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Disaggregation Methods to Expedite Product Line Forecasting

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ABSTRACT

This paper addresses the issue of forecasting individual items within a product line; where each line includes several independent but closely related products. The purpose of the research was to reduce the overall forecasting burden by developing and assessing schemes of disaggregating forecasts of a total product line to the related individual items. Measures were developed to determine appropriate disaggregated methodologies and to compare the forecast accuracy of individual product forecasts versus disaggregated totals. Several of the procedures used were based upon extensions of the combination of forecast research and applied to disaggregations of total forecasts of product lines. The objective was to identify situations when it was advantageous to produce disaggregated forecasts, and if advantageous, which method of disaggregation to utilize. This involved identification of the general conceptual characteristics within a set of product line data that might cause a disaggregation method to produce relatively accurate forecasts. These conceptual characteristics provided guidelines for forecasters on how to select a disaggregation method and under what conditions a particular method is applicable.

KEY WORDS Product line Forecasting Disaggregational methods Time series analysis Composite root mean square error differential

As an organization expands, its sales forecasting activities often escalate in scope to become a very burdensome set of tasks. Various forecasts may be required, including: multiple forecasts under different planning horizons—the immediate, short, medium and long term (Wheelwright and Makridakis, 1985); several aggregate forecasts—for entire product lines, divisions, the total organization and for the entire industry(s) in which the firm competes (Armstrong, 1985); and numerous disaggregate forecasts at the individual product and product/market segment levels for inventory management, product scheduling, advertising planning and similar short-range decision making (Willis, 1987; Hurwood *et al.*, 1978; Day, 1977).

Perhaps the greatest forecasting effort stems from the generation of these disaggregate forecasts due to the very large number of forecasts potentially involved. To illustrate, a firm

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that markets ten product lines, with each containing 50 products and with each being marketed to five different markets, requires that $10 \times 50 \times 5 = 2500$ separate forecasts are needed each period if the disaggregate level is to be estimated.

To effectively manage the voluminous forecasting task, managers have the following options; listed in order of increasing burden (Dalrymple, 1987):

(1) Forecast the total aggregates, then disaggregate these totals to the individual products and/or markets according to some predetermined weighting scheme. This option involves the least amount of effort.
(2) Forecast individual disaggregates for each unit, then sum them to provide the aggregate forecasts. This option portrays a middle ground in terms of the required forecasting work effort.
(3) Provide separate forecasts for all levels of concern, and ignore whether or not each of the separate forecasts sum to the aggregates. This option involves the greatest level of work, but it is likely to lead to the greatest level of forecasting accuracy due to the customization of individual models.

In choosing which option to follow, management must assess the trade-off between (a) the perceived benefits derived from generating separate forecasts (usually defined in terms of greater forecast accuracy) and (b) the economies derived from some disaggregation scheme that, presumably, involves lowering forecast accuracy.

This paper examines whether or not the extensive literature which combines forecasts from different models, with each forecasting a single data series, may be extended to the application of accurately disaggregating forecasts within a product line. The purpose is to examine possible methods of reducing the overall forecasting burden by focusing on the first alternative—the development of reasonably accurate disaggregation methods.

DATA AND VARIABLES

Data were obtained from the Metal Products Division of USG Industries, Inc. (a subsidiary of US Gypsum, Inc.). Three major industrial galvanized steel product lines were examined, which were distributed internationally. 'Grip strut' was a type of steel industrial flooring material that contained stamped, diamond-patterned perforations which provided a non-slip surface for areas such as catwalks, platforms and decks. There were seven product categories within this line (referred herein as GS1, GS2, ..., GS7), each with differing dimensions. 'Channel' comprised heavy-gauged steel tracks used primarily to house electrical cable in industrial areas. There were three product categories within channel (CH1, CH2 and CH3), each with varying dimensions. Finally, 'Accessories' (ACC1, ACC2, ..., ACC5) comprised various connectors and fittings used to assemble and install channel products.

Data collected were for the period January 1983 to May 1987, which provided a total of 18 monthly time series with 53 observations each. Unit sales were used in place of dollar sales to eliminate any price-related exogenous effects (Schnaars, 1986). An initial examination revealed very low correlations among the sales of all products, both within and between the product lines. Thus, all products were treated as independent in the analysis, suggesting the construction of separate forecasting models for each time series.

METHOD

The study compared the results of various disaggregation methods that were applied to each product line with individually generated, optimal forecasts to assess the extent of any loss in forecast accuracy. Procedures used to evaluate the question of using total versus individual forecasts are developed in this section. Results for all the product lines examined are presented in the Empirical Results section below.

Individual forecasting procedures

While several error measurements were available (Wheelwright and Makridakis, 1985), the root mean square error (RMSE) was considered the most meaningful to compare forecasting models on the raw series because of its simplicity and ability to weight heavily the magnitude of error. Further, use of the mean absolute percentage error (MAPE) was rejected. Several of the individual products contained zero or near-zero reported sales for various periods (i.e. MAPE would be undefined) due to a combination of low reported sales in some periods and the practice of netting sales returns with new sales. No attempt was made to adjust sales in previous periods for such sales returns as the details were irretrievable from the company's records.

Several forecasting models were considered for each series, including: exponential smoothing (ES) (single, double, Holt's two parameter, Winters' additive and multiplicative, and quadratic), time series regression (linear, quadratic and logarithmic), and two deseasonalization schemes (classical—method 1, modified Census II—method 2; as described in Gross and Peterson, 1983). Least squares procedures were used to initialize ES models (derived in Brown, 1963; detailed in Gross and Peterson, 1983). All the forecasting models were also designed to calculate and use optimal weights. These models were chosen because they appeared to be the best predictors for relatively short-term time series (Granger and Newbold, 1977), were relatively simple to use and they have been shown to be mathematically equivalent to several Box–Jenkins specifications (Bowerman and O'Connell, 1987; McKenzie, 1984; Ledolter and Abraham, 1984). The forecasting models used are summarized in Appendix 1.

Reducing RMSE was the objective established for selecting which particular forecasting model to choose for a given series, though additional criteria were also used. Each identified model was applied to two distinct phases to test model stability against known actual values. The models were first applied to periods 1 through 42 of each series and used to forecast the actual values for periods 43 through 48. The models were also applied to periods 1 through 48 and used to forecast values for periods 49 through 53. Each identified model was examined to assure that an appreciable inflation in the RMSE did not occur within the forecast phases. Finally, each model was applied to all 53 periods both to assess the reasonableness of the RMSEs and to update the coefficients. Further, the autocorrelations of the errors were examined during testing within each phase to assure that they resembled 'white noise' Table 1 provides a list of the final models used and their associated RMSEs at various stability-testing phases. Thus, the models used were generally quite stable.

