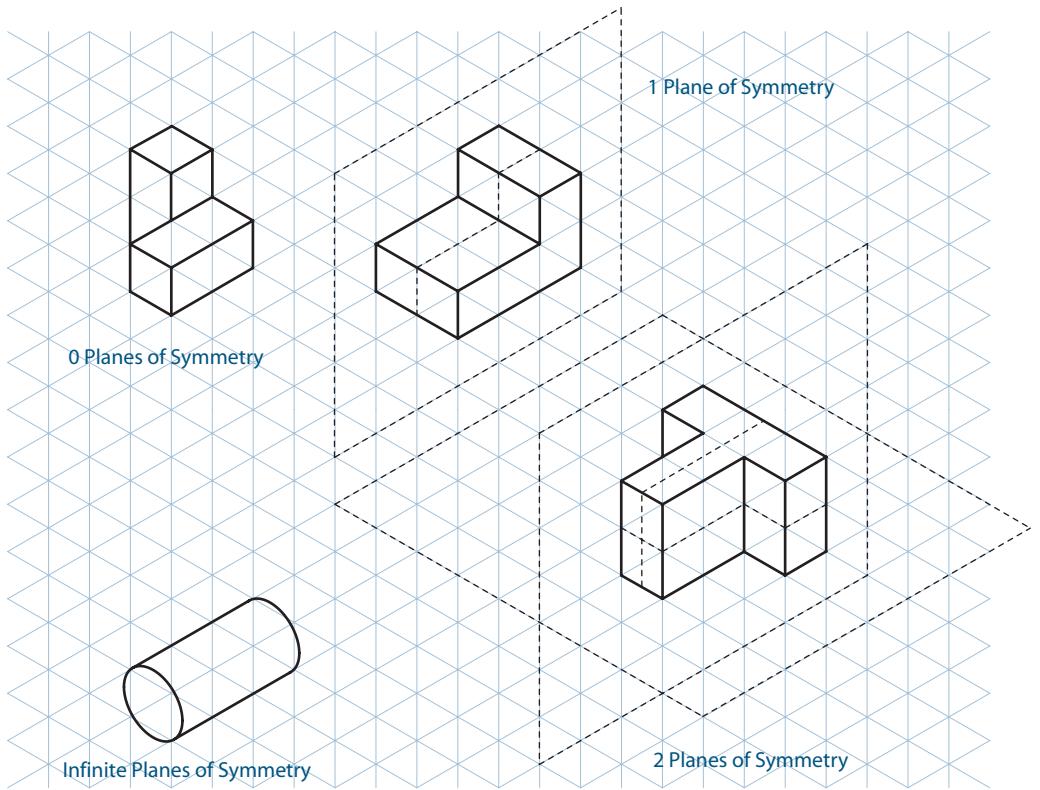
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### 3.10.01 Symmetry

Your job as an engineer may be easier if you can recognize planes of symmetry within an object. A plane of **symmetry** is an imaginary plane that cuts through an object such that the two parts, one on either side of the plane, are reflections of each other. Not all objects have inherent symmetry. The human body is roughly symmetrical and has one plane of symmetry—a vertical plane through the tip of the nose and the belly button. The left side is a reflection of the right side. Some objects contain no planes of symmetry, some contain only one plane of symmetry, and still others contain an infinite number of planes of symmetry. Figure 3.33 shows several objects and their planes of symmetry: one object contains no planes of symmetry, one object has just one plane of symmetry, one object has two planes of symmetry, and the last object contains an infinite number of planes of symmetry.

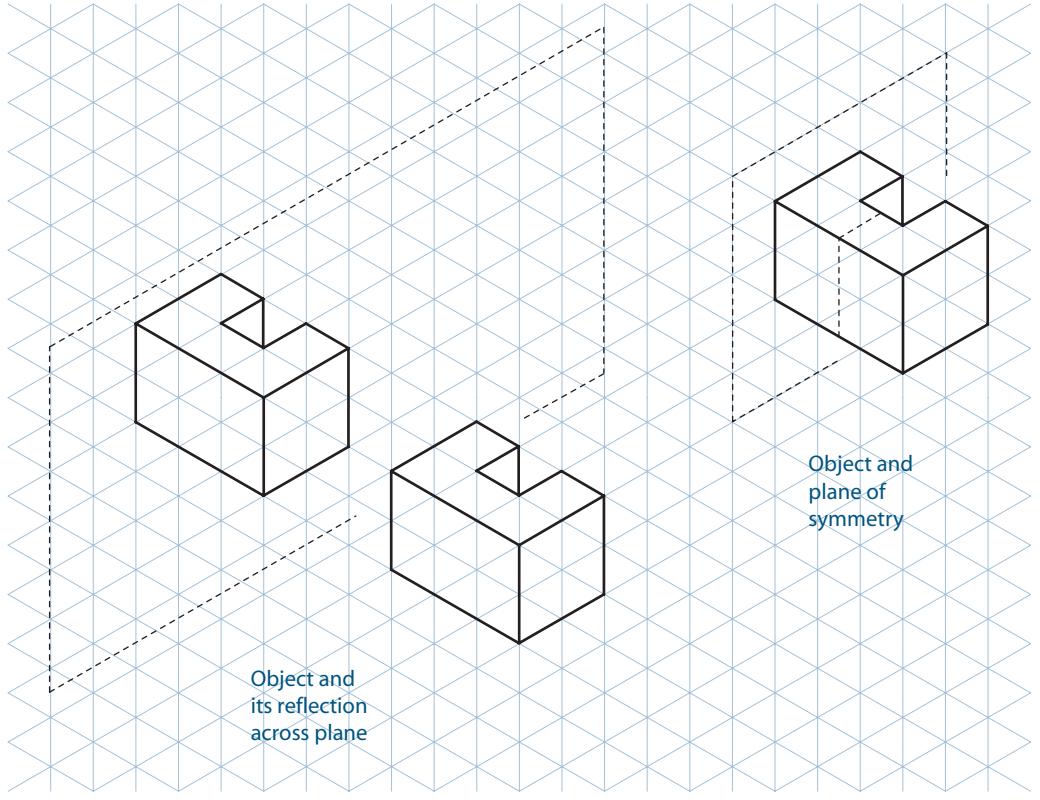
There is one major difference between object reflection and object symmetry. With reflections, you end up with two separate objects (the original and its reflected image); with symmetry, you have a single object that you imagine is being sliced by a plane to form two symmetrical halves. Figure 3.34 illustrates the difference between the two.

**FIGURE 3.33.** Objects and their planes of symmetry.



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**FIGURE 3.34.** A comparison of object reflection and symmetry.

