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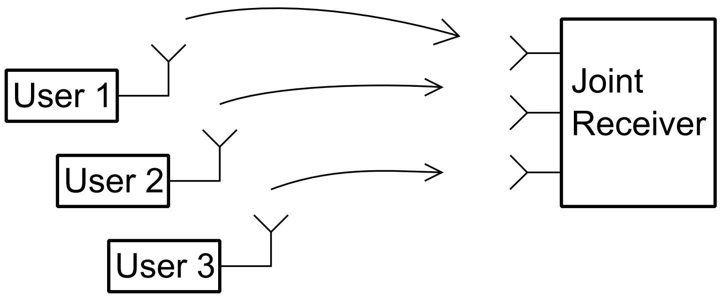
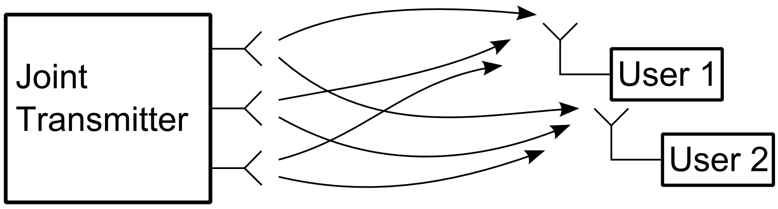
3 Multiuser MIMO

- We distinguish two cases:
  - multipoint - to - point transmission
  - point - to - multipoint transmission
- Multipoint - to - point transmission
  - typical uplink scenario in cellular systems
  - information theoretical channel model: Multiple Access Channel (MAC)



- Point - to -multipoint transmission
  - typical downlink scenarion in cellular systems
  - information theoretical channel model: Broadcast Channel (BC)



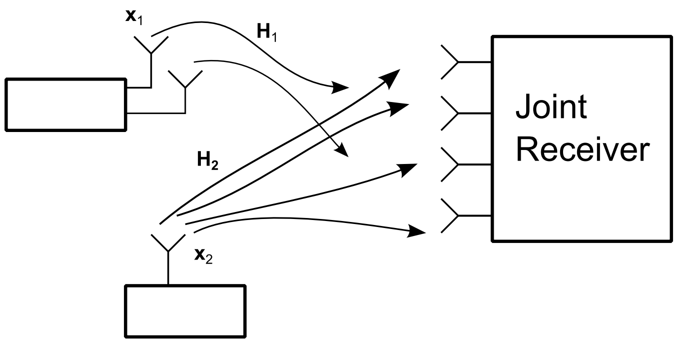
- Advantage of multiuser MIMO compared to point - to - point MIMO
  - multiplexing gain can be exploited even if users have only single antenna
  - users are spatially distributed in cell → channels to different users are independent

3.1 Multiple Access Channel (MAC)

We consider two aspects:

- Detector structures
- Rate region

3.1.1 Detector structures



Channel model: $\rightarrow$ general MAC: $\mathbf{y} = \sum\limits_{k=1}^{K} \mathbf{H}_k \mathbf{x}_k + \mathbf{n}$

with:

- K users
- user k has $N_{T,k}$ transmit antennas
- N_R receive antennas
- $\mathbf{H}_k \in \mathbb{C}^{N_R \times N_{T,k}}$

$$\mathbf{y} = \underbrace{\begin{bmatrix}\mathbf{H}_1 & \mathbf{H}_2 & \dots & \mathbf{H}_k\end{bmatrix}}_{\mathbf{H}} \cdot \underbrace{\begin{bmatrix}\mathbf{x}_1 \\ \vdots \\ \mathbf{x}_k\end{bmatrix}}_{\mathbf{x}} + \mathbf{n}$$

Observation:

- same equivalent channel model as for a point-to-point MIMO system transmitting $N_T = \sum_{k=1}^{K} N_{T,k}$ independent signal streams *(Anmerkung: kein Unterschied für Empfänger, ob Signale von einem Nutzer oder von mehreren)*
- the receiver (e.g. base station) can use detection schemes as for point-to-point MIMO systems
  - linear receiver
  - DFG
  - sphere decoder