There were a total of twelve different models chosen on the basis of the above criteria to forecast the eighteen series, as illustrated in Table 1. While such diversity might not be expected in many applications, in this case the variety of models used to produce the individual forecasts was considered reasonable. Many of the individual products were targeted toward specific, independent market segments and, naturally, were expected to evidence different sales patterns. Since the markets were quite dissimilar, it appeared reasonable that different models would be useful for forecasting sales. It is worth noting, however, that all the models chosen were members of three basic families of models and situations; no trend, linear trend and additive

Table 1. Optimal models identified for individual forecasts

Product	Forecasting model	RMSE[a] 1–42	RMSE[b] 43–48	RMSE[a] 1–48	RMSE[b] 49–53	RMSE[a] 1–53
Grip Strut (GS):						
GS1	Winters' Additive	1485	1134	1429	884	1383
GS2	Holt's ES (no deseas.)	1841	1693	1782	1287	1751
GS3	Holt's ES (deseas., method 1)	923	834	755	1095	763
GS4	Linear time series regression (deseas., method 1)	3594	3804	3450	4721	3562
GS5	Holt's ES (deseas., method 1)	356	648	378	560	376
GS6	Mean (deseas., method 2)	1590	1406	1436	1581	1457
GS7	Single ES (deseas., method 1)	1581	1927	1643	807	1563
Total GS	Linear time series regression Line (deseas., method 1)	6496	6715	6128	5648	6070
Channel (CH):						
CH1	Linear time series regression (deseas., method 1)	17058	16409	15264	17847	15397
CH2	Linear time series regression (deseas., method 2)	7803	9397	7377	15456	7756
CH3	Double ES (deseas., method 1)	6202	3435	5918	20715	8007
Total Line	CH Holt's ES (deseas., method 1)	25000	17354	21381	34875	22497
Accessories (ACC):						
ACC1	Holt's ES (no deseas.)	32156	7620	29890	3282	29369
ACC2	Single ES (no deseas.)	554	520	548	692	563
ACC3	Holt's ES (deseas., method 2)	591	615	565	1211	604
ACC4	Decomp.-(deseas., method 1, 10 mo. cycle)	7339	8797	7790	4696	7128
ACC5	Mean (deseas, method 1)	3305	1799	2499	3169	2526
Total Line	ACC Single ES (deseas., method 1)	28062	11533	25429	22595	24716

[a] Root mean square error for within-data forecasts.
[b] Root mean square error for actual forecasts.

trend. Thus, the models used were not unrelated to each other, nor were they assigned by chance.

While models were carefully selected according to the stated criteria, the resulting forecasts were viewed as givens. Thus, the primary focus of the study was directed to the various disaggregation methods evaluated below.

Proportional disaggregation methods

Twenty-one different proportional disaggregation schemes were examined in the study and included simple averages of the sales proportions, lagged proportions and combined lagged proportions. The methods used are summarized in Appendix 2. The procedures used had their foundation in the forecasting combination literature (for a summarized review of various combination methods, see Mahmoud, 1984). That literature addressed the issue of evaluating methods of combining forecasts, both subjective and objective, for a single time series. In this study these combination methods were extended to product line forecasting.

Methods A and F were based on the simple average (equal weights) of a product's share of total product line sales, over the entire historical period (Makridakis and Winkler, 1983). Method A was constructed to reflect the mean proportion that a given product represented of the total product line's sales. Method F was structured to represent the proportion that a given product's sales represented of the overall product line's mean sales.

Methods B1 through B4 were based upon the conjecture that the proportions of previous time periods were the best predictors for the next period's proportion of total sales. Thus, method B1 used a product's proportion in period $t-1$ as the disaggregation measure for period t. Similarly, methods B2 through B4 utilized two-, three- and four-period lags, respectively, for disaggregating a product line forecast to individual products. Method C attempted to capture trends in the proportions through a moving average of two-, three- and four-period lagged proportions (methods C2, C3 and C4).

The remaining disaggregation schemes investigated the use of differential weights. Methods D2 through D4 combined lagged proportions (lags of two, three and four periods, respectively) using weights based upon the correlations between the lag and the current period's proportion. These weights were formed by dividing each lag's correlation by the sum of all the correlations under consideration, with lags having the highest correlation receiving the maximum weight, the next highest the next largest weight, etc. (Ashton and Ashton, 1985).

In method E a regression model was used to determine the weights assigned to each lag (Bopp, 1985; Granger and Newbold, 1977). A restriction was imposed that the regression coefficients sum to 1.0 to maintain consistency between the differential weighting methods.

Methods G and H considered differential weights based upon the variance and the covariance of the errors (Bates and Granger, 1969; Newbold and Granger, 1974). In the context of our research, the error for lag one was defined as the difference between the actual sales proportion in period t and $t-1$. Likewise, for lag two, the difference between actual sales proportion in period t and $t-2$ was considered the lag-two error. Methods G2 and G3 (lags one and two, and lags one, two and three, respectively) combined the lagged proportions with differential weights based on the variance and covariance of the error terms. The lagged pair with the smallest variance–covariance structure was given the largest weight, with decreasing weights assigned accordingly. Methods H2 and H3 were similar to method G, except that the weights were based on the variance of the errors.

Method I was similar to method D with differential weights based on the correlation between a lag's proportion and the current period's proportion. However, in method I2 (for example), the two lags having the highest correlation with the current period's lag were combined, with the differential weights based on the value of the correlation, and with the weights then normalized. Method I3 considered the three lags with the largest correlations and method I4 combined four lags. Thus, method I chose the best predictors of the current proportion and used a differential moving average to combine the weights as a means of identifying trends.

Comparison procedure

Let P denote the proposition used for disaggregating the total forecast. Then, to obtain a prorated forecast from the total:

$$F_{it} = P_{it} T_t$$

where

F_{it} = forecast of product i for period t, using P_{it} as the weighting scheme,
T_t = total forecast for all products in the line for period t,
P_{it} = the proportion used to allocate product i's share of the total forecast at time t,

$t = 1, 2, 3, \ldots, n,$
n = total number of time periods,
$i = 1, 2, 3, \ldots, N,$
N = total number of products within the product line.

The corresponding RMSE for the prorated forecast (RMSEP) for product i is:

$$\text{RMSEP}_i = \left\{ \frac{\sum_{t=1}^{n} (X_{it} - F_{it})^2}{n} \right\}^{1/2}$$

where

X_{it} = actual sales for product i in period t.

To compare the RMSE from a given weighting scheme with the RMSE for the identified optimal individual product forecasting model, we defined:

$$\text{D}_i = \text{RMSEP}_i - \text{RMSE}_i$$

where

RMSE_i = the root mean square error for the individual product i model,
D_i = the root mean square error differential.

