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Probabilistic Forecast Reconciliation: Properties, Evaluation and Score Optimisation --Manuscript Draft--

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Abstract:	We develop a framework for forecasting multivariate data that follow known linear constraints. This is particularly common in forecasting where some variables are aggregates of others, commonly referred to as hierarchical time series, but also arises in other prediction settings. For point forecasting, an increasingly popular technique is reconciliation, whereby forecasts are made for all series (so-called base forecasts) and subsequently adjusted to cohere with the constraints. We extend reconciliation from point forecasting to probabilistic forecasting. A novel definition of reconciliation is developed and used to construct densities and draw samples from a reconciled probabilistic forecast. In the elliptical case, we prove that true predictive distributions can be recovered using reconciliation even when the location and scale of base predictions are chosen arbitrarily. Reconciliation weights are estimated to optimise energy or variogram score. The log score is not considered since it is improper when comparing unreconciled to reconciled forecasts, a result also proved in this paper. Due to randomness in the objective function, optimisation uses stochastic gradient descent. This method improves upon base forecasts in simulated and empirical data, particularly when the base forecasting models are severely misspecified. For milder misspecification, extending popular reconciliation methods for point forecasting results in similar performance to score optimisation.
Suggested Reviewers:	Feng Li feng.li@cufe.edu.cn Expert and thorough reviewer
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Opposed Reviewers:	

From: Professor George Athanasopoulos Econometrics and Business Statistics Monash University Caulfield, VIC, 3145, Australia

To: Professor Rudd H. Teunter Editor, European Journal of Operational Research

May 18, 2022

Dear Ruud,

Please find attached our revised manuscript, EJOR-D-21-02522, entitled "Probabilistic Forecast Reconciliation: Properties, Evaluation and Score Optimisation" by Anastasios Panagiotelis, Puwasala Gamakumara, George Athanasopoulos, and Rob Hyndman for your consideration for publication in the *European Journal of Operational Research*. We also enclose a point by point response addressing the comments of the referees.

For the reviewers convenience, our responses to referees are provided in blue text and all new parts of the paper (in both the manuscript and response letter) are provided in red text.

Please note that in response to Reviewer 2, in terms of providing EJOR readers a non-technical pathway and intuition, and in order to not dilute the mathematical rigour of the paper important from a technical perspective, we have added a new section, Section 2, entitled "Outline of Main Results". This section provides the reader less interested in technical details, with sufficient information in order to skip the technical sections that follow.

We look forward to hearing from you.

With kind regards and best wishes,

George Athanasopoulos

Editor

It has been decided that your paper should be reconsidered for publication in the EUROPEAN JOURNAL OF OPERATIONAL RESEARCH after a major revision taking into account the enclosed referees' comments. (You may wish to argue that some comments are invalid). Please also ensure that you have cited recent and relevant publications in EJOR and other OR journals.

We thank all reviewers for their invaluable/positive comments. All responses are in <u>Blue</u> font, any additions/changes to the paper are marked in <u>Red</u>. To the best of our knowledge, we have now added all recent and relevant publications in EJOR and other OR journals. This is particularly aided by the requests of the Reviewers.

Reviewer #1:

The manuscript provides a novel approach to probabilistic reconciliation using score optimization. It is concise, clearly structured and has a strong motivation, pointing out the necessary gaps in the literature. Following a detailed derivation of the SGD algorithm, the reconciled projections are evaluated using simulated and empirical data against a number of benchmark models. As the paper is very comprehensive, provides code as well as documentation, and appears quite mature, I have only a few minor comments.

- In the simulations, what are the assumptions on the covariance matrix in the joint Gaussian base forecasts?

In Section 6.2 we previously stated that we use "the variance covariance matrix of the residuals". We now modify the statement to make sure that this is clear to now read: "the variance covariance matrix of the residuals of the fitted models". In principle different choices could be used depending on assumptions made about how the covariance process should be modelled (e.g. time varying correlation), however such an issue lies beyond the scope of the simulation.

Why is it that the evaluated methods produce better scores for jointly distributed prediction errors?

In general, all methods score better for jointly distributed errors. This is due to the fact that assuming independence is likely to represent a (more severe) misspecification. The difference is substantial in the empirical application, especially with assuming Gaussian independent errors, as shown in Figure 8. We highlight this with the correlation heatmap we present in Figure 7. We also show some departures from normality in Figure 6. We comment on this stating "Therefore, independent Gaussian probabilistic forecasts are likely to represent severe misspecification" just before Figure 6.

In the simulations, all DGPs assume a joint error process and hence we expect that there is less misspecification error when probabilistic forecasts are generated using a joint distribution rather than assuming independence. As these differences are less pronounced we opt not to comment on these here and distract the reader from the main feature which is the performance evaluation across the reconciliation approaches.

- How well does the model scale to large, complex (grouped) hierarchies? How large is the computational demand compared to benchmark methods?

Relative to OLS and MinT, Score Optimisation is slower taking 2-3 minutes, mainly since it requires finding base forecasts over a rolling window (see line 2 of Algorithm 1). We note that implementing methods such as OLS and MinT for probabilistic rather than point forecasting using Theorem 3.5, is in itself novel and a significant contribution of the paper.

In an operational setting, where forecasts are made every period and kept, then these forecasts will already be available. The computational cost can also be mitigated by exploiting parallelisation (in our case we parallelised over different simulation/empirical scenarios rather than in the SGD itself). The stochastic gradient descent itself converges very quickly in most cases in less than 20 iterations.

- The authors might expand the discussion of the findings, in particular in section 7.2. How come the OLS approach performs so well given that it has been shown to lead to mediocre results in point forecasts?

A crucial point to make here is that OLS/MinT applied in the point forecasting setting are different and will thus have different properties to OLS/MinT applied in the probabilistic forecasting setting. In particular, for point forecasting, MinT will minimise the expected total mean squared error, although it should be noted that OLS also has some desirable properties for a loss function based on the L2 norm (see Panagiotelis et al 2021 for details). In the probabilistic setting we use scoring rules to evaluate forecast quality and these previous results do not necessarily apply. We now add the following statement to the paper (see Page 26, Lines 43-50):

"We suggest two possible reasons for the good performance of OLS in the probabilistic case. First, the energy score depends on the L2 norm of the difference between realizations and draws from the probabilistic forecast, which is similar to the setting for which OLS has optimal properties for point forecasts (see Panagiotelis et al, 2021). Second, for OLS there is less estimation uncertainty as fewer parameters need to be estimated."

In the electricity example, how is the joint distribution of the base forecasts estimated if they are generated using independent neural networks?

For both generating joint Gaussian and joint bootstrap probabilistic forecasts we use the residual covariance matrix of the fitted neural networks (fitted using the NNETAR function in the fable package, references are provided in the paper we do not repeat them here). This is identical to how the joint base forecasts are generated from the models in the simulations in Section 6. We have now added this information on Page 24, lines 40-42, by stating:

"Four situations were considered where base forecasts are assumed to be either Gaussian or bootstrapped from residuals, and either independent or dependent (we use the residual covariance matrix of the fitted neural networks, in a similar fashion as in Section 7)."

- A matter of preference, but the figures might look better when using a lighter theme than the out-of-the-box ggplot theme.

We acknowledge that this is a matter of taste. Having tried a few lighter themes our preference is to use the default ggplot theme.

Reviewer #2:

We thank the reviewer for the positive and constructive feedback on our paper. We respond below in detail to the comments that require a response.

Summary and General points

1. The paper is relevant to EJOR and continues a lineage of hierarchical forecasting papers recently published in the journal

No response required. Thank you for your positive view/comment.

2. The objective of the paper is to extend hierarchical forecasting methodology to the probabilistic setting.

No response required.

3. The paper is a difficult read due to the level of mathematical knowledge assumed. Whilst it is undoubtedly mathematically sophisticated, a good deal more effort could be made to explain the reasoning behind the linear algebra and borel sets. This sets a high bar for the empirical results - which in terms of the score optimisation algorithm in particular provide nothing by way of a reward for struggling through to the end. I'm sure the authors mathematical credentials are impeccable, but this is of little benefit to the journal if the reader is strongly inclined to give up reading on page 2.

We have now added a new section, after the introduction but before the main discussion of theoretical results (See Section 2 Outline of Main Results). The purpose of this section is to give a more intuitive and non-technical description of the main theoretical results. We

believe this new section provides the reader less interested in technical details sufficient information to skip the relevant sections.

Something that we now also emphasise in this new section (and in the introduction) is that the OLS and MinT methods are used in a novel way in this paper that relies on the way the theoretical results extend point reconciliation to probabilistic reconciliation. As a consequence, the assertion that the score optimisation algorithm offers little reward on the basis that it is outperformed by OLS and MinT in some settings, does not fairly represent the contributions of the paper. "OLS" and "MinT" as used here are not the existing point forecasting techniques, but probabilistic extensions of these techniques. These are introduced in this paper and are made possible by our definition of probabilistic coherence.

- 4. The paper makes significant and novel contributions in two main areas:
 - a. Firstly, in formally establishing how scoring rules can and should be applied in multivariate hierarchical contexts.

No response required.

b. Secondly the paper posits a valuable and far-reaching result that sampling from underlying base forecast distributions, and reconciling the samples delivers valid probabilistic forecasts.

No response required.

These two contributions are important and worthwhile, and the paper should focus on explaining and exploring them in more detail, and making them accessible to the forecasting and OR community.

Both of these results are highlighted in the new section summarising the key theoretical results of the paper.

- 6. Additionally, the authors present a HF algorithm which seeks to produce probabilistic forecasts by optimising a scoring rule. This part of the paper is much less successful:
 - a. The empirical results are weak, with the somewhat complex methodology presented only outperforming simpler methods when the base forecast models are clearly and obviously very badly miss-specified. The reader is left somewhat short-changed in that the level off effort required to understand and deploy the algorithm is in no way justified by improved empirical results compared to simply sampling from the base forecast distributions.

We respectfully disagree with the referee here. Please allow us to make a few points which we now include in the paper in order to highlight the contributions of the paper.

- 1. The novel contribution of the paper is not only using score optimisation in forecast reconciliation. What is labelled OLS and MinT Shrink in the empirical application (and of course in the simulation setting) is also novel and goes beyond the point forecast setting (which has been previously explored by us or other authors). (Please see added text in the Introduction Page 4, lines 43-50)
- 2. From our analysis we find that there are two possible sources of misspecification: (i) assuming independence and (ii) assuming Gaussian errors. Such misspecifications are not unusual in practice. For example, organisational sections or silos generating their forecasts independently, is common practice. We argue that one of the novel contributions of the hierarchical literature, including this paper, is to overcome such situations using forecast reconciliation. (Please see added text in the Introduction Page 3, lines 19-36).
- 3. Despite an increasing recognition of the importance of probabilistic forecasts, it is not always the case in practice that organisations produce probabilistic forecasts. On the other hand it is more common for organisations to at least provide a predicted mean and a predicted variance. Having only a mean and variance available is also common with judgementally adjusted forecasts. Where only a mean and variance are available the logical parametric assumption is a normal distribution and bootstrapping (due to the lack of residuals) is not possible. Therefore the assumption of normality, while a misspecification for our empirical example, is not unrealistic in practice. We have now added discussion regarding the importance of the Gaussian setting in Section 4.2, Page 13, lines 8-15.
- 4. Our results clearly show that score optimisation in forecast reconciliation is worth considering in such commonly observed misspecification settings. We believe this is a very strong, and also useful, result in practice.
- b. The empirical approach has a major conceptual weakness in that it fails to account for parameter uncertainty in the reconciliation process. HF processes are not parsimonious most reconciliation models require the estimation of a vast dimensional covariance matrix. It is well known that such estimation exercises are fraught with difficulty, for example in the similar context of VAR estimation, it is well established that accounting for parameter uncertainty is critical, and the vast majority of published research in the area adopts a Bayesian approach. The fact that the author's own (full covariance) MinT approach only works well when shrinkage based estimation is adopted supports this viewpoint. The authors make this point in their conclusion, but it is of critical importance, and in my view undermines their empirical results here.

We agree with the referee to some extent here, but believe that much of what is being proposed goes beyond the scope of our paper. We agree with the importance of shrinkage whether it be used for estimating the parameters of a large variance covariance matrix (as required for MinT) or in regularising the reconciliation coefficients in the score optimisation algorithm. Regarding the latter, we certainly believe that this is a worthwhile avenue for research, and for this reason discuss it in the conclusion. Given that this paper has a

number of theoretical results, uses these results to extend OLS and MinT into the probabilistic setting (in a novel way) and proposes the score optimisation algorithm, we feel that considering shrinkage in the score optimisation setting is best handled in another paper. We note that we comment on this as "A promising future research avenue" in the conclusion of the paper (see Page 27, lines 58-61 and Page 28 lines 1-2).

c. In my view the authors should focus on applying their own simple and robust idea of sampling from probabilistic forecasts and reconciling. The additional complexity of score optimisation without accounting for parameter uncertainty is certainly not justified by the results presented here.

We would contend that we do apply "a simple and robust idea of sampling from probabilistic forecasts and reconciling". This method is used to produce joint probabilistic base forecasts either by assuming Gaussianity or via joint bootstrapping and then reconciling via popular point forecasting methods such as OLS and MinT. As argued elsewhere, these are novel extensions of existing point forecasting methods motivated by the definitions of probabilistic reconciliation proposed in this paper.

The novel score optimisation algorithm may not work in all cases. However, we have shown an important practical situation (as we argue above in our response to 6a) where it does work and works significantly better than the other reconciliation approaches. This is a novel contribution that one may choose to implement in a different setting and may possibly get better results. As discussed in our response to 6b we agree that shrinkage/regularisation is a potentially fruitful area of future research.

7. For publication in a Journal where real world application and decision making is important, the authors should make the effort to provide much more in the way of explanation and general reasoning behind their results.

We believe that this has been addressed in our response to the other points raised by the referee. In particular the new Section 2 as well as further discussion in the section on the empirical application provide further explanation.

Detailed Comments

1. Abstract... 'This method improves on base forecasts...' this is hardly a contribution... I think the same authors prove elsewhere that reconciliation always improves on base forecasts...

The results referred to here from our other work apply to the point forecast setting. The proposed manuscript explores the probabilistic setting, therefore the results here, even for MinT and OLS are all new. Even in the point forecasting setting, the statement made here by the referee is only partially true. In Panagiotelis et al. (2021) we do present general

results but for two objectives: (i) to guarantee that reconciled forecasts improve upon base forecasts and (ii) to find the reconciliation method that is best on average. To guarantee objective (i) the loss function has to be matched with the weights matrix used in the projection reconciliation approach.

