Industrial Management & Data Systems

Hierarchical forecasting: issues and use guidelines

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Keywords
Forecasting, Time series, Hierarchy, Product management

Abstract
In order to provide the appropriate demand forecast information given various managerial levels and functional disciplines within organizations, reliance on family-based forecasting is increasing. The family-based approach, sometimes referred to as hierarchical forecasting (HF), is based on a strategy of aggregating items into families. HF systems are capable of providing forecasts for items and their respective families. The objectives of HF systems, include improved forecast performance and a reduction in the overall forecasting burden. To date, several studies have offered practical guidelines for the structural design of HF systems. The primary purpose of this paper is to summarize these guidelines. First, an explanation of the HF process is provided. In this explanation, important system parameters and strategic choices, which allow for the custom configuration of HF systems are identified. Second, the relevant family-based forecast research is reviewed. The important issues addressed and the conclusions presented in this research are identified. Third, practical guidelines regarding the use of a HF approach that have been reported in the research literature are clearly delineated. With much still unknown regarding the performance impact of various system parameter and strategic process choices, the paper concludes with suggestions for future research.

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101/1 [2001] 5–12

[ISSN 0263-5577]

Introduction

Individuals representing various management levels and functional disciplines within an organization have a variety of forecast information needs. Upper, middle, and lower level managers have different forecast information requirements. Likewise, operations, finance, and marketing managers each have different forecast information requirements. For many firms, their product lines can be used to portray these different forecast information needs. For example, L.L. Bean, Inc., a major cataloger and retailer of outdoor apparel, has the hierarchical product line structure illustrated in Figure 1. One can easily envision how forecast information could be useful for various users and various planning activities. For instance, long-term forecasts of company-wide dollar sales for all merchandise groups could be useful for financial budgeting and capacity planning purposes. Intermediate-term forecasts of semi-aggregate unit sales for demand centers could be useful for reserving supplier capacity and workforce size decisions. Short-term forecasts of individual item demands could be useful for inventory planning and control decisions.

Given the variety of forecast information needs in large organizations, a centralized forecast system capable of satisfying all users' information requirements could be an important information system. Hierarchical forecasting (HF), a family-based forecast methodology, is a centralized forecast approach capable of satisfying the variety of forecast information requirements. HF is able to provide decision support information to many users, each representing different management levels and organizational functions (Fliedner and Mabert, 1992). HF would be an integral element and easily integrated within the framework of the enterprise resource planning (ERP) system.

In addition to the ability of providing forecast information for numerous users, the potential of HF is important for two additional reasons. First, it has the potential to improve forecast accuracy and support improved decision making (Fliedner, 1989). Second, given the proliferation of product lines in order to enhance customer satisfaction, HF systems are being used more extensively in industry as firms attempt to reduce the magnitude of the forecast problem. This suggests a need to understand its capabilities.

As firms pursue cost-cutting measures to enhance competitiveness, the economic incentive for research to provide practical use guidelines regarding HF systems has become increasingly important. To date, several studies have offered practical guidelines for the structural design of HF systems. The primary purpose of this paper is to summarize these guidelines. First, an explanation of the HF process is provided. In this explanation, important system parameters and strategic choices, which allow for the custom configuration of HF systems, are identified. Second, the relevant family-based forecast research is reviewed. The important issues addressed and the conclusions presented in this research are identified. Third, practical guidelines regarding the use of a HF approach that have been reported in the research literature are clearly delineated. With much still unknown regarding the performance impact of various system parameters and strategic process choices, the paper concludes with suggestions for future research.