A positive D_i indicated that, for product i, the RMSE from the individual forecast was lower than the RMSE for the disaggregated total and, thus, implied that better forecasting was possible using a model fit specifically to product i sales data, rather than using total sales and then disaggregating. In contrast, a negative score indicated the opposite: that a better result was possible by first forecasting the total, then disaggregating according to the applicable proportional scheme (Mathews and Diamantapoulos, 1986).

Composite measure

A composite measure was designed to provide a summary statistic for an entire product line and for a given decomposition procedure. This was obtained by weighting each different RMSE by the average proportion of an individual product's sales to the total sales of that product line. Specifically,

$$CD = \sum_{i=1}^{N} D_i \overline{PR_i}$$

where

CD = composite root mean square error differential,
$\overline{PR_i}$ = average proportion of total product line sales for product i

For the composite measure, a negative CD suggested that relatively accurate forecasts were possible through a procedure of first forecasting the total product line and then disaggregating such a total according to the relative proportional scheme. A positive CD suggested the opposite: that more accurate forecasts resulted from separately forecasting each product within the line as well as the line's total sales. A CD of zero (or close to zero) suggested indifference between the two approaches in terms of accuracy. The CD coefficients also provided a means of evaluating various proportional allocation schemes under a disaggregation strategy. The most negative (or least positive) CD corresponded to the disaggregation scheme resulting in the lowest RMSE. Thus, the selection of a proportional method was not relegated to some arbitrary choice and the composite measure provided a means for management to assess the impact on forecast accuracy if a disaggregation scheme were selected over forecasting at the individual product level.

EMPIRICAL RESULTS

Tables 2 and 3 summarize the results of the study. Three sets of *CD* scores for each product line are shown in Table 2; for periods 1 through 42, 1 through 48 and 1 through 53. It is apparent in the table that the *CD* scores remained relatively stable between each of the phases used to test model stability. As expected, because reducing RMSE was a primary objective used in their selection, the individual forecasting models were superior to the disaggregation models for all product lines. This is indicated in Table 2 by the generally positive *CD* scores.

Table 3 presents the results of directly examining the forecasts produced by the disaggregation methods. The RMSEs for each model relative to forecasts of periods 43 through 48 based on actual data from periods 1 through 42 are presented in Part A of the table. Part B contains the results of forecasting periods 49 through 53 on the basis of actual data in periods 1 through 48. For comparison, the RMSEs for the corresponding time period, based upon using the individual optimal model for each product (Table 1), are presented at the top portion of each table. The measures presented in Table 3, therefore, illustrate the net forecast accuracy of each disaggregation method for each product over two forecast phases. Again, there is a high

Table 2. Composite differential (*CD*) results

	Grip strut			Channel			Accessories		
Method	1–42	1–48	1–53	1–42	1–48	1–53	1–42	1–48	1–53
A	188	94	76	1159	1771	2419	3383	1525	1626
B1	665	583	572	2173	2708	3070	5733	3368	3546
B2	711	586	609	1808	2127	3188	2274	884	1062
B3	970	864	803	1874	2342	2214	3572	2174	2387
B4	980	906	831	1537	2235	3099	3000	1764	1828
C2	430	327	346	1062	1442	1958	2862	1100	1260
C3	439	347	339	505	981	1094	2424	846	1008
C4	443	349	325	455	938	1202	2219	685	796
D2	675	558	459	1032	1410	1971	2265	874	1054
D3	687	550	456	407	893	1127	2378	973	1146
D4	624	490	454	336	819	1174	1893	590	709
E	407	301	294	392	868	1252	1807	438	520
F	187	93	74	1159	1770	2414	1456	264	507
G2	428	326	341	1017	1393	1943	2425	826	993
G3	412	312	308	450	916	1221	2049	578	695
H2	426	325	343	1021	1392	1944	2583	925	1099
H3	425	332	327	428	910	1093	2274	735	898
I1	597	549	498	1808	2127	2827	1856	726	681
I2	421	356	262	1072	1389	1883	1859	317	319
I3	368	279	190	443	892	1346	1551	241	−12
I4	327	248	168	109	833	1084	1572	−54	−47

Note: A negative score signals a reduced RMSE-based *CD* score through proportional allocation.

Table 3. RMSEs for individual products resulting from disaggregated forecasting methods
(A) Forecast of periods 43–48 based upon periods 1 through 42

	GS1	GS2	GS3	G34	GS5	GS6	GS7
Individual model	1134	1693	834	3804	648	1406	1927
A	857	1095	775	3235	317	1513	1636
B1	1174	779	301	2961	918	2983	1923
B2	1477	697	306	3462	328	3645	1672
B3	926	1450	563	2942	261	1686	2857
B4	1420	705	825	2837	585	5197	1964
C2	869	727	303	2893	428	3312	1771
C3	885	898	355	3116	339	2745	2049
C4	972	798	303	2995	393	3345	1761
D2	1477	713	306	3462	328	3495	1672
D3	1477	1110	342	3462	285	3495	1672
D4	1477	1019	342	3156	261	3495	1672
E	1067	887	325	2942	321	3562	1653
F	856	1123	805	3262	309	1484	1639
G2	918	725	304	2928	370	3397	1773
G3	947	961	373	3109	287	3288	1912
H2	892	726	303	2909	398	3365	1772
H3	901	922	357	3116	321	2913	2006
I1	1031	1450	306	2839	511	3645	1835
I2	846	1068	360	4251	523	2353	1792
I3	853	965	377	4319	447	2388	1765
I4	875	921	334	4126	414	2455	1726

	CH1	CH2	CH3	ACC1	ACC2	ACC3	ACC4	ACC5
Individual model	16409	9397	3435	7620	520	615	8797	1799
A	18547	10642	5186	5528	567	732	7622	1252
B1	14365	9645	2311	5617	800	594	7686	1467
B2	21201	10949	2588	5219	620	924	7776	1224
B3	23058	10589	2648	5361	846	616	7623	1207
B4	28776	13902	4517	5676	657	634	7598	4147
C2	16975	7919	2434	5361	702	747	7602	1247
C3	18775	8423	2183	5361	747	701	7607	1222
C4	2103	9392	2320	5211	723	683	7603	1658
D2	17178	7972	2588	5219	722	594	7776	1224
D3	18111	8223	2493	5226	722	605	7776	1215
D4	20035	8884	2493	5239	722	605	7649	1215
E	19757	8896	2488	5193	713	618	7620	1401
F	18497	10605	5206	7102	624	855	8270	1247
G2	17377	8143	2496	5308	710	707	7625	1226
G3	17968	8555	2274	5293	727	628	7643	1209
H2	17200	8042	2473	5329	706	728	7613	1232
H3	18598	8498	2215	5338	741	687	7616	1215
I1	21201	10949	2588	5219	672	944	8795	1213
I2	17178	7972	3172	5252	632	799	8227	1218
I3	19532	8207	3357	5282	664	749	7925	1214
I4	19256	8104	3442	5364	658	720	8159	1217

