MIMO Skript - Wintersemester 2013
Kapitel 4

Contents

4 Distributed MIMO

- This research topic emerged ca. 10 years ago and is still a very active area of research
- Simple relaying schemes have been included in recent standards such as IEEE 802.16 (WiMAX) and LTE - Advanced
- Advantages: relay - assisted communications:
  - Relays can help to reduce the effective overall pathloss
  - Relays can also combat small - scale fading effects
  - Relays can help to realize MIMO gains with single - antenna nodes
- Challenges in relays - assisted communication:
  - Network architectures are becoming more complex
  - Synchronization across different nodes may be necessary *(Anm.: untersch. Trägerfrequenzen der Relays → Offset, Fehler, etc.)*
  - Exchange of channel state information (CSI) across nodes may be required

4.1 Half - Duplex One - Way Relaying

Basic Relay Network

- Relay R assists source S in communication with destination D
- Two basic nodes of transmission (at the relay):

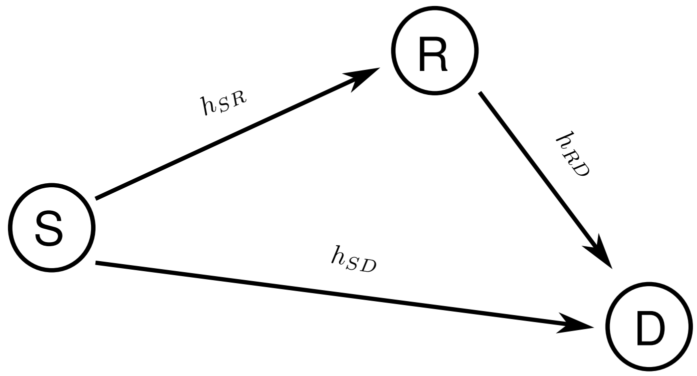

Figure 1: Basic Relay Network

Full - Duplex relaying: R can receive and transmit at the same time and in the same frequency band *(Anm.: effizient, da restliche Zeit und restliche Frequenzband von anderen genutzt werden kann)*

→ Since the TX signal power is orders of magnitude larger than the RX power, there is self - interference (at the relay)

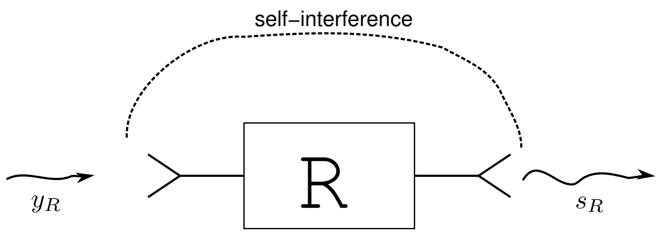

Figure 2: Relay with self-interference

→ Full - duplex relays are difficult to implement. The design of full - duplex relays is an active area of research.

→ Majority of existing literature assumes half - duplex relaying.

Half - duplex relaying: R transmits and receives in different time slots and/ or different frequency bands. Typically, a two - phase protocoll is used:

Phase 1: S transmits, R and D receive

Phase 2: R transmits, D receives, S may or may not transmit

There are different relaying strategies that differ in the processing applied at the relay. The most popular are:

- Decode - and - Forward
- Amplify - and - Forward
- (Compress - and - Forward)

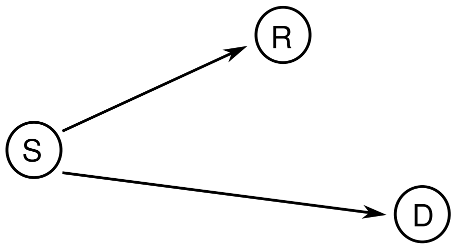

Figure 3: Half-duplex Relaying: Phase 1

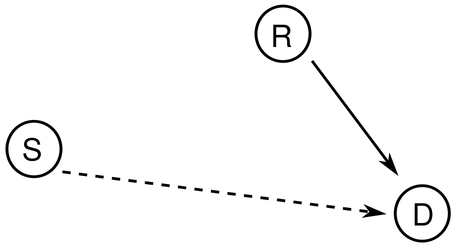

Figure 4: Half-duplex Relaying: Phase 2

4.1.1 Decode - and - Forward (DF) Relaying

In DF relaying, the relay detects and decodes the signal received from the source before encoding it and forwarding it to the destination.

