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Assigment 2:
Numerical Integration

1. What is need of numerical integration? & Explain various situation where such need arises

Ans whenever antiderivative is known, infinite integral is known.
The difficulty arises in computing definate integral in many situation for different reasons

Situation :

Evaluation $\int_{a}^{b} f(x)\,dx$ is anti-derivative

i) $\int_{0}^{1} e^{x^2}$ & 2) $\int_{1} \frac{dx}{\log x}$

Many such examples are seen in statistical and engineering problems.

$$H(x) = \frac{4 l \lambda}{l^2 - x^2} \int_{0}^{\pi/2} \left(1 - \left(\frac{x}{l}\right)^2 \sin^2\theta\right)^{\frac{1}{2}} d\theta$$

If x, l and l given, the integral occuring in equation is called integral and can't be expressed in term of standard form

Situation 2:

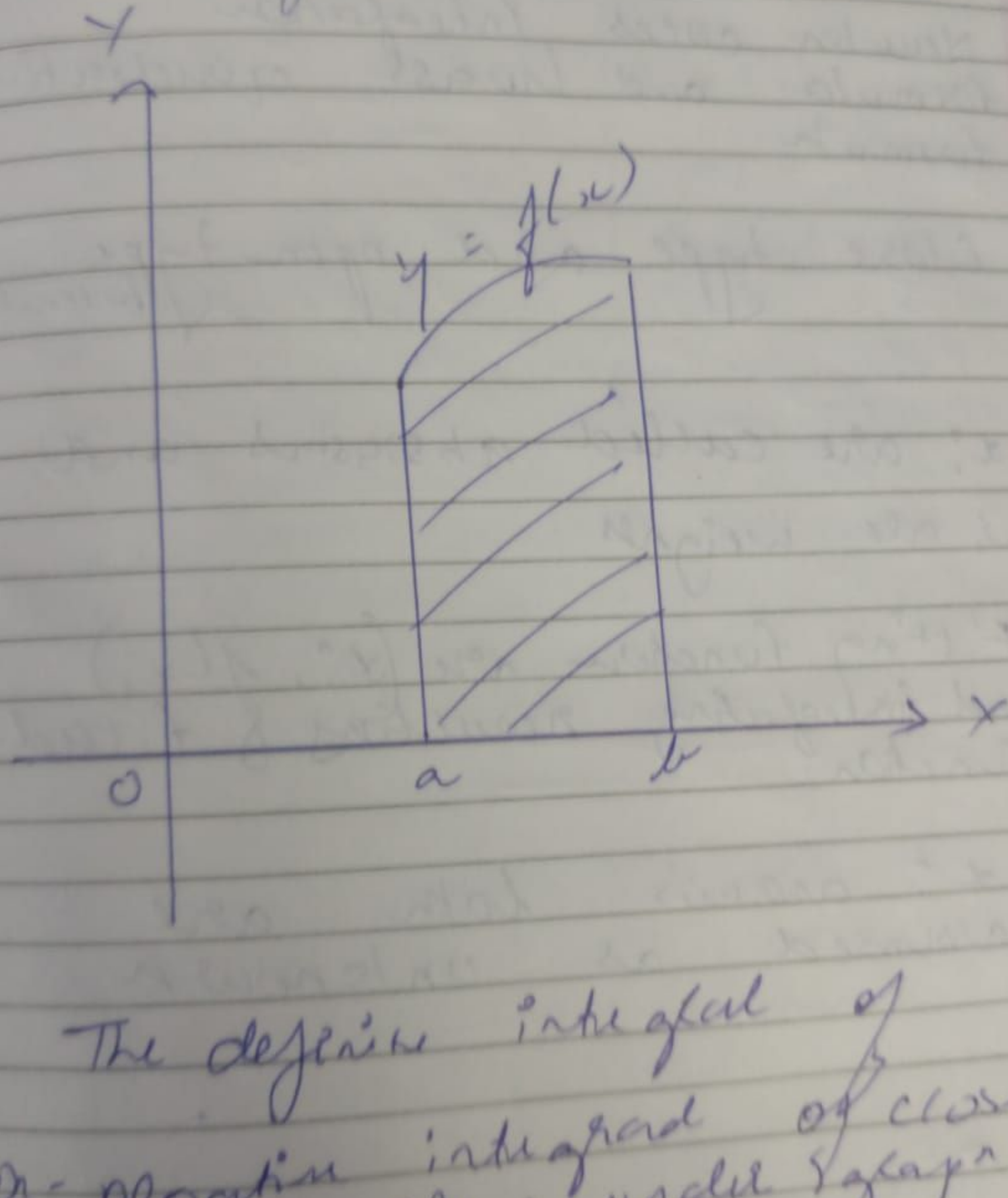
Rather than evaluate antiderative a typical situation would be full expression is in the form.