Hierarchical forecasting process explained

HF systems are used to provide forecast information based on a strategy of grouping

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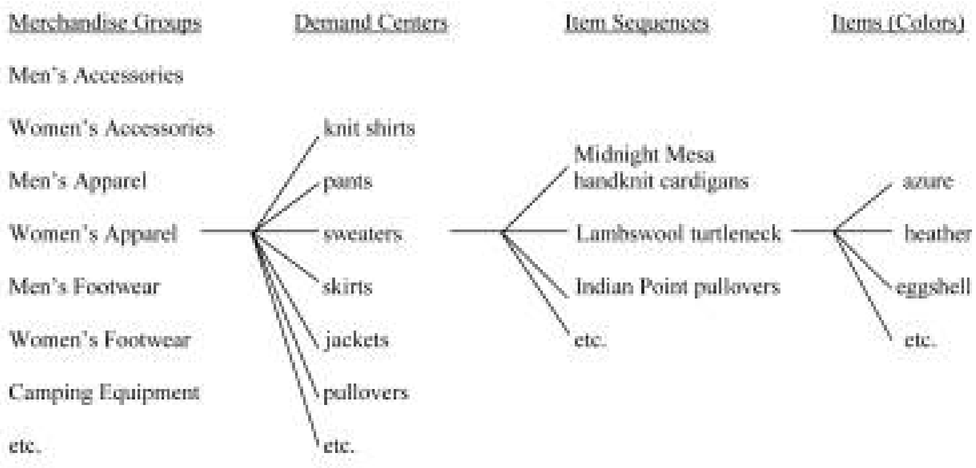
Figure 1
L.L. Bean's product line hierarchical structure



Source: Schliefer, Jr., Arthur. "L. L. Bean, Inc.: Item Forecasting and Inventory Management", Harvard Business School Case (9-893-003), 1992.

items into product families (like that depicted in Figure 1). Often times, this strategy puts similar items in naturally occurring groups similar to L.L. Bean's hierarchical product line structure. The HF process combines "bottom-up" and "top-down" forecast processes. Forecasts of item and family demands are produced using these combined processes. The process of HF is explained here by illustrative example using a three-level, hierarchical structure depicted in Figure 2. A numerical example of the HF process is also provided in the Appendix.

For family i in the current period t, define the family demand, $D_{i,t}$, as the contemporaneous sum of n item time series, where $d_{j,t}$ is the demand of item j. Namely, a family time series is formed as:

$$D_{i,t} = \sum\nolimits_j d_{j,t}, \quad j = 1 \text{ to } n \qquad (1)$$

Similarly, the company-wide aggregate demand, AD_t, for m families is defined items as the contemporaneous sum of family time series as:

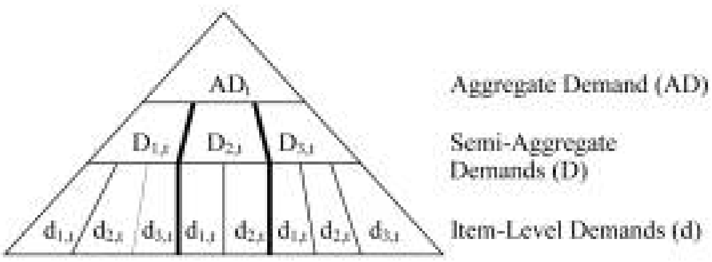
$$AD_t = \sum\nolimits_i D_{i,t}, \quad i = 1 \text{ to } m \qquad (2)$$

HF begins with a two-step "bottom-up" forecast process. These two steps are successively repeated for each level (items, families, families of families, etc.) comprising the hierarchical structure. In step one, a forecast ($f_{i,t+p}$) for each item i, is determined for any future period (t+p). This is done for each of the n items comprising the family and for all of the families that define the lowest level of the hierarchical structure. These forecasts are referred to as direct forecasts as the respective time series for each item is used to directly compute it.

Within commercial HF systems, averaging techniques are often used to determine these direct forecasts due to the large number of items and the frequency of forecast generation. Typically, an averaging technique such as exponential smoothing (Winters, 1960) is used to average k historical observations of demand $d_{j,t}, \alpha_{j,t-1}, \ldots, d_{j,t-k+1}$ ($j = 1, \ldots, n$) to determine these respective independent, item-level forecasts. In step two, item-level demand data are aggregated following equation (1).