(B) Forecast of periods 49–53 based upon periods 1 through 48

	GS1	GS2	GS3	G34	GS5	GS6	GS7
Individual model	884	1287	1095	4721	560	1581	807
A	1048	1415	876	4205	516	712	935
B1	2082	1035	858	4214	527	863	1559
B2	1278	1108	851	4217	571	1742	799
B3	1055	1678	1025	4672	479	1218	1637
B4	1440	899	848	4270	458	979	1204
C2	1643	1071	854	4216	549	1259	707
C3	1376	1250	887	4297	523	1246	978
C4	1127	1142	869	4290	504	1175	648
D2	1278	1097	851	4217	571	1453	799
D3	1278	1461	890	4217	480	1453	799
D4	1278	1378	890	4217	472	1453	799
E	1189	1266	888	4263	501	1171	575
F	1049	1438	878	4207	523	714	986
G2	1603	1073	853	4216	549	1346	698
G3	1434	1316	893	4269	519	1351	861
H2	1622	1072	853	4216	549	1309	702
H3	1391	1273	888	4288	521	1286	945
I1	1380	1678	851	5300	513	1742	4129
I2	1419	1656	866	5036	462	1457	2530
I3	1306	1521	888	4697	523	996	2535
I4	1225	1408	908	4629	514	728	2346

	CH1	CH2	CH3	ACC1	ACC2	ACC3	ACC4	ACC5
Individual model	17847	15456	20715	3282	692	1211	4696	3169
A	20541	22272	14679	6707	867	1227	14364	2215
B1	22952	28044	16360	9643	1576	1408	13466	1026
B2	21396	13578	16908	5468	955	1246	16024	955
B3	22352	10614	20284	6502	756	1284	16064	822
B4	25580	10457	16994	1030	856	2562	16957	3062
C2	20565	20253	16633	7462	1240	1311	14657	807
C3	20524	16059	17813	7142	1005	1302	15108	811
C4	21075	13799	17607	5547	854	1519	15548	1045
D2	20494	19603	16896	5468	1371	1408	16024	955
D3	20480	16390	17345	5571	1371	1347	16024	917
D4	20887	14181	17345	3508	1371	1347	16442	1171
E	20968	13396	17649	4685	1038	1424	15908	1012
F	20555	22461	14636	11377	777	1224	12389	1989
G2	20468	18753	16747	6939	1289	1336	14981	809
G3	20496	15399	17562	6554	1212	1334	15384	807
H2	20501	19487	16705	7161	1261	1322	14834	805
H3	20525	15819	17670	6965	1060	1308	15187	807
I1	21396	13578	16908	5468	4579	1253	21481	4102
I2	20494	19603	16966	3048	2730	1282	18710	5687
I3	20715	15613	17327	2368	1976	1311	18194	5562
I4	20878	14181	17243	3193	1896	1255	17107	4756

degree of model stability between periods, as well as consistency with the methods identified as performing well on the basis of *CD* scores, as reported in Table 2.

The remaining question was whether any incremental error was large compared to the efficiencies gained by adopting a disaggregation strategy. For the grip strut product line, methods A and F performed reasonably well, with the *CD* scores close to zero, and forecasts nearly as good as the optimal individual model forecasts. Methods I3 and I4 produced slightly higher *CD* scores then did methods A and F, though their results were also quite good. On the basis of forecast accuracy alone, therefore, one would have been relatively indifferent between between using one of these four proportional schemes and preparing separate forecasts. Considering the effort required to generate individual forecasts on an ongoing basis, however, one of the four proportional schemes (A, F, I3 or I4) would have likely been selected. The routine forecasting task would then have been reduced from detailed analyses of eight separate series (one for each product plus one for the total line) to one series (the total), which would then have been disaggregated on the basis of the selected proportional scheme.

Disaggregating procedures also appeared promising for the accessory products. As Table 2 illustrates, method I3 and I4 generated slightly negative *CD* measures for periods 1 through 48 and 1 through 53. Similarly, their resulting forecasts were relatively accurate as compared with the individual optimal models (Table 3). These factors suggested that disaggregation would have been favorable for accessory products. While the results by no means imply that forecasting accuracy could generally be improved by using disaggregational methods, nonetheless they did indicate that the errors might not deteriorate much, if at all, by so doing, and at a significantly reduced computational effort.

The opposite conclusion was reached for the channel product line. As Tables 2 and 3 illustrate, the *CD* measures were both large and positive for each of the proportional schemes examined, and the RMSEs were generally much larger than those derived from the individual optimal models. Thus, routinely performing separate product forecasts (for individual products as well as for the line's total sales) substantially improved overall forecast accuracy for channel products. Accordingly, management would not likely have chosen any proportional disaggregating scheme for forecasting channel products.

WHY DISAGGREGATION IS APPLICABLE IN SOME CASES

The results of this study represented the findings of three particular cases of one company. As a result, it was difficult to generalize the findings to other cases. However, attention was directed to understanding why disaggregation was applicable for the grip strut and accessories product lines but not for the channel line. This involved an in-depth analysis of the data sets to discover the common characteristics of those series where disaggregation appeared to work well. First, we considered the general conceptual characteristics within a set of product line data which might cause a disaggregation method to become relatively accurate. Second, we examined the results of this study in light of the general conceptual characteristics to learn if they were consistent with the findings.

Conceptual characteristics favoring disaggregation

In general, the applicability of a disaggregation procedure was considered a function of three factors: (1) the accuracy of the total product line forecast, (2) the accuracy of the proportion used in conjunction with the size of the individual product's share of the total product line's sales, and (3) the degree of accuracy attained in forecasting at the individual product level.

Although these factors were interconnected, an apparent hierarchical structure emerged to provide guidelines for determining whether or not disaggregation would seem operationally feasible.

At the first level, both the direction and the magnitude of the total product line forecast accuracy was considered. Given an accurate total forecast, the second level was to examine the magnitude of the errors of the estimated proportions used to allocate a product's share of the total forecast and the individual product's average share of the total product line's sales. The larger a product's share, the greater the accuracy required for the estimated proportions. If a total forecasting model consistently under- or overestimated the total product line's sales, then a disaggregation method whose estimated proportions was biased in the direction opposite to that of the total model should have tended to yield accurate disaggregated forecasts. This bias should have also been most prominent for those products with the largest share of the total product line's sales. For cases where the total forecast was inaccurate with no distinguished direction for the bias there appeared to be little hope for a disaggregation procedure to yield promising results. Specifically, for the second level to have provided accurate forecasts, the estimated proportions for the major products would have needed to have been accurate, at a minimum, to avoid compounding the effect of a poor total forecast, or have been biased in a compensating direction to have mitigated the effect of a poor total forecast. In general, the chances of this optimal matching appeared to be minimal.

