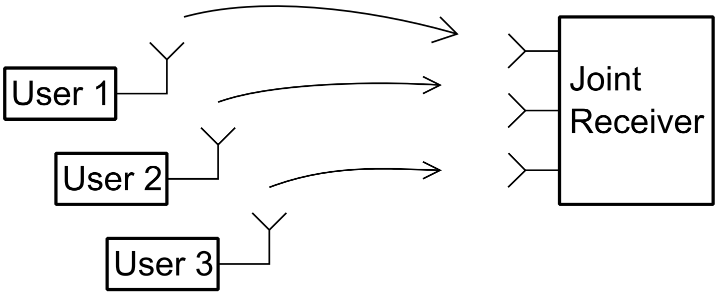
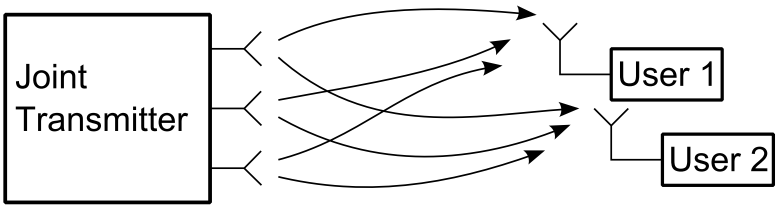
3 Multiuser MIMO

- We distinguish two cases:
  - multipoint - to - point transmission
  - point - to - multipoint transmission
- Multipoint - to - point transmission
  - typical uplink scenario in cellular systems
  - information theoretical channel model: Multiple Access Channel (MAC)



- Point - to -multipoint transmission
  - typical downlink scenarion in cellular systems
  - information theoretical channel model: Broadcast Channel (BC)

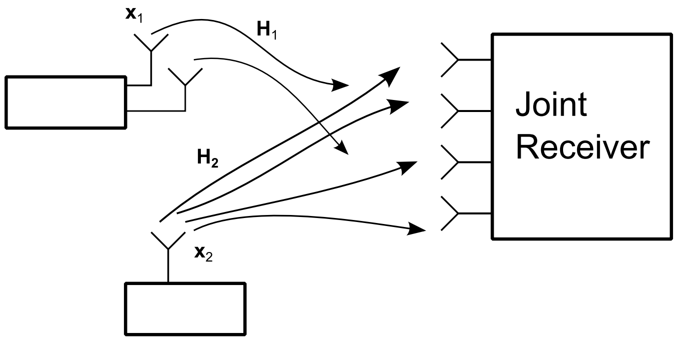


- Advantage of multiuser MIMO compared to point - to - point MIMO
  - multiplexing gain can be exploited even if users have only single antenna
  - users are spatially distributed in cell $\rightarrow$ channels to different users are independent

3.1 Multiple Access Channel (MAC)

We consider two aspects:

- Detector structures
- Rate region



3.1.1 Detector structures

Channel model: $\rightarrow$ general MAC: $\mathbf{y} = \sum\limits_{k=1}^{K} \mathbf{H}_k \mathbf{x}_k + \mathbf{n}$

with:

- K users
- user k has $N_{T,k}$ transmit antennas
- N_R receive antennas
- $\mathbf{H}_k \in \mathbb{C}^{N_R \times N_{T,k}}$

$$\mathbf{y} = \underbrace{\begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 & \dots & \mathbf{H}_k \end{bmatrix}}_{\mathbf{H}} \cdot \underbrace{\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_k \end{bmatrix}}_{\mathbf{x}} + \mathbf{n}$$

Observation:

- same equivalent channel model as for a point-to-point MIMO system transmitting $N_T = \sum_{k=1}^{K} N_{T,k}$ independent signal streams *(Anmerkung: kein Unterschied für Empfänger, ob Signale von einem Nutzer oder von mehreren)*
- the receiver (e.g. base station) can use detection schemes as for point-to-point MIMO systems
  - linear receiver
  - DFG
  - sphere decoder

Typical problems in uplink multiuser MIMO For given receiver structure:

- calculate SNR_k for all users k based on the expressions developed in Chapter 2.4
- optimize transmit power of users, $E_k = \mathcal{E}\{||x_k||^2\}$ for maximization of the sumrate or maximization of the minimum SNR_k *(Anmerkung: Maximierung der sumrate kann*

durch Maximierung des SNR des Users mit bestem Kanal erfolgen, aber: unfair anderen Usern gegenüber $\Rightarrow$ starving)