Panagiotelis, A., G. Athanasopoulos, P. Gamakumara, and R. J. Hyndman (2021). Forecast reconciliation: A geometric view with new insights on bias correction. International Journal of Forecasting 37(1), 343–359.

2. Section 2.2 I think this definition will be completely meaningless to many readers. Surely there should be a way of setting this more clearly without recourse to borel sets etc? Of course the definition is important (and therefore merits more explanation) but In a journal focused on applications this would be better off in an appendix?

As well as the discussion added in the new Section 2, we have added further explanation to this definition (see Page 9 lines 5-15, following Definition 3.1). In particular we talk about assigning probabilities to "intervals", "rectangles" or "regions", since these are all cases of Borel sets when giving intuitive explanations. We continue to use Borel sets in the definitions only to ensure full rigour.

3. Section 2.3 - Again much more effort could be made to explain what is going on here

We now summarise this result more succinctly in the new Section 2. We believe that the additional discussion around Section 3.2 (what was previously 2.2) as well as the use of Figure 2, makes the rest of this section clear.

4. Theorem 3.5 - I think this is a really important result (and one of the key contributions of the paper). A proof is of course provided, and while the theorem makes intuitive sense, no attempt is made to clarify and explain the logic of what is going on, and as the proof builds on the earlier results, and many readers will have to take it on trust...

We have added additional explanation here (see the new text following Theorem 4.5 - previously Theorem 3.5). This proof also leans heavily on the concepts from Definitions 2.1 and 2.2 (now 3.1 and 3.2) which we believe are now more clearly explained.

5. Page 16, line 37. 'bottom level series have lower signal to noise than higher level'? It is a little surprising that this does not occur automatically, in practice the noise level in the higher level series would normally reduce via a diversification effect?

Yes we agree. The bottom-level series did automatically have a lower signal-to-noise ratio. The additional noise was added to the bottom-level to make this difference more pronounced and replicate a realistic/practical setting. We have revised the statement now to read:

"After simulating from the ARIMA models, additional noise is added to ensure bottom-level series have a considerably lower signal-to-noise ratio than upper-level series with details provided in Appendix D of the online supplement."

Note we have also corrected a typo in the paper changing the second instance of the word "bottom" to "upper".

6. Section 7.2 - As noted by the authors, the assumptions of independence and gaussian errors are clearly both badly violated in the data. The score optimisation algorithm generates 'statistically significant' improvements to base forecasts generated using naïve and inaccurate base forecasts! Is it really worth going to the trouble of reconciling such poor forecasts? When more sensible base forecast assumptions are made, simpler and more robust methods based on Theorem 3.5 perform more strongly.

As we argue in our response to comment 6a above, and we argue again here, it is absolutely worth reconciling such forecasts, or at least it is worth having the knowledge and a tool, such as score optimisation, that can successfully reconcile these:

- 1. We never know the quality of our forecasts a priori. Any evaluation of these will always be in sample and many times such evaluation is not possible, e.g., when these are judgmentally generated or in general not model based or the model is not made available.
- 2. In commonly observed organisational silos, independence is the only assumption possible.
- 3. A parametric assumption of Gaussianity for the errors is the usual alternative when only the mean and variance are available and/or when bootstrapping is not a possibility.

Reviewer #3:

This paper is concerned with the highly relevant problem of forecast reconciliation in a probabilistic setting. The authors (i) present a theoretical framework and theoretical results for this problem, (ii) propose an algorithm for probabilistic forecast reconciliation based on score optimization, and (iii) present results on both simulated and real data. In general, the paper is well written and of high quality. Considering all these elements, in my opinion, this paper is suitable for publication in EJOR after a minor revision.

We thank the reviewer for the positive and constructive feedback on our paper. We respond below in detail to the comments that require a response.

Please find below my comments.

General comments:

I very much like that the paper is concise and to the point. This is true for both the theoretical and empirical parts. The empirical sections are very good: nice presentation + insightful discussion for both the simulation and case study results. However, the introduction and the theoretical sections (2 to 5) are hard to follow at times, which I think is a consequence of these sections being somewhat too concise. Extra context is needed in some cases to make the text more comprehensible for a more general audience.

We have now added a new section (Section 2) giving a non-technical explanation of the main theoretical results. We have provided more context to some of the theorems at the request of Reviewer 2. We believe the changes have made the more challenging sections of the paper much more comprehensible to a general audience.

Some examples:

 P2L53: "Prior to the development of forecast reconciliation, the focus was on finding a subset of variables that could be subsequently aggregated or disaggregated to find forecasts for all series." Please clarify.

We have now revised the statement as follows:

"Prior to the development of forecast reconciliation, the focus was on forecasting a subset of variables at some selected level of aggregation, and subsequently aggregating or disaggregating these to generate forecasts for all series."

 P3L1: "These papers formulated reconciliation as a regression model, however subsequent work has formulated reconciliation as an optimisation problem where weights are chosen to minimise a loss." Please clarify.

Let us please note that this statement does not end where the reviewer indicates it ends. We reproduce below the full statement:

"These papers formulated reconciliation as a regression model, however subsequent work has formulated reconciliation as an optimisation problem where weights are chosen to minimise a loss, such as a weighted squared error (Van Erven and Cugliari, 2015; Nystrup et al., 2020), a penalised version thereof (Ben Taieb and Koo, 2019), or the trace of the forecast error covariance (Wickramasuriya et al., 2019)."

In order to further clarify we break the statement into two sentences and also add to it. The revised text now reads as follows:

"These papers formulated reconciliation as a regression model, reconciling the base forecasts by projecting them onto a subspace for which aggregation constraints hold. Subsequent work has formulated reconciliation as an optimisation problem where weights are chosen to minimise a loss, such as a weighted squared error (Van Erven and Cugliari,

2015; Nystrup et al., 2020), a penalised version thereof (Ben Taieb and Koo, 2019), or the trace of the forecast error covariance (Wickramasuriya et al., 2019)."

 P3L9: "The accuracy and popularity of forecast reconciliation methods can be attributed to a number of factors". However, only one factor is discussed, i.e., breaking down organizational silos. Please also discuss/mention other factors.

We have now revised the paragraph to read as follows:

"The popularity of forecast reconciliation methods can be attributed to a number of factors. Forecasts across different aggregation levels may be generated by different departments or 'silos' within an organisation, using different sets of predictors, modelling approaches, or expert judgement. Potentially, these are viewed as optimal within these divisions. Reconciliation represents a way to combine information via the sharing of forecasts, thus breaking down these silos. Although it may be difficult to share forecasting processes and associated information across different parts of a large organisation, the forecasts themselves are much easier to share and reconcile. In contrast to bottom-up and top-down approaches, which effectively discard the forecasts of all but one level, the combination of forecasts across all levels also leads to improved forecast accuracy."

Ben Taieb et al. (2020) explanation: Either discuss it in more detail or in less detail, but now it is rather unclear what this method is exactly about. What is meant by 'reordering' base forecasts? To some extent this comment is also true for the discussion of Jeon et al. (2019): "ranking draws from independent base probabilistic forecasts before reconciliation is effective"?

To elaborate on "reordering", both JPP and BTTH obtain draws from the probabilistic forecast from each variable independently (let's think of these as vectors of length L). To reconcile these methods we take a single draw from each variable and combine these in an n-vector. In this case, the ordering of the draws within each vector of length L becomes critical and both BTTH and JPP attempt to do this in a way that captures dependence.

JPP do this in a way that is equivalent to reconciling quantiles (which is how we have described on Page 3 lines 56-57, Page 4 lines 21-23, and Page 19 lines 45-55). We have rewritten the sentence "ranking draws…is effective" since the connection between this and reconciling quantiles may not be clear. Regarding BTTH, we have expanded on our discussion of their method adding the following on Page 4 lines 1-6

"In particular, Ben Taieb et al.(2020) draw a sample of size L from the probabilistic forecasts of univariate models for the m bottom-level series and stack these in an L x m matrix. To induce dependence, the columns of this matrix are reordered so that the copula of the data matrix created, matches the empirical copula of the residuals. Samples of the aggregate series are obtained in a bottom-up fashion."

- P14L44: "These are typically high dimensional problems, deep neural networks handle millions of parameters, so this tool is well suited to our problem." What do you mean exactly by this and 'these'? Gamma is not necessarily high-dimensional, right?

The above sentence follows on from "There is also a recent but growing literature on using SGD to optimise scoring rules (see Gasthaus et al., 2019; Janke and Steinke, 2020; Hofert et al., 2020, and references therein for examples)." which highlights the recent literature using SGD to optimise scoring rules. "These...problems" refers to the problems tackled within these papers.

We have now rewritten the sentence to make it clear. The sentence now reads (see Page 16 lines 35-36):

"These papers typically deal with high dimensional problems, deep neural networks handle millions of parameters, so this tool is well suited to our problem."

Also although Gamma does not necessarily contain millions of parameters (although in some applications it may do so), we simply make this point to assuage any concerns a reader may have about the applicability of our methods to larger hierarchies.

P14L61: Discuss in more detail how the proposed score optimization algorithm differs from the method proposed by Rangapuram et al. (2021). At first glance, an obvious one is that your approach allows using general techniques (including ets() for example) to generate probabilistic base forecasts, which is not the case for Rangapuram et al. (2021). However, how is your method different from the one presented in Rangapuram et al. (2021) as for the reconciliation procedure (inclusion of translation, orthogonality of projection(?)...), and what are the implications? Just to be clear: I do not expect you to add this method to the empirical results in Sections 6 and 7.

We have now extended the discussion of Rangapuram et al. (2021) as follows (see Page 16 lines 60 and Page 61 lines 38-43):

"Rangapuram et al. (2021) use a similar approach in their end-to-end forecasting process. Their method is more restrictive than what we propose here in that the projection must be orthogonal, base forecasts are not translated, and base forecasts must be generated by a DeepVAR."

The conclusion could be improved a bit by adding some nuance to the discussion:

"Since the scores are approximated by Monte Carlo simulation, stochastic gradient descent is used for optimisation." One could also use batch gradient descent using all Monte Carlo simulations in one go, isn't it?

There is some subtlety to this point. In many contexts, (e.g. training neural networks) what makes stochastic gradient descent, "stochastic", is that the training data are subsampled for

computational reasons. In our setting, the Energy (and Variogram) Scores cannot be computed in closed form and must be estimated using a Monte Carlo estimate. This is what requires us to use SGD even when all data are used. We have now added the following (see Page 16, lines 39-46):

"An important distinction is that the use of SGD, rather than gradient descent in these contexts, arises due to computational considerations as it is not efficient to use all data. In contrast we use all data and the 'stochastic' nature of our gradient descent arises since the score functions contain integrals that must be estimated by Monte Carlo."

"This method is shown to lead to significant improvements over base forecasts, bottom-up methods and existing probabilistic reconciliation approaches across a wide variety of simulated and empirical examples." Refer also here to the degree of misspecification of the base forecasts.

We have now revised the statement and have added the suggestion. The statement now reads as follows (see Page 27 line 49):

This method is shown to lead to significant improvements over base forecasts, bottom-up methods and existing probabilistic reconciliation approaches across a wide variety of simulated and empirical examples, particularly when the base forecasting models are severely misspecified.

Minor comments:

Add references to the first sentence of the Intro.

Thank you for the suggestion. We have now added references to these.

P10L60: Add dot at the end of footnote 1.

Done.

P12L18: Replace 'and' by 'are'?

Done.

P14L60: Is 'projection' the correct term here (also see P14L2)?

Yes 'projection' is the correct term here.

P17L35: Replace period by colon.

Done.

- P18L10: Variogram score is only introduced by referring to Scheuerer and Hamill (2015), but no definition or explanation is provided.

We have now added the definition and appropriate additional references and discussion on the discrimination ability of scoring rules (see Page 15 lines 1-13).

P24L56: Remove dot after footnote 3 in text + add dot at the end of footnote 3.

Done.

- How many errors do you use for the construction of bootstrapped probabilistic forecasts? I think that this info is missing.

We have added the following on Page 19 lines 22-25.

"The number of bootstrap samples is set equal to the sample size both here and in Section 8."

- Can you provide an explanation for the deviant performance of BTTH for the Gaussian DGP in terms of variogram scores? The fact that a definition of the variogram score (and a discussion of how this score is different from the energy score) is missing makes it impossible for the reader to reason about this observation.

We are unable to explain the deviant performance of BTTH in this setting. However, we have now added the definition of the variogram score. We also provide references that discuss scenarios in which the variogram score has good discriminatory power (see Page 15 lines 1-13).

Highlights (for review)

- Forecast reconciliation in the probabilistic setting is rigorously developed.
- Point forecast reconciliation is extended in a novel way to the probabilistic setting.
- Results are derived for the Gaussian and non-Gaussian case.
- Theorems on scoring rules are derived with recommendations for forecast evaluation.
- A new reconciliation method based on score optimisation and stochastic gradient descent is proposed.
- The new methods are shown to improve forecast accuracy in a simulated and empirical setting.

