Supplementary Results for "Forecast reconciliation: A geometric view with new insights on bias correction"

February 1, 2020

1 Results for $h = 2$ to $h = 6$ for Section 5.2

Figures 1 through to 5 show results for $h = 2$ to $h = 6$ corresponding to the application studied in Section 5.2. These demonstrate the distance reducing properties of Theorem 3.1 and Theorem 3.2. The loss function TSE refers to total squared error (squared Euclidean distance). The other two loss functions are weighted squared errors (squared generalised Euclidean distance) with the weights either determined by the structure of the hierarchy (Structural-WSE) or the average spend per region (Spend-WSE). The reconciliation methods used are Base, bottom up, OLS, MinT WLS with the weights corresponding to either the 'structural' or 'spend weights.'

Tables 1 through to 5 demonstrate Theorem 3.3; namely that MinT minimises expected loss irrespective of the choice of the weighting matrix. For these results, an average loss is taken over all replications. These averages are all expressed relative to base.

Loss Function	Base	Bottom-up	OLS	Structural-WLS	Spend-WLS	MinT
TSE	1.00	1.27	0.97	1.17	0.98	0.97
Structural WSE	1.00	1.01	0.96	0.98	0.99	0.93
Spend WSE	1.00	1.25	0.97	1.16	0.98	0.96

Table 1: Means of different loss functions for 2-step ahead forecasts using different reconciliation methods in the tourism application. All figures are reported relative to base forecasts.

Loss Function	Base	Bottom-up	OLS	Structural-WLS	Spend-WLS	MinT
TSE	1.00	1.25	0.97	1.16	0.98	0.97
Structural WSE	1.00	1.00	0.96	0.98	1.00	0.93
Spend WSE	1.00	1.23	0.97	1.15	0.98	0.96

Table 2: Means of different loss functions for 3-step ahead forecasts using different reconciliation methods in the tourism application. All figures are reported relative to base forecasts.