Situation 3

Expression of function to be integrated is not available

Example, speed of an object may have been measured at different time intervals and distance needs to be computed.

Q2 Give geometrical interpretation with $f(x) \geq 0$ and $a \leq x \leq b$



The definite integral of non-negative integrand of closed interval is area under graph of f.

Q3 Differenciate:
i

(a) Newton cotes integration formula and Guass quadrati-c formula

(b) Close type and open type formulas

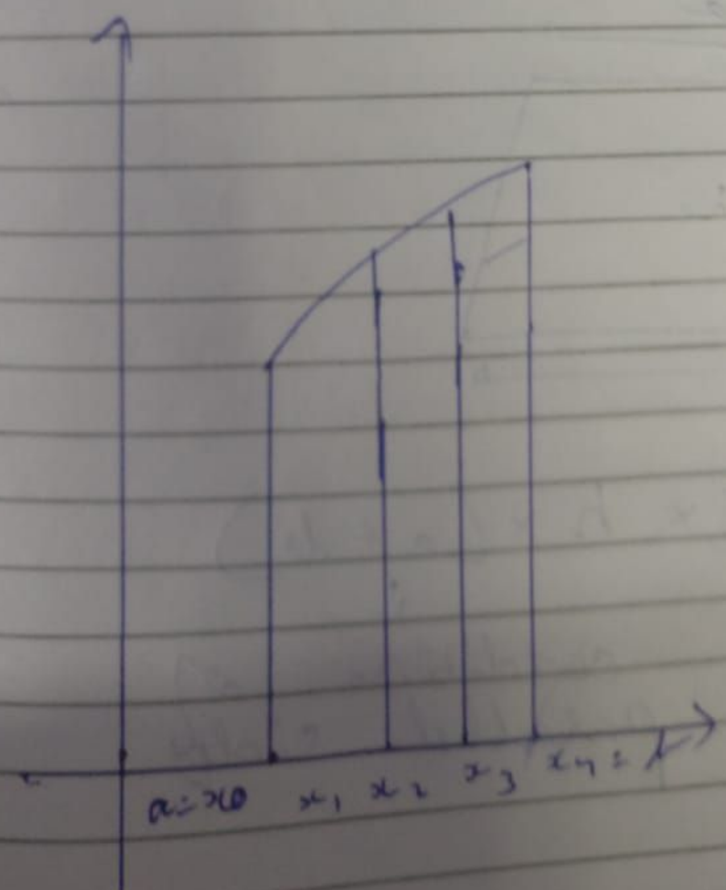
(a) x_i are called absissas and w_i are weights
Fitting function are $(x_i, f(x_i))$ and integrating resulting of fitted function.

(b) x_i and w_i's both are assumed as unknown

Integration Formula

x_i' fixed w_i unknown Newton cotes integration formula	x_i's unknown w_i's unknown Guass Quadratic formula

If $a < x_0 < x_1 < x_2 \cdots < x_{n-1} < x_n < b$
integration is open type integration



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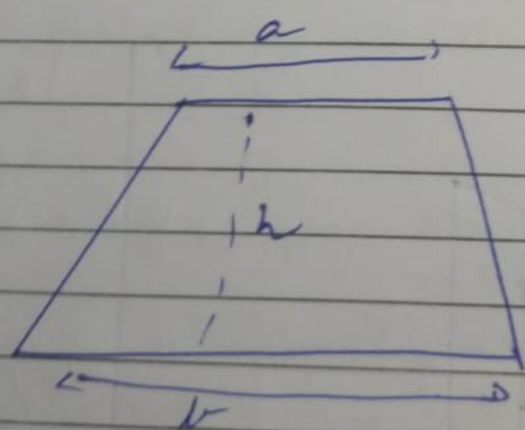
PAGE NO.: DATE