3.1.2 Rate region

For point - to - point links, we can decode error free, if the rate, R, meets

a) SISO $R < \log_2\left(1 + \frac{\mathcal{E}_s}{\sigma_n^2}\right)$

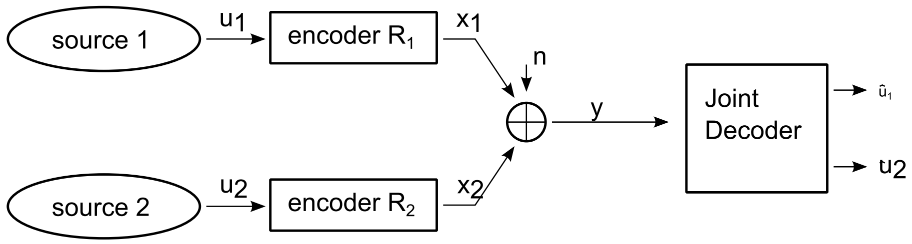
b) MIMO $R < \log_2 \underbrace{\left|\mathbf{I} + \frac{\mathcal{E}_s}{N_T \sigma_n^2}\mathbf{H}\mathbf{H}^H\right|}_{\text{det}}$

Questions: What happens if there are multiple users?

Rate Region for Single Antenna Users and Receivers

- Gaussian channel
- $N_R = N_{T,k} = 1 \forall k$
- received signal:

$$y = \sum_{k=1}^{K} x_k + n$$
$$*\mathcal{E}_k = \mathcal{E}\left\{||x_k||^2\right\}$$
$$*\sigma_n^2 = \left\{||n||^2\right\}$$



Example: 2 Users

- How should we choose R_1 and R_2 to ensure error free decoding of <u>both</u> signal streams?
- It is no longer sufficient to maximizie a single rate. Instead we have to consider rate pairs (R_1, R_2)
- All possible rate points, that allows error free decoding, define the rate region $\underline{C}$
- Possible desing goals of the system:
  - maximized sumrate $R_{\text{sum}} = \max\limits_{(R_1,R_2)\in\underline{C}} R_1 + R_2$
  - maximize minimum user rate: $R_{\text{max-min}} = \max\limits_{(R_1,R_2)\in\underline{C}} \min\limits_{i\in\{1,2\}} R_i$

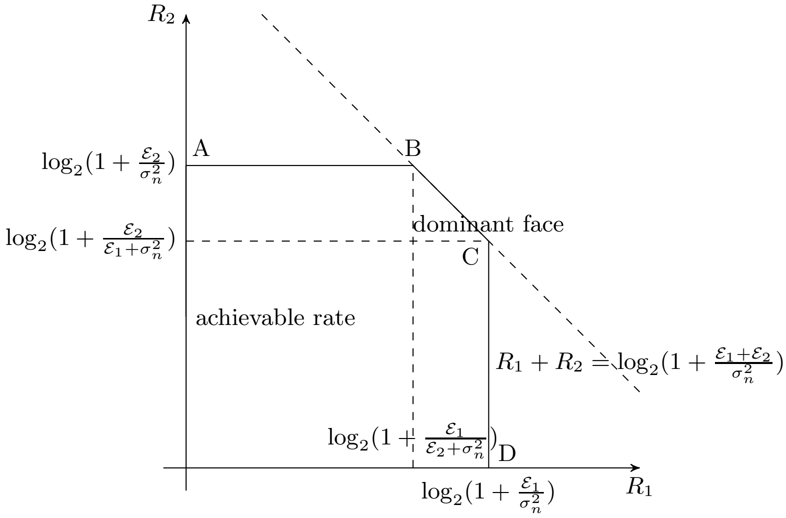
- Rate Region of two user Gaussian MAC *Anmerkung: Einschränkungen*

$$R_1 < \log_2\big(1 + \frac{\mathcal{E}_1}{\sigma_n^2}\big) \tag{1}$$

$$R_2 < \log_2\big(1 + \frac{\mathcal{E}_2}{\sigma_n^2}\big) \tag{2}$$

$$R_1 + R_2 < \log_2\big(1 + \frac{\mathcal{E}_1 + \mathcal{E}_2}{\sigma_n^2}\big) \tag{3}$$

- Interpretation:
  - (1) and (2) (= single-to user constraint) are the "single-user bounds, i.e., the maximum rates of user 1 and 2, if the other user was not there
  - (3) can be interpreted as the maximum rate if streams of users 1 and 2 were jointly encoded. The separate encoding in the MAC cannot yield a better performance
  - Graphical represantation:



- Observations:
  - A - B is defined by (2)
  - C - D is defined by (1)
  - B - C is defined by (3)
  - A - B suggests that even if user 2 transmits with the same max. rate as in the single user case, user 1 can transmit with non-zero rate! $\rightarrow$ Multiuser communication enables "free rate gains!
  - Which point on A - B - C,- D we choose, depends on the design criterion
- How do we achieve points on A - B - C,- D?
  - Both user use Gaussian codebooks
  - B:

* signal of user 1, x_1, is decoded first and x_2 is treated as noise:

$$y = x_1 + \underbrace{x_2 + n}_{\text{treat as noise}}$$

$$\rightarrow R_1 < \log_2\big(1 + \frac{\mathcal{E}_1}{\mathcal{E}_2 + \sigma_n^2}\big)$$

* once x_1 is known, we form

$$y - x_1 = x_2 + n$$

$$\rightarrow R_2 < \log_2\big(1 + \frac{\mathcal{E}_s}{\sigma_n^2}\big)$$

* this approach is referred to as successive interference cancellation (SIC) and is a direct result of the chain rule in information theory:
$I(X_1, X_2, Y) = I(X_1, Y) + I(X_2; Y|X_1)$

- C: same as B, but X_1 and X_2 change rules
- Points on A - B, C - D can be achieved by decreasing the rate of users 1 and 2 respectively (not desirable)
- Points on B - C (dominant face): Achievable by "time-sharing, i.e., $\theta \cdot 100\%$ of the time we decode user 1 first and $(1-\theta)100\%$ of the time we decode user 2 first, $0 \le \theta \le 1$

$$R_1 < \theta \log_2\big(1 + \frac{\mathcal{E}_1}{\mathcal{E}_2 + \sigma_n^2}\big) + \big(1-\theta\big)\log_2\big(1 + \frac{\mathcal{E}_1}{\sigma_n^2}\big)$$

$$R_2 < \theta \log_2\big(1 + \frac{\mathcal{E}_2}{\sigma_n^2}\big) + \big(1-\theta\big)\log_2\big(1 + \frac{\mathcal{E}_2}{\mathcal{E}_1 + \sigma_n^2}\big)$$

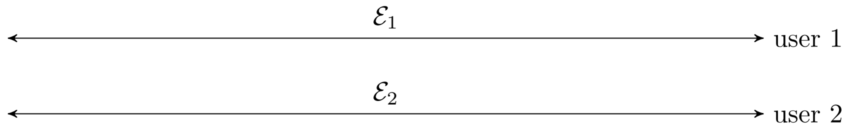
$$\begin{aligned}\rightarrow R_1 + R_2 < {} & \theta\Big(\log_2\big(1 + \frac{\mathcal{E}_1}{\mathcal{E}_2 + \sigma_n^2}\big) + \log_2\big(1 + \frac{\mathcal{E}_2}{\sigma_n^2}\big)\Big) + \\ & + \big(1-\theta\big)\Big(\log_2\big(1 + \frac{\mathcal{E}_1}{\sigma_n^2}\big) + \log_2\big(1 + \frac{\mathcal{E}_2}{\mathcal{E}_1 + \sigma_n^2}\big)\Big) = \\ & = \theta \log_2\Big(\frac{\mathcal{E}_1 + \mathcal{E}_2 + \sigma_n^2}{\mathcal{E}_2 + \sigma_n^2} \cdot \frac{\mathcal{E}_2 + \sigma_n^2}{\sigma_n^2}\Big) \cdot \big(1-\theta\big)\log_2\Big(\frac{\mathcal{E}_1 + \sigma_n^2}{\sigma_n^2} \cdot \frac{\mathcal{E}_1 + \mathcal{E}_2 + \sigma_n^2}{\mathcal{E}_1 + \sigma_n^2}\Big) = \\ & = \log_2\Big(1 + \frac{\mathcal{E}_1 + \mathcal{E}_2}{\sigma_n^2}\Big)\end{aligned}$$

- Comparison with orthogonal transmission
  - User 1 transmits for $\theta \cdot 100\%$ of the time and user 2 transmits for $(1-\theta) \cdot 100\%$ of the time, $0 \le \theta \le 1$
  - to keep average transmit power independent of θ, the users transmit with powers $\frac{\mathcal{E}_1}{\theta}$ and $\frac{\mathcal{E}_2}{1-\theta}$
  - Rates:

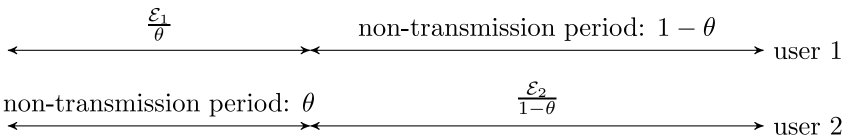
$$R_1 < \theta \log_2\Big(1 + \frac{\mathcal{E}_1}{\theta \sigma_n^2}\Big)$$

$$R_2 < \big(1-\theta\big)\log_2\Big(1 + \frac{\mathcal{E}_2}{(1-\theta)\sigma_n^2}\Big)$$

multiuser:



orthogonal:



– sumrate:

$$R_1 + R_2 < \theta \log_2\Big(1 + \frac{\mathcal{E}_1}{\theta\sigma_n^2}\Big) + \big(1-\theta\big)\log_2\Big(1 + \frac{\mathcal{E}_2}{(1-\theta)\sigma_n^2}\Big) = R_{\text{sum}}$$

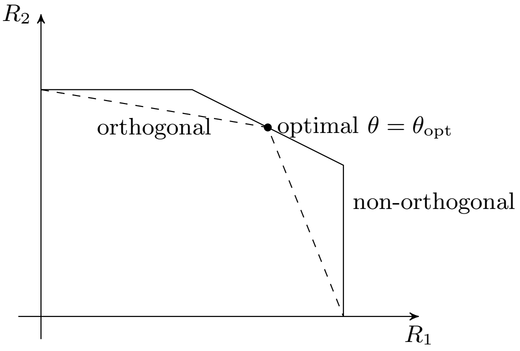
– Which θ maximizes sumrate?

$$\frac{\delta R_{\text{sum}}}{\delta \theta} \overset{!}{=} 0 \text{ leads to } \theta_{\text{opt}} = \frac{\mathcal{E}_1}{\mathcal{E}_1 + \mathcal{E}_2}$$

– Maximum sumrate

$$\begin{aligned} R_{\text{sum}} &= \frac{\mathcal{E}_1}{\mathcal{E}_1+\mathcal{E}_2}\log_2\Big(1+\frac{\mathcal{E}_1+\mathcal{E}_2}{\sigma_n^2}\Big) + \frac{\mathcal{E}_2}{\mathcal{E}_1+\mathcal{E}_2}\log_2\Big(1+\frac{\mathcal{E}_1+\mathcal{E}_2}{\sigma_n^2}\Big) = \\ &= \log_2\Big(1+\frac{\mathcal{E}_1+\mathcal{E}_2}{\sigma_n^2}\Big) \\ &\rightarrow \text{ same value as for general non-orthogonal transmission!} \end{aligned}$$

– <u>But:</u> In general, orthogonal transmission is suboptimal!



- 3 users case:

$$R_1 < \log_2\Big(1 + \frac{\mathcal{E}_1}{\sigma_n^2}\Big)$$
$$R_2 < \log_2\Big(1 + \frac{\mathcal{E}_2}{\sigma_n^2}\Big)$$
$$R_3 < \log_2\Big(1 + \frac{\mathcal{E}_3}{\sigma_n^2}\Big)$$
$$R_i + R_j < \log_2\Big(1 + \frac{\mathcal{E}_i + \mathcal{E}_j}{\sigma_n^2}\Big), \quad i \neq j$$
$$R_1 + R_2 + R_3 < \log_2\Big(1 + \frac{\mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3}{\sigma_n^2}\Big)$$

$\rightarrow$ rate region $\mathcal{C}$ has $3! = 6$ corner points

- general case of K users
  - define all non-empty subsets of $\mathbf{K} = \big\{1, \ldots, K\big\}$ as $\mathbf{S} \in \mathbf{K}$, e.g. $K = 2$: $\mathbf{K} = \big\{1, 2\big\}, \mathbf{S} = \big\{\{1\}, \{2\}, \{1, 2\}\big\}$
- rate region $\mathcal{C}$ is defined by

$$\sum_{k \in \mathbf{S}} R_k < \log_2\Big(1 + \frac{\sum_{k \in \mathbf{S}} \mathcal{E}_k}{\sigma_n^2}\Big) \quad \forall\, \mathbf{S}$$

$\rightarrow$ $\mathcal{C}$ has $K!$ corner points which can all be achieved by successive interference cancellation (SIC)