The typical HF structure consists of many levels: items, families, families of families, etc. Therefore, this two-step "bottom-up" process is repeated iteratively for each level up the HF system structure until a direct forecast of company-wide demand is determined. Following the hierarchical structure illustrated in Figure 2, an averaging technique could be used to average k historical observations of family demand ($D_{i,t}, D_{i,t-1}, \ldots, D_{i,t-k+1}$) for each of family associated with equation (1) to determine

Figure 2
Illustrative hierarchical structure



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these respective direct and independent family-level forecasts, F_{t+p}. A direct, company-wide aggregate forecast would then determined using the time series depicted in equation (2). This forecast is typically computed in overall dollar sales, whereas lower-level forecasts are often computed in unit values. A simple conversion process may be used to convert the forecast of company-wide dollar demand into a forecast of company-wide unit demand (e.g. division by a company-wide average unit sales price). For simplicity, unit forecasts are assumed throughout this paper.

The direct forecasts determined in the "bottom-up" process are subsequently used within the "top-down" process. In the "top-down" process, with the exception of the top-most level, a derived forecast for any family or family member is determined by prorating the forecast determined at the immediate higher (parent) hierarchical level. These forecasts are referred to as derived forecasts as lower-level (child) forecasts are ultimately derived from parent forecasts. The process begins with the direct forecast of the aggregate company-wide demand determined as the last step in the "bottom-up" process. It is used to determine derived child forecasts with a proration procedure.

Proration may be accomplished in various ways as demonstrated by Gross and Sohl (1990). One common procedure calculates the ratio of the direct child forecast divided by the sum of the direct child forecasts comprising its family, determined in the "bottom-up" process. The parent forecast is multiplied by this ratio. Specifically, derived forecasts for any child i, in a family of n children may be determined as:

$$f_{i,t+p} = F_{t+p}(\text{Direct}\, f_{i,t+p} / \textstyle\sum_i \text{Direct}\, f_{i,t+p}) \quad \text{for } i = 1, \ldots, n \text{ in family.} \tag{3}$$

One other common procedure multiplies the parent forecast, F_{t+p}, by the ratio of respective subaggregate demands to aggregate demand. For example, derived forecasts for forecasts for any item, i, in a family of n items may be determined as:

$$f_{i,t+p} = F_{t+p}(d_{i,t}/D_t) \text{ for } i = 1, \ldots, n \text{ in family.} \tag{4}$$

This process is repeated down the hierarchical product line structure for the number of levels that define the HF system structure. One objective of the "top-down" proration process is to have the resultant sum of n group member (subaggregate) forecasts equal the aggregate forecast between any two adjacent hierarchical levels. This objective may be defined as:

$$F_{t+p} = \textstyle\sum_i f_{i,t+p} \quad \text{for } i = 1, \ldots, n \text{ in family.} \tag{5}$$

Resultant forecasts of this HF process are consistent with forecasts at either a higher or lower hierarchical level, equation (5). It is these derived forecasts determined in the "bottom-down" process that are used for planning and decision-making purposes. Therefore, individuals among the various managerial echelons or across functional disciplines use consistent forecast information.

Attainment of a second objective of improved forecasts will always be situation dependent. Muir (1979) identifies the rational supporting the argument that HF may lead to improved forecast accuracy. The author argues that there is a stabilizing effect from combining demand data of two or more homogenous items. This rational may best be explained by a simple numerical example. Assume four items each have an identical sales pattern of 100 units of average monthly demand with a standard deviation 10. Let these four items comprise a family. Assuming normal and independent demand distributions, the statistical values for the family may be calculated, as shown in Table I. As shown in the table, the standard deviation of monthly or annual demand is proportionately smaller for a family of four items than for the individual item.

Hierarchical forecasting literature review

A large number of choices exist for the design of the HF system. Examples of some important HF system design parameters include:

- the number of system levels;
- the number of families at any level; and
- the number of children within any family.

Examples of some important HF design strategies include:

- the forecast method(s) used to generate direct forecasts in the "bottom-up" process;
- the proration procedure used to disaggregate parent family forecasts in the "top-down" process; and
- the criterion used to identify family members from a pool.

These parameters and strategies are especially important, as each firm will customize its HF system due to its unique characteristics. In this section, HF research literature is reviewed.