Q4 Explain trapezodial rule for estimating

Ans $\int_a^b f(x)\,dx = \frac{\Delta x}{2}\left[f(a) + f(b)\right]$

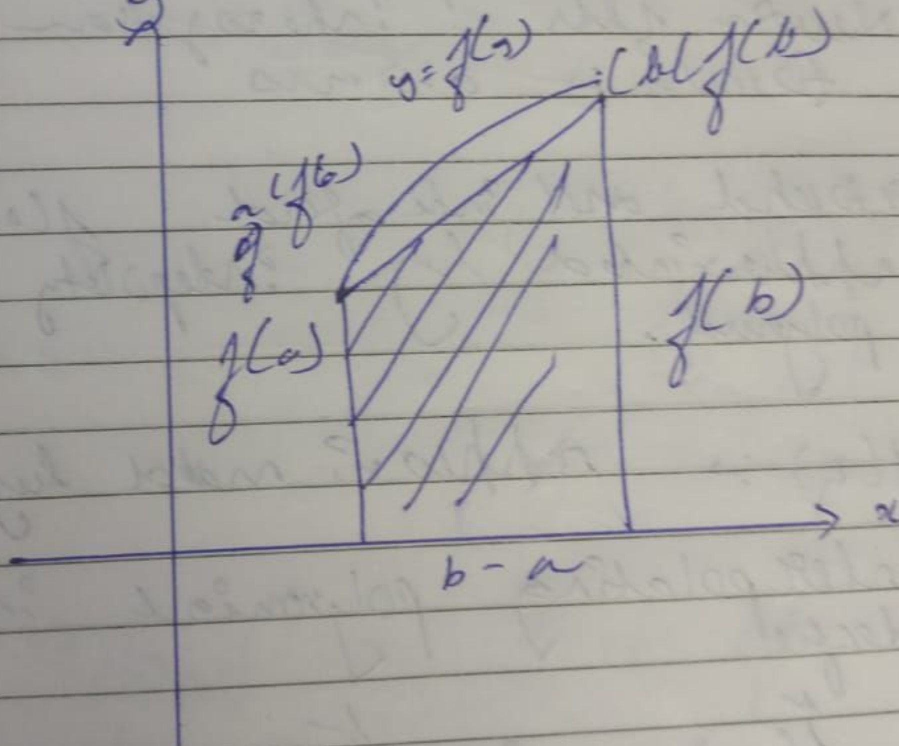
→ This rule is trapezodial rule as it gives area of trapezium formed from joining points by straight line and vertical lines $x=a$ $x=b$



Area $= \frac{1}{2} \times h \times (a+b)$

addition of parallel sides

So if we join points a(f a)
and b(f b) it forms a trapezium



Q5. Explain basic principle in deriving Newton Gate integration formula.

Ans Basic principle in obtaining Newton gate interogation formula is obvious

befored and integrad $f(x)$ is approximated by interpolating polynomial.

$f(x)$ is approximated by interpolating polynomial in case of degree

$$\int_a^b f(x)\,dx \cong \int_a^b (P_n(x)\,dx)$$

NEW INDIA PAGE NO.: DATE

$$\int_a^b P_n(x)\,dx = \int_a^b \left(\sum_{i=0}^{n} l_i(x)\, y_i\right) dx$$

$$= \sum_{i=0}^{n} \left(\int_a^b l_i(x)\,dx\right) y_i$$

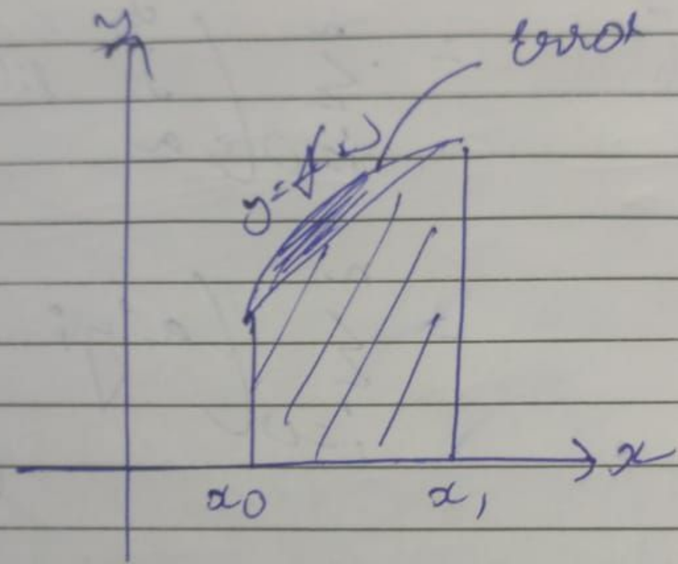
$$= \sum_{i=0}^{n} \left(a_i y_i \right. \text{; giving}$$

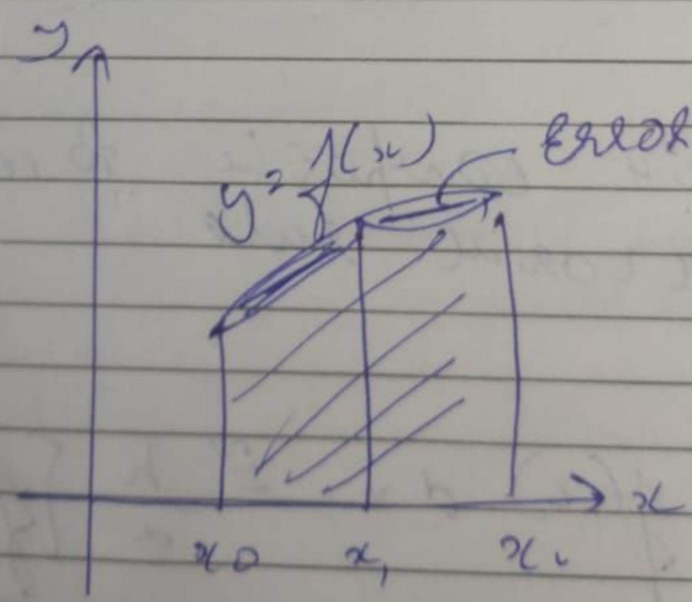
$$\left(a_i = \int_a^b l_i(x)\,dx\right)$$

