MIMO Skript - Wintersemester 2013
Kapitel 4

Contents

4 Distributed MIMO

- This research topic emerged ca. 10 years ago and is still a very active area of research
- Simple relaying schemes have been included in recent standards such as IEEE 802.16 (WiMAX) and LTE - Advanced
- Advantages: relay - assisted communications:
  - Relays can help to reduce the effective overall pathloss
  - Relays can also combat small - scale fading effects
  - Relays can help to realize MIMO gains with single - antenna nodes
- Challenges in relays - assisted communication:
  - Network architectures are becoming more complex
  - Synchronization across different nodes may be necessary *(Anm.: untersch. Trägerfrequenzen der Relays → Offset, Fehler, etc.)*
  - Exchange of channel state information (CSI) across nodes may be required

4.1 Half - Duplex One - Way Relaying

Basic Relay Network

- Relay R assists source S in communication with destination D
- Two basic nodes of transmission (at the relay):

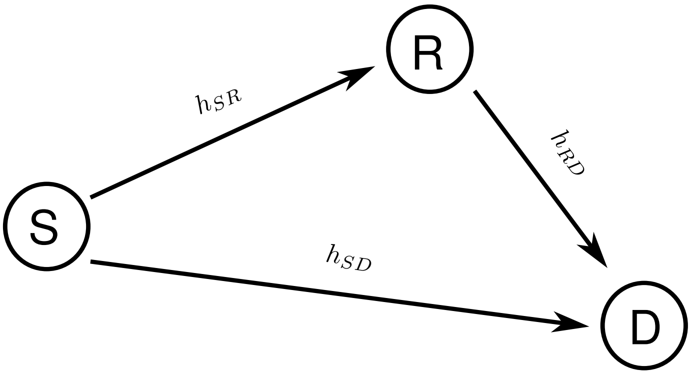

Figure 1: Basic Relay Network

Full - Duplex relaying: R can receive and transmit at the same time and in the same frequency band *(Anm.: effizient, da restliche Zeit und restliche Frequenzband von anderen genutzt werden kann)*

→ Since the TX signal power is orders of magnitude larger than the RX power, there is self - interference (at the relay)

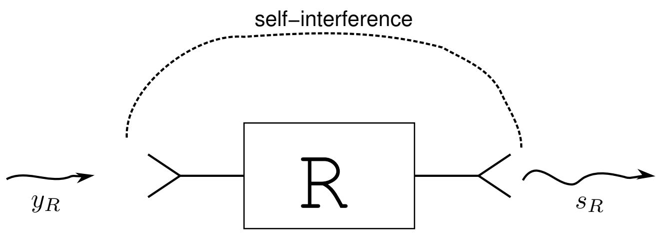

Figure 2: Relay with self-interference

→ Full - duplex relays are difficult to implement. The design of full - duplex relays is an active area of research.

→ Majority of existing literature assumes half - duplex relaying.

Half - duplex relaying: R transmits and receives in different time slots and/ or different frequency bands. Typically, a two - phase protocoll is used:

Phase 1: S transmits, R and D receive

Phase 2: R transmits, D receives, S may or may not transmit

There are different relaying strategies that differ in the processing applied at the relay. The most popular are:

- Decode - and - Forward
- Amplify - and - Forward
- (Compress - and - Forward)

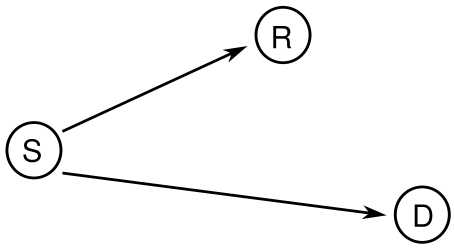

Figure 3: Half-duplex Relaying: Phase 1

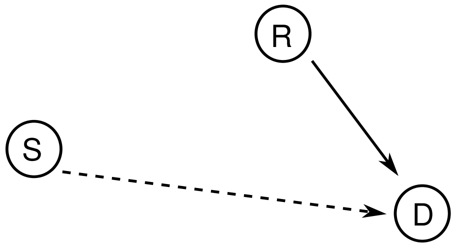

Figure 4: Half-duplex Relaying: Phase 2

4.1.1 Decode - and - Forward (DF) Relaying

In DF relaying, the relay detects and decodes the signal received from the source before encoding it and forwarding it to the destination.