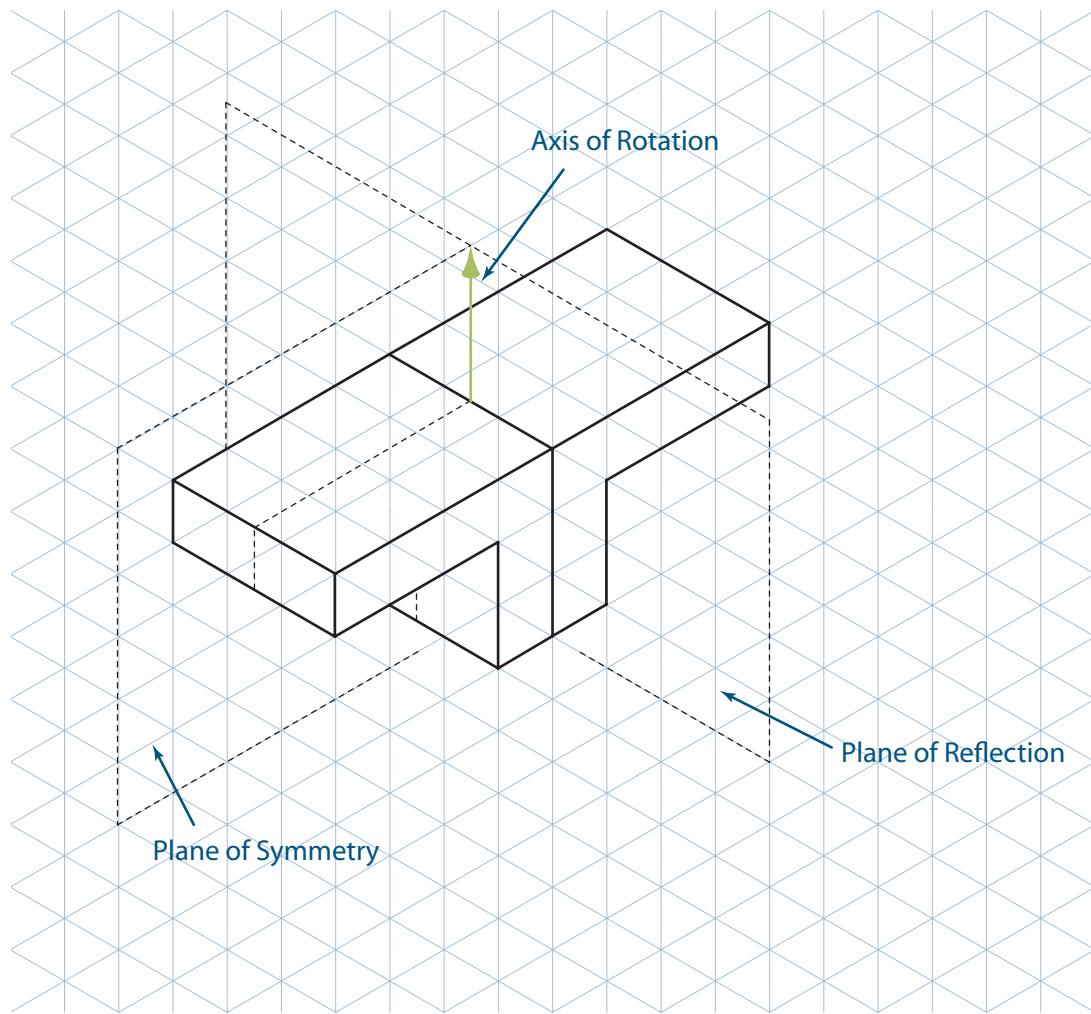


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For an object that is symmetrical about a plane, you can sometimes obtain its reflection by rotating the object 180 degrees. To do this, the axis of rotation must be the intersection between the plane of reflection and the plane of symmetry (two planes intersect to form a line). This concept is illustrated in Figure 3.35. Note that a reflection of an object that is not symmetrical cannot be achieved through a simple 180-degree rotation of the object. Hold up your hands in front of you to obtain an object (left hand) and its reflected image (right hand). Note that because your hands have no planes of symmetry, it is impossible to rotate one of them in space to obtain the other one.

### 3.11 Cross Sections of Solids

Visualizing cross sections enables an engineer to figure out how a building or a mechanical device is put together. Visualizing cross sections enables an electrical engineer to think about how circuit boards stack together within the housing that contains them. Chemical engineers and materials engineers think about the cross sections of molecules and the way those molecules combine with other molecules. Geological engineers and mining engineers visualize cross sections of the earth to determine where veins of rock and ore may be located. Most of the skills described in these examples are at an advanced



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**FIGURE 3.35.** Object reflection through rotation.

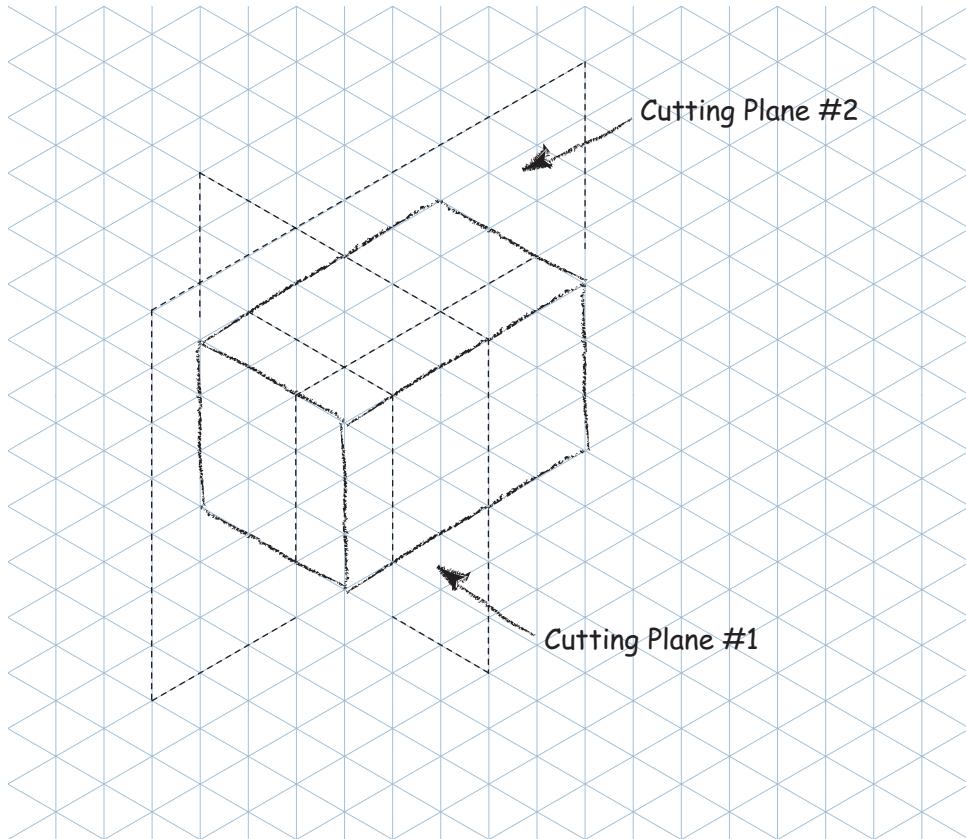
level; in this section, you will learn about cross sections of solids from a fundamental level. Then you can apply the principles to the visualization of more complex parts and systems in later courses and, of course, in your professional work.

Simply stated, a cross section is defined as “the intersection between a solid object and a cutting plane.” Because a plane is infinitely thin, the resulting intersection of the two entities is a planar section. The limits of the cross-sectional plane are the edges and the surfaces where the plane cuts through the object. Consider a loaf of bread. Imagine a single slice of infinitely thin bread. One slice of bread would represent the cross section obtained by slicing a vertical plane through the loaf. Because most loaves of bread are not “constant” in shape along their lengths, the cross section changes as you go along the loaf. You know from experience that the cross sections, or slices, on the ends of the loaf are typically smaller than the slices in the middle.

The cross section obtained by intersecting a cutting plane with an object depends on two things: (1) the orientation of the cutting plane with respect to the object and (2) the shape of the original object.

Consider the square prism shown in Figure 3.36. It is cut first by a vertical cutting plane perpendicular to its long axis to obtain the square cross section shown. If the

**FIGURE 3.36.** Cross sections from a square prism.



Cross Section #1



Cross Section #2

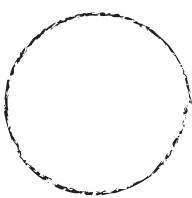
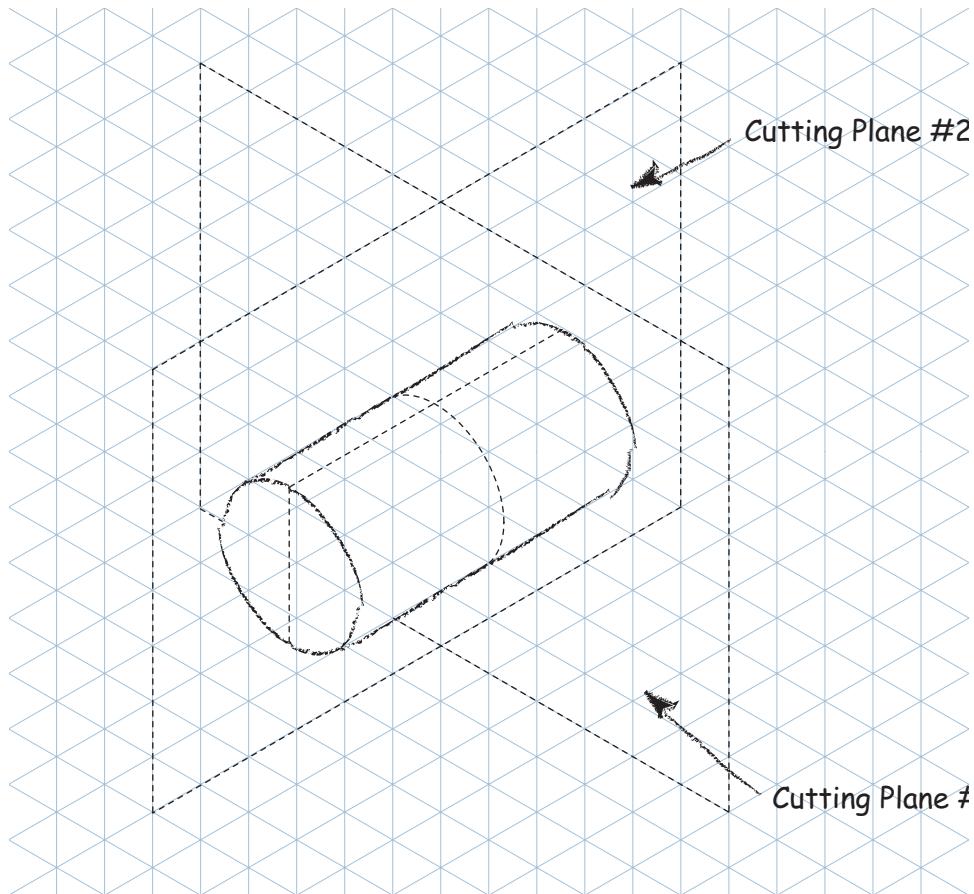
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cutting plane is rotated 90 degrees about a vertical axis, the result is the rectangular cross section shown in the figure. The two cross sections are obtained from the same object. The difference in the resulting cross sections is determined by changing the orientation of the cutting plane with respect to the object.