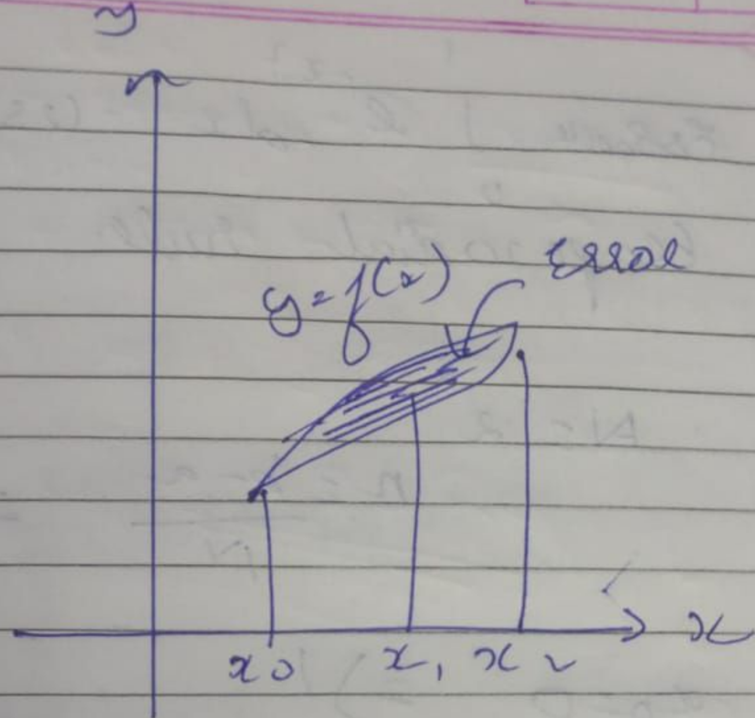
Q6 Write composite form of trapezoidal rule.

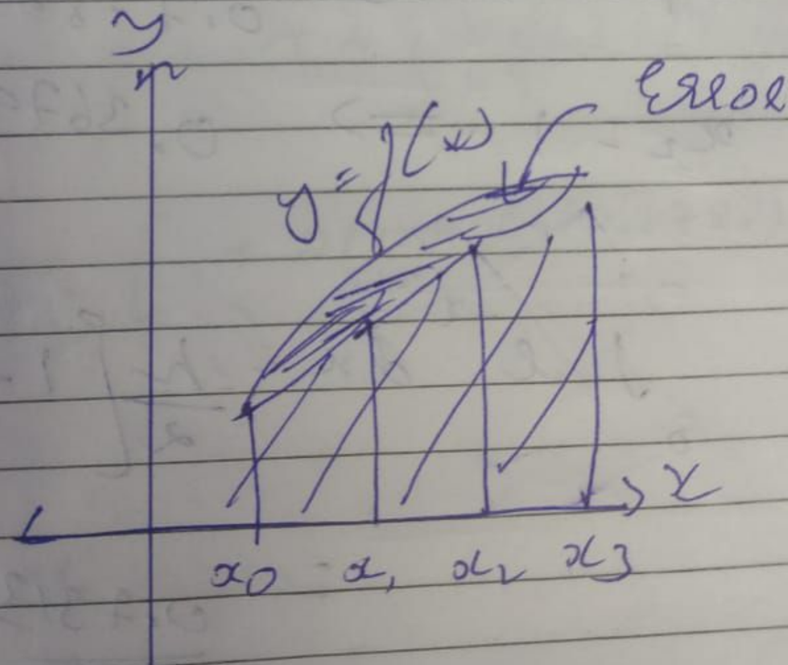
Ans $$\int_a^b f(x)\,dx \cong \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{N} f(x_i) + f(x_N)\right]$$

In following figure, intervals are shown reduces truncation error.






y
y = f(x)
Error
x
x0
x1, x2


y
y = f(x)
Error
x
x0
x1, x2, x3

$$= \frac{0.1667}{2}\left[1+2\left(\cancel{.992}^{\,9726} + .8948 + .7787 + 0.6412 + .4994 + .3679\right.\right.$$

$$= 0.8334\,[\cancel{0.89}\ 8.9415]$$

$$= \underline{\underline{0.7452}}$$

Q8 Calculate the integral value of $\int_0^2 f(x)\,dx$ when $f(x)$ is x^2 using (2) Trapezoidal rule

$$\int_a^b f(x)\,dx = \frac{h}{2}[f(a)+f(b)]$$

$$h = \frac{b-a}{n} = \frac{2-0}{1} = \underline{\underline{2}}$$

$$\int_0^2 x^2\,dx = \frac{2}{2}[f(0)+f(2)]$$

$$= 1[0+4] = 4.$$

(ii) Simpson rule

$n = 2$ $\quad \frac{2-0}{2} = 1$

$= \frac{1}{3}\left[f(0) + 4\left(f(0+h) + f(0+2h)\right]\right)$

$= \frac{1}{3}\left[0 + 4 + 4\right]$

$= \frac{1}{3}(8) = 2.667$

(iii) Simpson 3/8 rule.