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The practice of HF may trace its roots back to research conducted by Theil (1954). In that investigation, two strategies for determining an aggregate forecast are examined:

1 a direct strategy (averaging the aggregate demand observations, D_t); and
2 a derived, "bottom-up" strategy (summing the contemporaneous subaggregate forecasts, $f_{i,t+p}$).

Since 1954, additional investigations of direct and derived strategies for determining aggregate and subaggregate forecasts following "bottom-up" and "top-down" strategies have been reported. Some of these investigations are cited in Table II. Within these noted investigations, the author(s) assumed an economic (theoretical) framework, whereby the properties of the demand generating process were known. Complete (perfect) knowledge of the time series generating process implies that the statistical properties of the demand (the orders and/or coefficients of the subaggregate series) are known prior to determining aggregate forecasts. Regression analysis or univariate, autoregressive-integrated-moving average (ARIMA) forecast models were utilized in these studies.

Economic framework

Whether under conditions of complete or incomplete prior knowledge of the time series generating process, two important observations are apparent in the literature assuming an economic framework. First, as noted by Weatherby (1984), a survey of the economic literature regarding direct and derived ("bottom-up") forecast strategies for an aggregate variable revealed that no general consensus favors direct or derived strategies for determining aggregate demand forecasts. For example, Dunn *et al.* (1976) found the use of a derived forecasting strategy resulted in superior forecast performance. In contrast, Lutkepohl (1984) showed it might be preferable to forecast the aggregate variable using a direct strategy. However, the majority of the results seem to favor a derived strategy by forecasting an aggregate series based on the structural information found in the item-level series. It is surmised that an information loss may exist when summing the contemporaneous subaggregate time series to construct the aggregate time series that is subsequently used to determine the aggregate forecasts. Although, Weatherby (1984) observed that sufficient results do exist to demonstrate that a direct forecast strategy will be preferred in some situations. Second, Wei and Abraham (1981) observed that a direct strategy used to generate a demand forecast of an aggregate variable will not always be superior to a derived forecast strategy and vice versa. Rather, particular modeling assumptions, such as assumed relationships among item-level series, forecast method parameter-estimating procedures, or sampling bias may lead to favoring one strategy over another. Considerable disagreement exists within the economic research literature regarding a preference for a direct or a derived strategy for determining an aggregate variable forecast.

More recently, over the past 25 years, several empirical reports in support of HF have appeared in the literature. The authors of these reports typically assumed a production-planning framework without *a priori* knowledge of the time series statistical properties. Both "top-down" and "bottom-up" strategies have been examined, as forecasts are needed for planning purposes at aggregate, semi-aggregate, and item levels. Smoothing models have been commonly applied within these empirical reports. Several of these studies are addressed below.

Dalrymple (1987) reported on a widely-used product family and item forecasting software package in a US survey of forecast practices. Kuehne and Leach (1982) and Stratton (1979) reported on the implementation of HF systems that are comprised of multiple family levels. Muir (1979) described a HF system that relies on the basic premise of forecasting for families of products. Plossl (1973) argued that forecasts are more accurate for families of products, especially those sold in similar markets. Miller *et al.* (1976) and Barnea and Lakonishok (1980) demonstrated that forecast performance is

Table I
Item versus family demand patterns

Demand statistic	Item	Family
Monthly demand	100 units	4(100) = 400.00 units
Monthly demand standard deviation	10 units	$\sqrt{4(10^2)}$ = 20.00 units
Annual demand	12(100) = 1,200 units	12(100)4 = 4,800 units
Annual demand standard deviation	$\sqrt{12(10^2)}$ = 34.64 units	$\sqrt{12(10^2)4}$ = 69.28 units

Table II
"Direct" versus "derived" forecasting within a theoretical economic framework

Theil (1954)	Shlifer and Wolff (1979)
Grunfeld and Griliches (1960)	Tiao and Guttman (1980)
Orcutt, Watts and Edwards (1968)	Wei and Abraham (1981)
Dunn, Williams and DeChaine (1976)	Kohn (1982)
Rose (1977)	Weatherby (1984)
Ang (1979)	Lutkepohl (1984)

dependent on (1) the magnitude of the correlation between the subaggregate forecast variables, (2) the magnitude of the correlation between forecast errors of subaggregate variables, and (3) the quality of the forecasting technique(s). Miller *et al.* (1976) developed a predictive equation (based on Theil's (1966) U_2' statistic) which may be used to express the difference between forecast error variances associated with direct and derived forecast strategies. Having known or assumed values of the forecast error variances and demand variances for both direct and derived strategies, one may determine theoretical conditions under which one strategy would be preferable.