Now consider the cylinder shown in Figure 3.37. If a cutting plane is oriented perpendicular to the axis of the cylinder, a circular cross section results; if the plane is located along the axis of the cylinder, a rectangular cross section is obtained. Observe that this rectangular cross section through the cylinder is identical to the cross section obtained by slicing the rectangular prism along its long axis in Figure 3.36.

Because a resulting cross section through an object depends on the orientation of the cutting plane with respect to the object, most objects may have several cross sections associated with them. Figure 3.38 shows a cylinder with four possible cross sections. Can you imagine the orientation of the cutting plane with respect to the cylinder for each cross section?

**FIGURE 3.37.** Cross sections of a cylinder.



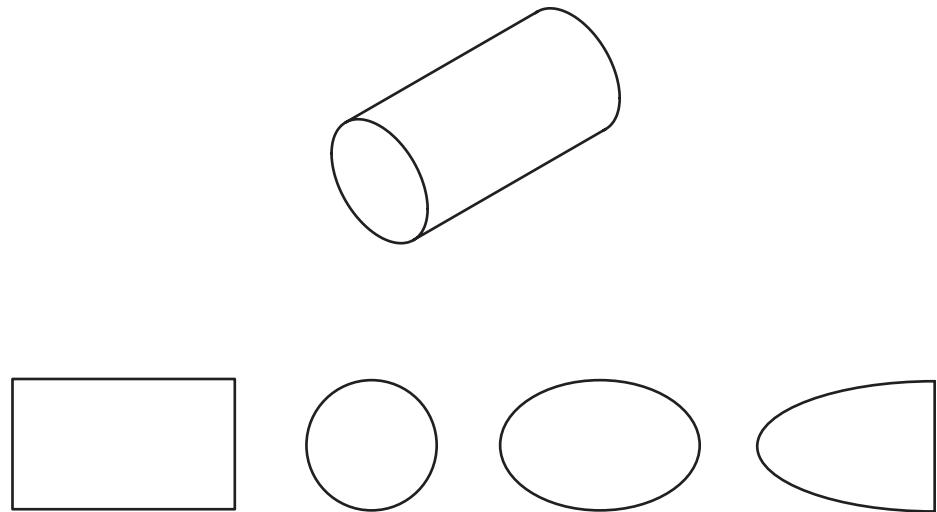
Cross Section #1



Cross Section #2

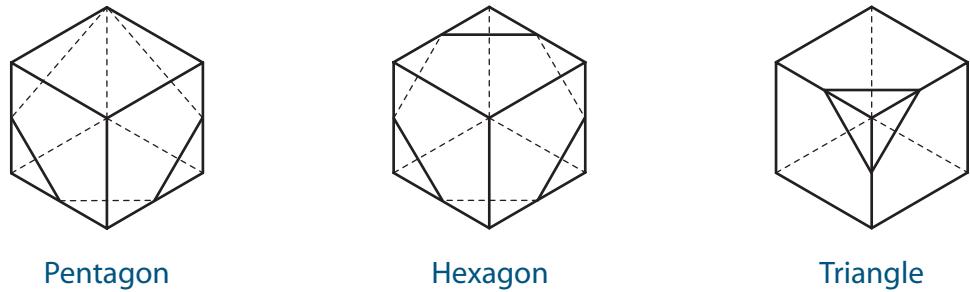
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**FIGURE 3.38.** Various cross sections of a cylinder.



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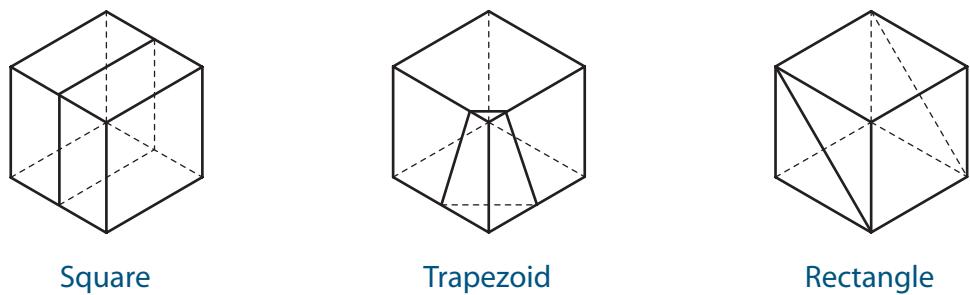
**FIGURE 3.39.** Various cross sections of a cube.



Pentagon

Hexagon

Triangle



Square

Trapezoid

Rectangle

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You already know that the first two cross sections, rectangle and circle, were obtained by orienting the cutting plane perpendicular to and along the long axis of the cylinder, respectively.

What about the third cross section? It was obtained by orienting the cutting plane at an angle with respect to the axis of the cylinder.

The fourth cross section was also obtained by angling the cutting plane with respect to the cylinder axis, but the angle was such that a portion of the cutting plane went through the flat circular end surface of the cylinder.

Figure 3.39 shows several cross sections obtained by slicing a cube with cutting planes at different orientations.

## 3.12 Combining Solids

Another skill that will be helpful to you as an engineer or technologist is the ability to visualize how two solids combine to form a third solid. The ability to visualize **combining solids** will be helpful as you learn how to use solid modeling software. In early versions of 3-D CAD software, commands used to combine solids were sometimes known as **Boolean operations**. This terminology was borrowed from mathematics set theory operations, called Booleans, where basic operations include unions, intersections, and complements between sets of numbers. Boolean logic is now the foundation of many modern innovations. In fact, if you have performed a search on the Web using an AND or an OR operator, you have used Boolean logic to help you narrow or expand your search. In terms of 3-D CAD, the Boolean set operations typically correspond to software commands of Join, Intersect, and Cut. To help you become familiar with the terminology since you probably will be building 3-D computer models, this section will use the same terminology.

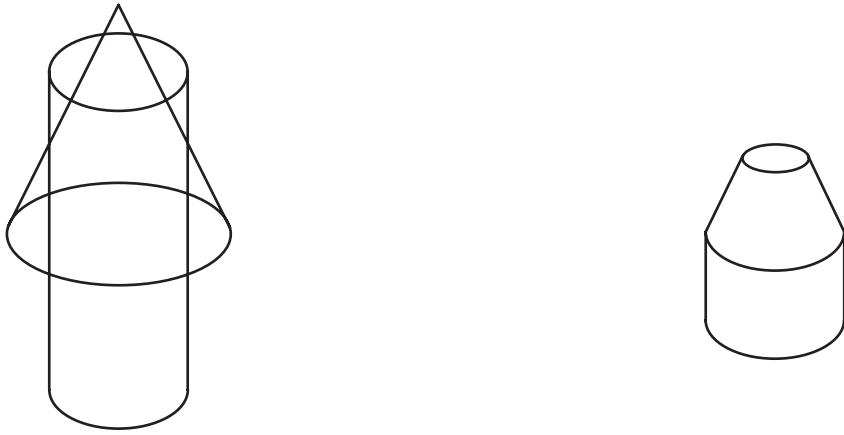
Two overlapping objects can be combined to form a third object with characteristics of each original object apparent in the final result. To perform any Cut, Join, or Intersect operation to combine objects, the objects must be overlapping initially. What is meant by overlapping is that they share a common volume in 3-D space—called the **volume of interference**. Figure 3.40a shows two objects that overlap; Figure 3.40b shows the volume of interference between the two objects. Notice that the volume of interference takes shape and size characteristics from each of the two initial objects.

When two objects are **joined**, the volume of interference is absorbed into the combined object. The result is a single object that does not have “double” volume in the region of interference. The Boolean Join operation is illustrated in Figure 3.41.