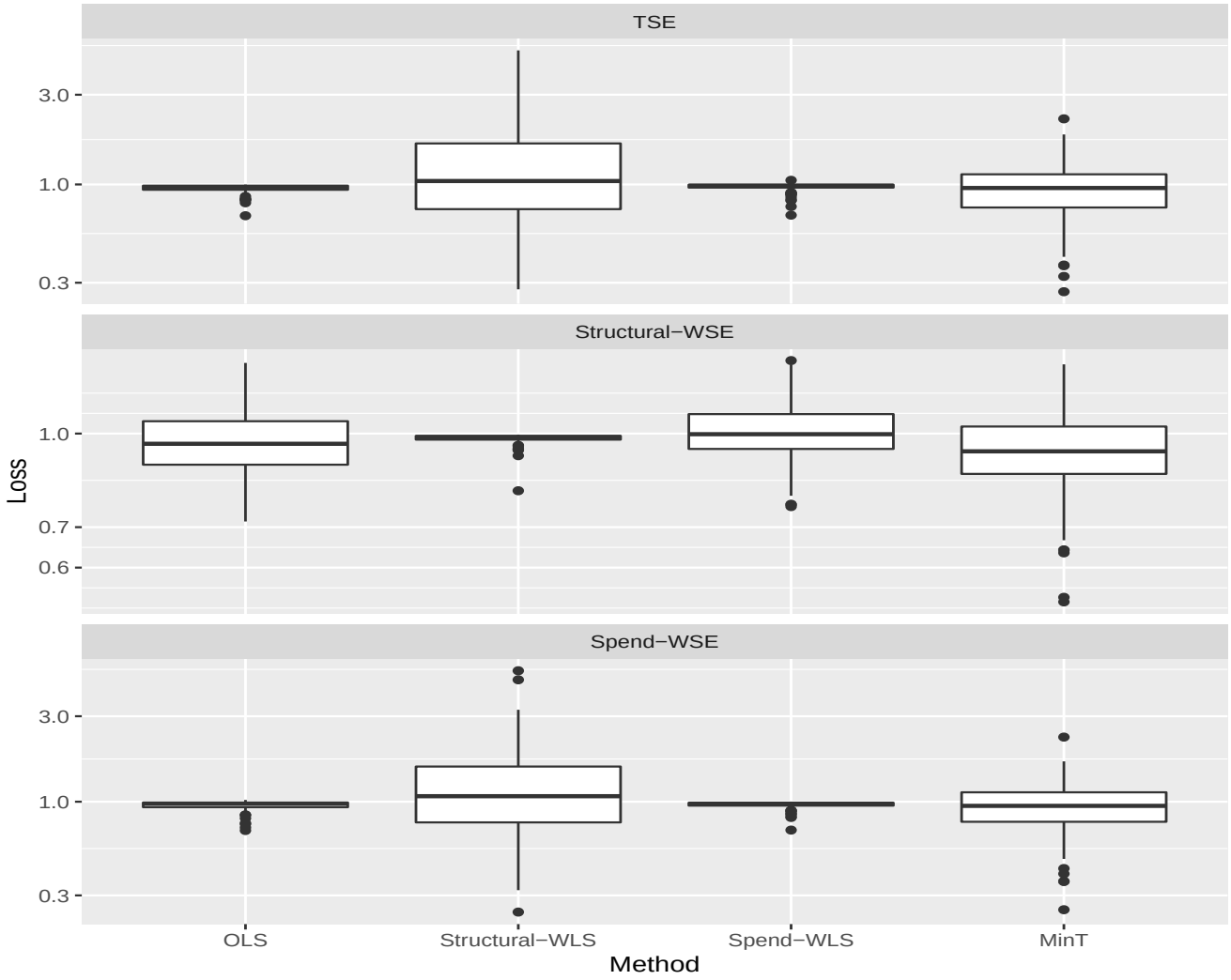

Figure 1: Ratio of loss of reconciled forecast to loss of base forecast for $h = 1$. A value less than 1 indicates that the reconciled forecasts improve upon base forecasts. A log scale is used for the y axis.

2 Results for all forecast horizons for Section 5.3

Table 6 shows results for $h = 2$ to $h = 6$ corresponding to the study in Section 5.3. For conciseness results for Method 2 which were shown to not adequately address the bias issue are omitted.