Given satisfactory results at the first two levels, the third level addressed the degree of accuracy attained in forecasting at the individual product level. If an individual product forecasting model demonstrated a poor forecasting performance (a high RMSE relative to other products in the product line), then a disaggregated forecast needed only to perform better than this benchmark. The implication was that if an individual series was inherently difficult to forecast, the difficulty should have been considered when assessing the forecasting performance of the disaggregation procedure.

In summary, the critical issues for conceptually determining the usefulness of pursuing a disaggregation strategy to product line forecasting was an identification of the situation when it appeared advantageous to use a disaggregation method and, if advantageous, which method of disaggregation to have utilized. The three-level structure discussed above provided conceptual guidelines for assessing the appropriateness of disaggregation. Given that the hierarchical structure identified candidates for disaggregation, the composite differential measure was used to identify which of the disaggregation procedures appeared most applicable.

Empirical assessment

The results were examined in light of the above three conceptual determinants (factors). At the first level, Table 4 summarizes both the direction and the magnitude of the total product line forecast accuracy for comparable periods examined in previous tables. The total sales of the grip strut product line was forecast relatively accurately, with between 60% and 74% of the total forecasts within 20% of actual sales during corresponding time periods. This implied that for grip strut, the appropriateness of disaggregation was largely determined by the accuracy of the proportions for those individual products with a substantial share of the total product line's sales (level two). For disaggregation procedures A and F, over 90% of the forecasted proportions were within approximately 10% of the actual individual products' market share over all time periods. Thus, the accurate disaggregation achieved by methods A and F combined with a stable and consistent total forecast resulted in the low composite measures. In addition, each of the disaggregated models generally compared favorably to the benchmark RMSEs of the individual models (Tables 1 and 3).

Table 4. Total product line forecast accuracy (frequency)
pe = (forecast − actual)/actual
(A) Grip strut

pe	1–42[a]	43–48[b]	1–48[a]	49–53[b]	1–53[a]
$pe \leqslant -0.30$	0%	0%	0%	0%	0%
$-0.30 < pe \leqslant -0.20$	14	0	10	20	11
$-0.20 < pe \leqslant -0.10$	5	17	12.5	0	11
$-0.10 < pe \leqslant 0.10$	55	50	52	40	50
$0.10 < pe \leqslant 0.20$	14	0	4	20	9
$0.20 < pe \leqslant 0.30$	7	0	12.5	20	13
$pe > 0.30$	5	33	8	0	6

(B) Channel

pe	1–42[a]	43–48[b]	1–48[a]	49–53[b]	1–53[a]
$pe \leqslant -0.30$	14%	0%	8%	40%	8%
$-0.30 < pe \leqslant -0.20$	12	0	12.5	20	13
$-0.20 < pe \leqslant -0.10$	19	0	10	20	13
$-0.10 < pe \leqslant 0.10$	24	66	37.5	20	34
$0.10 < pe \leqslant 0.20$	5	0	8	0	9
$0.20 < pe \leqslant 0.30$	7	17	4	0	4
$pe > 0.30$	19	17	19	0	19

(C) Accessories

pe	1–42[a]	43–48[b]	1–48[a]	49–53[b]	1–53[a]
$pe \leqslant -0.30$	14%	33%	12.5%	0%	11%
$-0.30 < pe \leqslant -0.20$	12	0	6	0	4
$-0.20 < pe \leqslant -0.10$	7	17	2	0	13
$-0.10 < pe \leqslant 0.10$	9.5	0	12.5	40	15
$0.10 < pe \leqslant 0.20$	9.5	17	12.5	0	8
$0.20 < pe \leqslant 0.30$	5	0	4	0	6
$pe > 0.30$	43	33	44	60	43

[a] Within-data forecasts
[b] Actual forecasts

For accessories, Table 4 indicates that the total forecasting model consistently overestimated the actual total product line sales. Since the bias was unidirectional, a disaggregation method whose estimated proportions were biased in the opposite direction for those products with the largest share of total product line sales should have produced accurate disaggregated forecasts. Two accessory products (one and four) accounted for, on average, 89% of the product line's sales. For disaggregation methods I3 and I4, the proportions for product four were underestimated when the total forecast was above the actual, and accurate (within 10% of the actual) when the difference between the actual and the total forecast was small. These optimal matches of estimated proportions and total product line sales forecasts combined to produce relatively accurate disaggregated forecasts. For accessory product one, the estimated proportions derived from methods I3 and I4 did not demonstrate the clear bias that existed for product four. However, product one's sales was an inherently difficult series to forecast, as evidenced by the high RMSE for the individual model (Table 1). Thus, in a relative sense, product one's disaggregated model produced more accurate results for a difficult series to forecast than did the individual forecasting model.

For the channel product line, Table 4 indicates that the total forecasting model was a relatively poor predictor of total sales, with approximately an equal number of forecasts being above, below or within 10% of the actual. In addition, for the disaggregated models, the products with the largest share of total sales yielded the least accurate estimated proportions. Thus, this combination of poor total forecasts and inaccurate disaggregation models offered little hope for a disaggregation procedure to yield promising results for the channel product line.

CONCLUSIONS

This paper has focused on the problems associated with forecasting sales within companies that sell several products within many product lines. The large number of items to forecast, plus the fact that there are often different forecast requirements within the firm itself, can lead to an unmanageable set of forecasting tasks or the adoption of relatively unsophisticated, inaccurate disaggregation techniques.

The purpose of the study was to identify ways of reducing the overall forecasting burden by developing and assessing schemes of disaggregating forecasts of the total related product line to the individual items without a significant deterioration in forecast accuracy. Several of the procedures used were based upon extensions of the combination of forecast research to disaggregations of total forecasts within a product line. Measures were developed to determine appropriate disaggregation methodologies and to compare the forecast accuracy of individual product forecasts versus disaggregated totals.

Three disaggregation procedures appeared most promising in this particular application: A, F and I. Two of the methods, A and F, were based on the simple average (equal weights) of the products' share of total product line sales over the entire historical period. They were also simple to calculate, implement and update as more data became available. The third method, I, was based on a differential weighting procedure, with weights corresponding to the correlation between the lag sales proportion and the current period's proportion. The method chose the best predictors of the current proportion and used a differential moving average to combine the weights as a means of identifying trends. Although method I involved more calculation than A or F, it possessed intuitive appeal, was readily updated and could be automated to require minimal user interaction.