Typical problems in uplink multiuser MIMO For given receiver structure:

- calculate SNR_k for all users k based on the expressions developed in Chapter 2.4
- optimize transmit power of users, $E_k = \mathcal{E}\{||x_k||^2\}$ for maximization of the sumrate or maximization of the minimum SNR_k *(Anmerkung: Maximierung der sumrate kann durch Maximierung des SNR des Users mit bestem Kanal erfolgen, aber: unfair anderen Usern gegenüber $\Rightarrow$ starving)*

3.1.2 Rate region

For point - to - point links, we can decode error free, if the rate, R, meets

a) SISO $R < \log_2\left(1 + \frac{\mathcal{E}_s}{\sigma_n^2}\right)$

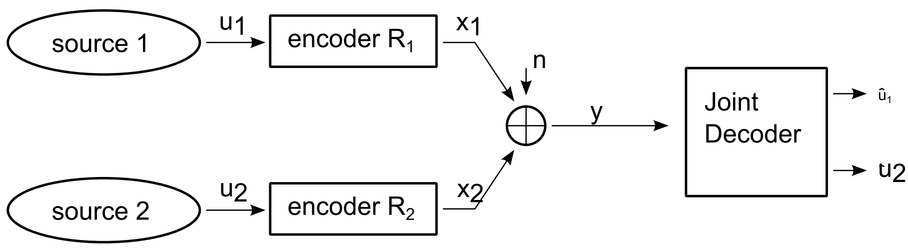
b) MIMO $R < \log_2 \underbrace{\left|\mathbf{I} + \frac{\mathcal{E}_s}{N_T \sigma_n^2}\mathbf{H}\mathbf{H}^H\right|}_{\det}$

Questions: What happens if there are multiple users?

Rate Region for Single Antenna Users and Receivers

- Gaussian channel
- $N_R = N_{T,k} = 1 \forall k$
- received signal:

$$y = \sum_{k=1}^{K} x_k + n$$
$$*\mathcal{E}_k = \mathcal{E}\{||x_k||^2\}$$
$$*\sigma_n^2 = \{||n||^2\}$$



Example: 2 Users

- How should we choose R_1 and R_2 to ensure error free decoding of <u>both</u> signal streams?
- It is no longer sufficient to maximizie a single rate. Instead we have to consider rate pairs (R_1, R_2)
- All possible rate points, that allows error free decoding, define the rate region $\underline{C}$

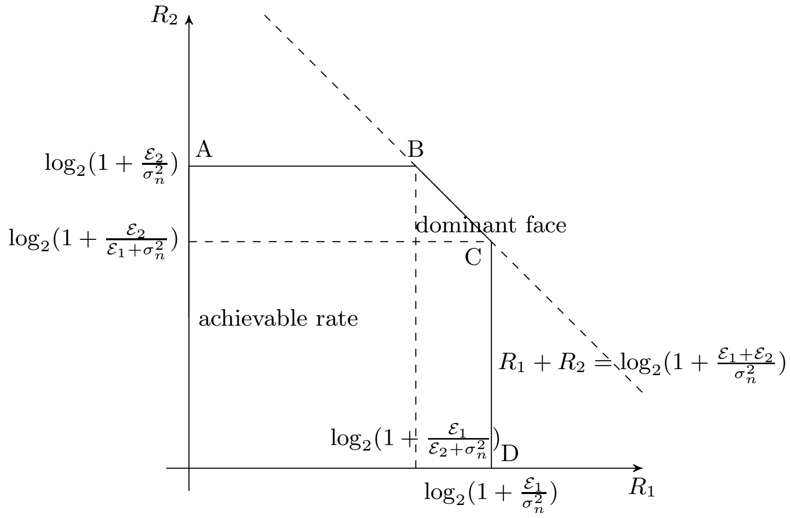
- Possible desing goals of the system:
  - maximized sumrate $R_{\text{sum}} = \max\limits_{(R_1,R_2)\in \underline{C}} R_1 + R_2$
  - maximize minimum user rate: $R_{\text{max-min}} = \max\limits_{(R_1,R_2)\in \underline{C}} \min\limits_{i\in\{1,2\}} R_i$
- Rate Region of two user Gaussian MAC *Anmerkung: Einschränkungen*