In the past ten years, four particular investigations of specific parameters and strategies concerning HF system design have appeared in the research literature. Each of these investigations offers important practical guidelines for HF system design. Gross and Sohl (1990) investigated forecast performance at the subaggregate level for a direct item forecast strategy versus a derived item ("top-down") strategy. These authors examined 21 different disaggregation (proration) techniques within a two-level HF system structure. They found that a simple linear average representing an item's fair share of the total group's demand (similar to equation 3) works relatively well.

Fliedner and Mabert (1992) reported on an investigation of two factors (family size and criteria used to identify family members) examined within a two-level HF system. These authors examined performance differences attributable to family size (varied as a consequence of controlling the number of families at values of two, five and ten for a fixed number of items) and four family grouping criteria (unit volume, dollar volume, demand seasonality, and historical forecast performance). Winters' (1960) exponential smoothing was used as the forecast method. Fliedner and Mabert (1992) found grouping criterion significantly impacts HF system performance while the number of families and consequential family size did not.

Fliedner and Lawrence (1995) examined two HF design parameters (grouping criteria and number of families) and two HF process strategies (forecast process and grouping process). Simulating a two-level HF system, these authors examined:

1 Two grouping criteria (unit and dollar volume).
2 Three levels for the number of families (two, five and ten) for a fixed number of items, (3) three short-term forecast methods (simple moving average, simple exponential smoothing, and Winters' (1960) exponential smoothing), and (4) three statistical clustering techniques (Ward's minimum variance, centroid, and average linkage methods).

Fliedner and Lawrence (1995) observed that the alternative grouping criteria, the number of families, and the statistical clustering grouping techniques did not lead to improved forecast performance. These authors observed that simple exponential smoothing performed better than a simple moving average or Winters' (1960) exponential smoothing. They also observed that sophisticated statistical clustering techniques used to identify family members did not lead to improved forecast performance when compared to families that were randomly determined.

Most recently, Fliedner (1999) investigated aggregate-level forecast performance for systematically controlled statistical correlation between two subaggregate time series for a two-level HF structure. Forecast performance for two forecast smoothing techniques (simple moving average and simple exponential smoothing) was examined for five levels (high-negative, low-negative, neutral, low-positive, and high-positive) of systematically-controlled, statistical correlation between subaggregate (item) time series comprising the family. Fliedner (1999) observed that high-positive and high-negative correlation afforded improved forecast performance at the aggregate level. An additional observation suggested that direct forecasts of the aggregate variable were more accurate than derived forecasts of the aggregate variable.

Hierarchical forecasting use guidelines

To date, the research literature concerning a HF system and its structure offers several guidelines for its use in industry. Gross and Sohl (1990) suggested that a simple linear average representing an item's fair share of the total group's demand (similar to equation 4) works relatively well when prorating a parent forecast. Fliedner and Mabert (1992) suggested that unit volume and dollar volume criteria work well for identifying candidate children to form the basis of the HF system structure, the parent group. Subsequently, Fliedner and Lawrence (1995) suggested that the method used to identify group candidates is not significant.

Fliedner (1999) suggested HF system families might be comprised of items possessing similar or polar demand characteristics. For example, items possessing complimentary or contra-cyclical demand patterns may be considered for inclusion within a family. Fliedner (1999) argued that the aggregate time series would reflect less volatility as random, positive noise values of one subaggregate time series cancel with random, negative noise values of the second subaggregate time series. The lower volatility leads to improved forecast performance measured at the aggregate level. The importance of this observation may be compounded while using the "bottom-up" and "top-down" forecasting strategies embodied within commercial HF systems where families of items are comprised of potentially thousands of items.

