

MIMO Skript - Wintersemester 2013

Kapitel 4

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4 Distributed MIMO

- This research topic emerged ca. 10 years ago and is still a very active area of research
- Simple relaying schemes have been included in recent standards such as IEEE 802.16 (WiMAX) and LTE-Advanced
- Advantages: relay-assisted communications:
 - Relays can help to reduce the effective overall pathloss
 - Relays can also combat small-scale fading effects
 - Relays can help to realize MIMO gains with single-antenna nodes
- Challenges in relays-assisted communication:
 - Network architectures are becoming more complex
 - Synchronization across different nodes may be necessary
 - Exchange of channel state information (CSI) across nodes may be required

4.1 Half-Duplex One-Way Relaying

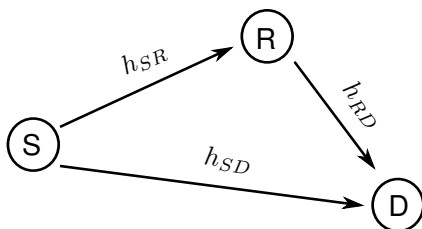


Figure 1: Basic Relay Network

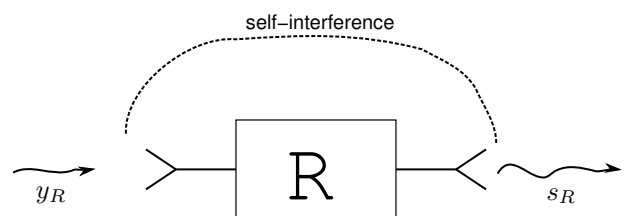


Figure 2: Relay with self-interference

Basic Relay Network

- Relay R assists source S in communication with destination D, see figure 1.
- Two basic nodes of transmission (at the relay):

Full - Duplex relaying: R can receive and transmit at the same time and in the same frequency band

- Since the TX signal power is orders of magnitude larger than the RX power, there is self-interference (at the relay), see figure 2.
- Full - duplex relays are difficult to implement. The design of full - duplex relays is an active area of research.
- Majority of existing literature assumes half - duplex relaying.

Half - duplex relaying: R transmits and receives in different time slots and/or different frequency bands. Typically, a two - phase protocol is used:

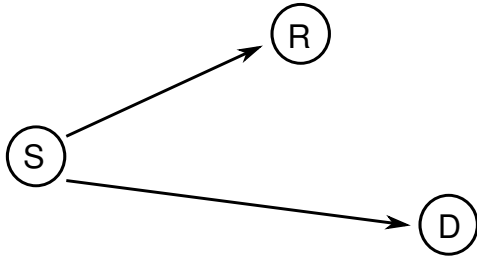


Figure 3: Half-duplex Relaying: Phase 1

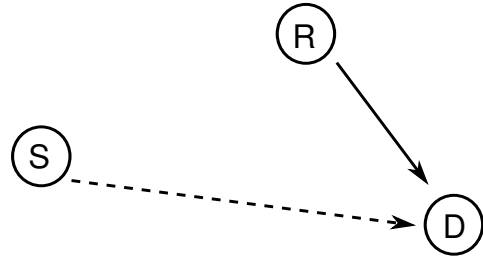


Figure 4: Half-duplex Relaying: Phase 2

Phase 1: S transmits, R and D receive, see figure 3.

Phase 2: R transmits, D receives, S may or may not transmit, see figure 4.

There are different relaying strategies that differ in the processing applied at the relay. The most popular are:

- Decode - and - Forward
- Amplify - and - Forward
- (Compress - and - Forward)

4.1.1 Decode - and - Forward (DF) Relaying

In DF relaying, the relay detects and decodes the signal received from the source before encoding it and forwarding it to the destination.