$h = \frac{2-0}{3} = \frac{2}{3}$

$\int_0^2 x^2\,dx = \frac{3}{8} \times \frac{2}{3}\left[f(0) + 3 \times \frac{4}{9} + 3f\left(0 + 2\cdot\frac{2}{3}\right) + f\left(0 + 3\cdot\frac{2}{3}\right)\right]$

$= \frac{1}{4}\left[0 + \frac{4}{3} + \frac{3 \times 16}{9} + 4\right)$

$= \frac{1}{4}\left[\frac{4}{3} + \frac{16}{3} + 4\right]$

$= 2.667$

Q7 Estimate $\int_0^1 e^{-x^2} dx$ using trapezoidal rule.

Ans

$N = 2$

$h = \frac{b-a}{N} = \frac{1-0}{2} = 0.5$

$x_0 = 0 \Rightarrow 1$

$x_1 = 0.5 \Rightarrow 0.7788$

$x_2 = 1 \Rightarrow 0.3679$

$\int_0^1 e^{-x^2} dx = \frac{h}{2}\left[1 + 2(0.7788 + \cdots 0.3679\right.$

$= \underline{0.7313}$

$1/(x+1)$

(1) <u>Trapezoidal</u> $n=1$
$h=2$

$$\int_0^2 1/(x+1)\,dx = 2/_2\,(f(0) + f(2))$$

$$= 1[1+0.33]$$

$$= \underline{1.33}$$

(2) <u>Simpson 1/3 rule:</u> $h=2/3$ $n=3$

$$\frac{3h}{8}\left[f(0)+3f(0+2/3)+3f(0+2\times 2/3)+f(0+3\times 2/3)\right)$$

$$= 1/4\,(1+3\times 3/5 + 3\times(3/7) + 1/3)$$

$$= \underline{1.1048}$$

NEW INDIA PAGE NO.: DATE

$$f(4/3) = \frac{1}{4/3+2} = 3/7$$

$$f\left(\frac{6}{3}\right) = f(2) = 1/3 = 0.33$$

sin x

(1) Trapezoidal $n=1$ $h=2$

$$\int_0^2 \sin x \, dx = h/2 \left[f(0) + f(2) \right]$$

$$f(0) = \sin 0 = 0, \quad f(2) = \sin 2 = 0.9093$$

$$= 1\left[0 + 0.9093\right]$$

$$= 0.9093$$

(2) Simpson 1/3 rule

$$\int_0^2 \sin x \, dx = h/3 \left[f(0) + 4(f(0.8)) + f(0.7) \right]$$

$\int e^{x}$

(i) Trapezoidal rule

$n = 1 \qquad h = 2$

$$\int_0^2 e^x\,dx = \frac{h}{2}\left[f(0) + f(2)\right]$$

$f(0) = e^0 = 1$

$f(2) = e^2 = 7.3891$

$$\int_0^2 e^x\,dx = 1\left[1 + 7.3891\right] = 8.3891$$

(ii) Simpson 1/3 rule

$$\int_0^2 e^x\,dx = h/3\left(f(0) + 4h(x_1) + f(x_2)\right]$$

$$= 1/3\left[1 + (4 \times 2.7183) + 7.3891\right]$$

$$= 1/3\left[19.2623\right]$$

$$= 6.4208$$

$$\int_0^2 \sin x \, dx = \frac{1}{3}\left[0 + 4 \times 0.8415 + 0.7093\right]$$

$$= \frac{1}{3}\left[4.2793\right]$$

$$= \underline{\underline{1.425}}$$

(iii) Simpson 3/8 rule

$n = 3 \quad h = 2/3$

$f(2/3) = \sin(2/3) = 0.6184$

$f(4/3) = \sin(4/3) \quad 0.9720$

$$\int_0^2 \sin x \, dx = \frac{1}{4}\left[0 + 3 \times 0.6184 + 3 \times 0.9720 + 0.9091\right]$$

$$= \frac{1}{4}\left[5.6805\right]$$

$$= \underline{\underline{1.420}}$$

(iii) Simpson 3/8 rule

$\int^{2} e^{x} dx$

$\frac{3h}{8}\left[f(0) + 3h\left(0+\frac{2}{3}\right) + 3h\left(0+\frac{2\cdot 2}{3}\right) + f\left(0+3\frac{2}{3}\right)\right.$