When two objects are combined by **intersecting**, the combined object that results from the intersection is the volume of interference between them, as shown in Figure 3.42.

In the **cutting** of two objects, the combined object that results from the cutting depends on which object serves as the cutting tool and which object is cut by the other object. The result of a cutting operation is that the volume of interference is removed from the object that is cut, as illustrated in Figure 3.43.

**FIGURE 3.40.** Overlapping objects and volume of interference.

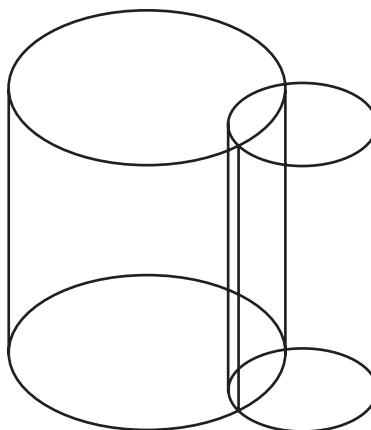


Overlapping  
Objects

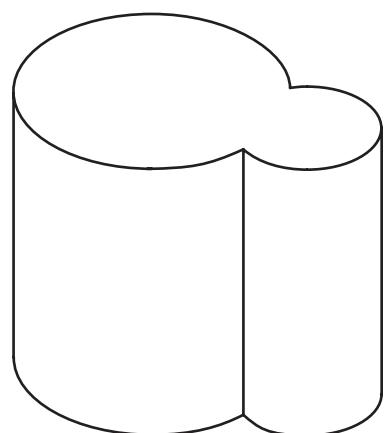
Volume of  
Interference

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**FIGURE 3.41.** Result of two objects joined.



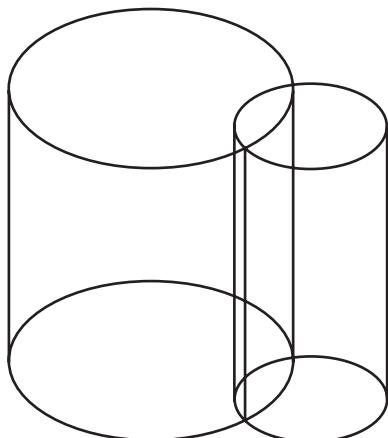
Overlapping Objects



Objects Joined

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**FIGURE 3.42.** Result of two objects intersected.

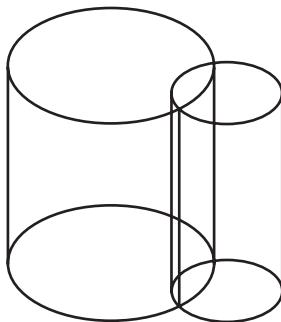


Overlapping Objects

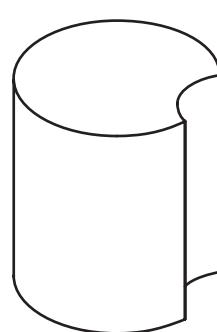
Objects Intersected

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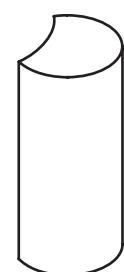
**FIGURE 3.43.** Result of two objects cutting.



Overlapping Objects



Small Cylinder Cuts Large Cylinder



Large Cylinder Cuts Small Cylinder

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## 3.13 Chapter Summary

In this chapter, you learned about Gardner's definitions of basic human intelligences (including spatial intelligence) and the way spatial intelligence is developed and assessed. Spatial intelligence is important for engineering success, especially in engineering graphics and solid modeling courses. The chapter outlined several exercises that help develop spatial skills, including:

- Constructing isometric sketches from different corner views.
- Rotating 3-D objects about one or more axes.
- Reflecting objects across a plane and recognizing planes of symmetry.
- Defining cross sections obtained between cutting planes and objects.
- Combining two objects to form a third object by cutting, joining, or intersecting.

### 3.14

### GLOSSARY OF KEY TERMS

**Boolean operations:** In early versions of 3-D CAD software, commands used to combine solids.

**combining solids:** The process of cutting, joining, or intersecting two objects to form a third object.

**corner views:** An isometric view of an object created from the perspective at a given corner of the object.

**cross section:** The intersection between a cutting plane and a 3-D object.

**cut:** To remove the volume of interference between two objects from one of the objects.

**cutting plane:** An imaginary plane that intersects with an object to form a cross section.

**intersect:** To create a new object that consists of the volume of interference between two objects.

**join:** To absorb the volume of interference between two objects to form a third object.

**mental rotations:** The ability to mentally turn an object in space.

**reflection:** The process of obtaining a mirror image of an object from a plane of reflection.

**spatial orientation:** The ability of a person to mentally determine his own location and orientation within a given environment.

**spatial perception:** The ability to identify horizontal and vertical directions.

**spatial relations:** The ability to visualize the relationship between two objects in space, i.e., overlapping or nonoverlapping.

**spatial visualization:** The ability to mentally transform (rotate, translate, or mirror) or to mentally alter (twist, fold, or invert) 2-D figures and/or 3-D objects.

**symmetry:** The characteristic of an object in which one half of the object is a mirror image of the other half.

**volume of interference:** The volume that is common between two overlapping objects.

### 3.15

### QUESTIONS FOR REVIEW

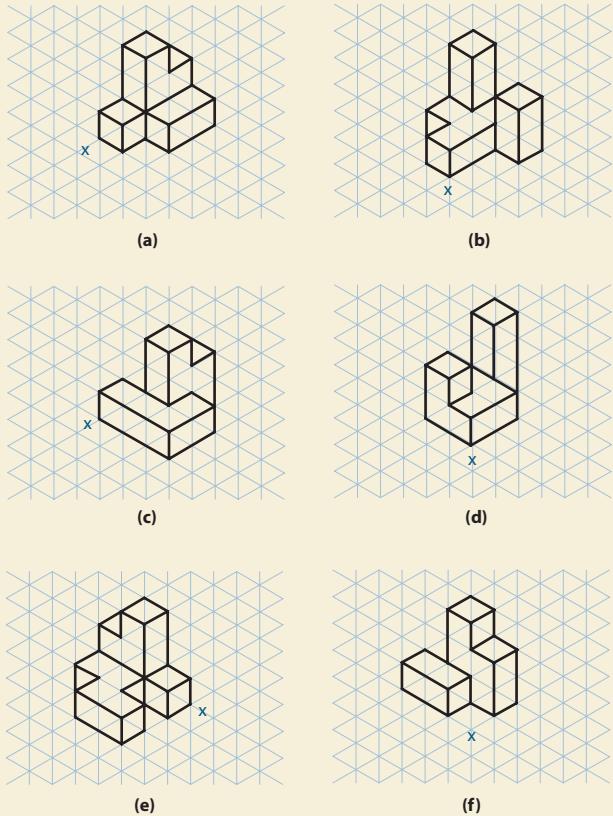
1. What are some of the basic human intelligences as defined by Gardner?
2. What are the stages of development for spatial intelligence?
3. What are some of the basic spatial skill types?
4. What do the numbers on a coded plan represent?
5. What are some general rules to follow when creating isometric sketches from coded plans?
6. When a person is looking down a coordinate axis, are positive rotations CW or CCW?
7. Describe the right-hand rule in your own words.
8. Are object rotations about two or more axes commutative? Why or why not?
9. What is one difference between object reflection and object symmetry?
10. Are all objects symmetrical about at least one plane? Explain.

- 11.** The shape of a cross section depends on two things. Name them.
- 12.** What is the effect on the resulting cross section of a cutting plane that is tilted?

- 13.** What are the three basic ways to combine solids?
- 14.** In the cutting of two objects, does it matter which object is doing the cutting?