Phase 1:

- R receives: $y_R = h_{SR}x + n_R$
- D receives: $y_{D1} = h_{SD}x + n_{D1}$
- with:
 - transmit signal $x, \mathcal{E}_s = \mathcal{E}\{|x|^2\}$

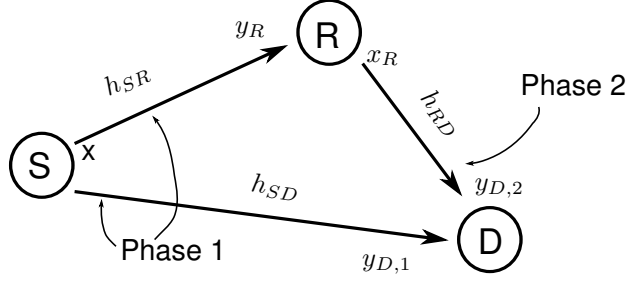


Figure 5: Block diagram Decode- and -forward Relaying

- AWGN n_R and n_{D1} , $\sigma_n^2 = \mathcal{E}\{|n_R|^2\} = \mathcal{E}\{|n_{D1}|^2\}$

Phase 2:

- R decodes and forwards x_R (estimate of x)
- D receives: $y_{D2} = h_{RD}x_R + n_{D2}$
 - x_R is estimate of x after decoding at R
 - $\sigma_n^2 = \mathcal{E}\{|n_{D2}|^2\}$; $\mathcal{E}_R = \mathcal{E}\{|x_R|^2\}$
 - we assume: S is silent in Phase 2

Performance analysis

- The capacity at the three node relay channel is not known!
- We provide an achievable rate under the following simplifying assumption: The direct source-destination link is not used/exploited.
- Achievable rate without S-D link:

$$C_{DF} = \frac{1}{2} \min \left\{ \log_2 \left(1 + \frac{\mathcal{E}_S |h_{SR}|^2}{\sigma_n^2} \right), \log_2 \left(1 + \frac{\mathcal{E}_R |h_{RD}|^2}{\sigma_n^2} \right) \right\}$$

- factor $\frac{1}{2}$ is due to the fact that we use two time slots to transmit one packet
- $\min\{\dots\}$ means we are limited by the weaker link (bottle-neck)
- If power allocation is possible, the total power $\mathcal{E} = \mathcal{E}_S + \mathcal{E}_R$ should be divided between S and R to guarantee:

$$\begin{aligned} \frac{\mathcal{E}_S |h_{SR}|^2}{\sigma_n^2} &= \frac{\mathcal{E}_R |h_{RD}|^2}{\sigma_n^2}, \\ \mathcal{E}_R &= \frac{|h_{SR}|^2}{|h_{SR}|^2 + |h_{RD}|^2} \cdot \mathcal{E}, \\ \mathcal{E}_S &= \frac{|h_{RD}|^2}{|h_{SR}|^2 + |h_{RD}|^2} \cdot \mathcal{E} \end{aligned}$$

- Outage-probability in fading:
 - We transmit with fixed rate R

- An outage occurs, if:

$$\frac{1}{2} \log_2 \left(1 + \underbrace{\frac{\mathcal{E}_S |h_{SR}|^2}{\sigma_n^2}}_{=\gamma_{SR}} \right) < R \quad \text{or}$$

$$\frac{1}{2} \log_2 \left(1 + \underbrace{\frac{\mathcal{E}_R |h_{RD}|^2}{\sigma_n^2}}_{=\gamma_{RD}} \right) < R$$

$$\boxed{\gamma_{SR} < \underbrace{2^{2R} - 1}_{\gamma_T} \quad \text{or} \quad \gamma_{RD} < 2^{2R} - 1}$$