Because the results were based upon the findings of three particular cases of one company, they were difficult to generalize to other situations. However, attention was directed to understanding why disaggregation was applicable to two of the product lines examined and not for the third. This involved analyzing the data set to uncover the common characteristics of those series where disaggregation worked well. The general conceptual characteristics were considered and a three-level hierarchical structure emerged as a means for providing guidelines for the applicability of disaggregation. The results of the study were examined in light of the general conceptual characteristics and were discovered to be consistent with these findings.

To assure the generalizability of the results, the study would have to be replicated for several additional product/industry combinations. Clearly, generating forecasts with disaggregation methods that both reduce the forecasting burden and enable reasonably accurate results will increase the application of forecasting procedures among large-scale, multi-product organizations.

APPENDIX 1: FORECASTING MODELS USED

General notation

X_t = actual sales value at time period t
F_t = forecast of sales at time period t
n = the number of time periods in the series
a = intercept estimate
b = slope estimate
c = rate of change estimate
s_t = a seasonal factor relating to time t
L = number of seasons in one year
l = a particular season in a year (e.g. March, quarter 3, etc.), and $l = 1$ to L
α = a weighting constant, where $0 \leqslant \alpha \leqslant 1.0$, and optimal (based on min. RMSE)
$\beta = 1.0 - \alpha$
OLSQ = ordinary least squares

(A) Horizontal (no) trend

(1) Mean

$$F_t = \Sigma X_t/N \qquad \text{For } t = 1, 2, \ldots, n$$

(2) Single exponential smoothing:

$$F_{t+1} = \alpha_{\text{opt}} X_t + \beta_{\text{opt}} F_t$$

Initialization at $t = 0$:

$X_0 = F_0$ = mean of first $n/3$ actual sales (rounded up to integer) periods.

(B) Linear trend

(1) Linear time series regression:

$$F_t = a + b_t$$

where a and b are parameters estimated with OLSQ.

(2) Double exponential smoothing:

$$\begin{aligned} F_{t+p} &= a_t + b_t p \\ Y'_t &= \alpha_{\text{opt}} X_t + \beta_{\text{opt}} Y'_{t-1} \\ Y''_t &= \alpha_{\text{opt}} Y'_t + \beta_{\text{opt}} Y''_{t-1} \\ a_t &= 2Y'_t - Y''_t \\ b_t &= (\alpha_{\text{opt}}/\beta_{\text{opt}})(Y'_t - Y''_t) \end{aligned}$$

where F_{t+p} = sales forecast p periods ahead made at time t.

Initialization at t = 0:

a_0 and b_0 parameter estimates obtained with OLSQ over the first $n/3$ (rounded up to integer) periods of sales. Then,

$$\begin{aligned} Y'_0 &= a_0 - (\beta_{\text{opt}}/\alpha_{\text{opt}}) b_0 \\ Y''_0 &= a_0 - 2(\beta_{\text{opt}}/\alpha_{\text{opt}}) b_0 \end{aligned}$$

(3) Holt's two-parameter exponential smoothing. Same as Winters' multiplicative exponential smoothing (below), except that the terms for the seasonal factors were deleted.

(C) Linear trend with seasonal component

(1) Winters' multiplicative exponential smoothing:

$$F_t = (a + bt)s_t$$

where α, β, γ are separately defined weighting constants, with each constrained to fall between 0 and 1.0,

$$a_t = \alpha_{\text{opt}}(X_t/s_{t-L}) + (1 - \alpha_{\text{opt}})(a_{t-1} + b_{t-1})$$
$$b_t = \beta_{\text{opt}}(a_t - a_{t-1}) + (1 - \beta_{\text{opt}})b_{t-1}$$
$$s_t = \gamma_{\text{opt}}(X_t/a_t) + (1 - \gamma_{\text{opt}})s_{t-L}$$

and all s_l adjusted such that $\Sigma\, s_l = L$ for $l = 1$ to L, at each and every iteration. The forecast, made at time t, for any future period $t + \tau$, is:

$$F_{t+\tau} = (a + \tau b_t)s_{t+\tau-L}$$

Initialization at $t = 0$:

$$b_0 = \left(\frac{\sum_{i=L+1}^{2L} X_i}{L} - \frac{\sum_{i=1}^{L} X_i}{L}\right)\Bigg/ L$$

$$a_0 = \frac{\sum_{i=L+1}^{2L} X_i}{2L} - b_0\,\frac{2L+1}{2}$$

$$s(i)_j = \left(\frac{X_j}{a_0 + b_0 t_j} + \frac{X_{j+L}}{a_0 + b_0 t_{j+L}}\right)\Bigg/ 2$$

where

$j =$ the jth index,
$\Sigma\, j = L$, and
$i =$ initial estimate.

(D) Additive trend

(1) Winters' additive exponential smoothing. Same as Winters' multiplicative exponential smoothing, except that addition and subtraction were substituted for multiplication and division, respectively.

(E) Linear trend with seasonal component and cycle component

(1) *Decomposition—Method 1.* An iterative model which uses one of the two deseasonalization schemes presented below, linear time series for assessing trend and the Winters' multiplication technique for assessing cycle.
(2) *Decomposition—Method 2.* An iterative model which uses one of the two deseasonalization schemes, the linear time series model for assessing trend and any combination of procedures otherwise listed to assess periodic cycle.

(F) Higher level trend

(1) Quadratic time series regression:

$$F_t = a + bt + ct^2$$
$$an + b\,\Sigma\, t_i + c\,\Sigma\, t_i^2 = \Sigma\, X_i$$
$$a\,\Sigma\, t_i + b\,\Sigma\, t_i^2 + c\,\Sigma\, t_i^3 = \Sigma\, t_i X_i$$
$$a\,\Sigma\, t_i^2 + c\,\Sigma\, t_i^3 + c\,\Sigma\, t_i^4 = \Sigma\, t_i^2 X_i$$

for $i = 1$ to n

Then, solve the last three above equations simultaneously for a, b and c.