$$R_1 < \log_2\left(1 + \frac{\mathcal{E}_1}{\sigma_n^2}\right) \tag{1}$$

$$R_2 < \log_2\left(1 + \frac{\mathcal{E}_2}{\sigma_n^2}\right) \tag{2}$$

$$R_1 + R_2 < \log_2\left(1 + \frac{\mathcal{E}_1 + \mathcal{E}_2}{\sigma_n^2}\right) \tag{3}$$

- Interpretation:
  - (1) and (2) (= single-to user constraint) are the "single-user bounds, i.e., the maximum rates of user 1 and 2, if the other user was not there
  - (3) can be interpreted as the maximum rate if streams of users 1 and 2 were jointly encoded. The separate encoding in the MAC cannot yield a better performance
  - Graphical represantation:



- Observations:
  - A - B is defined by (2)
  - C - D is defined by (1)
  - B - C is defined by (3)
  - A - B suggests that even if user 2 transmits with the same max. rate as in the single user case, user 1 can transmit with non-zero rate! → Multiuser communication enables "free rate gains!
  - Which point on A - B - C,- D we choose, depends on the design criterion

- How do we achieve points on A - B - C,- D?
  - Both user use Gaussian codebooks
  - B:
    * signal of user 1, x_1, is decoded first and x_2 is treated as noise:

$$y = x_1 + \underbrace{x_2 + n}_{\text{treat as noise}}$$
$$\to R_1 < \log_2\big(1 + \frac{\mathcal{E}_1}{\mathcal{E}_2 + \sigma_n^2}\big)$$

    * once x_1 is known, we form

$$y - x_1 = x_2 + n$$
$$\to R_2 < \log_2\big(1 + \frac{\mathcal{E}_s}{\sigma_n^2}\big)$$

    * this approach is referred to as successive interference cancellation (SIC) and is a direct result of the chain rule in information theory:
      $I(X_1, X_2, Y) = I(X_1, Y) + I(X_2; Y|X_1)$
  - C: same as B, but X_1 and X_2 change rules
  - Points on A - B, C - D can be achieved by decreasing the rate of users 1 and 2 respectively (not desirable)
  - Points on B - C (dominant face): Achievable by "time-sharing, i.e., $\theta \cdot 100\%$ of the time we decode user 1 first and $(1-\theta)100\%$ of the time we decode user 2 first, $0 \le \theta \le 1$

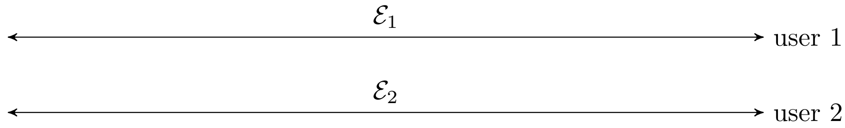
$$\begin{aligned}
R_1 &< \theta \log_2\big(1 + \frac{\mathcal{E}_1}{\mathcal{E}_2 + \sigma_n^2}\big) + \big(1-\theta\big)\log_2\big(1 + \frac{\mathcal{E}_1}{\sigma_n^2}\big) \\
R_2 &< \theta \log_2\big(1 + \frac{\mathcal{E}_2}{\sigma_n^2}\big) + \big(1-\theta\big)\log_2\big(1 + \frac{\mathcal{E}_2}{\mathcal{E}_1 + \sigma_n^2}\big) \\
\to R_1 + R_2 &< \theta\Big(\log_2\big(1 + \frac{\mathcal{E}_1}{\mathcal{E}_2 + \sigma_n^2}\big) + \log_2\big(1 + \frac{\mathcal{E}_2}{\sigma_n^2}\big)\Big) + \\
&+ \big(1-\theta\big)\Big(\log_2\big(1 + \frac{\mathcal{E}_1}{\sigma_n^2}\big) + \log_2\big(1 + \frac{\mathcal{E}_2}{\mathcal{E}_1 + \sigma_n^2}\big)\Big) = \\
&= \theta \log_2\Big(\frac{\mathcal{E}_1 + \mathcal{E}_2 + \sigma_n^2}{\mathcal{E}_2 + \sigma_n^2} \cdot \frac{\mathcal{E}_2 + \sigma_n^2}{\sigma_n^2}\Big) \cdot \big(1-\theta\big)\log_2\Big(\frac{\mathcal{E}_1 + \sigma_n^2}{\sigma_n^2} \cdot \frac{\mathcal{E}_1 + \mathcal{E}_2 + \sigma_n^2}{\mathcal{E}_1 + \sigma_n^2}\Big) = \\
&= \log_2\Big(1 + \frac{\mathcal{E}_1 + \mathcal{E}_2}{\sigma_n^2}\Big)
\end{aligned}$$