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\c@table=\count174
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\belowcaptionskip=\skip48
\bibindent=\dimen134
) (c:/TeXLive/2020/texmf-dist/tex/latex/amsmath/amsmath.sty
Package: amsmath 2020/01/20 v2.17e AMS math features
\@mathmargin=\skip49
For additional information on amsmath, use the `?' option.
(c:/TeXLive/2020/texmf-dist/tex/latex/amsmath/amstext.sty
Package: amstext 2000/06/29 v2.01 AMS text
(c:/TeXLive/2020/texmf-dist/tex/latex/amsmath/amsgen.sty
File: amsgen.sty 1999/11/30 v2.0 generic functions
\@emptytoks=\toks15
\ensuremath{\mbox{ex@=}\mbox{dimen135}}
)) (c:/TeXLive/2020/texmf-dist/tex/latex/amsmath/amsbsy.sty
Package: amsbsy 1999/11/29 v1.2d Bold Symbols
\pmbraise@=\dimen136
) (c:/TeXLive/2020/texmf-dist/tex/latex/amsmath/amsopn.sty
Package: amsopn 2016/03/08 v2.02 operator names
\inf@bad=\count175
LaTeX Info: Redefining \frac on input line 227.
\uproot@=\count176
\leftroot@=\count177
LaTeX Info: Redefining \overline on input line 389.
\classnum@=\count178
\DOTSCASE@=\count179
LaTeX Info: Redefining \ldots on input line 486.
LaTeX Info: Redefining \dots on input line 489.
LaTeX Info: Redefining \cdots on input line 610.
\Mathstrutbox@=\box45
\strutbox@=\box46
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\big@size=\dimen137
LaTeX Font Info:
                    Redeclaring font encoding OML on input line 733.
LaTeX Font Info:
                    Redeclaring font encoding OMS on input line 734.
\macc@depth=\count180
\c@MaxMatrixCols=\count181
\dotsspace@=\muskip16
\c@parentequation=\count182
\dspbrk@lvl=\count183
\tag@help=\toks16
\row@=\count184
\column@=\count185
\maxfields@=\count186
\andhelp@=\toks17
\eqnshift@=\dimen138
\alignsep@=\dimen139
\tagshift@=\dimen140
\tagwidth@=\dimen141
\totwidth@=\dimen142
\lineht@=\dimen143
\@envbody=\toks18
\multlinegap=\skip50
\multlinetaggap=\skip51
\mathdisplay@stack=\toks19
LaTeX Info: Redefining \[ on input line 2859.
LaTeX Info: Redefining \] on input line 2860.
) (c:/TeXLive/2020/texmf-dist/tex/latex/tools/bm.sty
Package: bm 2019/07/24 v1.2d Bold Symbol Support (DPC/FMi)
LaTeX Info: Redefining \bm on input line 209.
) (c:/TeXLive/2020/texmf-dist/tex/latex/amsfonts/amssymb.sty
Package: amssymb 2013/01/14 v3.01 AMS font symbols
(c:/TeXLive/2020/texmf-dist/tex/latex/amsfonts/amsfonts.sty
Package: amsfonts 2013/01/14 v3.01 Basic AMSFonts support
\symAMSa=\mathgroup4
\symAMSb=\mathgroup5
LaTeX Font Info:
                    Redeclaring math symbol \hbar on input line 98.
LaTeX Font Info:
                    Overwriting math alphabet `\mathfrak' in version
`bold'
(Font)
                        U/euf/m/n --> U/euf/b/n on input line 106.
)) (c:/TeXLive/2020/texmf-dist/tex/latex/amscls/amsthm.sty
Package: amsthm 2017/10/31 v2.20.4
\thm@style=\toks20
\t \
\thm@headfont=\toks22
\thm@notefont=\toks23
\thm@headpunct=\toks24
\thm@preskip=\skip52
\thm@postskip=\skip53
\theta = \
\dth@everypar=\toks25
) (c:/TeXLive/2020/texmf-dist/tex/latex/jknapltx/mathrsfs.sty
Package: mathrsfs 1996/01/01 Math RSFS package v1.0 (jk)
\symrsfs=\mathgroup6
) (c:/TeXLive/2020/texmf-dist/tex/latex/bbm-macros/bbm.sty
Package: bbm 1999/03/15 V 1.2 provides fonts for set symbols - TH
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LaTeX Font Info:
                    Overwriting math alphabet `\mathbbm' in version
`bold'
(Font)
                        U/bbm/m/n \longrightarrow U/bbm/bx/n on input line 33.
LaTeX Font Info:
                    Overwriting math alphabet `\mathbbmss' in version
`bold'
                        U/bbmss/m/n --> U/bbmss/bx/n on input line 35.
(Font)
) (c:/TeXLive/2020/texmf-dist/tex/latex/mathalpha/mathalfa.sty)
(c:/TeXLive/202
0/texmf-dist/tex/latex/mathalpha/mathalpha.sty
Package: mathalpha 2019/10/05 - 1.13mathalpha (License LPPL) Michael
Sharpe
`mathalpha' v1.13, 2019/10/05, a renaming of mathalfa (msharpe)
(c:/TeXLive/202
0/texmf-dist/tex/latex/xkeyval/xkeyval.sty
Package: xkeyval 2014/12/03 v2.7a package option processing (HA)
(c:/TeXLive/2020/texmf-dist/tex/generic/xkeyval/xkeyval.tex
(c:/TeXLive/2020/te
xmf-dist/tex/generic/xkeyval/xkvutils.tex
\XKV@toks=\toks26
\XKV@tempa@toks=\toks27
(c:/TeXLive/2020/texmf-dist/tex/generic/xkeyval/keyval.tex))
\XKV@depth=\count187
File: xkeyval.tex 2014/12/03 v2.7a key=value parser (HA)
LaTeX Font Info:
                    Overwriting math alphabet `\mathbb' in version `bold'
(Font)
                        U/BOONDOX-ds/m/n --> U/BOONDOX-ds/b/n on input
line 142
) (c:/TeXLive/2020/texmf-dist/tex/latex/paralist/paralist.sty
Package: paralist 2017/01/22 v2.7 Extended list environments
\pltopsep=\skip55
\plpartopsep=\skip56
\plitemsep=\skip57
\plparsep=\skip58
\pl@lab=\toks28
) (c:/TeXLive/2020/texmf-dist/tex/latex/natbib/natbib.sty
Package: natbib 2010/09/13 8.31b (PWD, AO)
\bibhang=\skip59
\bibsep=\skip60
LaTeX Info: Redefining \cite on input line 694.
\c@NAT@ctr=\count188
) (c:/TeXLive/2020/texmf-dist/tex/latex/url/url.sty
\Urlmuskip=\muskip17
Package: url 2013/09/16 ver 3.4 Verb mode for urls, etc.
(c:/TeXLive/2020/texmf-dist/tex/latex/geometry/geometry.sty
Package: geometry 2020/01/02 v5.9 Page Geometry
(c:/TeXLive/2020/texmf-dist/tex/generic/iftex/ifvtex.sty
Package: ifvtex 2019/10/25 v1.7 ifvtex legacy package. Use iftex instead.
(c:/TeXLive/2020/texmf-dist/tex/generic/iftex/iftex.sty
Package: iftex 2020/03/06 v1.0d TeX engine tests
\Gm@cnth=\count189
\Gm@cntv=\count190
\c@Gm@tempcnt=\count191
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\Gm@bindingoffset=\dimen144
\Gm@wd@mp=\dimen145
\Gm@odd@mp=\dimen146
\Gm@even@mp=\dimen147
\Gm@layoutwidth=\dimen148
\Gm@layoutheight=\dimen149
\Gm@layouthoffset=\dimen150
\Gm@layoutvoffset=\dimen151
\Gm@dimlist=\toks29
) (c:/TeXLive/2020/texmf-dist/tex/latex/placeins/placeins.sty
Package: placeins 2005/04/18 v 2.2
) (c:/TeXLive/2020/texmf-dist/tex/latex/multirow/multirow.sty
Package: multirow 2019/05/31 v2.5 Span multiple rows of a table
\multirow@colwidth=\skip61
\multirow@cntb=\count192
\multirow@dima=\skip62
\bigstrutjot=\dimen152
) (c:/TeXLive/2020/texmf-dist/tex/latex/booktabs/booktabs.sty
Package: booktabs 2020/01/12 v1.61803398 Publication quality tables
\heavyrulewidth=\dimen153
\lightrulewidth=\dimen154
\cmidrulewidth=\dimen155
\belowrulesep=\dimen156
\belowbottomsep=\dimen157
\aboverulesep=\dimen158
\abovetopsep=\dimen159
\cmidrulesep=\dimen160
\cmidrulekern=\dimen161
\defaultaddspace=\dimen162
\@cmidla=\count193
\@cmidlb=\count194
\@aboverulesep=\dimen163
\@belowrulesep=\dimen164
\@thisruleclass=\count195
\@lastruleclass=\count196
\@thisrulewidth=\dimen165
) (c:/TeXLive/2020/texmf-dist/tex/latex/float/float.sty
Package: float 2001/11/08 v1.3d Float enhancements (AL)
\c@float@type=\count197
\float@exts=\toks30
\float@box=\box47
\@float@everytoks=\toks31
\@floatcapt=\box48
) (c:/TeXLive/2020/texmf-dist/tex/latex/amsrefs/textcmds.sty
Package: textcmds 2012/08/02 v2.00
) (c:/TeXLive/2020/texmf-dist/tex/latex/siunitx/siunitx.sty
(c:/TeXLive/2020/te
xmf-dist/tex/latex/l3kernel/expl3.sty
Package: expl3 2020-05-05 L3 programming layer (loader)
(c:/TeXLive/2020/texmf-dist/tex/latex/l3backend/13backend-pdfmode.def
File: 13backend-pdfmode.def 2020-05-05 L3 backend support: PDF mode
\l kernel color stack int=\count198
)) (c:/TeXLive/2020/texmf-dist/tex/latex/l3packages/xparse/xparse.sty
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Package: xparse 2020-03-06 L3 Experimental document command parser
\l xparse current arg int=\count199
\g xparse grabber int=\count266
\l xparse_m_args_int=\count267
\lambda xparse v nesting_int=\count268
)
Package: siunitx 2020/02/25 v2.8b A comprehensive (SI) units package
(c:/TeXLive/2020/texmf-dist/tex/latex/tools/array.sty
Package: array 2019/08/31 v2.41 Tabular extension package (FMi)
\col@sep=\dimen166
\ar@mcellbox=\box50
\extrarowheight=\dimen167
\NC@list=\toks32
\extratabsurround=\skip63
\backup@length=\skip64
\ar@cellbox=\box51
) (c:/TeXLive/2020/texmf-dist/tex/latex/l3packages/l3keys2e/l3keys2e.sty
Package: 13keys2e 2020-03-06 LaTeX2e option processing using LaTeX3 keys
\l siunitx tmp box=\box52
\l siunitx_tmp_dim=\dimen168
\l siunitx tmp int=\count269
\l siunitx number mantissa length int=\count270
\l siunitx number uncert length int=\count271
\l siunitx round int=\count272
\l siunitx process decimal int=\count273
\l siunitx process uncertainty int=\count274
\l siunitx process fixed int=\count275
\l siunitx process_integer_min_int=\count276
\l siunitx process precision int=\count277
\l siunitx group min int=\count278
\l siunitx angle marker box=\box53
\l_ siunitx angle_unit_box=\box54
\l siunitx angle marker dim=\dimen169
\l siunitx angle unit dim=\dimen170
\l siunitx unit int=\count279
\l siunitx unit denominator int=\count280
\l siunitx unit numerator int=\count281
\l siunitx unit prefix int=\count282
\l__siunitx_unit_prefix_base_int=\count283
\l__siunitx_unit_prefix_gram_int=\count284
\l__siunitx_number_product_int=\count285
\c siunitx one fill skip=\skip65
\l siunitx table unit align skip=\skip66
\l siunitx table exponent dim=\dimen171
\l siunitx table integer dim=\dimen172
\l siunitx table mantissa dim=\dimen173
\l__siunitx_table_marker_dim=\dimen174
\l__siunitx_table_result_dim=\dimen175
\l siunitx table uncert dim=\dimen176
\l siunitx table fill pre dim=\dimen177
\l siunitx table fill post dim=\dimen178
\l__siunitx_table_fill_mid_dim=\dimen179
\l_siunitx_table pre box=\box55
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\l siunitx table post box=\box56
\l siunitx table mantissa box=\box57
\l siunitx table result box=\box58
\l siunitx table number align skip=\skip67
\l siunitx table text align skip=\skip68
(c:/TeXLive/2020/texmf-dist/tex/latex/translator/translator.sty
Package: translator 2019-05-31 v1.12a Easy translation of strings in
)) (c:/TeXLive/2020/texmf-dist/tex/latex/todonotes/todonotes.sty
Package: todonotes 2019/01/24 v1.1.2 Todonotes source and documentation.
Package: todonotes 2018/11/22
(c:/TeXLive/2020/texmf-dist/tex/latex/base/ifthen.sty
Package: ifthen 2014/09/29 v1.1c Standard LaTeX ifthen package (DPC)
) (c:/TeXLive/2020/texmf-dist/tex/latex/xcolor/xcolor.sty
Package: xcolor 2016/05/11 v2.12 LaTeX color extensions (UK)
(c:/TeXLive/2020/texmf-dist/tex/latex/graphics-cfg/color.cfg
File: color.cfg 2016/01/02 v1.6 sample color configuration
)
Package xcolor Info: Driver file: pdftex.def on input line 225.
(c:/TeXLive/2020/texmf-dist/tex/latex/graphics-def/pdftex.def
File: pdftex.def 2018/01/08 v1.01 Graphics/color driver for pdftex
Package xcolor Info: Model `cmy' substituted by `cmy0' on input line
Package xcolor Info: Model `hsb' substituted by `rgb' on input line 1352.
Package xcolor Info: Model `RGB' extended on input line 1364.
Package xcolor Info: Model `HTML' substituted by `rgb' on input line
Package xcolor Info: Model `Hsb' substituted by `hsb' on input line 1367.
Package xcolor Info: Model `tHsb' substituted by `hsb' on input line
Package xcolor Info: Model `HSB' substituted by `hsb' on input line 1369.
Package xcolor Info: Model `Gray' substituted by `gray' on input line
1370.
Package xcolor Info: Model `wave' substituted by `hsb' on input line
) (c:/TeXLive/2020/texmf-dist/tex/latex/pgf/frontendlayer/tikz.sty
(c:/TeXLive/
2020/texmf-dist/tex/latex/pgf/basiclayer/pgf.sty (c:/TeXLive/2020/texmf-
x/latex/pgf/utilities/pgfrcs.sty (c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/ut
ilities/pgfutil-common.tex
\pgfutil@everybye=\toks33
\pgfutil@tempdima=\dimen180
\pgfutil@tempdimb=\dimen181
(c:/TeXLive/2020/texmf-dist/tex/generic/pgf/utilities/pgfutil-common-
lists.tex)
) (c:/TeXLive/2020/texmf-dist/tex/generic/pgf/utilities/pgfutil-latex.def
\pgfutil@abb=\box59
(c:/TeXLive/2020/texmf-dist/tex/latex/ms/everyshi.sty
Package: everyshi 2001/05/15 v3.00 EveryShipout Package (MS)
)) (c:/TeXLive/2020/texmf-dist/tex/generic/pgf/utilities/pgfrcs.code.tex
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XLive/2020/texmf-dist/tex/generic/pgf/pgf.revision.tex)
Package: pgfrcs 2020/01/08 v3.1.5b (3.1.5b)
Package: pgf 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-dist/tex/latex/pgf/basiclayer/pgfcore.sty
(c:/TeXLive/20
20/texmf-dist/tex/latex/graphics/graphicx.sty
Package: graphicx 2019/11/30 v1.2a Enhanced LaTeX Graphics (DPC, SPQR)
(c:/TeXLive/2020/texmf-dist/tex/latex/graphics/graphics.sty
Package: graphics 2019/11/30 v1.4a Standard LaTeX Graphics (DPC, SPQR)
(c:/TeXLive/2020/texmf-dist/tex/latex/graphics/trig.sty
Package: trig 2016/01/03 v1.10 sin cos tan (DPC)
) (c:/TeXLive/2020/texmf-dist/tex/latex/graphics-cfg/graphics.cfg
File: graphics.cfg 2016/06/04 v1.11 sample graphics configuration
Package graphics Info: Driver file: pdftex.def on input line 105.
\Gin@req@height=\dimen182
\Gin@req@width=\dimen183
) (c:/TeXLive/2020/texmf-dist/tex/latex/pqf/systemlayer/pqfsys.sty
(c:/TeXLive/