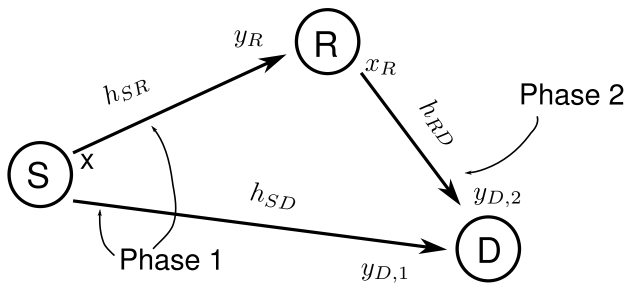

Figure 5: Block diagramm Decode - and - forward Relaying

Phase 1:

- R receives: $y_R = h_{SR}x + n_R$
- D receives: $y_{D1} = h_{SD}x + n_{D1}$
- with:
  - transmit signal $x, \mathcal{E}_s = \mathcal{E}\{|x|^2\}$
  - AWGN n_R and n_{D1}, $\sigma_n^2 = \mathcal{E}\{|n_R|^2\} = \mathcal{E}\{|n_{D1}|^2\}$

Phase 2:

- R decodes and forwards x_R (estimate of x)
- D receives: $y_{D2} = h_{RD} x_R + n_{D2}$
  - x_R is estimate of x after decoding at R
  - $\sigma_n^2 = \mathcal{E}\{|n_{D2}|^2\}\,; \quad \mathcal{E}_R = \mathcal{E}\{|x_R|^2\}$
  - weassume: S is silent in Phase 2

- The capacity at the three node relay channel is not known!
- We provide an achievable rate under the following simplifying assumption: The direct source - relay link is not used/ exploited.
- Achievable rate without S - D link:

$$\boxed{C_{DF} = \frac{1}{2}\min\Big\{\log_2\big(1+\frac{\mathcal{E}_S|h_{SR}|^2}{\sigma_n^2}\big),\, \log_2\big(1+\frac{\mathcal{E}_R|h_{RD}|^2}{\sigma_n^2}\big)\Big\}}$$

  - factor $\frac{1}{2}$ is due to the fact that we use two time slots to transmit one packet
  - $\min\{\ldots\}$ means we are limited by the weaker link (bottle - neck)
  - If power allocation is possible, the total power $\mathcal{E} = \mathcal{E}_S + \mathcal{E}_R$ should be divided between S and R to guarantee:

$$\frac{\mathcal{E}_S|h_{SR}|^2}{\sigma_n^2} = \frac{\mathcal{E}_R|h_{RD}|^2}{\sigma_n^2},$$

$$\mathcal{E}_R = \frac{|h_{SR}|^2}{|h_{SR}|^2 + |h_{RD}|^2}\cdot\mathcal{E},$$

$$\mathcal{E}_S = \frac{|h_{RD}|^2}{|h_{SR}|^2 + |h_{RD}|^2}\cdot\mathcal{E}$$

- Outage - probability in fading:
  - We transmit with fixed rate R
  - An outage occurs, if:

$$\frac{1}{2}\log_2\Big(1+\underbrace{\frac{\mathcal{E}_S|h_{SR}|^2}{\sigma_n^2}}_{=\gamma_{SR}}\Big) < R \quad \text{or}$$

$$\frac{1}{2}\log_2\Big(1+\underbrace{\frac{\mathcal{E}_R|h_{RD}|^2}{\sigma_n^2}}_{=\gamma_{RD}}\Big) < R$$