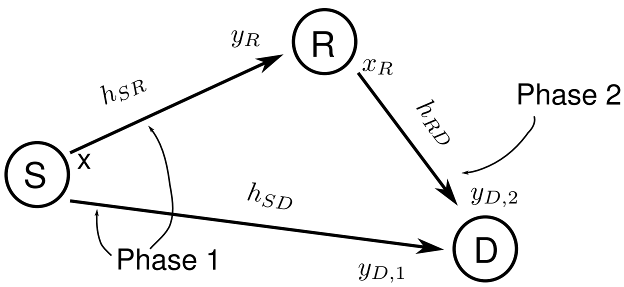

Figure 5: Block diagramm Decode - and - forward Relaying

Phase 1:

- R receives: $y_R = h_{SR}x + n_R$
- D receives: $y_{D1} = h_{SD}x + n_{D1}$
- with:
  - transmit signal $x, \mathcal{E}_s = \mathcal{E}\{|x|^2\}$
  - AWGN n_R and n_{D1}, $\sigma_n^2 = \mathcal{E}\{|n_R|^2\} = \mathcal{E}\{|n_{D1}|^2\}$

Phase 2:

- R decodes and forwards x_R (estimate of x)
- D receives: $y_{D2} = h_{RD} x_R + n_{D2}$
  - x_R is estimate of x after decoding at R
  - $\sigma_n^2 = \mathcal{E}\{|n_{D2}|^2\}$; $\quad \mathcal{E}_R = \mathcal{E}\{|x_R|^2\}$
  - weassume: S is silent in Phase 2

- The capacity at the three node relay channel is not known!
- We provide an achievable rate under the following simplifying assumption: The direct source - relay link is not used/ exploited.
- Achievable rate without S - D link:

$$\boxed{C_{DF} = \frac{1}{2}\min\big\{\log_2\big(1 + \frac{\mathcal{E}_S|h_{SR}|^2}{\sigma_n^2}\big),\, \log_2\big(1 + \frac{\mathcal{E}_R|h_{RD}|^2}{\sigma_n^2}\big)\big\}}$$

  - factor $\frac{1}{2}$ is due to the fact that we use two time slots to transmit one packet
  - $\min\{\ldots\}$ means we are limited by the weaker link (bottle - neck)
  - If power allocation is possible, the total power $\mathcal{E} = \mathcal{E}_S + \mathcal{E}_R$ should be divided between S and R to guarantee:

$$\frac{\mathcal{E}_S|h_{SR}|^2}{\sigma_n^2} = \frac{\mathcal{E}_R|h_{RD}|^2}{\sigma_n^2},$$

$$\mathcal{E}_R = \frac{|h_{SR}|^2}{|h_{SR}|^2 + |h_{RD}|^2} \cdot \mathcal{E},$$

$$\mathcal{E}_S = \frac{|h_{RD}|^2}{|h_{SR}|^2 + |h_{RD}|^2} \cdot \mathcal{E}$$

- Outage - probability in fading:
  - We transmit with fixed rate R
  - An outage occurs, if:

$$\frac{1}{2}\log_2\Big(1 + \underbrace{\frac{\mathcal{E}_S|h_{SR}|^2}{\sigma_n^2}}_{=\gamma_{SR}}\Big) < R \quad \text{or}$$

$$\frac{1}{2}\log_2\Big(1 + \underbrace{\frac{\mathcal{E}_R|h_{RD}|^2}{\sigma_n^2}}_{=\gamma_{RD}}\Big) < R$$

$$\boxed{\gamma_{SR} < \underbrace{2^{2R} - 1}_{\gamma_T} \quad \text{or} \quad \gamma_{RD} < 2^{2R} - 1}$$