- Comparison with orthogonal transmission
  - User 1 transmits for $\theta \cdot 100\%$ of the time and user 2 transmits for $(1-\theta)\cdot 100\%$ of the time, $0 \le \theta \le 1$
  - to keep average transmit power independent of θ, the users transmit with powers $\frac{\mathcal{E}_1}{\theta}$ and $\frac{\mathcal{E}_2}{1-\theta}$

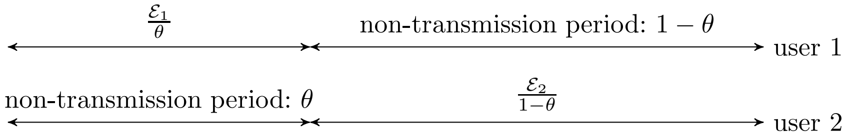
– Rates:

$$R_1 < \theta \log_2\Big(1 + \frac{\mathcal{E}_1}{\theta\sigma_n^2}\Big)$$
$$R_2 < \big(1-\theta\big)\log_2\Big(1 + \frac{\mathcal{E}_2}{(1-\theta)\sigma_n^2}\Big)$$

multiuser:



orthogonal:



– sumrate:

$$R_1 + R_2 < \theta\log_2\Big(1 + \frac{\mathcal{E}_1}{\theta\sigma_n^2}\Big) + \big(1-\theta\big)\log_2\Big(1 + \frac{\mathcal{E}_2}{(1-\theta)\sigma_n^2}\Big) = R_{\text{sum}}$$

– Which θ maximizes sumrate?

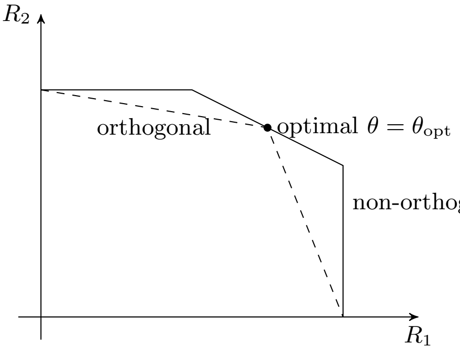
$$\frac{\delta R_{\text{sum}}}{\delta\theta} \overset{!}{=} 0 \text{ leads to } \theta_{\text{opt}} = \frac{\mathcal{E}_1}{\mathcal{E}_1 + \mathcal{E}_2}$$

– Maximum sumrate

$$R_{\text{sum}} = \frac{\mathcal{E}_1}{\mathcal{E}_1+\mathcal{E}_2}\log_2\Big(1 + \frac{\mathcal{E}_1+\mathcal{E}_2}{\sigma_n^2}\Big) + \frac{\mathcal{E}_2}{\mathcal{E}_1+\mathcal{E}_2}\log_2\Big(1 + \frac{\mathcal{E}_1+\mathcal{E}_2}{\sigma_n^2}\Big) =$$
$$= \log_2\Big(1 + \frac{\mathcal{E}_1+\mathcal{E}_2}{\sigma_n^2}\Big)$$

$\rightarrow$ same value as for general non-orthogonal transmission!

- But: In general, orthogonal transmission is suboptimal!