$$\boxed{\gamma_{SR} < \underbrace{2^{2R}-1}_{\gamma_T} \quad \text{or} \quad \gamma_{RD} < 2^{2R}-1}$$

$$P_{\text{out}} = \Pr\{\gamma_{SR} < \gamma_T \vee \gamma_{RD} < \gamma_T\} = \Pr\{\underbrace{\min\{\gamma_{SR}, \gamma_{RD}\}}_{=\gamma_{eq}} < \gamma_T\}$$
$$= 1 - \Pr\{\gamma_{SR} > \gamma_T \wedge \gamma_{RD} > \gamma_T\} = 1 - \Pr\{\gamma_{SR} > \gamma_T\} \Pr\{\gamma_{RD} > \gamma_T\} =$$
$$= 1 - \big(1 - F_{\gamma_{SR}}(\gamma_T)\big)\big(1 - F_{\gamma_{RD}}(\gamma_T)\big) =$$
$$= \underline{F_{\gamma_{SR}}(\gamma_T) + F_{\gamma_{RD}}(\gamma_T) - F_{\gamma_{SR}}(\gamma_T) \cdot F_{\gamma_{RD}}(\gamma_T)}$$

with CDFs: $F_{\gamma_{SR}}(\cdot)$ and $F_{\gamma_{RD}}(\cdot)$

– Rayleigh Fading:

$$\rightarrow F_{\gamma_{SR}}(\gamma) = 1 - \exp(\frac{-\gamma}{\bar{\gamma}_{SR}})\,; \quad \bar{\gamma}_{SR} = \mathcal{E}\{\gamma_{SR}\}$$
$$F_{\gamma_{RD}}(\gamma) = 1 - \exp(\frac{-\gamma}{\bar{\gamma}_{RD}})\,; \quad \bar{\gamma}_{RD} = \mathcal{E}\{\gamma_{RD}\}$$
$$\rightarrow P_{\text{out}} = 1 - \exp(\frac{-\gamma_T}{\bar{\gamma}_{SR}}) + 1 - \exp(\frac{-\gamma_T}{\bar{\gamma}_{RD}}) - \Big(1 - \exp(\frac{-\gamma_T}{\bar{\gamma}_{SR}})\Big)\Big(1 - \exp(\frac{-\gamma_T}{\bar{\gamma}_{RD}})\Big) =$$
$$= 1 - \exp\Big(-\frac{\bar{\gamma}_{SR} + \bar{\gamma}_{RD}}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}} \cdot \gamma_T\Big)$$

$\rightarrow$ equivalent SNR $\gamma_{eq} = \min\{\gamma_{SR}, \gamma_{RD}\}$ is also exponentially distributed with $\bar{\gamma}_{eq} = \frac{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}{\bar{\gamma}_{SR}+\bar{\gamma}_{RD}}$

– Diversity gain: Assume $\bar{\gamma}_{SR} = \alpha\bar{\gamma}$

$$\rightarrow P_{out} \xrightarrow{\bar{\gamma}\rightarrow\text{inf}} 1 - \Big(1 - \frac{\alpha+\beta}{\alpha\beta} \cdot \frac{\gamma_T}{\bar{\gamma}}\Big) + \mathcal{O}\big(\bar{\gamma}^{-1}\big) =$$
$$= \frac{\alpha+\beta}{\alpha\beta} \cdot \frac{\gamma_T}{\bar{\gamma}} + \mathcal{O}\big(\bar{\gamma}^{-1}\big) =$$
$$\rightarrow \boxed{G_d = 1}$$