$\int_0^2 e^x dx = \frac{1}{4}[1 + 3 \times 1.9477 + 3 \times 3.7937 + 7.3891$

$= \frac{1}{4}(28.6133)$

$= 6.4033$

Q9/2 Given function f at following value

x	1.8	2.0	2.2	2.4
$f(x)$	3.1804	4.42569	6.04241	8.03014

(a) Trapezoidal rule.

$$\int_a^b f(x)dx = \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right]$$

$$= \frac{0.2}{2}\left[3.12014 + 2(4.42569 + 6.04241 + 8.03014) + 10.46617\right)$$

$$= 0.1[50.58337]$$

$$= \underline{5.05834}$$

(b) Simpsons 1/3 rule

$$\int_a^b f(x)\,d(x) = \frac{h}{3}\left[f(x_0) + 4[f(x_1) + f(x_3)] + 3[f(x_2)\ f(x_4)]\right]$$

$$\int_{1.8}^{2.6} f(x) = \frac{0.2}{3}\left[3.12014 + 4[4.42569 + 8.03014] + 2(4.42569) + 10.46673\right]$$

$$= \underline{5.11274}$$

Q10 Given form of closed type Newton Cote's integration formula

$$\int_a^b f(x)\,dx \simeq nh \sum_{i=0}^{n} C_i^n f(x_i) \text{ with}$$

$$h = \frac{b-a}{n} \quad x_i = x_0$$

State 2 properties of it.

Ans C_i^n has 2 properties

(1) $\sum_{i=0}^{n} C_i^n = 1$

(2) $C_i^n = C_{n-i}^n$

Q11 Derive

Trapezoidal Rule

Ans
$$\int_a^b f(x)\,dx \simeq nh \sum_{i=0}^{n} C_i^n f(x_i)$$

with $h = \frac{b-a}{n}$, $x_i = x_0 + ih$, $i = 0, 1, 2, n$

a

let $n = 1$

$$\int_a^b f(x)\,dx = h \sum_{i=0}^{1} C_i^1 f(x_i)$$

$$= h C_0^1 f(x_0) + h C_1^1 f(x_1)$$

By two propertes

$$C_0^1 = C_1^1 \text{ and } C_0^1 + C_1^1 = 1$$

gives

$$C_0^1 = C_1^1 = 1/2$$

NEW INDIA PAGE NO.: DATE

$C_0^1 = C_1^1$ and $C_0^1 + C_1^1$ gives $C_0^1 = C_1^1 = 1/2$

Simpson 1/3 Rule

$$\int_a^b f(x)\,dx \simeq nh \sum_{i=0}^{n} c_i^n f(x_i)$$

with $h = \frac{b-a}{2}$ $x_i = x_0 + ih$

$i = 0, 1, 2, 2$

Let $n = 2$

$$\int_a^b f(x)\,dx \simeq 2h \sum_{i=0} c_i^n f(x_i) \cdots (1)$$

Here $c_0^2 = c_2^2$ and $c_0^2 + c_1^2 + c_2^2 = 1$

By evaluating C_i^2

$$c_i^n = \frac{(-1)^{n-i}}{n\,i!(n-i)!} \int_0^n \frac{t(t-1)\cdots(t-n)}{(t-i)}\,dt$$

NEW INDIA
PAGE NO.: DATE

$$c_\alpha^2 = \frac{(-1)^{2-2}}{2\cdot 2!\,(2-2)!}\int_0^2 t\,\frac{(t-1)(t-2)}{(t-2)}\,dt$$

$$= \frac{1}{4}\int_0^2 t(t-1)\,dt$$

$$= \frac{1}{4}\int_0^2 (t^2-t)\,dt = \frac{1}{4}\left[\frac{t^3}{3} - \frac{t^2}{2}\right]_0^2$$

$$= \frac{1}{4}\left[\frac{8}{3} - \frac{4}{2}\right] = \frac{1}{4}\left(\frac{8}{3} - 2\right)$$

$$= \frac{1}{4}\left(\frac{8-6}{3}\right)$$

$$= \frac{2}{4\times 3} = \frac{1}{6}$$

$$c_\alpha^2 = \frac{1}{6} \Rightarrow c_0^2 = c_2^2 = \frac{1}{6}$$

$$\int_a^b f(x)\,dx \cong 2h\,C_0^2 f(x_0) + 2h\,C_1^2 f(x_1) + 2h\,C_2^2 f(x_2)$$

$$\int_a^{2h} f(x)\,dx \cong \frac{h}{3}\left[f(x_0) + 4f(x_1)\right.$$

$$= \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right.$$

it can also be expressed as

$$\int_a^b f(x)\,dx \cong \frac{h}{3}\left[f(a) + 4f(a+h) + f(a+2h)\right]$$

This is Simpson $\frac{1}{3}$.

