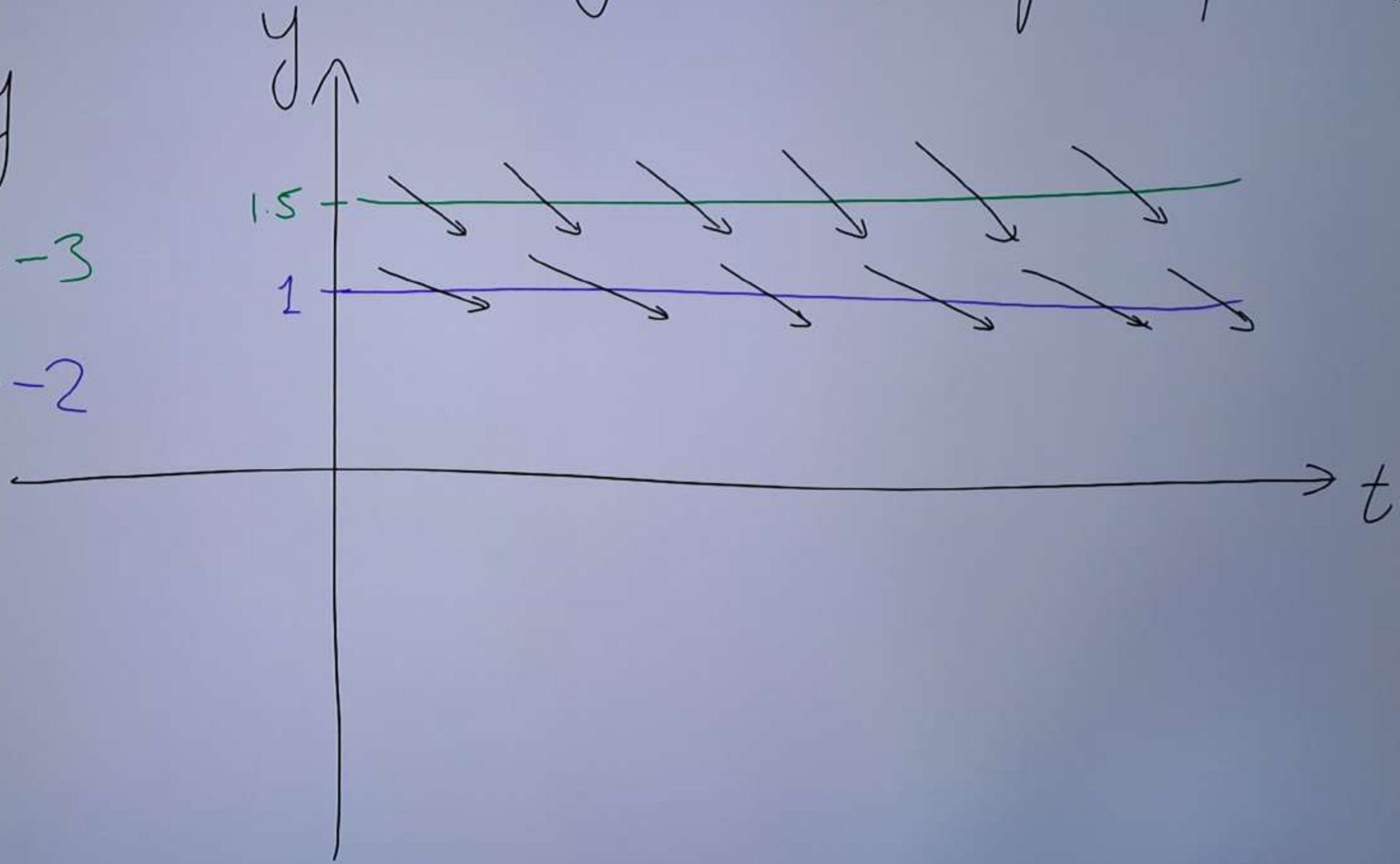
direction field (slope field)

$$\frac{dy}{dt} = -2y$$



$y = 1.5$, slope $= -3$

$y = 1$, slope $= -2$

$$\frac{(1+2y^2)\,dy}{y} = \cos x \, dx$$

$$w(x) = \frac{G M_e m}{(R+x)^2}$$

$$\frac{G M_e m}{R^2} = mg$$

$$\boxed{\cancel{m}\frac{dv}{dt} = -\frac{\cancel{m} g R^2}{(R+x)^2}}$$

$$v(0) = v_0$$

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

$$\boxed{v\frac{dv}{dx} = -\frac{gR^2}{(R+x)^2}}$$

$$v\,dv = -\frac{gR^2}{(R+x)^2}dx \Rightarrow \frac{v^2}{2} = \frac{gR^2}{(R+x)} + c$$

So, $$\frac{v_0^2}{2} = gR + c \Rightarrow c = \frac{v_0^2}{2} - gR$$

Hence $$v = \sqrt{\frac{2gR^2}{(R+x)} + v_0^2 - 2gR}$$

$$0 = \frac{2gR^2}{R+H} + v_0^2 - 2gR \Rightarrow v_0^2 = 2g\left(R - \frac{R^2}{R+H}\right)$$

$$= 2g\left(\frac{RH}{R+H}\right)$$

$$v_e^2 = 2gR \Rightarrow v_e = \sqrt{2gR}$$

$$y' = 3y^{2/3} \Rightarrow \frac{dy}{y^{2/3}} = 3\,dx$$

$$\Rightarrow 3y^{\frac{1}{3}} = 3x + 3c$$

$$\Rightarrow \boxed{y = (x+c)^3}$$

$$\boxed{y \equiv 0}$$

$$y(0) = 0$$

$$\boxed{y = x^3}$$

$$\phi_k(x) = \begin{cases} 0 & -\infty < x \leq k \\ (x-k)^3 & k < x < \infty \end{cases}$$

$$k \geq 0$$

$$f(tx_1, tx_2, \ldots, tx_n) = t^d f(x_1, x_2, \ldots, x_n)$$

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0, \quad y = vx$$

$$M(x, vx) + N(x, vx)\left(v + x\frac{dv}{dx}\right) = 0$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow \cancel{x^d} M(1, v) + \cancel{x^d} N(1, v)\left(v + x\frac{dv}{dx}\right) = 0$$

$$(y^2 - x^2)\frac{dy}{dx} + 2xy = 0 \qquad y = vx$$

$$\Rightarrow (v^2 - 1)\left(v + x\frac{dv}{dx}\right) + 2v = 0 \Rightarrow (v^3 - v) + (v^2x - x)\frac{dv}{dx} + 2v = 0$$

$$\Rightarrow (v^3 + v) + x(v^2 - 1)\frac{dv}{dx} \Rightarrow \frac{dx}{x} + \frac{(v^2 - 1)}{(v^3 + v)}dv = 0$$

$$\frac{v^2-1}{v(v^2+1)} = \frac{a+bv}{v^2+1} + \frac{c}{v}$$

$$v^2-1 = (a+bv)v + c(v^2+1)$$

$$\Rightarrow \quad c = -1, \quad 1 = (b+c), \quad 0 = a$$

$$b = 2$$

$$\int\left(\frac{2v}{v^2+1} - \frac{1}{v}\right) dv + \int \frac{dx}{x} = c_1$$

$$\Rightarrow \quad \ln|v^2+1| - \ln|v| + \ln|x| = \ln|c|$$