2020/texmf-dist/tex/generic/pgf/systemlayer/pgfsys.code.tex
Package: pgfsys 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-dist/tex/generic/pgf/utilities/pgfkeys.code.tex
\pgfkeys@pathtoks=\toks34
\pgfkeys@temptoks=\toks35
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/utilities/pgfkeysfiltered.code.tex
\pgfkeys@tmptoks=\toks36
) )
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\neq \
\pgf@xa=\dimen186
\pgf@ya=\dimen187
\pgf@xb=\dimen188
\pgf@yb=\dimen189
\pgf@xc=\dimen190
\pgf@yc=\dimen191
\pqf@xd=\dimen192
\pgf@yd=\dimen193
\w@pgf@writea=\write3
\r@pgf@reada=\read2
\c@pgf@counta=\count286
\c@pgf@countb=\count287
\c@pqf@countc=\count288
\c@pgf@countd=\count289
\t@pqf@toka=\toks37
\t@pgf@tokb=\toks38
\t@pqf@tokc=\toks39
\pgf@sys@id@count=\count290
(c:/TeXLive/2020/texmf-dist/tex/generic/pgf/systemlayer/pgf.cfg
File: pgf.cfg 2020/01/08 v3.1.5b (3.1.5b)
Driver file for pgf: pgfsys-pdftex.def
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File: pgfsys-pdftex.def 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-dist/tex/generic/pgf/systemlayer/pgfsys-common-
pdf.def
File: pgfsys-common-pdf.def 2020/01/08 v3.1.5b (3.1.5b)
)))
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/systemlayer/pgfsyssoftpath.code.tex
File: pgfsyssoftpath.code.tex 2020/01/08 v3.1.5b (3.1.5b)
\pgfsyssoftpath@smallbuffer@items=\count291
\pgfsyssoftpath@bigbuffer@items=\count292
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/systemlayer/pgfsysprotocol.code.tex
File: pgfsysprotocol.code.tex 2020/01/08 v3.1.5b (3.1.5b)
)) (c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/basiclayer/pgfcore.code.tex
Package: pgfcore 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-dist/tex/generic/pgf/math/pgfmath.code.tex
(c:/TeXLive/2
020/texmf-dist/tex/generic/pgf/math/pgfmathcalc.code.tex
(c:/TeXLive/2020/texmf
-dist/tex/generic/pgf/math/pgfmathutil.code.tex) (c:/TeXLive/2020/texmf-
x/generic/pgf/math/pgfmathparser.code.tex
\pgfmath@dimen=\dimen194
\pgfmath@count=\count293
\pgfmath@box=\box60
\pgfmath@toks=\toks40
\pgfmath@stack@operand=\toks41
\pgfmath@stack@operation=\toks42
) (c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/math/pgfmathfunctions.code.tex
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/math/pgfmathfunctions.basic.code.te
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/math/pgfmathfunctions.trigonometric
.code.tex)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/math/pgfmathfunctions.random.code.t
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/math/pgfmathfunctions.comparison.co
de.tex)
(c:/TeXLive/2020/texmf-
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etics.code.tex))) (c:/TeXLive/2020/texmf-
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.code.tex
\c@pgfmathroundto@lastzeros=\count294
)) (c:/TeXLive/2020/texmf-dist/tex/generic/pgf/math/pgfint.code.tex)
(c:/TeXLiv
e/2020/texmf-dist/tex/generic/pgf/basiclayer/pgfcorepoints.code.tex
File: pgfcorepoints.code.tex 2020/01/08 v3.1.5b (3.1.5b)
\pgf@picminx=\dimen195
\pgf@picmaxx=\dimen196
\pgf@picminy=\dimen197
\pgf@picmaxy=\dimen198
\pgf@pathminx=\dimen199
\pgf@pathmaxx=\dimen256
\pgf@pathminy=\dimen257
\pgf@pathmaxy=\dimen258
\pgf@xx=\dimen259
\py=\dim 260
\py=\dimen261
\pgf@yy=\dimen262
\pgf@zx=\dimen263
\pgf@zy=\dimen264
)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/basiclayer/pgfcorepathconstruct.cod
e.tex
File: pgfcorepathconstruct.code.tex 2020/01/08 v3.1.5b (3.1.5b)
\pgf@path@lastx=\dimen265
\pgf@path@lasty=\dimen266
)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/basiclayer/pgfcorepathusage.code.te
File: pgfcorepathusage.code.tex 2020/01/08 v3.1.5b (3.1.5b)
\pgf@shorten@end@additional=\dimen267
\pgf@shorten@start@additional=\dimen268
) (c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/basiclayer/pgfcorescopes.code.tex
File: pgfcorescopes.code.tex 2020/01/08 v3.1.5b (3.1.5b)
\pgfpic=\box61
\pgf@hbox=\box62
\pgf@layerbox@main=\box63
\pgf@picture@serial@count=\count295
)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/basiclayer/pgfcoregraphicstate.code
File: pgfcoregraphicstate.code.tex 2020/01/08 v3.1.5b (3.1.5b)
\pgflinewidth=\dimen269
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/basiclayer/pgfcoretransformations.c
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ode.tex
File: pgfcoretransformations.code.tex 2020/01/08 v3.1.5b (3.1.5b)
\pgf@pt@x=\dimen270
\pgf@pt@y=\dimen271
\pgf@pt@temp=\dimen272
) (c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/basiclayer/pgfcorequick.code.tex
File: pgfcorequick.code.tex 2020/01/08 v3.1.5b (3.1.5b)
) (c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/basiclayer/pgfcoreobjects.code.te
File: pgfcoreobjects.code.tex 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/basiclayer/pgfcorepathprocessing.co
de.tex
File: pgfcorepathprocessing.code.tex 2020/01/08 v3.1.5b (3.1.5b)
) (c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/basiclayer/pgfcorearrows.code.tex
File: pgfcorearrows.code.tex 2020/01/08 v3.1.5b (3.1.5b)
\pgfarrowsep=\dimen273
) (c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/basiclayer/pgfcoreshade.code.tex
File: pgfcoreshade.code.tex 2020/01/08 v3.1.5b (3.1.5b)
\pgf@max=\dimen274
\pgf@sys@shading@range@num=\count296
\pgf@shadingcount=\count297
) (c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/basiclayer/pgfcoreimage.code.tex
File: pgfcoreimage.code.tex 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/basiclayer/pgfcoreexternal.code.tex
File: pgfcoreexternal.code.tex 2020/01/08 v3.1.5b (3.1.5b)
\pgfexternal@startupbox=\box64
)) (c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/basiclayer/pgfcorelayers.code.te
File: pgfcorelayers.code.tex 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/basiclayer/pgfcoretransparency.code
File: pgfcoretransparency.code.tex 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/basiclayer/pgfcorepatterns.code.tex
File: pgfcorepatterns.code.tex 2020/01/08 v3.1.5b (3.1.5b)
) (c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/basiclayer/pgfcorerdf.code.tex
File: pgfcorerdf.code.tex 2020/01/08 v3.1.5b (3.1.5b)
))) (c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/modules/pgfmoduleshapes.code.te
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File: pgfmoduleshapes.code.tex 2020/01/08 v3.1.5b (3.1.5b)
\pgfnodeparttextbox=\box65
) (c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/modules/pgfmoduleplot.code.tex
File: pgfmoduleplot.code.tex 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-dist/tex/latex/pgf/compatibility/pgfcomp-version-
0-65.st
Package: pgfcomp-version-0-65 2020/01/08 v3.1.5b (3.1.5b)
\pgf@nodesepstart=\dimen275
\pgf@nodesepend=\dimen276
(c:/TeXLive/2020/texmf-dist/tex/latex/pgf/compatibility/pgfcomp-version-
1-18.st
Package: pgfcomp-version-1-18 2020/01/08 v3.1.5b (3.1.5b)
)) (c:/TeXLive/2020/texmf-dist/tex/latex/pgf/utilities/pgffor.sty
(c:/TeXLive/2
020/texmf-dist/tex/latex/pqf/utilities/pqfkeys.sty
(c:/TeXLive/2020/texmf-dist/
tex/generic/pgf/utilities/pgfkeys.code.tex)) (c:/TeXLive/2020/texmf-
dist/tex/la
tex/pgf/math/pgfmath.sty (c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/math/pgfma
th.code.tex)) (c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/utilities/pgffor.code
.tex
Package: pgffor 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-dist/tex/generic/pgf/math/pgfmath.code.tex)
\pgffor@iter=\dimen277
\pgffor@skip=\dimen278
\pgffor@stack=\toks43
\pqffor@toks=\toks44
)) (c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/frontendlayer/tikz/tikz.code.tex
Package: tikz 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/libraries/pgflibraryplothandlers.co
de.tex
File: pgflibraryplothandlers.code.tex 2020/01/08 v3.1.5b (3.1.5b)
\pgf@plot@mark@count=\count298
\pgfplotmarksize=\dimen279
\tikz@lastx=\dimen280
\tikz@lasty=\dimen281
\tikz@lastxsaved=\dimen282
\tikz@lastysaved=\dimen283
\tikz@lastmovetox=\dimen284
\tikz@lastmovetoy=\dimen285
\tikzleveldistance=\dimen286
\tikzsiblingdistance=\dimen287
\tikz@figbox=\box66
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\tikz@figbox@bg=\box67
\tikz@tempbox=\box68
\tikz@tempbox@bg=\box69
\tikztreelevel=\count299
\tikznumberofchildren=\count300
\tikznumberofcurrentchild=\count301
\tikz@fig@count=\count302
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/modules/pgfmodulematrix.code.tex
File: pgfmodulematrix.code.tex 2020/01/08 v3.1.5b (3.1.5b)
\pgfmatrixcurrentrow=\count303
\pgfmatrixcurrentcolumn=\count304
\pgf@matrix@numberofcolumns=\count305
\tikz@expandcount=\count306
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/frontendlayer/tikz/libraries/tikzli
brarytopaths.code.tex
File: tikzlibrarytopaths.code.tex 2020/01/08 v3.1.5b (3.1.5b)
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(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/frontendlayer/tikz/libraries/tikzli
brarypositioning.code.tex
File: tikzlibrarypositioning.code.tex 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/frontendlayer/tikz/libraries/tikzli
braryshadows.code.tex
File: tikzlibraryshadows.code.tex 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/frontendlayer/tikz/libraries/tikzli
braryfadings.code.tex
File: tikzlibraryfadings.code.tex 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/libraries/pgflibraryfadings.code.te
File: pgflibraryfadings.code.tex 2020/01/08 v3.1.5b (3.1.5b)
pdfTeX warning (Object streams): \pdfobjcompresslevel > 0 requires PDF-
1.5 or g
reater. Object streams disabled now.
))) (c:/TeXLive/2020/texmf-dist/tex/latex/tools/calc.sty
Package: calc 2017/05/25 v4.3 Infix arithmetic (KKT,FJ)
\calc@Acount=\count307
\calc@Bcount=\count308
\calc@Adimen=\dimen288
\calc@Bdimen=\dimen289
\calc@Askip=\skip69
\calc@Bskip=\skip70
LaTeX Info: Redefining \setlength on input line 80.
LaTeX Info: Redefining \addtolength on input line 81.
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\calc@Ccount=\count309
\calc@Cskip=\skip71
)
\c@@todonotes@numberoftodonotes=\count310
) (c:/TeXLive/2020/texmf-dist/tex/latex/qtree/qtree.sty
Package: 2008/12/12 v.3.1bQtree: tree-drawing for linguistics
Package: qtree 2008/12/12 v.3.1bQtree: tree-drawing for linguistics
(c:/TeXLive/2020/texmf-dist/tex/latex/pict2e/pict2e.sty
Package: pict2e 2019/08/20 v0.3c Improved picture commands (HjG,RN,JT)
(c:/TeXLive/2020/texmf-dist/tex/latex/pict2e/pict2e.cfg
File: pict2e.cfg 2016/02/05 v0.1u pict2e configuration for teTeX/TeXLive
Package pict2e Info: Driver file: pdftex.def on input line 119.
Package pict2e Info: Driver file for pict2e: p2e-pdftex.def on input line
121.
(c:/TeXLive/2020/texmf-dist/tex/latex/pict2e/p2e-pdftex.def
File: p2e-pdftex.def 2016/02/05 v0.1u Driver-dependant file (RN,HjG,JT)
)
\pIIe@GRAPH=\toks45
\@arclen=\dimen290
\@arcrad=\dimen291
\@tempdimd=\dimen292
\c@treecount=\count311
\c@branchcount=\count312
\parentbox=\box70
\treebox=\box71
\treeboxone=\box72
\treeboxtwo=\box73
\treeboxthree=\box74
\treeboxfour=\box75
\treeboxfive=\box76
\treeboxsix=\box77
\treeboxseven=\box78
\treeboxeight=\box79
\treeboxnine=\box80
\treeboxten=\box81
\treeboxeleven=\box82
\treeboxtwelve=\box83
\treeboxthirteen=\box84
\treeboxfourteen=\box85
\treeboxfifteen=\box86
\treeboxsixteen=\box87
\treeboxseventeen=\box88
\treeboxeighteen=\box89
\treeboxnineteen=\box90
\treeboxtwenty=\box91
\treeoffsetone=\skip72
\treeoffsettwo=\skip73
\treeoffsetthree=\skip74
\treeoffsetfour=\skip75
\treeoffsetfive=\skip76
\treeoffsetsix=\skip77
\treeoffsetseven=\skip78
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\treeoffseteight=\skip79 \treeoffsetnine=\skip80 \treeoffsetten=\skip81 \treeoffseteleven=\skip82 \treeoffsettwelve=\skip83 \treeoffsetthirteen=\skip84 \treeoffsetfourteen=\skip85 \treeoffsetfifteen=\skip86 \treeoffsetsixteen=\skip87 \treeoffsetseventeen=\skip88 \treeoffseteighteen=\skip89 \treeoffsetnineteen=\skip90 \treeoffsettwenty=\skip91 \treeshiftone=\skip92 \treeshifttwo=\skip93 \treeshiftthree=\skip94 \treeshiftfour=\skip95 \treeshiftfive=\skip96 \treeshiftsix=\skip97 \treeshiftseven=\skip98 \treeshifteight=\skip99 \treeshiftnine=\skip100 \treeshiftten=\skip101 \treeshifteleven=\skip102 \treeshifttwelve=\skip103 \treeshiftthirteen=\skip104 \treeshiftfourteen=\skip105 \treeshiftfifteen=\skip106 \treeshiftsixteen=\skip107 \treeshiftseventeen=\skip108 \treeshifteighteen=\skip109 \treeshiftnineteen=\skip110 \treeshifttwenty=\skip111 \treewidthone=\skip112 \treewidthtwo=\skip113 \treewidththree=\skip114 \treewidthfour=\skip115 \treewidthfive=\skip116 \treewidthsix=\skip117 \treewidthseven=\skip118 \treewidtheight=\skip119 \treewidthnine=\skip120 \treewidthten=\skip121 \treewidtheleven=\skip122 \treewidthtwelve=\skip123 \treewidththirteen=\skip124 \treewidthfourteen=\skip125 \treewidthfifteen=\skip126 \treewidthsixteen=\skip127 \treewidthseventeen=\skip128 \treewidtheighteen=\skip129 \treewidthnineteen=\skip130 \treewidthtwenty=\skip131