$$\begin{aligned} P_{\text{out}} &= \Pr\{\gamma_{SR} < \gamma_T \vee \gamma_{RD} < \gamma_T\} = \Pr\{\underbrace{\min\{\gamma_{SR}, \gamma_{RD}\}}_{=\gamma_{eq}} < \gamma_T\} \\ &= 1 - \Pr\{\gamma_{SR} > \gamma_T \wedge \gamma_{RD} > \gamma_T\} = 1 - \Pr\{\gamma_{SR} > \gamma_T\} \Pr\{\gamma_{RD} > \gamma_T\} = \\ &= 1 - (1 - F_{\gamma_{SR}}(\gamma_T))(1 - F_{\gamma_{RD}}(\gamma_T)) = \\ &= \underline{F_{\gamma_{SR}}(\gamma_T) + F_{\gamma_{RD}}(\gamma_T) - F_{\gamma_{SR}}(\gamma_T) \cdot F_{\gamma_{RD}}(\gamma_T)} \end{aligned}$$

with CDFs: $F_{\gamma_{SR}}(\cdot)$ and $F_{\gamma_{RD}}(\cdot)$

- Rayleigh Fading:

$$\begin{aligned} \rightarrow F_{\gamma_{SR}}(\gamma) &= 1 - \exp\left(\frac{-\gamma}{\bar{\gamma}_{SR}}\right); \quad \bar{\gamma}_{SR} = \mathcal{E}\{\gamma_{SR}\} \\ F_{\gamma_{RD}}(\gamma) &= 1 - \exp\left(\frac{-\gamma}{\bar{\gamma}_{RD}}\right); \quad \bar{\gamma}_{RD} = \mathcal{E}\{\gamma_{RD}\} \\ \rightarrow P_{\text{out}} &= 1 - \exp\left(\frac{-\gamma_T}{\bar{\gamma}_{SR}}\right) + 1 - \exp\left(\frac{-\gamma_T}{\bar{\gamma}_{RD}}\right) - \left(1 - \exp\left(\frac{-\gamma_T}{\bar{\gamma}_{SR}}\right)\right)\left(1 - \exp\left(\frac{-\gamma_T}{\bar{\gamma}_{RD}}\right)\right) = \\ &= 1 - \exp\left(-\frac{\bar{\gamma}_{SR} + \bar{\gamma}_{RD}}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}} \cdot \gamma_T\right) \end{aligned}$$

→ equivalent SNR $\gamma_{eq} = \min\{\gamma_{SR}, \gamma_{RD}\}$ is also exponentially distributed with $\bar{\gamma}_{eq} = \frac{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}{\bar{\gamma}_{SR} + \bar{\gamma}_{RD}}$

- Diversity gain: Assume $\bar{\gamma}_{SR} = \alpha\bar{\gamma}$, $\bar{\gamma}_{RD} = \beta\bar{\gamma}$

$$\begin{aligned} \rightarrow P_{\text{out}} &\xrightarrow{\bar{\gamma} \rightarrow \text{inf}} 1 - \left(1 - \frac{\alpha + \beta}{\alpha\beta} \cdot \frac{\gamma_T}{\bar{\gamma}}\right) + \mathcal{O}(\bar{\gamma}^{-1}) = \\ &= \frac{\alpha + \beta}{\alpha\beta} \cdot \frac{\gamma_T}{\bar{\gamma}} + \mathcal{O}(\bar{\gamma}^{-1}) = \\ &\rightarrow \boxed{G_d = 1} \end{aligned}$$

- Bit error rate (BER) of BPSK (uncoded)

$$\begin{aligned} - \text{BER}(\gamma_{SR}, \gamma_{RD}) &= \left(1 - \text{BER}_{SR}(\gamma_{SR})\right) \text{BER}_{RD}(\gamma_{RD}) + \left(1 - \text{BER}_{RD}(\gamma_{RD})\right) \text{BER}_{SR}(\gamma_{SR}) \\ &* \text{ with BER of the S - R link, } \text{BER}_{SR}(\gamma_{SR}) \text{ and BER of the R - D link } \text{BER}_{RD}(\gamma_{RD}) \\ &* \text{ for sufficiently high SNR } \rightsquigarrow \text{BER}_{SR}(\gamma_{SR}), \text{BER}_{RD}(\gamma_{RD}) \ll \text{BER}_{SR}(\gamma_{SR}) + \text{BER}_{RD}(\gamma_{RD}) \\ \rightarrow &\underline{\text{BER}(\gamma_{SR}, \gamma_{RD}) \approx \text{BER}_{SR}(\gamma_{SR}) + \text{BER}_{RD}(\gamma_{RD})} \end{aligned}$$