- 3 users case:

$$
\begin{aligned}
R_1 &< \log_2\Big(1 + \frac{\mathcal{E}_1}{\sigma_n^2}\Big) \\
R_2 &< \log_2\Big(1 + \frac{\mathcal{E}_2}{\sigma_n^2}\Big) \\
R_3 &< \log_2\Big(1 + \frac{\mathcal{E}_3}{\sigma_n^2}\Big) \\
R_i + R_j &< \log_2\Big(1 + \frac{\mathcal{E}_i + \mathcal{E}_j}{\sigma_n^2}\Big), \quad i \neq j \\
R_1 + R_2 + R_3 &< \log_2\Big(1 + \frac{\mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3}{\sigma_n^2}\Big) \\
&\to \text{rate region } \mathcal{C} \text{ has } 3! = 6 \text{ corner points}
\end{aligned}
$$

- general case of K users
  - define all non-empty subsets of $\mathbf{K} = \big\{1, \dots, K\big\}$ as $\mathbf{S} \in \mathbf{K}$, e.g. $K = 2$: $\mathbf{K} = \big\{1, 2\big\}, \mathbf{S} = \big\{\{1\}, \{2\}, \{1, 2\}\big\}$
- rate region $\mathcal{C}$ is defined by

$$
\sum_{k \in \mathbf{S}} R_k < \log_2\Big(1 + \frac{\sum_{k \in \mathbf{S}} \mathcal{E}_k}{\sigma_n^2}\Big) \quad \forall \mathbf{S}
$$

$\to$ $\mathcal{C}$ has $K!$ corner points which can all be achieved by successive interference cancellation (SIC)

Rate region for MIMO Users and Receivers

- Channel Model: $\mathbf{y} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{x}_k + \mathbf{n}$, with:
  - User k has $N_{T,k}$ transmit antennas
  - N_R receive antennas
  - $\mathbf{n}$: AWGN vector $\mathcal{N}(\mathbf{0}, \sigma_n^2 \mathbf{I})$

- 2 Users case:

$$\mathbf{y} = \mathbf{H}_1\mathbf{x}_1 + \mathbf{H}_2\mathbf{x}_2 + \mathbf{n} \tag{4}$$

  - Covariance matrix of the TX signal of user k: $\mathbf{Q}_k = \mathcal{E}\{\mathbf{x}_k\mathbf{x}_k^H\}$
  - transmit power: $\mathcal{E}_k = \text{tr}\{\mathbf{Q}_k\}$
- rate region for 2 user case and given $\mathbf{Q}_k$
  - $\mathbf{Q}_k$ given, for example
    a) $\mathbf{Q}_k$ optimal for single user case $\rightarrow \mathbf{Q}_k = \mathbf{U}_k\mathbf{\Lambda}_k\mathbf{U}_k^H$, where:
      · $\mathbf{U}_k$ is an unitary matrix
      · obtained from $\mathbf{H}_k = \mathbf{U}_k\mathbf{\Sigma}_k\mathbf{V}_k^H$
      · $\mathbf{\Lambda}_k = \text{diag}\{\mathcal{E}_{k,1}, \mathcal{E}_{k,2}, \dots, \mathcal{E}_{k,N_T}\}$ with $\mathcal{E}_{l,i}$ obtained from waterfilling and $\sum_{i=1}^{N_{Tk}} \mathcal{E}_{k,i} = \mathcal{E}_k$

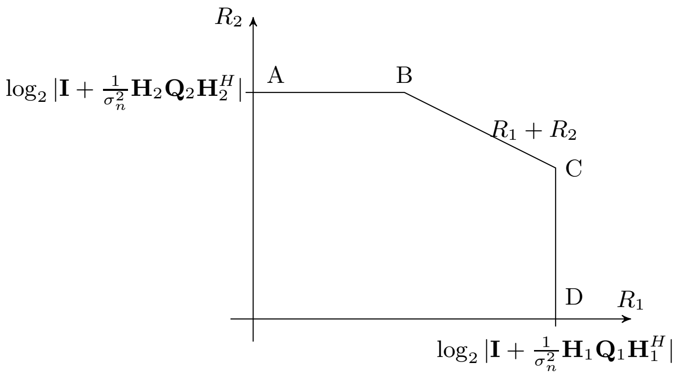
    b) $\mathbf{Q}_k = \frac{\mathcal{E}_k}{N_{T,k}}\mathbf{I}_{N_{T,k}}$ if $\mathbf{H}_k$ is not known at transmitter
  - for given $\mathbf{Q}_1$ and $\mathbf{Q}_2$ we can obtain the rate region as direct extension of the SISO case