~~This~~ is

Q2.

Estimate $\int_0^4 e^x \, dx$ by,

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a)+4f(a+h)+f(a+2h)\right]$$

here $n=2$, $h = \frac{4-0}{2} = 2$

$$\int_0^4 e^x\,dx = \frac{2}{3}\left[f(0)+4f(0+2)+f(0+2.2)\right]$$

$$= \frac{2}{3}\left[1+4[7.38906]+54.59815\right]$$

$$= \frac{2}{3}\left[85.0154\right]$$

$$= 56.769$$

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PAGE NO.: DATE

$x_0 = 0 \Rightarrow f(x_0) = e^0 = 1$

$x_1 = 1 \Rightarrow e^1 = 7.38906$

$x_2 = 2 \Rightarrow 54.59815$

$= \frac{h}{3}[f(x_0) + 4(f(x_1) + f(x_2)]$

$= \frac{2}{3}[1 + 4(7.38906]$

$= \frac{h}{3} f(x_0) + 4(f(x_1) + f(x_2)]$

$= \frac{2}{3}[1 + 4(7.38906) + 54.59815]$

$= 56.76959\underline{3}$

NEW INDIA
PAGE NO.: DATE

$n = 4$

$$h = \frac{4-0}{4} = 1$$

$x_0 = 0 \quad = 1$
$x_1 = 1 \quad = 2.71828$
$x_2 = 2 \quad = 7.38936$
$x_3 = 3 \quad = 20.08554$
$x_4 = 4 \quad = 34.59715$

$$\int_0^4 e^x \, dx = \frac{h}{3}\left[f(x_0) + 4(f(x_1) + f(x_3)) + 2(f(x_2)) + f(x_4)\right.$$

$$= \frac{1}{3}\left[161.59135\right]$$

$$= 53.86785$$

Error in Simpsons 1/3 rule

error = True − approx.
$= 53.59815 - 56.76959$

$= -3.17145$

NEW INDIA
PAGE NO.: DATE

Comboste simpson rule with $N=2$

$53.59815 - 96.76959\,3$

$= -3.17155\,3$

(c) Composite simpson rule $N=4$

$53.59815 - 53.86785$

$= -0.2657$

Q13 Write pseudocode for following integration formulas to obtain result

~~step 1:- input a, b ; f(x) MD~~

~~M = max iteration~~

~~Step 1: Input a, b ; f(x)~~

~~m = max no. of~~

Step 1: Input a, b f(x), M = max iteration

2: $h = (b-a)/2 * 1$

sum = 0, sum 1 = f(a+h)

K = 0 N = 1 old sum = 0

3. DO

FOR i = 1 to N - 1

x(2 * i) = a + h * 2 * i ;

x(2 * i + 1)

a + h * 2 * i + 1)

~~sum = sum + f(x) 2 * i)~~

~~sum 2 = sum 1 + 1 f(x 2 * i + 1))~~

End do

4. Sum = 2 * sum + 4 * sum 1 + f(a) + f(b)

5. ~~Output, sum = h * sum / 3~~ ; Print sum

6. if (sum - old sum) < ε ; Output : Ans = sum.

6. if |sum - oldsum| < E
{
7. Else
$k = k+1$ $x = a * x$, old sum = sum

8. Else print not give desired accuracy in
m iterations
}

Q14 Car laps a race track in 84 secs.

The speed of car at each 6 second interval is determined using radar gun from begining of lap. in feet/second by intrins

Time	0	6	18	18	24	30	36	42	48
Speed	124	134	148	156	147	133	121	109	99

54	60	66	72	78	84
85	78	89	104	116	123

How long is track?

7. $K = K+1$, $N = 2*N$ old sum = sum;
if $K <= M$, goto step 3.

8. Else print does not give desired accuracy in iterations

(b) Trapezoidal

1. INPUT N,

2. $sum = 0$ $K = 0$ $N = 1$

3. DO
$i = 1$ to $N-1$
$x = a + h*i$
$sum += 2*f(x)$

End DO

4. $sum += f(a) + f(b)$
5. $sum = (h/2) * sum$;
6. print sum

$x_0 = 0 \Rightarrow 0$

$x_1 = 0.05 = 0.00377$

$2 = 0.50 = 0.05579$

$3 = 0.75 = 0.25104$

$4 = 1 = 0.69315$

$5 = 1.25 = 1.47029$

$6 = 1.50 = 2.65147$

$7 = 1.75 = 4.29701$

$8 = 2 = 6.43775$

$$= \frac{0.05}{2}\Big[0 + 0\,(0.00377 + 0.05579 + 0.25104 + 0.69315 + 1.47029 + 2.65147 + 4.29701) + 6.43775\Big]$$