* to average BER (Rayleigh Fading):

$$\text{BER} = \mathcal{E}_{\gamma_{SR}, \gamma_{RD}} \left\{ \text{BER}(\gamma_{SR}, \gamma_{RD}) \right\} = \frac{1}{2} \left(1 - \sqrt{\frac{1}{1 + \frac{1}{\bar{\gamma}_{SR}}}} \right) + \frac{1}{2} \left(1 - \sqrt{\frac{1}{1 + \frac{1}{\bar{\gamma}_{RD}}}} \right)$$

* high SNR:

$$\begin{aligned} \text{BER} &\approx \frac{1}{2} \left(1 - 1 + \frac{1}{2} \frac{1}{\bar{\gamma}_{SR}} \right) + \frac{1}{2} \left(1 - 1 + \frac{1}{2} \frac{1}{\bar{\gamma}_{RD}} \right) = \\ &= \frac{1}{4} \left(\frac{1}{\bar{\gamma}_{SR}} + \frac{1}{\bar{\gamma}_{RD}} \right) \end{aligned}$$

\rightsquigarrow also indicates diversity gain $G_d = 1$

4.1.2 Amplify - and - Forward (AF) Relaying

- Relay does not decode signal received from source but only amplifies it before forwarding it to the destination

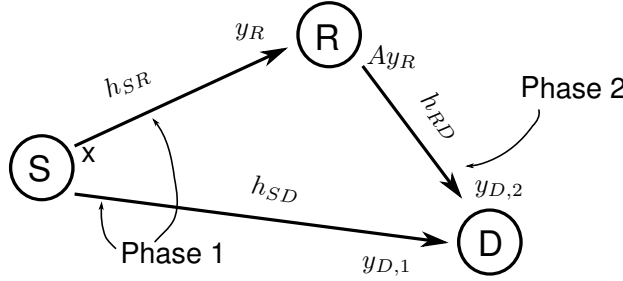


Figure 6: Block diagram AF - Relaying

- Amplification gain A may be constant or channel dependent and ensures a certain (average) transmit power

Phase 1:

- R receives: $y_R = h_{SR}x + n_R$
- D receives: $y_{D,1} = h_{SD}x + n_{D,1}$

Phase 2:

- R transmits: $s_R = Ay_R = A(h_{SR}x + n_R)$
- D receives: $y_{D,2} = h_{RD}Ay_R + n_{D,2}$

- We can use MRC to combine $y_{D,1}$ and $y_{D,2}$ at D: $y_{D,2} = Ah_{RD}h_{SR}x + h_{RD}An_R + n_{D,2}$, where: $h_{RD}An_R + n_{D,2}$ is effective noise n_{eff} with variance $\sigma_{n_{\text{eff}}}^2 = \sigma_n^2(|h_{RD}|^2 A^2 + 1)$

\rightarrow make noise variances of both branches equal

$$\begin{aligned} \bar{y}_{D,2} &= \frac{1}{\sqrt{|h_{RD}|^2 A^2 + 1}} \cdot y_{D,2} = \frac{Ah_{RD}h_{SR}}{\sqrt{A^2|h_{RD}|^2 + 1}} \cdot x + \tilde{n}_{\text{eff}} \\ \text{MRC: } r &= h_{SD}^* y_{D,1} + \underbrace{\frac{Ah_{RD}^* h_{SR}^*}{\sqrt{A^2|h_{RD}|^2 + 1}} \cdot \bar{y}_{D,2}}_{=\text{decision variable!}} = h_{SD}^* y_{D,1} + \frac{Ah_{RD}^* h_{SR}^*}{A^2|h_{RD}|^2 + 1} \cdot y_{D,2} \end{aligned}$$

