$$f : (0, \infty) \to \mathbb{R}$$

$$F(s) = \mathcal{L}(f)(s) = \int_0^\infty e^{-st} f(t)\, dt, \qquad s > 0.$$

1) Linearity

2) $\mathcal{L}(e^{at} f(t)) = F(s-a)$

3) $\mathcal{L}(f(ct)) = \frac{1}{c} F\left(\frac{s}{c}\right)$

4) $\mathcal{L}(f') = s\mathcal{L}(f) - f(0)$

$$\mathcal{L}(f^{(n)}) = s^n \mathcal{L}(f) - s^{n-1} f(0) - \dots - f^{(n-1)}(0)$$

$$y'' + 2y' + y = \cos x, \quad y(0) = 1$$

$$y'(0) = 0$$

$$L(y'') + 2L(y') + L(y) = L(\quad)$$

$$L(y) = \frac{s}{(s+1)^2}$$

$$s^2 L(y) - s f(0) - f'(0)$$

$$(s^2 L(y) - s + 0) - (s L(y) - 1) - 2L(y) = 0$$

$$(s^2 - s - 2) L(y) = s - 1$$

$$L(e^{2t}) = \frac{1}{s-2}$$

$$L(y) = L\left(\frac{1}{3}e^{2t} + \frac{2}{3}e^{-t}\right)$$

Apply Lerch thm

$$m^2 - m - 2 = 0$$

$$L(\sin at) = \frac{a}{s^2+a^2}, \quad L(\cos at) = \frac{s}{s^2+a^2}$$

$$s^2 - 2s + 5 = (s-1)^2 + 4$$

$$L^{-1}\left(\frac{s-1}{(s-1)^2+2^2}\right) = e^t \cos 2t$$

$$L\left(\int_0^t f(\tau)\,d\tau\right) = \frac{F(s)}{s}$$

$$F(s) = L(f), \quad |g(t)| \leq \int_0^t |f(\tau)|\,d\tau \leq \int_0^t K e^{\alpha\tau}\,d\tau$$

$$= \frac{K}{\alpha}\left(e^{\alpha t} - 1\right)$$

$$L\left(\int_0^t f(\tau)\,d\tau\right) = L(g) = \frac{L(f)}{s} = \frac{F(s)}{s}$$

$$L(f') = sL(f) - f(0)$$

$$L\left(\int_0^t f(\tau)\,d\tau\right) = \frac{F(s)}{s}$$

$$L\left(tf(t)\right) = -F'(s)$$