- Bit error rate (BER) of BPSK (uncoded)
  - $\text{BER}\big(\gamma_{SR}, \gamma_{RD}\big) = \Big(1 - \text{BER}_{SR}\big(\gamma_{SR}\big)\Big)\text{BER}_{RD}\big(\gamma_{RD}\big) + \Big(1 - \text{BER}_{RD}\big(\gamma_{RD}\big)\Big)\text{BER}_{SR}\big(\gamma_{SR}\big)$
    - with BER of the S - R link, $\text{BER}_{SR}(\gamma_{SR})$ and BER of the R - D link $\text{BER}_{RD}(\gamma_{RD})$
    - for sufficiently high SNR $\rightsquigarrow$ $\text{BER}_{SR}(\gamma_{SR}), \text{BER}_{RD}(\gamma_{RD}) \ll \text{BER}_{SR}(\gamma_{SR}) + \text{BER}_{RD}(\gamma_{RD})$

    $\rightarrow$ $\underline{\text{BER}(\gamma_{SR}, \gamma_{RD}) \approx \text{BER}_{SR}(\gamma_{SR}) + \text{BER}_{RD}(\gamma_{RD})}$

    - to average BER (Rayleigh Fading):

$$\text{BER} = \mathcal{E}_{\gamma_{SR},\gamma_{RD}}\Big\{\text{BER}\big(\gamma_{SR}, \gamma_{RD}\big)\Big\} = \frac{1}{2}\Big(1 - \sqrt{\frac{1}{1+\frac{1}{\bar{\gamma}_{SR}}}} + \frac{1}{2}\Big(1 - \sqrt{\frac{1}{1+\frac{1}{\bar{\gamma}_{RD}}}}\Big)$$

    - high SNR:

$$\text{BER} \approx \frac{1}{2}(1 - 1 + \frac{1}{2}\frac{1}{\bar{\gamma}_{SR}}) + \frac{1}{2}(1 - 1 + \frac{1}{2}\frac{1}{\bar{\gamma}_{RD}}) =$$
$$= \frac{1}{4}(\frac{1}{\bar{\gamma}_{SR}} + \frac{1}{\bar{\gamma}_{RD}})$$

$\rightsquigarrow$ also indicates diversity gain $G_d = 1$

4.1.2 Amplify - and - Forward (AF) Relaying

- Relay does not decode signal received from source but only amplifies it before forwarding it to the destination

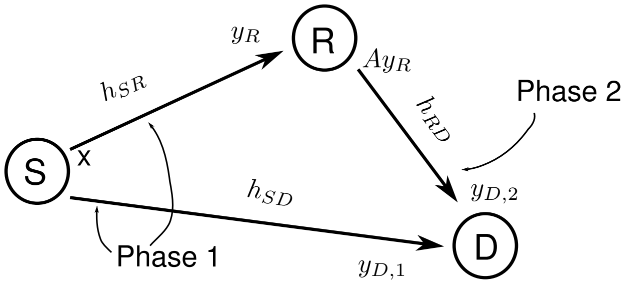

Figure 6: Block diagramm AF - Relaying

- Amplification gain A may be constant on channel dependent and ensures a certain (average) transmit power

  Phase 1:

  - R receives: $y_D = h_{SR}x + n_R$
  - D receives: $y_{D,1} = h_{SD}x + n_{D,1}$

  Phase 2:

  - R transmits: $s_R = Ay_R = A(h_{SR}x + n_R)$
  - D receives: $y_{D,2} = h_{RD}Ay_R + n_{D,2}$

- We can use MRC to combine $y_{D,1}$ and $y_{D,2}$ at D: $y_{D,2} = Ah_{RD}h_{SR}x + h_{RD}An_R + n_{D,2}$, where: $h_{RD}An_R + n_{D,2}$ is effective noise n_{eff} with variance $\sigma^2_{n_{\text{eff}}} = \sigma^2_n\big(|h_{RD}|^2A^2 + 1\big)$