- Choice of A : The goal is to ensure an (average) transmit power of \mathcal{E}_R

a) Variable gain relaying: In this case we introduce an instantaneous power constraint.

$$\begin{aligned}\mathcal{E}_{x,n}\{|S_R|^2\} &= \mathcal{E}_{x,n}\{A^2(|h_{SR}|^2|x|^2 + |n_R|^2)\} = \\ &= A^2(|h_{SR}|^2\mathcal{E}_S + \sigma_n^2) \stackrel{!}{=} \mathcal{E}_R \\ \rightarrow A^2 &= \frac{\mathcal{E}_R}{|h_{SR}|^2\mathcal{E}_S + \sigma_n^2}\end{aligned}$$

- A is channel dependent
- Instantaneous transmit power is not channel dependent

b) Fixed gain relaying: In this case, we introduce an average (with respect to the channel) power constraint

$$\begin{aligned}\mathcal{E}\{|S_R|^2\} &= \mathcal{E}\{A^2(|h_{SR}|^2|x|^2 + |n_R|^2)\} = \\ &= A^2(\underbrace{\mathcal{E}\{|h_{SR}|^2\}}_{\sigma_{S_R}^2}\mathcal{E}_S + \sigma_n^2) \stackrel{!}{=} \mathcal{E}_R \\ \rightarrow A^2 &= \frac{\mathcal{E}_R}{\mathcal{E}_S\sigma_{S_R}^2 + \sigma_n^2}\end{aligned}$$

- A is not channel dependent
- Instantaneous power of S_R depends on channel and may actually vary widely

Equivalent SNR for **variable gain AF relaying** (only relayed link)

$$y_{D,2} = Ah_{RD}h_{SR}x + h_{RD}An_R + n_{D,2}$$

$$\begin{aligned}\text{SNR: } \gamma_{eq}^{AF} &= \frac{A^2|h_{SR}|^2|h_{RD}|^2\mathcal{E}_S}{A^2|h_{RD}|^2\sigma_n^2 + \sigma_n^2} = \frac{\frac{\mathcal{E}_S}{\sigma_n^2}|h_{SR}|^2|h_{RD}|^2}{|h_{RD}|^2 + \frac{1}{\mathcal{E}_R}(|h_{SR}|^2\mathcal{E}_S + \sigma_n^2)} = \\ &= \frac{\frac{\mathcal{E}_S}{\sigma_n^2}|h_{SR}|^2 \cdot \frac{\mathcal{E}_R}{\sigma_n^2}|h_{RD}|^2}{\frac{\mathcal{E}_R}{\sigma_n^2}|h_{RD}|^2 + \frac{\mathcal{E}_S}{\sigma_n^2}|h_{SR}|^2 + 1} = \frac{\gamma_{SR}\gamma_{RD}}{\gamma_{SR} + \gamma_{RD} + 1}\end{aligned}$$

high SNR: $\gamma_{SR}, \gamma_{RD} \gg 1$

$$\boxed{\gamma_{eq}^{AF} = \frac{\gamma_{SR}\gamma_{RD}}{\gamma_{SR} + \gamma_{RD}}} \quad (1)$$

Comparison with equivalent SNR of DF:

$$\boxed{\gamma_{eq}^{DF} = \min\{\gamma_{SR}, \gamma_{RD}\}} \quad (2)$$

3 cases:

$$\text{a) } \gamma_{SR} = \gamma_{RD} = \gamma \rightarrow \gamma_{eq}^{AF} = \frac{1}{2}\gamma = \frac{1}{2}\gamma_{eq}^{DF} \quad (3)$$

$$\text{b) } \gamma_{SR} \gg \gamma_{RD} \rightarrow \gamma_{eq}^{AF} \approx \gamma_{RD} = \gamma_{eq}^{DF} \quad (4)$$

$$\text{c) } \gamma_{SR} \ll \gamma_{RD} \rightarrow \gamma_{eq}^{AF} \approx \gamma_{SR} = \gamma_{eq}^{DF} \quad (5)$$

Decision errors mostly occur if one of the two link SNRs is much smaller than the other. The probability, that both SNRs are small at the same time is much smaller, than the probability, that just one link SNR is small.