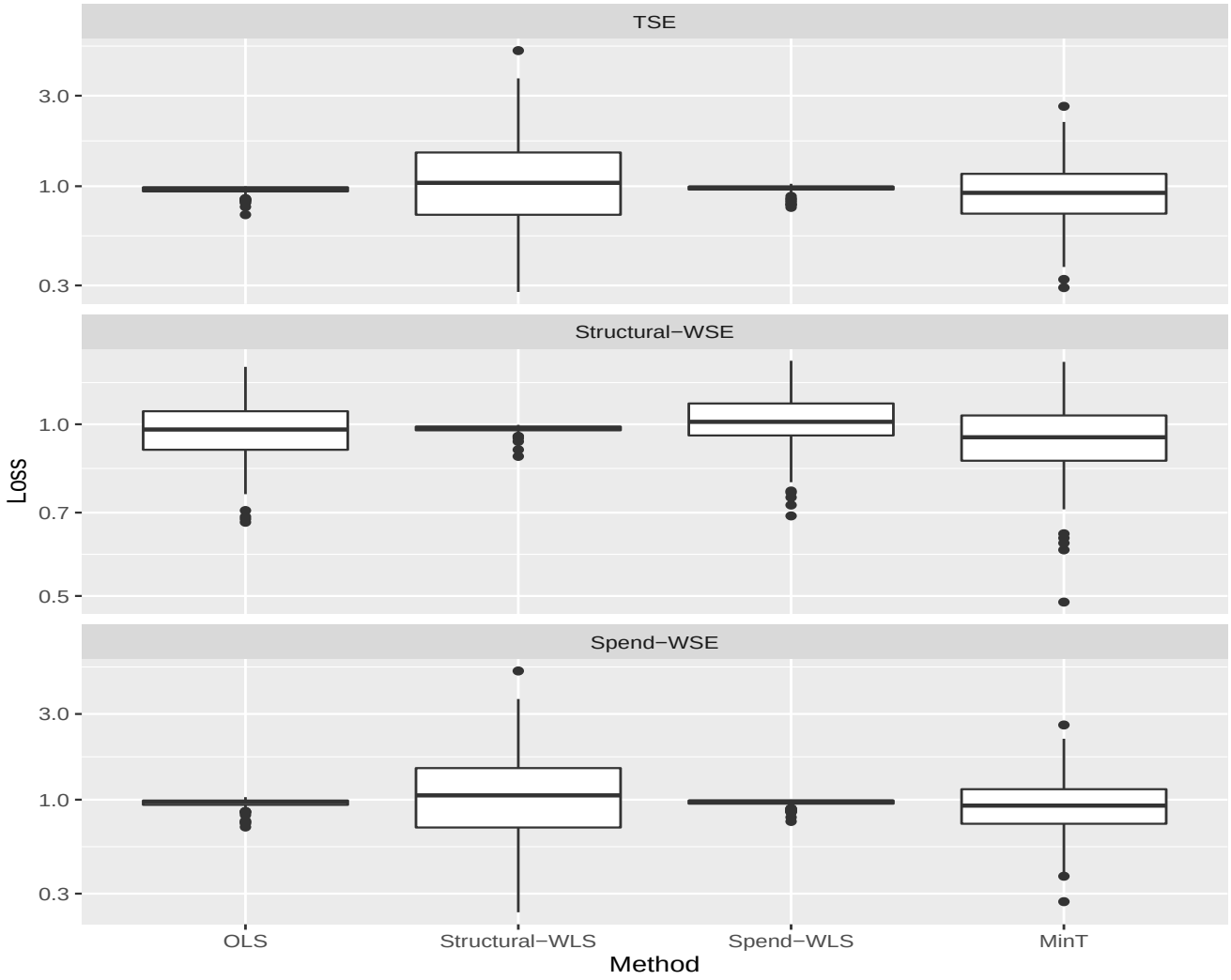

Figure 2: Ratio of loss of reconciled forecast to loss of base forecast for $h = 1$. A value less than 1 indicates that the reconciled forecasts improve upon base forecasts. A log scale is used for the y axis.

Loss Function	Base	Bottom-up	OLS	Structural-WLS	Spend-WLS	MinT
TSE	1.00	1.27	0.98	1.17	0.98	0.97
Structural WSE	1.00	1.01	0.96	0.98	0.99	0.92
Spend WSE	1.00	1.26	0.97	1.16	0.98	0.97

Table 3: Means of different loss functions for 4-step ahead forecasts using different reconciliation methods in the tourism application. All figures are reported relative to base forecasts.

Loss Function	Base	Bottom-up	OLS	Structural-WLS	Spend-WLS	MinT
TSE	1.00	1.28	0.98	1.18	0.98	0.97
Structural WSE	1.00	1.01	0.96	0.98	0.99	0.92
Spend WSE	1.00	1.26	0.97	1.17	0.98	0.96

Table 4: Means of different loss functions for 5-step ahead forecasts using different reconciliation methods in the tourism application. All figures are reported relative to base forecasts.