$\rightarrow$ make noise variances of both branches equal

$$\bar{y}_{D,2} = \frac{1}{\sqrt{|h_{RD}|^2A^2+1}} \cdot y_{D,2} = \frac{Ah_{RD}h_{SR}}{\sqrt{A^2|h_{RD}|^2+1}} \cdot x + \tilde{n}_{\text{eff}}$$

$$\text{MRC:} \quad r = h^*_{SD}y_{D,1} + \frac{Ah^*_{RD}h^*_{SR}}{\sqrt{A^2|h_{RD}|^2+1}} \cdot \bar{y}_{D,2} = \underbrace{h^*_{SD}y_{D,1} + \frac{Ah^*_{RD}h^*_{SR}}{A^2|h_{RD}|^2+1} \cdot y_{D,2}}_{=\text{decision variable!}}$$

- Choice of A: The goal is to ensure en (average) transmit power of $\mathcal{E}_R$

a) Variable gain relaying: In this case we introduce an instantaneous power constraint. *Anm.: A muss abhängig von h_{SR} sein, um es kompensieren zu können.*

$$\begin{aligned}\mathcal{E}_{x,n}\big\{|S_R|^2\big\} &= \mathcal{E}_{x,n}\big\{A^2\big(|h_{SR}|^2|x|^2 + |n_R|^2\big)\big\} = \\ &= A^2\big(|h_{SR}|^2\mathcal{E}_S + \sigma_n^2\big) \overset{!}{=} \mathcal{E}_R \\ \rightarrow A^2 &= \frac{\mathcal{E}_R}{|h_{SR}|^2\mathcal{E}_S + \sigma_n^2}\end{aligned}$$

- A is channel dependent
- Instantaneous transmit power is <u>not</u> channel dependent

b) Fixed gain relaying: In this case, we introduce an average (with respect to the channel) power constraint

$$\begin{aligned}\mathcal{E}\big\{|S_R|^2\big\} &= \mathcal{E}\big\{A^2\big(|h_{SR}|^2|x|^2 + |n_R|^2\big)\big\} = \\ &= A^2\big(\underbrace{\mathcal{E}\big\{|h_{SR}|^2\big\}}_{\sigma_{SR}^2}\mathcal{E}_S + \sigma_n^2\big) \overset{!}{=} \mathcal{E}_R \\ \rightarrow A^2 &= \frac{\mathcal{E}_R}{\mathcal{E}_S\sigma_{SR}^2 + \sigma_n^2}\end{aligned}$$

- A is not channel dependent
- Instantaneous power of S_R depends on channel and may actually vary widely

Equivalent SNR for variable gain AF relaying (inly relayed link)

$$y_{D,2} = Ah_{RD}h_{SR}x + h_{RD}An_R + n_{D,2}$$

$$\text{SNR:} \quad \gamma_{eq}^{AF} = \frac{A^2|h_{SR}|^2|h_{RD}|^2\mathcal{E}_S}{A^2|h_{RD}|^2\sigma_n^2 + \sigma_n^2} = \frac{\frac{\mathcal{E}_S}{\sigma_n^2}|h_{SR}|^2|h_{RD}|^2}{|h_{RD}|^2 + \frac{1}{\mathcal{E}_R}(|h_{SR}|^2\mathcal{E}_S + \sigma_n^2)} =$$

$$= \frac{\frac{\mathcal{E}_S}{\sigma_n^2}|h_{SR}|^2 \cdot \frac{\mathcal{E}_R}{\sigma_n^2}|h_{RD}|^2}{\frac{\mathcal{E}_R}{\sigma_n^2}|h_{RD}|^2 + \frac{\mathcal{E}_S}{\sigma_n^2}|h_{SR}|^2 + 1} =$$

$$= \frac{\gamma_{SR}\gamma_{RD}}{\gamma_{SR} + \gamma_{RD} + 1}$$

high SNR: $\gamma_{SR}, \gamma_{RD} \gg 1$

$$\boxed{\gamma_{eq}^{AF} = \frac{\gamma_{SR}\gamma_{RD}}{\gamma_{SR} + \gamma_{RD}}}$$

Comparison with equivalent SNF of DF:

$$\boxed{\gamma_{eq}^{DF} = \min\big\{\gamma_{SR}, \gamma_{RD}\big\}}$$