$$R_1 < \log_2\left|\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{H}_1\mathbf{Q}_1\mathbf{H}_1^H\right| \tag{5}$$

$$R_2 < \log_2\left|\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{H}_2\mathbf{Q}_2\mathbf{H}_2^H\right| \tag{6}$$

$$R_1 + R_2 < \log_2\left|\mathbf{I} + \frac{1}{\sigma_n^2}\sum_{i=1}^{2}\mathbf{H}_i\mathbf{Q}_i\mathbf{H}_i^H\right| \tag{7}$$

    * equation 5 and equation 6 are the single user bounds,
    * equation 7 is the bound for the joint encoding of both users
  - graphical represantation



  - Points on A - B - C - D can be achieved in a similar manner as for SISO case
  - e.g. bound C can be achieved by SIC

– At B we have

$$R_2 = \log_2 \left| \mathbf{I} + \frac{1}{\sigma_n^2} \mathbf{H}_2 \mathbf{Q}_2 \mathbf{H}_2^H \right|$$

$$R_1 = \log_2 \left| \mathbf{I} + \frac{1}{\sigma_n^2} \sum_{i=1}^{2} \mathbf{H}_i \mathbf{Q}_i \mathbf{H}_i^H \right| - R_2$$

$\rightarrow$ user 1 transmits with rate

$$R_1 = \log_2 \left| \mathbf{I} + \frac{1}{\sigma_n^2} (\mathbf{I} + \frac{1}{\sigma_n^2} \mathbf{H}_2 \mathbf{Q}_2 \mathbf{H}_2^H)^{-1} \mathbf{H}_1 \mathbf{Q}_1 \mathbf{H}_1^H \right|$$

– How to achieve rates at B? $\rightarrow$ Treat $\mathbf{H}_2\mathbf{x}_2 + \mathbf{n}$ in equation 4 as noise with covariance matrix $\mathbf{Q}_N = \mathbf{H}_2 \mathbf{Q}_2 \mathbf{H}_2^H + \sigma_n^2 \mathbf{I}$

$\rightarrow$ equivalent channel matrix with white noise: $\mathbf{r} = \mathbf{Q}_N^{-\frac{1}{2}} \mathbf{y} = \mathbf{Q}_N^{-\frac{1}{2}} \mathbf{H}_1 \mathbf{x}_1 + \tilde{\mathbf{n}}$ where $\tilde{\mathbf{n}}$ is white noise with covariance $\mathbf{I}$.
Anmerkung: Rauschen war vorher farbig, muss "geweißt" werden.

$$\rightarrow R_1 = \log_2 \left| \mathbf{I} + \underbrace{\mathbf{Q}_N^{-\frac{1}{2}} \mathbf{H}_1}_{\mathbf{H}} \mathbf{Q}_1 \underbrace{\mathbf{H}_1^H \mathbf{Q}_N^{-\frac{1}{2}}}_{\mathbf{H_{eq}^H}} \right| = \log_2 \left(\left| \mathbf{Q}_N^{\frac{1}{2}} + \mathbf{Q}_N^{-\frac{1}{2}} \mathbf{H}_1 \mathbf{Q}_1 \mathbf{H}_1^H \right| \cdot \left| \mathbf{Q}_N^{-\frac{1}{2}} \right| \right)$$

$$= \log_2 \left| \mathbf{I} + \mathbf{Q}_N^{-1} \mathbf{H}_1 \mathbf{Q}_1 \mathbf{H}_1^H \right| =$$

$$= \log_2 \left| \mathbf{I} + \frac{1}{\sigma_n^2} \left(\mathbf{I} + \frac{1}{\sigma_n^2} \mathbf{H}_2 \mathbf{Q}_2 \mathbf{H}_2^H \right)^{-1} \mathbf{H}_1 \mathbf{Q}_1 \mathbf{H}_1^H \right|$$