(2) Quadratic exponential smoothing:

$$F_{t+p} = a_t + b_t p + c_t p$$
$$Y_t' = \alpha_{\text{opt}} X_t + \beta_{\text{opt}} Y_{t-1}'$$
$$Y_t'' = \alpha_{\text{opt}} Y_t' + \beta_{\text{opt}} Y_{t-1}''$$
$$Y_t''' = \alpha_{\text{opt}} Y_t'' + \beta_{\text{opt}} Y_{t-1}'''$$
$$a_t = 3Y_t' - 3Y_t'' + Y_t'''$$
$$b_t = (\alpha/2\beta^2)[(6 - 5\alpha_{\text{opt}})Y_t' - 2(5 - 4\alpha_{\text{opt}})Y_t'' + (4 - 3\alpha_{\text{opt}})Y_t'''$$
$$c_t = (\alpha_{\text{opt}}/\beta_{\text{opt}})^2 (Y_t' - 2Y_t'' + Y_t''')$$

where

F_{t+p} = sales forecast p periods ahead made at time t.

Initialization at $t = 0$:

Use quadratic regression on first third of data to determine a_0, b_0 and c_0. Then, use the following to solve for Y_0', Y_0'' and Y_0''':

$$y_t = \begin{bmatrix} y_t' \\ y_t'' \\ y_t''' \end{bmatrix} \quad d = \begin{bmatrix} a_t \\ b_t \\ c_t \end{bmatrix} \qquad \text{and } M = \begin{bmatrix} 1 & -\dfrac{\beta}{\alpha} & \dfrac{\beta(2-\alpha)}{2\alpha^2} \\ 1 & -\dfrac{2\beta}{\alpha} & \dfrac{2\beta(3-2\alpha)}{2\alpha^2} \\ 1 & -\dfrac{3\beta}{\alpha} & \dfrac{3\beta(4-3\alpha)}{2\alpha^2} \end{bmatrix}$$

Then, $Y_0 = Md$.

(3) Logarithmic time series regression. Same as linear time series regression, except that the natural logarithm of all data were used as inputs, then antilogs were used to convert forecasts to original units.

(G) General deseasonalization schemes for models

(1) Classical deseasonalization:

$$s_l = \frac{\displaystyle\sum_{k=1}^{[(n-L)/L]^*} \left[X_{lk} \div \left(\sum_{j=l-(L/2)}^{l+(L/2)-1} X_j + \sum_{j=l-(L/2)+1}^{l+(L/2)} X_j \right) \Big/ 2L \right]}{[(n-L)/L]^*}$$

*rounded up to integer

$l = 1, 2, \ldots, L$, and all s_l adjusted such that $\Sigma\, s_l = L$, for all l.

(2) Modified Census II deseasonalization. Corrects for trend bias in seasonal calculations and eliminates highest and lowest ratios as outliers for each seasonal calculation. Let:

$I = n/L$, and integer; $IN = (I)(L)$; $B = n - IN$; $j = I - 1$. Then, slope:

$$b_1 = \left[\left(\sum_{i=L+B+1}^{2L+B} X_i \Big/ L\right) - \left(\sum_{i=B+1}^{L+B} X_i \Big/ L\right)\right] \Big/ L$$

$$b_2 = \left[\left(\sum_{i=2L+B+1}^{3L+B} X_i \Big/ L\right) - \left(\sum_{i=L+B+1}^{2L+B} X_i \Big/ L\right)\right] \Big/ L$$

$$b_j = \left[\left(\sum_{i=IN-L+B+1}^{IN+B} X_i \Big/ L\right) - \left(\sum_{i=(I-2)L+B+1}^{(I-1)L+B} X_i \Big/ L\right)\right] \Big/ L$$

Overall slope:

$$b = \frac{(b_1 + b_2 + \cdots + b_j)}{(I-1)}$$

Intercept:

$$a = \left(\sum_{j=B+1}^{n} X - \sum_{j=B+1}^{n} X\right) \Big/ IN$$

For trend line (tr):

(a) For an even number of observations:

at time $t = 1$, $tr_1 = a + 1/2b$; for remaining t, $tr_t = tr_{t-1} + b$

(b) For an odd number of observations:

at time $t = 1$, $tr_1 = a + b$; for remaining t, $tr_t = tr_{t-1} + b$

Raw seasonal ratio:

$$(sr) = a_t / tr_t$$

Seasonal indices:

average of $sr_t = a_t/tr_t$ for each l of L, with the min and max ratio excluded, then all seasonal indices (s_l) adjusted such that $\Sigma\, s_l = L$, for $l = 1$ to L.

APPENDIX 2: DISAGGREGATION METHODS

General notation

For a given product line:

P_{it} = the proportion used to allocate product i's share of the total forecast at time t
$i = 1, 2, \ldots, N$
N = total number of products within the product line

$t = 1, 2, \ldots, n$
n = total number of time periods
X_{it} = actual sales for product i in period t
TO_t = total sales of all products within the product line in period t

$$= \sum_{i=1}^{N} X_{it} \qquad \text{For } t = 1, 2, \ldots, n$$

$$PR_{it} = \frac{X_{it}}{TO_t}$$

Disaggregation methods

Method A:

$$P_{it} = \sum_{t=1}^{n} \frac{X_{it}}{TO_t} \Big/ n$$

Method B1:

$$P_{it} = \frac{X_{i,t-1}}{TO_{t-1}} = PR_{i,t-1}$$

Method B2:

$$P_{it} = PR_{i,t-2}$$

Method B3:

$$P_{it} = PR_{i,t-3}$$

Method B4:

$$P_{it} = PR_{i,t-4}$$

Method C2:

$$P_{it} = \left\{\frac{X_{i,t-1}}{TO_{t-1}} + \frac{X_{i,t-2}}{TO_{t-2}}\right\} \Big/ 2 = \left\{\frac{PR_{i,t-1} + PR_{i,t-2}}{2}\right\}$$

Method C3:

$$P_{it} = \left\{\frac{PR_{i,t-1} + PR_{i,t-2} + PR_{i,t-3}}{3}\right\}$$

Method C4:

$$P_{it} = \left\{\frac{PR_{i,t-1} + PR_{i,t-2} + PR_{i,t-3} + PR_{i,t-4}}{4}\right\}$$

Method D2:

$$P_{it} = \sum_{k=1}^{2} w_k PR_{i,t-k}$$

$$w_k = \frac{\operatorname{corr}(PR_{it}, PR_{i,t-k})}{\sum_{j=1}^{2} \operatorname{corr}(PR_{it}, PR_{i,t-j})}$$

Method D3:

$$P_{it} = \sum_{k=1}^{3} w_k PR_{i,t-k}$$

$$w_k = \frac{\text{corr}(PR_{it}, PR_{i,t-k})}{\sum_{j=1}^{3} \text{corr}(PR_{it}, PR_{i,t-j})}$$

Method D4:

$$P_{it} = \sum_{k=1}^{4} w_k PR_{i,t-k}$$

$$w_k = \frac{\text{corr}(PR_{it}, PR_{i,t-k})}{\sum_{j=1}^{4} \text{corr}(PR_{it}, PR_{i,t-j})}$$