$$
\begin{aligned}
P_{\text{out}} &= \Pr\big\{\gamma_{SR} < \gamma_T \vee \gamma_{RD} < \gamma_T\big\} = \Pr\big\{\underbrace{\min\big\{\gamma_{SR}, \gamma_{RD}\big\}}_{=\gamma_{eq}} < \gamma_T\big\} \\
&= 1 - \Pr\big\{\gamma_{SR} > \gamma_T \wedge \gamma_{RD} > \gamma_T\big\} = 1 - \Pr\big\{\gamma_{SR} > \gamma_T\big\}\Pr\big\{\gamma_{RD} > \gamma_T\big\} = \\
&= 1 - \big(1 - F_{\gamma_{SR}}(\gamma_T)\big)\big(1 - F_{\gamma_{RD}}(\gamma_T)\big) = \\
&= \underline{F_{\gamma_{SR}}(\gamma_T) + F_{\gamma_{RD}}(\gamma_T) - F_{\gamma_{SR}}(\gamma_T) \cdot F_{\gamma_{RD}}(\gamma_T)}
\end{aligned}
$$

with CDFs: $F_{\gamma_{SR}}(\cdot)$ and $F_{\gamma_{RD}}(\cdot)$

– Rayleigh Fading:

$$
\begin{aligned}
&\to F_{\gamma_{SR}}(\gamma) = 1 - \exp\big(\frac{-\gamma}{\bar{\gamma}_{SR}}\big)\,; \quad \bar{\gamma}_{SR} = \mathcal{E}\{\gamma_{SR}\} \\
&\quad F_{\gamma_{RD}}(\gamma) = 1 - \exp\big(\frac{-\gamma}{\bar{\gamma}_{RD}}\big)\,; \quad \bar{\gamma}_{RD} = \mathcal{E}\{\gamma_{RD}\} \\
&\quad \to P_{\text{out}} = 1 - \exp\big(\frac{-\gamma_T}{\bar{\gamma}_{SR}}\big) + 1 - \exp\big(\frac{-\gamma_T}{\bar{\gamma}_{RD}}\big) - \Big(1 - \exp\big(\frac{-\gamma_T}{\bar{\gamma}_{SR}}\big)\Big)\Big(1 - \exp\big(\frac{-\gamma_T}{\bar{\gamma}_{RD}}\big)\Big) = \\
&\qquad\quad = 1 - \exp\Big(-\frac{\bar{\gamma}_{SR} + \bar{\gamma}_{RD}}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}} \cdot \gamma_T\Big)
\end{aligned}
$$

→ equivalent SNR $\gamma_{eq} = \min\big\{\gamma_{SR}, \gamma_{RD}\big\}$ is also exponentially distributed with $\bar{\gamma}_{eq} = \frac{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}{\bar{\gamma}_{SR}+\bar{\gamma}_{RD}}$

– Diversity gain: Assume $\bar{\gamma}_{SR} = \alpha\bar{\gamma}$

$$
\begin{aligned}
\to P_{out} &\xrightarrow{\bar{\gamma}\to\text{inf}} 1 - \Big(1 - \frac{\alpha+\beta}{\alpha\beta} \cdot \frac{\gamma_T}{\bar{\gamma}}\Big) + \mathcal{O}\big(\bar{\gamma}^{-1}\big) = \\
&= \frac{\alpha+\beta}{\alpha\beta} \cdot \frac{\gamma_T}{\bar{\gamma}} + \mathcal{O}\big(\bar{\gamma}^{-1}\big) = \\
&\to \boxed{G_d = 1}
\end{aligned}
$$

- Bit error rate (BER) of BPSK (uncoded)
  - $\text{BER}\big(\gamma_{SR}, \gamma_{RD}\big) = \Big(1 - \text{BER}_{SR}\big(\gamma_{SR}\big)\Big)\text{BER}_{RD}\big(\gamma_{RD}\big) + \Big(1 - \text{BER}_{RD}\big(\gamma_{RD}\big)\Big)\text{BER}_{SR}\big(\gamma_{SR}\big)$
    * with BER of the S - R link, $\text{BER}_{SR}(\gamma_{SR})$ and BER of the R - D link $\text{BER}_{RD}(\gamma_{RD})$
    * for sufficiently high SNR $\rightsquigarrow \text{BER}_{SR}(\gamma_{SR}), \text{BER}_{RD}(\gamma_{RD}) \ll \text{BER}_{SR}(\gamma_{SR}) + \text{BER}_{RD}(\gamma_{RD})$