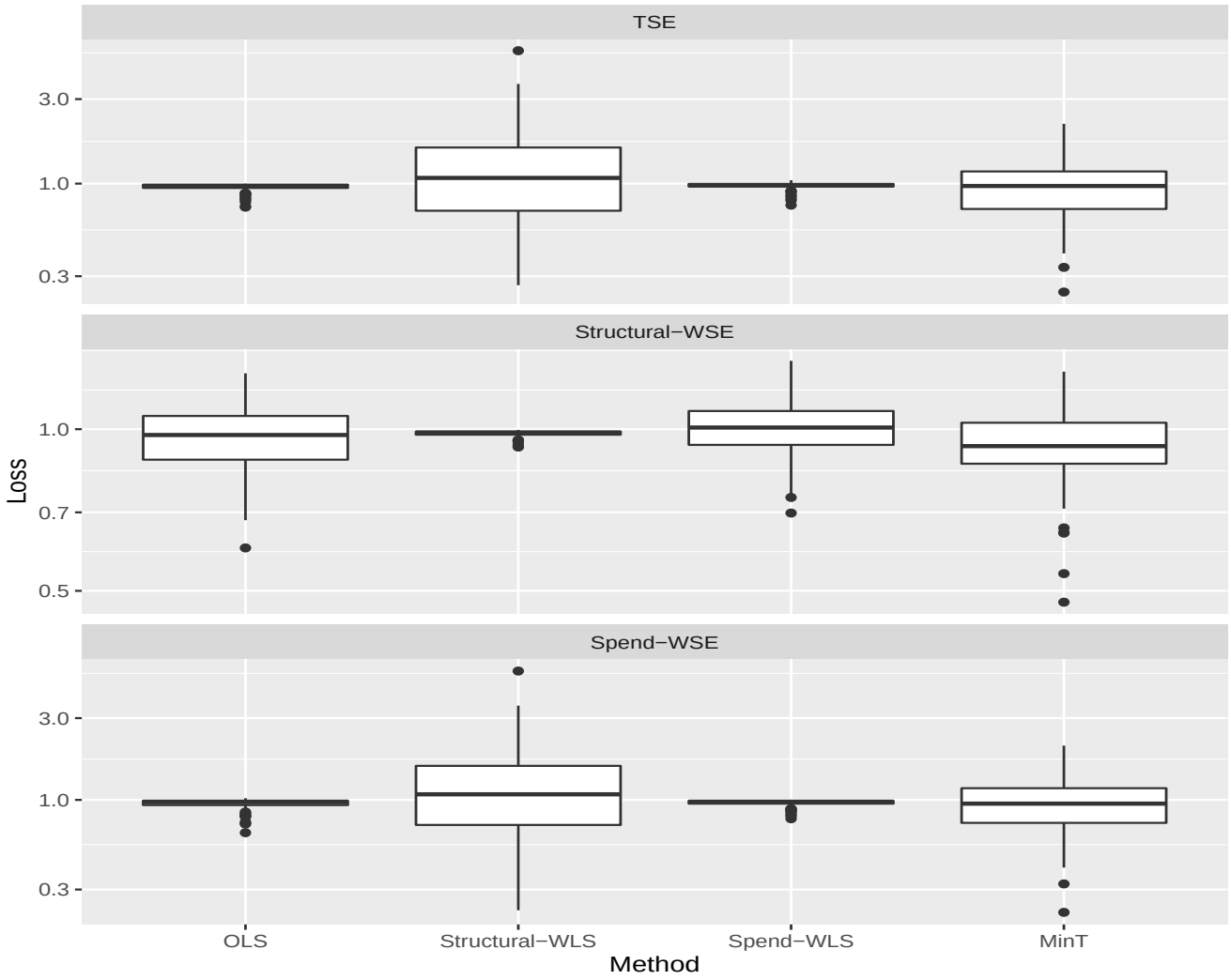

Figure 3: Ratio of loss of reconciled forecast to loss of base forecast for $h = 1$. A value less than 1 indicates that the reconciled forecasts improve upon base forecasts. A log scale is used for the y axis.

Loss Function	Base	Bottom-up	OLS	Structural-WLS	Spend-WLS	MinT
TSE	1.00	1.21	0.98	1.13	0.99	0.97
Structural WSE	1.00	1.00	0.97	0.98	1.00	0.93
Spend WSE	1.00	1.20	0.98	1.12	0.98	0.97

Table 5: Means of different loss functions for 6-step ahead forecasts using different reconciliation methods in the tourism application. All figures are reported relative to base forecasts.