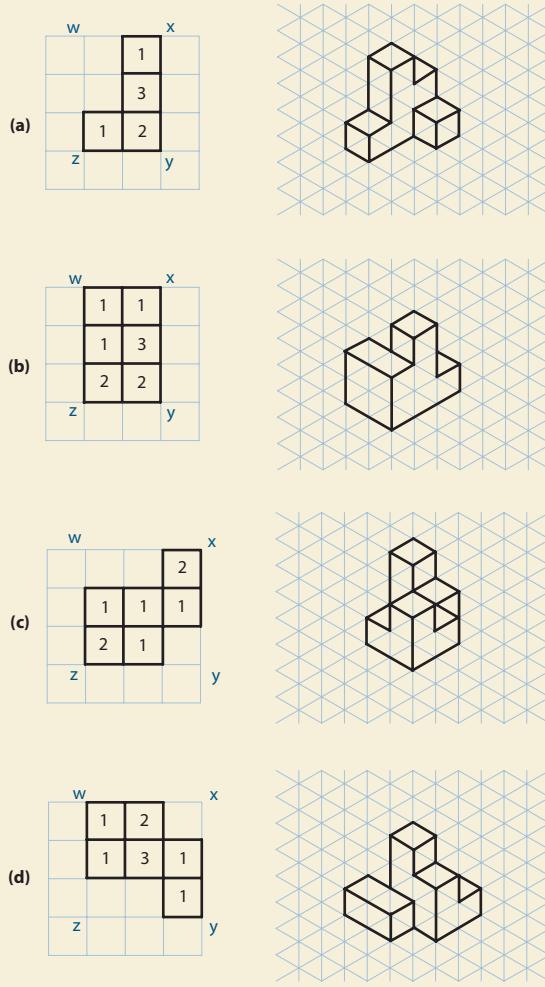
### 3.16 PROBLEMS

- 1.** For the following objects, sketch a coded plan, labeling the corner marked with an x properly.
- 2.** Indicate the coded plan corner view that corresponds to the isometric sketch provided.



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**FIGURE P3.1.**



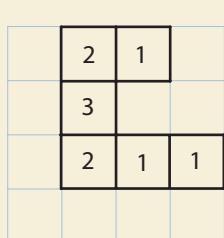
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**FIGURE P3.2.**

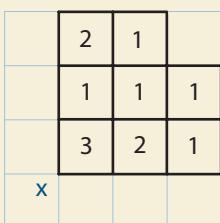
## 3.16

## PROBLEMS (CONTINUED)

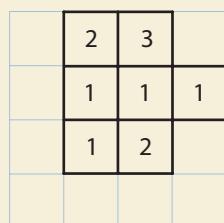
3. Use isometric grid paper to sketch the indicated corner view (marked with an *x*) for the coded plan.



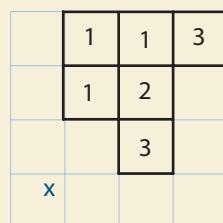
(a)



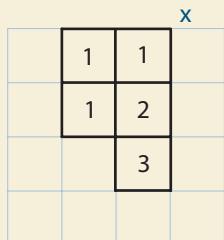
(b)



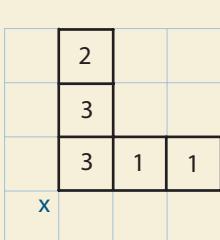
(c)



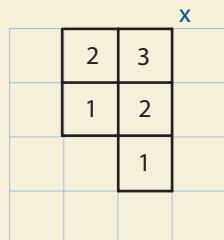
(d)



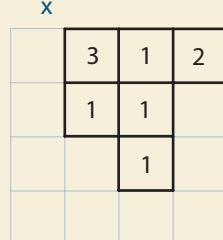
(e)



(f)



(g)



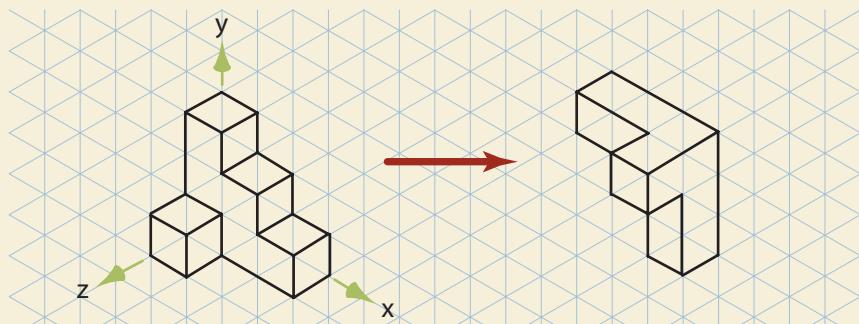
(h)

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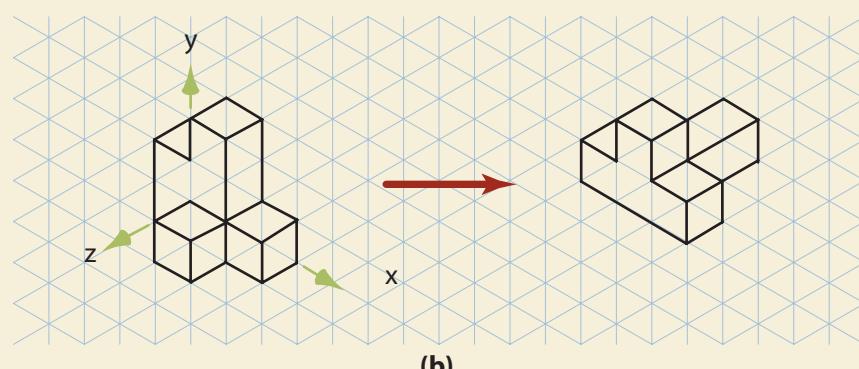
**FIGURE P3.3.**

## 3.16 PROBLEMS (CONTINUED)

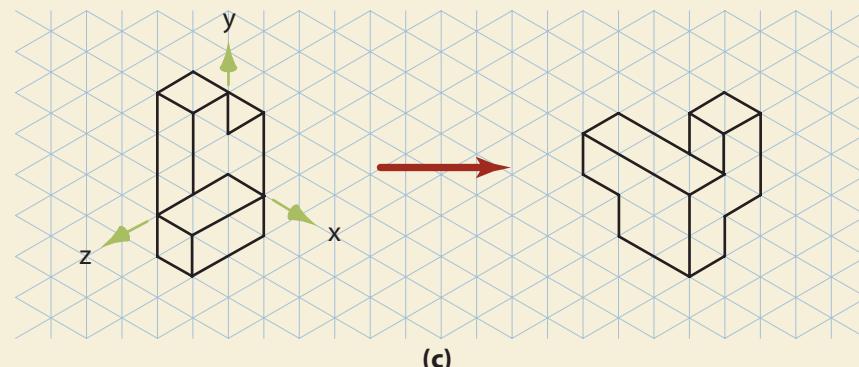
4. Using the notation developed in this chapter, indicate the rotation the following objects have experienced.



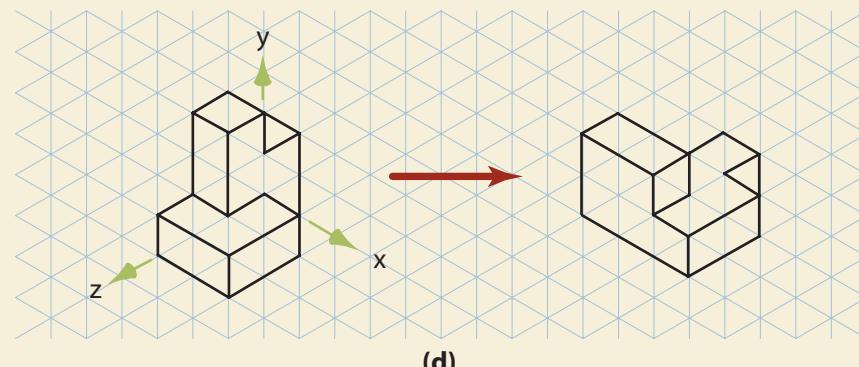
(a)



(b)



(c)