Future extensions

A number of interesting extensions to the research literature may be proposed. First, the investigations cited in this paper dealt with single- or two-level HF systems. To date, no formal investigation of a HF system comprised of more than two levels has been reported in the literature. Only descriptions of actual implementations of multi-level systems have been described in the literature, by Kuehne and Leach (1982) and Stratton (1979).

Second, characteristics such as item function (similar to Figure 1) that may be used to form groups should be investigated. This represents a convenient criterion that is often selected by default in industry to define the family hierarchy. As observed by Fliedner and Mabert (1992), the criterion used to define groups has a significant impact on HF performance. To date, performance results attributable to families based on item function have not been reported in the literature.

Third, as forecasts are needed for various management echelons and functional disciplines, another extension could examine several forecast approaches within the HF system. For example, qualitative methods (e.g. executive consensus) used at aggregate levels and quantitative forecast methods (e.g. exponential smoothing) used at semi-aggregate and item levels may be more appropriate. This may be more reflective of the types of forecast techniques used for longer-term, strategic planning at the company-wide level and shorter-term, tactical planning at middle and lower hierarchical levels.

Fourth, assuming a sufficient number of empirical time series possessing the necessary correlation could be identified, HF system performance with the use of a large number of empirical time series would afford additional insight regarding the implications of time series correlation on family composition. Fliedner (1999) observed that correlation led to significant performance differences for direct and derived forecasts of an aggregate variable.

Finally, determining the costs and benefits attributable to the use of a centralized HF system versus the more common use of decentralized forecast systems within a single firm would also be interesting. Although this would be challenging, it would clearly prove worthy given the increasing use of HF systems in industry.

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Appendix

A simple numerical example can illustrate the Hierarchical Forecasting (HF) process. In this example, exponential smoothing with an alpha (α) value of 0.10 is used to calculate forecasts for all items or groups of items in the hierarchical structure. This process used is depicted by equation (6) which depicts the forecast for a future period, F_{t+p}, being determined as a weighted summation of historical demand values, D_t.

$$F_{t+p} = \alpha(D_t) + (1 - \alpha)F_t \qquad (6)$$

The hierarchical structure used in this example consists of three levels. The item level has one three-member family and one two-member family. This structure is represented in Figure A1.

For this example, assume at the end of the preceding period, forecasts for the current period (F_t) were calculated as shown in Figure A1. Then, at the end of the current period, demand data (D_t) is recorded. Item demand values are aggregated to the respective semi-aggregate level and semi-aggregate demands are summed to the aggregate level. These values are depicted in Figure A2. Using the posted demand values along with the previous forecasts, forecasts for future periods ($t+p$) can be determined. These forecasts are determined as shown in Table AI and are also represented in Figure A2.

As shown in Figure A2, the forecasts determined in the "bottom-up" process demonstrate that a forecast determined for a family of items does not equal the sum of the forecasts for the items comprising the family. The "top-down" process of HF adjusts the "bottom-up" forecasts such that the family forecast at any level equals the sum of the forecasts for the items comprising the family. In order to accomplish this, the forecasts determined in the "bottom-up" process are used within the "top-down" process.

In the "top-down" process, with the exception of the top-most level, a forecast for any family or family member is determined by prorating the forecast determined at the immediate higher (parent) hierarchical level. The process begins with the forecast of the aggregate company-wide demand determined as the last step in the "bottom-up" process. It is used to determine child forecasts with a proration procedure. One common procedure

Table AI
Forecast value determination

Item level forecasts

Group 1: $f_{1,t+p} = 0.1(15) + 0.9(18) = 17.7$
$f_{2,t+p} = 0.1(18) + 0.9(15) = 15.3$
$f_{3,t+p} = 0.1(17) + 0.9(16) = 16.1$
Group 2: $f_{1,t+p} = 0.1(80) + 0.9(75) = 75.5$
$f_{2,t+p} = 0.1(90) + 0.9(96) = 95.4$

Semi-aggregate forecasts

Group 1: $F_{1,t+p} = 0.1(50) + 0.9(55) = 54.5$
$F_{2,t+p} = 0.1(170) + 0.9(166) = 166.4$

Aggregate forecast

$F_{t+p} = 0.1(220) + 0.9(222) = 221.8$

calculates the ratio of the "bottom-up" family forecast divided by the sum of the "bottom-up" child forecasts comprising its family, equation (2). The parent forecast, F_{t+p}, is then multiplied by this ratio.

