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CONM

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Solution of Ordinary Numerical Methods.

Assign - 3

Define a differential equation. What is meant by solution of differential equation? & Verify.

1. $y = C_1 e^x + C_2 e^x x$ is a solution of differential. $(y'' - 2y' + y = 0)$

(ii) $y = 2\sqrt{c-x}$ is solution of diff. equation $y + 1/y = 0$

Define a differential eq.

Ans A differential equation is an equation independent variable, dependent variable & its one or more derivative.

eg. $y'' + ay = 0$

Solution of differential equation

e.g. $y'' - 2y' + y = 0$

$y(x) = C_1 e^x + C_2 e^x x$

$= C_1 e^x + C_2 \frac{d}{dx}[x e^x]$

$= C_1 e^x + C_2 \left(x \frac{d}{dx} e^x + e^x \frac{d}{dx} x \right)$ (2)

$= C_1 e^x + C_2 (x e^x + e^x)$

(3)

$y'(x) = C_1 e^x + C_2 e^x + C_2 e^x$

Substitute 1, (2),

$0 = C_1 e^x + 2C_1 e^x + C_2 x e^x$
$+ C_1 e^x + C_2 e^x$

$= 0$

Here diff eq. $y' + 1/y = 0$

$$\frac{-1}{\sqrt{c-x}} + \frac{1}{\alpha\sqrt{c-x}} \neq 0$$

Q2 How is ordinary differential equation differt from partial differential equation?

A3 When independent variable is function only one variable then all derivatives involved on equation are ordinary & equation is called ordinary diff. equation.

$$y'' + gy = 0$$

$$y' + 1/y = 0$$

Q3. Differentiate initial value problem & boundary value problem. Classify following differential equations in initial value problem & boundary value problem.

Ans. If conditions are specified at single point. These conditions are called initial conditions.

→ If conditions are specified at more than one point. These are boundary conditions.

i) $y' = t - y \qquad y(0) = 1$

→ Initial value problem

(ii) $y'' + 9y = e^{x} \sin x, \quad y(0) = 1$
$y(1) = 2$

→ Boundary value problem

(iii) $y'' + y = 0$, $y(0) = 0$ $y'(0) = 2$

→ Boundary value problem

(iv) $y' = t^2 + y^2$ $y(0) = 1$

→ Initial value problem

Q4 Determine order & degree of following differential

i) $y'' + 4y = e^x$

→ Order $= 2$, degree $= 1$

(ii) $y'' + 4(y')^3 + y^2 = x^3 + x^2$

Order $= 2$ Degree $= 1$

(iii) $(y'')^2 + (y')^3 + 3y = 5x$

Order $= 2$ Degree $= 2$

(iv) $y' + \alpha y^2 = x^2$

Degree $= 1$

Order $= 1$

Q5/ What are characteristics of single step numerical methods to find solutions of first order.

→ It is direct
→ It is non-iterative
→ Practically, error can not be estimated at given step?

[illegible] [illegible]

→ Practically, error can not be estimated.

→ ~~It can also be used~~

a A a a

Q6

Single step	Multistep
• Direct	• More than one prev. step
• Not iterative	• Iteration
• Based on Taylor series method	• Predictor corrector formulas
• Self starting	• Not self-starting
• Probably accuracy and be estimated	• Practically, accuracy can be estimated