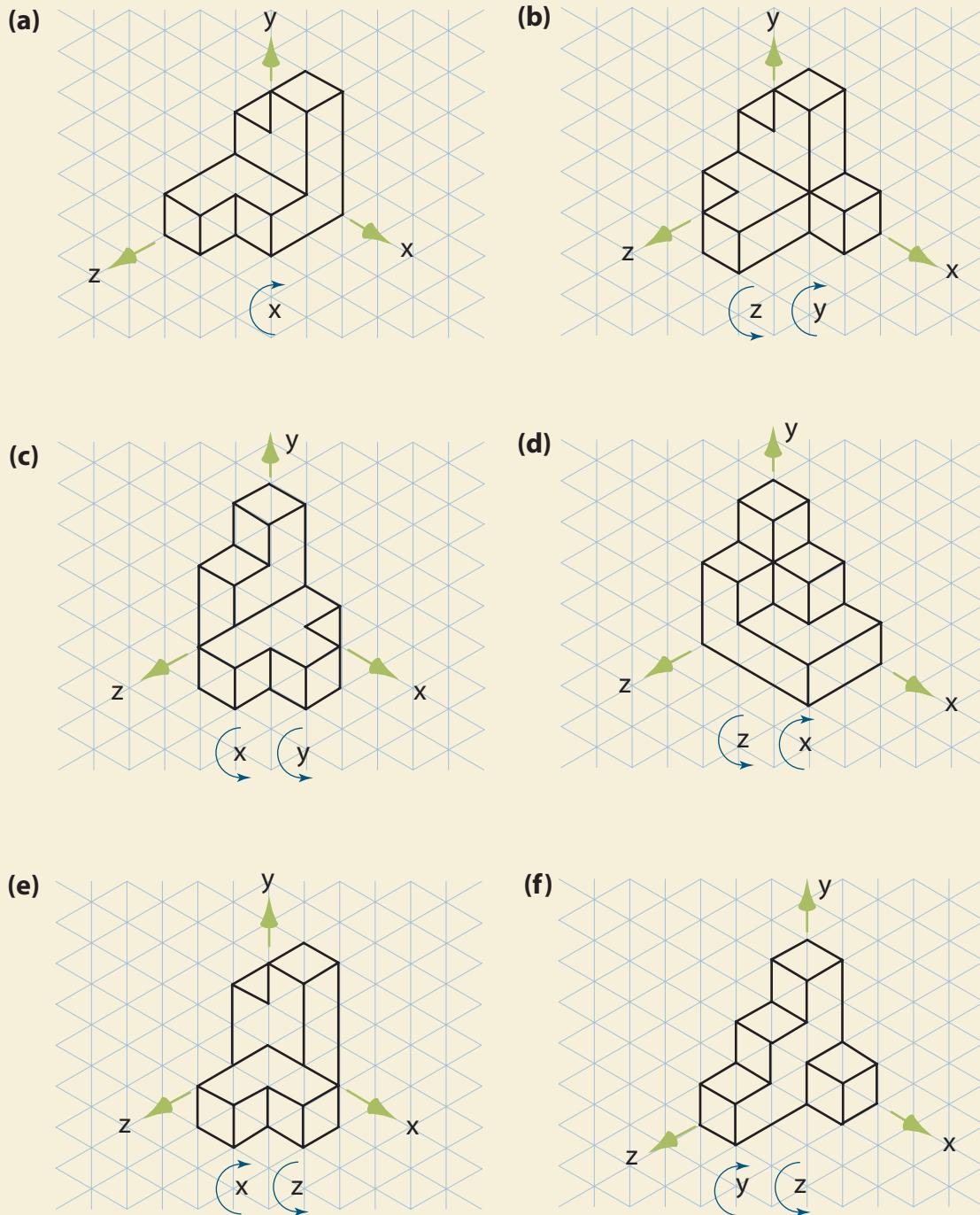
(d)

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**FIGURE P3.4.**

## 3.16 PROBLEMS (CONTINUED)

5. Rotate the following objects by the indicated amount and sketch the results on isometric grid paper.

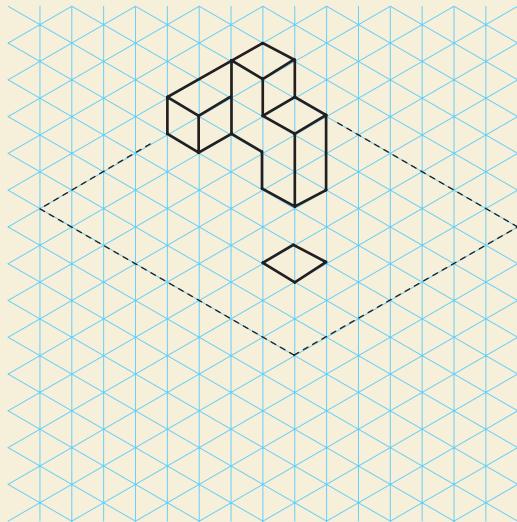


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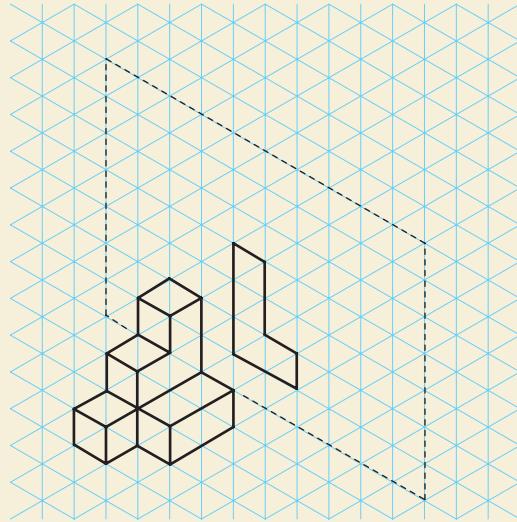
**FIGURE P3.5.**

## 3.16 PROBLEMS (CONTINUED)

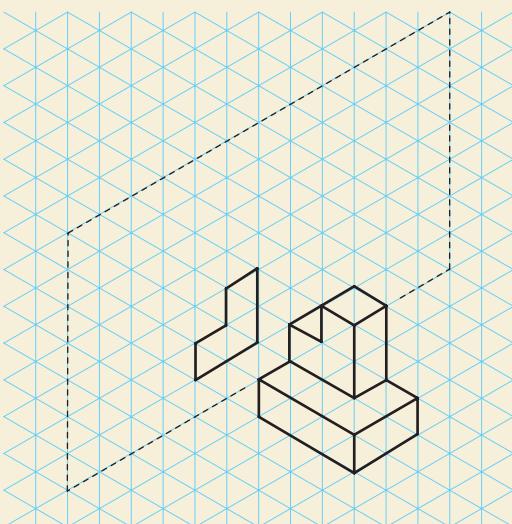
6. Copy the following object on isometric grid paper and sketch its reflection across the indicated plane. Note that the sketch of the reflection has been started for you.



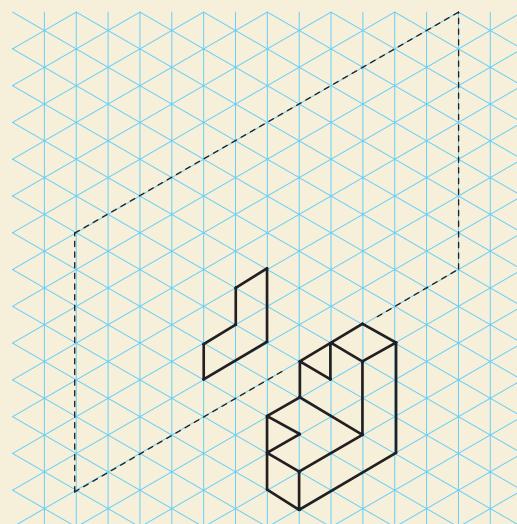
(a)



(b)



(c)



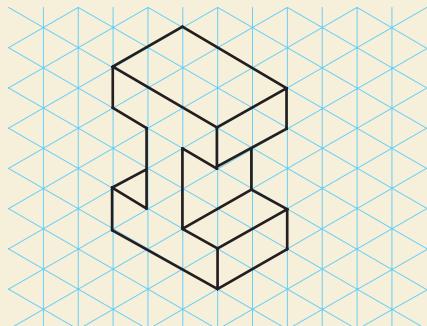
(d)

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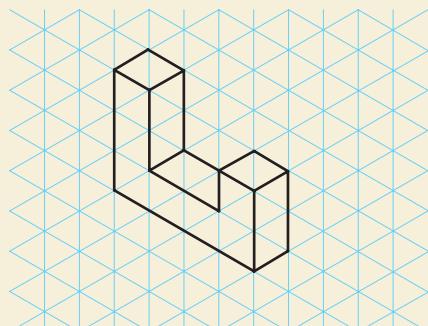
**FIGURE P3.6.**

## 3.16 PROBLEMS (CONTINUED)

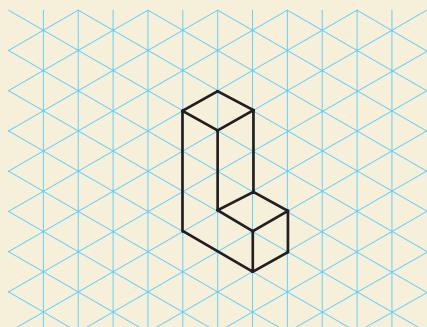
7. How many planes of symmetry does each of the following objects have?