$\rightarrow \gamma_{eq}^{AF} = \gamma_{eq}^{DF}$ holds most of the time

\rightarrow AF relaying with variable gain has the same performance as DF relaying in high SNR, vgl Plot vom 07.02.13

4.1.3 Buffer - aided DF Relaying

- For conventional relaying, the performance is always limited by the weaker (bottleneck) link since the relay has to immediately retransmit

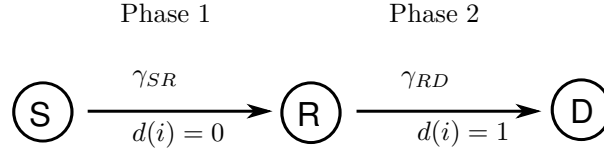


Figure 7: Buffer - aided DF Relaying

- In practice, the nodes in the network have buffers. Thus, we can use the stronger link and wait until the channel conditions of the weaker link have sufficiently improved.
- To avoid buffer over- or underflow at the relay, we demand that the average rate of the source relay channel (S - R) is equal to the average rate of the R - D channel
- We introduce a binary selection variable $d(i)$ for time slot $i = \{1, 2, \dots\}$, where:

$$\begin{aligned} d(i) = 0 &\Rightarrow \text{S transmits, R receives} \\ d(i) = 1 &\Rightarrow \text{R transmits, D receives} \end{aligned}$$

- Note: For conventional relaying we have $d(1) = 0, d(2) = 1, d(3) = 0, d(4) = 1, \dots$
- The average rate in the S - R link is:

$$R_{SR} = \frac{1}{N} \sum_{i=1}^N (1 - d(i)) \log_2(1 + \gamma_{SR}(i))$$

and that of the R - D link is:

$$R_{RD} = \frac{1}{N} \sum_{i=1}^N d(i) \log_2(1 + \gamma_{RD}(i))$$

where N denotes the total number of time slots.

- $\gamma_{SR}(i)$ and $\gamma_{RD}(i)$ change from one time slot to the next following e.g. a Rayleigh distribution
- At the relay, we have the constraint $R_{SR} = R_{RD}$ to avoid buffer over- / underflow
- To maximize the achievable throughput, we formulate an optimization problem:

$$\begin{aligned} \max_{d(i) \forall i} R_{RD}; \quad \text{subject to: C1: } R_{RD} &= R_{SR} \\ \text{C2: } d(i) &\in \{0, 1\} \end{aligned}$$

- For finite N, this problem is very difficult to solve.
- For infinite N, a simple solution exists \rightarrow Solution can be found by Lagrange method
- Solution (for $N \rightarrow \infty$): The optimal $d(i)$ is given by:

$$d(i) = \begin{cases} 1 & \text{if } \log_2(1 + \gamma_{RD}(i)) \geq \rho \log_2(1 + \gamma_{SR}(i)) \\ 0 & \text{otherwise} \end{cases}$$

where ρ is a constant, that only depends on the statistics of $\gamma_{SR}(i)$ and $\gamma_{RD}(i)$ and can be obtained from (numerical search needed):

$$\mathcal{E}_{\gamma_{SR}(i)}\{(1 - d(i)) \log_2(1 + \gamma_{SR}(i))\} = \mathcal{E}_{\gamma_{RD}(i)}\{d(i) \cdot \log_2(1 + \gamma_{RD}(i))\}$$

- Since always: the „best“ of two links is selected, this scheme can achieve a diversity gain of $G_d = 2$ (Rayleigh Fading)