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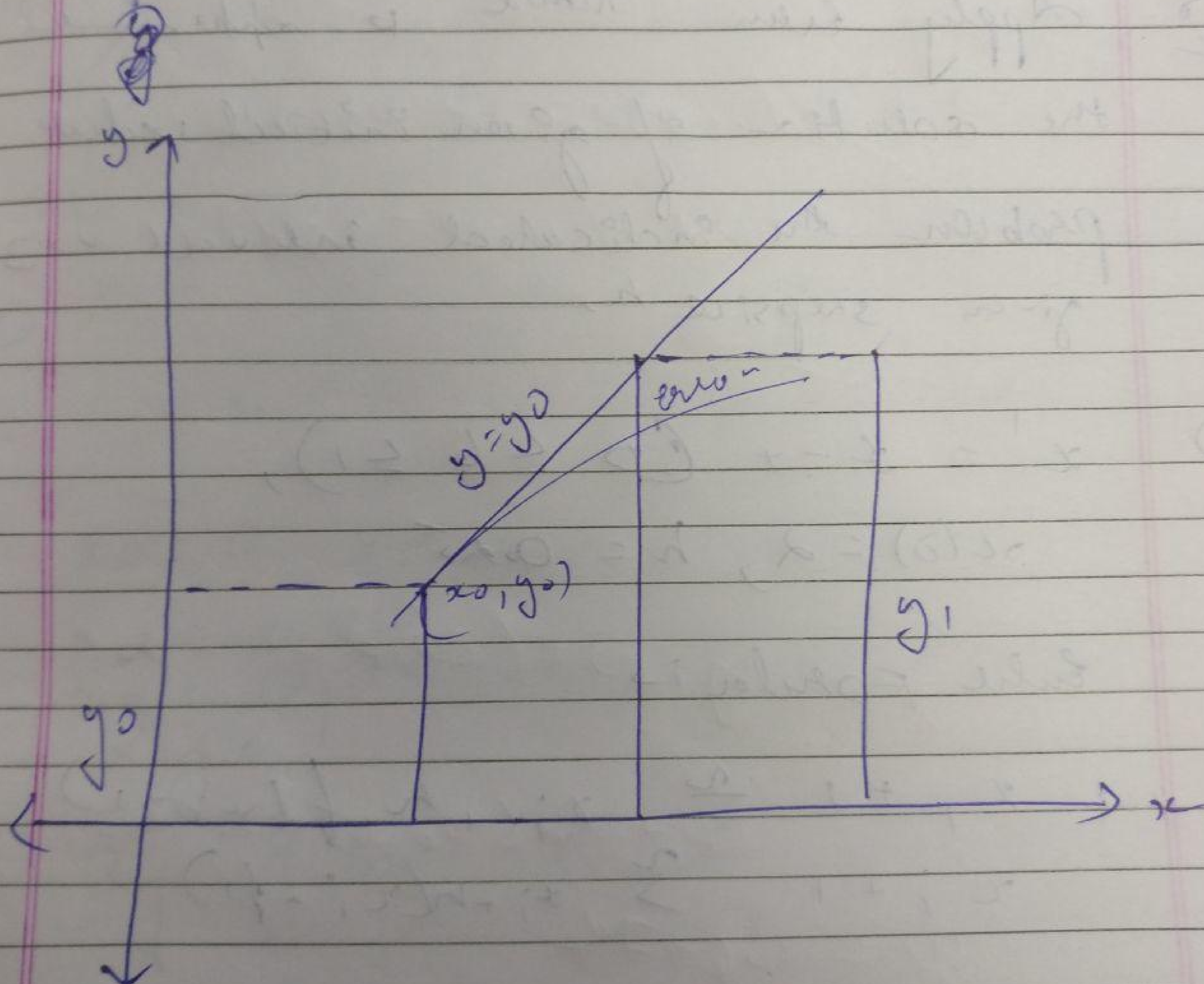
Q7 What is Euler Formula to solve IVP?

Ans Euler Formula is Taylor series of order 1. when $n=1$ second and higher powers of h are ignored.

→ The Taylor formula for say ...

→ The Taylor Formula for say one IVP $\frac{dy}{dx} = f(x, y(x))$

$$y_{i+1} = y_i + h y_i' + O(h^2)$$


y
y = y0
error
(x0, y0)
y0
y1
x

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Q8 Apply Euler method to approximate the solution of given initial value problem the indicated interval using given stepsize h.

i) $x' = x - t \quad (0 \le t \le 1)$,

$x(0) = 2, \; h = 0.25$

Euler Formula is

$x_{i+1} \simeq x_i + h\, f(x_i, t_i)$

$x_{i+1} \simeq x_i + h(x_i - t_i)$

$x_2 = 1.25(x_1) - 0.2 x(t_1)$

$x\ 1.25(1.5) - 0.25(0.25)$

$= 3.125 - 0.0625$

$x_2 = 3.0625$

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(ii) $y' = xy^3 - y \quad (0 \le x \le 1)$

$y(0) = 1$, $h = 0.25$

$\Rightarrow$ Here, $f(x, y) = xy^3 - y$

$0 \le x \le 1$, $y(0) = 1$ $h = 0.25$

$x_0 = 0.$ $x_1 = 0.25$

$x_2 = 0.50$

$x_3 = 0.75$

$x_4 = 1$

By Euler Formula

$y_{i+1} \cong y_i + h(x_i y_i)$

$0.75 + 0.25(x_1 + y_1)$

$y = 0.1$

t_1

$y_1 = y_0 + 0.2(\cos 2 t_0 - \sin 3 t_0)$

$y_1 = 1 + 0.2(1 - 0) - 1 + 2?$

$= 1.2$

t_2

$y_2 = y_1 + 0.2(\cos 2 t_1 + \sin 3 t_1)$

$= 1.2 + 0.2(9210) + 0.5664$

$y_2 = 1.49714$

t_3

$y_2 + 0.2(\cos 2 t_2 + \sin 3 t_2)$

$1.49714 + 0.2(\cos 0.8 + \sin 1.2)$

$y_3 = 1.82289$

x_4

$y_3 + 0.2(\cos^2 x_3 + \sin^2 x_3)$

$= 1.82287 + 0.2(\cos^2 1.2 + \sin^2 1.4)$

$= \underline{\underline{2.09013}}$

x_5

$y_5 = y_4 + 0.2(\cos^2 x_4 + \sin^2 x_4)$

$= 2.09013 + 0.2(\cos 1.6 + \sin 2.4)$

$= \underline{\underline{2.21939}}$

Solution $\rightarrow$ $\boxed{2.21939}$