    → $\underline{\text{BER}(\gamma_{SR}, \gamma_{RD}) \approx \text{BER}_{SR}(\gamma_{SR}) + \text{BER}_{RD}(\gamma_{RD})}$
    * to average BER (Rayleigh Fading):

$$
\text{BER} = \mathcal{E}_{\gamma_{SR},\gamma_{RD}}\Big\{\text{BER}\big(\gamma_{SR}, \gamma_{RD}\big)\Big\} = \frac{1}{2}\Big(1 - \sqrt{\frac{1}{1 + \frac{1}{\bar{\gamma}_{SR}}}} + \frac{1}{2}\Big(1 - \sqrt{\frac{1}{1 + \frac{1}{\bar{\gamma}_{RD}}}}\Big)
$$

    * high SNR:

$$
\begin{aligned}
\text{BER} &\approx \frac{1}{2}\big(1 - 1 + \frac{1}{2}\frac{1}{\bar{\gamma}_{SR}}\big) + \frac{1}{2}\big(1 - 1 + \frac{1}{2}\frac{1}{\bar{\gamma}_{RD}}\big) = \\
&= \frac{1}{4}\big(\frac{1}{\bar{\gamma}_{SR}} + \frac{1}{\bar{\gamma}_{RD}}\big)
\end{aligned}
$$

$\rightsquigarrow$ also indicates diversity gain $G_d = 1$

4.1.2 Amplify - and - Forward (AF) Relaying

- Relay does not decode signal received from source but only amplifies it before forwarding it to the destination

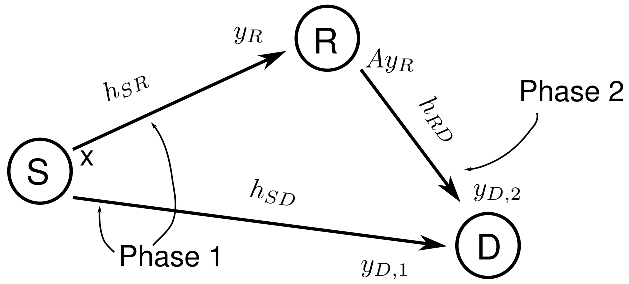

Figure 6: Block diagramm AF - Relaying

- Amplification gain A may be constant on channel dependent and ensures a certain (average) transmit power

  Phase 1:

  - R receives: $y_D = h_{SR}x + n_R$
  - D receives: $y_{D,1} = h_{SD}x + n_{D,1}$

  Phase 2:

  - R transmits: $s_R = Ay_R = A(h_{SR}x + n_R)$
  - D receives: $y_{D,2} = h_{RD}Ay_R + n_{D,2}$

- We can use MRC to combine $y_{D,1}$ and $y_{D,2}$ at D: $y_{D,2} = Ah_{RD}h_{SR}x + h_{RD}An_R + n_{D,2}$, where: $h_{RD}An_R + n_{D,2}$ is effective noise n_{eff} with variance $\sigma^2_{n_{\text{eff}}} = \sigma^2_n\big(|h_{RD}|^2A^2+1\big)$

$\rightarrow$ make noise variances of both branches equal

$$\bar{y}_{D,2} = \frac{1}{\sqrt{|h_{RD}|^2A^2+1}} \cdot y_{D,2} = \frac{Ah_{RD}h_{SR}}{\sqrt{A^2|h_{RD}|^2+1}} \cdot x + \tilde{n}_{\text{eff}}$$

$$\text{MRC:} \quad r = h^*_{SD}y_{D,1} + \frac{Ah^*_{RD}h^*_{SR}}{\sqrt{A^2|h_{RD}|^2+1}} \cdot \bar{y}_{D,2} = \underbrace{h^*_{SD}y_{D,1} + \frac{Ah^*_{RD}h^*_{SR}}{A^2|h_{RD}|^2+1} \cdot y_{D,2}}_{=\text{decision variable!}}$$