\daughteroffsetone=\skip132

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\daughteroffsettwo=\skip133
\daughteroffsetthree=\skip134
\daughteroffsetfour=\skip135
\branchwidthone=\skip136
\branchwidthtwo=\skip137
\branchwidththree=\skip138
\branchwidthfour=\skip139
\parentoffset=\skip140
\treeoffset=\skip141
\daughteroffset=\skip142
\branchwidth=\skip143
\parentwidth=\skip144
\treewidth=\skip145
\qta=\toks46
\abstyle \text{ } \text{toks47}
\nbranches=\count313
\qroofx=\count314
\qroofy=\count315
\@qrscratchbox=\box92
)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/frontendlayer/tikz/libraries/tikzli
braryarrows.code.tex
File: tikzlibraryarrows.code.tex 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/libraries/pgflibraryarrows.code.tex
File: pgflibraryarrows.code.tex 2020/01/08 v3.1.5b (3.1.5b)
\arrowsize=\dimen293
))
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/frontendlayer/tikz/libraries/tikzli
braryshapes.code.tex
File: tikzlibraryshapes.code.tex 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/frontendlayer/tikz/libraries/tikzli
braryshapes.geometric.code.tex
File: tikzlibraryshapes.geometric.code.tex 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/libraries/shapes/pgflibraryshapes.g
eometric.code.tex
File: pgflibraryshapes.geometric.code.tex 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/frontendlayer/tikz/libraries/tikzli
braryshapes.misc.code.tex
File: tikzlibraryshapes.misc.code.tex 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/libraries/shapes/pgflibraryshapes.m
isc.code.tex
File: pgflibraryshapes.misc.code.tex 2020/01/08 v3.1.5b (3.1.5b)
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))
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dist/tex/generic/pgf/frontendlayer/tikz/libraries/tikzli
braryshapes.symbols.code.tex
File: tikzlibraryshapes.symbols.code.tex 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/libraries/shapes/pgflibraryshapes.s
ymbols.code.tex
File: pgflibraryshapes.symbols.code.tex 2020/01/08 v3.1.5b (3.1.5b)
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(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/frontendlayer/tikz/libraries/tikzli
braryshapes.arrows.code.tex
File: tikzlibraryshapes.arrows.code.tex 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/libraries/shapes/pgflibraryshapes.a
rrows.code.tex
File: pgflibraryshapes.arrows.code.tex 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/frontendlayer/tikz/libraries/tikzli
braryshapes.callouts.code.tex
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/libraries/shapes/pgflibraryshapes.c
allouts.code.tex))
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/frontendlayer/tikz/libraries/tikzli
braryshapes.multipart.code.tex
File: tikzlibraryshapes.multipart.code.tex 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/libraries/shapes/pgflibraryshapes.m
ultipart.code.tex
File: pgflibraryshapes.multipart.code.tex 2020/01/08 v3.1.5b (3.1.5b)
\pgfnodepartlowerbox=\box93
\pgfnodeparttwobox=\box94
\pgfnodepartthreebox=\box95
\pgfnodepartfourbox=\box96
\pgfnodeparttwentybox=\box97
\pgfnodepartnineteenbox=\box98
\pgfnodeparteighteenbox=\box99
\pgfnodepartseventeenbox=\box100
\pgfnodepartsixteenbox=\box101
\pgfnodepartfifteenbox=\box102
\pgfnodepartfourteenbox=\box103
\pgfnodepartthirteenbox=\box104
\pgfnodeparttwelvebox=\box105
\pgfnodepartelevenbox=\box106
\pgfnodeparttenbox=\box107
\pgfnodepartninebox=\box108
\pgfnodeparteightbox=\box109
\pgfnodepartsevenbox=\box110
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\pgfnodepartsixbox=\box111
\pgfnodepartfivebox=\box112
))))
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/frontendlayer/tikz/libraries/tikzli
braryfit.code.tex
File: tikzlibraryfit.code.tex 2020/01/08 v3.1.5b (3.1.5b)
(c:/TeXLive/2020/texmf-
dist/tex/generic/pgf/frontendlayer/tikz/libraries/tikzli
brarycalc.code.tex
File: tikzlibrarycalc.code.tex 2020/01/08 v3.1.5b (3.1.5b)
) (c:/TeXLive/2020/texmf-dist/tex/latex/algorithms/algorithm.sty
Package: algorithm 2009/08/24 v0.1 Document Style `algorithm' - floating
enviro
nment
\@float@every@algorithm=\toks48
\c@algorithm=\count316
) (c:/TeXLive/2020/texmf-dist/tex/latex/algorithmicx/algorithmicx.sty
Package: algorithmicx 2005/04/27 v1.2 Algorithmicx
Document Style algorithmicx 1.2 - a greatly improved `algorithmic' style
\c@ALG@line=\count317
\c@ALG@rem=\count318
\c@ALG@nested=\count319
\ALG@tlm=\skip146
\ALG@thistlm=\skip147
\c@ALG@Lnr=\count320
\c@ALG@blocknr=\count321
\c@ALG@storecount=\count322
\c@ALG@tmpcounter=\count323
\ALG@tmplength=\skip148
) (c:/TeXLive/2020/texmf-dist/tex/latex/algorithmicx/algpseudocode.sty
Package: algpseudocode
Document Style - pseudocode environments for use with the `algorithmicx'
) (c:/TeXLive/2020/texmf-dist/tex/latex/newtx/newtxtext.sty
Package: newtxtext 2020/03/02 v1.625
`newtxtext' v1.625, 2020/03/02 Text macros taking advantage of TeX-Gyre
Termes
fonts (msharpe) (c:/TeXLive/2020/texmf-
dist/tex/latex/fontaxes/fontaxes.sty
Package: fontaxes 2014/03/23 v1.0d Font selection axes
LaTeX Info: Redefining \upshape on input line 29.
LaTeX Info: Redefining \itshape on input line 31.
LaTeX Info: Redefining \slshape on input line 33.
LaTeX Info: Redefining \swshape on input line 35.
LaTeX Info: Redefining \scshape on input line 37.
LaTeX Info: Redefining \sscshape on input line 39.
LaTeX Info: Redefining \ulcshape on input line 41.
LaTeX Info: Redefining \textsw on input line 47.
LaTeX Info: Redefining \textssc on input line 48.
LaTeX Info: Redefining \textulc on input line 49.
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\ntx@fs=\dimen294 (c:/TexLive/2020/texmf-dist/tex/latex/etoolbox/etoolbox.sty Package: etoolbox 2019/09/21 v2.5h e-TeX tools for LaTeX (JAW) \etb@tempcnta=\count324) (c:/TeXLive/2020/texmf-dist/tex/latex/base/fontenc.sty Package: fontenc 2020/02/11 v2.0o Standard LaTeX package LaTeX Font Info: Trying to load font information for T1+ntxtlf on input line 112. (c:/TeXLive/2020/texmf-dist/tex/latex/newtx/t1ntxtlf.fd File: t1ntxtlf.fd 2015/01/17 v1.0 font definition file for T1/ntx/tlf LaTeX Font Info: Font shape `T1/ntxtlf/m/n' will be (Font) scaled to size 12.0pt on input line 112.) (c:/TeXLive/2020/texmf-dist/tex/latex/base/textcomp.sty Package: textcomp 2020/02/02 v2.0n Standard LaTeX package LaTeX Font Info: Changing ? sub-encoding to TS1/0 on input line 75.) \ntxt@cntz=\count325 Redeclaring math symbol \mathsterling on input line LaTeX Font Info: 263.) (c:/TeXLive/2020/texmf-dist/tex/latex/newtx/newtxmath.sty Package: newtxmath 2020/05/02 v1.626 `newtxmath' v1.626, 2020/05/02 Math macros based originally on txfonts (msharpe) (c:/TeXLive/2020/texmf-dist/tex/generic/iftex/ifxetex.sty Package: ifxetex 2019/10/25 v0.7 ifxetex legacy package. Use iftex instead.) (c:/TeXLive/2020/texmf-dist/tex/generic/iftex/ifluatex.sty Package: ifluatex 2019/10/25 v1.5 ifluatex legacy package. Use iftex instead.) (c:/TeXLive/2020/texmf-dist/tex/latex/oberdiek/centernot.sty Package: centernot 2016/05/16 v1.4 Centers the not symbol horizontally (HO)) \tx@cntz=\count326 (c:/TeXLive/2020/texmf-dist/tex/generic/kastrup/binhex.tex) \tx@Isdigit=\count327 \tx@IsAlNum=\count328 \tx@tA=\toks49 $\tx@tB=\toks50$ \tx@su=\read3 LaTeX Font Info: Redeclaring symbol font `operators' on input line 320. LaTeX Font Info: Overwriting symbol font `operators' in version `normal' (Font) OT1/cmr/m/n --> OT1/ntxtlf/m/n on input line 320. Overwriting symbol font `operators' in version `bold' LaTeX Font Info: OT1/cmr/bx/n --> OT1/ntxtlf/m/n on input line (Font) 320. LaTeX Font Info: Overwriting symbol font `operators' in version `bold' (Font) OT1/ntxtlf/m/n --> OT1/ntxtlf/b/n on input line 321.

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Redeclaring math alphabet \mathsf on input line 329.
LaTeX Font Info:
LaTeX Font Info:
                     Overwriting math alphabet `\mathsf' in version
`normal'
(Font)
                         OT1/cmss/m/n \longrightarrow T1/qhv/m/n on input line 329.
LaTeX Font Info:
                     Overwriting math alphabet `\mathsf' in version `bold'
                         OT1/cmss/bx/n \longrightarrow T1/qhv/m/n on input line 329.
(Font)
LaTeX Font Info:
                     Redeclaring math alphabet \mathit on input line 330.
LaTeX Font Info:
                     Overwriting math alphabet `\mathit' in version
`normal'
(Font)
                         OT1/cmr/m/it --> OT1/ntxtlf/m/it on input line
330.
                     Overwriting math alphabet `\mathit' in version `bold'
LaTeX Font Info:
(Font)
                         OT1/cmr/bx/it --> OT1/ntxtlf/m/it on input line
330.
LaTeX Font Info:
                     Redeclaring math alphabet \mathtt on input line 331.
                     Overwriting math alphabet `\mathtt' in version
LaTeX Font Info:
`normal'
                         OT1/cmtt/m/n --> T1/ntxtt/m/n on input line 331.
(Font)
LaTeX Font Info:
                     Overwriting math alphabet `\mathtt' in version `bold'
                         OT1/cmtt/m/n \longrightarrow T1/ntxtt/m/n on input line 331.
(Font)
LaTeX Font Info:
                     Redeclaring math alphabet \mathbf on input line 333.
                     Overwriting math alphabet `\mathbf' in version
LaTeX Font Info:
`normal'
(Font)
                         OT1/cmr/bx/n --> OT1/ntxtlf/b/n on input line
333.
                     Overwriting math alphabet `\mathbf' in version `bold'
LaTeX Font Info:
(Font)
                         OT1/cmr/bx/n --> OT1/ntxtlf/b/n on input line
LaTeX Font Info:
                     Overwriting math alphabet `\mathit' in version `bold'
(Font)
                         OT1/ntxtlf/m/it --> OT1/ntxtlf/b/it on input line
334.
LaTeX Font Info:
                     Overwriting math alphabet `\mathsf' in version `bold'
                         T1/qhv/m/n \longrightarrow T1/qhv/b/n on input line 335.
(Font)
                     Overwriting math alphabet `\mathtt' in version `bold'
LaTeX Font Info:
                         T1/ntxtt/m/n \longrightarrow T1/ntxtt/b/n on input line 336.
(Font)
LaTeX Font Info:
                     Redeclaring symbol font `letters' on input line 425.
                     Overwriting symbol font `letters' in version `normal'
LaTeX Font Info:
(Font)
                         OML/cmm/m/it --> OML/ntxmi/m/it on input line
425.
LaTeX Font Info:
                     Overwriting symbol font `letters' in version `bold'
(Font)
                         OML/cmm/b/it --> OML/ntxmi/m/it on input line
425.
                     Overwriting symbol font `letters' in version `bold'
LaTeX Font Info:
(Font)
                         OML/ntxmi/m/it --> OML/ntxmi/b/it on input line
426.
LaTeX Info: Redefining \mathscr on input line 458.
\symlettersA=\mathgroup7
LaTeX Font Info:
                     Overwriting symbol font `lettersA' in version `bold'
(Font)
                         U/ntxmia/m/it --> U/ntxmia/b/it on input line
479.
LaTeX Font Info:
                     Redeclaring math alphabet \mathfrak on input line
Now handling font encoding LMS ...
... no UTF-8 mapping file for font encoding LMS
```