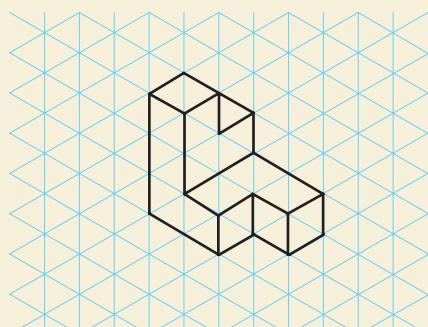
(a)



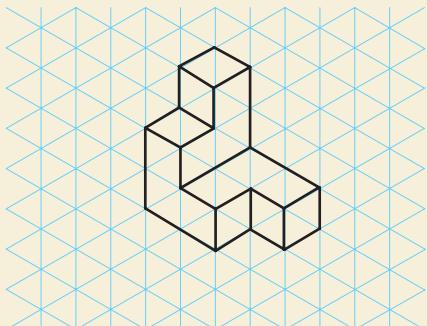
(b)



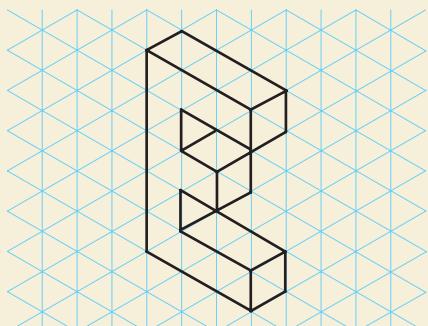
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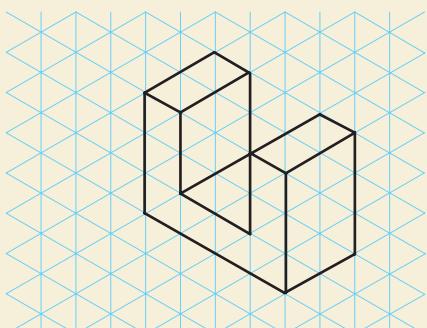
(d)



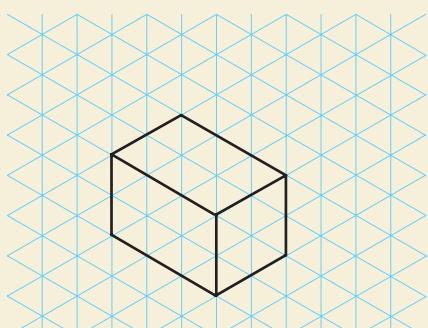
(e)



(f)



(g)



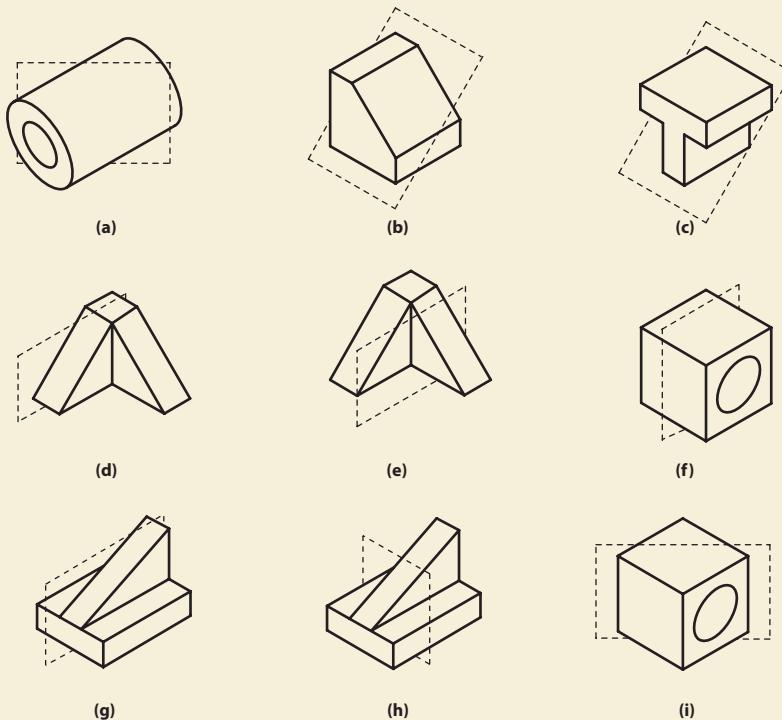
(h)

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**FIGURE P3.7.**

## 3.16 PROBLEMS (CONTINUED)

8. Sketch the cross section obtained between the intersection of the object and the cutting plane.

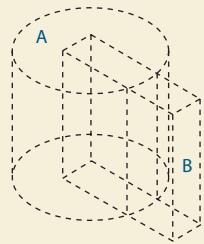


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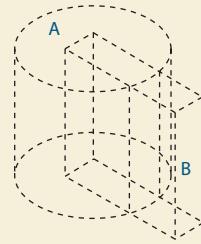
**FIGURE P3.8.**

9. Sketch the result of combining the following objects by the indicated method.

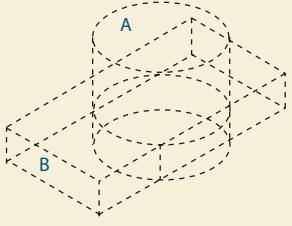
(a) A joined with B



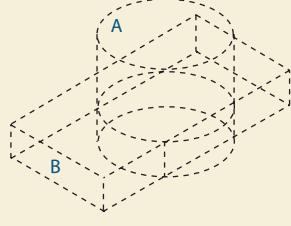
(b) B cuts A



(c) Intersection of A and B



(d) A cuts B

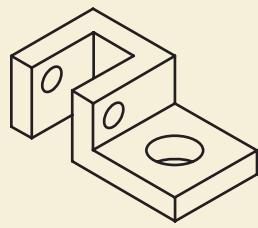


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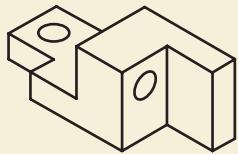
**FIGURE P3.9.**

**3.16 PROBLEMS (CONTINUED)**

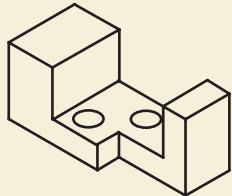
- 10.** Describe by words and sketches how you would create the following objects by combining basic 3-D shapes.
- 11.** Create isometric sketches from these coded plans using the corner view that is circled or the corner prescribed by your instructor.



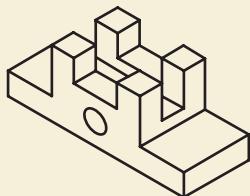
(a)



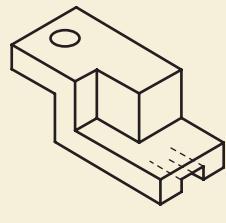
(b)



(c)

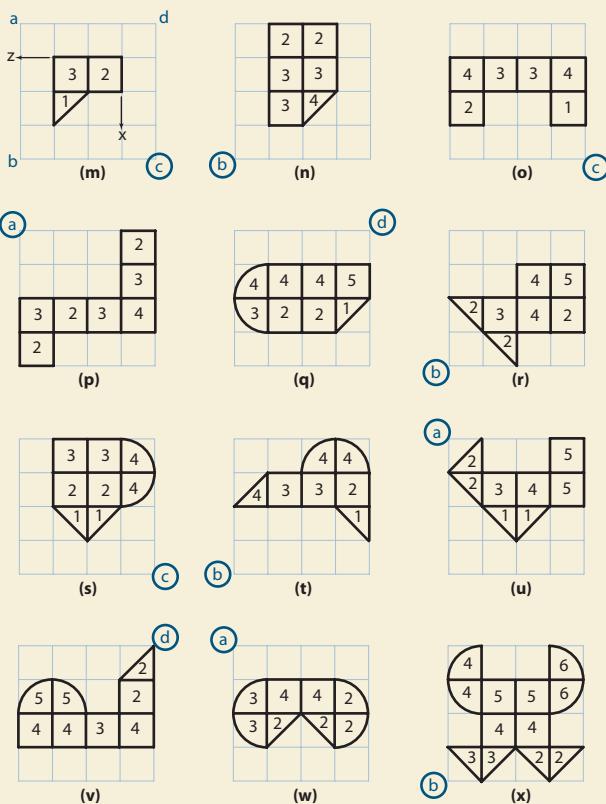


(d)

