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CONM

Assignment 4

Assignment - 4

A ball at 1200 k is allowed to cool down in air at an ambient temperature at 300k.

Temperature at given by $\frac{d\theta}{dt} = -2.2067 \times 10^{-12}$

when θ is in k and t in seconds. Find temp t = 480 secs
Assume h = 240

Ans: Runge kutta formula at order 2

$$\frac{dy}{dx} = f(x, y(x)) = y(x_0) = y_0$$

$$a \le x \le b$$

$$k_1 = h f(x_i + h, y_i + k_0)$$

for $i = 0, 1, 2, n$

$t_1 = 400$ s

$\theta_2 = \theta_1 + \frac{1}{2}(k_0 + k_1)$

$k_0 = 240 \times 2.02067 \times 10^{-12}$

$= -93.2856$

$k_1 = hf(t_1 + h, \theta_1 + k_0)$

$= 240 + f(480, 561.8731)$

$= 240 \times -2.0207 \times 10^{-12}$

$= -48.4948$

Q. Use Taylor's method of order 2 to approximate solution of each of problem

Ans. at $x_2 = 1$

$y_2 = y_1 + hy_1' + \frac{h^2}{2!} y_1''$

$= 1.25 + 0.5(1.25 - (0.5)^2 + 1) + \frac{0.25}{2}(-2(0.5))$

$y_2 = 2.125$

→ at $x_3 = 1.50$

$= 2.125 + 0.5(y_2 - x_2^2 + 1) + \frac{(0.5)^2}{2!}(-2x_2)$

$y_3 = 2.9375$ ---- (3)

(6)

at $x_4 = 1$

$$x_4 = y_3 + h y_3' + \frac{h^2}{2!} y_3''$$

$$= 2.9375 + 0.5\left(2.9375 - (1.5)^2 + 1\right)$$

$$+ 0.125\left(-2(1.5)\right)$$

$$= 3.40625$$

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(iii) $\frac{dy}{dt} = \cos 2t + \sin 3t$

Here $\frac{dy}{dt} = y' = \cos 2t + \sin 3t$

$y_0 = 1, \quad t_0 = 0 \quad h = 0.25$

$y' = \cos 2t + \sin 3t$

$y'' = -2\sin 2t + 3\cos 3t$

$= 1 + 0.25(\cos 2t_0 + \sin 3t_0)$

$= y_1 = 1.34375$

(c) $y' = \frac{1+t}{1+y}$, $1 \le t \le 2$

Here,

$y' = \frac{1+t}{1+y}, \quad y'' =$

Using Taylor method

at $t_1 = 1.5$

$$y_1 = y_0 + h y_0' + \frac{h^2}{2!} y_0''$$

$$= 2 + 0.5\left(\frac{1+t_0}{1+y_0}\right) + \frac{(0.5)^2}{2}\left(\frac{1}{1+y_0}\right)$$

$$= 2 + 0.5\left(\frac{1+1}{1+2}\right) + \frac{0.25}{2}\left(\frac{1}{3}\right)$$

$$= 2.375$$

(d) $y' = 1 + \frac{y}{t}$, $y(1) = 1$

$h = 0.25$

$y' = 1 + \frac{y}{t}$ $\quad y'' = y\frac{d}{dt}\left(\frac{1}{t}\right)$

$y'' = \frac{-y}{t^2}$

Order 2 Taylor Method :->

$y_1 = y_0 + h y_0' + \frac{h^2}{2!} y_0''$

$1 + (0.25)\left(1 + \frac{y_0}{t_0}\right) + \frac{0.25^2}{2}\left(\frac{-y_0}{t_0^2}\right)$

$y_1 = 1.46875$

at $t_2 = 1.50$

$y_2 = y_1 + h y_1' + \frac{h^2}{2!} y_1''$

$1.46875 + 0.25 \times \left(1 + \frac{y_1}{t_1}\right) + \frac{0.0625}{2}\left(\frac{-y_1}{t_1^2}\right)$

$= 1.98312 5$

Q) Compare performance of Euler method, second Taylor, RK of order 2, and 4.

(1) Euler

$$y_{i+1} = y_i + h f(x_i, y_i)$$

at $x_0 = 0$, $y_0 = 1$

at $x_1 = 0.5$

$$y_1 = y_0 + 0.5 (x_0 / y_0)$$

$$= 1 + 0.5 (0/1) = 1$$

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Second Taylor

$x+x_1=0.5$

$y_2=1.125+0.5\left(\frac{0.3}{1.25}\right)+0.125\left(\frac{1}{1.25}\right)$
$=1.4583$

$\rightarrow x+x_2=1.5$

$y_3=1.4583+0.5\left(\frac{1}{1.4377}\right)$
$+0.125\left(\frac{1}{1.4583}\right)=\underline{1.7869}$

$x\ \underline{y=2}$
$y_4=1.8769+0.5\left(\frac{1.70}{1.8869}\right)+0.125$
$\left(\frac{1}{1.8867}\right)$
$=\underline{2.3505}$

