$$(x^2 D^2 + p x D + q)\, y = r(x)$$

$x^2 D^2 \quad , \quad (xD)^2$

$$D^m e^{ax} = a^m e^{ax}$$

$$P(D)\, e^{ax} = P(a)\, e^{ax}$$

$(D-a)\, x e^{ax}$

$= e^{ax}$

$(xD-a)(\ln x)\, x^a$

$= x^a$

$f:(0,\infty) \to \mathbb{R}, \quad L(f)(s) = \int_0^\infty e^{-st} f(t)\, dt, \quad s>0$

(1) Linearity

(2) Shifting thm $\quad L\left(e^{at}f(t)\right) = F(s-a)$

$$L\left(t^n e^{at}\right) = \frac{n!}{(s-a)^{n+1}}$$

$$L(\cosh t \cos at) = L\left(\frac{e^{at}+e^{-at}}{2}\cos at\right)$$

$$L\left(e^{-t}\sin^2 t\right) = L\left(e^{-t}\,\frac{1-\cos 2t}{2}\right)$$

(3) Scaling $\quad L\left(f(ct)\right) = \frac{1}{c}F\left(\frac{s}{c}\right)$

$L(f)(s)$

$= F(s)$

$e^{-\frac{s\varepsilon}{c}}$

e^{-st}

$F\left(\frac{s}{c}\right)$

$$L(e^{at}) = \frac{1}{a} \frac{1}{\frac{s}{a} - 1}$$

(4) $$L(f') = sL(f) - f(0)$$

$$L(f^n) = s^n L(f) - s^{n-1} f(0) \ldots - f^{n-1}(0)$$