- Choice of A: The goal is to ensure en (average) transmit power of $\mathcal{E}_R$

a) Variable gain relaying: In this case we introduce an instantaneous power constraint. *Anm.: A muss abhängig von h_{SR} sein, um es kompensieren zu können.*

$$
\begin{aligned}
\mathcal{E}_{x,n}\{|S_R|^2\} &= \mathcal{E}_{x,n}\{A^2(|h_{SR}|^2|x|^2 + |n_R|^2)\} = \\
&= A^2(|h_{SR}|^2\mathcal{E}_S + \sigma_n^2) \stackrel{!}{=} \mathcal{E}_R \\
\rightarrow A^2 &= \frac{\mathcal{E}_R}{|h_{SR}|^2\mathcal{E}_S + \sigma_n^2}
\end{aligned}
$$

- A is channel dependent
- Instantaneous transmit power is <u>not</u> channel dependent

b) Fixed gain relaying: In this case, we introduce an average (with respect to the channel) power constraint

$$
\begin{aligned}
\mathcal{E}\{|S_R|^2\} &= \mathcal{E}\{A^2(|h_{SR}|^2|x|^2 + |n_R|^2)\} = \\
&= A^2(\underbrace{\mathcal{E}\{|h_{SR}|^2\}}_{\sigma_{SR}^2}\mathcal{E}_S + \sigma_n^2) \stackrel{!}{=} \mathcal{E}_R \\
\rightarrow A^2 &= \frac{\mathcal{E}_R}{\mathcal{E}_S\sigma_{SR}^2 + \sigma_n^2}
\end{aligned}
$$

- A is not channel dependent
- Instantaneous power of S_R depends on channel and may actually vary widely

Equivalent SNR for variable gain AF relaying (inly relayed link)

$$y_{D,2} = Ah_{RD}h_{SR}x + h_{RD}An_R + n_{D,2}$$

$$
\begin{aligned}
\text{SNR:} \quad \gamma_{eq}^{AF} &= \frac{A^2|h_{SR}|^2|h_{RD}|^2\mathcal{E}_S}{A^2|h_{RD}|^2\sigma_n^2 + \sigma_n^2} = \frac{\frac{\mathcal{E}_S}{\sigma_n^2}|h_{SR}|^2|h_{RD}|^2}{|h_{RD}|^2 + \frac{1}{\mathcal{E}_R}(|h_{SR}|^2\mathcal{E}_S + \sigma_n^2)} = \\
&= \frac{\frac{\mathcal{E}_S}{\sigma_n^2}|h_{SR}|^2 \cdot \frac{\mathcal{E}_R}{\sigma_n^2}|h_{RD}|^2}{\frac{\mathcal{E}_R}{\sigma_n^2}|h_{RD}|^2 + \frac{\mathcal{E}_S}{\sigma_n^2}|h_{SR}|^2 + 1} = \\
&= \frac{\gamma_{SR}\gamma_{RD}}{\gamma_{SR} + \gamma_{RD} + 1}
\end{aligned}
$$

high SNR: $\gamma_{SR}, \gamma_{RD} \gg 1$

$$\boxed{\gamma_{eq}^{AF} = \frac{\gamma_{SR}\gamma_{RD}}{\gamma_{SR} + \gamma_{RD}}} \tag{1}$$

Anm.: Vgl. Formel 1 mit Berechnung zweier paralleler Widerstände. Comparison with equivalent SNF of DF:

$$\boxed{\gamma_{eq}^{DF} = \min\{\gamma_{SR}, \gamma_{RD}\}} \tag{2}$$

3 cases:

$$\text{a)} \quad \gamma_{SR} = \gamma_{RD} = \gamma \quad \rightarrow \quad \gamma_{eq}^{AF} = \frac{1}{2}\gamma = \frac{1}{2}\gamma_{eq}^{DF} \tag{3}$$

$$\text{b)} \quad \gamma_{SR} \gg \gamma_{RD} \quad \rightarrow \quad \gamma_{eq}^{AF} = \gamma_{RD} = \gamma_{eq}^{DF} \tag{4}$$

$$\text{c)} \quad \gamma_{SR} \ll \gamma_{RD} \quad \rightarrow \quad \gamma_{eq}^{AF} = \gamma_{SR} = \gamma_{eq}^{DF} \tag{5}$$

Anm.: Fälle 4 und 5 sind am wahrscheinlichsten. Decision errors mostly occur if one of the two link SNRs is much smaller than the other. The probability, that both SNRs are small at the same time is much smaller, than the probability, that just one link SNR is small.

