# Second Order linear diff. eq$^n$s.

Existence & uniqueness for IVP.

$$y'' + py' + qy = 0, \quad y(x_0) = a, \; y'(x_0) = b$$

$p$ & $q$ are continuous on an open interval $I \ni x_0$.

$$E(x) = y^2 + (y')^2 \quad \Big/ \quad E'(x) \leq (1+3M)E(x)$$

$y'' + p y' + q y = 0$. Let $y_1$ & $y_2$ be sd$^n$

(1) $y_1$ & $y_2$ lin dep. iff $\exists x_0$ s.t. $W(x_0) = 0$

p iff q

(2) $q \Rightarrow p \overset{\text{proof of}}{\Rightarrow} q$ $\quad W(x) = 0 \ \forall x$

(3) First part of the proof of (1)

$p \Rightarrow q$ $\quad y_1$ & $y_2$ l.d $\Rightarrow W(x) = 0$ $\forall x$

$\sim q \Rightarrow \sim p$

$$x^2, \quad x^3$$

$$y' = 2x, \quad y'' = 2$$

$$x^2(2) - 4x(2x) + 6x^2 = 0$$

$$W(x^2, x^3)(0) = \begin{vmatrix} x^2 & x^3 \\ 2x & 3x^2 \end{vmatrix}(0) = 0$$

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$$y'' + py' + qy = 0$$

$$y'' - \frac{4}{x}y' + \frac{6}{x^2}y = 0$$

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = 0$$

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$$c_1 y_1 + c_2 y_2 = 0$$

$$\left.\begin{aligned} x_1 = 1, \qquad & c_1 + c_2 = 0 \\ x_2 = -1, \qquad & c_1 - c_2 = 0 \end{aligned}\right\} \quad c_1 = c_2 = 0$$

$$x^2 y'' - 4xy' + 6y = 0$$

$$y_1 = x^2, \quad y_2 = x^3, \quad y_3 = \begin{cases} x^3, & x \geq 0 \\ -x^3, & x < 0 \end{cases}$$

$$x^2 y'' - 6xy' + 12y = 0$$

$x^3, x^4$

$$W(x_0) = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$y_1, y_2$ are lin ind sol$^n$s

Let $Y$ be another sol$^n$.

$$Y(x) = c_1 y_1(x) + c_2 y_2(x)$$

$x_0 \in I$

$$\begin{cases} Y(x_0) = c_1 y_1(x_0) + c_2 y_2(x_0) \\ Y'(x_0) = c_1 y_1'(x_0) + c_2 y_2'(x_0) \end{cases}$$

$$u(x_0) = u'(x_0) = 0$$

$$y'' + py' + y = 0$$

$$y'' + p y' + q y = 0$$

$y_1$ known , $y_2(x) = v(x)\, y_1(x)$

$$(v y_1)'' + p (v y_1)' + q (v y_1) = 0$$

$$v = \int \frac{e^{-\int p\, dx}}{y_1^2}\, dx$$

$$y_2 = y_1 v$$
$$= x\left(x - \frac{1}{x}\right)$$
$$= x^2 - 1$$