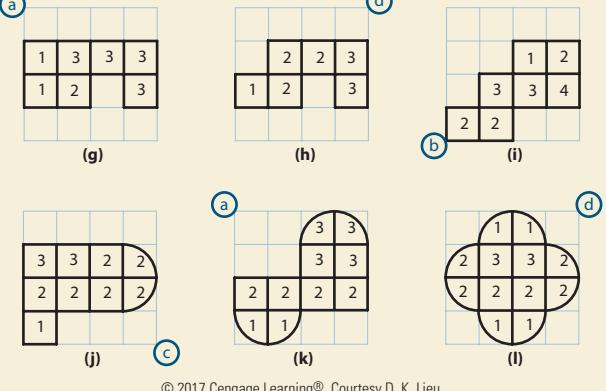
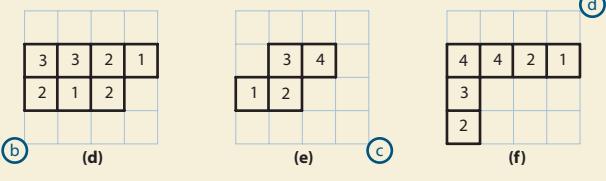
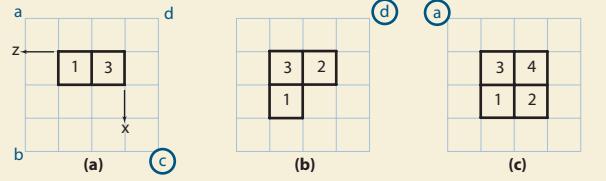


(e)

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**FIGURE P3.10.**

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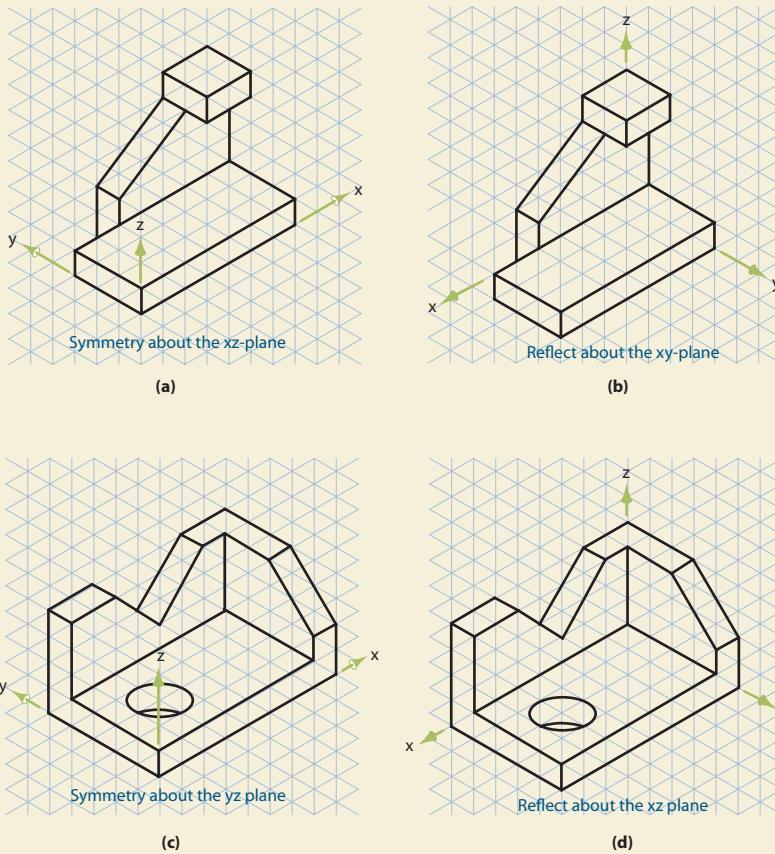


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**FIGURE P3.11.**

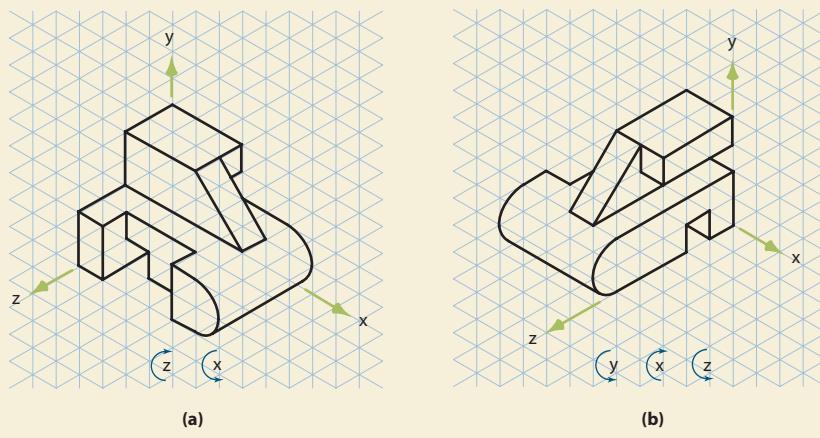
## 3.16 PROBLEMS (CONTINUED)

12. Add the reflected images or redraw these objects with symmetry using the  $xy$ -,  $yz$ -, or  $xz$ -planes as indicated or the planes prescribed by your instructor.
13. The object shown in (a) is show again in (b) rotated by  $-90^\circ$  degrees about the  $y$ -axis to reveal more detail. Rotate the object sequentially about the axes indicated or about the axes prescribed by your instructor.



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**FIGURE P3.12.**

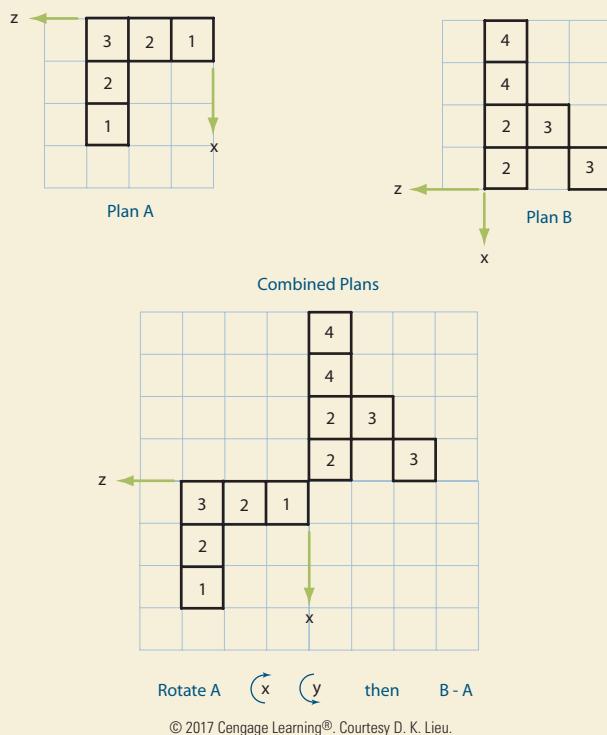


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**FIGURE P3.13.**

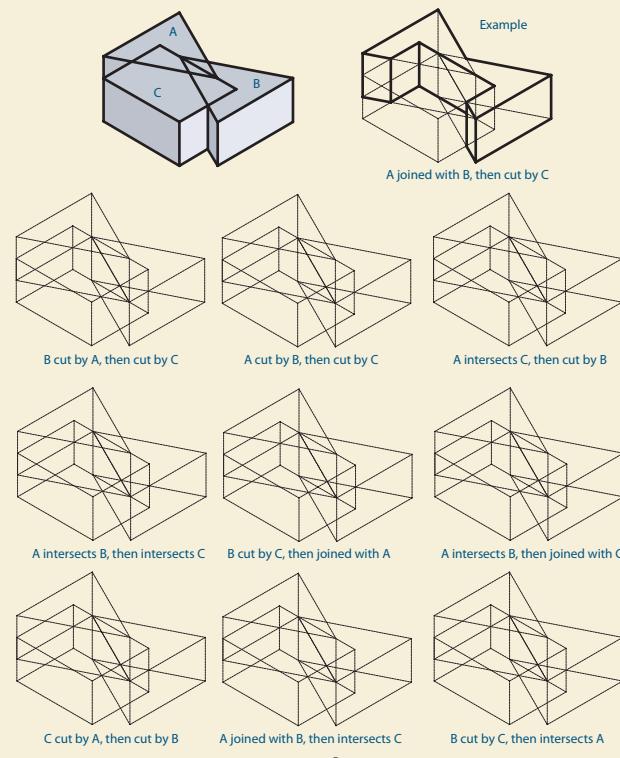
**3.16 PROBLEMS (CONTINUED)**

14. Create an isometric sketch of the objects created from coded plans A and B. Rotate object A sequentially about the axes indicated or about the axes prescribed by your instructor. Show the new object created by the indicated Boolean combination of object A and object B or the Boolean operation prescribed by your instructor when the coordinate axes of A and B are aligned.
15. Triangular volume A, triangular volume B, and rectangular volume C are shown intersecting in space. On the dashed outline drawings, darken and add edges to show all visible edges of the final volume created by the indicated Boolean operations.



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**FIGURE P3.14.**



**FIGURE P3.15.**