Second Taylor

$x + x_1 = 0.5$

$y_2 = 1.125 + 0.5\left(\frac{0.3}{1.25}\right) + 0.125\left(\frac{1}{1.25}\right)$

$= 1.4583$

$\rightarrow x + x_2 = 1.5$

$y_3 = 1.4583 + 0.5\left(\frac{1}{1.4583}\right)$

$+ 0.125\left(\frac{1}{1.4583}\right) = \underline{\underline{1.8869}}$

$x\ \underline{y = 2}$

$y_4 = 1.8869 + 0.5\left(\frac{1.70}{1.8869}\right) + 0.125$

$\left(\frac{1}{1.8869}\right)$

$= \underline{\underline{2.3506}}$

RK order 4

$k_0 = h f(x_i, y_i)$ $k_1 = h f(x_i + h/2,$

$k_2 = h f(x_i + h/2, y_i + k_1/2)$

$k_3 = h f(x_i + h, y_i + k_2)$

at $x_2 = 1$

k_0 $0.5\left(\frac{0.5}{1.100}\right) = 0.2272$

k_1 $0.5 f(0.75, 1.2136) = 0.3054$

k_2 $0.5 f(0.75, 1.2527) = 0.2987$

k_3 $0.5 f(1, 1.3974) = 0.3173$

k_4 $1 + \frac{1}{6}(0.2272) + 0.3054 + 0.2987) +$
0.3173

$y_2 = 1.3$

$a + 2z = 1.5$

$k_0 = 0.5\left(\frac{1}{1.3}\right) = 1.2692$

$k_1 = 0.5\left(\frac{1.25}{1.4346}\right) = 0.3231$

$k_2 = 0.5\left(\frac{1.25}{1.4155}\right) = 0.427$

$k_3 = 0.5\left(\frac{1.5}{1.7226}\right) = 0.4347$

$y_1 = 1.3 + \frac{1}{6}\left(1.2672 + 2(0.3231) + 2(0.427 + 0.435)\right)$

$y_1 = 1.8341$

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Q4 Explain Taylor method of order n to approximate solution of differential equation
$y'(x) = f(x, y(x))$

List Advantages. Why is it used seldom in real life?

Ans

$y_{i+1} = y_i + h y_i' + \frac{h^2}{2!} y_i'' + \frac{h^n}{n!} (y_i^n$

→ Taylor series is seldom used in practical for practical due to requirement of derivation of expression and tables

→ It is Backbone of single step formulas

Q. Perform Question with Range Kutta formula

$$y_{i+1} = y_i + \frac{1}{6}(k_0 + 2k_1 + 2k_2 + k_3)$$

$k_0 = hf(x_i, y_i)$

$k_1 = hf(x_i, y_i)$

$k_1 = hf(x_i + h/2), y_i + k_0/2$

$k_2 = hf(x_i + h/2, y_i + k_1/2)$

$k_3 = hf(x_i + h, y_i + k_2)$

$f(0,1) = 0.5$

$k = 0.5 \times f(0,0.5)$

$= 0.5 (0.5 + 0^2 + 1)$

$= 0.75$

$0.5 f(0.25, 0.5 + 0.9021)$

$= 0.5 + f(0.25, 0.5)$

$= 0.9453$

$0.5 + \frac{1}{6} (0.75 + 2(0.90625)$

$+ 2(0.945) + 0.1076)$

$y = 1.4251$

At $x_2 = 1$

$k_0 = 0.5 \times f(0.5, 1.5251) = 1.08755$

$k_1 = 0.5 \times f(0.75, 1.9687) = 1.0032$

$k_2 = 0.5 \times f(0.75, 2.0167) = 1.2321$

$k_3 = 0.5 f(1.2, 2.1\ldots) = 1.3280$

$y_2 = 2.6376$

At point x_4

$k_0 = 0.5 f(1.5, 4.0668) = 1.3707$

$k_1 = 0.5 f(1.75, 4.696) = 1.31675$

$k_2 = 0.5 f(1.75, 4.6652) = 1.3015$

$k_3 = 0.5 f(2, 5.3082) = 1.1541$

$x_4 = 4.0068 + \frac{1}{6}\left(1.3787 + 2(1.31675)\right.$
$\left. + 2(1.3005) + 1.1541\right)$