```
Redeclaring symbol font `symbols' on input line 500.
LaTeX Font Info:
LaTeX Font Info:
                    Encoding `OMS' has changed to `LMS' for symbol font
                    `symbols' in the math version `normal' on input line
(Font)
500.
LaTeX Font Info:
                    Overwriting symbol font `symbols' in version `normal'
                        OMS/cmsy/m/n --> LMS/ntxsy/m/n on input line 500.
(Font)
                    Encoding `OMS' has changed to `LMS' for symbol font
LaTeX Font Info:
(Font)
                    `symbols' in the math version `bold' on input line
500.
LaTeX Font Info:
                    Overwriting symbol font `symbols' in version `bold'
                        OMS/cmsy/b/n --> LMS/ntxsy/m/n on input line 500.
(Font)
LaTeX Font Info:
                    Overwriting symbol font `symbols' in version `bold'
(Font)
                        LMS/ntxsy/m/n --> LMS/ntxsy/b/n on input line
501.
\symAMSm=\mathgroup8
LaTeX Font Info:
                    Overwriting symbol font `AMSm' in version `bold'
(Font)
                        U/ntxsym/m/n --> U/ntxsym/b/n on input line 524.
LaTeX Font Info:
                    Redeclaring math alphabet \mathbb on input line 526.
\symsymbolsC=\mathgroup9
                    Overwriting symbol font `symbolsC' in version `bold'
LaTeX Font Info:
                        U/ntxsyc/m/n --> U/ntxsyc/b/n on input line 545.
(Font)
Now handling font encoding LMX ...
... no UTF-8 mapping file for font encoding LMX
LaTeX Font Info:
                   Redeclaring symbol font `largesymbols' on input line
558.
LaTeX Font Info:
                    Encoding `OMX' has changed to `LMX' for symbol font
(Font)
                    `largesymbols' in the math version `normal' on input
line 5
58.
LaTeX Font Info:
                    Overwriting symbol font `largesymbols' in version
`normal'
(Font)
                        OMX/cmex/m/n --> LMX/ntxexx/m/n on input line
558.
LaTeX Font Info:
                    Encoding `OMX' has changed to `LMX' for symbol font
(Font)
                    `largesymbols' in the math version `bold' on input
line 558
LaTeX Font Info:
                    Overwriting symbol font `largesymbols' in version
`bold'
(Font)
                        OMX/cmex/m/n --> LMX/ntxexx/m/n on input line
558.
LaTeX Font Info:
                    Overwriting symbol font `largesymbols' in version
`bold'
(Font)
                        LMX/ntxexx/m/n --> LMX/ntxexx/b/n on input line
559.
\symlargesymbolsTXA=\mathgroup10
LaTeX Font Info: Overwriting symbol font `largesymbolsTXA' in version
`bold'
                        U/ntxexa/m/n --> U/ntxexa/b/n on input line 573.
(Font)
\tx@sbptoks=\toks51
LaTeX Font Info:
                   Redeclaring math delimiter \lfloor on input line 795.
                    Redeclaring math delimiter \rfloor on input line 796.
LaTeX Font Info:
LaTeX Font Info:
                    Redeclaring math delimiter \lceil on input line 797.
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LaTeX Font Info:
                    Redeclaring math delimiter \rceil on input line 798.
LaTeX Font Info:
                    Redeclaring math delimiter \lbrace on input line 799.
                    Redeclaring math delimiter \rbrace on input line 800.
LaTeX Font Info:
LaTeX Font Info:
                    Redeclaring math delimiter \langle on input line 801.
LaTeX Font Info:
                    Redeclaring math delimiter \rangle on input line 803.
                    Redeclaring math delimiter \arrowvert on input line
LaTeX Font Info:
807.
LaTeX Font Info:
                    Redeclaring math delimiter \vert on input line 808.
LaTeX Font Info:
                    Redeclaring math accent \dot on input line 879.
                    Redeclaring math accent \ddot on input line 880.
LaTeX Font Info:
                    Redeclaring math accent \vec on input line 1943.
LaTeX Font Info:
\ntx@numstart=\count329
\ntx@UCstart=\count330
\ntx@lcstart=\count331
\ntx@tmpc=\count332
\ntx@imath=\count333
\ntx@jmath=\count334
LaTeX Info: Redefining \Bbbk on input line 2452.
LaTeX Info: Redefining \not on input line 2601.
LaTeX Info: Redefining \textsquare on input line 2631.
LaTeX Info: Redefining \openbox on input line 2633.
) (c:/TeXLive/2020/texmf-dist/tex/latex/comment/comment.sty
\CommentStream=\write4
Excluding comment 'comment') (c:/TeXLive/2020/texmf-
dist/tex/generic/ulem/ulem.
sty
\UL@box=\box113
\UL@hyphenbox=\box114
\UL@skip=\skip149
\UL@hook=\toks52
\UL@height=\dimen295
\UL@pe=\count335
\UL@pixel=\dimen296
\ULC@box=\box115
Package: ulem 2019/11/18
\ULdepth=\dimen297
) (c:/TeXLive/2020/texmf-dist/tex/latex/titlesec/titlesec.sty
Package: titlesec 2019/10/16 v2.13 Sectioning titles
\ttl@box=\box116
\beforetitleunit=\skip150
\aftertitleunit=\skip151
\ttl@plus=\dimen298
\ttl@minus=\dimen299
\ttl@toksa=\toks53
\titlewidth=\dimen300
\titlewidthlast=\dimen301
\titlewidthfirst=\dimen302
) (c:/TeXLive/2020/texmf-dist/tex/latex/microtype/microtype.sty
Package: microtype 2019/11/18 v2.7d Micro-typographical refinements (RS)
\MT@toks=\toks54
\MT@count=\count336
LaTeX Info: Redefining \textls on input line 790.
\MT@outer@kern=\dimen303
LaTeX Info: Redefining \textmicrotypecontext on input line 1354.
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\MT@listname@count=\count337
(c:/TeXLive/2020/texmf-dist/tex/latex/microtype/microtype-pdftex.def
File: microtype-pdftex.def 2019/11/18 v2.7d Definitions specific to
pdftex (RS)
LaTeX Info: Redefining \lsstyle on input line 914.
LaTeX Info: Redefining \lslig on input line 914.
\MT@outer@space=\skip152
Package microtype Info: Loading configuration file microtype.cfg.
(c:/TeXLive/2020/texmf-dist/tex/latex/microtype/microtype.cfg
File: microtype.cfg 2019/11/18 v2.7d microtype main configuration file
(RS)
)) (c:/TeXLive/2020/texmf-dist/tex/latex/caption/caption.sty
Package: caption 2020/01/03 v3.4h Customizing captions (AR)
(c:/TeXLive/2020/texmf-dist/tex/latex/caption/caption3.sty
Package: caption3 2020/01/03 v1.8h caption3 kernel (AR)
Package caption3 Info: TeX engine: e-TeX on input line 61.
\captionmargin=\dimen304
\captionmargin@=\dimen305
\captionwidth=\dimen306
\caption@tempdima=\dimen307
\caption@indent=\dimen308
\caption@parindent=\dimen309
\caption@hangindent=\dimen310
Package caption Info: Standard document class detected.
\c@caption@flags=\count338
\c@continuedfloat=\count339
Package caption Info: float package is loaded.
\c@theo=\count340
\c@definition=\count341
(./ProbabilisticReconciliationR1.aux)
\openout1 = `ProbabilisticReconciliationR1.aux'.
                    Checking defaults for OML/ntxmi/m/it on input line
LaTeX Font Info:
60.
LaTeX Font Info:
                    Trying to load font information for OML+ntxmi on
input line
 60.
(c:/TeXLive/2020/texmf-dist/tex/latex/newtx/omlntxmi.fd
File: omlntxmi.fd 2015/08/25 Fontinst v1.933 font definitions for
OML/ntxmi.
LaTeX Font Info:
                    ... okay on input line 60.
LaTeX Font Info:
                    Checking defaults for OMS/cmsy/m/n on input line 60.
LaTeX Font Info:
                    ... okay on input line 60.
LaTeX Font Info:
                    Checking defaults for OT1/cmr/m/n on input line 60.
LaTeX Font Info:
                    ... okay on input line 60.
                    Checking defaults for T1/cmr/m/n on input line 60.
LaTeX Font Info:
LaTeX Font Info:
                    ... okay on input line 60.
LaTeX Font Info:
                    Checking defaults for TS1/cmr/m/n on input line 60.
LaTeX Font Info:
                    ... okay on input line 60.
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LaTeX Font Info:
                    Checking defaults for OMX/cmex/m/n on input line 60.
LaTeX Font Info:
                    ... okay on input line 60.
                    Checking defaults for U/ntxexa/m/n on input line 60.
LaTeX Font Info:
LaTeX Font Info:
                    Trying to load font information for U+ntxexa on input
line
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(c:/TeXLive/2020/texmf-dist/tex/latex/newtx/untxexa.fd
File: untxexa.fd 2012/04/16 Fontinst v1.933 font definitions for
U/ntxexa.
LaTeX Font Info:
                    ... okay on input line 60.
LaTeX Font Info:
                    Checking defaults for LMS/ntxsy/m/n on input line 60.
LaTeX Font Info:
                    Trying to load font information for LMS+ntxsy on
input line
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(c:/TeXLive/2020/texmf-dist/tex/latex/newtx/lmsntxsy.fd
File: lmsntxsy.fd 2016/07/02 Fontinst v1.933 font definitions for
LMS/ntxsy.
)
LaTeX Font Info:
                    ... okay on input line 60.
LaTeX Font Info:
                    Checking defaults for LMX/ntxexx/m/n on input line
LaTeX Font Info:
                    Trying to load font information for LMX+ntxexx on
input lin
e 60.
(c:/TeXLive/2020/texmf-dist/tex/latex/newtx/lmxntxexx.fd
File: lmxntxexx.fd 2016/07/03 Fontinst v1.933 font definitions for
LMX/ntxexx.
LaTeX Font Info:
                    ... okay on input line 60.
*geometry* driver: auto-detecting
*geometry* detected driver: pdftex
*geometry* verbose mode - [ preamble ] result:
* driver: pdftex
* paper: a4paper
* layout: <same size as paper>
* layoutoffset: (h, v) = (0.0pt, 0.0pt)
* modes:
* h-part: (L,W,R) = (78.33022pt, 440.84743pt, 78.33023pt)
* v-part: (T,H,B) = (48.93872pt, 722.7pt, 73.40813pt)
* \paperwidth=597.50787pt
* \paperheight=845.04684pt
* \textwidth=440.84743pt
* \textheight=722.7pt
* \oddsidemargin=6.06023pt
* \evensidemargin=6.06023pt
* \topmargin=-60.33127pt
* \headheight=12.0pt
* \headsep=25.0pt
* \topskip=12.0pt
* \footskip=30.0pt
* \marginparwidth=35.0pt
* \marginparsep=10.0pt
* \columnsep=10.0pt
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* \skip\footins=10.8pt plus 4.0pt minus 2.0pt
* \hoffset=0.0pt
* \voffset=0.0pt
* \mag=1000
* \ensuremath{\setminus} \texttt{@twocolumnfalse}
* \@twosidefalse
* \@mparswitchfalse
* \@reversemarginfalse
* (1in=72.27pt=25.4mm, 1cm=28.453pt)
(c:/TeXLive/2020/texmf-dist/tex/latex/translator/translator-basic-
dictionary-En
glish.dict
Dictionary: translator-basic-dictionary, Language: English
) (c:/TeXLive/2020/texmf-dist/tex/latex/siunitx/siunitx-abbreviations.cfg
File: siunitx-abbreviations.cfg 2017/11/26 v2.7k siunitx: Abbreviated
units
) (c:/TeXLive/2020/texmf-dist/tex/context/base/mkii/supp-pdf.mkii
[Loading MPS to PDF converter (version 2006.09.02).]
\scratchcounter=\count342
\scratchdimen=\dimen311
\scratchbox=\box117
\nofMPsegments=\count343
\nofMParguments=\count344
\everyMPshowfont=\toks55
\MPscratchCnt=\count345
\MPscratchDim=\dimen312
\MPnumerator=\count346
\makeMPintoPDFobject=\count347
\everyMPtoPDFconversion=\toks56
) (c:/TeXLive/2020/texmf-dist/tex/latex/epstopdf-pkg/epstopdf-base.sty
Package: epstopdf-base 2020-01-24 v2.11 Base part for package epstopdf
Package epstopdf-base Info: Redefining graphics rule for `.eps' on input
line 4
85.
(c:/TeXLive/2020/texmf-dist/tex/latex/latexconfig/epstopdf-sys.cfg
File: epstopdf-sys.cfg 2010/07/13 v1.3 Configuration of (r)epstopdf for
TeX Liv
)) ABD: EveryShipout initializing macros
Package todonotes Warning: The length marginparwidth is less than 2cm and
will
most likely cause issues with the appearance of inserted todonotes. The
issue c
an be solved by adding a line like \setlength {\marginparwidth }{2cm}
prior to
loading the todonotes package. on input line 60.
\c@mv@tabular=\count348
\c@mv@boldtabular=\count349
LaTeX Info: Redefining \microtypecontext on input line 60.
Package microtype Info: Generating PDF output.
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Package microtype Info: Character protrusion enabled (level 2).
Package microtype Info: Using default protrusion set `alltext'.
Package microtype Info: Automatic font expansion enabled (level 2),
(microtype)
                        stretch: 20, shrink: 20, step: 1, non-selected.
Package microtype Info: Using default expansion set `basictext'.
LaTeX Info: Redefining \showhyphens on input line 60.
Package microtype Info: No adjustment of tracking.
Package microtype Info: No adjustment of interword spacing.
Package microtype Info: No adjustment of character kerning.
(c:/TeXLive/2020/texmf-dist/tex/latex/microtype/mt-ptm.cfg
File: mt-ptm.cfg 2006/04/20 v1.7 microtype config. file: Times (RS)
Package caption Info: Begin \AtBeginDocument code.
Package caption Info: End \AtBeginDocument code.
                 Trying to load font information for OT1+ntxtlf on
LaTeX Font Info:
input lin
e 60.
(c:/TeXLive/2020/texmf-dist/tex/latex/newtx/ot1ntxtlf.fd
File: otlntxtlf.fd 2015/01/17 v1.0 font definition file for OTl/ntx/tlf
LaTeX Font Info:
                    Font shape `OT1/ntxtlf/m/n' will be
                    scaled to size 12.0pt on input line 60.
(Font)
LaTeX Font Info:
                    Font shape `OT1/ntxtlf/m/n' will be
(Font)
                    scaled to size 8.8pt on input line 60.
LaTeX Font Info:
                  Font shape `OT1/ntxtlf/m/n' will be
(Font)
                    scaled to size 6.6pt on input line 60.
LaTeX Font Info:
                  Trying to load font information for U+msa on input
line 60.
(c:/TeXLive/2020/texmf-dist/tex/latex/amsfonts/umsa.fd
File: umsa.fd 2013/01/14 v3.01 AMS symbols A
) (c:/TeXLive/2020/texmf-dist/tex/latex/microtype/mt-msa.cfg
File: mt-msa.cfg 2006/02/04 v1.1 microtype config. file: AMS symbols (a)
(RS)
LaTeX Font Info:
                    Trying to load font information for U+msb on input
line 60.
(c:/TeXLive/2020/texmf-dist/tex/latex/amsfonts/umsb.fd
File: umsb.fd 2013/01/14 v3.01 AMS symbols B
) (c:/TeXLive/2020/texmf-dist/tex/latex/microtype/mt-msb.cfg
File: mt-msb.cfg 2005/06/01 v1.0 microtype config. file: AMS symbols (b)
(RS)
LaTeX Font Info:
                    Trying to load font information for U+rsfs on input
line 60
(c:/TeXLive/2020/texmf-dist/tex/latex/jknapltx/ursfs.fd
File: ursfs.fd 1998/03/24 rsfs font definition file (jk)
)
LaTeX Font Info:
                    Trying to load font information for U+ntxmia on input
line
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(c:/TeXLive/2020/texmf-dist/tex/latex/newtx/untxmia.fd
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File: untxmia.fd 2018/04/14 Fontinst v1.933 font definitions for
U/ntxmia.
LaTeX Font Info:
                    Trying to load font information for U+ntxsym on input
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File: untxsym.fd 2015/03/20 Fontinst v1.933 font definitions for
U/ntxsym.
LaTeX Font Info:
                    Trying to load font information for U+ntxsyc on input
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(c:/TeXLive/2020/texmf-dist/tex/latex/newtx/untxsyc.fd
File: untxsyc.fd 2012/04/12 Fontinst v1.933 font definitions for
U/ntxsyc.
LaTeX Font Info:
                    Trying to load font information for T1+qhv on input
line 60
(c:/TeXLive/2020/texmf-dist/tex/latex/tex-gyre/t1qhv.fd
File: tlqhv.fd 2009/09/25 v1.2 font definition file for T1/qhv
LaTeX Font Info:
                    Font shape `T1/qhv/m/n' will be
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                    scaled to size 10.79993pt on input line 60.
Package microtype Info: Loading generic protrusion settings for font
family
                        `qhv' (encoding: T1).
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                        For optimal results, create family-specific
(microtype)
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                        See the microtype manual for details.
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                    Font shape `T1/qhv/m/n' will be
                    scaled to size 7.91994pt on input line 60.
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                  Font shape `T1/qhv/m/n' will be
(Font)
                    scaled to size 5.93996pt on input line 60.
LaTeX Font Info:
                    Trying to load font information for T1+ntxtt on input
line
60.
(c:/TeXLive/2020/texmf-dist/tex/latex/newtx/t1ntxtt.fd
File: t1ntxtt.fd 2012/04/20 v3.1
)
LaTeX Font Info:
                    Font shape `T1/ntxtt/m/n' will be
                    scaled to size 12.0pt on input line 60.
(Font)
Package microtype Info: Loading generic protrusion settings for font
family
                        `ntxtt' (encoding: T1).
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                        For optimal results, create family-specific
settings.
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(Font)
                    scaled to size 8.8pt on input line 60.
LaTeX Font Info:
                    Font shape `T1/ntxtt/m/n' will be
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                    scaled to size 6.6pt on input line 60.
LaTeX Font Info:
                   Font shape `T1/ntxtlf/m/n' will be
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scaled to size 10.95pt on input line 65.
(Font)
                    Font shape `T1/ntxtlf/m/n' will be
LaTeX Font Info:
                    scaled to size 20.74pt on input line 92.
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                    Font shape `T1/ntxtlf/b/n' will be
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                    scaled to size 7.9pt on input line 92.
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LaTeX Font Warning: Font shape `U/rsfs/m/n' in size <10.5> not available
(Font)
                    size <10.95> substituted on input line 92.
LaTeX Font Info:
                    Font shape `T1/ntxtlf/m/n' will be
                    scaled to size 10.5pt on input line 92.
LaTeX Font Info:
                    Trying to load font information for TS1+ntxtlf on
input lin
e 92.
(c:/TeXLive/2020/texmf-dist/tex/latex/newtx/ts1ntxtlf.fd
File: ts1ntxtlf.fd 2015/01/18 v1.0 fd file for TS1/ntxtlf
                    Font shape `TS1/ntxtlf/m/n' will be
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                    Font shape `T1/ntxtlf/m/n' will be
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LaTeX Font Warning: Font shape `U/rsfs/m/n' in size <5.5> not available