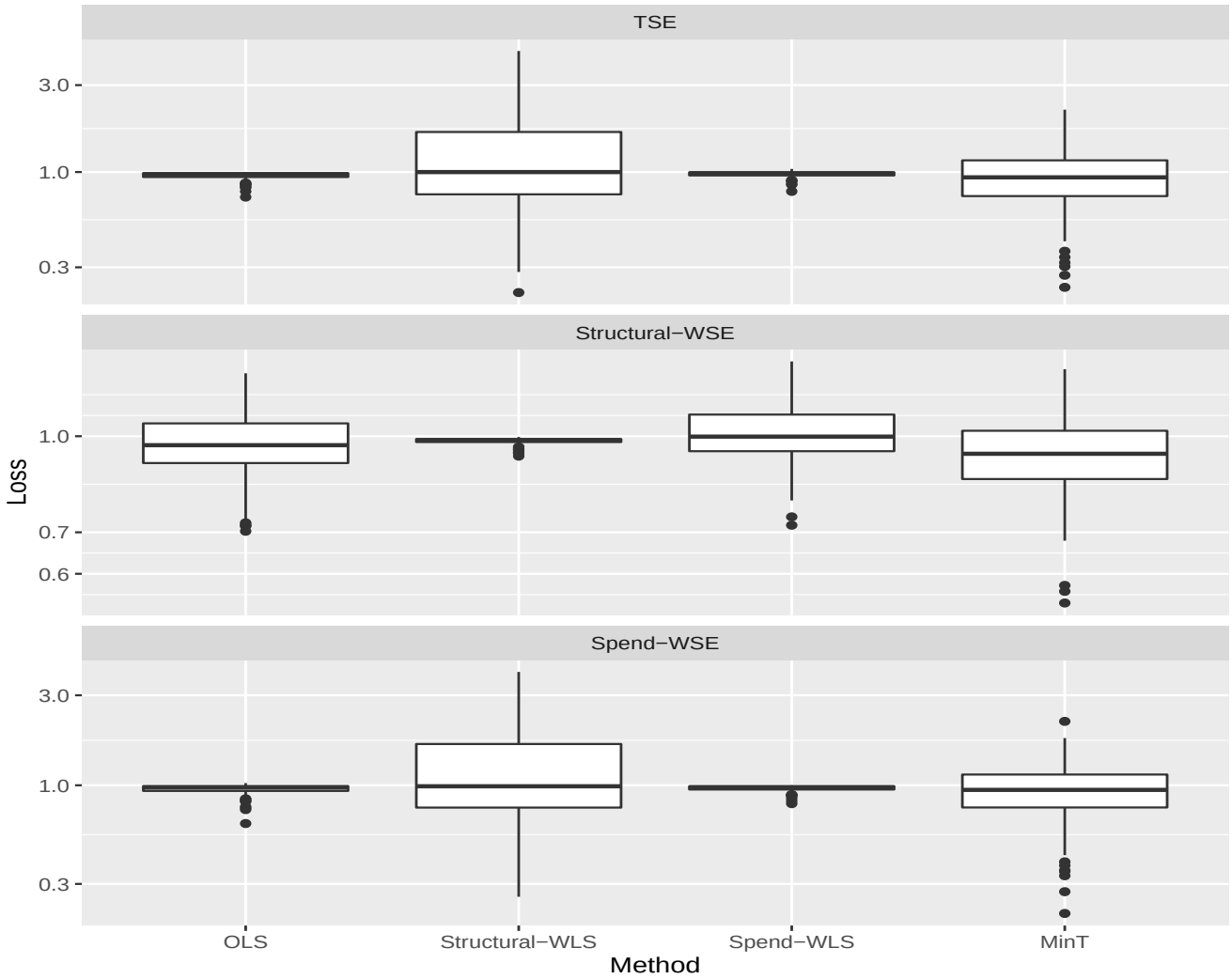

Figure 4: Ratio of loss of reconciled forecast to loss of base forecast for $h = 1$. A value less than 1 indicates that the reconciled forecasts improve upon base forecasts. A log scale is used for the y axis.