$\rightarrow$ we can achieve R_1 in B by treating user 2 as noise

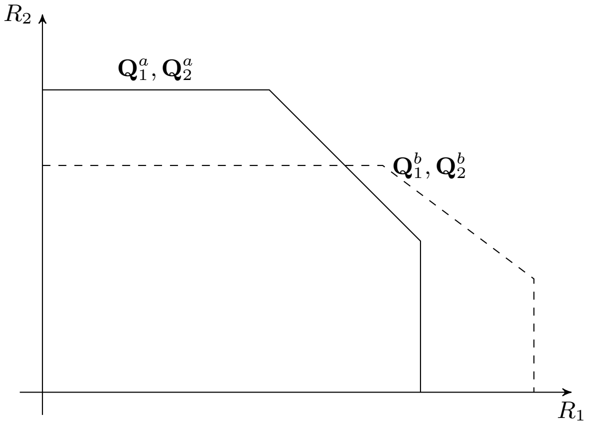
$\rightarrow$ once user 1 is detected, we can subtract its contribution from the received signal and detect user 2

$\Rightarrow$ user 2 can transmit with maximum single user rate

$\rightarrow$ bound C can be achieved by SIC similar to SISO case

– points on B - C are achieved through time sharing

- extension to K user case $\rightarrow$ analogous to SISO case
- Note: Different choices for Q_k will lead to different rate regions



- Example: $K = 2$

→ $\mathbf{Q}_1$ and $\mathbf{Q}_2$ can be optimized to achieve desired trade-off between performance of users 1 and 2

3.2 Broadcast Channel

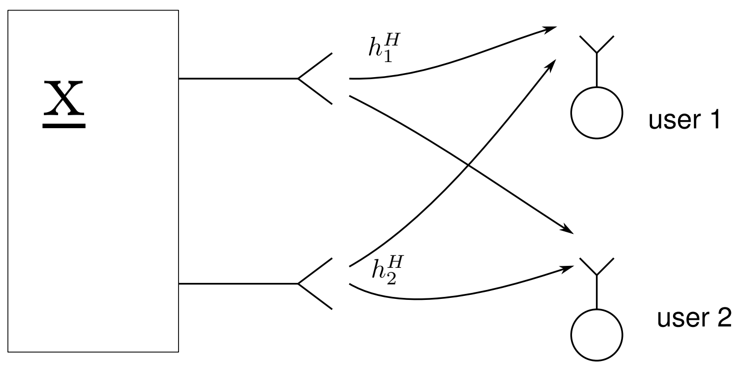
We consider:

- uplink - downlink duality
- rate region

3.2.1 Multiplexing Gain - Degrees of freedom

Downlink scenarios:

- N_R antennas at transmitter, single antennas at the users
- user k receives: $\mathbf{y}_k = \mathbf{h}_k^H \mathbf{x} + \mathbf{n}_k$, with:
  - N_R dimensional channel vector of user k: $\mathbf{h}_k^H$
  - n_k: AWGN at user k
  - $\mathbf{x}$ transmit vector



- How many independent signal streams can we transmit?
  - Consider transmit signal: $\mathbf{x} = \sum_{k=1}^{K} \mathbf{h}_k \mathbf{x}_k$, with TX transmit vector $\mathbf{n}_k$ and symbol x_k is intended for user k
- received signal of user k: $y_k = \sum_{k=1}^{K} (\mathbf{h}_k^H \mathbf{n}_i) \mathbf{x}_i + n_k$
- if all $\mathbf{h}_k$ were orthogonal and we chose $\mathbf{n}_k = \mathbf{h}_k$, the received signal would be: $y_k = ||h_k||^2 \mathbf{x}_k + n_k$
- if $N_R \geq K$, we can transmit simultaneously and interference free to all K users ⇒ multiplexing gain = $\min\{K, N_k\}$