$N=4$

$n = \log_2 4 = 2$

$0 \le p \le n-1$

i.e. $0 \le p \le 2-1$

$= 0 \le p \le 1$

$\therefore p = 0, 1$

For, $p=0$,

$q = 0$ and 1

and for,

$p=1$.

$1 \le q \le 2^p$

$1 \le q \le 2^1$

$1 \le q \le 2$

$\therefore q = 1, 2$

Now,

$k = 2^p + q - 1$

p	q	k
0	0	0
0	1	1
1	1	2
1	2	3

Determine z,

$z \Rightarrow \{ 0/4, 1/4, 2/4, 3/4 \}$

If $k=0$, then $H_0(z) = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{4}}$

When $k=1$,

$$H_1(z) = \frac{1}{\sqrt{N}} \begin{cases} 2^{p/2}, & \frac{q-1}{2^p} \le z < \frac{q-\frac{1}{2}}{2^p} \\ -2^{p/2}, & \frac{q-\frac{1}{2}}{2^p} \le z < \frac{q}{2^p} \\ 0, & \text{otherwise} \end{cases}$$

$$= \frac{1}{\sqrt{4}} \begin{cases} 2^0, & 0 \le z < \frac{1}{2} \\ -2^0, & \frac{1}{2} \le z < 1 \\ 0, & \text{otherwise} \end{cases}$$

$$\cancel{= \frac{1}{\sqrt{4}} \begin{cases} 1 \\ -1 \\ 0 \end{cases}} = \frac{1}{\sqrt{4}} \begin{cases} 1, & 0 \le z < \frac{1}{2} \\ -1, & \frac{1}{2} \le z < 1 \\ 0, & \text{otherwise} \end{cases}$$

~~Now,~~ Now,

$\cancel{H_1(0/4)}$ $H_1(0/4) = 1/\sqrt{4}$

$H_1(1/4) = 1/\sqrt{4}$

$H_2(2/4) = -1/\sqrt{4}$

$H_3(3/4) = -1/\sqrt{4}$

When $k=2$

$$H_2(z) = \frac{1}{\sqrt{4}} \begin{cases} 2^{1/2}, & \frac{1-1}{2^1} \le z < \left(\frac{1-1/2}{2^1}\right) \\ -2^{1/2}, & \frac{1-1/2}{2^1} \le z < \frac{1}{2^1} \\ 0, & \text{otherwise} \end{cases}$$

$$= \frac{1}{\sqrt{4}} \begin{cases} \sqrt{2}, & 0 \le z < 1/4 \\ -\sqrt{2}, & \frac{1}{4} \le z < 1/2 \\ 0, & \text{otherwise} \end{cases}$$

Now,

$H_2(0/4) = \frac{1}{\sqrt{4}} \cdot \sqrt{2}$

$H_2(1/4) = \frac{1}{\sqrt{4}} \cdot -\sqrt{2}$

$H_3(2/4) = 0$

$H_4(3/4) = 0$

$$(p, q) = (1, 2)$$

When $k = 3$

$$H_3(z) = \frac{1}{\sqrt{4}} \begin{cases} 2^{1/2}, & \frac{2-1}{2^1} \le z < 2-\frac{1}{2} \\ -2^{1/2}, & \frac{2-\frac{1}{2}}{2^1} \le z < \frac{2}{2^1} \\ 0, & \text{otherwise} \end{cases}$$

$$= \frac{1}{\sqrt{4}} \begin{cases} \sqrt{2}, & \frac{1}{2} \le z < 3/4 \\ -\sqrt{2}, & \frac{3}{4} \le z < 1 \\ 0, & \text{otherwise} \end{cases}$$

Now,

$H_3(0/4) = 0$

$H_3(1/4) = \cancel{\frac{1}{\sqrt{4}} \cdot \frac{-1}{\sqrt{2}}} \quad \cancel{\frac{1}{\sqrt{4}} \cdot -\sqrt{2}} = 0$

$H_3(2/4) = \cancel{\frac{1}{\sqrt{4}} \cdot \frac{1}{\sqrt{2}}} \quad 1/\sqrt{4} \cdot \sqrt{2}$

$H_4(3/4) = \cancel{\frac{1}{\sqrt{4}} \cdot \frac{-1}{\sqrt{2}}} \quad \frac{1}{\sqrt{4}} \cdot -\sqrt{2}$

Now, matrix for haar transform is

$$A_H = \begin{bmatrix} H_0(0/4) & H_0(1/4) & H_0(2/4) & H_0(3/4) \\ H_1(0/4) & H_1(1/4) & H_1(2/4) & H_1(3/4) \\ H_2(0/4) & H_2(1/4) & H_2(2/4) & H_2(3/4) \\ H_3(0/4) & H_3(1/4) & H_3(2/4) & H_3(3/4) \end{bmatrix}$$

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$$A_H = \begin{bmatrix} 1/\sqrt{4} & 1/\sqrt{4} & 1/\sqrt{4} & 1/\sqrt{4} \\ 1/\sqrt{4} & 1/\sqrt{4} & -1/\sqrt{4} & -1/\sqrt{4} \\ 1/\sqrt{4}\cdot\sqrt{2} & \frac{1}{\sqrt{4}}\cdot -\sqrt{2} & 0 & 0 \\ 0 & 0\ \cancel{\frac{1}{\sqrt{4}}\cdot -\sqrt{2}} & \frac{1}{\sqrt{4}}\cdot\sqrt{2} & \frac{1}{\sqrt{4}}\cdot -\sqrt{2} \end{bmatrix}$$

$$\therefore A_H = \frac{1}{\sqrt{4}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ \sqrt{2} & -\sqrt{2} & 0 & 0 \\ 0 & \cancel{-\sqrt{2}}\ 0 & \sqrt{2} & -\sqrt{2} \end{bmatrix}$$