→ $\gamma_{eq}^{AF} = \gamma_{eq}^{DF}$ holds most of the time

→ AF relaying with variable gain has the same performance as DF relaying in high SNR, vgl Plot vom 07.02.13

4.1.3 Buffer - aided DF Relaying

- For conventional relaying, the performance is always limited by the weaker (bottleneck) link since the relay has to immediately retransmit

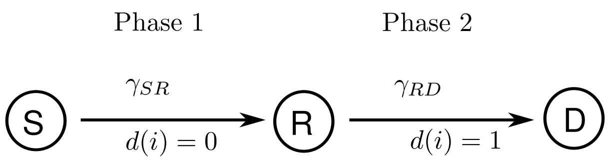

Figure 7: Buffer - aided DF Relaying

- In practice, the nodes in the network have buffers. Thus, we can use the stronger link and wait until the channel conditions of the weaker link have sufficiently improved.
- To avoid buffer over - or underflow at the relay, we demand that the average rate of the source relay channel (S - R) is equal to the average rate of the R - D channel
- We introduce a binary selection variable $d(i)$ for time slot $i = \{1, 2, \ldots\}$, where:

$$\begin{aligned} d(i) = 0 \quad &\Rightarrow \quad \text{S transmits, R receives} \\ d(i) = 1 \quad &\Rightarrow \quad \text{R transmits, D receives} \end{aligned}$$

- Note: For conventional relaying we have $d(1) = 0$, $d(2) = 1$, $d(3) = 0$, $d(4) = 1, \ldots$
- The average rate in the S - R link is:

$$R_{SR} = \frac{1}{N}\sum_{i=1}^{N}\big(1 - d(i)\big)\log_2\big(1 + \gamma_{SR}(i)\big)$$

  and that of the R - D link is:

$$R_{RD} = \frac{1}{N}\sum_{i=1}^{N} d(i)\log_2\big(1 + \gamma_{RD}(i)\big)$$

  where N denotes the total number of time slots.

- $\gamma_{SR}(i)$ and $\gamma_{RD}(i)$ change from one time slot to the next following e.g. a Rayleigh distribution
- At the relay, we have the constraint $R_{SR} = R_{RD}$ to avoid buffer over- / underflow
- To maximize the achievable throughput, we formulate an optimization problem:

$$\begin{aligned} &\max_{d(i)\forall i} R_{RD} \\ \text{subject to:}\ &\text{C1:} \quad R_{RD} = R_{SR} \\ &\text{C2:} \quad d(i) \in \{0,\, 1\} \end{aligned}$$

- For finite N, this problem is very difficult to solve.
- For infinte N, a simple solution exists.

$\rightarrow$ Solution can be found by Langrange method

- Solution (for $N \rightarrow \inf$): The optimal $d(i)$ is given by:

$$d(i) = \begin{cases} 1 & \text{if} \quad \log_2\bigl(1 + \gamma_{RD}(i)\bigr) \geq \rho \log_2\bigl(1 + \gamma_{SR}(i)\bigr) \\ 0 & \text{otherwise} \end{cases}$$

  where ρ is a constant, that only depends on the statistics of $\gamma_{SR}(i)$ and $\gamma_{RD}(i)$ and can be obtained from (numerical search needed):

$$\mathcal{E}_{\gamma_{SR}(i)}\bigl\{\bigl(1 - d(i)\bigr) \log_2\bigl(1 + \gamma_{SR}(i)\bigr)\bigr\} = \mathcal{E}_{\gamma_{RD}(i)}\bigl\{d(i) \cdot \log_2\bigl(1 + \gamma_{RD}(i)\bigr)\bigr\}$$

- Since always: the "best" of two links is selected, this scheme can achieve a diversity gain of $G_d = 2$ (Rayleigh Fading)