$= 5.3016$

At point $t_2 = 0.50$

$k_0 = 0.25 f(0.25, 1.7292) = 0.3977$

$k_1 = 0.25 f(0.375, 1.5241) = 0.4085$

$k_2 = 0.25 f(0.375, 1.5335) = 0.4083$

$y_2 = (1.7992 + \frac{1}{6}(0.3958 + 2.0405 + 2(0.4083) + 0.3097)$

$= 1.7706$

(c) $\frac{dy}{dt} = \frac{1+t}{1+y}$

$h = t_0 = 1 \quad y_0 = 2$

$t_1 = 1.5$

$k_1 = 0.5 f(1, 2) = \cancel{0.333}$ 0.333

$k_2 = 0.5 f(1.25, 2.1667) = 0.3551$

$k_3 = 0.5 f(1.25, 2.1777) = 0.3540$

$k_4 = 0.5 f(1.5, 2.354) = 0.37727$

$y_1 = 2 + \frac{1}{6} [0.3333 + 2(0.3551)$

$+ 2.03540 + 0.37727$

$y = \underline{2.3541}$

(d) $y' = 1/t + y/t \quad y(1) = 1$

$h = 0.5$

At $t_0 = 1$

$k_0 = 0.25\, f(1, 1) = 0.5$

$k_1 = 0.25\, f(1.25, 1.05) = 0.5278$

$k_2 = 0.5\, f(1.5, 2.079$

$= y_1 + \frac{1}{6}(0.5 + 2.6 + 2.278 + 2.05135$
$+ 2.5562)$

$y_1 = .5204$

$t_2 = 1.5$

$k = 0.25\, f(1.25, 1.5281) = 0.5558$

$k_1 = 0.25\, f(1.375, 1.7068) = 0.5785$

$k_2 = 0.25\, f(1.375, 1.7182) = 0.5800$

$k_3 = 0.25\, f(1.5, 2.1071) = 0.6016$

$y_2 = 1.5291 + \frac{1}{6}\left(0.5558 + 2(0.5785)\right.$
$\left. + 2(0.5800) + 0.6016\right)$

$y_2 = 2.1082$

c) $y' = xe^{y+x} - 1, \quad y(0) = -1$
$0 < x < 7$

$h = 0.5$

$y' = y^2\, y^{+x} - 1$
$y(0) = 1$ $\quad 0 \le x \le 2$
$h = 0.5$

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$x_1 = 0.5$

$k_0 = 0.5 f(0, -1) = 0.5$

$k_1 = 0.5 f(0.25, -1.25) = 0.454$

$k_2 = 0.5 f(0.25, -1.220) = -0.45$

$k_3 = 0.5 f(0.5, 1.433) = -0.4036$

$y_1 = -1 + \frac{1}{6}(-0.5 (2(-0.454) + 2(-0.45)) + 0.4076)$

$y_1 = -1.4529$

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Q7 Give answers of following in one-2 sentences

a) What is advantage of RK method over Taylor series methods of same order?

Ans RK method and Taylor method of same order are equally efficient but RK methods have advantage of no requirement of doing the expression of derivatives

(b) What is order of local truncation error in RK methods?

Ans In RK of order 2 the order of local truncation error is $O(h^3)$ and order of global truncation error is $O(h^2)$

e) What is drawback of multistep methods?

Ans Not self starting

f) Advantage of predictor corrector method over RK methods?

Ans In RK 4 function evaluation per step required.

Q8 Consider IVP

$y' = y - t^2 + 1$

taking $h = 0.2$

M.D. Milne Simpson predictor corrector formula to obtain value at $y(0.8)$ & $y(0)$

A) Exact soln $(t+1)^2 - 0.5e^t$

$t_0 = 0$	$y_0 = 0.5$
$t_1 = 0.2$	$y_1 = 0.8293$
$t_2 = 0.4$	$y_2 = 1.2141$
$t_3 = 0.6$	$y_3 = 1.6487$
$t_4 = 0.8$	$y_4 = 2.1272$

$$y_4 = y_0 + \frac{4 \times 0.2}{3}\left(2y_3' - y_2' + 2y_1'\right)$$

$$= 0.5 + 0.2667\left(2f(t_3, y_3) - f(t_2, y_2) + f(t_1, y_1)\right)$$

$$= 0.5 + 0.2667\left(2(1.6648) - (0.6) + \right)$$

$$= 0.5 + 0.2667\left(2 \times 2.8039 - 2.05472 + 1.787\right)$$

$$y_4 = 2.1275$$

$y_4 = 1.2141 + 0.6667 (f(0.8, 2.1277) + 4f(0.6, 1.6489) + f(0.4, 1.2, 141))$

$y_4 = 2.5277$

Correction Formula

$y_4 = y_3 + \frac{0.2}{3} (f_4 + 4f_3 + f_2)$

$= 1.6489 + 0.0667 (f(1, 2.5877) + 4f(0.8, 2.1277) + f(0.6, 1.6489))$

Again,

$y_5 = 1.6489 + 0.0667 (f(1, 2.6, 377) + 4f(0.8, 2.172) + f(0.6, 1.6489))$

$y_5 = 2.6413$

Compare ~~sults~~ results for exact solution

	Milne-Simpson	Exact Sol	Error
y(0.1)	2.1277	2.1272	0.0005
y(0.2)	2.6413	2.6407	0.0325

	Adams Moulton	Exact Soln	Error
y(0.1)	2.1272	2.1272	0.0
y(0.2)	2.6407	2.6407	0.0001