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LaTeX Font Info: Font shape `T1/ntxtlf/m/n' will be (Font) scaled to size 7.3pt on input line 92.

LaTeX Font Info: Font shape `TS1/ntxtlf/m/n' will be (Font) scaled to size 7.3pt on input line 92.

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LaTeX Font Info: Font shape `T1/ntxtlf/b/n' will be

(Font) scaled to size 10.95pt on input line 107.

LaTeX Font Info: Font shape `T1/ntxtlf/m/it' will be

(Font) scaled to size 12.0pt on input line 115.

LaTeX Font Info: Font shape `T1/ntxtlf/m/n' will be

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LaTeX Font Info: Font shape `T1/ntxtlf/b/n' will be

(Font) scaled to size 17.28pt on input line 120.

LaTeX Font Info: Font shape `T1/ntxtlf/b/n' will be

(Font) scaled to size 12.0pt on input line 122.

Package natbib Warning: Citation `BabEtAl2021' on page 2 undefined on input lin e 122.

Package natbib Warning: Citation `KouAth2021' on page 2 undefined on input line 122.

Package natbib Warning: Citation `Taieb2017' on page 2 undefined on input line 122.

Package natbib Warning: Citation `nystrup2020' on page 2 undefined on input lin e 122.

Package natbib Warning: Citation `LiHyn2021' on page 2 undefined on input line 122.

Package natbib Warning: Citation `EckEtAl2021' on page 2 undefined on input \lim

e 122.

Package natbib Warning: Citation `AthEtAl2020_MacroBook' on page 2 undefined on input line 122.

Package natbib Warning: Citation `AthEtAl2022' on page 2 undefined on input lin e 122.

Package natbib Warning: Citation `KouAth2019' on page 2 undefined on input line 122.

Package natbib Warning: Citation `FPP2018' on page 2 undefined on input line 12
2.

Package natbib Warning: Citation `Dunn1976' on page 2 undefined on input line 1 24.

Package natbib Warning: Citation `Gross1990' on page 2 undefined on input line 124.

Package natbib Warning: Citation `AthEtAl2009' on page 2 undefined on input lin e 126.

Package natbib Warning: Citation `HynEtAl2011' on page 2 undefined on input lin e 126.

Package natbib Warning: Citation `VanErven2015a' on page 2 undefined on input 1 ine 126.

Package natbib Warning: Citation `nystrup2020' on page 2 undefined on input lin e 126.

Package natbib Warning: Citation `bentaiebkoo' on page 2 undefined on input lin e 126.

Package natbib Warning: Citation `WicEtAl2019' on page 2 undefined on input lin e 126.

[2]

Package natbib Warning: Citation `Gneiting2014' on page 3 undefined on input line 130.

Package natbib Warning: Citation `rossi2014' on page 3 undefined on input line 130.

Package natbib Warning: Citation `mclean2013' on page 3 undefined on input line 130.

Package natbib Warning: Citation `BenTaieb2017' on page 3 undefined on input line 130.

Package natbib Warning: Citation `bose2017' on page 3 undefined on input line 1 30.

Package natbib Warning: Citation `ShaHyn2017' on page 3 undefined on input line 130.

Package natbib Warning: Citation `JeoEtAl2019' on page 3 undefined on input lin e 130.

Package natbib Warning: Citation `Taieb2017' on page 3 undefined on input line 130.

Package natbib Warning: Citation `Taieb2017' on page 3 undefined on input line 130.

Package natbib Warning: Citation `WicEtAl2019' on page 3 undefined on input lin e 130. [3] Package natbib Warning: Citation `JeoEtAl2019' on page 4 undefined on input lin e 132. Package natbib Warning: Citation `Taieb2017' on page 4 undefined on input line 132. Package natbib Warning: Citation `JeoEtAl2019' on page 4 undefined on input lin e 132. Package natbib Warning: Citation `Taieb2017' on page 4 undefined on input line 132. Package natbib Warning: Citation `Taieb2017' on page 4 undefined on input line 132. Package natbib Warning: Citation `PanEtAl2020 Geometry' on page 4 undefined on input line 134. Package natbib Warning: Citation `RProbReco' on page 4 undefined on input line 134. [4] Font shape `TS1/ntxtlf/m/n' will be LaTeX Font Info: (Font) scaled to size 12.0pt on input line 144. [5] Package natbib Warning: Citation `PanEtAl2020 Geometry' on page 6 undefined on input line 159.

LaTeX Font Info: Font shape `T1/ntxtlf/b/n' will be (Font) scaled to size 14.4pt on input line 161.

LaTeX Font Info: Font shape `OT1/ntxtlf/b/n' will be

(Font) scaled to size 12.0pt on input line 163.

LaTeX Font Info: Font shape `OT1/ntxtlf/b/n' will be scaled to size 8.8pt on input line 163.

LaTeX Font Info: Font shape `OT1/ntxtlf/b/n' will be scaled to size 6.6pt on input line 163.

[6]

Package natbib Warning: Citation `Dunn1976' on page 7 undefined on input line 1 89.

Package natbib Warning: Citation `HynEtAl2011' on page 7 undefined on input lin e 189.

Package natbib Warning: Citation `AthEtAl2017' on page 7 undefined on input lin e 189.

Package natbib Warning: Citation `WicEtAl2019' on page 7 undefined on input lin e 189.

[7]

LaTeX Font Info: Font shape `T1/ntxtlf/m/it' will be (Font) scaled to size 8.8pt on input line 192.

LaTeX Font Info: Font shape `T1/ntxtlf/m/it' will be (Font) scaled to size 6.6pt on input line 192.

Package natbib Warning: Citation `Schafer2005' on page 8 undefined on input lin e 192.

Package natbib Warning: Citation `Schafer2005' on page 8 undefined on input lin e 192.

LaTeX Font Info: Font shape `T1/ntxtlf/m/n' will be scaled to size 8.8pt on input line 200.

LaTeX Font Info: Font shape `T1/ntxtlf/m/n' will be (Font) scaled to size 6.6pt on input line 200.

Package natbib Warning: Citation `Taieb2017' on page 8 undefined on input line 220.

[8]

LaTeX Warning: File `probforerec_schematic.pdf' not found on input line 239.

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(microtype)

! Package pdftex.def Error: File `probforerec schematic.pdf' not found: using d raft setting. See the pdftex.def package documentation for explanation. Type H <return> for immediate help. 1.2393\textheight] {probforerec schematic.pdf} Try typing <return> to proceed. If that doesn't work, type X <return> to quit. [9] [10] LaTeX Font Info: Trying to load font information for U+bbm on input line 273 (c:/TeXLive/2020/texmf-dist/tex/latex/bbm-macros/ubbm.fd File: ubbm.fd 1999/03/15 V 1.2 Font definition for bbm font - TH) [11] LaTeX Font Info: Trying to load font information for T1+ntxsups on input lin e 305. (c:/TeXLive/2020/texmf-dist/tex/latex/newtx/t1ntxsups.fd File: t1ntxsups.fd 2015/01/17 v1.0 LaTeX Font Info: Font shape `T1/ntxsups/m/n' will be (Font) scaled to size 12.0pt on input line 305. Package microtype Info: Loading generic protrusion settings for font family `ntxsups' (encoding: T1). (microtype) (microtype) For optimal results, create family-specific settings. (microtype) See the microtype manual for details. Package microtype Info: Character `textendash ' is missing in font `T1/ntxsups/m/n/12'. (microtype) Ignoring protrusion settings for this character. (microtype) Package microtype Info: Character `textemdash ' is missing in font `T1/ntxsups/m/n/12'. (microtype) (microtype) Ignoring protrusion settings for this character. Package microtype Info: Character `textquotedblleft ' is missing in font `T1/ntxsups/m/n/12'. (microtype) Ignoring protrusion settings for this character. (microtype) Package microtype Info: Character `textquotedblright ' is missing (microtype) in font `T1/ntxsups/m/n/12'. (microtype) Ignoring protrusion settings for this character. Package microtype Info: Character `AE ' is missing in font `T1/ntxsups/m/n/12'. (microtype) (microtype) Ignoring protrusion settings for this character. Package microtype Info: Character `quotesinglbase ' is missing (microtype) in font `T1/ntxsups/m/n/12'.

Ignoring protrusion settings for this character.

Package microtype Info: Character `quotedblbase ' is missing (microtype) in font `T1/ntxsups/m/n/12'. Ignoring protrusion settings for this character. (microtype) Package microtype Info: Character `guilsinglleft ' is missing in font `T1/ntxsups/m/n/12'. (microtype) Ignoring protrusion settings for this character. (microtype) Package microtype Info: Character `guilsinglright ' is missing in font `T1/ntxsups/m/n/12'. (microtype) Ignoring protrusion settings for this character. (microtype) Package microtype Info: Character `guillemotleft ' is missing in font `T1/ntxsups/m/n/12'. (microtype) Ignoring protrusion settings for this character. (microtype) Package microtype Info: Character `guillemotright ' is missing in font `T1/ntxsups/m/n/12'. (microtype) Ignoring protrusion settings for this character. (microtype) Package microtype Info: Character `textquestiondown ' is missing in font `T1/ntxsups/m/n/12'. (microtype) Ignoring protrusion settings for this character. (microtype) LaTeX Font Info: Font shape `T1/ntxsups/m/n' will be (Font) scaled to size 10.0pt on input line 305. Package microtype Info: Character `textendash ' is missing in font `T1/ntxsups/m/n/10'. (microtype) Ignoring protrusion settings for this character. (microtype) Package microtype Info: Character `textemdash ' is missing in font `T1/ntxsups/m/n/10'. (microtype) (microtype) Ignoring protrusion settings for this character. Package microtype Info: Character `textquotedblleft ' is missing in font `T1/ntxsups/m/n/10'. (microtype) Ignoring protrusion settings for this character. (microtype) Package microtype Info: Character `textquotedblright ' is missing in font `T1/ntxsups/m/n/10'. (microtype) (microtype) Ignoring protrusion settings for this character. Package microtype Info: Character `AE ' is missing in font `T1/ntxsups/m/n/10'. (microtype) (microtype) Ignoring protrusion settings for this character. Package microtype Info: Character `quotesinglbase ' is missing in font `T1/ntxsups/m/n/10'. (microtype) (microtype) Ignoring protrusion settings for this character. Package microtype Info: Character `quotedblbase ' is missing in font `T1/ntxsups/m/n/10'. (microtype) (microtype) Ignoring protrusion settings for this character. Package microtype Info: Character `guilsinglleft ' is missing in font `T1/ntxsups/m/n/10'. (microtype) Ignoring protrusion settings for this character. (microtype) Package microtype Info: Character `quilsinglright ' is missing (microtype) in font `T1/ntxsups/m/n/10'. (microtype) Ignoring protrusion settings for this character. Package microtype Info: Character `guillemotleft ' is missing in font `T1/ntxsups/m/n/10'. (microtype) Ignoring protrusion settings for this character. (microtype) Package microtype Info: Character `guillemotright ' is missing in font `T1/ntxsups/m/n/10'. (microtype) (microtype) Ignoring protrusion settings for this character.