$$= 0.125\,[25.275]$$

$$= \underline{\underline{3.1595}}$$

NEW INDIA
PAGE NO.: DATE:

$$\int_0^{84} f(x)\,dx = \frac{h}{3}\left[f(x_0) + 4\left(f(x_1) + f(x_3) + f(x_5) + f(x_7) + f(x_9) + f(x_{11}) + f(x_{13})\right) + 2\left(x_2 + x_4 + f(x_8) + f(x_{10}) + f(x_{12})\right) + f(x_{14})\right]$$

$$= \frac{6}{3}\left[124 + 4(134 + 137 + 107 + 85 + 84 + 76) + 2(148 + 147 + 101 + 99 + 78 + 107 + 102)\right]$$

$$= 2[4929]$$

$$= 9858 \text{ feet}$$

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PAGE NO.: DATE

Q15 Approx: $\int_0^2 x^2 \ln|x^2+1| \, dx$

using $h = 0.05$ usl.

(i) Compile trapezoidal rule

Ans $h = 0.05$

$N = \frac{b-a}{0.25} = \frac{2}{0.25} = 8$

$\int_0^2 x^2 \ln(x+1) \, dx = \frac{h}{2} [f(x_0) + 2(f(x_1)$

$+ f(x_2) + f(x_3) + f(x_4) + f(x_5)$

$+ f(x_6) + f(x_7)) + f(x_8)]$

NEW INDIA
PAGE NO.: DATE

Approximate

$$\int_0^2 x^2 e^{-x^2} \quad \text{using} \quad h=0.25$$

i) Composite Trapezoidal rule

$h=0.05 \quad N=8$

$x_0 = 0 \Rightarrow 0$
$x_1 = 0.25 \Rightarrow 0.05871$
$x_2 = 0.50 \Rightarrow 0.1947$
$x_3 = 0.75 \Rightarrow 0.3205$
$x_4 = 1 \Rightarrow 0.36788$
$x_5 = 1.25 \Rightarrow 0.3275$
$x_6 = 1.50 \Rightarrow 0.23715$
$x_7 = 1.75 \Rightarrow 0.14324$
$x_8 = 2 \Rightarrow 0.07326$

$= \frac{0.25}{2}[0 + 2(0.05871 + 0.1947 + 0.3205 + 0.36788 + 0.3275 + 0.23715 + 0.14324) + 0.07326$

$= 0.42158$

NEW INDIA
PAGE NO.: | DATE

(5) Composite mid point rule

$= 0.25\, f(0.125)$

$= \underline{\underline{0.00385}}$

4)

$$\int_{0.25}^{0.50} x^2 e^{-x^2}\,dx = 0.25 \times f(0.375)$$

$= 0.25 \times 0.12213$

$= \underline{0.3054}$

$$\int_{0.50}^{0.75} x^2 e^{-x^2}\,dx \quad 0.25 \times f(0.875)$$

$= 0.25 \times 0.35605$

$= \underline{0.08701}$

$$\int_{0.75}^{1} x^2 e^{-x}\,dx = 0.05 \times f(1.125)$$

$= 0.25 \times 0.35699$

$= \underline{\underline{0.08925}}$

$$\int_{1}^{1.25} x^2 e^{-x^2}\,dx = 0.25 \times f(1.375)$$

$= 0.25 \times 0.28544$

$= \underline{\underline{0.07136}}$

$$\int_{1.25}^{1.50} x^2 e^{-x^2}\,dx = 0.25 \times f(1.375)$$

$= 0.25 \times 0.2844$

$= 0.07136$

(5) Composite simpson 1/3 rule

$$= \frac{h}{3}\Big[f(x_0) + 4\big(f(x_1) + f(x_3) + f(x_5) + f(x_7) \big] + 2\big(f(x_2) + f(x_4) + f(x_6) \big) + f(x_7) \Big]$$

$$= \frac{0.25}{3} \Big[0 + 4(0.05871) + 0.3205 + 0.3275 + 0.14324 + 0.1947$$
$$0.36788 + 0.23715 + 0.07326 \Big]$$

$$= 0.083 [5.07252]$$

$$= \underline{\underline{0.42102}}$$

NEW INDIA
PAGE NO.: | DATE

$$\int_{1.50}^{1.75} x^2 e^{-x^2} dx = 0.05 \times f(1.625)$$
$$0.05 \times 0.18832$$
$$= 0.0708$$

$$\int_{1.75}^{2} x^2 e^{-x^2} dx = 0.05 \times f(1.875)$$
$$0.05 \times 0.1652$$
$$= 0.02613$$

$$\int_0^2 x^2 e^{-x^2} dx = 0.00385 + 0.03054 + 0.06608$$
$$+ 0.08901 + 0.08905 +$$
$$0.07136 + 0.004708 +$$
$$0.02613$$
$$= 0.4233$$