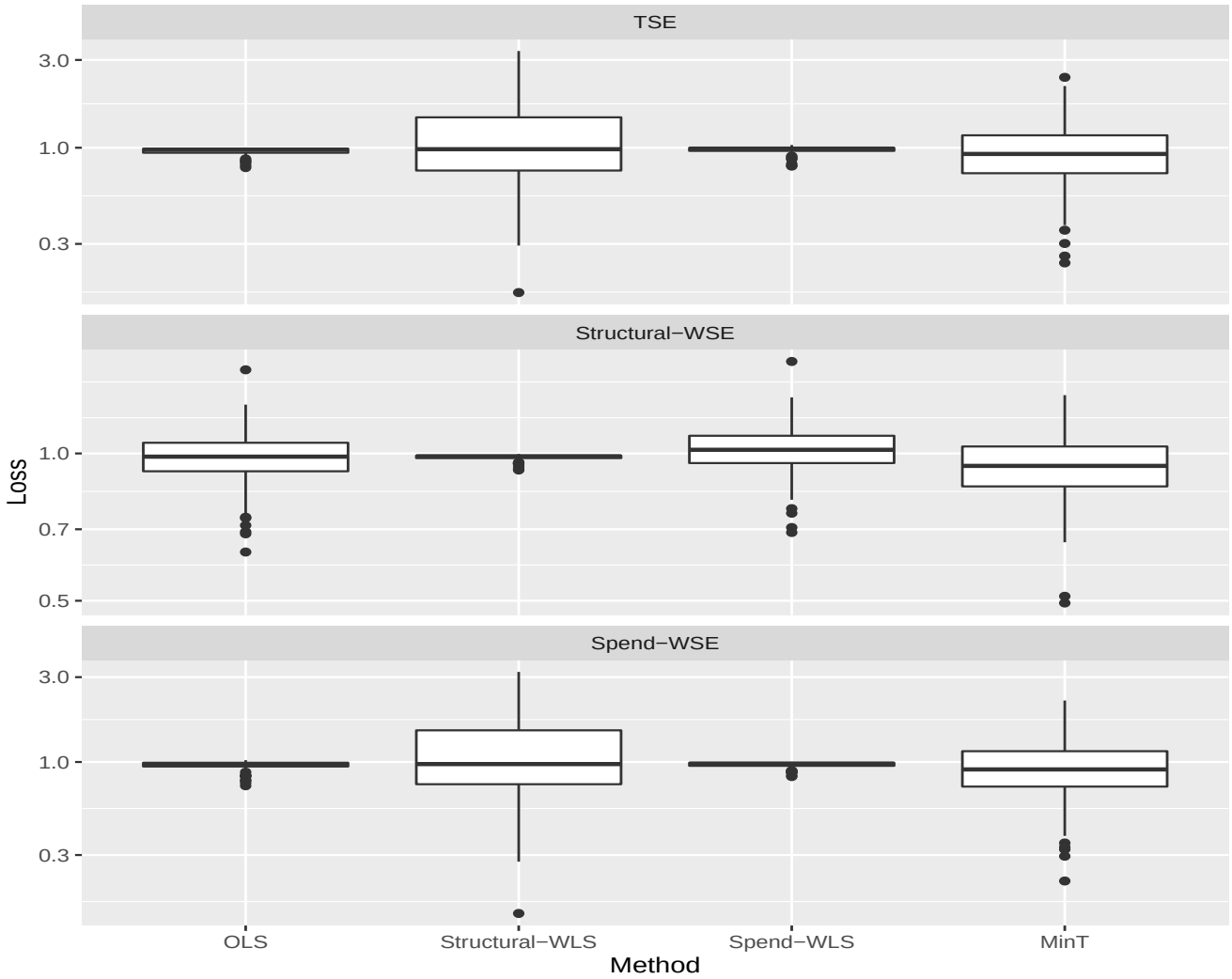

Figure 5: Ratio of loss of reconciled forecast to loss of base forecast for $h = 1$. A value less than 1 indicates that the reconciled forecasts improve upon base forecasts. A log scale is used for the y axis.

Table 6: Summary of results for all forecast horizons. All unbiased methods bias-correct the back-transformed forecasts before reconciling using Method 1 as described in the paper. The reconciliation methods used are Bottom-up, MinT and OLS. RMATE refers to relative mean absolute total error and RMTSE refers to relative mean total squared error.

	Log Transformation						Box-Cox Transformation					
Forecast horizon	1	2	3	4	5	6	1	2	3	4	5	6
	RMATE											
Biased Base	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Biased Bottom-up	1.76	1.69	1.73	1.72	1.64	1.60	1.73	1.69	1.67	1.66	1.59	1.55
Biased MinT	0.77	0.81	0.79	0.78	0.80	0.81	0.77	0.78	0.77	0.75	0.79	0.80
Biased OLS	0.63	0.65	0.65	0.65	0.68	0.70	0.65	0.67	0.67	0.68	0.70	0.72
Unbiased Base	0.58	0.62	0.64	0.65	0.68	0.69	0.73	0.75	0.76	0.77	0.78	0.79
Unbiased Bottom-up	0.69	0.73	0.79	0.82	0.81	0.81	0.84	0.86	0.88	0.90	0.88	0.87
Unbiased MinT	0.57	0.62	0.61	0.61	0.65	0.66	0.70	0.71	0.71	0.70	0.74	0.75
Unbiased OLS	0.54	0.56	0.56	0.58	0.61	0.63	0.67	0.69	0.69	0.70	0.71	0.72
	RMTSE											
Biased Base	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Biased Bottom-up	1.42	1.48	1.56	1.53	1.52	1.50	1.35	1.37	1.39	1.39	1.37	1.32
Biased MinT	0.97	0.98	1.02	0.98	0.98	0.98	0.93	0.95	0.96	0.96	0.96	0.94
Biased OLS	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97
Unbiased Base	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Unbiased Bottom-up	1.18	1.21	1.27	1.24	1.23	1.20	1.16	1.15	1.17	1.17	1.14	1.11
Unbiased MinT	0.93	0.94	0.97	0.94	0.94	0.93	0.91	0.92	0.93	0.93	0.93	0.91
Unbiased OLS	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96