Method E:

$$P_{it} = \sum_{k=1}^{4} B_k PR_{i,t-k}$$

B_k = OLSQ parameter estimates, subject to:

$$\sum_{k=1}^{4} B_k = 1$$

Method F:

$$P_{it} = \frac{\sum_{t=1}^{n} X_{it}}{n} \Bigg/ \frac{\sum_{t=1}^{n} TO_t}{n}$$

Method G2:

$$P_{it} = \sum_{k=1}^{2} W_k PR_{i,t-k}$$

$$e_l = PR_{it} - PR_{i,t-l} \qquad l = 1, 2$$

$$W_1 = \frac{\text{var}(e_2) - \text{cov}(e_1, e_2)}{\text{var}(e_1) + \text{var}(e_2) - 2\,\text{cov}(e_1, e_2)}$$

$$W_2 = \frac{\text{var}(e_1) - \text{cov}(e_1, e_2)}{\text{var}(e_1) + \text{var}(e_2) - 2\text{cov}(e_1, e_2)}$$

Method G3:

$$P_{it} = \sum_{k=1}^{3} W_k PR_{i,t-k}$$

$$e_l = PR_{it} - PR_{i,t-l} \qquad l = 1, 2, 3$$

$$W = (\hat{\Sigma}^{-1}1)/(1'\hat{\Sigma}^{-1}1)$$

W = vector of weights

$\hat{\Sigma}$ = variance–covariance structure of the errors, e_l

1 = column vector of unity

Method H2:

$$P_{it} = \sum_{k=1}^{2} W_k PR_{i,t-k}$$

$$e_l = PR_{it} - PR_{i,t-l} \qquad l = 1, 2$$

$$W_1 = \mathrm{var}(e_2)/\{\mathrm{var}(e_1) + \mathrm{var}(e_2)\}$$

$$W_2 = \mathrm{var}(e_1)/\{\mathrm{var}(e_1) + \mathrm{var}(e_2)\}$$

Method H3:

$$P_{it} = \sum_{k=1}^{3} W_k PR_{i,t-k}$$

$$e_l = PR_{it} - PR_{i,t-l} \qquad l = 1, 2, 3$$

$$W_k = \{\mathrm{var}(e_k)\}^{-1} / \sum_{j=1}^{3} \{\mathrm{var}(e_j)\}^{-1}$$

Method I1:

$$P_{it} = PR_{i,t-q}$$

where

$$\max_{r=1 \text{ to } n} \{\mathrm{corr}(PR_{it}, PR_{i,t-r})\} = \mathrm{corr}(PR_{i,t-q})$$

Method I2:

$$P_{it} = \sum_{\substack{k=1 \\ l=q,s}}^{2} W_k PR_{i,t-l}$$

$$W_1 = \mathrm{corr}(PR_{it}, PR_{i,t-q}) / \sum_{j=q,s} \mathrm{corr}(PR_{it}, PR_{i,t-j})$$

$$W_2 = \mathrm{corr}(PR_{it}, PR_{i,t-s}) / \sum_{j=q,s} \mathrm{corr}(PR_{it}, PR_{j,t-j})$$

where:

$$\max_{r=1 \text{ to } n} \{\mathrm{corr}(PR_{it}, PR_{i,t-r})\} = \mathrm{corr}(PR_{i,t}, PR_{i,t-q})$$

$$\max_{\substack{r=1 \text{ to } n \\ r \neq q}} \{\mathrm{corr}(PR_{it}, PR_{i,t-r})\} = \mathrm{corr}(PR_{i,t}, PR_{i,t-s})$$

Method I3:

$$P_{it} = \sum_{\substack{k=1 \\ l=q,s,u}}^{3} W_k PR_{i,t-l}$$

$$W_1 = \mathrm{corr}(PR_{it}, PR_{i,t-q}) / \sum_{j=q,s,u} \mathrm{corr}(PR_{it}, PR_{i,t-j})$$

$$W_2 = \mathrm{corr}(PR_{it}, PR_{i,t-s}) / \sum_{j=q,s,u} \mathrm{corr}(PR_{it}, PR_{i,t-j})$$

$$W_3 = \mathrm{corr}(PR_{it}, PR_{i,t-u}) / \sum_{j=q,s,u} \mathrm{corr}(PR_{it}, PR_{i,t-j})$$

where:

$$\max_{r=1 \text{ to } n} \{\text{corr}(PR_{it}, PR_{i,t-r})\} = \text{corr}(PR_{i,t}, PR_{i,t-q})$$

$$\max_{\substack{r=1 \text{ to } n \\ r \neq q}} \{\text{corr}(PR_{it}, PR_{i,t-r})\} = \text{corr}(PR_{i,t}, PR_{i,t-s})$$

$$\max_{\substack{r=1 \text{ to } n \\ r \neq q,s}} \{\text{corr}(PR_{it}, PR_{i,t-r})\} = \text{corr}(PR_{i,t}, PR_{i,t-u})$$

Method I4:

$$P_{it} = \sum_{\substack{k=1 \\ l=q,s,u,v}}^{4} W_k PR_{i,t-l}$$

$$W_1 = \text{corr}(PR_{it}, PR_{i,t-q}) / \sum_{j=q,s,u,v} \text{corr}(PR_{it}, PR_{i,t-j})$$

$$W_2 = \text{corr}(PR_{it}, PR_{i,t-s}) / \sum_{j=q,s,u,v} \text{corr}(PR_{it}, PR_{i,t-j})$$

$$W_3 = \text{corr}(PR_{it}, PR_{i,t-u}) / \sum_{j=q,s,u,v} \text{corr}(PR_{it}, PR_{i,t-j})$$

$$W_4 = \text{corr}(PR_{it}, PR_{i,t-v}) / \sum_{j=q,s,u,v} \text{corr}(PR_{it}, PR_{i,t-j})$$

where:

$$\max_{r=1 \text{ to } n} \{\text{corr}(PR_{it}, PR_{i,t-r})\} = \text{corr}(PR_{i,t}, PR_{i,t-q})$$

$$\max_{\substack{r=1 \text{ to } n \\ r \neq q}} \{\text{corr}(PR_{it}, PR_{i,t-r})\} = \text{corr}(PR_{i,t}, PR_{i,t-s})$$

$$\max_{\substack{r=1 \text{ to } n \\ r \neq q,s}} \{\text{corr}(PR_{it}, PR_{i,t-r})\} = \text{corr}(PR_{i,t}, PR_{i,t-u})$$

$$\max_{\substack{r=1 \text{ to } n \\ r \neq q,s,u}} \{\text{corr}(PR_{it}, PR_{i,t-r})\} = \text{corr}(PR_{i,t}, PR_{i,t-v})$$

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