This process is repeated down the hierarchical product line structure for the number of levels that define the HF system structure. Prorating the "bottom-up" forecasts in the "top-down" process using the process of equation (3) yields the forecasts depicted in Table AII. The reader will note that the sum of the semi-aggregate forecasts equals the aggregate forecast. Similarly, the sum of the item-level forecasts by group equals the semi-aggregate forecast for its group.

Figure A1
Preceding period forecasts

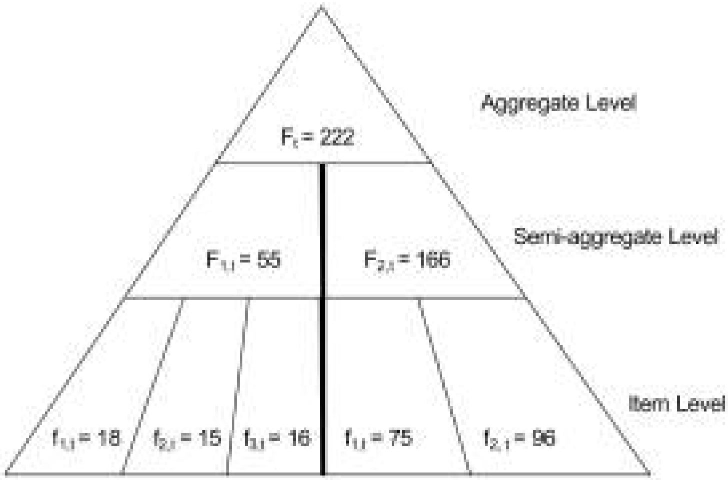
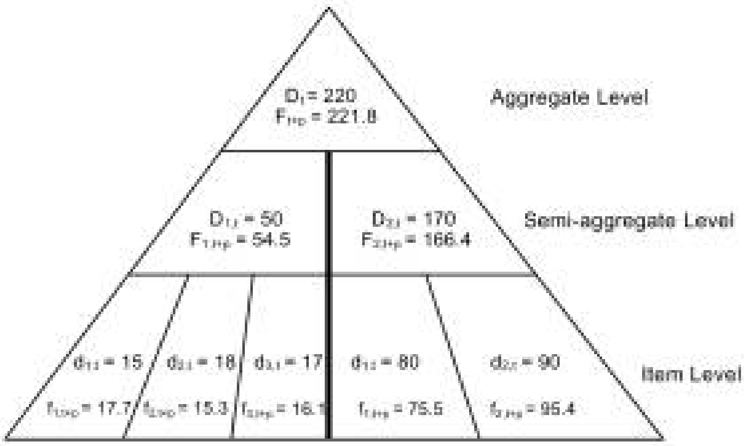


Table AII
"Top-down" hierarchical forecast value determination

Aggregate forecast:		F_{t+p} = 221.80
Semi-aggreate forecasts:		Group 1: $F_{1,t+p}$ = 221.8(54.5/(54.5+166.4) = 54.72
		Group 2: $F_{2,t+p}$ = 221.8(54.5/(54.5+166.4) = 167.08
Item level forecasts:		
	Group 1,	Item 1: $f_{1,t+p}$ = 54.72(17.7/(17.7+15.3+16.1) = 19.73
		Item 2: $f_{2,t+p}$ = 54.72(15.3/(17.7+15.3+16.1) = 17.05
		Item 3: $f_{3,t+p}$ = 54.72(16.1/(17.7+15.3+16.1) = 17.94
	Group 2,	Item 1: $f_{1,t+p}$ = 167.08(75.5/(75.5+95.4) = 73.81
		Item 2: $f_{2,t+p}$ = 167.08(95.4/(75.5+95.4) = 93.26

Figure A2
Current demands and future forecasts



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