Package microtype Info: Character `textquestiondown ' is missing

(microtype) in font `T1/ntxsups/m/n/10'.
(microtype) Ignoring protrusion settings for this character.

LaTeX Font Warning: Font shape `U/bbm/m/n' in size <5.5> not available (Font) size <5> substituted on input line 305.

LaTeX Font Info: Font shape `OT1/ntxtlf/b/n' will be scaled to size 10.0pt on input line 305.

LaTeX Font Info: Font shape `OT1/ntxtlf/b/n' will be scaled to size 7.3pt on input line 305.

LaTeX Font Info: Font shape `OT1/ntxtlf/b/n' will be

(Font) scaled to size 5.5pt on input line 305.

[12]

Package natbib Warning: Citation `JeoEtAl2019' on page 13 undefined on input li ne 341.

Package natbib Warning: Citation `RanEtAl2021' on page 13 undefined on input li ne 341.

Package natbib Warning: Citation `Gneiting2007' on page 13 undefined on input 1 ine 346.

Package natbib Warning: Citation `Gneiting2014' on page 13 undefined on input 1 ine 346.

[13]

Package natbib Warning: Citation `Gneiting2007' on page 14 undefined on input 1 ine 350.

Package natbib Warning: Citation `SCHEUERER2015' on page 14 undefined on input line 356.

Package natbib Warning: Citation `AleEtAl2021' on page 14 undefined on input line 361.

Package natbib Warning: Citation `BjeEtAl2021' on page 14 undefined on input li ne 361.

[14] [15]

Package natbib Warning: Citation `bottou2010' on page 16 undefined on input lin e 394.

Package natbib Warning: Citation `kingma2013' on page 16 undefined on input lin e 394.

Package natbib Warning: Citation `GasEtAl2019' on page 16 undefined on input line 394.

Package natbib Warning: Citation `JanSte2020' on page 16 undefined on input lin e 394.

Package natbib Warning: Citation `HofEtAl2020' on page 16 undefined on input line 394.

Package natbib Warning: Citation `carpenter2015' on page 16 undefined on input line 396.

Package natbib Warning: Citation `kingma2014' on page 16 undefined on input lin e 396.

Package natbib Warning: Citation `RProbReco' on page 16 undefined on input line 396.

LaTeX Font Info: Font shape `T1/ntxtlf/m/sc' will be (Font) scaled to size 10.95pt on input line 402.

Package microtype Info: Character `029' is missing (microtype) in font `T1/ntxtlf/m/sc/10.95'.

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LaTeX Font Info: Font shape `OT1/ntxtlf/m/n' will be (Font) scaled to size 10.95pt on input line 402.

LaTeX Font Info: Font shape `OT1/ntxtlf/m/n' will be (Font) scaled to size 8.0pt on input line 402.

LaTeX Font Info: Font shape `OT1/ntxtlf/m/n' will be (Font) scaled to size 6.1pt on input line 402.

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Package natbib Warning: Citation `gneiting2005' on page 16 undefined on input 1 ine 423.

Package natbib Warning: Citation `RanEtAl2021' on page 16 undefined on input line 423.

[16]

Package natbib Warning: Citation `kingma2014' on page 17 undefined on input lin e 429.

[17]

Package natbib Warning: Citation `Rfable' on page 18 undefined on input line 44 0.

Package natbib Warning: Citation `Rcore' on page 18 undefined on input line 440

Package natbib Warning: Citation `HynKha2008' on page 18 undefined on input lin e 440.

[18]

Package natbib Warning: Citation `FPP2018' on page 19 undefined on input line 4 50. Package natbib Warning: Citation `JeoEtAl2019' on page 19 undefined on input li ne 457. Package natbib Warning: Citation `Taieb2017' on page 19 undefined on input line 458. Package natbib Warning: Citation `WicEtAl2019' on page 19 undefined on input li ne 458. Package natbib Warning: Citation `WicEtAl2019' on page 19 undefined on input li ne 461. [19] Package natbib Warning: Citation `JeoEtAl2019' on page 20 undefined on input li ne 468. ! LaTeX Error: File `energy meanscore' not found. See the LaTeX manual or LaTeX Companion for explanation. Type H <return> for immediate help. . . . 1.476 ...height=0.45\textheight]{energy meanscore} I could not locate the file with any of these extensions: .pdf,.png,.jpg,.mps,.jpeg,.jbig2,.jb2,.PDF,.PNG,.JPG,.JPEG,.JBIG2,.JB2,.e ps Try typing <return> to proceed. If that doesn't work, type X <return> to quit. Package natbib Warning: Citation `HolEtAl2013' on page 20 undefined on input li ne 481.

Package natbib Warning: Citation `tsutilspackage' on page 20 undefined on input line 481.

LaTeX Warning: File `gse.pdf' not found on input line 485. ! Package pdftex.def Error: File `gse.pdf' not found: using draft setting. See the pdftex.def package documentation for explanation. Type H <return> for immediate help. 1.485 ...degraphics[height=.5\textheight] {gse.pdf} Try typing <return> to proceed. If that doesn't work, type X <return> to quit. [20] [21] LaTeX Warning: File `selected.pdf' not found on input line 513. ! Package pdftex.def Error: File `selected.pdf' not found: using draft setting. See the pdftex.def package documentation for explanation. Type H <return> for immediate help. 1.513 ...aphics[width=.75\textwidth] {selected.pdf} Try typing <return> to proceed. If that doesn't work, type X <return> to quit. Package natbib Warning: Citation `bigvar' on page 22 undefined on input line 52 1. [22] [23] LaTeX Warning: File `densities.pdf' not found on input line 527. ! Package pdftex.def Error: File `densities.pdf' not found: using draft setting See the pdftex.def package documentation for explanation. Type H <return> for immediate help. 1.527 ...aphics[width=.7\textwidth] {densities.pdf}

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Try typing <return> to proceed. If that doesn't work, type X <return> to quit. LaTeX Warning: File `corr.pdf' not found on input line 534. ! Package pdftex.def Error: File `corr.pdf' not found: using draft setting. See the pdftex.def package documentation for explanation. Type H <return> for immediate help. . . . 1.534 ...degraphics[width=.55\textwidth] {corr.pdf} Try typing <return> to proceed. If that doesn't work, type X <return> to quit. [24] ! LaTeX Error: File `meanenergyscore' not found. See the LaTeX manual or LaTeX Companion for explanation. Type H <return> for immediate help. 1.548 ...cs[width=.35\textheight] {meanenergyscore} I could not locate the file with any of these extensions: .pdf,.png,.jpg,.mps,.jpeg,.jbig2,.jb2,.PDF,.PNG,.JPG,.JPEG,.JBIG2,.JB2,.e Try typing <return> to proceed. If that doesn't work, type X <return> to quit. LaTeX Warning: File `nemenyi ig.pdf' not found on input line 553. ! Package pdftex.def Error: File `nemenyi_ig.pdf' not found: using draft settin g. See the pdftex.def package documentation for explanation. Type H <return> for immediate help. 1.553 ...hics[width=.4\textwidth] {nemenyi jb.pdf}} Try typing <return> to proceed. If that doesn't work, type X <return> to quit.

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LaTeX Warning: File `nemenyi jb.pdf' not found on input line 553. ! Package pdftex.def Error: File `nemenyi jb.pdf' not found: using draft settin g. See the pdftex.def package documentation for explanation. Type H <return> for immediate help. 1.553 ...hics[width=.4\textwidth] {nemenyi jb.pdf}} Try typing <return> to proceed. If that doesn't work, type X < return> to quit. Package natbib Warning: Citation `PanEtAl2020_Geometry' on page 25 undefined on input line 558. [25] Package natbib Warning: Citation `BuhYu2003' on page 26 undefined on input line 567. [26] No file ProbabilisticReconciliationR1.bbl. [27] Underfull \hbox (badness 10000) in paragraph at lines 583--584 [] $\T1/ntxtlf/m/n/12$ (+20) Consider the re-gion $\LMS/ntxsy/m/n/12$ I\$ \T1/ntxtlf /m/n/12 (+20) given by the Carte-sian prod-uct of in-ter-vals [] [28 Package natbib Warning: Citation `PanEtAl2020_Geometry' on page 29 undefined on input line 640. [29] [30] [31] Package natbib Warning: Citation `Szekely2013' on page 32 undefined on input li ne 710. Package natbib Warning: Citation `WicEtAl2019' on page 32 undefined on input li ne 737.

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[32]
Package natbib Warning: Citation `snpackage' on page 33 undefined on
input line
 762.
! LaTeX Error: File `variogram meanscore' not found.
See the LaTeX manual or LaTeX Companion for explanation.
Type H <return> for immediate help.
1.772 ...dth=0.45\textheight]{variogram meanscore}
I could not locate the file with any of these extensions:
.pdf,.png,.jpg,.mps,.jpeg,.jbig2,.jb2,.PDF,.PNG,.JPG,.JPEG,.JBIG2,.JB2,.e
Try typing <return> to proceed.
If that doesn't work, type X <return> to quit.
LaTeX Warning: File `gsv.pdf' not found on input line 779.
! Package pdftex.def Error: File `gsv.pdf' not found: using draft
setting.
See the pdftex.def package documentation for explanation.
Type H <return> for immediate help.
 . . .
1.779 ...degraphics[height=.4\textheight] {gsv.pdf}
Try typing <return> to proceed.
If that doesn't work, type X <return> to quit.
LaTeX Warning: File `nsv.pdf' not found on input line 787.
! Package pdftex.def Error: File `nsv.pdf' not found: using draft
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See the pdftex.def package documentation for explanation.
Type H <return> for immediate help.
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1.787 ...degraphics[height=.4\textheight] {nsv.pdf}
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If that doesn't work, type X <return> to quit.
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[33] [34]
! LaTeX Error: File `meanenergyscore VAR' not found.
See the LaTeX manual or LaTeX Companion for explanation.
Type H <return> for immediate help.
1.800 ...width=.4\textheight] {meanenergyscore VAR}
I could not locate the file with any of these extensions:
.pdf,.png,.jpg,.mps,.jpeg,.jbig2,.jb2,.PDF,.PNG,.JPG,.JPEG,.JBIG2,.JB2,.e
Try typing <return> to proceed.
If that doesn't work, type X < return> to quit.
LaTeX Warning: File `nemenyi_ig_VAR.pdf' not found on input line 805.
! Package pdftex.def Error: File `nemenyi ig VAR.pdf' not found: using
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tting.
See the pdftex.def package documentation for explanation.
Type H <return> for immediate help.
 . . .
1.805 ...[width=.3\textwidth] {nemenyi jb VAR.pdf}}
Try typing <return> to proceed.
If that doesn't work, type X <return> to quit.
LaTeX Warning: File `nemenyi jb VAR.pdf' not found on input line 805.
! Package pdftex.def Error: File `nemenyi jb VAR.pdf' not found: using
draft se
tting.
See the pdftex.def package documentation for explanation.
Type H <return> for immediate help.
 . . .
1.805 ... [width=.3\textwidth] {nemenyi jb VAR.pdf}}
Try typing <return> to proceed.
If that doesn't work, type X <return> to quit.
Package natbib Warning: There were undefined citations.
[35
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1 (./ProbabilisticReconciliationR1.aux)
LaTeX Font Warning: Size substitutions with differences
(Font)
                    up to 0.5pt have occurred.
 )
Here is how much of TeX's memory you used:
 27474 strings out of 480681
 549353 string characters out of 5908536
 798282 words of memory out of 5000000
 42372 multiletter control sequences out of 15000+600000
 759699 words of font info for 490 fonts, out of 8000000 for 9000
 1141 hyphenation exceptions out of 8191
 68i,18n,96p,2045b,651s stack positions out of
5000i,500n,10000p,200000b,80000s
{c:/TeXLive/2020/texmf-dist/fonts/enc/dvips/newtx/ntx-ec-tlf-
sc.enc}{c:/TeXLi
ve/2020/texmf-dist/fonts/enc/dvips/newtx/ntxsups t1.enc}
<c:/Users/asynch/.texl
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-dist/fonts/enc/dvips/newtx/ntx-ot1-tlf.enc}{c:/TeXLive/2020/texmf-
dist/fonts/e
nc/dvips/newtx/ntx-ec-tlf.enc}{c:/TeXLive/2020/texmf-
dist/fonts/enc/dvips/tex-q
yre/q-ts1.enc}<c:/TeXLive/2020/texmf-</pre>
dist/fonts/type1/public/newtx/NewTXBMI.pfb
><c:/TeXLive/2020/texmf-</pre>
dist/fonts/type1/public/newtx/NewTXBMI7.pfb><c:/TeXLive</pre>
/2020/texmf-
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c:/TeXLive/2020/texmf-
dist/fonts/type1/public/txfonts/txsyb.pfb><c:/TeXLive/202
0/texmf-dist/fonts/type1/public/newtx/txsys.pfb><c:/TeXLive/2020/texmf-</pre>
dist/fon
ts/type1/public/newtx/ztmb.pfb><c:/TeXLive/2020/texmf-
dist/fonts/type1/public/n
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ewtx/ztmr.pfb><c:/TeXLive/2020/texmfdist/fonts/type1/public/newtx/ztmri.pfb>
Output written on ProbabilisticReconciliationR1.pdf (35 pages, 362842 bytes).
PDF statistics:
 229 PDF objects out of 1000 (max. 8388607)
 0 named destinations out of 1000 (max. 500000)
 70253 words of extra memory for PDF output out of 74296 (max. 10000000)

Click here to access/download **LaTeX Source Files**nemenyi_jb.pdf

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probforerec_schematic.pdf

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LaTeX Source Files

selected.pdf

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Response to Technical Check Results

Dear Ruud

We have now addressed the following technical problem.

1. Please remove the PDF version of manuscript and upload the editable source file (TEX or DOC).

We have now done that.

Cheers, George Supplementary Material

Click here to access/download **Supplementary Material**ProbabilisticReconciliationR1-Appendices.pdf